

Multivariate Meta-Analysis of Vector Autoregressive Model Coefficients: A Two-Step Structural Equation Modeling Approach

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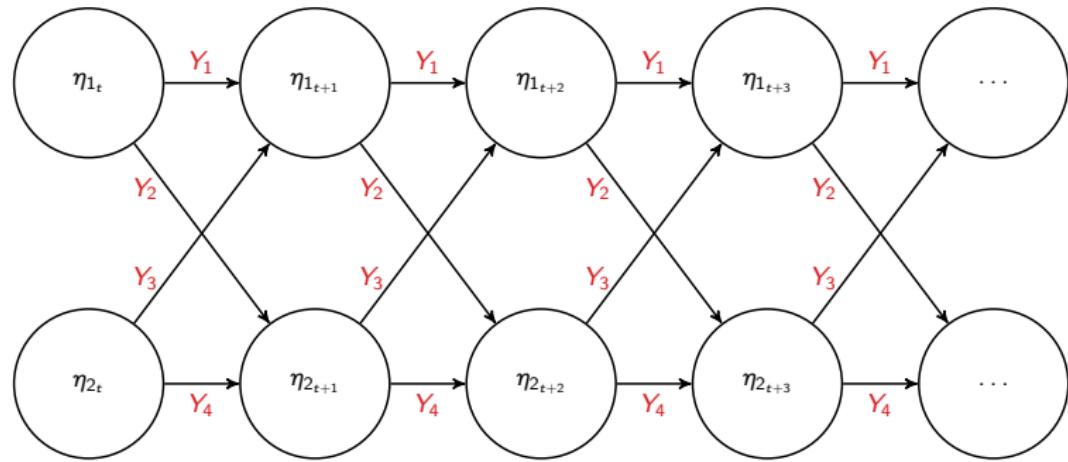
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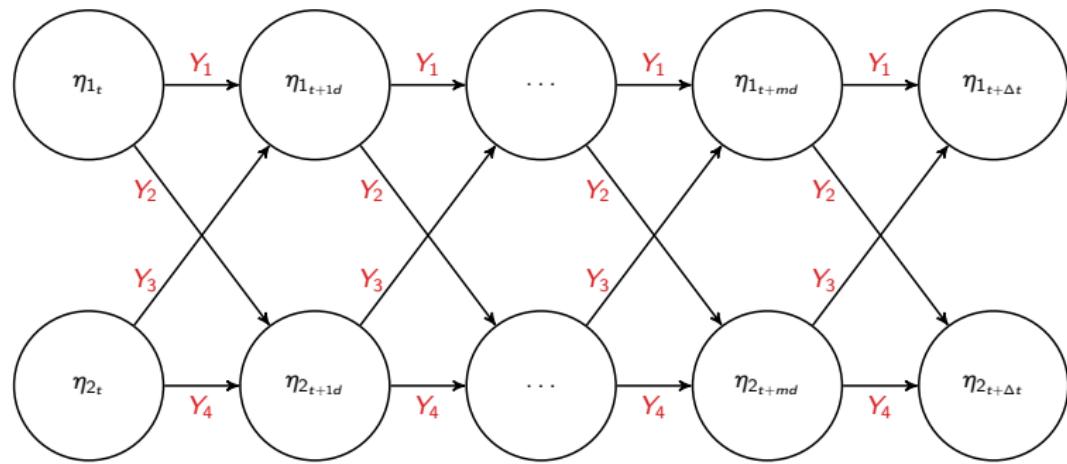


First-Order Discrete-Time Vector Autoregressive Model



$$\boldsymbol{\eta}_{i,t} = \beta \boldsymbol{\eta}_{i,t-1} + \zeta_{i,t} \quad (1)$$

