

Package ‘betaDelta’

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Title Confidence Intervals for Standardized Regression Coefficients

Version 1.0.5.9000

Description Generates confidence intervals for standardized regression coefficients using delta method standard errors for models fitted by `lm()` as described in Yuan and Chan (2011) <[doi:10.1007/s11336-011-9224-6](https://doi.org/10.1007/s11336-011-9224-6)> and Jones and Waller (2015) <[doi:10.1007/s11336-013-9380-y](https://doi.org/10.1007/s11336-013-9380-y)>. The package can also be used to generate confidence intervals for differences of standardized regression coefficients and as a general approach to performing the delta method. A description of the package and code examples are presented in Pesigan, Sun, and Cheung (2023) <[doi:10.1080/00273171.2023.2201277](https://doi.org/10.1080/00273171.2023.2201277)>.

URL <https://github.com/jeksterslab/betaDelta>,
<https://jeksterslab.github.io/betaDelta/>

BugReports <https://github.com/jeksterslab/betaDelta/issues>

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| | |
|-----------|---|
| BetaDelta | <i>Estimate Standardized Regression Coefficients and the Corresponding Sampling Covariance Matrix</i> |
|-----------|---|

Description

Estimate Standardized Regression Coefficients and the Corresponding Sampling Covariance Matrix

Usage

```
BetaDelta(object, type = "mvn", alpha = c(0.05, 0.01, 0.001))
```

Arguments

| | |
|--------|---|
| object | Object of class <code>lm</code> . |
| type | Character string. If <code>type = "mvn"</code> , use the multivariate normal-theory approach. If <code>type = "adf"</code> , use the asymptotic distribution-free approach. |
| alpha | Numeric vector. Significance level α . |

Value

Returns an object of class `betadelta` which is a list with the following elements:

call Function call.

args Function arguments.

lm_process Processed `lm` object.

gamma Asymptotic covariance matrix of the sample covariance matrix.

acov Asymptotic covariance matrix of the standardized slopes.

vcov Sampling covariance matrix of the standardized slopes.

est Vector of standardized slopes.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Jones, J. A., & Waller, N. G. (2015). The normal-theory and asymptotic distribution-free (ADF) covariance matrix of standardized regression coefficients: Theoretical extensions and finite sample behavior. *Psychometrika*, 80(2), 365–378. doi:[10.1007/s113360139380y](https://doi.org/10.1007/s113360139380y)

Pesigan, I. J. A., Sun, R. W., & Cheung, S. F. (2023). `betaDelta` and `betaSandwich`: Confidence intervals for standardized regression coefficients in R. *Multivariate Behavioral Research*. doi:[10.1080/00273171.2023.2201277](https://doi.org/10.1080/00273171.2023.2201277)

Yuan, K.-H., & Chan, W. (2011). Biases and standard errors of standardized regression coefficients. *Psychometrika*, 76(4), 670–690. doi:[10.1007/s1133601192246](https://doi.org/10.1007/s1133601192246)

See Also

Other Beta Delta Functions: [DiffBetaDelta\(\)](#)

Examples

```
object <- lm(QUALITY ~ NARTIC + PCTGRT + PCTSUPP, data = nas1982)
std <- BetaDelta(object)
# Methods -----
print(std)
summary(std)
coef(std)
vcov(std)
confint(std, level = 0.95)
```

| | |
|----------------|---------------------------------------|
| coef.betadelta | <i>Standardized Regression Slopes</i> |
|----------------|---------------------------------------|

Description

Standardized Regression Slopes

Usage

```
## S3 method for class 'betadelta'
coef(object, ...)
```

Arguments

| | |
|--------|----------------------------|
| object | Object of class betadelta. |
| ... | additional arguments. |

Value

Returns a vector of standardized regression slopes.

Author(s)

Ivan Jacob Agaloos Pesigan

Examples

```
object <- lm(QUALITY ~ NARTIC + PCTGRT + PCTSUPP, data = nas1982)
std <- BetaDelta(object)
coef(std)
```

| | |
|------------------|------------------|
| coef.deltamethod | <i>Estimates</i> |
|------------------|------------------|

Description

Estimates

Usage

```
## S3 method for class 'deltamethod'
coef(object, ...)
```

Arguments

object Object of class deltamethod.
 ... additional arguments.

Value

Returns a vector of estimates.

