Reticular Action Model (RAM) Notation

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Contents

Chapter 1

Description

This is a collection of my personal notes on the Reticular Action Model (RAM) notation that accompanies the `ram` package. You can install the released version of `ram` from GitHub with:

```
remotes::install_github("jeksterslab/ram")
```

These notes are based on the following resources:

- Boker and McArdle (2005)
- McArdle and McDonald (1984)
- McArdle (2005)

See GitHub Pages for the html deployment.

Chapter 2

Reticular Action Model (RAM) Matrix Notation

$$\mathbf{v} = \mathbf{A}\mathbf{v} + \mathbf{u} \tag{2.1}$$

where

- $\mathbf{v}$ is a $t \times 1$ vector of random variables
- u_i represent the residual of v_i
- $\mathbf{A}$ is a $t \times t$ matrix of *directed* or *asymmetric* relationship from column variable v_j to row variable v_i
  - regression of each of the t variables on the other $t - 1$ variables
  - diagonal $a_{i,i}$ is zero
  - if all regression coefficients on other variables are zero, then the variable v_i is considered the same as its own residual u_i

$$\Omega = \mathbb{E}\left(\mathbf{u}\mathbf{u}'\right) \tag{2.2}$$

where

- Ω is a $t \times t$ matrix of *undirected* or *symmetric* relationship

$$\Sigma\left(\theta\right) = \left(\mathbf{I} - \mathbf{A}\right)^{-1} \Omega \left[\left(\mathbf{I} - \mathbf{A}\right)^{-1}\right]^{\mathsf{T}} \tag{2.3}$$

- $\Sigma\left(\theta\right)$ is a $t \times t$ symmetric matrix of associations between v_i and v_j

$$\mathbf{v}^{\intercal} = [\mathbf{m}, \mathbf{l}]^{\intercal} \tag{2.4}$$

where

- $\mathbf{m}$ are observed or manifest variables of j components
- $\mathbf{l}$ are observed or manifest variables of k components
- $t = j + k$

$$\mathbf{F} = \left[\mathbf{I}_j : \mathbf{O}_{j \times k}\right] \tag{2.5}$$

- the $\mathbf{F}$ matrix acts as a *filter* to select the manifest variables out of the full set of manifest and latent variables

2.1 Model-Implied Matrices

The model-implied mean vector $\mu(\theta)$ as a function of Reticular Action Model (RAM) matrices is given by

$$\mu(\theta) = \mathbf{F}\left(\mathbf{I} - \mathbf{A}\right)^{-1} \mathbf{m}. \tag{2.6}$$

The `ram::mutheta()` function can be used to derive the model-implied mean vector.

The model-implied variance-covariance matrix $\Sigma(\theta)$ as a function of Reticular Action Model (RAM) matrices is given by

$$\Sigma(\theta) = \mathbf{F}\left(\mathbf{I} - \mathbf{A}\right)^{-1} \Omega \left[\left(\mathbf{I} - \mathbf{A}\right)^{-1}\right]^{\intercal} \mathbf{F}^{\intercal}. \tag{2.7}$$

The `ram::Sigmatheta()` function can be used to derive the model-implied variance-covariance matrix.

2.2 Parameters

2.2.1 Mean Structure

$$\mathbf{m} = \left[\mathbf{F}\left(\mathbf{I} - \mathbf{A}\right)^{-1}\right]^{-1} \mu(\theta) \tag{2.8}$$

The `ram::m()` function can be used to derive the mean structure vector.

2.2.2 Covariance Structure

$$\Omega = \left(\mathbf{I} - \mathbf{A} \right) \Sigma \left(\theta \right) \left(\mathbf{I} - \mathbf{A} \right)^{\mathsf{T}} \tag{2.9}$$

The `ram::Omega()` function can be used to derive the *symmetric* matrix Ω.

