

# Package ‘bootStateSpace’

January 21, 2025

**Title** Bootstrap for State Space Models

**Version** 1.0.1

**Description** Provides a streamlined and user-friendly framework for bootstrapping in state space models, particularly when the number of subjects/units (n) exceeds one, a scenario commonly encountered in social and behavioral sciences. For an introduction to state space models in social and behavioral sciences, refer to Chow, Ho, Hamaker, and Dolan (2010) <[doi:10.1080/10705511003661553](https://doi.org/10.1080/10705511003661553)>.

**URL** <https://github.com/jeksterslab/bootStateSpace>,  
<https://jeksterslab.github.io/bootStateSpace/>

**BugReports** <https://github.com/jeksterslab/bootStateSpace/issues>

**License** GPL (>= 3)

**Encoding** UTF-8

**Roxygen** list(markdown = TRUE)

**Depends** R (>= 3.5.0)

**Imports** stats, simStateSpace, dynr

**Suggests** knitr, rmarkdown, testthat

**RoxygenNote** 7.3.2

**NeedsCompilation** no

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coef.bootstatespace	<i>Estimated Parameter Method for an Object of Class bootstatespace</i>
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**Description**

Estimated Parameter Method for an Object of Class bootstatespace

**Usage**

```
## S3 method for class 'bootstatespace'  
coef(object, ...)
```

**Arguments**

- object            Object of Class bootstatespace.
- ...              additional arguments.

**Value**

Returns a vector of estimated parameters.

**Author(s)**

Ivan Jacob Agaloos Pesigan

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confint.bootstatespace	<i>Confidence Intervals Method for an Object of Class bootstatespace</i>
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**Description**

Confidence Intervals Method for an Object of Class bootstatespace

**Usage**

```
## S3 method for class 'bootstatespace'  
confint(object, parm = NULL, level = 0.95, type = "pc", ...)
```

**Arguments**

object	Object of Class bootstatespace.
parm	a specification of which parameters are to be given confidence intervals, either a vector of numbers or a vector of names. If missing, all parameters are considered.
level	the confidence level required.
type	Character string. Confidence interval type, that is, type = "pc" for percentile; type = "bc" for bias corrected.
...	additional arguments.

**Value**

Returns a matrix of confidence intervals.

**Author(s)**

Ivan Jacob Agaloos Pesigan

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extract	<i>Extract Generic Function</i>
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---

**Description**

A generic function for extracting elements from objects.

**Usage**

```
extract(object, what)
```

**Arguments**

object	An object.
what	Character string.

**Value**

A value determined by the specific method for the object's class.

---

```
extract.bootstatespace
```

*Extract Method for an Object of Class bootstatespace*

---

### Description

Extract Method for an Object of Class bootstatespace

### Usage

```
## S3 method for class 'bootstatespace'
extract(object, what = NULL)
```

### Arguments

object	Object of Class bootstatespace.
what	Character string. What specific matrix to extract. If what = NULL, extract all available matrices.

### Value

Returns a list. Each element of the list is a list of bootstrap estimates in matrix format.

### Author(s)

Ivan Jacob Agaloos Pesigan

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PBSSMFixed

*Parametric Bootstrap for the State Space Model (Fixed Parameters)*

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### Description

This function simulates data from a state-space model and fits the model using the dynr package. The process is repeated R times. It assumes that the parameters remain constant across individuals and over time. At the moment, the function only supports type = 0.

### Usage

```
PBSSMFixed(
  R,
  path,
  prefix,
  n,
  time,
  delta_t = 1,
```

```

    mu0,
    sigma0_l,
    alpha,
    beta,
    psi_l,
    nu,
    lambda,
    theta_l,
    type = 0,
    x = NULL,
    gamma = NULL,
    kappa = NULL,
    mu0_fixed = FALSE,
    sigma0_fixed = FALSE,
    alpha_level = 0.05,
    optimization_flag = TRUE,
    hessian_flag = FALSE,
    verbose = FALSE,
    weight_flag = FALSE,
    debug_flag = FALSE,
    perturb_flag = FALSE,
    xtol_rel = 1e-07,
    stopval = -9999,
    ftol_rel = -1,
    ftol_abs = -1,
    maxeval = as.integer(-1),
    maxtime = -1,
    ncores = NULL,
    seed = NULL
)

```

### Arguments

R	Positive integer. Number of bootstrap samples.
path	Path to a directory to store bootstrap samples and estimates.
prefix	Character string. Prefix used for the file names for the bootstrap samples and estimates.
n	Positive integer. Number of individuals.
time	Positive integer. Number of time points.
delta_t	Numeric. Time interval. The default value is 1.0 with an option to use a numeric value for the discretized state space model parameterization of the linear stochastic differential equation model.
mu0	Numeric vector. Mean of initial latent variable values ( $\mu_{\eta 0}$ ).
sigma0_l	Numeric matrix. Cholesky factorization ( $t(\text{chol}(\text{sigma0}))$ ) of the covariance matrix of initial latent variable values ( $\Sigma_{\eta 0}$ ).
alpha	Numeric vector. Vector of constant values for the dynamic model ( $\alpha$ ).

