betaDelta: Methods

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1 Multivariate Normal-Theory Approach

```
object <- lm(QUALITY ~ NARTIC + PCTGRT + PCTSUPP, data = nas1982)
```

```
out <- BetaDelta(object, type = "mvn")</pre>
str(out)
#> List of 8
#> $ call: language BetaDelta(object = object, type = "mvn")
#> $ lm :List of 12
   ..$ coefficients : Named num [1:4] 10.3592 0.0842 0.216 0.1126
   ...- attr(*, "names")= chr [1:4] "(Intercept)" "NARTIC" "PCTGRT" "PCTSUPP"
    ..$ residuals : Named num [1:46] -3.068 -0.688 2.675 2.052 4.039 ...
    ....- attr(*, "names")= chr [1:46] "1" "2" "3" "4" ...
    ..$ effects
                  : Named num [1:46] -187.4 51.49 27.59 -16.11 4.51 ...
    ... - attr(*, "names")= chr [1:46] "(Intercept)" "NARTIC" "PCTGRT" "PCTSUPP" ...
#>
    ..$ rank
                   : int 4
    ...$ fitted.values: Named num [1:46] 15.1 23.7 26.3 33.9 40 ...
    ....- attr(*, "names")= chr [1:46] "1" "2" "3" "4" ...
    ..$ assign : int [1:4] 0 1 2 3
#>
    ..$ qr
                   :List of 5
    ....$ qr : num [1:46, 1:4] -6.782 0.147 0.147 0.147 0.147 ...
    ..... attr(*, "dimnames")=List of 2
    .....$: chr [1:46] "1" "2" "3" "4" ...
    .....$ : chr [1:4] "(Intercept)" "NARTIC" "PCTGRT" "PCTSUPP"
#>
    ..... attr(*, "assign")= int [1:4] 0 1 2 3
    ....$ qraux: num [1:4] 1.15 1.04 1.06 1.09
    ....$ pivot: int [1:4] 1 2 3 4
    ....$ tol : num 1e-07
#>
    .. ..$ rank : int 4
   .. ..- attr(*, "class")= chr "qr"
    ..$ df.residual : int 42
    ..$ xlevels : Named list()
#>
    ..$ call
                   : language lm(formula = QUALITY ~ NARTIC + PCTGRT + PCTSUPP, data = nas1982)
#>
                  :Classes 'terms', 'formula' language QUALITY ~ NARTIC + PCTGRT + PCTSUPP
   ..$ terms
```

```
....- attr(*, "variables")= language list(QUALITY, NARTIC, PCTGRT, PCTSUPP)
   .... attr(*, "factors")= int [1:4, 1:3] 0 1 0 0 0 0 1 0 0 0 ...
    ..... attr(*, "dimnames")=List of 2
    ..... s: chr [1:4] "QUALITY" "NARTIC" "PCTGRT" "PCTSUPP"
#>
#>
    ..... : chr [1:3] "NARTIC" "PCTGRT" "PCTSUPP"
    ..... attr(*, "term.labels")= chr [1:3] "NARTIC" "PCTGRT" "PCTSUPP"
    ..... attr(*, "order")= int [1:3] 1 1 1
#>
    .. .. ..- attr(*, "intercept")= int 1
    .. .. - attr(*, "response")= int 1
    ..... attr(*, ".Environment")=<environment: 0x55a166db3d48>
    .... attr(*, "predvars")= language list(QUALITY, NARTIC, PCTGRT, PCTSUPP)
#>
    ..... attr(*, "dataClasses")= Named chr [1:4] "numeric" "numeric" "numeric" "numeric"
    ..... attr(*, "names")= chr [1:4] "QUALITY" "NARTIC" "PCTGRT" "PCTSUPP"
#>
    ..$ model
                  :'data.frame': 46 obs. of 4 variables:
#>
    ....$ QUALITY: int [1:46] 12 23 29 36 44 21 40 42 24 30 ...
    .. ..$ NARTIC : int [1:46] 14 61 68 49 130 65 79 187 32 50 ...
    ....$ PCTGRT : int [1:46] 8 3 13 63 53 29 35 40 19 8 ...
    ....$ PCTSUPP: int [1:46] 16 67 66 52 64 59 81 65 87 43 ...
    ...- attr(*, "terms")=Classes 'terms', 'formula' language QUALITY ~ NARTIC + PCTGRT + PCTSUI
#>
#>
    ..... attr(*, "variables")= language list(QUALITY, NARTIC, PCTGRT, PCTSUPP)
    ..... attr(*, "factors")= int [1:4, 1:3] 0 1 0 0 0 0 1 0 0 0 ...
#>
#>
    ..... attr(*, "dimnames")=List of 2
    #>
   ..... : chr [1:3] "NARTIC" "PCTGRT" "PCTSUPP"
    ..... attr(*, "term.labels")= chr [1:3] "NARTIC" "PCTGRT" "PCTSUPP"
    .. .. .. - attr(*, "order")= int [1:3] 1 1 1
#>
    ..... attr(*, "intercept")= int 1
#>
    .. .. .. - attr(*, "response")= int 1
#>
   ..... attr(*, ".Environment")=<environment: 0x55a166db3d48>
    ..... attr(*, "predvars")= language list(QUALITY, NARTIC, PCTGRT, PCTSUPP)
#>
   ..... attr(*, "dataClasses")= Named chr [1:4] "numeric" "numeric" "numeric" "numeric"
   ..- attr(*, "class")= chr "lm"
#> $ type: chr "mvn"
#> $ beta: Named num [1:3] 0.495 0.391 0.263
#> ..- attr(*, "names")= chr [1:3] "NARTIC" "PCTGRT" "PCTSUPP"
#> $ vcov: num [1:3, 1:3] 0.00575 -0.00336 -0.00217 -0.00336 0.00593 ...
#>
   ..- attr(*, "dimnames")=List of 2
   ....$ : chr [1:3] "NARTIC" "PCTGRT" "PCTSUPP"
   ....$ : chr [1:3] "NARTIC" "PCTGRT" "PCTSUPP"
#> $ n : int 46
#> $ p : num 3
#> $ df : int 42
#> - attr(*, "class")= chr [1:2] "betadelta" "list"
BetaDelta(object, type = "mvn")
```

```
#> Call:
#> BetaDelta(object = object, type = "mvn")
#>
#> Standardized regression slopes with MVN standard errors:
#> est se t p 0.05% 0.5% 2.5% 97.5% 99.5% 99.95%
#> NARTIC 0.4951 0.0759 6.5272 0.000 0.2268 0.2905 0.3421 0.6482 0.6998 0.7635
#> PCTGRT 0.3915 0.0770 5.0824 0.000 0.1190 0.1837 0.2360 0.5469 0.5993 0.6640
#> PCTSUPP 0.2632 0.0747 3.5224 0.001 -0.0011 0.0616 0.1124 0.4141 0.4649 0.5276
```

