UNIVERSITÀ DEGLI STUDI DI GENOVA

Robotics Engineering

AY. 2021/2022

Robot dynamics and control second assignment's report:

Recursive inverse dynamics

0 Introduction:

This report's purpose is to explain the basics theory concepts used in the static equilibrium analysis of a mechanical chain, how it has been implemented in a MATLAB program, using also the data obtained from the Cad model of the manipulator, and a brief analysis of the results.

1 Theory concepts:

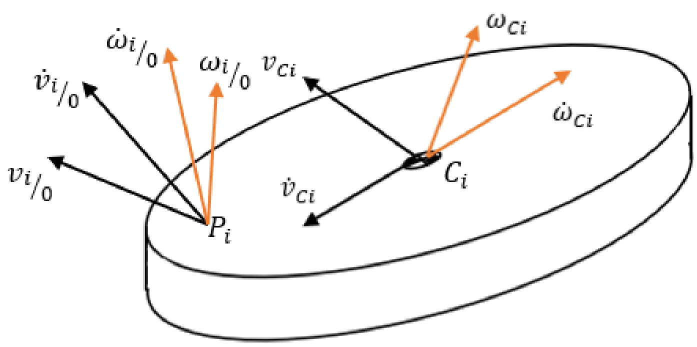
The inverse dynamic problem (IDP) is an analysis of the robot/manipulator that starting from the desired behavior of the robot, specified by the external generalized noise force τ_{ext}, the desired joint configuration $q(t)$, speeds $\dot{q}(t)$ and acceleration $\ddot{q}(t)$, evaluate the necessary generalized actuation force $\tau(t)$.

In order to do that, two main steps must be followed:

1.1 forward recursion:

The aim of the forward recursion Is to obtain the positions, velocities and accelerations of the joint point P_i and center of mass C_i of each link starting from the joint configuration $q(t)$, speeds $\dot{q}(t)$, acceleration $\ddot{q}(t)$ and the geometry of the manipulator, considering to be in a fixed instant in time, starting from the root of the mechanism and moving to his tip.

Let we consider a generic body i in the space that belongs to a mechanical chain, so which is connected with other bodies by a revolute joint or a prismatic joint:



we can start by evaluating the vector from the point P_i to the point Q_i^+ as:

$$r_{i/_{i-1}} = (Q_{i-1}^+ - P_{i-1}) = \begin{cases} (Q_{i-1} - P_{i-1}) & if\ i \in RJ \\ (Q_{i-1} - P_{i-1}) + k_i q_i & if\ i \in TJ \end{cases}$$

Than we can compute the linear and angular velocity of the point P_i as:

$$v_{i/_0} = \begin{cases} v_{i-1/_0} + \omega_{i-1/_0} \times r_{i/_{i-1}} & if\ i \in RJ \\ v_{i-1/_0} + \omega_{i-1/_0} \times r_{i/_{i-1}} + k_i \dot{q}_i & if\ i \in TJ \end{cases}$$

$$\omega_{i/_0} = \begin{cases} \omega_{i-1/_0} + k_i \dot{q}_i & if\ i \in RJ \\ \omega_{i-1/_0} & if\ i \in TJ \end{cases}$$

For completeness we can now evaluate the speed of the center of mass C_i, even if we will not use it in the inverse recursion as:

$$v_{Ci/0} = v_{i/0} + \omega_{i/0} \times r_{Ci/i}$$

After that we can compute the linear and angular accelerations as:

$$\dot{v}_{i/0} = \begin{cases} \dot{v}_{i-1/0} + \dot{\omega}_{i-1/0} \times r_{i/i-1} + \omega_{i-1/0} \times (\omega_{i-1/0} \times r_{i/i-1}) & if\ i \in RJ \\ \dot{v}_{i-1/0} + \dot{\omega}_{i-1/0} \times r_{i/i-1} + \omega_{i-1/0} \times \left(\omega_{i-1/0} \times r_{i/i-1}\right) + 2\left(\omega_{i-1/0} \times k_i\right)\dot{q}_i + k_i\ddot{q}_i & if\ i \in TJ \end{cases}$$

$$\dot{\omega}_{i/0} = \begin{cases} \dot{\omega}_{i-1/0} + (\omega_{i-1/0} \times k_i)\dot{q}_i + k_i\ddot{q}_i & if\ i \in RJ \\ \dot{\omega}_{i-1/0} & if\ i \in TJ \end{cases}$$

In the end we can evaluate the acceleration of the center of mass as:

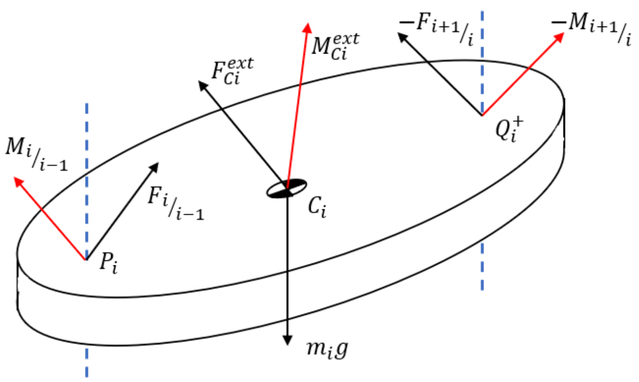
$$\dot{v}_{Ci/0} = \dot{v}_{i/0} + \dot{\omega}_{i/0} \times r_{Ci/i} + \omega_{i/0} \times (\omega_{i/0} \times r_{Ci/i})$$

Pay attention: all the quantities in the formulas are geometric vectors and so to use these equations we've to project them on a reference frame, in our case the best option is to choose the local reference frame of each link.