beta	Numeric matrix. Transition matrix relating the values of the latent variables at the previous to the current time point ( $\beta$ ).
psi_l	Numeric matrix. Cholesky factorization ( $t(chol(psi))$ ) of the covariance matrix of the process noise ( $\Psi$ ).
nu	Numeric vector. Vector of intercept values for the measurement model ( $\nu$ ).
lambda	Numeric matrix. Factor loading matrix linking the latent variables to the observed variables ( $\Lambda$ ).
theta_l	Numeric matrix. Cholesky factorization ( $t(chol(theta))$ ) of the covariance matrix of the measurement error ( $\Theta$ ).
type	Integer. State space model type. See Details for more information.
x	List. Each element of the list is a matrix of covariates for each individual $i$ in $n$ . The number of columns in each matrix should be equal to time.
gamma	Numeric matrix. Matrix linking the covariates to the latent variables at current time point ( $\Gamma$ ).
kappa	Numeric matrix. Matrix linking the covariates to the observed variables at current time point ( $\kappa$ ).
mu0_fixed	Logical. If <code>mu0_fixed = TRUE</code> , fix the initial mean vector to <code>mu0</code> . If <code>mu0_fixed = FALSE</code> , <code>mu0</code> is estimated.
sigma0_fixed	Logical. If <code>sigma0_fixed = TRUE</code> , fix the initial covariance matrix to <code>tcrossprod(sigma0_l)</code> . If <code>sigma0_fixed = FALSE</code> , <code>sigma0</code> is estimated.
alpha_level	Numeric vector. Significance level $\alpha$ .
optimization_flag	a flag (TRUE/FALSE) indicating whether optimization is to be done.
hessian_flag	a flag (TRUE/FALSE) indicating whether the Hessian matrix is to be calculated.
verbose	a flag (TRUE/FALSE) indicating whether more detailed intermediate output during the estimation process should be printed
weight_flag	a flag (TRUE/FALSE) indicating whether the negative log likelihood function should be weighted by the length of the time series for each individual
debug_flag	a flag (TRUE/FALSE) indicating whether users want additional dynr output that can be used for diagnostic purposes
perturb_flag	a flag (TRUE/FLASE) indicating whether to perturb the latent states during estimation. Only useful for ensemble forecasting.
xtol_rel	Stopping criteria option for parameter optimization. See <code>dynr::dynr.model()</code> for more details.
stopval	Stopping criteria option for parameter optimization. See <code>dynr::dynr.model()</code> for more details.
ftol_rel	Stopping criteria option for parameter optimization. See <code>dynr::dynr.model()</code> for more details.
ftol_abs	Stopping criteria option for parameter optimization. See <code>dynr::dynr.model()</code> for more details.
maxeval	Stopping criteria option for parameter optimization. See <code>dynr::dynr.model()</code> for more details.

maxtime	Stopping criteria option for parameter optimization. See <code>dynr::dynr.model()</code> for more details.
ncores	Positive integer. Number of cores to use. If ncores = NULL, use a single core. Consider using multiple cores when number of bootstrap samples R is a large value.
seed	Random seed.

## Details

### Type 0:

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \boldsymbol{\Lambda}\boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t}, \quad \text{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Theta})$$

where  $\mathbf{y}_{i,t}$ ,  $\boldsymbol{\eta}_{i,t}$ , and  $\boldsymbol{\varepsilon}_{i,t}$  are random variables and  $\boldsymbol{\nu}$ ,  $\boldsymbol{\Lambda}$ , and  $\boldsymbol{\Theta}$  are model parameters.  $\mathbf{y}_{i,t}$  represents a vector of observed random variables,  $\boldsymbol{\eta}_{i,t}$  a vector of latent random variables, and  $\boldsymbol{\varepsilon}_{i,t}$  a vector of random measurement errors, at time  $t$  and individual  $i$ .  $\boldsymbol{\nu}$  denotes a vector of intercepts,  $\boldsymbol{\Lambda}$  a matrix of factor loadings, and  $\boldsymbol{\Theta}$  the covariance matrix of  $\boldsymbol{\varepsilon}$ .

An alternative representation of the measurement error is given by

$$\boldsymbol{\varepsilon}_{i,t} = \boldsymbol{\Theta}^{\frac{1}{2}} \mathbf{z}_{i,t}, \quad \text{with} \quad \mathbf{z}_{i,t} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$$

where  $\mathbf{z}_{i,t}$  is a vector of independent standard normal random variables and  $\left(\boldsymbol{\Theta}^{\frac{1}{2}}\right) \left(\boldsymbol{\Theta}^{\frac{1}{2}}\right)' = \boldsymbol{\Theta}$ .

The dynamic structure is given by

$$\boldsymbol{\eta}_{i,t} = \boldsymbol{\alpha} + \boldsymbol{\beta}\boldsymbol{\eta}_{i,t-1} + \boldsymbol{\zeta}_{i,t}, \quad \text{with} \quad \boldsymbol{\zeta}_{i,t} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Psi})$$

where  $\boldsymbol{\eta}_{i,t}$ ,  $\boldsymbol{\eta}_{i,t-1}$ , and  $\boldsymbol{\zeta}_{i,t}$  are random variables, and  $\boldsymbol{\alpha}$ ,  $\boldsymbol{\beta}$ , and  $\boldsymbol{\Psi}$  are model parameters. Here,  $\boldsymbol{\eta}_{i,t}$  is a vector of latent variables at time  $t$  and individual  $i$ ,  $\boldsymbol{\eta}_{i,t-1}$  represents a vector of latent variables at time  $t-1$  and individual  $i$ , and  $\boldsymbol{\zeta}_{i,t}$  represents a vector of dynamic noise at time  $t$  and individual  $i$ .  $\boldsymbol{\alpha}$  denotes a vector of intercepts,  $\boldsymbol{\beta}$  a matrix of autoregression and cross regression coefficients, and  $\boldsymbol{\Psi}$  the covariance matrix of  $\boldsymbol{\zeta}_{i,t}$ .

An alternative representation of the dynamic noise is given by

$$\boldsymbol{\zeta}_{i,t} = \boldsymbol{\Psi}^{\frac{1}{2}} \mathbf{z}_{i,t}, \quad \text{with} \quad \mathbf{z}_{i,t} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$$

where  $\left(\boldsymbol{\Psi}^{\frac{1}{2}}\right) \left(\boldsymbol{\Psi}^{\frac{1}{2}}\right)' = \boldsymbol{\Psi}$ .

### Type 1:

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \boldsymbol{\Lambda}\boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t}, \quad \text{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Theta}).$$

The dynamic structure is given by

$$\boldsymbol{\eta}_{i,t} = \boldsymbol{\alpha} + \boldsymbol{\beta}\boldsymbol{\eta}_{i,t-1} + \boldsymbol{\Gamma}\mathbf{x}_{i,t} + \boldsymbol{\zeta}_{i,t}, \quad \text{with} \quad \boldsymbol{\zeta}_{i,t} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Psi})$$

where  $\mathbf{x}_{i,t}$  represents a vector of covariates at time  $t$  and individual  $i$ , and  $\boldsymbol{\Gamma}$  the coefficient matrix linking the covariates to the latent variables.

**Type 2:**

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \boldsymbol{\Lambda}\boldsymbol{\eta}_{i,t} + \boldsymbol{\kappa}\mathbf{x}_{i,t} + \boldsymbol{\varepsilon}_{i,t}, \quad \text{with } \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Theta})$$

where  $\boldsymbol{\kappa}$  represents the coefficient matrix linking the covariates to the observed variables.