Author(s)

Ivan Jacob Agaloos Pesigan

Examples

```
object <- glm(
  formula = vs ~ wt + disp,
  family = "binomial",
  data = mtcars
)
def <- list("exp(wt)", "exp(disp)")
out <- DeltaGeneric(
  object = object,
  def = def,
  alpha = 0.05
)
coef(out)
```

coef.diffbetadelta *Differences of Standardized Regression Slopes*

Description

Differences of Standardized Regression Slopes

Usage

```
## S3 method for class 'diffbetadelta'
coef(object, ...)
```

Arguments

object Object of class diffbetadelta.
 ... additional arguments.

Value

Returns a vector of differences of standardized regression slopes.

Author(s)

Ivan Jacob Agaloos Pesigan

Examples

```
object <- lm(QUALITY ~ NARTIC + PCTGRT + PCTSUPP, data = nas1982)
std <- BetaDelta(object)
diff <- DiffBetaDelta(std)
coef(diff)
```

| | |
|-------------------|--|
| confint.betadelta | <i>Confidence Intervals for Standardized Regression Slopes</i> |
|-------------------|--|

Description

Confidence Intervals for Standardized Regression Slopes

Usage

```
## S3 method for class 'betadelta'
confint(object, parm = NULL, level = 0.95, ...)
```

Arguments

| | |
|--------|---|
| object | Object of class betadelta. |
| parm | a specification of which parameters are to be given confidence intervals, either a vector of numbers or a vector of names. If missing, all parameters are considered. |
| level | the confidence level required. |
| ... | additional arguments. |

Value

Returns a matrix of confidence intervals.

Author(s)

Ivan Jacob Agaloos Pesigan

Examples

```
object <- lm(QUALITY ~ NARTIC + PCTGRT + PCTSUPP, data = nas1982)
std <- BetaDelta(object)
confint(std, level = 0.95)
```

`confint.deltamethod` *Confidence Intervals*

Description

Confidence Intervals

Usage

```
## S3 method for class 'deltamethod'  
confint(object, parm = NULL, level = 0.95, ...)
```

Arguments

| | |
|---------------------|---|
| <code>object</code> | Object of class <code>deltamethod</code> . |
| <code>parm</code> | a specification of which parameters are to be given confidence intervals, either a vector of numbers or a vector of names. If missing, all parameters are considered. |
| <code>level</code> | the confidence level required. |
| <code>...</code> | additional arguments. |

Value

Returns a matrix of confidence intervals.

Author(s)

Ivan Jacob Agaloos Pesigan

Examples

```
object <- glm(  
  formula = vs ~ wt + disp,  
  family = "binomial",  
  data = mtcars  
)  
def <- list("exp(wt)", "exp(disp)")  
out <- DeltaGeneric(  
  object = object,  
  def = def,  
  alpha = 0.05  
)  
confint(out, level = 0.95)
```

confint.diffbetadelta *Confidence Intervals for Differences of Standardized Regression Slopes*

Description

Confidence Intervals for Differences of Standardized Regression Slopes

Usage

```
## S3 method for class 'diffbetadelta'  
confint(object, parm = NULL, level = 0.95, ...)
```

Arguments

| | |
|--------|---|
| object | Object of class diffbetadelta. |
| parm | a specification of which parameters are to be given confidence intervals, either a vector of numbers or a vector of names. If missing, all parameters are considered. |
| level | the confidence level required. |
| ... | additional arguments. |

Value

Returns a matrix of confidence intervals.

Author(s)

Ivan Jacob Agaloos Pesigan

Examples

```
object <- lm(QUALITY ~ NARTIC + PCTGRT + PCTSUPP, data = nas1982)  
std <- BetaDelta(object)  
diff <- DiffBetaDelta(std)  
confint(diff)
```


Delta

*Delta Method***Description**

Calculates delta method sampling variance-covariance matrix for a function of parameters using a numerical Jacobian.

Usage

```
Delta(
  coef,
  vcov,
  func,
  ...,
  theta = 0,
  alpha = c(0.05, 0.01, 0.001),
  z = TRUE,
  df = NULL
)
```

Arguments

| | |
|--------------------|---|
| <code>coef</code> | Numeric vector. Vector of parameters. |
| <code>vcov</code> | Numeric matrix. Matrix of sampling variance-covariance matrix of parameters. |
| <code>func</code> | R function. <ol style="list-style-type: none"> 1. The first argument <code>x</code> is the argument <code>coef</code>. 2. The function algebraically manipulates <code>coef</code> to return a new numeric vector. It is best to have a named vector as an output. 3. The function can take additional named arguments passed using <code>...</code> |
| <code>...</code> | Additional arguments to pass to <code>func</code> . |
| <code>theta</code> | Numeric vector. Parameter values when the null hypothesis is true. |
| <code>alpha</code> | Numeric vector. Significance level/s. |
| <code>z</code> | Logical. If <code>z = TRUE</code> , use the standard normal distribution. If <code>z = FALSE</code> , use the <code>t</code> distribution. |
| <code>df</code> | Numeric. Degrees of freedom if <code>z = FALSE</code> . |

