

Reticular Action Model (RAM) Notation Notes

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Contents

1	Description	5
2	Reticular Action Model (RAM) Matrix Notation	7
2.1	Full Model	7
2.2	Observed/Manifest/Given Variables vs. Unobserved/Latent/Hidden Variables	8
3	Reticular Action Model (RAM) Path Diagram	11
4	Student's t-test	17
4.1	Symbolic	17
4.2	Numerical Example	21
4.3	Equations to RAM	28
4.4	Equations to Expectations	30

Chapter 1

Description

This is a collection of my personal notes on the Reticular Action Model (RAM) notation that accompanies the **ramR** package (Pesigan, 2021) in the **R** statistical environment (R Core Team, 2020). You can install the released version of **ramR** from GitHub with:

```
remotes::install_github("jeksterslab/ramR")
```

These notes are based on the following resources:

- Boker and McArdle (2005)
- McArdle and McDonald (1984)
- McArdle (2005)

See GitHub Pages for the html deployment.

Chapter 2

Reticular Action Model (RAM) Matrix Notation

2.1 Full Model

Definition 2.1.

$$\mathbf{v} = \mathbf{A}\mathbf{v} + \mathbf{u} \quad (2.1)$$

where

- \mathbf{v} and \mathbf{u} are $t \times 1$ vectors of random variables
- \mathbf{A} is a $t \times t$ matrix of *directed* or *asymmetric* relationship from column variable v_j to row variable v_i
 - \mathbf{A} represent the regression of each of the t variables \mathbf{v} on the other $t - 1$ variables
 - diagonal $a_{i,i}$ is zero
 - u_i represent the residual of v_i
 - if all regression coefficients on other variables are zero, then the variable v_i is considered the same as its own residual u_i

Definition 2.2.

$$\mathbf{S} = \mathbb{E} \{ \mathbf{u}\mathbf{u}' \}, \quad (2.2)$$

where

- \mathbf{S} is a $t \times t$ matrix of *undirected* or *symmetric* relationship
 - the notation Ω is used in other sources for \mathbf{S}
- \mathbb{E} is the expectation operator

Definition 2.3.

$$\mathbf{C} = \mathbb{E} \{ \mathbf{v}\mathbf{v}' \}, \quad (2.3)$$

where

- \mathbf{C} is a $t \times t$ variance-covariance matrix
 - the notation Σ is used in other sources for \mathbf{C}

Definition 2.4.

$$\mathbf{v} = \mathbf{A}\mathbf{v} + \mathbf{u}$$

can be rewritten as

$$\begin{aligned}\mathbf{v} - \mathbf{A}\mathbf{v} &= \mathbf{u} \\ \mathbf{u} &= \mathbf{v} - \mathbf{A}\mathbf{v} \\ \mathbf{u} &= (\mathbf{I} - \mathbf{A})\mathbf{v}\end{aligned}\tag{2.4}$$

assuming that $(\mathbf{I} - \mathbf{A})$ is non-singular,

$$\mathbf{E} = (\mathbf{I} - \mathbf{A})^{-1}\tag{2.5}$$

then

$$\begin{aligned}\mathbf{v} &= (\mathbf{I} - \mathbf{A})^{-1}\mathbf{u} \\ &= \mathbf{E}\mathbf{u}.\end{aligned}\tag{2.6}$$

Using the definitions above, \mathbf{S} and \mathbf{C} are given by

$$\begin{aligned}\mathbf{S} &= (\mathbf{I} - \mathbf{A})\mathbf{C}(\mathbf{I} - \mathbf{A})^{-1} \\ &= \mathbf{E}^{-1}\mathbf{C}(\mathbf{E}^{-1})^{\top}\end{aligned}\tag{2.7}$$

$$\begin{aligned}\mathbf{C} &= (\mathbf{I} - \mathbf{A})^{-1}\mathbf{S}[(\mathbf{I} - \mathbf{A})^{-1}]^{\top} \\ &= \mathbf{E}\mathbf{S}\mathbf{E}^{\top}\end{aligned}\tag{2.8}$$

2.2 Observed/Manifest/Given Variables vs. Unobserved/Latent/Hidden Variables

Definition 2.5.

$$\mathbf{v} = \begin{bmatrix} \mathbf{g}_{p \times 1} \\ \mathbf{h}_{q \times 1} \end{bmatrix}\tag{2.9}$$

$$t = p + q\tag{2.10}$$

- \mathbf{g} may be considered observed, manifest or *given* variables
- \mathbf{h} may be considered unobserved, latent, or *hidden* variables

Definition 2.6.

$$\mathbf{F} = [\mathbf{I}_{p \times p} : \mathbf{0}_{p \times q}]\tag{2.11}$$

- the \mathbf{F} matrix acts as a *filter* to select the manifest variables out of the full set of manifest and latent variables

$$\mathbf{g} = \mathbf{F}\mathbf{v} \quad (2.12)$$

$$\begin{aligned} \mathbf{g} &= \mathbf{F}(\mathbf{I} - \mathbf{A})^{-1} \mathbf{u} \\ &= \mathbf{F}\mathbf{E}\mathbf{u} \end{aligned} \quad (2.13)$$

Definition 2.7.

$$\mathbf{M} = \mathbb{E} \{ \mathbf{g}\mathbf{g}^T \} \quad (2.14)$$

$$\begin{aligned} \mathbf{M} &= \mathbf{F}(\mathbf{I} - \mathbf{A})^{-1} \mathbf{S} [(\mathbf{I} - \mathbf{A})^{-1}]^T \mathbf{F}^T \\ &= \mathbf{F}\mathbf{E}\mathbf{S}\mathbf{E}^T \mathbf{F}^T \\ &= \mathbf{F}\mathbf{C}\mathbf{F}^T \end{aligned} \quad (2.15)$$

- when components of \mathbf{v} are permuted, the columns of \mathbf{F} can be correspondingly permuted
- the rows and columns of \mathbf{C} that are filtered out by \mathbf{F} contain useful information about the latent variable structure.