TODO: Figure out how to isolate the A matrix

Chapter 3

Simple Regression

Let v_1, v_2, and v_3 be random variables whose associations are given by the regression equation

$$\begin{aligned} v_1 &= m_1 + a_{1,2} v_2 + v_3 \\ &= -3.951208 + 1.269259 \cdot v_2 + v_3. \end{aligned} \tag{3.1}$$

v_1 and v_2 are observed variables and v_3 is a stochastic error term which is normally distributed around zero with constant variance across values of v_2

$$v_3 \sim \mathcal{N}\left(m_3 = 0, \omega_{3,3} = 47.659854\right). \tag{3.2}$$

v_2 has a mean of $m_2 = 13.038328$ and a variance of $\omega_{2,2} = 7.151261$.

3.0.1 Expectations

$$\begin{aligned} \mathbb{E}\left(v_3\right) &= m_3 \\ &= 0 \end{aligned} \tag{3.3}$$

$$\begin{aligned} \mathbb{E}\left(v_2\right) &= m_2 \\ &= 13.038328 \end{aligned} \tag{3.4}$$

$$\begin{aligned}\mathbb{E}\left(v_1\right) &= \mathbb{E}\left(m_1 + a_{1,2}v_2 + v_3\right) \\ &= \mathbb{E}\left(m_1\right) + \mathbb{E}\left(a_{1,2}v_2\right) + \mathbb{E}\left(v_3\right) \\ &= m_1 + a_{1,2}\mathbb{E}\left(v_2\right) + 0 \\ &= m_1 + a_{1,2}m_2 \\ &= -3.951208 + 1.269259 \times 13.038328 \\ &= 12.5978072\end{aligned} \tag{3.5}$$

$$\begin{aligned}\mathbb{E}\left(\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}\right) &= \begin{bmatrix} m_1 + a_{1,2}m_2 \\ m_2 \\ m_3 \end{bmatrix} \\ &= \begin{bmatrix} 12.5978072 \\ 13.038328 \\ 0 \end{bmatrix}\end{aligned} \tag{3.6}$$

$$\begin{aligned}\mathrm{Cov}\left(v_3, v_3\right) &= \mathrm{Var}\left(v_3\right) \\ &= \omega_{3,3} \\ &= 47.659854\end{aligned} \tag{3.7}$$

$$\begin{aligned}\mathrm{Cov}\left(v_1, v_3\right) &= \mathrm{Cov}\left(a_{1,2}v_2 + v_3, v_3\right) \\ &= \mathrm{Cov}\left(a_{1,2}v_2, v_3\right) + \mathrm{Cov}\left(v_3, v_3\right) \\ &= a_{1,2}^2\mathrm{Cov}\left(v_2, v_3\right) + \mathrm{Var}\left(v_3\right) \\ &= a_{1,2}^2 \cdot 0 + \omega_{3,3} \\ &= 0 + \omega_{3,3} \\ &= \omega_{3,3} \\ &= 47.659854\end{aligned} \tag{3.8}$$

$$\mathrm{Cov}\left(v_2, v_3\right) = 0 \tag{3.9}$$

$$\begin{aligned}
\mathrm{Cov}\left(v_1, v_1\right) &= \mathrm{Cov}\left(a_{1,2}v_2 + v_3, a_{1,2}v_2 + v_3\right) \\
&= \mathrm{Cov}\left(a_{1,2}v_2, a_{1,2}v_2\right) + \mathrm{Cov}\left(a_{1,2}v_2, v_3\right) + \mathrm{Cov}\left(a_{1,2}v_2, v_3\right) + \mathrm{Cov}\left(v_3, v_3\right) \\
&= a_{1,2}^2 \mathrm{Cov}\left(v_2, v_2\right) + a_{1,2}\mathrm{Cov}\left(v_2, v_3\right) + a_{1,2}\mathrm{Cov}\left(v_2, v_3\right) + \mathrm{Var}\left(v_3\right) \\
&= a_{1,2}^2 \mathrm{Var}\left(v_2\right) + a_{1,2} \cdot 0 + a_{1,2} \cdot 0 + \omega_{3,3} \\
&= a_{1,2}^2 \mathrm{Var}\left(v_2\right) + 0 + 0 + \omega_{3,3} \\
&= a_{1,2}^2 \omega_{2,2} + \omega_{3,3} \\
&= 1.269259^2 \times 7.151261 + 47.659854 \\
&= 59.1806671
\end{aligned} \tag{3.10}$$

