Making Mediation Dynamic: Inference in Continuous-Time Mediation Models

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February 2, 2026
https://jeksterslab.github.io/presentation-2026-02-02-berd/

AMERICAN PSYCHOLOGICAL ASSOCIATION

Psychological Methods

ISSN: 1082-989X

https://doi.org/10.1037/met0000779

Inferences and Effect Sizes for Direct, Indirect, and Total Effects in Continuous-Time Mediation Models

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PSYCHOMETRIKA—VOL. 87, NO. 1, 214–252
MARCH 2022
https://doi.org/10.1007/s11336-021-09767-0

TIME TO INTERVENE: A CONTINUOUS-TIME APPROACH TO NETWORK ANALYSIS AND CENTRALITY

OISÍN RYAN AND ELLEN L. HAMAKER

UTRECHT UNIVERSITY

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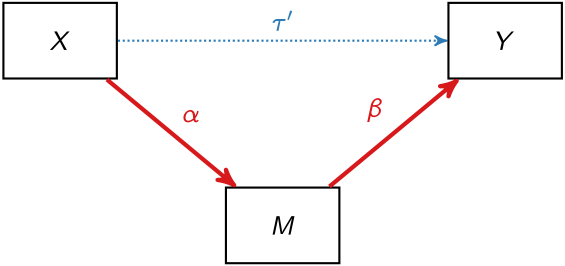
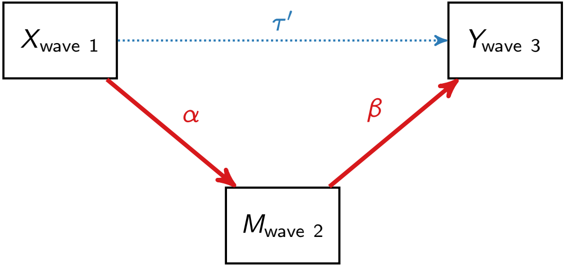
The Mediation Model

Cross-Sectional Mediation Model (Baron & Kenny, 1986)

