Multivariate Meta-Analysis of Vector Autoregressive Model Coefficients: A Two-Step Structural Equation Modeling Approach

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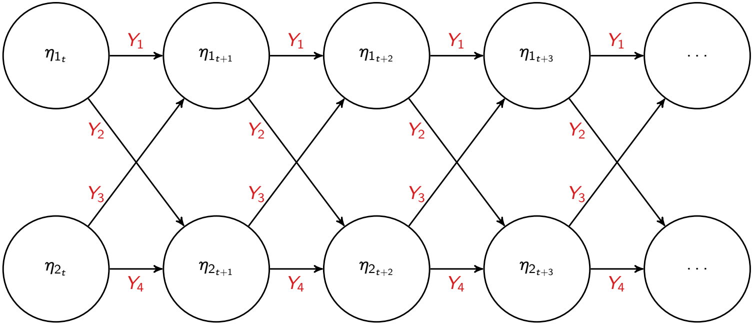
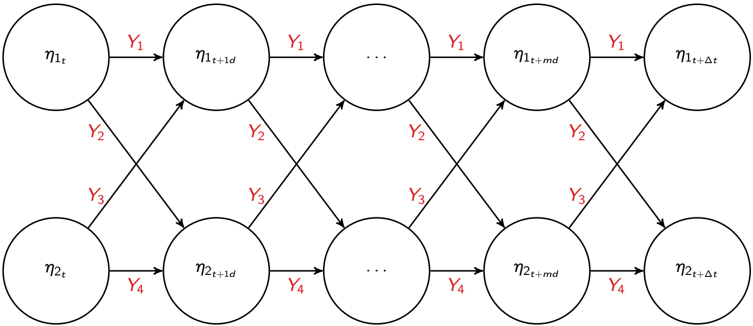
First-Order Discrete-Time Vector Autoregressive Model



$$\boldsymbol{\eta}_{i,t} = \boldsymbol{\beta}\boldsymbol{\eta}_{i,t-1} + \boldsymbol{\zeta}_{i,t} \tag{1}$$

First-Order Continuous-Time Vector Autoregressive Model



$$\mathrm{d}\boldsymbol{\eta}_{i,t} = \left(\boldsymbol{\Phi}\boldsymbol{\eta}_{i,t}\right)\mathrm{d}t + \boldsymbol{\Sigma}^{\frac{1}{2}}\mathrm{d}\mathbf{W}_{i,t} \tag{2}$$

Vector Autoregressive Model

- In Equations 1 and 2, $\boldsymbol{\beta}$ and $\boldsymbol{\Phi}$ do not have the subscript i, meaning we assume that the dynamics is invariant across individuals.
- However, this is such a strong assumption to make, it is more likely that the dynamics vary across individuals ($\boldsymbol{\beta}_i$ and $\boldsymbol{\Phi}_i$).
- Let $\mathbf{y}_i = \mathrm{Vec}\left(\boldsymbol{\beta}_i\right)$ or $\mathbf{y}_i = \mathrm{Vec}\left(\boldsymbol{\Phi}_i\right)$.
- We assume that the vector of coefficients for each individual, $\mathbf{y}_i$, come from a multivariate normal distribution with some mean and covariance matrix, $\mathbf{y} \sim \mathcal{N}\left(\boldsymbol{\mu}, \boldsymbol{\Sigma}\right)$.

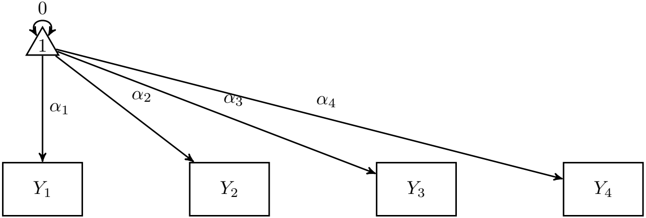
Meta-Analysis

- First Stage: Estimate the parameters ($\mathbf{y}_i$) and the sampling covariance matrix ($\mathbf{V}_i^2$) per individual i.
- Second Stage: Pool the coefficients using the corresponding sampling covariance matrix as a weight matrix.
  - The average/pooled estimate is given by $\boldsymbol{\alpha}$.
  - The variability around the pooled estimate is given by $\boldsymbol{\tau}^2$.
- Meta-analysis as a structural equation model (Cheung, 2015).