First-Order Continuous-Time Vector Autoregressive Model



$$d\eta_{i,t} = (\Phi \eta_{i,t}) dt + \Sigma^{\frac{1}{2}} dW_{i,t} \quad (2)$$

Vector Autoregressive Model

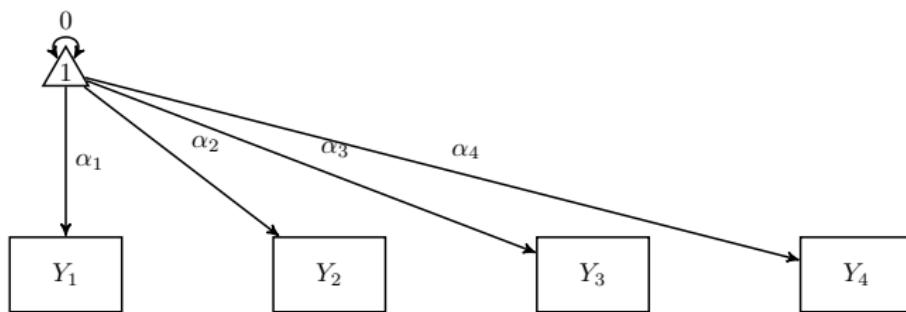
- ▶ In Equations 1 and 2, β and Φ do not have the subscript i , meaning we assume that the dynamics is invariant across individuals.
- ▶ However, this is such a strong assumption to make, it is more likely that the dynamics vary across individuals (β_i and Φ_i).
- ▶ Let $\mathbf{y}_i = \text{Vec}(\beta_i)$ or $\mathbf{y}_i = \text{Vec}(\Phi_i)$.
- ▶ We assume that the vector of coefficients for each individual, \mathbf{y}_i , come from a multivariate normal distribution with some mean and covariance matrix, $\mathbf{y} \sim \mathcal{N}(\mu, \Sigma)$.

Meta-Analysis

- ▶ First Stage: Estimate the parameters (\mathbf{y}_i) and the sampling covariance matrix (\mathbf{V}_i^2) per individual i .
- ▶ Second Stage: Pool the coefficients using the corresponding sampling covariance matrix as a weight matrix.
 - ▶ The average/pooled estimate is given by α .
 - ▶ The variability around the pooled estimate is given by τ^2 .
- ▶ Meta-analysis as a structural equation model (Cheung, 2015).

Meta-Analysis as a Structural Equation Model

Figure 1
Average/Pooled Coefficients

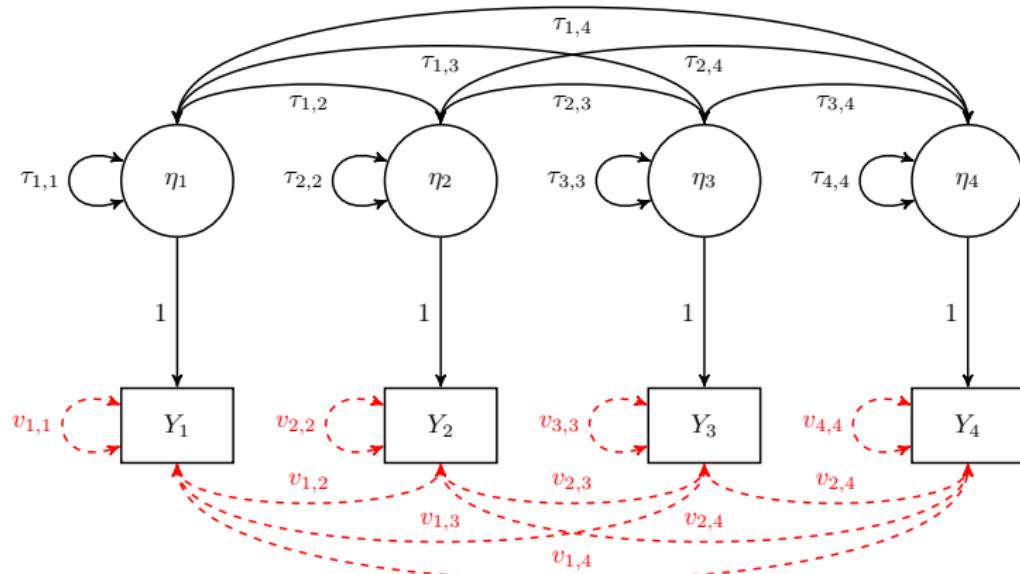


$$\mu_i(\theta) = \alpha$$

Meta-Analysis as a Structural Equation Model

Figure 2

Between- and Within-Level Variability



$$\Sigma_i(\theta) = \tau^2 + \mathbf{V}_i^2$$

(3)