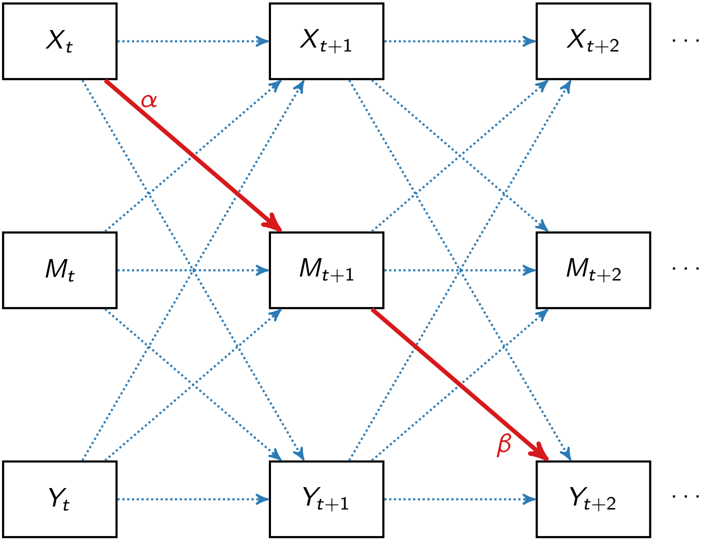


$$X = stress, M = coping, Y = mood$$

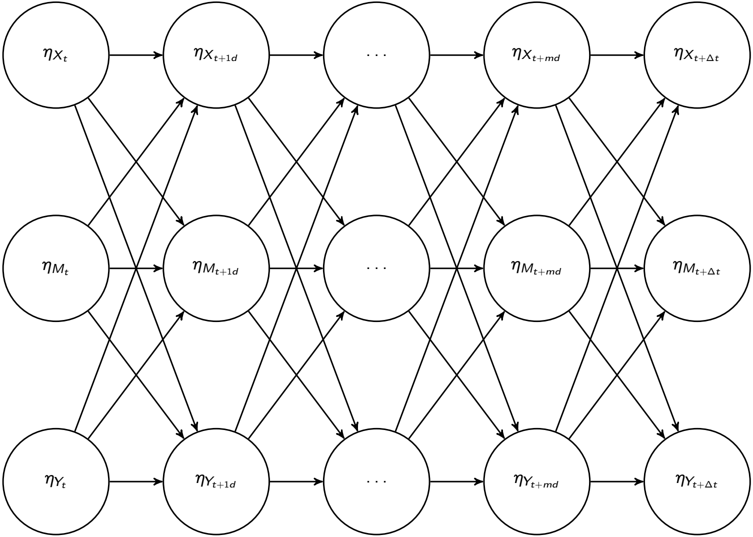
Three-Wave Longitudinal Mediation Model

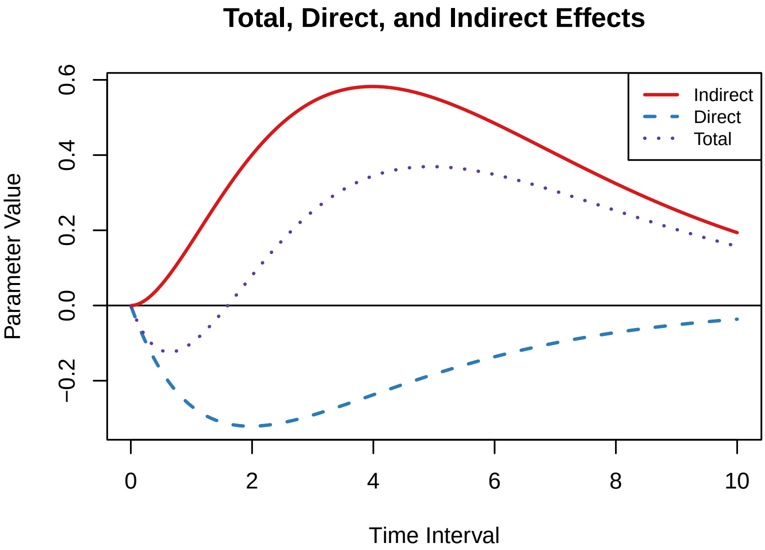


Discrete-Time Vector Autoregressive Model (DT-VAR; Cole & Maxwell, 2003; Wang & Zhang, 2020)



Continuous-Time Vector Autoregressive Model (CT-VAR; Deboeck & Preacher, 2015; Pesigan et al., 2025; Ryan & Hamaker, 2021)





Note: Generated by the `cTMed` package.

CT-VAR

CT-VAR Notation (Pesigan et al., 2025)

Measurement component:

$$\mathbf{y}_{i,t_m} = \boldsymbol{\nu} + \boldsymbol{\Lambda}\boldsymbol{\eta}_{i,t_m} + \boldsymbol{\varepsilon}_{i,t_m}, \boldsymbol{\varepsilon}_{i,t_m} \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Theta}\right) \tag{1}$$

Dynamic component:

$$\mathrm{d}\boldsymbol{\eta}_{i,t} = \boldsymbol{\Phi}\left(\boldsymbol{\eta}_{i,t} - \boldsymbol{\mu}\right)\mathrm{d}t + \boldsymbol{\Sigma}^{\frac{1}{2}}\mathrm{d}\mathbf{W}_{i,t} \tag{2}$$

Three-variable continuous-time mediation (CT-Med) model:

$$\mathbf{y} = \begin{pmatrix} X \\ M \\ Y \end{pmatrix}, \quad \text{and} \quad \boldsymbol{\eta} = \begin{pmatrix} \eta_X \\ \eta_M \\ \eta_Y \end{pmatrix} \tag{3}$$