1.2 Inverse recursion:

The aim of the inverse recursion Is to obtain the generalized actuation force $\tau(t)$ starting from the positions, velocities and accelerations evaluated in the forward recursion, starting from the tip of the mechanism and moving to the root of it.

Let we consider a generic body i in the space that belongs to a mechanical chain, so which is connected with other bodies by a revolute joint or a prismatic joint:



We can divide all the forces and moment acting on the body in five main groups:

- The gravity's force of the body $m_i g$, acting on the center of mass of the body.

- The forces and moments that the body exchange with the previous one through the joint ${F_i}_{/_{i-1}}, {M_i}_{/_{i-1}}$ thanks to the mechanical constraint of the joint and the actuation action, acting on the connection point P_i.
- The forces and moments that the body exchange with the following one through the joint ${F_{i+1}}_{/_i}, {M_{i+1}}_{/_i}$ thanks to the mechanical constraint of the joint and the actuation action, acting on the connection point Q_i^+.
- The external forces and moments acting on the body reconducted to the center of mass $F_{Ci}^{ext}, M_{Ci}^{ext}$, acting on the center of mass C_i.
- The inertial forces and moments acting on the center of mass C_i of each body, $m_i {\dot{v}_{Ci}}_{/_0}$, $I_{Ci} {\dot{\omega}_i}_{/_0} + {\omega_i}_{/_0} \times I_{Ci} {\omega_i}_{/_0}$.

To obtain the dynamic equilibrium of each the generic bodies and so of the manipulator we can introduce the recursive Newton-Euler algorithm: since we are looking for the dynamic equilibrium of the body we can start from the Newton-Euler equations:

$$\begin{cases} F_{Ci}^{tot} = F_{inertial} \\ M_{Ci}^{tot} = M_{inertial} \end{cases}$$

$$\begin{cases} F_{Ci}^{ext} + m_i g + {F_i}_{/_{i-1}} - {F_{i+1}}_{/_i} = m_i {\dot{v}_{Ci}}_{/_0} \\ M_{Ci}^{ext} + {M_i}_{/_{i-1}} - {M_{i+1}}_{/_i} + {r_{Pi}}_{/_{Ci}} \times {F_i}_{/_{i-1}} - {r_{Q_i^+}}_{/_{Ci}} \times {F_{i+1}}_{/_i} = I_{Ci} {\dot{\omega}_i}_{/_0} + {\omega_i}_{/_0} \times I_{Ci} {\omega_i}_{/_0} \end{cases}$$

Where:

$$Q_i^+ = \begin{cases} Q_i & if\ i \in RJ \\ Q_i + K_{i+1} q_{i+1} & if\ i \in TJ \end{cases}$$

Starting from these equations that describes the dynamic of the generic body we can compute the actuation forces and moments acting on the generic link starting from the last body (n) with the set of equations:

$$\begin{cases} {F_n}_{/_{n-1}} = m_n {\dot{v}_{Cn}}_{/_0} - m_n g - F_{Cn}^{ext} \\ {M_n}_{/_{n-1}} = I_{Cn} {\dot{\omega}_n}_{/_0} + {\omega_n}_{/_0} \times I_{Cn} {\omega_n}_{/_0} - M_{Cn}^{ext} - {r_{Pn}}_{/_{Cn}} \times {F_n}_{/_{n-1}} \end{cases}$$

Then we can pass to the following link where we need to take in account also the reaction forces and moments exchanged between the link n and the link $n-1$ which are the ones we've just evaluated

$$\begin{cases} {F_{n-1}}_{/_{n-2}} = m_n {\dot{v}_{Cn}}_{/_0} - m_{n-1} g - F_{Cn-1}^{ext} + {F_n}_{/_{n-1}} \\ {M_{n-1}}_{/_{n-2}} = I_{Cn} {\dot{\omega}_n}_{/_0} + {\omega_n}_{/_0} \times I_{Cn} {\omega_n}_{/_0} - M_{Cn-1}^{ext} - {r_{Pn-1}}_{/_{Cn-1}} \times (-F_{Cn-1}^{ext} - m_{n-1} g) + {r_{Q_{n-1}^+}}_{/_{Pn-1}} \times {F_n}_{/_{n-1}} + {M_n}_{/_{n-1}} \end{cases}$$

Then we can evaluate the general actuation forces and moments on each link with the equations:

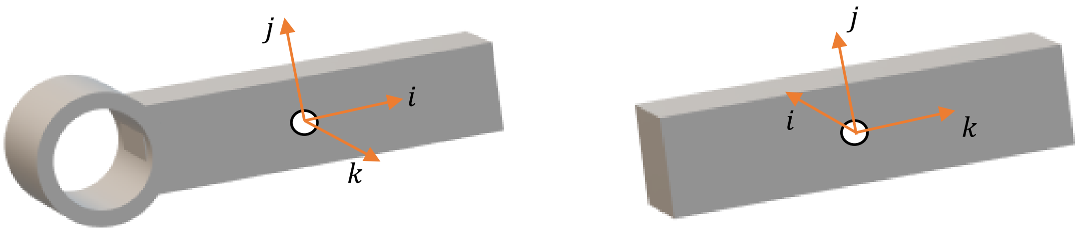
$$\tau_i = \begin{cases} {F_i}_{/_{i-1}} \cdot K_i & if\ i \in TJ \\ {M_i}_{/_{i-1}} \cdot K_i & if\ i \in RJ \end{cases}$$

Pay attention: all the quantities in the formulas are geometric vectors and so to use these equations we've to project them on a reference frame, in our case the best option is to choose the local reference frame of each link.

2 Exercises settings and results:

2.1 Settings:

Before starting to analyze the exercises and the relative results some word about the program modeling of the problem must be spend:
As we can see in the following picture the referance frame for each link has been choosen to obay the Denavit Hartenberg rule and follow the axis of inertia:



The program is designed to be as general as possible, in it any object along the mechanical chain of the manipulator is consider as a link with his own joint type and configuration, and each link is enumerated.

Then some attention must be taken on the configuration vector q, in it the i line refer to the i joint starting from the one between ground and first link.

In each line we've three columns:

- The first column contains the configuration angle in radiant θ_i (if it is a revolute joint) or distance in meters d_i (if it is a prismatic joint) or the fixed angle between the frames θ_i in case of a rigid joint.
- The second column contains the joint's type in particular the following numbers are used to specify the joint's type:
  - 0 = revolute joint
  - 1 = prismatic joint
  - 2 = rigid joint
- The last three columns are used to specify the angles, respectively around x,y and z, for the rotational matrix between the link l-1 and l (where l is the line number).

The joint speed vector, as the joint acceleration vector, is simply composed by the speed (acceleration) of each link, in which the line's number specify the joint.

Then it's also necessary to consider the external forces, moments and gravity and how they are implemented:

- The external forces array F_ext: in it each line contains all the information of an external force needed for the following evaluation, in each line we've seven columns:
  - The first three columns contain the components of the force along the x, y, z axis of the absolute reference frame in Newton.
  - The fourth column contain the number of the link on which the force act:
  - The last three column contain the position along the x, y, z axis of the local reference frame of the link on which the force act in meters.
- The external moments array M_ext: in it each line contains all the information of an external moment needed for the following evaluation, in each line we've four columns:
  - The first three columns contain the components of the moment along the x, y, z axis of the absolute reference frame in Newton meters.
  - The fourth column contain the number of the link on which the force act:
- The gravity flag g_flag is used to turn on/off the gravity: by assigning a value of 1 the gravity is turned on, by assigning a value of 0 the gravity is turned off.

2.2 Exercises and results:

Is important to notice that all the following results are evaluated in the local reference frame, which follow the Denavit Hartenberg rule, follow the principal inertial axis and has the origin in the center of mass of the link, of each link and are expressed in the corresponding I.S. measure unit.

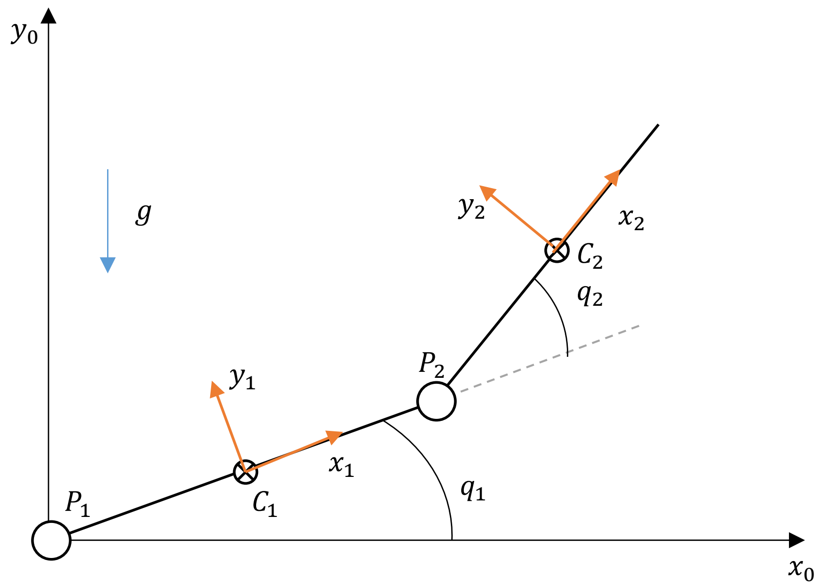
The absolute reference frame is considered with the j unit vector in a vertical position directed upward, the i unit vector in a horizontal position and the k unit vector exiting from the plane.