Value

Returns an object of class `deltamethod` which is a list with the following elements:

call Function call.

args Function arguments.

coef Estimates.

vcov Sampling variance-covariance matrix.

jacobian Jacobian matrix.

fun Function used ("Delta").

Author(s)

Ivan Jacob Agaloos Pesigan

See Also

Other Delta Method Functions: [DeltaGeneric\(\)](#)

Examples

```
object <- glm(
  formula = vs ~ wt + disp,
  family = "binomial",
  data = mtcars
)
func <- function(x) {
  y <- exp(x)
  names(y) <- paste0("exp", "(", names(x), ")")
  return(y[-1])
}
Delta(
  coef = coef(object),
  vcov = vcov(object),
  func = func,
  alpha = 0.05
)
```

DeltaGeneric

Delta Method (Generic Object Input)

Description

Calculates delta method sampling variance-covariance matrix for a function of parameters using a numerical Jacobian.

Usage

```
DeltaGeneric(
  object,
  def,
  theta = 0,
  alpha = c(0.05, 0.01, 0.001),
  z = TRUE,
  df = NULL
)
```

Arguments

| | |
|---------------|--|
| object | R object. Fitted model object with <code>coef</code> and <code>vcov</code> methods that return a named vector of estimated parameters and sampling variance-covariance matrix, respectively. |
| def | List of character strings. A list of defined functions of parameters. The string should be a valid R expression when parsed and should result a single value when evaluated. |
| theta | Numeric vector. Parameter values when the null hypothesis is true. |
| alpha | Numeric vector. Significance level/s. |
| z | Logical. If <code>z = TRUE</code> , use the standard normal distribution. If <code>z = FALSE</code> , use the <code>t</code> distribution. |
| df | Numeric. Degrees of freedom if <code>z = FALSE</code> . |

Value

Returns an object of class `deltamethod` which is a list with the following elements:

call Function call.

args Function arguments.

coef Estimates.

vcov Sampling variance-covariance matrix.

jacobian Jacobian matrix.

fun Function used ("DeltaGeneric").

Author(s)

Ivan Jacob Agaloos Pesigan

See Also

Other Delta Method Functions: [Delta\(\)](#)

Examples

```
object <- glm(
  formula = vs ~ wt + disp,
  family = "binomial",
  data = mtcars
)
def <- list("exp(wt)", "exp(disp)")
DeltaGeneric(
  object = object,
  def = def,
  alpha = 0.05
)
```

| | |
|---------------|---|
| DiffBetaDelta | <i>Estimate Differences of Standardized Slopes and the Corresponding Sampling Covariance Matrix</i> |
|---------------|---|

Description

Estimate Differences of Standardized Slopes and the Corresponding Sampling Covariance Matrix

Usage

```
DiffBetaDelta(object, alpha = c(0.05, 0.01, 0.001))
```

Arguments

| | |
|--------|--|
| object | Object of class <code>betadelta</code> , that is, the output of the <code>BetaDelta()</code> function. |
| alpha | Numeric vector. Significance level α . |

Value

Returns an object of class `diffbetadelta` which is a list with the following elements:

call Function call.

fit The argument `object`.

args Function arguments.

vcov Sampling covariance matrix of differences of standardized slopes.

est Vector of differences of standardized slopes.

Author(s)

Ivan Jacob Agaloos Pesigan

See Also

Other Beta Delta Functions: `BetaDelta()`

Examples

```
object <- lm(QUALITY ~ NARTIC + PCTGRT + PCTSUPP, data = nas1982)
std <- BetaDelta(object)
diff <- DiffBetaDelta(std)
# Methods -----
print(diff)
summary(diff)
coef(diff)
vcov(diff)
confint(diff, level = 0.95)
```

nas1982

*1982 National Academy of Sciences Doctoral Programs Data***Description**

1982 National Academy of Sciences Doctoral Programs Data

Usage

nas1982

Format

Ratings of 46 doctoral programs in psychology in the USA with the following variables:

QUALITY Program quality ratings.**NFACUL** Number of faculty members in the program.**NGRADS** Number of program graduates.**PCTSUPP** Percentage of program graduates who received support.**PCTGRT** Percent of faculty members holding research grants.**NARTIC** Number of published articles attributed to program faculty member.**PCTPUB** Percent of faculty with one or more published article.**References**

National Research Council. (1982). *An assessment of research-doctorate programs in the United States: Social and behavioral sciences*. doi:10.17226/9781. Reproduced with permission from the National Academy of Sciences, Courtesy of the National Academies Press, Washington, D.C.

print.betadelta

*Print Method for an Object of Class betadelta***Description**

Print Method for an Object of Class betadelta

Usage

```
## S3 method for class 'betadelta'
print(x, alpha = NULL, digits = 4, ...)
```

Arguments

| | |
|---------------------|---|
| <code>x</code> | Object of class <code>betadelta</code> . |
| <code>alpha</code> | Numeric vector. Significance level α . If <code>alpha = NULL</code> , use the argument <code>alpha</code> used in <code>x</code> . |
| <code>digits</code> | Digits to print. |
| <code>...</code> | additional arguments. |

Value

Returns a matrix of standardized regression slopes, standard errors, test statistics, degrees of freedom, p-values, and confidence intervals.

Author(s)

Ivan Jacob Agaloos Pesigan

Examples

```
object <- lm(QUALITY ~ NARTIC + PCTGRT + PCTSUPP, data = nas1982)
std <- BetaDelta(object)
print(std)
```

| | |
|--------------------------------|---|
| <code>print.deltamethod</code> | <i>Print Method for an Object of Class <code>deltamethod</code></i> |
|--------------------------------|---|

Description

Print Method for an Object of Class `deltamethod`

Usage

```
## S3 method for class 'deltamethod'
print(x, alpha = NULL, digits = 4, ...)
```

Arguments

| | |
|---------------------|---|
| <code>x</code> | Object of class <code>deltamethod</code> . |
| <code>alpha</code> | Numeric vector. Significance level α . If <code>alpha = NULL</code> , use the argument <code>alpha</code> used in <code>x</code> . |
| <code>digits</code> | Digits to print. |
| <code>...</code> | additional arguments. |

Value

Returns a matrix of coefficients, standard errors, test statistics, degrees of freedom (if `z = FALSE`), p-values, and confidence intervals.

Author(s)

Ivan Jacob Agaloos Pesigan

Examples

```
object <- glm(
  formula = vs ~ wt + disp,
  family = "binomial",
  data = mtcars
)
def <- list("exp(wt)", "exp(disp)")
out <- DeltaGeneric(
  object = object,
  def = def,
  alpha = 0.05
)
print(out)
```

print.diffbetadelta *Print Method for an Object of Class diffbetadelta*

Description

Print Method for an Object of Class diffbetadelta

Usage

```
## S3 method for class 'diffbetadelta'
print(x, alpha = NULL, digits = 4, ...)
```

Arguments

| | |
|---------------------|---|
| <code>x</code> | Object of class diffbetadelta. |
| <code>alpha</code> | Numeric vector. Significance level α . If <code>alpha = NULL</code> , use the argument <code>alpha</code> used in <code>x</code> . |
| <code>digits</code> | Digits to print. |
| <code>...</code> | additional arguments. |

Value

Returns a matrix of standardized regression slopes, standard errors, test statistics, degrees of freedom, p-values, and confidence intervals.

Author(s)

Ivan Jacob Agaloos Pesigan

Examples

```
object <- lm(QUALITY ~ NARTIC + PCTGRT + PCTSUPP, data = nas1982)
std <- BetaDelta(object)
diff <- DiffBetaDelta(std)
print(diff)
```

| | |
|-------------------|--|
| summary.betadelta | <i>Summary Method for an Object of Class betadelta</i> |
|-------------------|--|

Description

Summary Method for an Object of Class betadelta

Usage

```
## S3 method for class 'betadelta'
summary(object, alpha = NULL, digits = 4, ...)
```

Arguments

| | |
|--------|---|
| object | Object of class betadelta. |
| alpha | Numeric vector. Significance level α . If alpha = NULL, use the argument alpha used in object. |
| digits | Digits to print. |
| ... | additional arguments. |

Value

Returns a matrix of standardized regression slopes, standard errors, test statistics, degrees of freedom, p-values, and confidence intervals.