The equations above completely define RAM.

Chapter 3

Reticular Action Model (RAM) Path Diagram

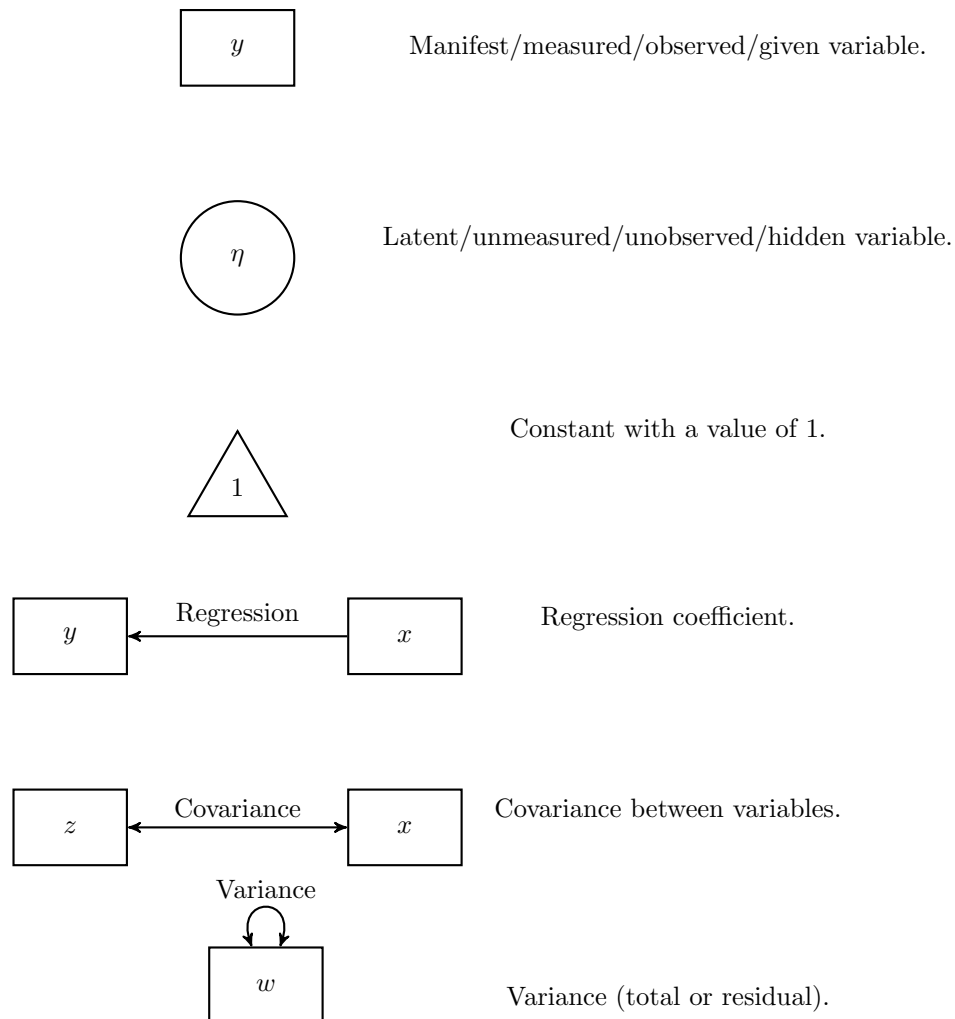
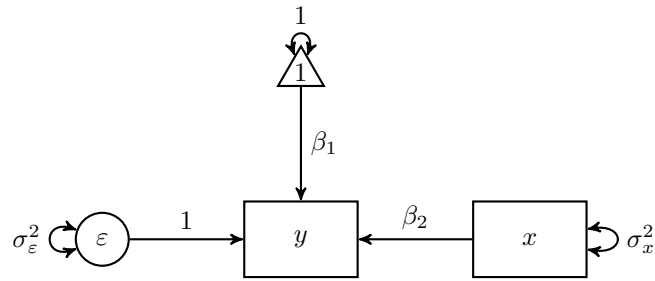
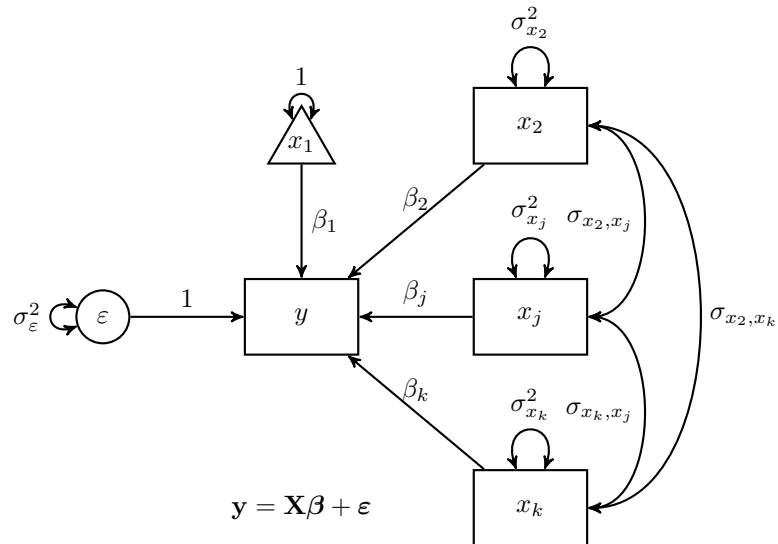


