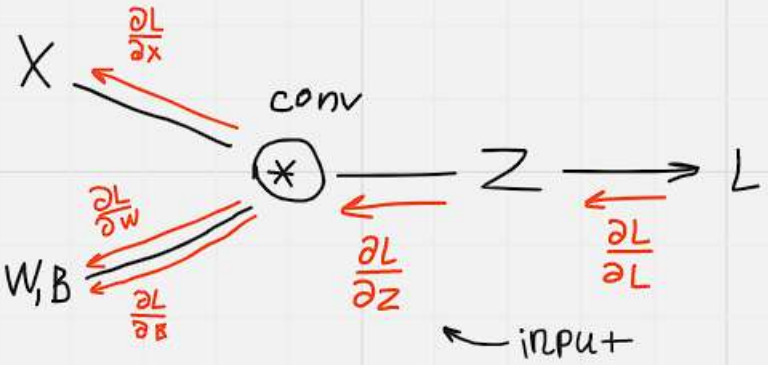
Conv2D Backprop



$$\begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} \circledast \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} + B = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$$

$$Z_{11} = x_{11} k_{11} + x_{12} k_{12} + x_{21} k_{21} + x_{22} k_{22} + B$$

$$Z_{12} = x_{12} k_{11} + x_{13} k_{12} + x_{22} k_{21} + x_{23} k_{22} + B$$

$$Z_{21} = x_{21} k_{11} + x_{22} k_{12} + x_{31} k_{21} + x_{32} k_{22} + B$$

$$Z_{22} = x_{22} k_{11} + x_{23} k_{12} + x_{32} k_{21} + x_{33} k_{22} + B$$

input

$$\frac{\partial L}{\partial K} = \frac{\partial L}{\partial Z} \times \frac{\partial Z}{\partial K}$$

$$\frac{\partial L}{\partial K} = \begin{bmatrix} \frac{\partial L}{\partial k_{11}} & \frac{\partial L}{\partial k_{12}} \\ \frac{\partial L}{\partial k_{21}} & \frac{\partial L}{\partial k_{22}} \end{bmatrix}$$

$$\frac{\partial L}{\partial k_{mn}} = \sum_{ij} \frac{\partial L}{\partial Z_{ij}} \cdot \frac{\partial Z_{ij}}{\partial k_{mn}}$$

$$\frac{\partial L}{\partial k_{11}} = \frac{\partial L}{\partial z_{11}} \frac{\partial z_{11}}{\partial k_{11}} + \frac{\partial L}{\partial z_{12}} \frac{\partial z_{12}}{\partial k_{11}} + \frac{\partial L}{\partial z_{21}} \frac{\partial z_{21}}{\partial k_{11}} + \frac{\partial L}{\partial z_{22}} \frac{\partial z_{22}}{\partial k_{11}}$$

$$\frac{\partial L}{k_{12}} = \frac{\partial L}{\partial z_{11}} \frac{\partial z_{11}}{\partial k_{12}} + \frac{\partial L}{\partial z_{12}} \frac{\partial z_{12}}{\partial k_{12}} + \frac{\partial L}{\partial z_{21}} \frac{\partial z_{11}}{\partial k_{12}} + \frac{\partial L}{\partial z_{22}} \frac{\partial z_{22}}{\partial k_{12}}$$

$$\frac{\partial L}{\partial k_{21}} = \frac{\partial L}{\partial z_{11}} \frac{\partial z_{11}}{\partial k_{21}} + \frac{\partial L}{\partial z_{12}} \frac{\partial z_{12}}{\partial k_{21}} + \frac{\partial L}{\partial z_{21}} \frac{\partial z_{21}}{\partial k_{21}} + \frac{\partial L}{\partial z_{22}} \frac{\partial z_{22}}{\partial k_{21}}$$

$$\frac{\partial L}{\partial k_{22}} = \frac{\partial L}{\partial z_{11}} \frac{\partial z_{11}}{\partial k_{22}} + \frac{\partial L}{\partial z_{12}} \frac{\partial z_{12}}{\partial k_{22}} + \frac{\partial L}{\partial z_{21}} \frac{\partial z_{21}}{\partial k_{22}} + \frac{\partial L}{\partial z_{22}} \frac{\partial z_{22}}{\partial k_{22}}$$