2.2.1 Exercise 1:

The recursive Newton-Euler algorithm for the inverse dynamic analysis has been implemented in the dynamic_processing file, as we can see the program take as input the data about the robot like the number of links, their length, mass, inertial matrix (referred to the reference frame along the principal inertial axis and located in the center of mass of each link), the configuration vector, the joint speed vector, joint acceleration vector, the presence or absence of the gravity and the external forces and moments acting on the robot.

Than the program start with the forward recursion which evaluate the position, speed and acceleration of the points P_i and C_i, then the results are used in the backward recursion in which the generalized actuation torques and forces are evaluated together with the vincular reaction for joint.

2.2.2 Exercise 2:



As we can see the mechanism is planar and completely kinematically described by two configuration angles, velocities and accelerations.

2.2.2.1 Exercise 2.1 without gravity:

In this case the settings for the exercises are:
The link's lengths are:

```
l = [1, 0.8]
```

The position of the centers of mass expressed in the local reference frame are:

```
C = [0.5, 0, 0;
     0.4, 0, 0]
```

The inertial matrices of each link are:

```
I_c(:,:,1) = [0, 0, 0;
              0, 0, 0;
              0, 0, 0.4]
I_c(:,:,2) = [0, 0, 0;
              0, 0, 0;
              0, 0, 0.3]
```

The masses of each link are:

```
m = [22; 19]
```

The configuration vector:

```
q = [0.349066, 0, 0, 0, 0.349066;
     0.698132, 0, 0, 0, 0.698132]
```

The joint velocities vector:

```
q_d = [0.2; 0.15]
```

The joint accelerations vector:

```
q_dd = [0.1; 0.085]
```

The gravity is turned off:

```
g_flag = 0
```

There isn't any external force or moment:

```
F_ext = [0 ,0 ,0 , 0 ,0 ,0 ,0;
         0 ,0 ,0 ,0 ,0 ,0 ,0;
        0 ,0 ,0 ,0 ,0 ,0 ,0]

M_ext = [0, 0, 0, 0;
         0, 0, 0, 0;
         0, 0, 0, 0]
```

Then the results are:
the generalized actuation torque and force vector is composed by:

tau 1 = 4.36 Nm
tau 2 = 1.40 Nm

while the vincular reaction forces and moment are:

vr 1 = [-2.82 N, 3.48 N, 0.00 N, 0.00 Nm, 0.00 Nm, 0.00 Nm]
vr 2 = [-0.29 N, 3.35 N, 0.00 N, 0.00 Nm, 0.00 Nm, 0.00 Nm]

as we can se since the moment of inertia of each link, as the angular accelerations are contained both the actuation torques and the vincular reaction as a small value.

Since both the accelerations and the velocities of the joints are positive, which means we're in an acceleration phase, both the torque are positives.

2.2.2.1 Exercise 2.1 with gravity:

In this case the settings for the exercises are:
The link's lengths are:

```
l = [1, 0.8]
```

The position of the centers of mass expressed in the local reference frame are:

```
C = [0.5, 0, 0;
     0.4, 0, 0]
```

The inertial matrices of each link are:

```
I_c(:,:,1) = [0, 0, 0;
              0, 0, 0;
              0, 0, 0.4]
I_c(:,:,2) = [0, 0, 0;
              0, 0, 0;
              0, 0, 0.3]
```

The masses of each link are:

```
m = [22; 19]
```

The configuration vector:

```
q = [0.349066, 0, 0, 0, 0.349066;
     0.698132, 0, 0, 0, 0.698132]
```

The joint velocities vector:

```
q_d = [0.2; 0.15]
```

The joint accelerations vector:

```
q_dd = [0.1; 0.085]
```

The gravity is turned on:

```
g_flag = 1
```

There isn't any external force or moment:

```
F_ext = [0 ,0 ,0 , 0 ,0 ,0 ,0;
         0 ,0 ,0 ,0 ,0 ,0 ,0;
         0 ,0 ,0 ,0 ,0 ,0 ,0]

M_ext = [0, 0, 0, 0;
         0, 0, 0, 0;
         0, 0, 0, 0]
```

Then the results are:
the generalized actuation torque and force vector is composed by:

tau 1 = 318.19 Nm
tau 2 = 38.67 Nm

while the vincular reaction forces and moment are:

vr 1 = [134.75 N, 381.43 N, 0.00 N, 0.00 Nm, 0.00 Nm, 0.00 Nm]
vr 2 = [161.13 N, 96.54 N, 0.00 N, 0.00 Nm, 0.00 Nm, 0.00 Nm]

as we can see with respect to the previous exercise the effect of the weight force of the links assume a predominant role that leads to a distinct increment of the actuation torques which maintain a positive sign since we're still in an acceleration phase.