The dynamic structure is given by

$$\boldsymbol{\eta}_{i,t} = \boldsymbol{\alpha} + \boldsymbol{\beta}\boldsymbol{\eta}_{i,t-1} + \boldsymbol{\Gamma}\mathbf{x}_{i,t} + \boldsymbol{\zeta}_{i,t}, \quad \text{with } \boldsymbol{\zeta}_{i,t} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Psi}).$$

**Value**

Returns an object of class `bootstatespace` which is a list with the following elements:

**call** Function call.

**args** Function arguments.

**thetahatstar** Sampling distribution of  $\hat{\boldsymbol{\theta}}$ .

**vcov** Sampling variance-covariance matrix of  $\hat{\boldsymbol{\theta}}$ .

**est** Vector of estimated  $\hat{\boldsymbol{\theta}}$ .

**fun** Function used ("PBSSMFixed").

**method** Bootstrap method used ("parametric").

**Author(s)**

Ivan Jacob Agaloos Pesigan

**References**

Chow, S.-M., Ho, M. R., Hamaker, E. L., & Dolan, C. V. (2010). Equivalence and differences between structural equation modeling and state-space modeling techniques. *Structural Equation Modeling: A Multidisciplinary Journal*, 17(2), 303–332. doi:10.1080/10705511003661553

**See Also**

Other Bootstrap for State Space Models Functions: [PBSSMLinSDEFixed\(\)](#), [PBSSMOUFixed\(\)](#), [PBSSMVARFixed\(\)](#)

**Examples**

```
# prepare parameters
set.seed(42)
## number of individuals
n <- 5
## time points
time <- 50
delta_t <- 1
## dynamic structure
p <- 3
mu0 <- rep(x = 0, times = p)
sigma0 <- 0.001 * diag(p)
```



```

sigma0_l <- t(chol(sigma0))
alpha <- rep(x = 0, times = p)
beta <- 0.50 * diag(p)
psi <- 0.001 * diag(p)
psi_l <- t(chol(psi))
## measurement model
k <- 3
nu <- rep(x = 0, times = k)
lambda <- diag(k)
theta <- 0.001 * diag(k)
theta_l <- t(chol(theta))

path <- tempdir()

pb <- PBSSMFixed(
  R = 10L, # use at least 1000 in actual research
  path = path,
  prefix = "ssm",
  n = n,
  time = time,
  delta_t = delta_t,
  mu0 = mu0,
  sigma0_l = sigma0_l,
  alpha = alpha,
  beta = beta,
  psi_l = psi_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 0,
  ncores = 1, # consider using multiple cores
  seed = 42
)
print(pb)
summary(pb)
confint(pb)
vcov(pb)
coef(pb)
print(pb, type = "bc") # bias-corrected
summary(pb, type = "bc")
confint(pb, type = "bc")

```

## Description

This function simulates data from a linear stochastic differential equation model using a state-space model parameterization and fits the model using the dynr package. The process is repeated  $R$  times. It assumes that the parameters remain constant across individuals and over time. At the moment, the function only supports  $\text{type} = 0$ .

## Usage

```
PBSSMLinSDEFixed(  
  R,  
  path,  
  prefix,  
  n,  
  time,  
  delta_t = 0.1,  
  mu0,  
  sigma0_l,  
  iota,  
  phi,  
  sigma_l,  
  nu,  
  lambda,  
  theta_l,  
  type = 0,  
  x = NULL,  
  gamma = NULL,  
  kappa = NULL,  
  mu0_fixed = FALSE,  
  sigma0_fixed = FALSE,  
  alpha_level = 0.05,  
  optimization_flag = TRUE,  
  hessian_flag = FALSE,  
  verbose = FALSE,  
  weight_flag = FALSE,  
  debug_flag = FALSE,  
  perturb_flag = FALSE,  
  xtol_rel = 1e-07,  
  stopval = -9999,  
  ftol_rel = -1,  
  ftol_abs = -1,  
  maxeval = as.integer(-1),  
  maxtime = -1,  
  ncores = NULL,  
  seed = NULL  
)
```

**Arguments**

R	Positive integer. Number of bootstrap samples.
path	Path to a directory to store bootstrap samples and estimates.
prefix	Character string. Prefix used for the file names for the bootstrap samples and estimates.
n	Positive integer. Number of individuals.
time	Positive integer. Number of time points.
delta_t	Numeric. Time interval ( $\Delta_t$ ).
mu0	Numeric vector. Mean of initial latent variable values ( $\mu_{\eta 0}$ ).
sigma0_l	Numeric matrix. Cholesky factorization ( $t(\text{chol}(\text{sigma0}))$ ) of the covariance matrix of initial latent variable values ( $\Sigma_{\eta 0}$ ).
iota	Numeric vector. An unobserved term that is constant over time ( $\iota$ ).
phi	Numeric matrix. The drift matrix which represents the rate of change of the solution in the absence of any random fluctuations ( $\Phi$ ).
sigma_l	Numeric matrix. Cholesky factorization ( $t(\text{chol}(\text{sigma}))$ ) of the covariance matrix of volatility or randomness in the process ( $\Sigma$ ).
nu	Numeric vector. Vector of intercept values for the measurement model ( $\nu$ ).
lambda	Numeric matrix. Factor loading matrix linking the latent variables to the observed variables ( $\Lambda$ ).
theta_l	Numeric matrix. Cholesky factorization ( $t(\text{chol}(\text{theta}))$ ) of the covariance matrix of the measurement error ( $\Theta$ ).
type	Integer. State space model type. See Details for more information.
x	List. Each element of the list is a matrix of covariates for each individual $i$ in $n$ . The number of columns in each matrix should be equal to time.
gamma	Numeric matrix. Matrix linking the covariates to the latent variables at current time point ( $\Gamma$ ).
kappa	Numeric matrix. Matrix linking the covariates to the observed variables at current time point ( $\kappa$ ).
mu0_fixed	Logical. If <code>mu0_fixed = TRUE</code> , fix the initial mean vector to <code>mu0</code> . If <code>mu0_fixed = FALSE</code> , <code>mu0</code> is estimated.
sigma0_fixed	Logical. If <code>sigma0_fixed = TRUE</code> , fix the initial covariance matrix to <code>tcrossprod(sigma0_l)</code> . If <code>sigma0_fixed = FALSE</code> , <code>sigma0</code> is estimated.
alpha_level	Numeric vector. Significance level $\alpha$ .
optimization_flag	a flag (TRUE/FALSE) indicating whether optimization is to be done.
hessian_flag	a flag (TRUE/FALSE) indicating whether the Hessian matrix is to be calculated.
verbose	a flag (TRUE/FALSE) indicating whether more detailed intermediate output during the estimation process should be printed
weight_flag	a flag (TRUE/FALSE) indicating whether the negative log likelihood function should be weighted by the length of the time series for each individual