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$$\frac{\partial z_{11}}{\partial k_{11}} = \frac{\partial}{\partial k_{11}} \left(x_{11} k_{11} + x_{12} k_{12} + x_{21} k_{21} + x_{22} k_{22} + B \right) = x_{11}$$

$$\Downarrow$$

$$\begin{cases} \dfrac{\partial L}{\partial k_{11}} = \dfrac{\partial L}{\partial z_{11}} x_{11} + \dfrac{\partial L}{\partial z_{12}} x_{12} + \dfrac{\partial L}{\partial z_{21}} x_{21} + \dfrac{\partial L}{\partial z_{22}} x_{22} \\ \dfrac{\partial L}{k_{12}} = \dfrac{\partial L}{\partial z_{11}} x_{12} + \dfrac{\partial L}{\partial z_{12}} x_{13} + \dfrac{\partial L}{\partial z_{21}} x_{22} + \dfrac{\partial L}{\partial z_{22}} x_{23} \\ \dfrac{\partial L}{\partial k_{21}} = \dfrac{\partial L}{\partial z_{11}} x_{21} + \dfrac{\partial L}{\partial z_{22}} x_{22} + \dfrac{\partial L}{\partial z_{21}} x_{31} + \dfrac{\partial L}{\partial z_{22}} x_{33} \\ \dfrac{\partial L}{\partial k_{22}} = \dfrac{\partial L}{\partial z_{11}} x_{22} + \dfrac{\partial L}{\partial z_{12}} x_{23} + \dfrac{\partial L}{\partial z_{21}} x_{32} + \dfrac{\partial L}{\partial z_{22}} x_{33} \end{cases}$$

$$\Downarrow$$

$$\begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} \circledast \begin{bmatrix} \dfrac{\partial L}{\partial z_{11}} & \dfrac{\partial L}{\partial z_{12}} \\ \dfrac{\partial L}{\partial z_{21}} & \dfrac{\partial L}{\partial z_{22}} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial L}{\partial k_{11}} & \dfrac{\partial L}{\partial k_{12}} \\ \dfrac{\partial L}{\partial k_{21}} & \dfrac{\partial L}{\partial k_{22}} \end{bmatrix}$$

$$X \circledast \frac{\partial L}{\partial Z} = \frac{\partial L}{\partial k}$$

$$\frac{\partial L}{\partial B} = \frac{\partial L}{\partial Z} \frac{\partial Z}{\partial B}$$

$$\frac{\partial L}{\partial B} = \sum \frac{\partial L}{\partial Z_{ij}} \frac{\partial Z_{ij}}{\partial B}$$

$$\frac{\partial L}{\partial B_k} = \frac{\partial L}{\partial Z_{11}} \frac{\partial Z_{11}}{\partial B_k} + \frac{\partial L}{\partial Z_{12}} \frac{\partial Z_{12}}{\partial B_k} + \frac{\partial L}{\partial Z_{21}} \frac{\partial Z_{21}}{\partial B_k} + \frac{\partial L}{\partial Z_{22}} \frac{\partial Z_{21}}{\partial B_k}$$

$$\frac{\partial Z_{ij}}{\partial B_k} = 1 \quad \forall i,j,k$$

$$\Downarrow$$

$$\frac{\partial L}{\partial B_k} = \sum_{ij} \frac{\partial L}{\partial Z}$$

$$\frac{\partial L}{\partial x_{mn}} = \sum \frac{\partial L}{\partial z_{ij}} \frac{\partial z_{ij}}{\partial x_{mn}}$$

$$\frac{\partial L}{\partial x_{11}} = \frac{\partial L}{\partial z_{11}} \frac{\partial z_{11}}{\partial x_{11}} + \frac{\partial L}{\partial z_{12}} \frac{\partial z_{12}}{\partial x_{11}}^{0} + \frac{\partial L}{\partial z_{21}} \frac{\partial z_{21}}{\partial x_{11}}^{0} + \frac{\partial L}{\partial z_{22}} \frac{\partial z_{22}}{\partial x_{11}}^{0}$$

