

# betaDelta: Methods

Ivan Jacob Agaloos Pesigan

## 1 Multivariate Normal-Theory Approach

```
object <- lm(QUALITY ~ NARTIC + PCTGRT + PCTSUPP, data = nas1982)
```

```
out <- BetaDelta(object, type = "mvn")
str(out)

#> List of 8
#> $ call: language BetaDelta(object = object, type = "mvn")
#> $ lm :List of 12
#> ..$ coefficients : Named num [1:4] 10.3592 0.0842 0.216 0.1126
#> .. ..- attr(*, "names")= chr [1:4] "(Intercept)" "NARTIC" "PCTGRT" "PCTSUPP"
#> ..$ residuals : Named num [1:46] -3.068 -0.688 2.675 2.052 4.039 ...
#> .. ..- attr(*, "names")= chr [1:46] "1" "2" "3" "4" ...
#> ..$ effects : Named num [1:46] -187.4 51.49 27.59 -16.11 4.51 ...
#> .. ..- attr(*, "names")= chr [1:46] "(Intercept)" "NARTIC" "PCTGRT" "PCTSUPP" ...
#> ..$ rank : int 4
#> ..$ fitted.values: Named num [1:46] 15.1 23.7 26.3 33.9 40 ...
#> .. ..- attr(*, "names")= chr [1:46] "1" "2" "3" "4" ...
#> ..$ assign : int [1:4] 0 1 2 3
#> ..$ qr :List of 5
#> .. ..$ qr : num [1:46, 1:4] -6.782 0.147 0.147 0.147 0.147 ...
#> .. .. ..- attr(*, "dimnames")=List of 2
#> .. .. ..$ : chr [1:46] "1" "2" "3" "4" ...
#> .. .. ..$ : chr [1:4] "(Intercept)" "NARTIC" "PCTGRT" "PCTSUPP"
#> .. .. ..- attr(*, "assign")= int [1:4] 0 1 2 3
#> .. ..$ qraux: num [1:4] 1.15 1.04 1.06 1.09
#> .. ..$ pivot: int [1:4] 1 2 3 4
#> .. ..$ tol : num 1e-07
#> .. ..$ rank : int 4
#> .. ..- attr(*, "class")= chr "qr"
#> ..$ df.residual : int 42
#> ..$ xlevels : Named list()
#> ..$ call : language lm(formula = QUALITY ~ NARTIC + PCTGRT + PCTSUPP, data = nas1982)
#> ..$ terms :Classes 'terms', 'formula' language QUALITY ~ NARTIC + PCTGRT + PCTSUPP
```

```

#> .. ..- attr(*, "variables")= language list(QUALITY, NARTIC, PCTGRT, PCTSUPP)
#> .. ..- attr(*, "factors")= int [1:4, 1:3] 0 1 0 0 0 0 1 0 0 0 ...
#> .. ..- attr(*, "dimnames")=List of 2
#> .. ..$ : chr [1:4] "QUALITY" "NARTIC" "PCTGRT" "PCTSUPP"
#> .. ..$ : chr [1:3] "NARTIC" "PCTGRT" "PCTSUPP"
#> .. ..- attr(*, "term.labels")= chr [1:3] "NARTIC" "PCTGRT" "PCTSUPP"
#> .. ..- attr(*, "order")= int [1:3] 1 1 1
#> .. ..- attr(*, "intercept")= int 1
#> .. ..- attr(*, "response")= int 1
#> .. ..- attr(*, ".Environment")=<environment: 0x56333c1f5638>
#> .. ..- attr(*, "predvars")= language list(QUALITY, NARTIC, PCTGRT, PCTSUPP)
#> .. ..- attr(*, "dataClasses")= Named chr [1:4] "numeric" "numeric" "numeric" "numeric"
#> .. ..- attr(*, "names")= chr [1:4] "QUALITY" "NARTIC" "PCTGRT" "PCTSUPP"
#> ..$ model : 'data.frame': 46 obs. of 4 variables:
#> ..$ QUALITY: int [1:46] 12 23 29 36 44 21 40 42 24 30 ...
#> ..$ NARTIC : int [1:46] 14 61 68 49 130 65 79 187 32 50 ...
#> ..$ PCTGRT : int [1:46] 8 3 13 63 53 29 35 40 19 8 ...
#> ..$ PCTSUPP: int [1:46] 16 67 66 52 64 59 81 65 87 43 ...
#> ..- attr(*, "terms")=Classes 'terms', 'formula' language QUALITY ~ NARTIC + PCTGRT + PCTSUPP
#> .. ..- attr(*, "variables")= language list(QUALITY, NARTIC, PCTGRT, PCTSUPP)
#> .. ..- attr(*, "factors")= int [1:4, 1:3] 0 1 0 0 0 0 1 0 0 0 ...
#> .. ..- attr(*, "dimnames")=List of 2
#> .. ..$ : chr [1:4] "QUALITY" "NARTIC" "PCTGRT" "PCTSUPP"
#> .. ..$ : chr [1:3] "NARTIC" "PCTGRT" "PCTSUPP"
#> .. ..- attr(*, "term.labels")= chr [1:3] "NARTIC" "PCTGRT" "PCTSUPP"
#> .. ..- attr(*, "order")= int [1:3] 1 1 1
#> .. ..- attr(*, "intercept")= int 1
#> .. ..- attr(*, "response")= int 1
#> .. ..- attr(*, ".Environment")=<environment: 0x56333c1f5638>
#> .. ..- attr(*, "predvars")= language list(QUALITY, NARTIC, PCTGRT, PCTSUPP)
#> .. ..- attr(*, "dataClasses")= Named chr [1:4] "numeric" "numeric" "numeric" "numeric"
#> .. ..- attr(*, "names")= chr [1:4] "QUALITY" "NARTIC" "PCTGRT" "PCTSUPP"
#> ..- attr(*, "class")= chr "lm"
#> $ type: chr "mvn"
#> $ beta: Named num [1:3] 0.495 0.391 0.263
#> ..- attr(*, "names")= chr [1:3] "NARTIC" "PCTGRT" "PCTSUPP"
#> $ vcov: num [1:3, 1:3] 0.00575 -0.00336 -0.00217 -0.00336 0.00593 ...
#> ..- attr(*, "dimnames")=List of 2
#> ..$ : chr [1:3] "NARTIC" "PCTGRT" "PCTSUPP"
#> ..$ : chr [1:3] "NARTIC" "PCTGRT" "PCTSUPP"
#> $ n : int 46
#> $ p : num 3
#> $ df : int 42
#> - attr(*, "class")= chr [1:2] "betadelta" "list"

BetaDelta(object, type = "mvn")