2.2.2.2 Exercise 2.2 without gravity:

In this case the settings for the exercises are:
The link's lengths are:

```
l = [1, 0.8]
```

The position of the centers of mass expressed in the local reference frame are:

```
C = [0.5, 0, 0;
     0.4, 0, 0]
```

The inertial matrices of each link are:

```
I_c(:,:,1) = [0, 0, 0;
              0, 0, 0;
              0, 0, 0.4]
I_c(:,:,2) = [0, 0, 0;
              0, 0, 0;
              0, 0, 0.3]
```

The masses of each link are:

```
m = [22; 19]
```

The configuration vector:

```
q = [pi/2, 0, 0, 0, pi/2;
   pi/4, 0, 0, 0, pi/4]
```

The joint velocities vector:

```
q_d = [-0.8; 0.35]
```

The joint accelerations vector:

```
q_dd = [-0.4; 0.1]
```

The gravity is turned off:

```
g_flag = 0
```

There isn't any external force or moment:

```
F_ext = [0 ,0 ,0 , 0 ,0 ,0 ,0;
         0 ,0 ,0 ,0 ,0 ,0 ,0;
         0 ,0 ,0 ,0 ,0 ,0 ,0]

M_ext = [0, 0, 0, 0;
         0, 0, 0, 0;
         0, 0, 0, 0]
```

Then the results are:
the generalized actuation torque and force vector is composed by:

tau 1 = -12.37 Nm
tau 2 = 0.29 Nm

while the vincular reaction forces and moment are:

vr 1 = [-18.68 N, -14.70 N, 0.00 N, 0.00 Nm, 0.00 Nm, 0.00 Nm]
vr 2 = [-15.51 N, 0.94 N, 0.00 N, 0.00 Nm, 0.00 Nm, 0.00 Nm]

in this case the second joint is in an acceleration phase while the first one is in a deceleration one, that leads to a negative first actuation torque and a positive second once, moreover since this situation the first actuation torque need to be bigger in absolute value since has to win a bigger couple generated by the kinematic of the second link.

2.2.2.2 Exercise 2.2 with gravity:

In this case the settings for the exercises are:
The link's lengths are:

```
l = [1, 0.8]
```

The position of the centers of mass expressed in the local reference frame are:

```
C = [0.5, 0, 0;
     0.4, 0, 0]
```

The inertial matrices of each link are:

```
I_c(:,:,1) = [0, 0, 0;
              0, 0, 0;
              0, 0, 0.4]
I_c(:,:,2) = [0, 0, 0;
              0, 0, 0;
              0, 0, 0.3]
```

The masses of each link are:

```
m = [22; 19]
```

The configuration vector:

```
q = [pi/2, 0, 0, 0, pi/2;
     pi/4, 0, 0, 0, pi/4]
```

The joint velocities vector:

```
q_d = [-0.8; 0.35]
```

The joint accelerations vector:

```
q_dd = [-0.4; 0.1]
```

The gravity is turned on:

```
g_flag = 1
```

There isn't any external force or moment:

```
F_ext = [0 ,0 ,0 , 0 ,0 ,0 ,0;
         0 ,0 ,0 ,0 ,0 ,0 ,0;
        0 ,0 ,0 ,0 ,0 ,0 ,0]

M_ext = [0, 0, 0, 0;
         0, 0, 0, 0;
         0, 0, 0, 0]
```

Then the results are:
the generalized actuation torque and force vector is composed by:

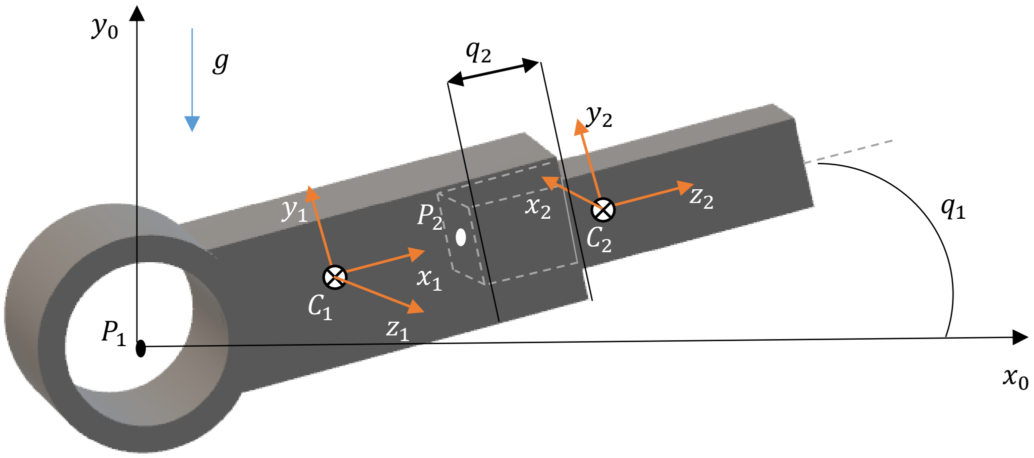
tau 1 = -65.09 Nm
tau 2 = -52.43 Nm

while the vincular reaction forces and moment are:

vr 1 = [383.53 N, -14.70 N, 0.00 N, 0.00 Nm, 0.00 Nm, 0.00 Nm]
vr 2 = [116.29 N, -130.85 N, 0.00 N, 0.00 Nm, 0.00 Nm, 0.00 Nm]

in this case with respect to the previous one we can see how the gravity effect is bigger than the kinematic situation effect of acceleration and deceleration of the links and leads to both the actuation torques to be negative, this because the second link needs to be slowed down to reduce the effect of the gravity and obtain the desired behavior of the manipulator.

