

Multivariate Meta-Analysis of Vector Autoregressive Model Coefficients

Ivan Jacob Agaloos Pesigan, Michael A. Russell, and Sy-Miin Chow
The Pennsylvania State University

Abstract

TODO

Keywords: vector autoregressive model, multilevel model

```
options(scipen = 999)
n <- seq(
  from = 50,
  to = 250,
  by = 50
)
n <- c(5, 15, 25, n)
time <- seq(
  from = 50,
  to = 250,
  by = 50
)
theta <- c(
  .2,
  .4,
  .6,
  .8
)
k <- p <- 2
iden <- diag(k)
null_vec <- rep(x = 0, times = k)
mu0 <- list(
  null_vec
)
sigma0 <- diag(p)
sigma0_l <- list(
```

```

    t(chol(sigma0))
  )
  alpha <- null_vec
  psi_d <- sqrt(
    c(1.3, 1.56)
  ) * iden
  psi_r <- matrix(
    data = c(1, 0.4, 0.4, 1),
    nrow = p
  )
  psi <- psi_d %*% psi_r %*% psi_d
  psi_l <- list(
    t(chol(psi))
  )
  beta_mu <- matrix(
    data = c(
      0.28, -0.035,
      -0.048, 0.26
    ),
    nrow = p
  )
  beta_d <- sqrt(
    c(
      0.0169,
      0.00810,
      0.000784,
      0.0256
    )
  ) * diag(4)
  beta_r <- matrix(
    data = c(
      1, 0.4, 0.4, 0.4,
      0.4, 1, 0.4, 0.4,
      0.4, 0.4, 1, 0.4,
      0.4, 0.4, 0.4, 1
    ),
    nrow = 4
  )
  beta_sigma <- beta_d %*% beta_r %*% beta_d

```

Monte Carlo Simulation Parameters

We based the population parameters on Bringmann et al. (2013).

We added a single-indicator measurement error model for the observed variables following Schuurman and Hamaker (2019).

Sample size ($n = \{5, 15, 25, 50, 100, 150, 200, 250\}$).

Time points ($m = \{50, 100, 150, 200, 250\}$).

$$\mathbf{y}_{i,t} = \mathbf{\Lambda}\boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}, \quad \boldsymbol{\varepsilon} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Theta}) \quad (1)$$

$$\mathbf{\Lambda} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (2)$$

$$\boldsymbol{\Theta} = \begin{pmatrix} \theta & 0 \\ 0 & \theta \end{pmatrix} \quad (3)$$

Unique variance of the manifest variables ($\theta = \{0.2, 0.4, 0.6, 0.8\}$).

$$\boldsymbol{\eta}_{i,t} = \boldsymbol{\beta}\boldsymbol{\eta}_{i,t-1} + \boldsymbol{\zeta}_{i,t}, \quad \boldsymbol{\zeta}_{i,t} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Psi}) \quad (4)$$

$$\boldsymbol{\mu}_{\boldsymbol{\eta}|0} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (5)$$

$$\boldsymbol{\Sigma}_{\boldsymbol{\eta}|0} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (6)$$

Let the transition matrix $\boldsymbol{\beta}$ be normally distributed with the following means

$$\boldsymbol{\mu}_{\boldsymbol{\beta}} = \begin{pmatrix} 0.28 & -0.048 \\ -0.035 & 0.26 \end{pmatrix} \quad (7)$$

and covariance matrix

$$\begin{aligned} \boldsymbol{\Sigma}_{\boldsymbol{\beta}} &= \begin{pmatrix} 0.13 & 0 & 0 & 0 \\ 0 & 0.09 & 0 & 0 \\ 0 & 0 & 0.028 & 0 \\ 0 & 0 & 0 & 0.16 \end{pmatrix} \begin{pmatrix} 1 & 0.4 & 0.4 & 0.4 \\ 0.4 & 1 & 0.4 & 0.4 \\ 0.4 & 0.4 & 1 & 0.4 \\ 0.4 & 0.4 & 0.4 & 1 \end{pmatrix} \begin{pmatrix} 0.13 & 0 & 0 & 0 \\ 0 & 0.09 & 0 & 0 \\ 0 & 0 & 0.028 & 0 \\ 0 & 0 & 0 & 0.16 \end{pmatrix} \\ \boldsymbol{\Sigma}_{\boldsymbol{\beta}} &= \begin{pmatrix} 0.0169 & 0.00468 & 0.001456 & 0.00832 \\ 0.00468 & 0.0081 & 0.001008 & 0.00576 \\ 0.001456 & 0.001008 & 0.000784 & 0.001792 \\ 0.00832 & 0.00576 & 0.001792 & 0.0256 \end{pmatrix} \end{aligned} \quad (8)$$

$$\begin{aligned} \boldsymbol{\Psi} &= \begin{pmatrix} 1.1401754 & 0 \\ 0 & 1.2489996 \end{pmatrix} \begin{pmatrix} 1 & 0.4 \\ 0.4 & 1 \end{pmatrix} \begin{pmatrix} 1.1401754 & 0 \\ 0 & 1.2489996 \end{pmatrix} \\ \boldsymbol{\Psi} &= \begin{pmatrix} 1.3 & 0.5696315 \\ 0.5696315 & 1.56 \end{pmatrix}. \end{aligned} \quad (9)$$

Data was generated using the **GenData** which is a wrapper function for the **SimSSMIVary** from the **simStateSpace** package.

References

- Bringmann, L. F., Vissers, N., Wichers, M., Geschwind, N., Kuppens, P., Peeters, F., Borsboom, D., & Tuerlinckx, F. (2013). A network approach to psychopathology: New insights into clinical longitudinal data (G. A. de Erausquin, Ed.). *PLoS ONE*, *8*(4), e60188. <https://doi.org/10.1371/journal.pone.0060188>
- Schuurman, N. K., & Hamaker, E. L. (2019). Measurement error and person-specific reliability in multilevel autoregressive modeling. *Psychological Methods*, *24*(1), 70–91. <https://doi.org/10.1037/met0000188>