Figure 3.1: Path Diagram Elements



$$y = \alpha + \beta x + \varepsilon$$

Figure 3.2: Two-Variable Regression Model

Figure 3.3: k -Variable Regression Model

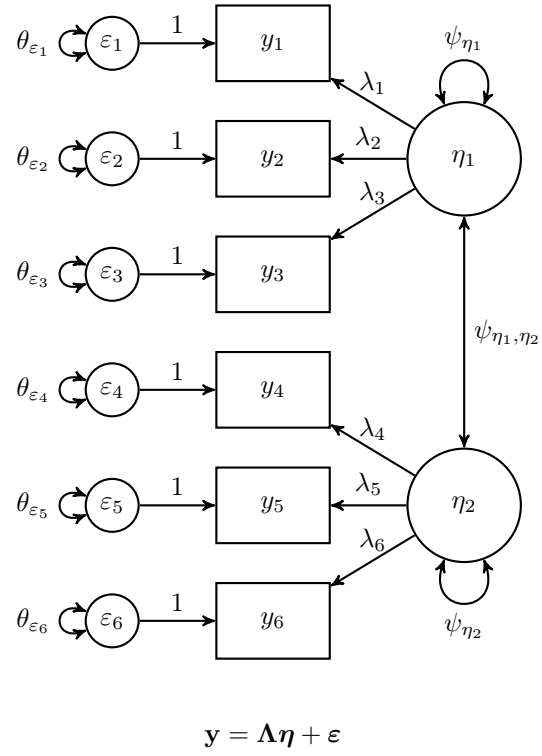


Figure 3.4: Two-Factor Confirmatory Factor Analysis Model

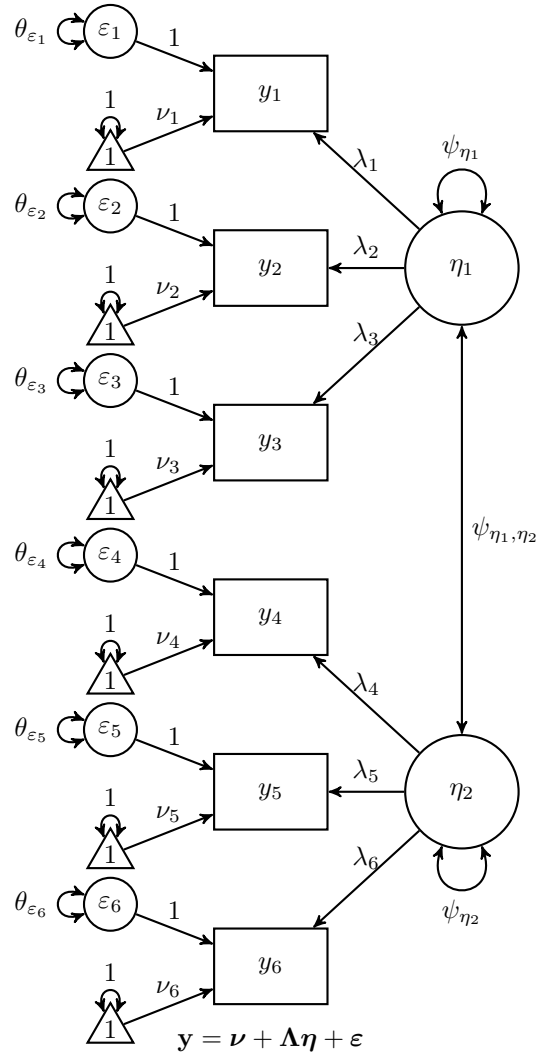
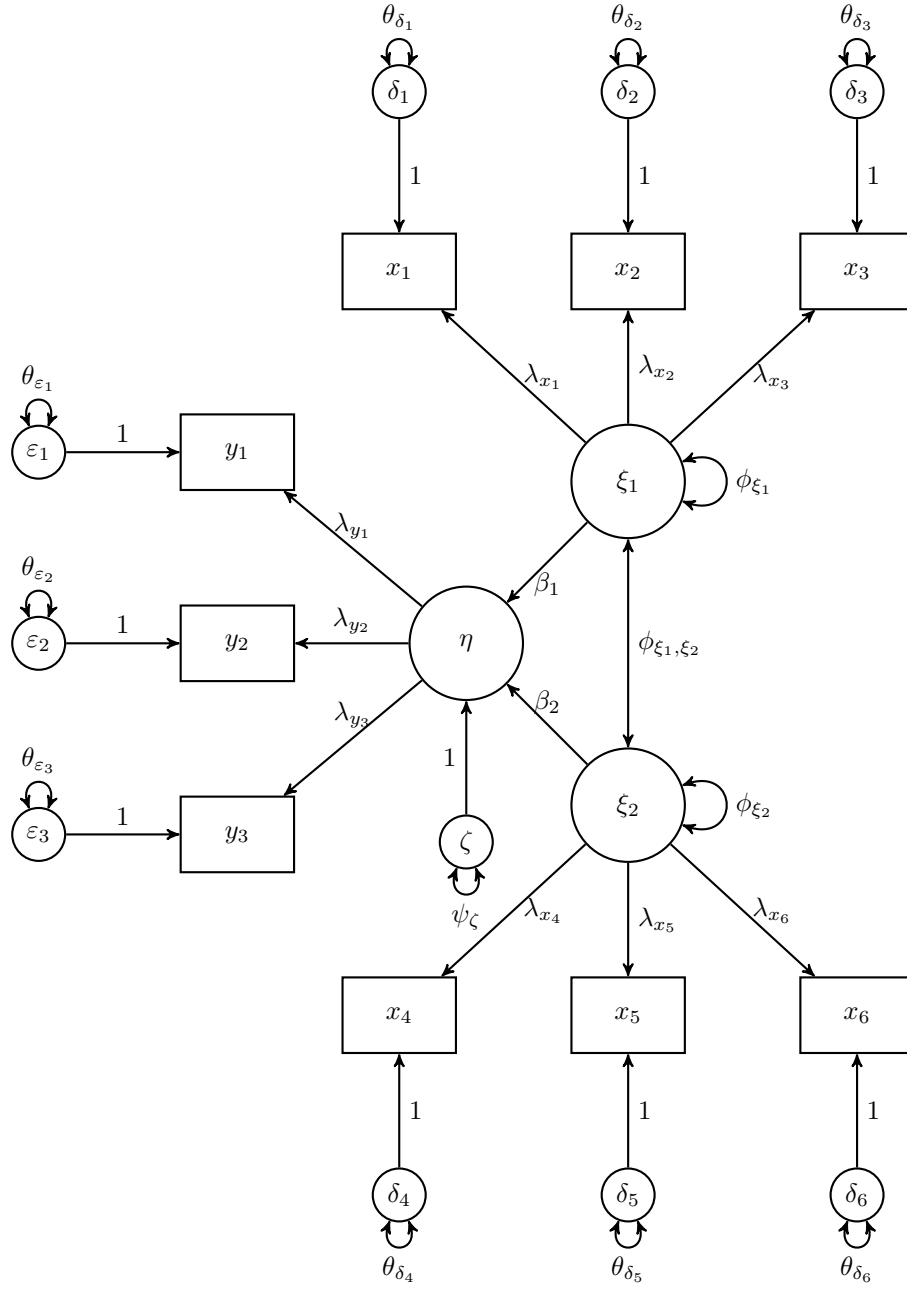