debug_flag	a flag (TRUE/FALSE) indicating whether users want additional dynr output that can be used for diagnostic purposes
perturb_flag	a flag (TRUE/FALSE) indicating whether to perturb the latent states during estimation. Only useful for ensemble forecasting.
xtol_rel	Stopping criteria option for parameter optimization. See <code>dynr::dynr.model()</code> for more details.
stopval	Stopping criteria option for parameter optimization. See <code>dynr::dynr.model()</code> for more details.
ftol_rel	Stopping criteria option for parameter optimization. See <code>dynr::dynr.model()</code> for more details.
ftol_abs	Stopping criteria option for parameter optimization. See <code>dynr::dynr.model()</code> for more details.
maxeval	Stopping criteria option for parameter optimization. See <code>dynr::dynr.model()</code> for more details.
maxtime	Stopping criteria option for parameter optimization. See <code>dynr::dynr.model()</code> for more details.
ncores	Positive integer. Number of cores to use. If ncores = NULL, use a single core. Consider using multiple cores when number of bootstrap samples R is a large value.
seed	Random seed.

## Details

### Type 0:

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \mathbf{\Lambda}\boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t}, \quad \text{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Theta})$$

where  $\mathbf{y}_{i,t}$ ,  $\boldsymbol{\eta}_{i,t}$ , and  $\boldsymbol{\varepsilon}_{i,t}$  are random variables and  $\boldsymbol{\nu}$ ,  $\mathbf{\Lambda}$ , and  $\boldsymbol{\Theta}$  are model parameters.  $\mathbf{y}_{i,t}$  represents a vector of observed random variables,  $\boldsymbol{\eta}_{i,t}$  a vector of latent random variables, and  $\boldsymbol{\varepsilon}_{i,t}$  a vector of random measurement errors, at time  $t$  and individual  $i$ .  $\boldsymbol{\nu}$  denotes a vector of intercepts,  $\mathbf{\Lambda}$  a matrix of factor loadings, and  $\boldsymbol{\Theta}$  the covariance matrix of  $\boldsymbol{\varepsilon}$ .

An alternative representation of the measurement error is given by

$$\boldsymbol{\varepsilon}_{i,t} = \boldsymbol{\Theta}^{\frac{1}{2}} \mathbf{z}_{i,t}, \quad \text{with} \quad \mathbf{z}_{i,t} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$$

where  $\mathbf{z}_{i,t}$  is a vector of independent standard normal random variables and  $\left(\boldsymbol{\Theta}^{\frac{1}{2}}\right) \left(\boldsymbol{\Theta}^{\frac{1}{2}}\right)' = \boldsymbol{\Theta}$ .

The dynamic structure is given by

$$d\boldsymbol{\eta}_{i,t} = (\boldsymbol{\iota} + \boldsymbol{\Phi}\boldsymbol{\eta}_{i,t}) dt + \boldsymbol{\Sigma}^{\frac{1}{2}} d\mathbf{W}_{i,t}$$

where  $\boldsymbol{\iota}$  is a term which is unobserved and constant over time,  $\boldsymbol{\Phi}$  is the drift matrix which represents the rate of change of the solution in the absence of any random fluctuations,  $\boldsymbol{\Sigma}$  is the matrix of volatility or randomness in the process, and  $d\mathbf{W}$  is a Wiener process or Brownian motion, which represents random fluctuations.

**Type 1:**

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \boldsymbol{\Lambda}\boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t}, \quad \text{with } \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Theta}).$$

The dynamic structure is given by

$$d\boldsymbol{\eta}_{i,t} = (\boldsymbol{\iota} + \boldsymbol{\Phi}\boldsymbol{\eta}_{i,t}) dt + \boldsymbol{\Gamma}\mathbf{x}_{i,t} + \boldsymbol{\Sigma}^{\frac{1}{2}}d\mathbf{W}_{i,t}$$

where  $\mathbf{x}_{i,t}$  represents a vector of covariates at time  $t$  and individual  $i$ , and  $\boldsymbol{\Gamma}$  the coefficient matrix linking the covariates to the latent variables.

**Type 2:**

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \boldsymbol{\Lambda}\boldsymbol{\eta}_{i,t} + \boldsymbol{\kappa}\mathbf{x}_{i,t} + \boldsymbol{\varepsilon}_{i,t}, \quad \text{with } \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Theta})$$

where  $\boldsymbol{\kappa}$  represents the coefficient matrix linking the covariates to the observed variables.

The dynamic structure is given by

$$d\boldsymbol{\eta}_{i,t} = (\boldsymbol{\iota} + \boldsymbol{\Phi}\boldsymbol{\eta}_{i,t}) dt + \boldsymbol{\Gamma}\mathbf{x}_{i,t} + \boldsymbol{\Sigma}^{\frac{1}{2}}d\mathbf{W}_{i,t}.$$

**State Space Parameterization:**

The state space parameters as a function of the linear stochastic differential equation model parameters are given by

$$\boldsymbol{\beta}_{\Delta t_{i_i}} = \exp(\Delta t \boldsymbol{\Phi})$$

$$\boldsymbol{\alpha}_{\Delta t_{i_i}} = \boldsymbol{\Phi}^{-1}(\boldsymbol{\beta} - \mathbf{I}_p)\boldsymbol{\iota}$$

$$\text{vec}(\boldsymbol{\Psi}_{\Delta t_{i_i}}) = [(\boldsymbol{\Phi} \otimes \mathbf{I}_p) + (\mathbf{I}_p \otimes \boldsymbol{\Phi})] [\exp([( \boldsymbol{\Phi} \otimes \mathbf{I}_p) + (\mathbf{I}_p \otimes \boldsymbol{\Phi})] \Delta t) - \mathbf{I}_{p \times p}] \text{vec}(\boldsymbol{\Sigma})$$

where  $p$  is the number of latent variables and  $\Delta t$  is the time interval.