$$\frac{\partial L}{\partial x_{11}} = \frac{\partial L}{\partial z_{11}} k_{11}$$

$$\frac{\partial L}{\partial x_{12}} = \frac{\partial L}{\partial z_{11}} k_{12} + \frac{\partial L}{\partial z_{12}} k_{11}$$

$$\frac{\partial L}{\partial x_{13}} = \frac{\partial L}{\partial z_{12}} k_{12}$$

$$\frac{\partial L}{\partial x_{21}} = \frac{\partial L}{\partial z_{11}} k_{21} + \frac{\partial L}{\partial z_{21}} k_{11}$$

$$\frac{\partial L}{\partial x_{22}} = \frac{\partial L}{\partial z_{11}} k_{22} + \frac{\partial L}{\partial z_{12}} k_{21} + \frac{\partial L}{\partial z_{21}} k_{12} + \frac{\partial L}{\partial z_{22}} k_{11}$$

$$\frac{\partial L}{\partial x_{23}} = \frac{\partial L}{\partial z_{12}} k_{22} + \frac{\partial L}{\partial z_{22}} k_{12}$$

$$\frac{\partial L}{\partial x_{31}} = \frac{\partial L}{\partial z_{21}} k_{21}$$

$$\frac{\partial L}{\partial x_{32}} = \frac{\partial L}{\partial z_{21}} k_{22} + \frac{\partial L}{\partial z_{22}} k_{21}$$

$$\frac{\partial L}{\partial x_{33}} = \frac{\partial L}{\partial z_{22}} k_{22}$$

$$\Downarrow$$

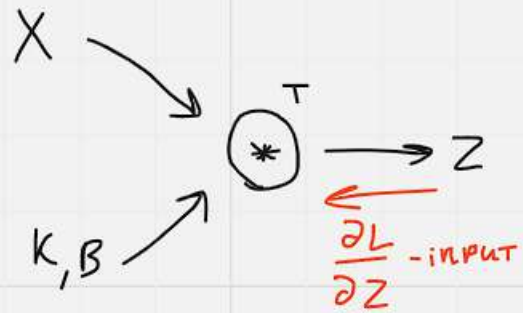
$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{\partial L}{\partial z_{11}} & \frac{\partial L}{\partial z_{12}} & 0 \\ 0 & \frac{\partial L}{\partial z_{21}} & \frac{\partial L}{\partial z_{22}} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \circledast \begin{bmatrix} k_{22} & k_{21} \\ k_{12} & k_{11} \end{bmatrix} = \begin{bmatrix} \frac{\partial L}{\partial x_{11}} & \frac{\partial L}{\partial x_{12}} & \frac{\partial L}{\partial x_{13}} \\ \frac{\partial L}{\partial x_{21}} & \frac{\partial L}{\partial x_{22}} & \frac{\partial L}{\partial x_{23}} \\ \frac{\partial L}{\partial x_{31}} & \frac{\partial L}{\partial x_{32}} & \frac{\partial L}{\partial x_{33}} \end{bmatrix}$$

$$\Downarrow$$

$$K \xrightarrow{180^\circ} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \xrightarrow{180^\circ}$$

Conv 2DTranspose

$$\left(\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} \circledast^{T} \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \right) + b = \begin{bmatrix} z_{11} & z_{12} & z_{13} \\ z_{21} & z_{22} & z_{23} \\ z_{31} & z_{32} & z_{33} \end{bmatrix}$$



$$z_{11} = x_{11} k_{11} + b$$

$$z_{12} = x_{11} k_{12} + x_{12} k_{11} + b$$

$$z_{13} = x_{12} k_{12} + b$$

$$z_{21} = x_{11} k_{21} + x_{21} k_{11} + b$$

$$z_{22} = x_{11} k_{22} + x_{12} k_{21} + x_{21} k_{12} + x_{22} k_{11} + b$$

$$z_{23} = x_{12} k_{22} + x_{22} k_{12} + b$$

$$z_{31} = x_{21} k_{21} + b$$

$$z_{32} = x_{21} k_{22} + x_{22} k_{21} + b$$

$$z_{33} = x_{22} k_{22} + b$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & x_{11} & x_{12} & 0 \\ 0 & x_{21} & x_{22} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \circledast \begin{bmatrix} k_{22} & k_{21} \\ k_{12} & k_{11} \end{bmatrix}$$