Figure 3.5: Two-Factor Confirmatory Factor Analysis Model with Mean Structure



$$\eta = \mathbf{B}\eta + \mathbf{\Gamma}\xi + \zeta, \mathbf{y} = \mathbf{\Lambda}_y\eta + \varepsilon, \mathbf{x} = \mathbf{\Lambda}_x\xi + \delta$$

Figure 3.6: Path Model with Latent Variables

Chapter 4

Student's t -test

In this section, the Student's t -test is presented as a structural equation model using the RAM notation. Let y be a continuous dependent variable, x be a dichotomous independent variable ($x = \{0, 1\}$), and ε be the stochastic error term with mean 0 and constant variance of σ_ε^2 across the values of x . The associations of the variables are given by

$$y = \alpha + \beta x + \varepsilon$$

where

- α is the expected value of y when $x = 0$
- β is the unit change in y for unit change in x
- $\alpha + \beta$ is the expected value of y when $x = 1$

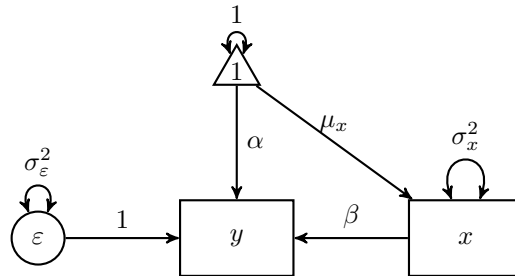


Figure 4.1: Student's t -test

4.1 Symbolic

Let $\{y, x, \varepsilon\}$ be the variables of interest.

$$\mathbf{A} = \begin{pmatrix} 0 & \beta & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{S} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \sigma_x^2 & 0 \\ 0 & 0 & \sigma_\varepsilon^2 \end{pmatrix}$$

$$\mathbf{C} = (\mathbf{I} - \mathbf{A})^{-1} \mathbf{S} [(\mathbf{I} - \mathbf{A})^{-1}]^\top$$

$$= \mathbf{E} \mathbf{S} \mathbf{E}^\top$$

$$\begin{aligned} &= \begin{pmatrix} 1 & \beta & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \sigma_x^2 & 0 \\ 0 & 0 & \sigma_\varepsilon^2 \end{pmatrix} \begin{pmatrix} 1 & \beta & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}^\top \\ &= \begin{pmatrix} \sigma_x^2 \beta^2 + \sigma_\varepsilon^2 & \beta \sigma_x^2 & \sigma_\varepsilon^2 \\ \sigma_x^2 \beta & \sigma_x^2 & 0 \\ \sigma_\varepsilon^2 & 0 & \sigma_\varepsilon^2 \end{pmatrix} \end{aligned}$$

$$\mathbf{F} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\mathbf{M} = \mathbf{F} (\mathbf{I} - \mathbf{A})^{-1} \mathbf{S} [(\mathbf{I} - \mathbf{A})^{-1}]^\top \mathbf{F}^\top$$

$$= \mathbf{F} \mathbf{E} \mathbf{S} \mathbf{E}^\top \mathbf{F}^\top$$

$$= \mathbf{F} \mathbf{C} \mathbf{F}^\top$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \sigma_x^2 \beta^2 + \sigma_\varepsilon^2 & \beta \sigma_x^2 & \sigma_\varepsilon^2 \\ \sigma_x^2 \beta & \sigma_x^2 & 0 \\ \sigma_\varepsilon^2 & 0 & \sigma_\varepsilon^2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}^\top$$

$$= \begin{pmatrix} \sigma_x^2 \beta^2 + \sigma_\varepsilon^2 & \beta \sigma_x^2 \\ \sigma_x^2 \beta & \sigma_x^2 \end{pmatrix}$$

$$\mathbf{v} = (\mathbf{I} - \mathbf{A})^{-1} \mathbf{u}$$

$$= \left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & \beta & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right]^{-1} \begin{pmatrix} \alpha \\ \mu_x \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} \alpha + \beta \mu_x \\ \mu_x \\ 0 \end{pmatrix}$$

$$\begin{aligned}
\mathbf{u} &= (\mathbf{I} - \mathbf{A}) \mathbf{v} \\
&= \left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & \beta & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right] \begin{pmatrix} \alpha + \beta\mu_x \\ \mu_x \\ 0 \end{pmatrix} \\
&= \begin{pmatrix} \alpha \\ \mu_x \\ 0 \end{pmatrix}
\end{aligned}$$

$$\begin{aligned}
\mathbf{g} &= \mathbf{F} (\mathbf{I} - \mathbf{A})^{-1} \mathbf{u} \\
&= \left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & \beta & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right]^{-1} \begin{pmatrix} \alpha \\ \mu_x \\ 0 \end{pmatrix} \\
&= \begin{pmatrix} \alpha + \beta\mu_x \\ \mu_x \\ 0 \end{pmatrix}
\end{aligned}$$