print

```
print(out)

#> Call:
#> BetaDelta(object = object, type = "mvn")
#>

#> Standardized regression slopes with MVN standard errors:
#> est se t p 0.05% 0.5% 2.5% 97.5% 99.5% 99.95%
#> NARTIC 0.4951 0.0759 6.5272 0.000 0.2268 0.2905 0.3421 0.6482 0.6998 0.7635
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#> PCTSUPP 0.2632 0.0747 3.5224 0.001 -0.0011 0.0616 0.1124 0.4141 0.4649 0.5276
```

coef

```
coef(out)
#> NARTIC PCTGRT PCTSUPP
#> 0.4951451 0.3914887 0.2632477
```

vcov

```
vcov(out)

#> NARTIC PCTGRT PCTSUPP

#> NARTIC 0.005754524 -0.003360334 -0.002166127

#> PCTGRT -0.003360334 0.005933462 -0.001769723

#> PCTSUPP -0.002166127 -0.001769723 0.005585256
```

confint

summary

```
summary(out)

#> Call:
#> BetaDelta(object = object, type = "mvn")
#>

#> Standardized regression slopes with MVN standard errors:
#> est se t p 0.05% 0.5% 2.5% 97.5% 99.5% 99.95%
#> NARTIC 0.4951 0.0759 6.5272 0.000 0.2268 0.2905 0.3421 0.6482 0.6998 0.7635
#> PCTGRT 0.3915 0.0770 5.0824 0.000 0.1190 0.1837 0.2360 0.5469 0.5993 0.6640
#> PCTSUPP 0.2632 0.0747 3.5224 0.001 -0.0011 0.0616 0.1124 0.4141 0.4649 0.5276
```