Meta-Analysis as a Structural Equation Model

Figure 1

Average/Pooled Coefficients

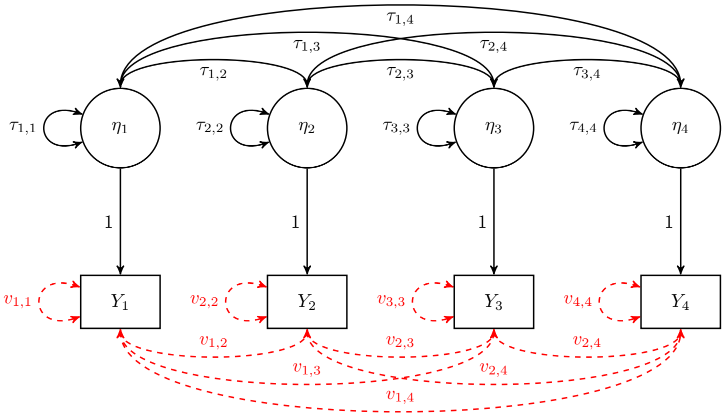


$$\boldsymbol{\mu}_i\left(\boldsymbol{\theta}\right) = \boldsymbol{\alpha}$$

Meta-Analysis as a Structural Equation Model

Figure 2

Between- and Within-Level Variability



$$\boldsymbol{\Sigma}_i\left(\boldsymbol{\theta}\right) = \boldsymbol{\tau}^2 + \mathbf{V}_i^2 \quad (3)$$

Meta-Analysis as a Structural Equation Model

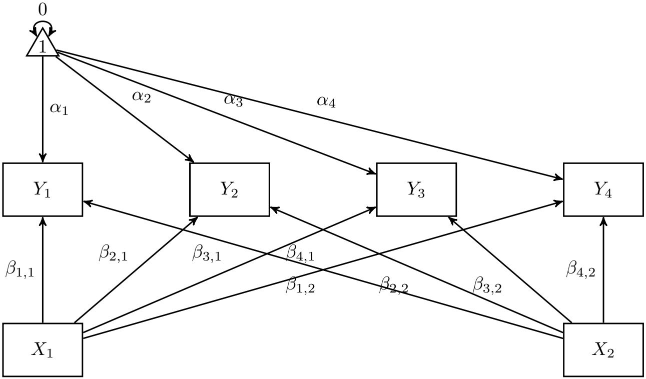
In meta-analysis, it is customary to quantify the heterogeneity of effect sizes using the two sources of variability ($\mathbf{I}^2$; Higgins & Thompson, 2002)

$$\mathbf{I}^2 = \frac{\hat{\boldsymbol{\tau}}^2}{\hat{\boldsymbol{\tau}}^2 + \tilde{\mathbf{V}}^2} \tag{4}$$

$\mathbf{I}^2$ is interpreted as the proportion of the total variation in effect sizes that is due to between-individual heterogeneity.

Covariates

Figure 3
Covariates

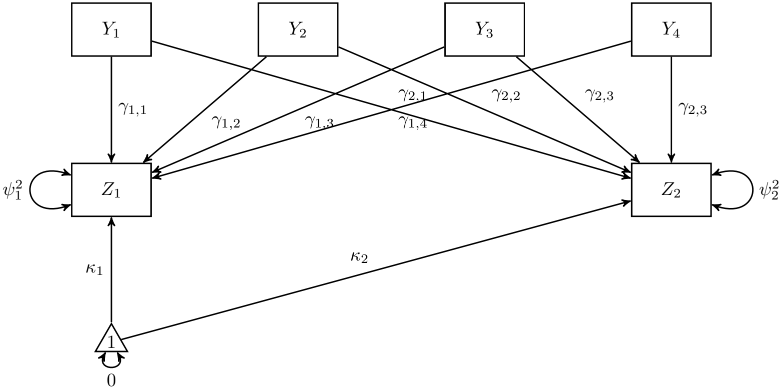


$$\boldsymbol{\mu}_i\left(\boldsymbol{\theta}\right) = \boldsymbol{\alpha} + \boldsymbol{\beta}\mathbf{x}_i$$

Distal Outcomes

Figure 4

Distal Outcomes



Mixture of Normal Distributions

$$\boldsymbol{\mu}_k\left(\boldsymbol{\theta}_k\right)$$

$$\boldsymbol{\Sigma}_k\left(\boldsymbol{\theta}_k\right)$$

where k represents the k^{th} normal distribution.

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References

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Cheung, M. W.-L. (2015). *Meta-analysis: A structural equation modeling approach*. Wiley. https://doi.org/10.1002/9781118957813

Higgins, J. P. T., & Thompson, S. G. (2002). Quantifying heterogeneity in a meta-analysis. *Statistics in Medicine, 21*(11), 1539–1558. https://doi.org/10.1002/sim.1186