betaDelta: Methods

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1 Multivariate Normal-Theory Approach

```
object <- lm(QUALITY ~ NARTIC + PCTGRT + PCTSUPP, data = nas1982)
```

```
out <- BetaDelta(object, type = "mvn")
str(out)

#> List of 8
#>  $ call: language BetaDelta(object = object, type = "mvn")
#>  $ lm  :List of 12
#>   ..$ coefficients : Named num [1:4] 10.3592 0.0842 0.216 0.1126
#>   .. ..- attr(*, "names")= chr [1:4] "(Intercept)" "NARTIC" "PCTGRT" "PCTSUPP"
#>   ..$ residuals    : Named num [1:46] -3.068 -0.688 2.675 2.052 4.039 ...
#>   .. ..- attr(*, "names")= chr [1:46] "1" "2" "3" "4" ...
#>   ..$ effects      : Named num [1:46] -187.4 51.49 27.59 -16.11 4.51 ...
#>   .. ..- attr(*, "names")= chr [1:46] "(Intercept)" "NARTIC" "PCTGRT" "PCTSUPP" ...
#>   ..$ rank         : int 4
#>   ..$ fitted.values: Named num [1:46] 15.1 23.7 26.3 33.9 40 ...
#>   .. ..- attr(*, "names")= chr [1:46] "1" "2" "3" "4" ...
#>   ..$ assign       : int [1:4] 0 1 2 3
#>   ..$ qr           :List of 5
#>   .. ..$ qr   : num [1:46, 1:4] -6.782 0.147 0.147 0.147 0.147 ...
#>   .. .. ..- attr(*, "dimnames")=List of 2
#>   .. .. .. ..$ : chr [1:46] "1" "2" "3" "4" ...
#>   .. .. .. ..$ : chr [1:4] "(Intercept)" "NARTIC" "PCTGRT" "PCTSUPP"
#>   .. .. ..- attr(*, "assign")= int [1:4] 0 1 2 3
#>   .. ..$ qraux: num [1:4] 1.15 1.04 1.06 1.09
#>   .. ..$ pivot: int [1:4] 1 2 3 4
#>   .. ..$ tol  : num 1e-07
#>   .. ..$ rank : int 4
#>   .. ..- attr(*, "class")= chr "qr"
#>   ..$ df.residual  : int 42
#>   ..$ xlevels      : Named list()
#>   ..$ call         : language lm(formula = QUALITY ~ NARTIC + PCTGRT + PCTSUPP, data = nas1982)
#>   ..$ terms        :Classes 'terms', 'formula'  language QUALITY ~ NARTIC + PCTGRT + PCTSUPP
```

```
#>    .. .. ..- attr(*, "variables")= language list(QUALITY, NARTIC, PCTGRT, PCTSUPP)
#>    .. .. ..- attr(*, "factors")= int [1:4, 1:3] 0 1 0 0 0 0 1 0 0 0 ...
#>    .. .. .. ..- attr(*, "dimnames")=List of 2
#>    .. .. .. .. ..$ : chr [1:4] "QUALITY" "NARTIC" "PCTGRT" "PCTSUPP"
#>    .. .. .. .. ..$ : chr [1:3] "NARTIC" "PCTGRT" "PCTSUPP"
#>    .. .. ..- attr(*, "term.labels")= chr [1:3] "NARTIC" "PCTGRT" "PCTSUPP"
#>    .. .. ..- attr(*, "order")= int [1:3] 1 1 1
#>    .. .. ..- attr(*, "intercept")= int 1
#>    .. .. ..- attr(*, "response")= int 1
#>    .. .. ..- attr(*, ".Environment")=<environment: 0x55eafcca99d8>
#>    .. .. ..- attr(*, "predvars")= language list(QUALITY, NARTIC, PCTGRT, PCTSUPP)