```

```
#> Call:
#> BetaDelta(object = object, type = "mvn")
#>
#> Standardized regression slopes with MVN standard errors:
#>      est      se      t      p  0.05%  0.5%  2.5% 97.5% 99.5% 99.95%
#> NARTIC  0.4951 0.0759 6.5272 0.000  0.2268 0.2905 0.3421 0.6482 0.6998 0.7635
#> PCTGRT  0.3915 0.0770 5.0824 0.000  0.1190 0.1837 0.2360 0.5469 0.5993 0.6640
#> PCTSUPP 0.2632 0.0747 3.5224 0.001 -0.0011 0.0616 0.1124 0.4141 0.4649 0.5276
```

## print

```
print(out)

#> Call:
#> BetaDelta(object = object, type = "mvn")
#>
#> Standardized regression slopes with MVN standard errors:
#>      est      se      t      p  0.05%  0.5%  2.5% 97.5% 99.5% 99.95%
#> NARTIC  0.4951 0.0759 6.5272 0.000  0.2268 0.2905 0.3421 0.6482 0.6998 0.7635
#> PCTGRT  0.3915 0.0770 5.0824 0.000  0.1190 0.1837 0.2360 0.5469 0.5993 0.6640
#> PCTSUPP 0.2632 0.0747 3.5224 0.001 -0.0011 0.0616 0.1124 0.4141 0.4649 0.5276
```