2 Asymptotic Distribution-Free Approach

```
object <- lm(QUALITY ~ NARTIC + PCTGRT + PCTSUPP, data = nas1982)
```

```
out <- BetaDelta(object, type = "adf")</pre>
str(out)
#> List of 8
#> $ call: language BetaDelta(object = object, type = "adf")
#> $ lm :List of 12
   ..$ coefficients : Named num [1:4] 10.3592 0.0842 0.216 0.1126
   ...- attr(*, "names")= chr [1:4] "(Intercept)" "NARTIC" "PCTGRT" "PCTSUPP"
   ..$ residuals : Named num [1:46] -3.068 -0.688 2.675 2.052 4.039 ...
   ....- attr(*, "names")= chr [1:46] "1" "2" "3" "4" ...
    ..$ effects : Named num [1:46] -187.4 51.49 27.59 -16.11 4.51 ...
   ...- attr(*, "names")= chr [1:46] "(Intercept)" "NARTIC" "PCTGRT" "PCTSUPP" ...
   ..$ rank : int 4
   ..$ fitted.values: Named num [1:46] 15.1 23.7 26.3 33.9 40 ...
    ....- attr(*, "names")= chr [1:46] "1" "2" "3" "4" ...
   ..$ assign : int [1:4] 0 1 2 3
#>
   ..$ qr
                   :List of 5
```

```
....$ qr : num [1:46, 1:4] -6.782 0.147 0.147 0.147 0.147 ...
    ..... attr(*, "dimnames")=List of 2
    .....$: chr [1:46] "1" "2" "3" "4" ...
    ..... : chr [1:4] "(Intercept)" "NARTIC" "PCTGRT" "PCTSUPP"
#>
#>
    .. .. - attr(*, "assign")= int [1:4] 0 1 2 3
    .. ..$ qraux: num [1:4] 1.15 1.04 1.06 1.09
    ....$ pivot: int [1:4] 1 2 3 4
#>
    ...$ tol : num 1e-07
#>
#>
    .. ..$ rank : int 4
    .. ..- attr(*, "class")= chr "qr"
    ..$ df.residual : int 42
#>
#>
    ..$ xlevels : Named list()
                   : language lm(formula = QUALITY ~ NARTIC + PCTGRT + PCTSUPP, data = nas1982)
#>
    ..$ call
                  :Classes 'terms', 'formula' language QUALITY ~ NARTIC + PCTGRT + PCTSUPP
    ..$ terms
#>
    .... - attr(*, "variables") = language list(QUALITY, NARTIC, PCTGRT, PCTSUPP)
    ..... attr(*, "factors")= int [1:4, 1:3] 0 1 0 0 0 0 1 0 0 0 ...
    .. .. .. - attr(*, "dimnames")=List of 2
    .....$: chr [1:4] "QUALITY" "NARTIC" "PCTGRT" "PCTSUPP"
    ..... : chr [1:3] "NARTIC" "PCTGRT" "PCTSUPP"
#>
#>
    ..... attr(*, "term.labels")= chr [1:3] "NARTIC" "PCTGRT" "PCTSUPP"
    .. .. - attr(*, "order")= int [1:3] 1 1 1
#>
    .. .. ..- attr(*, "intercept")= int 1
#>
    .. .. - attr(*, "response")= int 1
#>
    ..... attr(*, ".Environment")=<environment: 0x55a166db3d48>
    .... - attr(*, "predvars")= language list(QUALITY, NARTIC, PCTGRT, PCTSUPP)
    ....- attr(*, "dataClasses")= Named chr [1:4] "numeric" "numeric" "numeric" "numeric"
#>
    ..... attr(*, "names")= chr [1:4] "QUALITY" "NARTIC" "PCTGRT" "PCTSUPP"
#>
                    :'data.frame': 46 obs. of 4 variables:
#>
    ..$ model
    ....$ QUALITY: int [1:46] 12 23 29 36 44 21 40 42 24 30 ...
    ....$ NARTIC : int [1:46] 14 61 68 49 130 65 79 187 32 50 ...
#>
    ....$ PCTGRT : int [1:46] 8 3 13 63 53 29 35 40 19 8 ...
    ....$ PCTSUPP: int [1:46] 16 67 66 52 64 59 81 65 87 43 ...
    ...- attr(*, "terms")=Classes 'terms', 'formula' language QUALITY ~ NARTIC + PCTGRT + PCTSU
    ..... attr(*, "variables")= language list(QUALITY, NARTIC, PCTGRT, PCTSUPP)
#>
    ..... attr(*, "factors")= int [1:4, 1:3] 0 1 0 0 0 0 1 0 0 0 ...
#>
#>
    ..... attr(*, "dimnames")=List of 2
    ..... s: chr [1:4] "QUALITY" "NARTIC" "PCTGRT" "PCTSUPP"
    ..... : chr [1:3] "NARTIC" "PCTGRT" "PCTSUPP"
#>
    ..... attr(*, "term.labels")= chr [1:3] "NARTIC" "PCTGRT" "PCTSUPP"
#>
    .. .. .. - attr(*, "order")= int [1:3] 1 1 1
    .. .. .. - attr(*, "intercept")= int 1
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#>
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    ..... attr(*, "dataClasses")= Named chr [1:4] "numeric" "numeric" "numeric" "numeric"
#>
```

```
#> ..... attr(*, "names")= chr [1:4] "QUALITY" "NARTIC" "PCTGRT" "PCTSUPP"
#> ..- attr(*, "class")= chr "lm"
#> $ type: chr "adf"
#> $ beta: Named num [1:3] 0.495 0.391 0.263
#> ..- attr(*, "names")= chr [1:3] "NARTIC" "PCTGRT" "PCTSUPP"
#> $ vcov: num [1:3, 1:3] 0.00454 -0.00255 -0.00174 -0.00255 0.00504 ...
   ..- attr(*, "dimnames")=List of 2
#>
   ....$ : chr [1:3] "NARTIC" "PCTGRT" "PCTSUPP"
#> ....$ : chr [1:3] "NARTIC" "PCTGRT" "PCTSUPP"
#> $ n : int 46
#> $ p : num 3
#> $ df : int 42
#> - attr(*, "class")= chr [1:2] "betadelta" "list"
BetaDelta(object, type = "adf")
#> Call:
#> BetaDelta(object = object, type = "adf")
#> Standardized regression slopes with ADF standard errors:
            est se t p 0.05% 0.5% 2.5% 97.5% 99.5% 99.95%
#> NARTIC 0.4951 0.0674 7.3490 0.0000 0.2568 0.3134 0.3592 0.6311 0.6769 0.7335
#> PCTGRT 0.3915 0.0710 5.5164 0.0000 0.1404 0.2000 0.2483 0.5347 0.5830 0.6426
#> PCTSUPP 0.2632 0.0769 3.4231 0.0014 -0.0088 0.0558 0.1081 0.4184 0.4707 0.5353
```