imp. padd = $k - p - 1 = 2 - 0 - 1 = 1$

$$\frac{\partial L}{\partial K_{mn}} = \sum \frac{\partial L}{\partial Z_{ij}} \frac{\partial Z_{ij}}{\partial K_{mn}}$$

$$\frac{\partial L}{\partial K_{11}} = \frac{\partial L}{\partial Z_{11}}\frac{\partial Z_{11}}{\partial K_{11}} + \frac{\partial L}{\partial Z_{12}}\frac{\partial Z_{12}}{\partial K_{11}} + \frac{\partial L}{\partial Z_{13}}\frac{\partial Z_{13}}{\partial K_{11}} + \frac{\partial L}{\partial Z_{21}}\frac{\partial Z_{11}}{\partial K_{11}} + \frac{\partial L}{\partial Z_{22}}\frac{\partial Z_{22}}{\partial K_{11}} + \frac{\partial L}{\partial Z_{23}}\frac{\partial Z_{13}}{\partial K_{11}} + \frac{\partial L}{\partial Z_{31}}\frac{\partial Z_{31}}{\partial K_{11}} + \frac{\partial L}{\partial Z_{32}}\frac{\partial Z_{32}}{\partial K_{11}} + \frac{\partial L}{\partial Z_{33}}\frac{\partial Z_{33}}{\partial K_{11}}$$

$$\frac{\partial L}{\partial K_{11}} = \frac{\partial L}{\partial Z_{11}} X_{11} + \frac{\partial L}{\partial Z_{12}} X_{12} + \frac{\partial L}{\partial Z_{13}} 0 + \frac{\partial L}{\partial Z_{21}} X_{11} + \frac{\partial L}{\partial Z_{22}} X_{12} + \frac{\partial L}{\partial Z_{23}} 0 + \frac{\partial L}{\partial Z_{31}} 0 + \frac{\partial L}{\partial Z_{32}} 0 + \frac{\partial L}{\partial Z_{33}} 0$$

$$\frac{\partial L}{\partial K_{11}} = \frac{\partial L}{\partial Z_{11}} X_{11} \quad \frac{\partial L}{\partial Z_{12}} X_{12} + \frac{\partial L}{\partial Z_{21}} X_{21} + \frac{\partial L}{\partial Z_{22}} X_{22}$$

$$\frac{\partial L}{\partial K} = \begin{bmatrix} \frac{\partial L}{\partial Z_{11}} & \frac{\partial L}{\partial Z_{12}} & \frac{\partial L}{\partial Z_{13}} \\ \frac{\partial L}{\partial Z_{21}} & \frac{\partial L}{\partial Z_{22}} & \frac{\partial L}{\partial Z_{23}} \\ \frac{\partial L}{\partial Z_{31}} & \frac{\partial L}{\partial Z_{32}} & \frac{\partial L}{\partial Z_{33}} \end{bmatrix} \circledast \begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix}$$

$$\frac{\partial L}{\partial b} = \sum \frac{\partial L}{\partial z_{ij}} \frac{\partial z_{ij}}{\partial b} \quad , \quad \frac{\partial z_{ij}}{\partial b} = 1 \quad \forall i,j$$

$$\Downarrow$$

$$\frac{\partial L}{\partial b} = \sum_{ij} \frac{\partial L}{\partial z_{ij}}$$

$$\frac{\partial L}{\partial X_{mn}} = \sum_{ij} \frac{\partial L}{\partial Z_{ij}} \frac{\partial Z_{ij}}{\partial X_{mn}}$$

