Reticular Action Model (RAM) Notation

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Contents

Chapter 1

Description

This is a collection of my personal notes on the Reticular Action Model (RAM) notation that accompanies the `ram` package. You can install the released version of `ram` from GitHub with:

```
remotes::install_github("jeksterslab/ram")
```

See GitHub Pages for the html deployment.

Chapter 2

Simple Regression

Let v_1, v_2, and u be random variables whose associations are given by the regression equation

$$\begin{aligned} v_1 &= m_1 + a_{1,2} v_2 + u \\ &= -3.951208 + 1.269259 \cdot v_2 + u \end{aligned} \tag{2.1}$$

where v_1 and v_2 are observed variables and u is a stochastic error term which is normally distributed around zero with constant variance across values of v_2

$$u \sim \mathcal{N}\left(m_3 = 0, \omega_{3,3} = 47.659854\right). \tag{2.2}$$

v_2 has a mean of $m_2 = 13.038328$ and a variance of $\omega_{2,2} = 7.151261$.

Below are two ways of specifying this model. The first specification includes the error term u as a latent variable. The second specification only includes the observed variables.

2.1 Specification 1 - Includes Error Term as a Latent Variable

2.1.1 Matrix Notation

$$\text{variables} = \begin{bmatrix} v_1 \\ v_2 \\ u \end{bmatrix} \tag{2.3}$$

$$\mathbf{A} = \begin{bmatrix} 0 & a_{1,2} & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1.269259 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{2.4}$$

$$\Omega = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \omega_{2,2} & 0 \\ 0 & 0 & \omega_{3,3} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 7.151261 & 0 \\ 0 & 0 & 47.659854 \end{bmatrix} \tag{2.5}$$

$$\mathbf{m} = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} -3.951208 \\ 13.038328 \\ 0 \end{bmatrix} \tag{2.6}$$

To filter the observed variables, use the following filter matrix

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}. \tag{2.7}$$

To include all variables, use the following filter matrix

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{2.8}$$

2.1.2 Path Diagram

2.1.3 Expectations

$$\mathbb{E}(u) = m_3 = 0 \tag{2.9}$$

$$\mathbb{E}(v_2) = m_2 = 13.038328 \tag{2.10}$$

$$\begin{aligned} \mathbb{E}(v_1) &= \mathbb{E}\left(m_1 + a_{1,2}v_2 + \varepsilon\right) \\ &= \mathbb{E}(m_1) + \mathbb{E}\left(a_{1,2}v_2\right) + \mathbb{E}(\varepsilon) \\ &= m_1 + a_{1,2}\mathbb{E}(v_2) + \mathbb{E}(\varepsilon) \\ &= m_1 + a_{1,2}m_2 + 0 \\ &= m_1 + a_{1,2}m_2 \\ &= -3.951208 + 1.269259 \times 13.038328 \\ &= 12.5978072 \end{aligned} \tag{2.11}$$

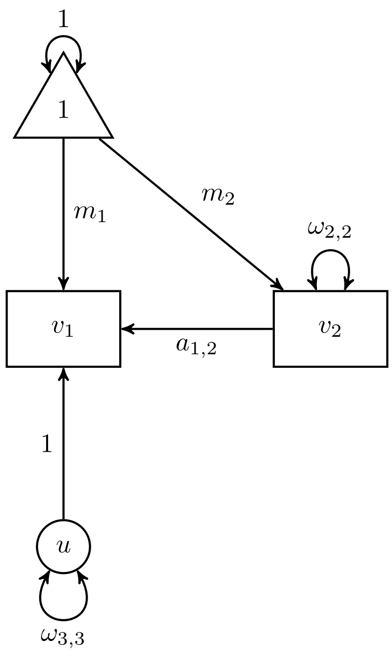

Figure 2.1: The Simple Linear Regression Model (with u)

$$\begin{aligned} \mathbb{E}\left(\begin{bmatrix} v_1 \\ v_2 \\ u \end{bmatrix}\right) &= \begin{bmatrix} m_1 + a_{1,2}m_2 \\ m_2 \\ m_3 \end{bmatrix} \\ &= \begin{bmatrix} 12.5978072 \\ 13.038328 \\ 0 \end{bmatrix} \end{aligned} \tag{2.12}$$

$$\begin{aligned} \mathrm{Cov}\left(u, u\right) &= \mathrm{Var}\left(u\right) \\ &= \omega_{3,3} \\ &= 47.659854 \end{aligned} \tag{2.13}$$

$$
\begin{aligned}
\operatorname{Cov}(v_1, u) &= \operatorname{Cov}\left(a_{1,2}v_2 + u, u\right) \\
&= \operatorname{Cov}\left(a_{1,2}v_2, u\right) + \operatorname{Cov}(u, u) \\
&= a_{1,2}^2 \operatorname{Cov}(v_2, u) + \operatorname{Var}(u) \\
&= a_{1,2}^2 \cdot 0 + \omega_{3,3} \\
&= 0 + \omega_{3,3} \\
&= \omega_{3,3} \\
&= 47.659854
\end{aligned} \tag{2.14}
$$