#>    .. .. ..- attr(*, "dataClasses")= Named chr [1:4] "numeric" "numeric" "numeric" "numeric"
#>    .. .. .. ..- attr(*, "names")= chr [1:4] "QUALITY" "NARTIC" "PCTGRT" "PCTSUPP"
#>   ..$ model        :'data.frame': 46 obs. of  4 variables:
#>    .. ..$ QUALITY: int [1:46] 12 23 29 36 44 21 40 42 24 30 ...
#>    .. ..$ NARTIC : int [1:46] 14 61 68 49 130 65 79 187 32 50 ...
#>    .. ..$ PCTGRT : int [1:46] 8 3 13 63 53 29 35 40 19 8 ...
#>    .. ..$ PCTSUPP: int [1:46] 16 67 66 52 64 59 81 65 87 43 ...
#>    .. ..- attr(*, "terms")=Classes 'terms', 'formula'  language QUALITY ~ NARTIC + PCTGRT + PCTSUP
#>    .. .. .. ..- attr(*, "variables")= language list(QUALITY, NARTIC, PCTGRT, PCTSUPP)
#>    .. .. .. ..- attr(*, "factors")= int [1:4, 1:3] 0 1 0 0 0 0 1 0 0 0 ...
#>    .. .. .. .. ..- attr(*, "dimnames")=List of 2
#>    .. .. .. .. .. ..$ : chr [1:4] "QUALITY" "NARTIC" "PCTGRT" "PCTSUPP"
#>    .. .. .. .. .. ..$ : chr [1:3] "NARTIC" "PCTGRT" "PCTSUPP"
#>    .. .. .. ..- attr(*, "term.labels")= chr [1:3] "NARTIC" "PCTGRT" "PCTSUPP"
#>    .. .. .. ..- attr(*, "order")= int [1:3] 1 1 1
#>    .. .. .. ..- attr(*, "intercept")= int 1
#>    .. .. .. ..- attr(*, "response")= int 1
#>    .. .. .. ..- attr(*, ".Environment")=<environment: 0x55eafcca99d8>
#>    .. .. .. ..- attr(*, "predvars")= language list(QUALITY, NARTIC, PCTGRT, PCTSUPP)
#>    .. .. .. ..- attr(*, "dataClasses")= Named chr [1:4] "numeric" "numeric" "numeric" "numeric"
#>    .. .. .. .. ..- attr(*, "names")= chr [1:4] "QUALITY" "NARTIC" "PCTGRT" "PCTSUPP"
#>   ..- attr(*, "class")= chr "lm"
#>  $ type: chr "mvn"
#>  $ beta: Named num [1:3] 0.495 0.391 0.263
#>   ..- attr(*, "names")= chr [1:3] "NARTIC" "PCTGRT" "PCTSUPP"
#>  $ vcov: num [1:3, 1:3] 0.00575 -0.00336 -0.00217 -0.00336 0.00593 ...
#>   ..- attr(*, "dimnames")=List of 2
#>   .. ..$ : chr [1:3] "NARTIC" "PCTGRT" "PCTSUPP"
#>   .. ..$ : chr [1:3] "NARTIC" "PCTGRT" "PCTSUPP"
#>  $ n   : int 46
#>  $ p   : num 3
#>  $ df  : int 42
#>  - attr(*, "class")= chr [1:2] "betadelta" "list"

BetaDelta(object, type = "mvn")
```

```
#> Call:
#> BetaDelta(object = object, type = "mvn")
#>
#> Standardized regression slopes with MVN standard errors:
#>            est     se      t      p    0.05%    0.5%    2.5%   97.5%   99.5%  99.95%
#> NARTIC   0.4951 0.0759 6.5272 0.000   0.2268  0.2905  0.3421  0.6482  0.6998  0.7635
#> PCTGRT   0.3915 0.0770 5.0824 0.000   0.1190  0.1837  0.2360  0.5469  0.5993  0.6640
#> PCTSUPP  0.2632 0.0747 3.5224 0.001  -0.0011  0.0616  0.1124  0.4141  0.4649  0.5276
```