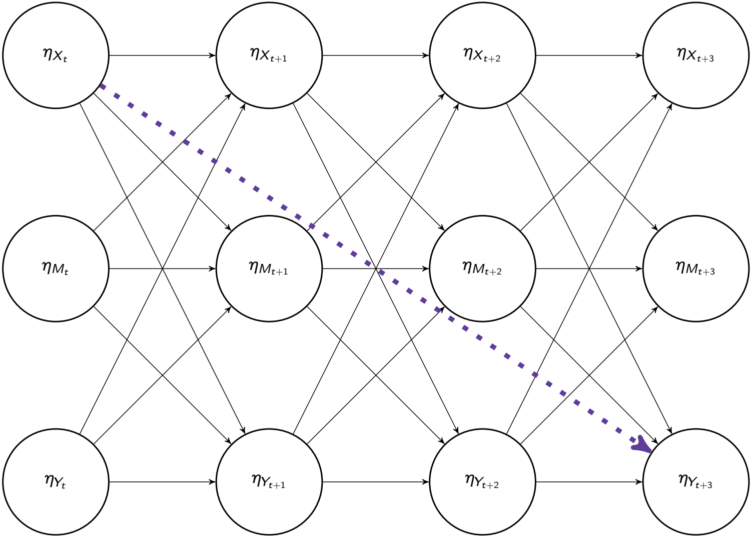
Integral form:

$$\boldsymbol{\eta}_{i,t_m} = \boldsymbol{\alpha}_{\Delta t_m} + \boldsymbol{\beta}_{\Delta t_m}\boldsymbol{\eta}_{i,t_{m-1}} + \boldsymbol{\zeta}_{i,t_m}, \boldsymbol{\zeta}_{i,t_m} \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Psi}_{\Delta t_m}\right) \tag{4}$$

$$\boldsymbol{\beta}_{\Delta t_m} = \exp\left(\Delta t \boldsymbol{\Phi}\right) \tag{5}$$

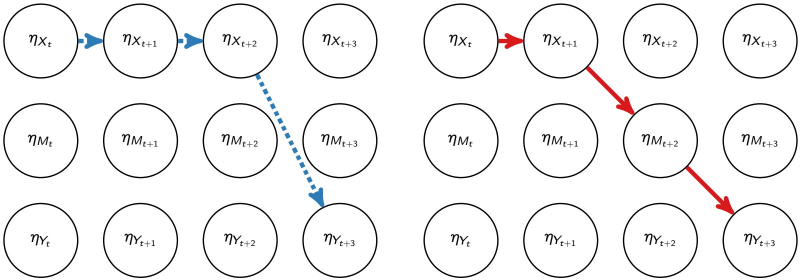
Total, Direct, and Indirect Effects in CT-Med

Total Effect Decomposition (Deboeck & Preacher, 2015; Ryan & Hamaker, 2021)



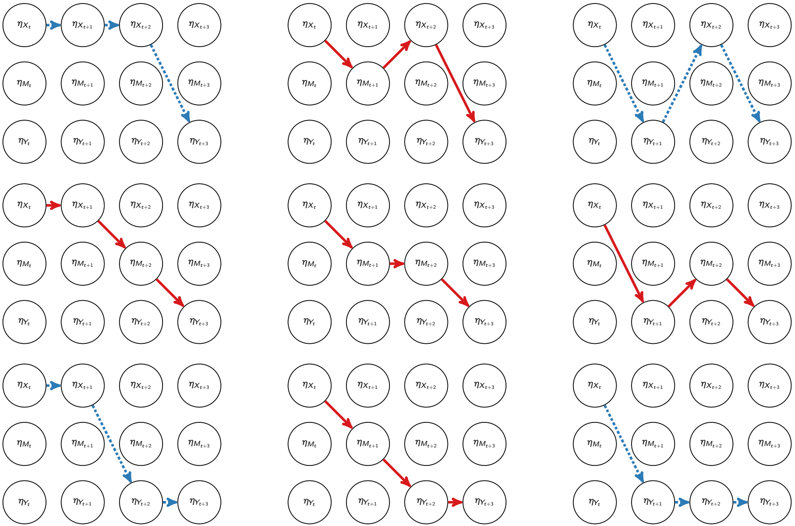
Total Effect Decomposition

Blue $=$ direct effect $(\beta_{11}\beta_{11}\beta_{31})$. Red $=$ indirect effect $(\beta_{11}\beta_{21}\beta_{32})$.