$$\begin{cases} \frac{\partial L}{\partial X_{11}} = \frac{\partial L}{\partial Z_{11}} \frac{\partial Z_{11}}{\partial X_{11}} + \frac{\partial L}{\partial Z_{12}} \frac{\partial Z_{12}}{\partial X_{11}} + \frac{\partial L}{\partial Z_{21}} \frac{\partial Z_{21}}{\partial X_{11}} + \frac{\partial L}{\partial Z_{22}} \frac{\partial Z_{22}}{\partial X_{11}} \\ \frac{\partial L}{\partial X_{12}} = \frac{\partial L}{\partial Z_{12}} \frac{\partial Z_{12}}{\partial X_{12}} + \frac{\partial L}{\partial Z_{13}} \frac{\partial Z_{13}}{\partial X_{12}} + \frac{\partial L}{\partial Z_{22}} \frac{\partial Z_{22}}{\partial X_{12}} + \frac{\partial L}{\partial Z_{23}} \frac{\partial Z_{23}}{\partial X_{12}} \\ \frac{\partial L}{\partial X_{21}} = \frac{\partial L}{\partial Z_{21}} \frac{\partial Z_{21}}{\partial X_{21}} + \frac{\partial L}{\partial Z_{22}} \frac{\partial Z_{22}}{\partial X_{21}} + \frac{\partial L}{\partial Z_{31}} \frac{\partial Z_{31}}{\partial X_{21}} + \frac{\partial L}{\partial Z_{32}} \frac{\partial Z_{32}}{\partial X_{21}} \\ \frac{\partial L}{\partial X_{22}} = \frac{\partial L}{\partial Z_{22}} \frac{\partial Z_{22}}{\partial X_{22}} + \frac{\partial L}{\partial Z_{23}} \frac{\partial Z_{23}}{\partial X_{22}} + \frac{\partial L}{\partial Z_{32}} \frac{\partial Z_{32}}{\partial X_{22}} + \frac{\partial L}{\partial Z_{33}} \frac{\partial Z_{33}}{\partial X_{22}} \end{cases}$$

$$
\begin{cases}
\dfrac{\partial L}{\partial x_{11}} = \dfrac{\partial L}{\partial z_{11}} k_{11} + \dfrac{\partial L}{\partial z_{12}} k_{12} + \dfrac{\partial L}{\partial z_{21}} k_{21} + \dfrac{\partial L}{\partial z_{22}} k_{22} \\
\dfrac{\partial L}{\partial x_{12}} = \dfrac{\partial L}{\partial z_{12}} k_{11} + \dfrac{\partial L}{\partial z_{13}} k_{12} + \dfrac{\partial L}{\partial z_{22}} k_{21} + \dfrac{\partial L}{\partial z_{23}} k_{22} \\
\dfrac{\partial L}{\partial x_{21}} = \dfrac{\partial L}{\partial z_{21}} k_{11} + \dfrac{\partial L}{\partial z_{22}} k_{12} + \dfrac{\partial L}{\partial z_{31}} k_{21} + \dfrac{\partial L}{\partial z_{32}} k_{22} \\
\dfrac{\partial L}{\partial x_{22}} = \dfrac{\partial L}{\partial z_{22}} k_{11} + \dfrac{\partial L}{\partial z_{23}} k_{12} + \dfrac{\partial L}{\partial z_{32}} k_{21} + \dfrac{\partial L}{\partial z_{33}} k_{22}
\end{cases}
$$

$$
\Downarrow
$$

$$
\begin{bmatrix}
\frac{\partial L}{\partial z_{11}} & \frac{\partial L}{\partial z_{12}} & \frac{\partial L}{\partial z_{13}} \\
\frac{\partial L}{\partial z_{21}} & \frac{\partial L}{\partial z_{22}} & \frac{\partial L}{\partial z_{23}} \\
\frac{\partial L}{\partial z_{31}} & \frac{\partial L}{\partial z_{32}} & \frac{\partial L}{\partial z_{33}}
\end{bmatrix}
\circledast
\begin{bmatrix}
k_{11} & k_{12} \\
k_{21} & k_{22}
\end{bmatrix}
=
\begin{bmatrix}
\frac{\partial L}{\partial x_{11}} & \frac{\partial L}{\partial x_{12}} \\
\frac{\partial L}{\partial x_{21}} & \frac{\partial L}{\partial x_{22}}
\end{bmatrix}
$$