$$\begin{aligned}
\mathrm{Cov}\left(v_2, v_1\right) &= \mathrm{Cov}\left(v_2, a_{1,2}v_2 + v_3\right) \\
&= \mathrm{Cov}\left(v_2, a_{1,2}v_2\right) + \mathrm{Cov}\left(v_2, v_3\right) \\
&= a_{1,2}\mathrm{Cov}\left(v_2, v_2\right) + 0 \\
&= a_{1,2}\mathrm{Var}\left(v_2\right) \\
&= a_{1,2}\omega_{2,2} \\
&= 1.269259 \times 7.151261 \\
&= 9.0768024
\end{aligned} \tag{3.11}$$

$$\begin{aligned}
\mathrm{Cov}\left(v_2, v_2\right) &= \mathrm{Var}\left(v_2\right) \\
&= \omega_{2,2} \\
&= 7.151261
\end{aligned} \tag{3.12}$$

$$\begin{aligned}
\mathrm{Cov}\left(\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}\right) &= \begin{bmatrix} a_{1,2}^2\omega_{2,2} + \omega_{3,3} & a_{1,2}\omega_{2,2} & \omega_{3,3} \\ a_{1,2}\omega_{2,2} & \omega_{2,2} & 0 \\ \omega_{3,3} & 0 & \omega_{3,3} \end{bmatrix} \\
&= \begin{bmatrix} 59.1806671 & 9.0768024 & 47.659854 \\ 9.0768024 & 7.151261 & 0 \\ 47.659854 & 0 & 47.659854 \end{bmatrix}
\end{aligned} \tag{3.13}$$

Below are two ways of specifying this model. The first specification includes the error term v_3 as a latent variable. The second specification only includes the observed variables.

3.1 Specification 1 - Includes Error Term as a Latent Variable

3.1.1 Matrix Notation

$$\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \tag{3.14}$$

$$\begin{aligned}\mathbf{A} &= \begin{bmatrix} 0 & a_{1,2} & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 1.269259 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\end{aligned} \tag{3.15}$$

$$\begin{aligned}\Omega &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & \omega_{2,2} & 0 \\ 0 & 0 & \omega_{3,3} \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 7.151261 & 0 \\ 0 & 0 & 47.659854 \end{bmatrix}\end{aligned} \tag{3.16}$$

$$\begin{aligned}\mathbf{m} &= \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} \\ &= \begin{bmatrix} -3.951208 \\ 13.038328 \\ 0 \end{bmatrix}\end{aligned} \tag{3.17}$$

To filter the observed variables, use the following filter matrix

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}. \tag{3.18}$$

To include all variables, use the following filter matrix

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{3.19}$$

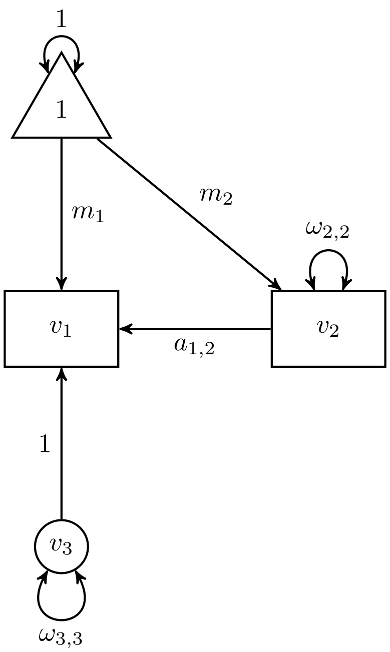

Figure 3.1: The Simple Linear Regression Model (with v_3)

Table 3.1: $\mu(\theta)$

	μ
v_1	12.59781
v_2	13.03833
v_3	0.00000

3.1.1.1 Using the `ram()` Package

```
knitr::kable(
  ram::mutheta(
    m,
    A = A,
    filter = filter
  ),
  col.names = "$\\boldsymbol{\\mu}$",
  caption = "$\\boldsymbol{\\mu} \\left( \\boldsymbol{\\theta} \\right)$",
  escape = FALSE
)
```