**Value**

Returns an object of class `bootstatespace` which is a list with the following elements:

**call** Function call.

**args** Function arguments.

**thetahatstar** Sampling distribution of  $\hat{\boldsymbol{\theta}}$ .

**vcov** Sampling variance-covariance matrix of  $\hat{\boldsymbol{\theta}}$ .

**est** Vector of estimated  $\hat{\boldsymbol{\theta}}$ .

**fun** Function used ("PBSSMLinSDEFixed").

**method** Bootstrap method used ("parametric").

**Author(s)**

Ivan Jacob Agaloos Pesigan

**References**

Chow, S.-M., Ho, M. R., Hamaker, E. L., & Dolan, C. V. (2010). Equivalence and differences between structural equation modeling and state-space modeling techniques. *Structural Equation Modeling: A Multidisciplinary Journal*, 17(2), 303–332. doi:[10.1080/10705511003661553](https://doi.org/10.1080/10705511003661553)

**See Also**

Other Bootstrap for State Space Models Functions: [PBSSMFixed\(\)](#), [PBSSMOUFixed\(\)](#), [PBSSMVARFixed\(\)](#)

**Examples**

```
# prepare parameters
## number of individuals
n <- 5
## time points
time <- 50
delta_t <- 0.10
## dynamic structure
p <- 2
mu0 <- c(-3.0, 1.5)
sigma0 <- 0.001 * diag(p)
sigma0_l <- t(chol(sigma0))
iota <- c(0.317, 0.230)
phi <- matrix(
  data = c(
    -0.10,
    0.05,
    0.05,
    -0.10
  ),
  nrow = p
)
sigma <- matrix(
  data = c(
    2.79,
    0.06,
    0.06,
    3.27
  ),
  nrow = p
)
sigma_l <- t(chol(sigma))
## measurement model
k <- 2
nu <- rep(x = 0, times = k)
lambda <- diag(k)
theta <- 0.001 * diag(k)
theta_l <- t(chol(theta))
```

```

path <- tempdir()

pb <- PBSSMLinSDEFixed(
  R = 10L, # use at least 1000 in actual research
  path = path,
  prefix = "lse",
  n = n,
  time = time,
  delta_t = delta_t,
  mu0 = mu0,
  sigma0_l = sigma0_l,
  iota = iota,
  phi = phi,
  sigma_l = sigma_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 0,
  ncores = 1, # consider using multiple cores
  seed = 42
)
print(pb)
summary(pb)
confint(pb)
vcov(pb)
coef(pb)
print(pb, type = "bc") # bias-corrected
summary(pb, type = "bc")
confint(pb, type = "bc")

```

PBSSMOUFixed

*Parametric Bootstrap for the Ornstein–Uhlenbeck Model using a State Space Model Parameterization (Fixed Parameters)*

## Description

This function simulates data from a Ornstein–Uhlenbeck (OU) model using a state-space model parameterization and fits the model using the dynr package. The process is repeated R times. It assumes that the parameters remain constant across individuals and over time. At the moment, the function only supports `type = 0`.

## Usage

```

PBSSMOUFixed(
  R,
  path,
  prefix,

```

```

n,
time,
delta_t = 0.1,
mu0,
sigma0_l,
mu,
phi,
sigma_l,
nu,
lambda,
theta_l,
type = 0,
x = NULL,
gamma = NULL,
kappa = NULL,
mu0_fixed = FALSE,
sigma0_fixed = FALSE,
alpha_level = 0.05,
optimization_flag = TRUE,
hessian_flag = FALSE,
verbose = FALSE,
weight_flag = FALSE,
debug_flag = FALSE,
perturb_flag = FALSE,
xtol_rel = 1e-07,
stopval = -9999,
ftol_rel = -1,
ftol_abs = -1,
maxeval = as.integer(-1),
maxtime = -1,
ncores = NULL,
seed = NULL
)

```

### Arguments

R	Positive integer. Number of bootstrap samples.
path	Path to a directory to store bootstrap samples and estimates.
prefix	Character string. Prefix used for the file names for the bootstrap samples and estimates.
n	Positive integer. Number of individuals.
time	Positive integer. Number of time points.
delta_t	Numeric. Time interval ( $\Delta_t$ ).
mu0	Numeric vector. Mean of initial latent variable values ( $\mu_{\eta 0}$ ).
sigma0_l	Numeric matrix. Cholesky factorization ( $t(\text{chol}(\text{sigma0}))$ ) of the covariance matrix of initial latent variable values ( $\Sigma_{\eta 0}$ ).



mu	Numeric vector. The long-term mean or equilibrium level ( $\mu$ ).
phi	Numeric matrix. The drift matrix which represents the rate of change of the solution in the absence of any random fluctuations ( $\Phi$ ). It also represents the rate of mean reversion, determining how quickly the variable returns to its mean.
sigma_l	Numeric matrix. Cholesky factorization ( $t(\text{chol}(\text{sigma}))$ ) of the covariance matrix of volatility or randomness in the process ( $\Sigma$ ).
nu	Numeric vector. Vector of intercept values for the measurement model ( $\nu$ ).
lambda	Numeric matrix. Factor loading matrix linking the latent variables to the observed variables ( $\Lambda$ ).
theta_l	Numeric matrix. Cholesky factorization ( $t(\text{chol}(\text{theta}))$ ) of the covariance matrix of the measurement error ( $\Theta$ ).
type	Integer. State space model type. See Details for more information.
x	List. Each element of the list is a matrix of covariates for each individual $i$ in $n$ . The number of columns in each matrix should be equal to time.
gamma	Numeric matrix. Matrix linking the covariates to the latent variables at current time point ( $\Gamma$ ).
kappa	Numeric matrix. Matrix linking the covariates to the observed variables at current time point ( $\kappa$ ).
mu0_fixed	Logical. If <code>mu0_fixed = TRUE</code> , fix the initial mean vector to <code>mu0</code> . If <code>mu0_fixed = FALSE</code> , <code>mu0</code> is estimated.
sigma0_fixed	Logical. If <code>sigma0_fixed = TRUE</code> , fix the initial covariance matrix to <code>tcrossprod(sigma0_l)</code> . If <code>sigma0_fixed = FALSE</code> , <code>sigma0</code> is estimated.
alpha_level	Numeric vector. Significance level $\alpha$ .
optimization_flag	a flag (TRUE/FALSE) indicating whether optimization is to be done.
hessian_flag	a flag (TRUE/FALSE) indicating whether the Hessian matrix is to be calculated.
verbose	a flag (TRUE/FALSE) indicating whether more detailed intermediate output during the estimation process should be printed
weight_flag	a flag (TRUE/FALSE) indicating whether the negative log likelihood function should be weighted by the length of the time series for each individual
debug_flag	a flag (TRUE/FALSE) indicating whether users want additional dynr output that can be used for diagnostic purposes
perturb_flag	a flag (TRUE/FLASE) indicating whether to perturb the latent states during estimation. Only useful for ensemble forecasting.
xtol_rel	Stopping criteria option for parameter optimization. See <a href="#">dynr::dynr.model()</a> for more details.
stopval	Stopping criteria option for parameter optimization. See <a href="#">dynr::dynr.model()</a> for more details.
ftol_rel	Stopping criteria option for parameter optimization. See <a href="#">dynr::dynr.model()</a> for more details.
ftol_abs	Stopping criteria option for parameter optimization. See <a href="#">dynr::dynr.model()</a> for more details.

maxeval	Stopping criteria option for parameter optimization. See <code>dynr::dynr.model()</code> for more details.
maxtime	Stopping criteria option for parameter optimization. See <code>dynr::dynr.model()</code> for more details.
ncores	Positive integer. Number of cores to use. If ncores = NULL, use a single core. Consider using multiple cores when number of bootstrap samples R is a large value.
seed	Random seed.