Author(s)

Ivan Jacob Agaloos Pesigan

Examples

```
object <- lm(QUALITY ~ NARTIC + PCTGRT + PCTSUPP, data = nas1982)
std <- BetaDelta(object)
summary(std)
```

| | |
|---------------------|--|
| summary.deltamethod | <i>Summary Method for an Object of Class deltamethod</i> |
|---------------------|--|

Description

Summary Method for an Object of Class deltamethod

Usage

```
## S3 method for class 'deltamethod'  
summary(object, alpha = NULL, digits = 4, ...)
```

Arguments

| | |
|--------|---|
| object | Object of class deltamethod. |
| alpha | Numeric vector. Significance level α . If alpha = NULL, use the argument alpha used in object. |
| digits | Digits to print. |
| ... | additional arguments. |

Value

Returns a matrix of standardized regression slopes, standard errors, test statistics, degrees of freedom, p-values, and confidence intervals.

Author(s)

Ivan Jacob Agaloos Pesigan

Examples

```
object <- glm(  
  formula = vs ~ wt + disp,  
  family = "binomial",  
  data = mtcars  
)  
def <- list("exp(wt)", "exp(disp)")  
out <- DeltaGeneric(  
  object = object,  
  def = def,  
  alpha = 0.05  
)  
summary(out)
```

summary.diffbetadelta *Summary Method for an Object of Class diffbetadelta*

Description

Summary Method for an Object of Class diffbetadelta

Usage

```
## S3 method for class 'diffbetadelta'  
summary(object, alpha = NULL, digits = 4, ...)
```

Arguments

| | |
|--------|---|
| object | Object of class diffbetadelta. |
| alpha | Numeric vector. Significance level α . If alpha = NULL, use the argument alpha used in object. |
| digits | Digits to print. |
| ... | additional arguments. |

Value

Returns a matrix of standardized regression slopes, standard errors, test statistics, degrees of freedom, p-values, and confidence intervals.

Author(s)

Ivan Jacob Agaloos Pesigan

Examples

```
object <- lm(QUALITY ~ NARTIC + PCTGRT + PCTSUPP, data = nas1982)  
std <- BetaDelta(object)  
diff <- DiffBetaDelta(std)  
summary(diff)
```

| | |
|----------------|---|
| vcov.betadelta | <i>Sampling Covariance Matrix of the Standardized Regression Slopes</i> |
|----------------|---|

Description

Sampling Covariance Matrix of the Standardized Regression Slopes

Usage

```
## S3 method for class 'betadelta'  
vcov(object, ...)
```

Arguments

| | |
|--------|----------------------------|
| object | Object of class betadelta. |
| ... | additional arguments. |

Value

Returns a matrix of the variance-covariance matrix of standardized slopes.

Author(s)

Ivan Jacob Agaloos Pesigan

Examples

```
object <- lm(QUALITY ~ NARTIC + PCTGRT + PCTSUPP, data = nas1982)  
std <- BetaDelta(object)  
vcov(std)
```

| | |
|------------------|-----------------------------------|
| vcov.deltamethod | <i>Sampling Covariance Matrix</i> |
|------------------|-----------------------------------|

Description

Sampling Covariance Matrix

Usage

```
## S3 method for class 'deltamethod'  
vcov(object, ...)
```

Arguments

object Object of class `deltamethod`.
 ... additional arguments.

Value

Returns a matrix of the variance-covariance matrix.

Author(s)

Ivan Jacob Agaloos Pesigan

Examples

```
object <- glm(
  formula = vs ~ wt + disp,
  family = "binomial",
  data = mtcars
)
def <- list("exp(wt)", "exp(disp)")
out <- DeltaGeneric(
  object = object,
  def = def,
  alpha = 0.05
)
vcov(out)
```

| | |
|--------------------|--|
| vcov.diffbetadelta | <i>Sampling Covariance Matrix of Differences of Standardized Regression Slopes</i> |
|--------------------|--|

Description

Sampling Covariance Matrix of Differences of Standardized Regression Slopes

Usage

```
## S3 method for class 'diffbetadelta'
vcov(object, ...)
```

Arguments

object Object of class `diffbetadelta`.
 ... additional arguments.

Value

Returns a matrix of the variance-covariance matrix of differences of standardized regression slopes.

Author(s)

Ivan Jacob Agaloos Pesigan

Examples

```
object <- lm(QUALITY ~ NARTIC + PCTGRT + PCTSUPP, data = nas1982)
std <- BetaDelta(object)
diff <- DiffBetaDelta(std)
vcov(diff)
```

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