print

```
print(out)
```

```
#> Call:
#> BetaDelta(object = object, type = "mvn")
#>
#> Standardized regression slopes with MVN standard errors:
#>            est     se      t      p    0.05%    0.5%    2.5%   97.5%   99.5%  99.95%
#> NARTIC   0.4951 0.0759 6.5272 0.000   0.2268  0.2905  0.3421  0.6482  0.6998  0.7635
#> PCTGRT   0.3915 0.0770 5.0824 0.000   0.1190  0.1837  0.2360  0.5469  0.5993  0.6640
#> PCTSUPP  0.2632 0.0747 3.5224 0.001  -0.0011  0.0616  0.1124  0.4141  0.4649  0.5276
```

coef

```
coef(out)
```

```
#>    NARTIC    PCTGRT   PCTSUPP
#> 0.4951451 0.3914887 0.2632477
```

vcov

```
vcov(out)
```

```
#>               NARTIC       PCTGRT      PCTSUPP
#> NARTIC   0.005754524 -0.003360334 -0.002166127
#> PCTGRT  -0.003360334  0.005933462 -0.001769723
#> PCTSUPP -0.002166127 -0.001769723  0.005585256
```

confint

```
confint(out, level = 0.95)

#>              2.5%      97.5%
#> NARTIC   0.3420563 0.6482339
#> PCTGRT   0.2360380 0.5469395
#> PCTSUPP  0.1124272 0.4140682
```

summary

```
summary(out)

#> Call:
#> BetaDelta(object = object, type = "mvn")
#>
#> Standardized regression slopes with MVN standard errors:
#>            est     se      t      p   0.05%    0.5%    2.5%  97.5%  99.5% 99.95%
#> NARTIC   0.4951 0.0759 6.5272 0.000  0.2268  0.2905  0.3421 0.6482 0.6998 0.7635
#> PCTGRT   0.3915 0.0770 5.0824 0.000  0.1190  0.1837  0.2360 0.5469 0.5993 0.6640
#> PCTSUPP  0.2632 0.0747 3.5224 0.001 -0.0011  0.0616  0.1124 0.4141 0.4649 0.5276
```

2 Asymptotic Distribution-Free Approach

```
object <- lm(QUALITY ~ NARTIC + PCTGRT + PCTSUPP, data = nas1982)
```

```
out <- BetaDelta(object, type = "adf")
str(out)

#> List of 8
#>  $ call: language BetaDelta(object = object, type = "adf")
#>  $ lm  :List of 12
#>   ..$ coefficients : Named num [1:4] 10.3592 0.0842 0.216 0.1126
#>   .. ..- attr(*, "names")= chr [1:4] "(Intercept)" "NARTIC" "PCTGRT" "PCTSUPP"
#>   ..$ residuals    : Named num [1:46] -3.068 -0.688 2.675 2.052 4.039 ...
#>   .. ..- attr(*, "names")= chr [1:46] "1" "2" "3" "4" ...
#>   ..$ effects      : Named num [1:46] -187.4 51.49 27.59 -16.11 4.51 ...
#>   .. ..- attr(*, "names")= chr [1:46] "(Intercept)" "NARTIC" "PCTGRT" "PCTSUPP" ...
#>   ..$ rank         : int 4
#>   ..$ fitted.values: Named num [1:46] 15.1 23.7 26.3 33.9 40 ...
#>   .. ..- attr(*, "names")= chr [1:46] "1" "2" "3" "4" ...
#>   ..$ assign       : int [1:4] 0 1 2 3
#>   ..$ qr           :List of 5
```

