# Vectorization

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```
library(linearAlgebra)
```

## Vectorize

The vectorization of a $m \times n$ matrix $\boldsymbol{A}$, given by $\text{vec}\left(\boldsymbol{A}\right)$, is the $mn \times 1$ vector obtained by stacking the elements of $\boldsymbol{A}$ column-wise.

For example, for

$$\boldsymbol{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix} \tag{1}$$

the vectorization is given by

$$\text{vec}\left(\boldsymbol{A}\right) = \begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \\ a_{12} \\ a_{22} \\ a_{32} \end{pmatrix}. \tag{2}$$

```
A <- matrix(c("a11", "a21", "a31", "a12", "a22", "a32"), ncol = 2)
A
```

```
##      [,1]  [,2]
## [1,] "a11" "a12"
## [2,] "a21" "a22"
## [3,] "a31" "a32"
```

```
vec(A)
```

```
## [1] "a11" "a21" "a31" "a12" "a22" "a32"
```

## Half-Vectorize

The half-vectorization of an $k \times k$ matrix $\boldsymbol{A}$, given by vech $(\boldsymbol{A})$, is the $\frac{1}{2}k\,(k+1) \times 1$ vector obtained from the vectorization of $\boldsymbol{A}$, given by vec $(\boldsymbol{A})$, where that all upper diagonal elements of $\boldsymbol{A}$ are eliminated.

For example, for

$$\boldsymbol{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \tag{3}$$

the vectorization is given by

$$\text{vec}\,(\boldsymbol{A}) = \begin{pmatrix} a_{11} \\ a_{21} \\ a_{12} \\ a_{22} \end{pmatrix} \tag{4}$$

and the half-vectorization is given by

$$\text{vech}\,(\boldsymbol{A}) = \begin{pmatrix} a_{11} \\ a_{21} \\ a_{22} \end{pmatrix}. \tag{5}$$

```
A <- matrix(c("a11", "a21", "a12", "a22"), ncol = 2)
A
```

```
##      [,1]  [,2]
## [1,] "a11" "a12"
## [2,] "a21" "a22"
```

```
vech(A)
```

```
## [1] "a11" "a21" "a22"
```

## Strict Half-Vectorize

The strict half-vectorization of a $k \times k$ matrix $\boldsymbol{A}$, given by $\text{vechs}(\boldsymbol{A})$, is the $\frac{1}{2}k(k+1)-k \times 1$ vector obtained from the vectorization of $\boldsymbol{A}$, given by $\text{vec}(\boldsymbol{A})$, where that all diagonal and upper diagonal elements of $\boldsymbol{A}$ are eliminated.

For example, for

$$\boldsymbol{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \tag{6}$$

the vectorization is given by

$$\text{vec}(\boldsymbol{A}) = \begin{pmatrix} a_{11} \\ a_{21} \\ a_{12} \\ a_{22} \end{pmatrix}, \tag{7}$$

the half-vectorization is given by

$$\text{vech}(\boldsymbol{A}) = \begin{pmatrix} a_{11} \\ a_{21} \\ a_{22} \end{pmatrix}, \tag{8}$$

and the strict half-vectorization is given by

$$\text{vechs}(\boldsymbol{A}) = \begin{pmatrix} a_{21} \end{pmatrix}. \tag{9}$$

```
A <- matrix(c("a11", "a21", "a12", "a22"), ncol = 2)
A
```

```
##      [,1]  [,2]
## [1,] "a11" "a12"
## [2,] "a21" "a22"
```

```
vechs(A)
```

```
## [1] "a21"
```

## Names for Half-Vectorization

```
A <- matrix(c("a11", "a21", "a12", "a22"), ncol = 2)
colnames(A) <- rownames(A) <- c("v1", "v2")
A
```

```
##    v1    v2
## v1 "a11" "a12"
## v2 "a21" "a22"
```

```
vechnames(colnames(A))
```

```
## [1] "v1.v1" "v1.v2" "v2.v2"
```

## Names for Strict Half-Vectorization

```
A <- matrix(c("a11", "a21", "a12", "a22"), ncol = 2)
colnames(A) <- rownames(A) <- c("v1", "v2")
A
```

```
##    v1    v2
## v1 "a11" "a12"
## v2 "a21" "a22"
```

```
vechsnames(colnames(A))
```

```
## [1] "v1.v2"
```

## Readings

See Magnus and Neudecker (2019) p. 56–57 and 444 and Abadir and Magnus (2005) ch. 10–11.

## References

Abadir, K. M., & Magnus, J. R. (2005, August). *Matrix algebra*. Cambridge University Press. https://doi.org/10.1017/cbo9780511810800

Magnus, J. R., & Neudecker, H. (2019, February). *Matrix differential calculus with applications in statistics and econometrics*. Wiley. https://doi.org/10.1002/9781119541219