print

```
print(out)

#> Call:
#> BetaDelta(object = object, type = "adf")
#>

#> Standardized regression slopes with ADF standard errors:
#> est se t p 0.05% 0.5% 2.5% 97.5% 99.5% 99.95%
#> NARTIC 0.4951 0.0674 7.3490 0.0000 0.2568 0.3134 0.3592 0.6311 0.6769 0.7335
#> PCTGRT 0.3915 0.0710 5.5164 0.0000 0.1404 0.2000 0.2483 0.5347 0.5830 0.6426
#> PCTSUPP 0.2632 0.0769 3.4231 0.0014 -0.0088 0.0558 0.1081 0.4184 0.4707 0.5353
```

coef

```
coef(out)
#> NARTIC PCTGRT PCTSUPP
#> 0.4951451 0.3914887 0.2632477
```

vcov

```
vcov(out)

#> NARTIC PCTGRT PCTSUPP

#> NARTIC 0.004539472 -0.002552698 -0.001742698

#> PCTGRT -0.002552698 0.005036538 -0.001906216

#> PCTSUPP -0.001742698 -0.001906216 0.005914088
```

confint

summary

```
summary(out)

#> Call:
#> BetaDelta(object = object, type = "adf")
#>

#> Standardized regression slopes with ADF standard errors:
#> est se t p 0.05% 0.5% 2.5% 97.5% 99.5% 99.95%
#> NARTIC 0.4951 0.0674 7.3490 0.0000 0.2568 0.3134 0.3592 0.6311 0.6769 0.7335
#> PCTGRT 0.3915 0.0710 5.5164 0.0000 0.1404 0.2000 0.2483 0.5347 0.5830 0.6426
#> PCTSUPP 0.2632 0.0769 3.4231 0.0014 -0.0088 0.0558 0.1081 0.4184 0.4707 0.5353
```

References

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