2.2.3 Exercise 3:



As we can see the mechanism is planar and completely kinematically described by one configuration angle and one linear displacement, two velocities and two accelerations respectively angular and linear.

2.2.3.1 Exercise 3.1 without gravity:

In this case the settings for the exercises are:
The link's lengths are:

```
l = [1, 0.8]
```

The position of the centers of mass expressed in the local reference frame are:

```
C = [0.5, 0, 0;
     0.4, 0, 0]
```

The inertial matrices of each link are:

```
I_c(:,:,1) = [0, 0, 0;
              0, 0, 0;
              0, 0, 0.4]
I_c(:,:,2) = [0, 0, 0;
              0, 0, 0;
              0, 0, 0.3]
```

The masses of each link are:

```
m = [10; 6]
```

The configuration vector:

```
q = [0.349066, 0, 0, 0, 0.349066;
     -0.2, 1, 0, pi/2, 0]
```

The joint velocities vector:

```
q_d = [0.08; 0.03]
```

The joint accelerations vector:

```
q_dd = [0.1; 0.01]
```

The gravity is turned off:

```
g_flag = 0
```

There isn't any external force or moment:

```
F_ext = [0 ,0 ,0 , 0 ,0 ,0 ,0;
         0 ,0 ,0 ,0 ,0 ,0 ,0;
         0 ,0 ,0 ,0 ,0 ,0 ,0]

M_ext = [0, 0, 0, 0;
         0, 0, 0, 0;
         0, 0, 0, 0]
```

Then the results are:
the generalized actuation torque and force vector is composed by:

tau 1 = 0.70 Nm
tau 2 = 0.03 N

while the vincular reaction forces and moment are:

vr 1 = [-0.00 N, 1.01 N, 0.00 N, 0.20 Nm, -0.01 Nm, 0.00 Nm]
vr 2 = [-0.00 N, 0.51 N, 0.00 N, 0.10 Nm, -0.01 Nm, 0.20 Nm]

we can notice that in this case both the links are in an acceleration phase, since the accelerations and speeds have the same signs, so both the actuation torque and force need to be positive to obtain this behavior.

Moreover the actuation force is very little since the rotation of the first link leads to an inertial force that forces the second link to move along the k axis.

2.2.3.1 Exercise 3.1 with gravity:

In this case the settings for the exercises are:
The link's lengths are:

```
l = [1, 0.8]
```

The position of the centers of mass expressed in the local reference frame are:

```
C = [0.5, 0, 0;
     0.4, 0, 0]
```

The inertial matrices of each link are:

```
I_c(:,:,1) = [0, 0, 0;
              0, 0, 0;
              0, 0, 0.4]
I_c(:,:,2) = [0, 0, 0;
              0, 0, 0;
              0, 0, 0.3]
```

The masses of each link are:

```
m = [10; 6]
```

The configuration vector:

```
q = [0.349066, 0, 0, 0, 0.349066;
     -0.2, 1, 0, pi/2, 0]
```

The joint velocities vector:

```
q_d = [0.08; 0.03]
```

The joint accelerations vector:

```
q_dd = [0.1; 0.01]
```

The gravity is turned on:

```
g_flag = 1
```

There isn't any external force or moment:

```
F_ext = [0 ,0 ,0 , 0 ,0 ,0 ,0;
         0 ,0 ,0 ,0 ,0 ,0 ,0;
         0 ,0 ,0 ,0 ,0 ,0 ,0]

M_ext = [0, 0, 0, 0;
         0, 0, 0, 0;
         0, 0, 0, 0]
```

Then the results are:
the generalized actuation torque and force vector is composed by:

tau 1 = 91.04 Nm
tau 2 = 20.16 N

while the vincular reaction forces and moment are:

vr 1 = [53.68 N, 148.50 N, 0.00 N, 22.33 Nm, -8.06 Nm, 0.00 Nm]
vr 2 = [0.00 N, 55.82 N, 0.00 N, 11.16 Nm, -8.06 Nm, 22.33 Nm]

in this case with respect to the previous one the effect of the gravity leads to a bigger actuation torque and force since the weights of the two links needs to be win by the actuation torque and force

2.2.3.2 Exercise 3.2 without gravity:

In this case the settings for the exercises are:
The link's lengths are:

```
l = [1, 0.8]
```

The position of the centers of mass expressed in the local reference frame are:

```
C = [0.5, 0, 0;
     0.4, 0, 0]
```

The inertial matrices of each link are:

```
I_c(:,:,1) = [0, 0, 0;
              0, 0, 0;
              0, 0, 0.4]
I_c(:,:,2) = [0, 0, 0;
              0, 0, 0;
              0, 0, 0.3]
```

The masses of each link are:

```
m = [10; 6]
```

The configuration vector:

```
q = [2.0944, 0, 0, 0, 2.0944;
     -0.6, 1, 0, pi/2, 0]
```

The joint velocities vector:

```
q_d = [-0.4; -0.08]
```

The joint accelerations vector:

```
q_dd = [-0.1; -0.01]
```

The gravity is turned off:

```
g_flag = 0
```

There isn't any external force or moment:

```
F_ext = [0 ,0 ,0 , 0 ,0 ,0 ,0;
         0 ,0 ,0 ,0 ,0 ,0 ,0;
         0 ,0 ,0 ,0 ,0 ,0 ,0]

M_ext = [0, 0, 0, 0;
         0, 0, 0, 0;
         0, 0, 0, 0]
```