Total Effect Decomposition

Blue = direct effects. Red = indirect effects.



Total Effect Decomposition

Total, **Direct**, and **Indirect** Effects:

$$\text{Total} = \exp\left(\Delta t \boldsymbol{\Phi}\right) = \boldsymbol{\beta}_{\Delta t} \tag{6}$$

$$\text{Direct} = \exp\left(\Delta t \mathbf{D_m} \boldsymbol{\Phi} \mathbf{D_m}\right) \tag{7}$$

$$\text{Indirect} = \text{Total} - \text{Direct} \tag{8}$$

Total Effect Decomposition

Standardized **Total**, **Direct**, and **Indirect** Effects:

$$\text{Total}^{*}_{i,j} = \text{Total}_{i,j} \left(\frac{\sigma_{x_j}}{\sigma_{y_i}} \right), \tag{9}$$

$$\text{Direct}^{*}_{i,j} = \text{Direct}_{i,j} \left(\frac{\sigma_{x_j}}{\sigma_{y_i}} \right), \quad \text{and} \tag{10}$$

$$\text{Indirect}^{*}_{i,j} = \text{Total}^{*}_{i,j} - \text{Direct}^{*}_{i,j} \tag{11}$$

Solving for $\mathbf{X}$ in the following equation returns the steady-state model-implied covariance matrix.

$$\boldsymbol{\Phi}\mathbf{X} + \mathbf{X}\boldsymbol{\Phi}' + \boldsymbol{\Sigma} = \mathbf{0} \tag{12}$$

Total Effect and Indirect Effect Centrality (Ryan & Hamaker, 2021)

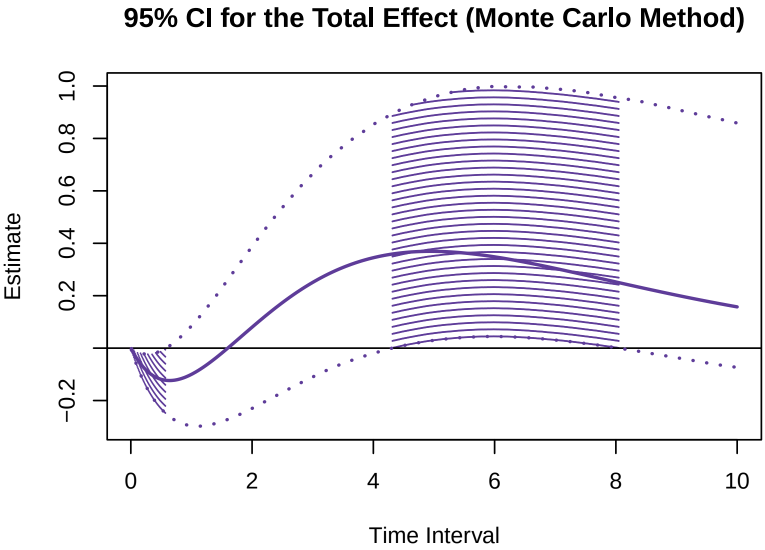
- **Total Effect Centrality:** The total effect centrality of a variable is the sum of the total effects of a variable on all other variables at a particular time interval.
- **Indirect Effect Centrality:** Indirect effect centrality is the sum of all possible indirect effects between different pairs of variables in which a specific variable serves as the only mediator.