## coef

```
coef(out)

#>      NARTIC      PCTGRT      PCTSUPP
#> 0.4951451 0.3914887 0.2632477
```

## vcov

```
vcov(out)

#>      NARTIC      PCTGRT      PCTSUPP
#> NARTIC  0.005754524 -0.003360334 -0.002166127
#> PCTGRT -0.003360334  0.005933462 -0.001769723
#> PCTSUPP -0.002166127 -0.001769723  0.005585256
```

## confint

```

confint(out, level = 0.95)

#>           2.5%      97.5%
#> NARTIC  0.3420563 0.6482339
#> PCTGRT  0.2360380 0.5469395
#> PCTSUPP 0.1124272 0.4140682

```

## summary

```

summary(out)

#> Call:
#> BetaDelta(object = object, type = "mvn")
#>
#> Standardized regression slopes with MVN standard errors:
#>      est      se      t      p  0.05%  0.5%  2.5%  97.5%  99.5% 99.95%
#> NARTIC  0.4951 0.0759 6.5272 0.000  0.2268 0.2905 0.3421 0.6482 0.6998 0.7635
#> PCTGRT  0.3915 0.0770 5.0824 0.000  0.1190 0.1837 0.2360 0.5469 0.5993 0.6640
#> PCTSUPP 0.2632 0.0747 3.5224 0.001 -0.0011 0.0616 0.1124 0.4141 0.4649 0.5276

```

## 2 Asymptotic Distribution-Free Approach

```

object <- lm(QUALITY ~ NARTIC + PCTGRT + PCTSUPP, data = nas1982)

```

```

out <- BetaDelta(object, type = "adf")
str(out)

#> List of 8
#> $ call: language BetaDelta(object = object, type = "adf")
#> $ lm :List of 12
#> ..$ coefficients : Named num [1:4] 10.3592 0.0842 0.216 0.1126
#> .. ..- attr(*, "names")= chr [1:4] "(Intercept)" "NARTIC" "PCTGRT" "PCTSUPP"
#> ..$ residuals : Named num [1:46] -3.068 -0.688 2.675 2.052 4.039 ...
#> .. ..- attr(*, "names")= chr [1:46] "1" "2" "3" "4" ...
#> ..$ effects : Named num [1:46] -187.4 51.49 27.59 -16.11 4.51 ...
#> .. ..- attr(*, "names")= chr [1:46] "(Intercept)" "NARTIC" "PCTGRT" "PCTSUPP" ...
#> ..$ rank : int 4
#> ..$ fitted.values: Named num [1:46] 15.1 23.7 26.3 33.9 40 ...
#> .. ..- attr(*, "names")= chr [1:46] "1" "2" "3" "4" ...
#> ..$ assign : int [1:4] 0 1 2 3
#> ..$ qr :List of 5