4.1.1 Using the ramR Package

A

```
##   y   x       e
## y "0" "beta" "1"
## x "0" "0"    "0"
## e "0" "0"    "0"
```

S

```
##   y   x       e
## y "0" "0"      "0"
## x "0" "sigma[x]^2" "0"
## e "0" "0"      "sigma[varepsilon]^2"
```

u

```
##   u
## y "alpha"
## x "mu[x]"
## e "0"
```

filter

```
##   y x e
## y 1 0 0
## x 0 1 0
```

The covariance expectations can be symbolically derived using the `ramR::C_sym()` function.

```
ramR::C_sym(A, S)
```

```
## {{sigma[x]^2*beta^2+sigma[varepsilon]^2,          beta*sigma[x]^2,
## {          sigma[x]^2*beta,                      sigma[x]^2,
## {          sigma[varepsilon]^2,                      0,
```

$$\mathbf{C} = \begin{pmatrix} \sigma_x^2 \beta^2 + \sigma_\varepsilon^2 & \beta \sigma_x^2 & \sigma_\varepsilon^2 \\ \sigma_x^2 \beta & \sigma_x^2 & 0 \\ \sigma_\varepsilon^2 & 0 & \sigma_\varepsilon^2 \end{pmatrix}$$

The covariance expectations for the observed variables can be symbolically derived using the `ramR::M_sym()` function.

```
ramR::M_sym(A, S, filter)
```

```
## {{sigma[x]^2*beta^2+sigma[varepsilon]^2,          beta*sigma[x]^2},
## {          sigma[x]^2*beta,                      sigma[x]^2}}
```

$$\mathbf{M} = \begin{pmatrix} \sigma_x^2 \beta^2 + \sigma_\varepsilon^2 & \beta \sigma_x^2 \\ \sigma_x^2 \beta & \sigma_x^2 \end{pmatrix}$$

The mean expectations can be symbolically derived using the `ramR::v_sym()` function.

```
ramR::v_sym(A, u)
```

```
## {{alpha+beta*mu[x]},
## {          mu[x]},
## {          0}}
```

$$\mathbf{v} = \begin{pmatrix} \alpha + \beta \mu_x \\ \mu_x \\ 0 \end{pmatrix}$$

The mean expectations for the observed variables can be symbolically derived using the `ramR::g_sym()` function.

```
ramR::g_sym(A, u, filter)
```

```
## {{alpha+beta*mu[x]},
## {          mu[x]}}
```

$$\mathbf{g} = \begin{pmatrix} \alpha + \beta \mu_x \\ \mu_x \end{pmatrix}$$

4.2 Numerical Example

Let \mathbf{df} be a random sample with the following parameters

Parameter	$x = 0$	$x = 1$
Sample Size	500	500
μ	0	1
σ^2	1	1

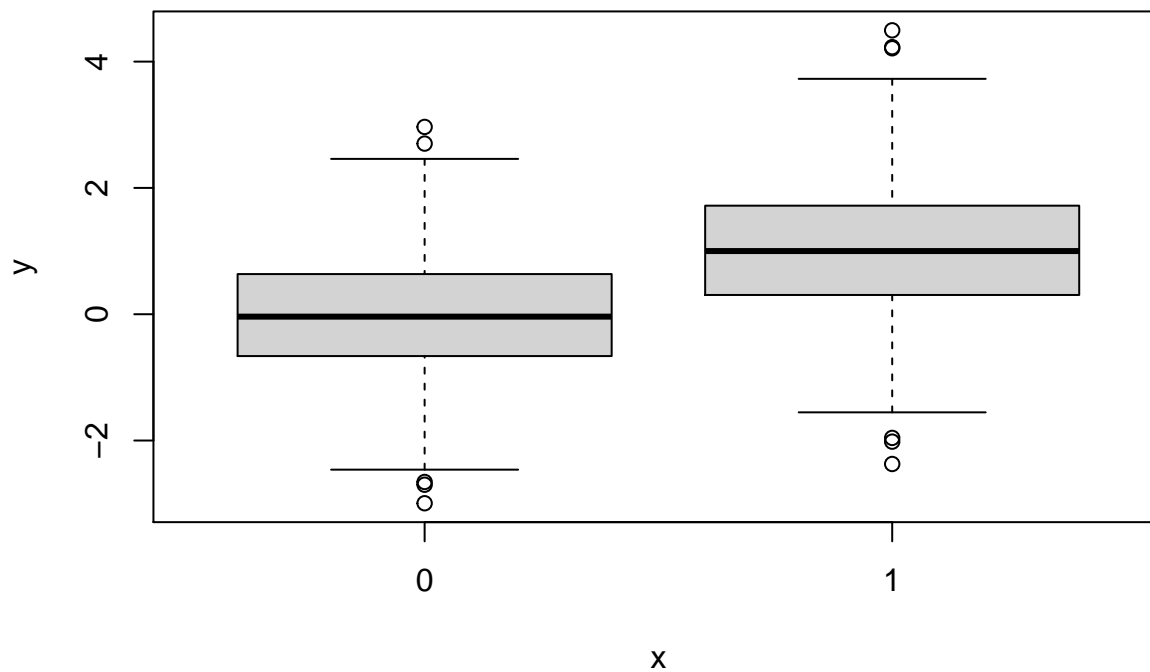
Parameter	Description	Estimate
α	Mean of $x = 0$.	0
β	Mean of $x = 1$ minus $x = 0$.	1