## Details

### Type 0:

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \boldsymbol{\Lambda}\boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t}, \quad \text{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Theta})$$

where  $\mathbf{y}_{i,t}$ ,  $\boldsymbol{\eta}_{i,t}$ , and  $\boldsymbol{\varepsilon}_{i,t}$  are random variables and  $\boldsymbol{\nu}$ ,  $\boldsymbol{\Lambda}$ , and  $\boldsymbol{\Theta}$  are model parameters.  $\mathbf{y}_{i,t}$  represents a vector of observed random variables,  $\boldsymbol{\eta}_{i,t}$  a vector of latent random variables, and  $\boldsymbol{\varepsilon}_{i,t}$  a vector of random measurement errors, at time  $t$  and individual  $i$ .  $\boldsymbol{\nu}$  denotes a vector of intercepts,  $\boldsymbol{\Lambda}$  a matrix of factor loadings, and  $\boldsymbol{\Theta}$  the covariance matrix of  $\boldsymbol{\varepsilon}$ .

An alternative representation of the measurement error is given by

$$\boldsymbol{\varepsilon}_{i,t} = \boldsymbol{\Theta}^{\frac{1}{2}} \mathbf{z}_{i,t}, \quad \text{with} \quad \mathbf{z}_{i,t} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$$

where  $\mathbf{z}_{i,t}$  is a vector of independent standard normal random variables and  $\left(\boldsymbol{\Theta}^{\frac{1}{2}}\right) \left(\boldsymbol{\Theta}^{\frac{1}{2}}\right)' = \boldsymbol{\Theta}$ .

The dynamic structure is given by

$$d\boldsymbol{\eta}_{i,t} = \boldsymbol{\Phi}(\boldsymbol{\eta}_{i,t} - \boldsymbol{\mu}) dt + \boldsymbol{\Sigma}^{\frac{1}{2}} d\mathbf{W}_{i,t}$$

where  $\boldsymbol{\mu}$  is the long-term mean or equilibrium level,  $\boldsymbol{\Phi}$  is the rate of mean reversion, determining how quickly the variable returns to its mean,  $\boldsymbol{\Sigma}$  is the matrix of volatility or randomness in the process, and  $d\mathbf{W}$  is a Wiener process or Brownian motion, which represents random fluctuations.

### Type 1:

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \boldsymbol{\Lambda}\boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t}, \quad \text{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Theta}).$$

The dynamic structure is given by

$$d\boldsymbol{\eta}_{i,t} = \boldsymbol{\Phi}(\boldsymbol{\eta}_{i,t} - \boldsymbol{\mu}) dt + \boldsymbol{\Gamma}\mathbf{x}_{i,t} + \boldsymbol{\Sigma}^{\frac{1}{2}} d\mathbf{W}_{i,t}$$

where  $\mathbf{x}_{i,t}$  represents a vector of covariates at time  $t$  and individual  $i$ , and  $\boldsymbol{\Gamma}$  the coefficient matrix linking the covariates to the latent variables.

### Type 2:

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \boldsymbol{\Lambda}\boldsymbol{\eta}_{i,t} + \boldsymbol{\kappa}\mathbf{x}_{i,t} + \boldsymbol{\varepsilon}_{i,t}, \quad \text{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Theta})$$

where  $\boldsymbol{\kappa}$  represents the coefficient matrix linking the covariates to the observed variables.

The dynamic structure is given by

$$d\boldsymbol{\eta}_{i,t} = \boldsymbol{\Phi}(\boldsymbol{\eta}_{i,t} - \boldsymbol{\mu}) dt + \boldsymbol{\Gamma}\mathbf{x}_{i,t} + \boldsymbol{\Sigma}^{\frac{1}{2}} d\mathbf{W}_{i,t}.$$

**The OU model as a linear stochastic differential equation model:**

The OU model is a first-order linear stochastic differential equation model in the form of

$$d\boldsymbol{\eta}_{i,t} = (\boldsymbol{\iota} + \boldsymbol{\Phi}\boldsymbol{\eta}_{i,t}) dt + \boldsymbol{\Sigma}^{\frac{1}{2}} d\mathbf{W}_{i,t}$$

where  $\boldsymbol{\mu} = -\boldsymbol{\Phi}^{-1}\boldsymbol{\iota}$  and, equivalently  $\boldsymbol{\iota} = -\boldsymbol{\Phi}\boldsymbol{\mu}$ .

**Value**

Returns an object of class `bootstatespace` which is a list with the following elements:

**call** Function call.

**args** Function arguments.

**thetahatstar** Sampling distribution of  $\hat{\boldsymbol{\theta}}$ .

**vcov** Sampling variance-covariance matrix of  $\hat{\boldsymbol{\theta}}$ .

**est** Vector of estimated  $\hat{\boldsymbol{\theta}}$ .

**fun** Function used ("PBSSMOUFixed").

**method** Bootstrap method used ("parametric").

**Author(s)**

Ivan Jacob Agaloos Pesigan

**References**

Chow, S.-M., Ho, M. R., Hamaker, E. L., & Dolan, C. V. (2010). Equivalence and differences between structural equation modeling and state-space modeling techniques. *Structural Equation Modeling: A Multidisciplinary Journal*, 17(2), 303–332. doi:10.1080/10705511003661553

**See Also**

Other Bootstrap for State Space Models Functions: [PBSSMFixed\(\)](#), [PBSSMLinSDEFixed\(\)](#), [PBSSMVARFixed\(\)](#)