Quantifying Uncertainty

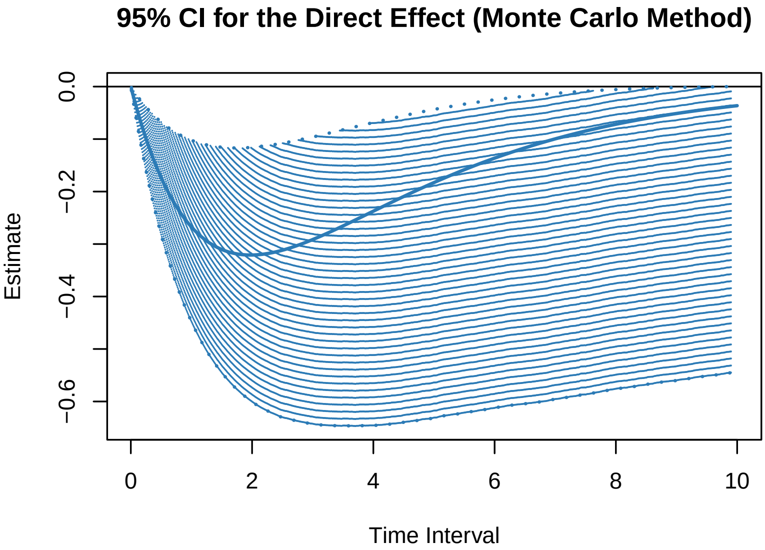
Quantifying Uncertainty: Standard Errors and Confidence Intervals

- **Delta Method** (Casella & Berger, 2002; Oehlert, 1992; Pesigan et al., 2023; Sobel, 1982; Ver Hoef, 2012; Yuan & Chan, 2011)
- **Monte Carlo Method** (MacKinnon et al., 2004; Pesigan & Cheung, 2024; Preacher & Selig, 2012)
- **Parametric bootstrap** (Efron, 1979; Zhang, 2018)

Regions of Significance



Note: Generated by the cTMed package.



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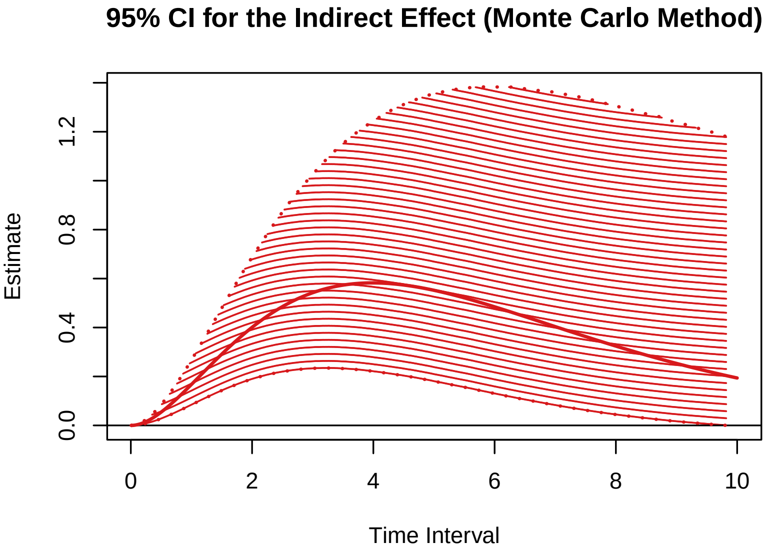

Note: Generated by the cTMed package.

Key Takeaways

Mediation Is a Temporal Process

- Mediation unfolds dynamically in time.
- Cross-sectional models give associations, not dynamics.
- DT-VAR approximates these dynamics at fixed intervals but depend on the chosen sampling rate.
- CT-Med generalizes this idea by recovering how effects emerge, propagate, and dissipate regardless of measurement spacing.

From snapshots to motion, from averages to mechanisms.

Total, Direct, and Indirect Effects Evolve

- CT-Med shows how total, direct, and indirect effects change with time, **when** mediation is strongest and **how long** effects last.
- **Standardized effect sizes** provide an interpretable metric analogous to standardized regression coefficients, enabling meaningful comparison of effects.

Continuous-time mediation adds rhythm and timing to the classic path model.

Importance of Inference and Regions of Significance

- **Inference** is central to understanding uncertainty in mediation effects, it tells us how confident we can be in our estimates of total, direct, and indirect effects.
- The **Regions of Significance (RoS)** tool in `cTMed` provides an intuitive way to visualize when effects are statistically significant across time intervals (Δt), offering insight into **how long** and **when** effects matter.

Inference and RoS help quantify and visualize uncertainty dynamically over time.

The cTMed Package

cTMed integrates effect sizes, inference methods, along with regions of significance, to provide a dynamic approach to continuous-time mediation analysis.

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