Meta-Analysis as a Structural Equation Model

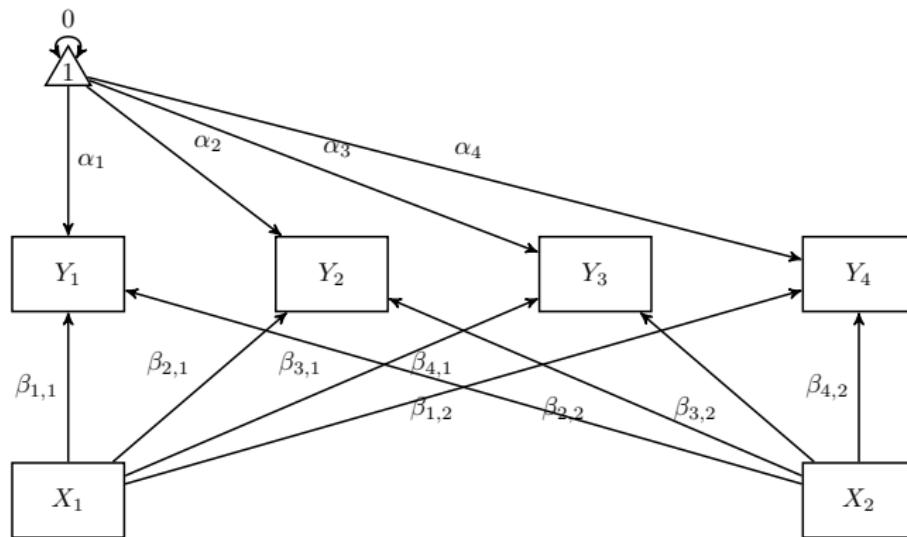
In meta-analysis, it is customary to quantify the **heterogeneity of effect sizes** using the two sources of variability (I^2 ; Higgins & Thompson, 2002)

$$I^2 = \frac{\hat{\tau}^2}{\hat{\tau}^2 + \tilde{V}^2} \quad (4)$$

I^2 is interpreted as the proportion of the total variation in effect sizes that is due to between-individual heterogeneity.

Covariates

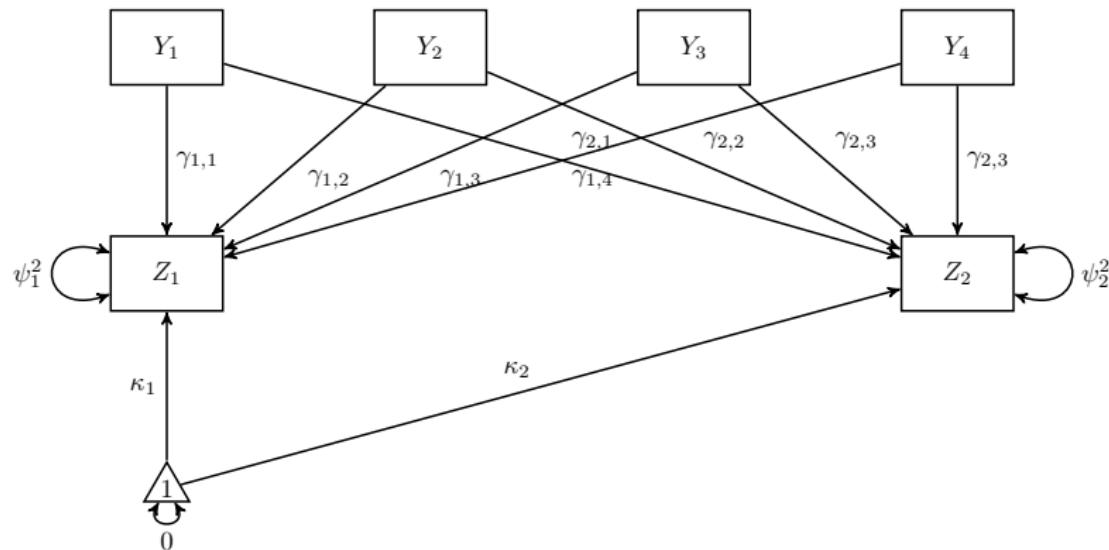
Figure 3
Covariates



$$\mu_i(\theta) = \alpha + \beta \mathbf{x}_i$$

Distal Outcomes

Figure 4
Distal Outcomes



Mixture of Normal Distributions

$$\mu_k(\theta_k)$$

$$\Sigma_k(\theta_k)$$

where k represents the k^{th} normal distribution.

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References

References |

-  Cheung, M. W.-L. (2015). *Meta-analysis: A structural equation modeling approach*. Wiley.
<https://doi.org/10.1002/9781118957813>
-  Higgins, J. P. T., & Thompson, S. G. (2002). Quantifying heterogeneity in a meta-analysis. *Statistics in Medicine*, 21(11), 1539–1558. <https://doi.org/10.1002/sim.1186>