**Examples**

```
# prepare parameters
## number of individuals
n <- 5
## time points
time <- 50
delta_t <- 0.10
## dynamic structure
p <- 2
mu0 <- c(-3.0, 1.5)
sigma0 <- 0.001 * diag(p)
sigma0_l <- t(chol(sigma0))
mu <- c(5.76, 5.18)
phi <- matrix(
  data = c(
```

```

      -0.10,
      0.05,
      0.05,
      -0.10
    ),
    nrow = p
  )
  sigma <- matrix(
    data = c(
      2.79,
      0.06,
      0.06,
      3.27
    ),
    nrow = p
  )
  sigma_l <- t(chol(sigma))
  ## measurement model
  k <- 2
  nu <- rep(x = 0, times = k)
  lambda <- diag(k)
  theta <- 0.001 * diag(k)
  theta_l <- t(chol(theta))

  path <- tempdir()

  pb <- PBSSMOUFixed(
    R = 10L, # use at least 1000 in actual research
    path = path,
    prefix = "ou",
    n = n,
    time = time,
    delta_t = delta_t,
    mu0 = mu0,
    sigma0_l = sigma0_l,
    mu = mu,
    phi = phi,
    sigma_l = sigma_l,
    nu = nu,
    lambda = lambda,
    theta_l = theta_l,
    type = 0,
    ncores = 1, # consider using multiple cores
    seed = 42
  )
  print(pb)
  summary(pb)
  confint(pb)
  vcov(pb)
  coef(pb)
  print(pb, type = "bc") # bias-corrected
  summary(pb, type = "bc")
  confint(pb, type = "bc")

```

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PBSSMVARFixed	<i>Parametric Bootstrap for the Vector Autoregressive Model (Fixed Parameters)</i>
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## Description

This function simulates data from a vector autoregressive model using a state-space model parameterization and fits the model using the dynr package. The process is repeated  $R$  times. It assumes that the parameters remain constant across individuals and over time. At the moment, the function only supports  $\text{type} = 0$ .

## Usage

```
PBSSMVARFixed(
  R,
  path,
  prefix,
  n,
  time,
  mu0,
  sigma0_l,
  alpha,
  beta,
  psi_l,
  type = 0,
  x = NULL,
  gamma = NULL,
  mu0_fixed = FALSE,
  sigma0_fixed = FALSE,
  alpha_level = 0.05,
  optimization_flag = TRUE,
  hessian_flag = FALSE,
  verbose = FALSE,
  weight_flag = FALSE,
  debug_flag = FALSE,
  perturb_flag = FALSE,
  xtol_rel = 1e-07,
  stopval = -9999,
  ftol_rel = -1,
  ftol_abs = -1,
  maxeval = as.integer(-1),
  maxtime = -1,
  ncores = NULL,
  seed = NULL
)
```

**Arguments**

R	Positive integer. Number of bootstrap samples.
path	Path to a directory to store bootstrap samples and estimates.
prefix	Character string. Prefix used for the file names for the bootstrap samples and estimates.
n	Positive integer. Number of individuals.
time	Positive integer. Number of time points.
mu0	Numeric vector. Mean of initial latent variable values ( $\mu_{\eta 0}$ ).
sigma0_l	Numeric matrix. Cholesky factorization ( $t(\text{chol}(\text{sigma0}))$ ) of the covariance matrix of initial latent variable values ( $\Sigma_{\eta 0}$ ).
alpha	Numeric vector. Vector of constant values for the dynamic model ( $\alpha$ ).
beta	Numeric matrix. Transition matrix relating the values of the latent variables at the previous to the current time point ( $\beta$ ).
psi_l	Numeric matrix. Cholesky factorization ( $t(\text{chol}(\text{psi}))$ ) of the covariance matrix of the process noise ( $\Psi$ ).
type	Integer. State space model type. See Details for more information.
x	List. Each element of the list is a matrix of covariates for each individual $i$ in $n$ . The number of columns in each matrix should be equal to time.
gamma	Numeric matrix. Matrix linking the covariates to the latent variables at current time point ( $\Gamma$ ).
mu0_fixed	Logical. If <code>mu0_fixed = TRUE</code> , fix the initial mean vector to <code>mu0</code> . If <code>mu0_fixed = FALSE</code> , <code>mu0</code> is estimated.
sigma0_fixed	Logical. If <code>sigma0_fixed = TRUE</code> , fix the initial covariance matrix to <code>tcrossprod(sigma0_l)</code> . If <code>sigma0_fixed = FALSE</code> , <code>sigma0</code> is estimated.
alpha_level	Numeric vector. Significance level $\alpha$ .
optimization_flag	a flag (TRUE/FALSE) indicating whether optimization is to be done.
hessian_flag	a flag (TRUE/FALSE) indicating whether the Hessian matrix is to be calculated.
verbose	a flag (TRUE/FALSE) indicating whether more detailed intermediate output during the estimation process should be printed
weight_flag	a flag (TRUE/FALSE) indicating whether the negative log likelihood function should be weighted by the length of the time series for each individual
debug_flag	a flag (TRUE/FALSE) indicating whether users want additional <code>dynr</code> output that can be used for diagnostic purposes
perturb_flag	a flag (TRUE/FLASE) indicating whether to perturb the latent states during estimation. Only useful for ensemble forecasting.
xtol_rel	Stopping criteria option for parameter optimization. See <code>dynr::dynr.model()</code> for more details.
stopval	Stopping criteria option for parameter optimization. See <code>dynr::dynr.model()</code> for more details.

ftol_rel	Stopping criteria option for parameter optimization. See <a href="#">dynr::dynr.model()</a> for more details.
ftol_abs	Stopping criteria option for parameter optimization. See <a href="#">dynr::dynr.model()</a> for more details.
maxeval	Stopping criteria option for parameter optimization. See <a href="#">dynr::dynr.model()</a> for more details.
maxtime	Stopping criteria option for parameter optimization. See <a href="#">dynr::dynr.model()</a> for more details.
ncores	Positive integer. Number of cores to use. If ncores = NULL, use a single core. Consider using multiple cores when number of bootstrap samples R is a large value.
seed	Random seed.

## Details

### Type 0:

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\eta}_{i,t}$$

where  $\mathbf{y}_{i,t}$  represents a vector of observed variables and  $\boldsymbol{\eta}_{i,t}$  a vector of latent variables for individual  $i$  and time  $t$ . Since the observed and latent variables are equal, we only generate data from the dynamic structure.