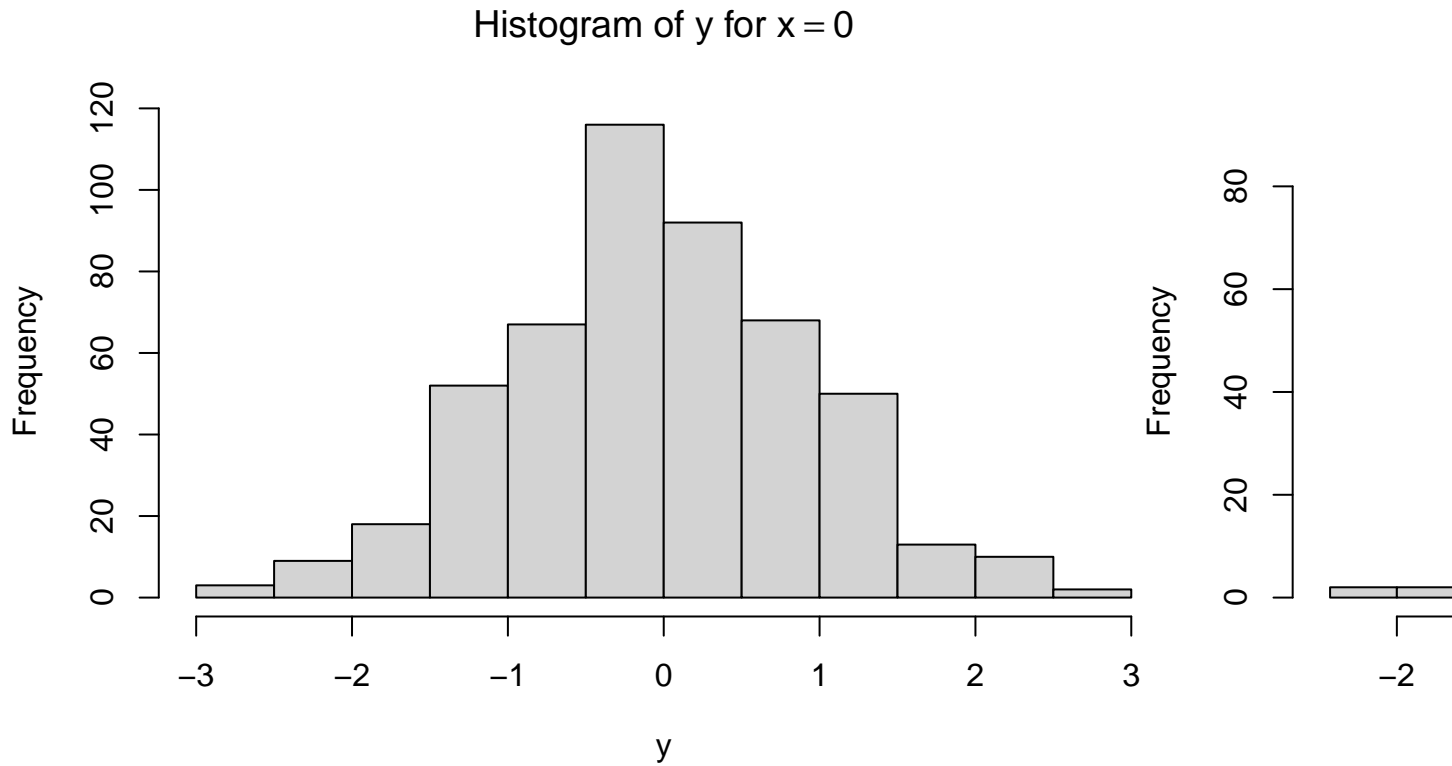
```
head(df)
```

```
##           y x
## 1  1.3709584 0
## 2 -0.5646982 0
## 3  0.3631284 0
## 4  0.6328626 0
## 5  0.4042683 0
## 6 -0.1061245 0
```

```
summary(df)
```

```
##           y           x
## Min.   :-2.9931  Min.   :0.0
## 1st Qu.: -0.2770  1st Qu.:0.0
## Median :  0.4503  Median :0.5
## Mean    :  0.4742  Mean    :0.5
## 3rd Qu.:  1.2492  3rd Qu.:1.0
## Max.    :  4.4953  Max.    :1.0
```





4.2.1 t-test

```
t.test(y ~ x, data = df)
```

```
##
## Welch Two Sample t-test
##
## data: y by x
## t = -15.897, df = 994.36, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
## -1.1329278 -0.8839594
## sample estimates:
## mean in group 0 mean in group 1
## -0.03004622 0.97839737
```

4.2.2 Linear Regression

```
summary(lm(y ~ x, data = df))
```

```
##
## Call:
## lm(formula = y ~ x, data = df)
##
```

```
## Residuals:
##      Min       1Q   Median       3Q      Max
## -3.3501 -0.6517  0.0086  0.6858  3.5169
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.03005    0.04486   -0.67   0.503
## x            1.00844    0.06344   15.90 <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.003 on 998 degrees of freedom
## Multiple R-squared:  0.2021, Adjusted R-squared:  0.2013
## F-statistic: 252.7 on 1 and 998 DF,  p-value: < 2.2e-16
```