Table 3.2: $\Sigma(\theta)$

	v_1	v_2	v_3
v_1	59.180667	9.076802	47.65985
v_2	9.076802	7.151261	0.00000
v_3	47.659854	0.000000	47.65985

```
knitr::kable(
  ram::Sigmatheta(
    A = A,
    Omega = Omega,
    filter = filter
  ),
  caption = "$\\boldsymbol{\\Sigma} \\left( \\boldsymbol{\\theta} \\right)$",
  escape = FALSE
)
```

3.2 Specification 2 - Observed Variables

3.2.1 Matrix Notation

$$\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{3.20}$$

$$\begin{aligned} \mathbf{A} &= \begin{bmatrix} 0 & a_{1,2} \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 1.269259 \\ 0 & 0 \end{bmatrix} \end{aligned} \tag{3.21}$$

$$\begin{aligned} \boldsymbol{\Omega} &= \begin{bmatrix} \omega_{1,1} & 0 \\ 0 & \omega_{2,2} \end{bmatrix} \\ &= \begin{bmatrix} 47.659854 & 0 \\ 0 & 7.151261 \end{bmatrix} \end{aligned} \tag{3.22}$$

$$\begin{aligned} \mathbf{m} &= \begin{bmatrix} m_1 \\ m_2 \end{bmatrix} \\ &= \begin{bmatrix} -3.951208 \\ 13.038328 \end{bmatrix} \end{aligned} \tag{3.23}$$

Table 3.3: $\mu\left(\theta\right)$

	μ
v_1	12.59781
v_2	13.03833

$$\mathbf{F} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{3.24}$$

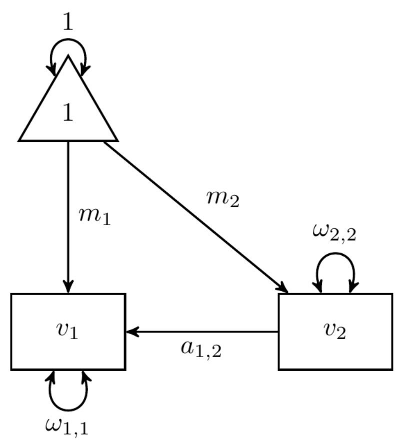

Figure 3.2: The Simple Linear Regression Model (without v_3)

3.2.1.1 Using the `ram()` Package

```
knitr::kable(
  ram::mutheta(
    m,
    A = A,
    filter = filter
  ),
  col.names = "$\\boldsymbol{\\mu}$",
  caption = "$\\boldsymbol{\\mu} \\left( \\boldsymbol{\\theta} \\right)$",
  escape = FALSE
)
```

Table 3.4: $\boldsymbol{\Sigma}\left(\boldsymbol{\theta}\right)$

	v_1	v_2
v_1	59.180667	9.076802
v_2	9.076802	7.151261

```r
knitr::kable(
  ram::Sigmatheta(
    A = A,
    Omega = Omega,
    filter = filter
  ),
  caption = "$\\boldsymbol{\\Sigma} \\left( \\boldsymbol{\\theta} \\right)$",
  escape = FALSE
)
```

Bibliography

Boker, S. M. and McArdle, J. J. (2005). Path analysis and path diagrams. In Everitt, B. S. and Howell, D. C., editors, *Encyclopedia of Statistics in Behavioral Science*, pages 1529–1531. John Wiley & Sons, Ltd, Chichester, UK.

McArdle, J. J. (2005). The development of the RAM rules for latent variable structural equation modeling. In Maydeu-Olivares, A. and McArdle, J. J., editors, *Contemporary psychometrics: A festschrift for Roderick P. McDonald*, Multivariate applications book series, pages 225–273. Lawrence Erlbaum Associates, Mahwah, NJ.

McArdle, J. J. and McDonald, R. P. (1984). Some algebraic properties of the reticular action model for moment structures. *British Journal of Mathematical and Statistical Psychology*, 37(2):234–251.