Then the results are:
the generalized actuation torque and force vector is composed by:

tau 1 = -0.23 Nm
tau 2 = -0.44 N

while the vincular reaction forces and moment are:

vr 1 = [-1.24 N, -0.36 N, -0.00 N, 0.06 Nm, 0.18 Nm, 0.00 Nm]
vr 2 = [-0.00 N, 0.14 N, 0.00 N, 0.09 Nm, 0.18 Nm, 0.06 Nm]

since both the acceleration and the speeds are negative in this case the mechanism is accelerating in a clockwise direction for the revolute joint and the second link is moving inside the first one, for both these reasons the actuation torque and force are negative.
Moreover since the gravity is turned off and the inertial matrices and accelerations are contained the value of both the actuation is contained as well.

2.2.3.2 Exercise 3.2 with gravity:

In this case the settings for the exercises are:
The link's lengths are:

```
l = [1, 0.8]
```

The position of the centers of mass expressed in the local reference frame are:

```
C = [0.5, 0, 0;
     0.4, 0, 0]
```

The inertial matrices of each link are:

```
I_c(:,:,1) = [0, 0, 0;
              0, 0, 0;
              0, 0, 0.4]
I_c(:,:,2) = [0, 0, 0;
              0, 0, 0;
              0, 0, 0.3]
```

The masses of each link are:

```
m = [10; 6]
```

The configuration vector:

```
q = [2.0944, 0, 0, 0, 2.0944;
     -0.6, 1, 0, pi/2, 0]
```

The joint velocities vector:

```
q_d = [-0.4; -0.08]
```

The joint accelerations vector:

```
q_dd = [-0.1; -0.01]
```

The gravity is turned on:

```
g_flag = 1
```

There isn't any external force or moment:

```
F_ext = [0 ,0 ,0 , 0 ,0 ,0 ,0;
         0 ,0 ,0 ,0 ,0 ,0 ,0;
         0 ,0 ,0 ,0 ,0 ,0 ,0]

M_ext = [0, 0, 0, 0;
         0, 0, 0, 0;
         0, 0, 0, 0]
```

Then the results are:
the generalized actuation torque and force vector is composed by:

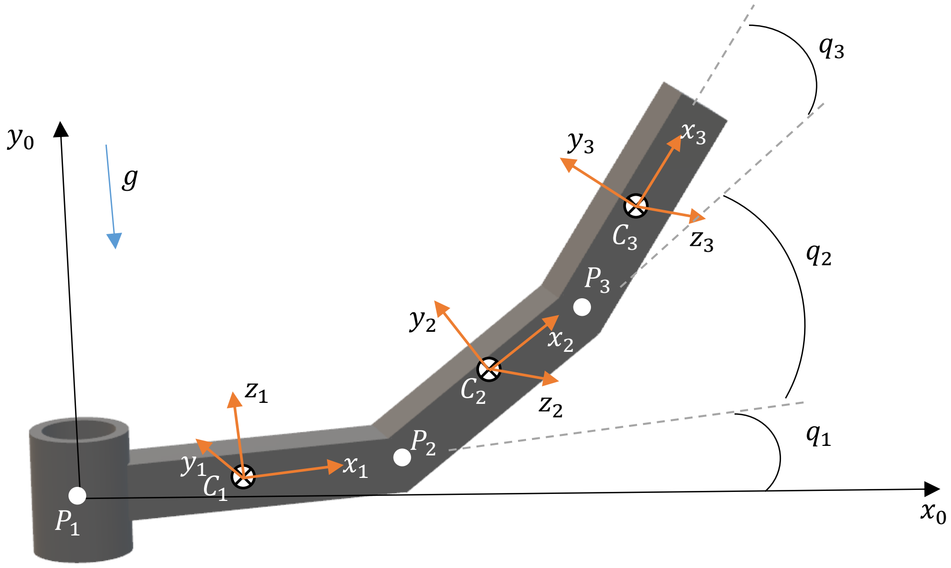
tau 1 = -36.53 Nm
tau 2 = 50.53 N

while the vincular reaction forces and moment are:

vr 1 = [134.69 N, -78.84 N, -0.00 N, -11.71 Nm, -20.21 Nm, 0.00 Nm]
vr 2 = [0.00 N, -29.29 N, 0.00 N, -17.57 Nm, -20.21 Nm, -11.71 Nm]

in this case the effect of the gravity affects the results of the actuation torque and force, in facts the weight of the second link leads to a higher acceleration and speed of the second link by itself so it's necessary to slow down the link by applying a positive actuation force.
While the effect of the gravity on the first link is limited since it doesn't produce a large moment due to a small arm of the force.

2.2.4 Exercise 4:



As we can see the mechanism is no longer planar but can be completely kinematically described by three configuration angles, velocities and accelerations.