$$
\operatorname{Cov}(v_2, u) = 0 \tag{2.15}
$$

$$
\begin{aligned}
\operatorname{Cov}(v_1, v_1) &= \operatorname{Cov}\left(a_{1,2}v_2 + u, a_{1,2}v_2 + u\right) \\
&= \operatorname{Cov}\left(a_{1,2}v_2, a_{1,2}v_2\right) + \operatorname{Cov}\left(a_{1,2}v_2, u\right) + \operatorname{Cov}\left(a_{1,2}v_2, u\right) + \operatorname{Cov}(u, u) \\
&= a_{1,2}^2 \operatorname{Cov}(v_2) + a_{1,2} \operatorname{Cov}(v_2, u) + a_{1,2} \operatorname{Cov}(v_2, u) + \operatorname{Var}(u, u) \\
&= a_{1,2}^2 \operatorname{Var}(v_2) + a_{1,2} 0 + a_{1,2} 0 + \omega_{3,3} \\
&= a_{1,2}^2 \omega_{2,2} + \omega_{3,3} \\
&= 1.269259^2 \times 7.151261 + 47.659854 \\
&= 59.1806671
\end{aligned} \tag{2.16}
$$

$$
\begin{aligned}
\operatorname{Cov}(v_2, v_1) &= \operatorname{Cov}\left(v_2, a_{1,2}v_2 + u\right) \\
&= \operatorname{Cov}\left(v_2, a_{1,2}v_2\right) + \operatorname{Cov}(v_2, u) \\
&= a_{1,2} \operatorname{Cov}(v_2, v_2) + 0 \\
&= a_{1,2} \operatorname{Var}(v_2) \\
&= a_{1,2} \omega_{2,2} \\
&= 1.269259 \times 7.151261 \\
&= 9.0768024
\end{aligned} \tag{2.17}
$$

$$
\begin{aligned}
\operatorname{Cov}(v_2, v_2) &= \operatorname{Var}(v_2) \\
&= \omega_{2,2} \\
&= 7.151261
\end{aligned} \tag{2.18}
$$

$$
\begin{aligned}
\mathrm{Cov}\left(\begin{bmatrix} v_1 \\ v_2 \\ u \end{bmatrix}\right) &= \begin{bmatrix} a_{1,2}^2\omega_{2,2} + \omega_{3,3} & a_{1,2}\omega_{2,2} & \omega_{3,3} \\ a_{1,2}\omega_{2,2} & \omega_{2,2} & 0 \\ \omega_{3,3} & 0 & \omega_{3,3} \end{bmatrix} \\
&= \begin{bmatrix} 59.1806671 & 9.0768024 & 47.659854 \\ 9.0768024 & 7.151261 & 0 \\ 47.659854 & 0 & 47.659854 \end{bmatrix}
\end{aligned} \tag{2.19}
$$

2.1.3.1 Using the `ram()` Package

```
knitr::kable(
  ram::mutheta(
    m,
    A = A,
    filter = filter
  ),
  col.names = "$\\boldsymbol{\\mu}$",
  escape = FALSE
)
```

	$\boldsymbol{\mu}$
v_1	12.59781
v_2	13.03833
u	0.00000

```
knitr::kable(
  ram::Sigmatheta(
    A = A,
    Omega = Omega,
    filter = filter
  ),
  escape = FALSE
)
```

	v_1	v_2	u
v_1	59.180667	9.076802	47.65985
v_2	9.076802	7.151261	0.00000
u	47.659854	0.000000	47.65985

2.2 Specification 2 - Observed Variables

2.2.1 Matrix Notation

$$\text{variables} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{2.20}$$

$$\mathbf{A} = \begin{bmatrix} 0 & a_{1,2} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1.269259 \\ 0 & 0 \end{bmatrix} \tag{2.21}$$

$$\Omega = \begin{bmatrix} \omega_{1,1} & 0 \\ 0 & \omega_{2,2} \end{bmatrix} = \begin{bmatrix} 47.659854 & 0 \\ 0 & 7.151261 \end{bmatrix} \tag{2.22}$$

$$\mathbf{m} = \begin{bmatrix} m_1 \\ m_2 \end{bmatrix} = \begin{bmatrix} -3.951208 \\ 13.038328 \end{bmatrix} \tag{2.23}$$

$$\mathbf{F} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{2.24}$$

2.2.2 Path Diagram

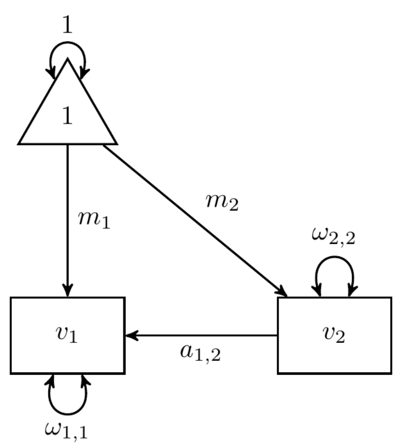

Figure 2.2: The Simple Linear Regression Model (without u)

2.2.2.1 Using the `ram()` Package

```r
knitr::kable(
  ram::mutheta(
    m,
    A = A,
    filter = filter
  ),
  col.names = "$\\boldsymbol{\\mu}$",
  escape = FALSE
)
```

	$\boldsymbol{\mu}$
v_1	12.59781
v_2	13.03833

```r
knitr::kable(
  ram::Sigmatheta(
    A = A,
    Omega = Omega,
    filter = filter
  ),
  escape = FALSE
)
```

	v_1	v_2
v_1	59.180667	9.076802
v_2	9.076802	7.151261