4.2.3 Structural Equation Modeling

4.2.3.1 lavaan (Rosseel, 2012)

```
model <- "
  y ~ x
"
fit <- lavaan::sem(
  model,
  data = df,
  meanstructure = TRUE,
  fixed.x = FALSE
)
lavaan::summary(fit)

## lavaan 0.6-7 ended normally after 12 iterations
##
##      Estimator                      ML
##      Optimization method          NLMINB
##      Number of free parameters      5
##
##      Number of observations          1000
##
## Model Test User Model:
##
##      Test statistic                  0.000
##      Degrees of freedom              0
##
## Parameter Estimates:
##
##      Standard errors                Standard
##      Information                    Expected
##      Information saturated (h1) model Structured
##
## Regressions:
##              Estimate Std.Err  z-value  P(>|z|)
## y ~
```

```
##      x              1.008    0.063    15.913    0.000
##
## Intercepts:
##              Estimate Std.Err z-value P(>|z|)
##      .y             -0.030   0.045   -0.671   0.503
##      x              0.500   0.016   31.623   0.000
##
## Variances:
##              Estimate Std.Err z-value P(>|z|)
##      .y              1.004   0.045   22.361   0.000
##      x              0.250   0.011   22.361   0.000
```

4.2.3.2 OpenMx (Boker et al., 2020)

RAM matrices can be used to specify models in `OpenMx`. Note, however, that the `u` vector in the RAM notation is `M` in the `OpenMx` notation.

```
mxData <- OpenMx::mxData(
  observed = df,
  type = "raw"
)
mxA <- OpenMx::mxMatrix(
  type = "Full",
  nrow = 3,
  ncol = 3,
  free = c(
    F, T, F,
    F, F, F,
    F, F, F
  ),
  values = c(
    0, 0.20, 1,
    0, 0, 0,
    0, 0, 0
  ),
  labels = c(
    NA, "beta", NA,
    NA, NA, NA,
    NA, NA, NA
  ),
  byrow = TRUE,
  name = "mxA"
)
mxS <- OpenMx::mxMatrix(
  type = "Symm",
  nrow = 3,
  ncol = 3,
  free = c(
    F, F, F,
    F, T, F,
    F, F, T
  ),
  values = c(
```



```

    0, 0, 0,
    0, 0.20, 0,
    0, 0, 0.20
  ),
  labels = c(
    NA, NA, NA,
    NA, "sigma2x", NA,
    NA, NA, "sigma2e"
  ),
  byrow = TRUE,
  name = "mxS"
)
mxM <- OpenMx::mxMatrix(
  type = "Full",
  nrow = 1,
  ncol = 3,
  free = c(
    T, T, F
  ),
  values = c(
    0.20,
    0.20,
    0
  ),
  labels = c(
    "alpha",
    "mux",
    NA
  ),
  byrow = TRUE,
  name = "mxM"
)
mxF <- OpenMx::mxMatrix(
  type = "Full",
  nrow = 2,
  ncol = 3,
  free = FALSE,
  values = c(
    1, 0, 0,
    0, 1, 0
  ),
  byrow = TRUE,
  name = "mxF"
)
expRAM <- OpenMx::mxExpectationRAM(
  A = "mxA",
  S = "mxS",
  F = "mxF",
  M = "mxM",
  dimnames = c(
    "y",
    "x",
    "e"
  )

```

```

)
)
objML <- OpenMx::mxFitFunctionML()
mxMod <- OpenMx::mxModel(
  name = "Model",
  data = mxData,
  matrices = list(
    mxA,
    mxS,
    mxF,
    mxM
  ),
  expectation = expRAM,
  fitfunction = objML
)
fit <- OpenMx::mxRun(mxMod)

```

```
## Running Model with 5 parameters
```

```
summary(fit)
```

```
## Summary of Model
##
## free parameters:
##      name matrix row col   Estimate Std.Error A
## 1  beta    mxA   1   2  1.00844356 0.06337369
## 2 sigma2x  mxS   2   2  0.25000000 0.01118034
## 3 sigma2e  mxS   3   3  1.00402596 0.04490152
## 4  alpha  mxM   1   y -0.03004621 0.04481202
## 5  mux    mxM   1   x  0.49999999 0.01581140
##
## Model Statistics:
##           | Parameters | Degrees of Freedom | Fit (-2lnL units)
##      Model:              5              1995          4293.478
##      Saturated:           5              1995              NA
##      Independence:        4              1996              NA
## Number of observations/statistics: 1000/2000
##
## Information Criteria:
##           | df Penalty | Parameters Penalty | Sample-Size Adjusted
##      AIC:      303.4776          4303.478          4303.538
##      BIC:     -9487.4941          4328.016          4312.136
##      CFI: NA
##      TLI: 1 (also known as NNFI)
##      RMSEA: 0 [95% CI (NA, NA)]
##      Prob(RMSEA <= 0.05): NA
## To get additional fit indices, see help(mxRefModels)
## timestamp: 2021-01-23 18:35:53
## Wall clock time: 0.166863 secs
## optimizer: SLSQP
## OpenMx version number: 2.18.1
## Need help? See help(mxSummary)
```

4.2.4 Using the ramR Package

A

```
##      y      x e
## y 0 1.008444 1
## x 0 0.000000 0
## e 0 0.000000 0
```

S

```
##      y      x      e
## y 0 0.0000000 0.000000
## x 0 0.2502503 0.000000
## e 0 0.0000000 1.006038
```

u

```
##      u
## y -0.03004622
## x 0.50000000
## e 0.00000000
```

filter

```
##      y x e
## y 1 0 0
## x 0 1 0
```

The covariance expectations can be numerically derived using the `ramR::C_num()` function.

```
ramR::C_num(A, S)
```

```
##      y      x      e
## y 1.2605321 0.2523633 1.006038
## x 0.2523633 0.2502503 0.000000
## e 1.0060380 0.0000000 1.006038
```

The covariance expectations for the observed variables can be numerically derived using the `ramR::M_num()` function.

```
ramR::M_num(A, S, filter)
```

```
##      y      x
## y 1.2605321 0.2523633
## x 0.2523633 0.2502503
```

The mean expectations can be numerically derived using the `ramR::v_num()` function.

```
ramR::v_num(A, u)
```

```
##           v
## y 0.4741756
## x 0.5000000
## e 0.0000000
```

