The Duplication Matrix

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The duplication matrix $\boldsymbol{D}_k$ is the $k^2 \times \frac{1}{2} k (k+1)$ matrix for a given $k \times k$ symmetric matrix $\boldsymbol{A}$ where

$$\boldsymbol{D}_k \text{vech}\left(\boldsymbol{A}\right) = \text{vec}\left(\boldsymbol{A}\right) \quad \left(\boldsymbol{A} = \boldsymbol{A}'\right) \tag{1}$$

$\text{vec}\left(\cdot\right)$ is the vectorization of a matrix, and $\text{vech}\left(\cdot\right)$ is the half-vectorization of a matrix.

Examples

```r
library(linearAlgebra)
```

```r
A <- matrix(
  data = c(
    1.0, 0.5, 0.4,
    0.5, 1.0, 0.6,
    0.4, 0.6, 1.0
  ),
  ncol = 3
)
k <- dim(A)[1]
```

```r
dcap(k)
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    1    0    0    0    0    0
## [2,]    0    1    0    0    0    0
## [3,]    0    0    1    0    0    0
## [4,]    0    1    0    0    0    0
## [5,]    0    0    0    1    0    0
## [6,]    0    0    0    0    1    0
## [7,]    0    0    1    0    0    0
## [8,]    0    0    0    0    1    0
## [9,]    0    0    0    0    0    1
```

```
dcap(k) %*% vech(A)
```

```
##        [,1]
##  [1,]  1.0
##  [2,]  0.5
##  [3,]  0.4
##  [4,]  0.5
##  [5,]  1.0
##  [6,]  0.6
##  [7,]  0.4
##  [8,]  0.6
##  [9,]  1.0
```

```
all.equal(
  c(dcap(k) %*% vech(A)),
  vec(A)
)
```

```
## [1] TRUE
```

Readings

See Magnus and Neudecker (2019) p. 56–57 and 444, Magnus and Neudecker (1980), and Abadir and Magnus (2005) ch. 11.

References

Abadir, K. M., & Magnus, J. R. (2005, August). *Matrix algebra*. Cambridge University Press. https://doi.org/10.1017/cbo9780511810800

Magnus, J. R., & Neudecker, H. (1980). The elimination matrix: Some lemmas and applications. *SIAM Journal on Algebraic Discrete Methods*, *1*(4), 422–449. https://doi.org/10.1137/0601049

Magnus, J. R., & Neudecker, H. (2019, February). *Matrix differential calculus with applications in statistics and econometrics*. Wiley. https://doi.org/10.1002/9781119541219