```

```

#> .. ..$ qr : num [1:46, 1:4] -6.782 0.147 0.147 0.147 0.147 ...
#> .. ..- attr(*, "dimnames")=List of 2
#> .. ..$ : chr [1:46] "1" "2" "3" "4" ...
#> .. ..$ : chr [1:4] "(Intercept)" "NARTIC" "PCTGRT" "PCTSUPP"
#> .. ..- attr(*, "assign")= int [1:4] 0 1 2 3
#> .. ..$ qraux: num [1:4] 1.15 1.04 1.06 1.09
#> .. ..$ pivot: int [1:4] 1 2 3 4
#> .. ..$ tol : num 1e-07
#> .. ..$ rank : int 4
#> .. ..- attr(*, "class")= chr "qr"
#> ..$ df.residual : int 42
#> ..$ xlevels : Named list()
#> ..$ call : language lm(formula = QUALITY ~ NARTIC + PCTGRT + PCTSUPP, data = nas1982)
#> ..$ terms :Classes 'terms', 'formula' language QUALITY ~ NARTIC + PCTGRT + PCTSUPP
#> .. ..- attr(*, "variables")= language list(QUALITY, NARTIC, PCTGRT, PCTSUPP)
#> .. ..- attr(*, "factors")= int [1:4, 1:3] 0 1 0 0 0 0 1 0 0 0 ...
#> .. ..- attr(*, "dimnames")=List of 2
#> .. ..$ : chr [1:4] "QUALITY" "NARTIC" "PCTGRT" "PCTSUPP"
#> .. ..$ : chr [1:3] "NARTIC" "PCTGRT" "PCTSUPP"
#> .. ..- attr(*, "term.labels")= chr [1:3] "NARTIC" "PCTGRT" "PCTSUPP"
#> .. ..- attr(*, "order")= int [1:3] 1 1 1
#> .. ..- attr(*, "intercept")= int 1
#> .. ..- attr(*, "response")= int 1
#> .. ..- attr(*, ".Environment")=<environment: 0x56333c1f5638>
#> .. ..- attr(*, "predvars")= language list(QUALITY, NARTIC, PCTGRT, PCTSUPP)
#> .. ..- attr(*, "dataClasses")= Named chr [1:4] "numeric" "numeric" "numeric" "numeric"
#> .. ..- attr(*, "names")= chr [1:4] "QUALITY" "NARTIC" "PCTGRT" "PCTSUPP"
#> ..$ model :'data.frame': 46 obs. of 4 variables:
#> .. ..$ QUALITY: int [1:46] 12 23 29 36 44 21 40 42 24 30 ...
#> .. ..$ NARTIC : int [1:46] 14 61 68 49 130 65 79 187 32 50 ...
#> .. ..$ PCTGRT : int [1:46] 8 3 13 63 53 29 35 40 19 8 ...
#> .. ..$ PCTSUPP: int [1:46] 16 67 66 52 64 59 81 65 87 43 ...
#> .. ..- attr(*, "terms")=Classes 'terms', 'formula' language QUALITY ~ NARTIC + PCTGRT + PCTSUPP
#> .. ..- attr(*, "variables")= language list(QUALITY, NARTIC, PCTGRT, PCTSUPP)
#> .. ..- attr(*, "factors")= int [1:4, 1:3] 0 1 0 0 0 0 1 0 0 0 ...
#> .. ..- attr(*, "dimnames")=List of 2
#> .. ..$ : chr [1:4] "QUALITY" "NARTIC" "PCTGRT" "PCTSUPP"
#> .. ..$ : chr [1:3] "NARTIC" "PCTGRT" "PCTSUPP"
#> .. ..- attr(*, "term.labels")= chr [1:3] "NARTIC" "PCTGRT" "PCTSUPP"
#> .. ..- attr(*, "order")= int [1:3] 1 1 1
#> .. ..- attr(*, "intercept")= int 1
#> .. ..- attr(*, "response")= int 1
#> .. ..- attr(*, ".Environment")=<environment: 0x56333c1f5638>
#> .. ..- attr(*, "predvars")= language list(QUALITY, NARTIC, PCTGRT, PCTSUPP)
#> .. ..- attr(*, "dataClasses")= Named chr [1:4] "numeric" "numeric" "numeric" "numeric"