The mean expectations for the observed variables can be numerically derived using the `ramR::v_num()` function.

```
ramR::g_num(A, u, filter)
```

```
##           g
## y 0.4741756
## x 0.5000000
```

4.3 Equations to RAM

The `ramR` package has a utility function to convert structural equations to RAM notation. The Student's *t*-test can be expressed in the following equations

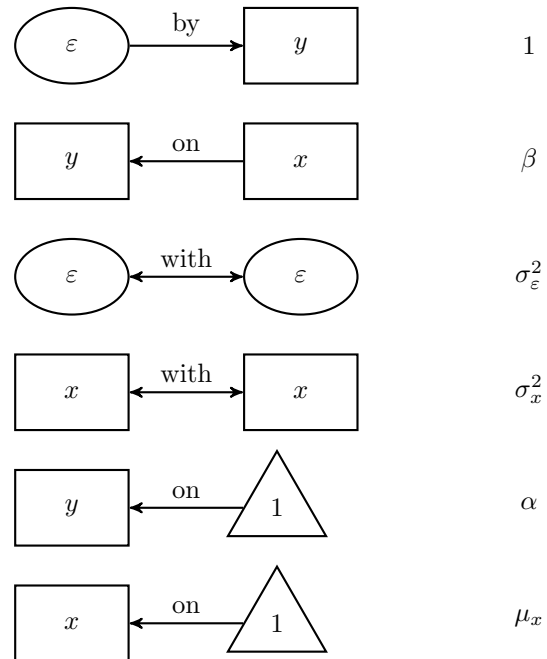
```
eq <- "
# VARIABLE1 OPERATION VARIABLE2 LABEL
e          by          y          1
y          on          x          beta
e          with         e      sigma[varepsilon]^2
x          with         x      sigma[x]^2
y          on          1          alpha
x          on          1          mu[x]
"
```

The error term is treated as a latent variable and defined with the operation `by`. It's value is constrained to 1. The regression of y on x is defined by operation `on`. It is labeled as `beta`. The variance of x and the error variance are defined using the operation `with`. These are labeled `sigma[x]^2` and `sigma[varepsilon]^2` respectively. The intercept and the mean of x are defined using the operation `on` 1. These are labeled `alpha` and `mu[x]` respectively.

The `ramR::eq2ram` converts the equations to RAM notation.

```
ramR::eq2ram(eq)
```

```
## $eq
##   var1  op var2          label
## 1    e  by   y           1
## 2    y  on   x          beta
## 3    e with  e sigma[varepsilon]^2
## 4    x with  x      sigma[x]^2
## 5    y  on   1          alpha
## 6    x  on   1          mu[x]
##
```

Figure 4.2: Student's t -test's Structural Equations

```
## $variables
## [1] "y" "x" "e"
##
## $A
##   y  x    e
## y "0" "beta" "1"
## x "0" "0"    "0"
## e "0" "0"    "0"
##
## $S
##   y  x          e
## y "0" "0"        "0"
## x "0" "sigma[x]^2" "0"
## e "0" "0"        "sigma[varepsilon]^2"
##
## $filter
##   y x e
## y 1 0 0
## x 0 1 0
##
## $u
##   u
## y "alpha"
## x "mu[x]"
## e "0"
```

4.4 Equations to Expectations

The `ramR` package has a utility function to convert structural equations to expectations both symbolically and numerically.

```
eq <- "
# VARIABLE1 OPERATION VARIABLE2 LABEL
e      by      y      1
y      on      x      beta
e      with    e      sigma[varepsilon]^2
x      with    x      sigma[x]^2
y      on      1      alpha
x      on      1      mu[x]
"
```

```
ramR::eq2exp_sym(eq)
```

```
## $variables
## [1] "y" "x" "e"
##
## $A
## {{ 0, beta, 1},
## { 0, 0, 0},
## { 0, 0, 0}}
##
## $S
## {{ 0, 0, 0},
## { 0, sigma[x]^2, 0},
## { 0, 0, sigma[varepsilon]^2}}
##
## $u
## {{alpha},
## {mu[x]},
## { 0}}
##
## $filter
## {{1, 0, 0},
## {0, 1, 0}}
##
## $v
## {{alpha+beta*mu[x]},
## { mu[x]},
## { 0}}
##
## $g
## {{alpha+beta*mu[x]},
## { mu[x]}}
##
## $C
## {{sigma[x]^2*beta^2+sigma[varepsilon]^2, beta*sigma[x]^2,
## { sigma[x]^2*beta, sigma[x]^2,
## { sigma[varepsilon]^2, 0,
##
```

```
## $M
## {{sigma[x]^2*beta^2+sigma[varepsilon]^2,          beta*sigma[x]^2},
## {          sigma[x]^2*beta,                      sigma[x]^2}}
```

```
eq <- "
# VARIABLE1 OPERATION VARIABLE2 VALUE
e          by          y          1.00
y          on          x          1.00
e          with        e          1.00
x          with        x          0.25
y          on          1          0.00
x          on          1          0.50
"
```

```
ramR::eq2exp_num(eq)
```

```
## $variables
## [1] "y" "x" "e"
##
## $A
##   y x e
## y 0 1 1
## x 0 0 0
## e 0 0 0
##
## $S
##   y   x e
## y 0 0.00 0
## x 0 0.25 0
## e 0 0.00 1
##
## $u
##   u
## y 0.0
## x 0.5
## e 0.0
##
## $filter
##   y x e
## y 1 0 0
## x 0 1 0
##
## $v
##   v
## y 0.5
## x 0.5
## e 0.0
##
## $g
##   g
## y 0.5
## x 0.5
##
```

```
## $C
##      y      x e
## y 1.25 0.25 1
## x 0.25 0.25 0
## e 1.00 0.00 1
##
## $M
##      y      x
## y 1.25 0.25
## x 0.25 0.25
```


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