4

```
#>   .. ..$ qr   : num [1:46, 1:4] -6.782 0.147 0.147 0.147 0.147 ...
#>   .. .. ..- attr(*, "dimnames")=List of 2
#>   .. .. .. ..$ : chr [1:46] "1" "2" "3" "4" ...
#>   .. .. .. ..$ : chr [1:4] "(Intercept)" "NARTIC" "PCTGRT" "PCTSUPP"
#>   .. .. ..- attr(*, "assign")= int [1:4] 0 1 2 3
#>   .. ..$ qraux: num [1:4] 1.15 1.04 1.06 1.09
#>   .. ..$ pivot: int [1:4] 1 2 3 4
#>   .. ..$ tol  : num 1e-07
#>   .. ..$ rank : int 4
#>   .. ..- attr(*, "class")= chr "qr"
#>   ..$ df.residual  : int 42
#>   ..$ xlevels      : Named list()
#>   ..$ call         : language lm(formula = QUALITY ~ NARTIC + PCTGRT + PCTSUPP, data = nas1982)
#>   ..$ terms        :Classes 'terms', 'formula'  language QUALITY ~ NARTIC + PCTGRT + PCTSUPP
#>   .. .. ..- attr(*, "variables")= language list(QUALITY, NARTIC, PCTGRT, PCTSUPP)
#>   .. .. ..- attr(*, "factors")= int [1:4, 1:3] 0 1 0 0 0 0 1 0 0 0 ...
#>   .. .. .. ..- attr(*, "dimnames")=List of 2
#>   .. .. .. .. ..$ : chr [1:4] "QUALITY" "NARTIC" "PCTGRT" "PCTSUPP"
#>   .. .. .. .. ..$ : chr [1:3] "NARTIC" "PCTGRT" "PCTSUPP"
#>   .. .. ..- attr(*, "term.labels")= chr [1:3] "NARTIC" "PCTGRT" "PCTSUPP"
#>   .. .. ..- attr(*, "order")= int [1:3] 1 1 1
#>   .. .. ..- attr(*, "intercept")= int 1
#>   .. .. ..- attr(*, "response")= int 1
#>   .. .. ..- attr(*, ".Environment")=<environment: 0x55eafcca99d8>
#>   .. .. ..- attr(*, "predvars")= language list(QUALITY, NARTIC, PCTGRT, PCTSUPP)
#>   .. .. ..- attr(*, "dataClasses")= Named chr [1:4] "numeric" "numeric" "numeric" "numeric"
#>   .. .. .. ..- attr(*, "names")= chr [1:4] "QUALITY" "NARTIC" "PCTGRT" "PCTSUPP"
#>   ..$ model        :'data.frame': 46 obs. of  4 variables:
#>   .. ..$ QUALITY: int [1:46] 12 23 29 36 44 21 40 42 24 30 ...
#>   .. ..$ NARTIC : int [1:46] 14 61 68 49 130 65 79 187 32 50 ...
#>   .. ..$ PCTGRT : int [1:46] 8 3 13 63 53 29 35 40 19 8 ...
#>   .. ..$ PCTSUPP: int [1:46] 16 67 66 52 64 59 81 65 87 43 ...
#>   .. ..- attr(*, "terms")=Classes 'terms', 'formula'  language QUALITY ~ NARTIC + PCTGRT + PCTSU
#>   .. .. .. ..- attr(*, "variables")= language list(QUALITY, NARTIC, PCTGRT, PCTSUPP)
#>   .. .. .. ..- attr(*, "factors")= int [1:4, 1:3] 0 1 0 0 0 0 1 0 0 0 ...
#>   .. .. .. .. ..- attr(*, "dimnames")=List of 2
#>   .. .. .. .. .. ..$ : chr [1:4] "QUALITY" "NARTIC" "PCTGRT" "PCTSUPP"
#>   .. .. .. .. .. ..$ : chr [1:3] "NARTIC" "PCTGRT" "PCTSUPP"
#>   .. .. .. ..- attr(*, "term.labels")= chr [1:3] "NARTIC" "PCTGRT" "PCTSUPP"
#>   .. .. .. ..- attr(*, "order")= int [1:3] 1 1 1
#>   .. .. .. ..- attr(*, "intercept")= int 1
#>   .. .. .. ..- attr(*, "response")= int 1
#>   .. .. .. ..- attr(*, ".Environment")=<environment: 0x55eafcca99d8>
#>   .. .. .. ..- attr(*, "predvars")= language list(QUALITY, NARTIC, PCTGRT, PCTSUPP)
#>   .. .. .. ..- attr(*, "dataClasses")= Named chr [1:4] "numeric" "numeric" "numeric" "numeric"
```

```
#>    .. .. .. .. ..- attr(*, "names")= chr [1:4] "QUALITY" "NARTIC" "PCTGRT" "PCTSUPP"
#>    ..- attr(*, "class")= chr "lm"
#>  $ type: chr "adf"
#>  $ beta: Named num [1:3] 0.495 0.391 0.263
#>    ..- attr(*, "names")= chr [1:3] "NARTIC" "PCTGRT" "PCTSUPP"
#>  $ vcov: num [1:3, 1:3] 0.00454 -0.00255 -0.00174 -0.00255 0.00504 ...
#>    ..- attr(*, "dimnames")=List of 2
#>    .. ..$ : chr [1:3] "NARTIC" "PCTGRT" "PCTSUPP"
#>    .. ..$ : chr [1:3] "NARTIC" "PCTGRT" "PCTSUPP"
#>  $ n   : int 46
#>  $ p   : num 3
#>  $ df  : int 42
#>  - attr(*, "class")= chr [1:2] "betadelta" "list"

BetaDelta(object, type = "adf")

#> Call:
#> BetaDelta(object = object, type = "adf")
#>
#> Standardized regression slopes with ADF standard errors:
#>           est     se      t      p   0.05%   0.5%   2.5%  97.5%  99.5% 99.95%
#> NARTIC  0.4951 0.0674 7.3490 0.0000  0.2568 0.3134 0.3592 0.6311 0.6769 0.7335
#> PCTGRT  0.3915 0.0710 5.5164 0.0000  0.1404 0.2000 0.2483 0.5347 0.5830 0.6426
#> PCTSUPP 0.2632 0.0769 3.4231 0.0014 -0.0088 0.0558 0.1081 0.4184 0.4707 0.5353
```

