

# Duplication Matrix

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The duplication matrix  $\mathbf{D}_k$  is the  $k^2 \times \frac{k(k+1)}{2}$  matrix for a given  $k \times k$  symmetric matrix  $\mathbf{A}$  where

$$\mathbf{D}_k \text{vech}(\mathbf{A}) = \text{vec}(\mathbf{A}) \quad (1)$$

$\text{vec}(\cdot)$  is the vectorization of a matrix, and  $\text{vech}(\cdot)$  is the half-vectorization of a matrix.

## Examples

```
library(linearAlgebra)
```

```
A <- matrix(
  data = c(
    1.0, 0.5, 0.4,
    0.5, 1.0, 0.6,
    0.4, 0.6, 1.0
  ),
  ncol = 3
)
k_i <- dim(A)[1]
dcap(k_i)

##           [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]      1    0    0    0    0    0
## [2,]      0    1    0    0    0    0
## [3,]      0    0    1    0    0    0
## [4,]      0    1    0    0    0    0
## [5,]      0    0    0    1    0    0
## [6,]      0    0    0    0    1    0
## [7,]      0    0    1    0    0    0
## [8,]      0    0    0    0    1    0
## [9,]      0    0    0    0    0    1
```

```
dcap(k_i) %*% vech(A)
```

```
##           [,1]
## [1,]      1.0
## [2,]      0.5
## [3,]      0.4
## [4,]      0.5
## [5,]      1.0
## [6,]      0.6
## [7,]      0.4
## [8,]      0.6
## [9,]      1.0
```

```
all.equal(
  c(dcap(k_i) %*% vech(A)),
  vec(A)
)
```

```
## [1] TRUE
```

## Readings

See Magnus and Neudecker (2019) p. 56–57, Magnus and Neudecker (1980), and Abadir and Magnus (2005) ch. 11.

## References

- Abadir, K. M., & Magnus, J. R. (2005, August). *Matrix algebra*. Cambridge University Press. <https://doi.org/10.1017/cbo9780511810800>
- Magnus, J. R., & Neudecker, H. (1980). The elimination matrix: Some lemmas and applications. *SIAM Journal on Algebraic Discrete Methods*, 1(4), 422–449. <https://doi.org/10.1137/0601049>
- Magnus, J. R., & Neudecker, H. (2019, February). *Matrix differential calculus with applications in statistics and econometrics*. Wiley. <https://doi.org/10.1002/9781119541219>