The dynamic structure is given by

$$\boldsymbol{\eta}_{i,t} = \boldsymbol{\alpha} + \boldsymbol{\beta}\boldsymbol{\eta}_{i,t-1} + \boldsymbol{\zeta}_{i,t}, \quad \text{with} \quad \boldsymbol{\zeta}_{i,t} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Psi})$$

where  $\boldsymbol{\eta}_{i,t}$ ,  $\boldsymbol{\eta}_{i,t-1}$ , and  $\boldsymbol{\zeta}_{i,t}$  are random variables, and  $\boldsymbol{\alpha}$ ,  $\boldsymbol{\beta}$ , and  $\boldsymbol{\Psi}$  are model parameters. Here,  $\boldsymbol{\eta}_{i,t}$  is a vector of latent variables at time  $t$  and individual  $i$ ,  $\boldsymbol{\eta}_{i,t-1}$  represents a vector of latent variables at time  $t-1$  and individual  $i$ , and  $\boldsymbol{\zeta}_{i,t}$  represents a vector of dynamic noise at time  $t$  and individual  $i$ .  $\boldsymbol{\alpha}$  denotes a vector of intercepts,  $\boldsymbol{\beta}$  a matrix of autoregression and cross regression coefficients, and  $\boldsymbol{\Psi}$  the covariance matrix of  $\boldsymbol{\zeta}_{i,t}$ .

An alternative representation of the dynamic noise is given by

$$\boldsymbol{\zeta}_{i,t} = \boldsymbol{\Psi}^{\frac{1}{2}} \mathbf{z}_{i,t}, \quad \text{with} \quad \mathbf{z}_{i,t} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$$

where  $\left(\boldsymbol{\Psi}^{\frac{1}{2}}\right) \left(\boldsymbol{\Psi}^{\frac{1}{2}}\right)' = \boldsymbol{\Psi}$ .

### Type 1:

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\eta}_{i,t}.$$

The dynamic structure is given by

$$\boldsymbol{\eta}_{i,t} = \boldsymbol{\alpha} + \boldsymbol{\beta}\boldsymbol{\eta}_{i,t-1} + \boldsymbol{\Gamma}\mathbf{x}_{i,t} + \boldsymbol{\zeta}_{i,t}, \quad \text{with} \quad \boldsymbol{\zeta}_{i,t} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Psi})$$

where  $\mathbf{x}_{i,t}$  represents a vector of covariates at time  $t$  and individual  $i$ , and  $\boldsymbol{\Gamma}$  the coefficient matrix linking the covariates to the latent variables.

**Value**

Returns an object of class `bootstatespace` which is a list with the following elements:

**call** Function call.

**args** Function arguments.

**thetahatstar** Sampling distribution of  $\hat{\theta}$ .

**vcov** Sampling variance-covariance matrix of  $\hat{\theta}$ .

**est** Vector of estimated  $\hat{\theta}$ .

**fun** Function used ("PBSSMVARFixed").

**method** Bootstrap method used ("parametric").

**Author(s)**

Ivan Jacob Agaloos Pesigan

**References**

Chow, S.-M., Ho, M. R., Hamaker, E. L., & Dolan, C. V. (2010). Equivalence and differences between structural equation modeling and state-space modeling techniques. *Structural Equation Modeling: A Multidisciplinary Journal*, 17(2), 303–332. doi:10.1080/10705511003661553

**See Also**

Other Bootstrap for State Space Models Functions: `PBSSMFixed()`, `PBSSMLinSDEFixed()`, `PBSSMOUFixed()`

**Examples**

```
# prepare parameters
## number of individuals
n <- 5
## time points
time <- 50
## dynamic structure
p <- 3
mu0 <- rep(x = 0, times = p)
sigma0 <- 0.001 * diag(p)
sigma0_l <- t(chol(sigma0))
alpha <- rep(x = 0, times = p)
beta <- 0.50 * diag(p)
psi <- 0.001 * diag(p)
psi_l <- t(chol(psi))

path <- tempdir()

pb <- PBSSMVARFixed(
  R = 10L, # use at least 1000 in actual research
  path = path,
  prefix = "var",
  n = n,
```



```

    time = time,
    mu0 = mu0,
    sigma0_l = sigma0_l,
    alpha = alpha,
    beta = beta,
    psi_l = psi_l,
    type = 0,
    ncores = 1, # consider using multiple cores
    seed = 42
)
print(pb)
summary(pb)
confint(pb)
vcov(pb)
coef(pb)
print(pb, type = "bc") # bias-corrected
summary(pb, type = "bc")
confint(pb, type = "bc")

```

---

print.bootstatespace    *Print Method for an Object of Class bootstatespace*

---

## Description

Print Method for an Object of Class bootstatespace

## Usage

```

## S3 method for class 'bootstatespace'
print(x, alpha = NULL, type = "pc", digits = 4, ...)

```

## Arguments

x	Object of Class bootstatespace.
alpha	Numeric vector. Significance level $\alpha$ . If alpha = NULL, use the argument alpha used in x.
type	Charater string. Confidence interval type, that is, type = "pc" for percentile; type = "bc" for bias corrected.
digits	Digits to print.
...	additional arguments.

## Value

Prints a matrix of estimates, standard errors, number of bootstrap replications, and confidence intervals.

**Author(s)**

Ivan Jacob Agaloos Pesigan

---

summary.bootstatespace

*Summary Method for an Object of Class bootstatespace*

---

**Description**

Summary Method for an Object of Class bootstatespace

**Usage**

```
## S3 method for class 'bootstatespace'  
summary(object, alpha = NULL, type = "pc", digits = 4, ...)
```

**Arguments**

object	Object of Class bootstatespace.
alpha	Numeric vector. Significance level $\alpha$ . If alpha = NULL, use the argument alpha used in object.
type	Charater string. Confidence interval type, that is, type = "pc" for percentile; type = "bc" for bias corrected.
digits	Digits to print.
...	additional arguments.

**Value**

Returns a matrix of estimates, standard errors, number of bootstrap replications, and confidence intervals.

**Author(s)**

Ivan Jacob Agaloos Pesigan

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vcov.bootstatespace	<i>Sampling Variance-Covariance Matrix Method for an Object of Class bootstatespace</i>
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**Description**

Sampling Variance-Covariance Matrix Method for an Object of Class bootstatespace

**Usage**

```
## S3 method for class 'bootstatespace'  
vcov(object, ...)
```

**Arguments**

object	Object of Class bootstatespace.
...	additional arguments.

**Value**

Returns the variance-covariance matrix of estimates.

**Author(s)**

Ivan Jacob Agaloos Pesigan

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