2.2.4.1 Exercise 4.1 without gravity:

In this case the settings for the exercises are:
The link's lengths are:

```
l = [1, 0.8, 0.35]
```

The position of the centers of mass expressed in the local reference frame are:

```
C = [0.5, 0, 0;
     0.4, 0, 0;
     0.175, 0, 0]
```

The inertial matrices of each link are:

```
I_c(:,:,1) = [0.2, 0, 0;
              0, 0.2, 0;
              0, 0, 0.8]
I_c(:,:,2) = [0.2, 0, 0;
              0, 0.2, 0;
              0, 0, 0.8]
I_c(:,:,3) = [0.08, 0, 0;
              0, 0.08, 0;
              0, 0, 0.1]
```

The masses of each link are:

```
m = [20; 20; 6]
```

The configuration vector:

```
q = [0.349066, 0, -pi/2, 0.349066, 0;
     0.698132, 0, pi/2, 0, 0.698132;
     0.1745, 0, 0, 0, 0.1745]
```

The joint velocities vector:

```
q_d = [0.2; 0.15; -0.2]
```

The joint accelerations vector:

```
q_dd = [0.1; 0.085; 0]
```

The gravity is turned off:

```
g_flag = 0
```

There isn't any external force or moment:

```
F_ext = [0 ,0 ,0 , 0 ,0 ,0 ,0;
         0 ,0 ,0 ,0 ,0 ,0 ,0;
         0 ,0 ,0 ,0 ,0 ,0 ,0]

M_ext = [0, 0, 0, 0;
         0, 0, 0, 0;
         0, 0, 0, 0]
```

Then the results are:
the generalized actuation torque and force vector is composed by:

tau 1 = 6.02 Nm
tau 2 = 0.84 Nm
tau 3 = 0.10 Nm

while the vincular reaction forces and moment are:

vr 1 = [-2.99 N, 4.11 N, 1.18 N, 0.53 Nm, -1.82 Nm, 0.00 Nm]
vr 2 = [0.02 N, 1.18 N, -4.05 N, -0.01 Nm, 2.33 Nm, 0.00 Nm]
vr 3 = [-0.07 N, 0.52 N, -1.20 N, 0.00 Nm, 0.22 Nm, 0.00 Nm]

since the inertial moments are limited, as the accelerations, the actuation torques result to be limited, moreover the last link is moving with uniform speed since the other two are in an acceleration phase and this leads to a positive value of the actuation torques.

2.2.4.1 Exercise 4.1 with gravity:

In this case the settings for the exercises are:
The link's lengths are:

```
l = [1, 0.8, 0.35]
```

The position of the centers of mass expressed in the local reference frame are:

```
C = [0.5, 0, 0;
     0.4, 0, 0;
     0.175, 0, 0]
```

The inertial matrices of each link are:

```
I_c(:,:,1) = [0.2, 0, 0;
              0, 0.2, 0;
              0, 0, 0.8]
I_c(:,:,2) = [0.2, 0, 0;
              0, 0.2, 0;
              0, 0, 0.8]
I_c(:,:,3) = [0.08, 0, 0;
              0, 0.08, 0;
              0, 0, 0.1]
```

The masses of each link are:

```
m = [20; 20; 6]
```

The configuration vector:

```
q = [0.349066, 0, -pi/2, 0.349066, 0;
     0.698132, 0, pi/2, 0, 0.698132;
```

```
0.1745, 0, 0, 0, 0.1745]
```

The joint velocities vector:

```
q_d = [0.2; 0.15; -0.2]
```

The joint accelerations vector:

```
q_dd = [0.1; 0.085; 0]
```

The gravity is turned on:

```
g_flag = 1
```

There isn't any external force or moment:

```
F_ext = [0 ,0 ,0 , 0 ,0 ,0 ,0;
         0 ,0 ,0 ,0 ,0 ,0 ,0;
        0 ,0 ,0 ,0 ,0 ,0 ,0]

M_ext = [0, 0, 0, 0;
         0, 0, 0, 0;
         0, 0, 0, 0]
```

Then the results are:
the generalized actuation torque and force vector is composed by:

tau 1 = 6.02 Nm
tau 2 = 136.55 Nm
tau 3 = 10.24 Nm

while the vincular reaction forces and moment are:

vr 1 = [-2.99 N, 4.11 N, 452.44 N, 87.76 Nm, -458.95 Nm, 0.00 Nm]
vr 2 = [0.02 N, 256.24 N, -4.05 N, -0.01 Nm, 2.33 Nm, 0.00 Nm]
vr 3 = [10.15 N, 58.48 N, -1.20 N, 0.00 Nm, 0.22 Nm, 0.00 Nm]

as we can see with respect to the previous case the first actuation torque remains the same, as we could expect, thanks to his vertical orientation while the vincular reaction forces and moments acting on it increase a lot due to the action of the force of gravity.
The same behavior can be seen on the last two actuation torque since they need to win also the weight forces.

3 Conclusions:

As we can see the effect of the gravity can be important under small velocities and accelerations, leading to a change of sign of the actuation forces and torques.
Moreover the dynamic forces and torques have an effect proportional to the velocities and accelerations, so in particular in case of quick accelerations/decelerations the actuation torque required to the engines that control the robot can blow up quickly.