print

```
print(out)

#> Call:
#> BetaDelta(object = object, type = "adf")
#>
#> Standardized regression slopes with ADF standard errors:
#>           est     se      t      p   0.05%   0.5%   2.5%  97.5%  99.5% 99.95%
#> NARTIC  0.4951 0.0674 7.3490 0.0000  0.2568 0.3134 0.3592 0.6311 0.6769 0.7335
#> PCTGRT  0.3915 0.0710 5.5164 0.0000  0.1404 0.2000 0.2483 0.5347 0.5830 0.6426
#> PCTSUPP 0.2632 0.0769 3.4231 0.0014 -0.0088 0.0558 0.1081 0.4184 0.4707 0.5353
```

coef

```
coef(out)

#>    NARTIC    PCTGRT   PCTSUPP
#> 0.4951451 0.3914887 0.2632477
```

vcov

```
vcov(out)

#>                NARTIC       PCTGRT       PCTSUPP
#> NARTIC    0.004539472 -0.002552698 -0.001742698
#> PCTGRT   -0.002552698  0.005036538 -0.001906216
#> PCTSUPP  -0.001742698 -0.001906216  0.005914088
```

confint

```
confint(out, level = 0.95)

#>              2.5%      97.5%
#> NARTIC   0.3591757 0.6311146
#> PCTGRT   0.2482683 0.5347091
#> PCTSUPP  0.1080509 0.4184444
```

summary

```
summary(out)

#> Call:
#> BetaDelta(object = object, type = "adf")
#>
#> Standardized regression slopes with ADF standard errors:
#>            est     se     t      p   0.05%   0.5%   2.5%  97.5%  99.5% 99.95%
#> NARTIC  0.4951 0.0674 7.3490 0.0000  0.2568 0.3134 0.3592 0.6311 0.6769 0.7335
#> PCTGRT  0.3915 0.0710 5.5164 0.0000  0.1404 0.2000 0.2483 0.5347 0.5830 0.6426
#> PCTSUPP 0.2632 0.0769 3.4231 0.0014 -0.0088 0.0558 0.1081 0.4184 0.4707 0.5353
```

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References

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R Core Team. (2022). *R: A language and environment for statistical computing*. R Foundation for Statistical Computing. Vienna, Austria. https://www.R-project.org/

Yuan, K.-H., & Chan, W. (2011). Biases and standard errors of standardized regression coefficients. *Psychometrika*, *76*(4), 670–690. https://doi.org/10.1007/s11336-011-9224-6