Multivariate Meta-Analysis of Vector Autoregressive Model Coefficients

Ivan Jacob Agaloos Pesigan, Michael A. Russell, and Sy-Miin Chow
The Pennsylvania State University

Abstract

TODO

Keywords: vector autoregressive model, multilevel model

```r
options(scipen = 999)
n <- seq(
  from = 50,
  to = 250,
  by = 50
)
n <- c(5, 15, 25, n)
time <- seq(
  from = 50,
  to = 250,
  by = 50
)
theta <- c(
  .2,
  .4,
  .6,
  .8
)
k <- p <- 2
iden <- diag(k)
null_vec <- rep(x = 0, times = k)
mu0 <- list(
  null_vec
)
sigma0 <- diag(p)
sigma0_l <- list(
```

Ivan Jacob Agaloos Pesigan https://orcid.org/0000-0003-4818-8420; Michael A. Russell https://orcid.org/0000-0002-3956-604X; Sy-Miin Chow https://orcid.org/0000-0003-1938-027X;

```
  t(chol(sigma0))
)
alpha <- null_vec
psi_d <- sqrt(
  c(1.3, 1.56)
) * iden
psi_r <- matrix(
  data = c(1, 0.4, 0.4, 1),
  nrow = p
)
psi <- psi_d %*% psi_r %*% psi_d
psi_l <- list(
  t(chol(psi))
)
beta_mu <- matrix(
  data = c(
    0.28, -0.035,
    -0.048, 0.26
  ),
  nrow = p
)
beta_d <- sqrt(
  c(
    0.0169,
    0.00810,
    0.000784,
    0.0256
  )
) * diag(4)
beta_r <- matrix(
  data = c(
    1, 0.4, 0.4, 0.4,
    0.4, 1, 0.4, 0.4,
    0.4, 0.4, 1, 0.4,
    0.4, 0.4, 0.4, 1
  ),
  nrow = 4
)
beta_sigma <- beta_d %*% beta_r %*% beta_d
```

Monte Carlo Simulation Parameters

We based the population parameters on Bringmann et al. (2013).

We added a single-indicator measurement error model for the observed variables following Schuurman and Hamaker (2019).

Sample size ($n = \{5, 15, 25, 50, 100, 150, 200, 250\}$).
Time points ($m = \{50, 100, 150, 200, 250\}$).

$$\mathbf{y}_{i,t} = \boldsymbol{\Lambda} \boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}, \quad \boldsymbol{\varepsilon} \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Theta}\right) \tag{1}$$

$$\boldsymbol{\Lambda} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{2}$$

$$\boldsymbol{\Theta} = \begin{pmatrix} \theta & 0 \\ 0 & \theta \end{pmatrix} \tag{3}$$

Unique variance of the manifest variables ($\theta = \{0.2, 0.4, 0.6, 0.8\}$).

$$\boldsymbol{\eta}_{i,t} = \boldsymbol{\beta} \boldsymbol{\eta}_{i,t-1} + \boldsymbol{\zeta}_{i,t}, \quad \boldsymbol{\zeta}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Psi}\right) \tag{4}$$

$$\boldsymbol{\mu}_{\boldsymbol{\eta}|0} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{5}$$

$$\boldsymbol{\Sigma}_{\boldsymbol{\eta}|0} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{6}$$

Let the transition matrix $\boldsymbol{\beta}$ be normally distributed with the following means

$$\boldsymbol{\mu}_{\boldsymbol{\beta}} = \begin{pmatrix} 0.28 & -0.048 \\ -0.035 & 0.26 \end{pmatrix} \tag{7}$$

and covariance matrix

$$\boldsymbol{\Sigma}_{\boldsymbol{\beta}} = \begin{pmatrix} 0.13 & 0 & 0 & 0 \\ 0 & 0.09 & 0 & 0 \\ 0 & 0 & 0.028 & 0 \\ 0 & 0 & 0 & 0.16 \end{pmatrix} \begin{pmatrix} 1 & 0.4 & 0.4 & 0.4 \\ 0.4 & 1 & 0.4 & 0.4 \\ 0.4 & 0.4 & 1 & 0.4 \\ 0.4 & 0.4 & 0.4 & 1 \end{pmatrix} \begin{pmatrix} 0.13 & 0 & 0 & 0 \\ 0 & 0.09 & 0 & 0 \\ 0 & 0 & 0.028 & 0 \\ 0 & 0 & 0 & 0.16 \end{pmatrix}$$

$$\boldsymbol{\Sigma}_{\boldsymbol{\beta}} = \begin{pmatrix} 0.0169 & 0.00468 & 0.001456 & 0.00832 \\ 0.00468 & 0.0081 & 0.001008 & 0.00576 \\ 0.001456 & 0.001008 & 0.000784 & 0.001792 \\ 0.00832 & 0.00576 & 0.001792 & 0.0256 \end{pmatrix} \tag{8}$$

$$\boldsymbol{\Psi} = \begin{pmatrix} 1.1401754 & 0 \\ 0 & 1.2489996 \end{pmatrix} \begin{pmatrix} 1 & 0.4 \\ 0.4 & 1 \end{pmatrix} \begin{pmatrix} 1.1401754 & 0 \\ 0 & 1.2489996 \end{pmatrix}$$

$$\boldsymbol{\Psi} = \begin{pmatrix} 1.3 & 0.5696315 \\ 0.5696315 & 1.56 \end{pmatrix}. \tag{9}$$

Data was generated using the `GenData` which is a wrapper function for the `SimSSMIVary` from the `simStateSpace` package.

References

Bringmann, L. F., Vissers, N., Wichers, M., Geschwind, N., Kuppens, P., Peeters, F., Borsboom, D., & Tuerlinckx, F. (2013). A network approach to psychopathology: New insights into clinical longitudinal data (G. A. de Erausquin, Ed.). *PLoS ONE*, *8*(4), e60188. https://doi.org/10.1371/journal.pone.0060188

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