```

```

#> .. .. .- attr(*, "names")= chr [1:4] "QUALITY" "NARTIC" "PCTGRT" "PCTSUPP"
#> ..- attr(*, "class")= chr "lm"
#> $ type: chr "adf"
#> $ beta: Named num [1:3] 0.495 0.391 0.263
#> ..- attr(*, "names")= chr [1:3] "NARTIC" "PCTGRT" "PCTSUPP"
#> $ vcov: num [1:3, 1:3] 0.00454 -0.00255 -0.00174 -0.00255 0.00504 ...
#> ..- attr(*, "dimnames")=List of 2
#> .. ..$ : chr [1:3] "NARTIC" "PCTGRT" "PCTSUPP"
#> .. ..$ : chr [1:3] "NARTIC" "PCTGRT" "PCTSUPP"
#> $ n : int 46
#> $ p : num 3
#> $ df : int 42
#> - attr(*, "class")= chr [1:2] "betadelta" "list"

BetaDelta(object, type = "adf")

#> Call:
#> BetaDelta(object = object, type = "adf")
#>
#> Standardized regression slopes with ADF standard errors:
#>      est      se      t      p  0.05%  0.5%  2.5%  97.5%  99.5% 99.95%
#> NARTIC  0.4951 0.0674 7.3490 0.0000  0.2568 0.3134 0.3592 0.6311 0.6769 0.7335
#> PCTGRT  0.3915 0.0710 5.5164 0.0000  0.1404 0.2000 0.2483 0.5347 0.5830 0.6426
#> PCTSUPP 0.2632 0.0769 3.4231 0.0014 -0.0088 0.0558 0.1081 0.4184 0.4707 0.5353

```

## print

```

print(out)

#> Call:
#> BetaDelta(object = object, type = "adf")
#>
#> Standardized regression slopes with ADF standard errors:
#>      est      se      t      p  0.05%  0.5%  2.5%  97.5%  99.5% 99.95%
#> NARTIC  0.4951 0.0674 7.3490 0.0000  0.2568 0.3134 0.3592 0.6311 0.6769 0.7335
#> PCTGRT  0.3915 0.0710 5.5164 0.0000  0.1404 0.2000 0.2483 0.5347 0.5830 0.6426
#> PCTSUPP 0.2632 0.0769 3.4231 0.0014 -0.0088 0.0558 0.1081 0.4184 0.4707 0.5353

```

## coef

```

coef(out)

#>      NARTIC      PCTGRT      PCTSUPP
#> 0.4951451 0.3914887 0.2632477

```

## vcov

```
vcov(out)

#>           NARTIC      PCTGRT      PCTSUPP
#> NARTIC  0.004539472 -0.002552698 -0.001742698
#> PCTGRT -0.002552698  0.005036538 -0.001906216
#> PCTSUPP -0.001742698 -0.001906216  0.005914088
```

## confint

```
confint(out, level = 0.95)

#>           2.5%      97.5%
#> NARTIC  0.3591757 0.6311146
#> PCTGRT  0.2482683 0.5347091
#> PCTSUPP 0.1080509 0.4184444
```

## summary

```
summary(out)

#> Call:
#> BetaDelta(object = object, type = "adf")
#>
#> Standardized regression slopes with ADF standard errors:
#>           est      se      t      p  0.05%  0.5%  2.5%  97.5%  99.5% 99.95%
#> NARTIC  0.4951 0.0674 7.3490 0.0000  0.2568 0.3134 0.3592 0.6311 0.6769 0.7335
#> PCTGRT  0.3915 0.0710 5.5164 0.0000  0.1404 0.2000 0.2483 0.5347 0.5830 0.6426
#> PCTSUPP 0.2632 0.0769 3.4231 0.0014 -0.0088 0.0558 0.1081 0.4184 0.4707 0.5353
```

## References

- Jones, J. A., & Waller, N. G. (2015). The normal-theory and asymptotic distribution-free (ADF) covariance matrix of standardized regression coefficients: Theoretical extensions and finite sample behavior. *Psychometrika*, 80(2), 365–378. <https://doi.org/10.1007/s11336-013-9380-y>
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- Yuan, K.-H., & Chan, W. (2011). Biases and standard errors of standardized regression coefficients. *Psychometrika*, 76(4), 670–690. <https://doi.org/10.1007/s11336-011-9224-6>