

Reticular Action Model (RAM) Notation Notes

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Chapter 1

Description

This is a collection of my notes on the Reticular Action Model (RAM) notation that accompanies the **ramR** package (Pesigan, 2021) in the R statistical environment (R Core Team, 2020). You can install the released version of **ramR** from GitHub with:

```
remotes::install_github("jeksterslab/ramR")
```

These notes are based on the following resources:

- Boker and McArdle (2005)
- McArdle and McDonald (1984)
- McArdle (2005)

See GitHub Pages for the html deployment.

Chapter 2

Reticular Action Model (RAM) Matrix Notation

2.1 Full Model

Definition 2.1.

$$\mathbf{v} = \mathbf{A}\mathbf{v} + \mathbf{u} \quad (2.1)$$

where

- \mathbf{v} and \mathbf{u} are $t \times 1$ vectors of random variables
- \mathbf{A} is a $t \times t$ matrix of *directed* or *asymmetric* relationship from column variable v_j to row variable v_i
 - \mathbf{A} represent the regression of each of the t variables \mathbf{v} on the other $t - 1$ variables
 - diagonal $a_{i,i}$ is zero
 - u_i represent the residual of v_i
 - if all regression coefficients on other variables are zero, then the variable v_i is considered the same as its own residual u_i

Definition 2.2.

$$\mathbf{S} = \mathbb{E} \{ \mathbf{u}\mathbf{u}' \}, \quad (2.2)$$

where

- \mathbf{S} is a $t \times t$ matrix of *undirected* or *symmetric* relationship
 - the notation Ω is used in other sources for \mathbf{S}
- \mathbb{E} is the expectation operator

Definition 2.3.

$$\mathbf{C} = \mathbb{E} \{ \mathbf{v}\mathbf{v}' \}, \quad (2.3)$$

where

- \mathbf{C} is a $t \times t$ variance-covariance matrix
 - the notation Σ is used in other sources for \mathbf{C}

Definition 2.4.

$$\mathbf{v} = \mathbf{A}\mathbf{v} + \mathbf{u}$$

can be rewritten as

$$\begin{aligned} \mathbf{v} - \mathbf{A}\mathbf{v} &= \mathbf{u} \\ \mathbf{u} &= \mathbf{v} - \mathbf{A}\mathbf{v} \\ \mathbf{u} &= (\mathbf{I} - \mathbf{A})\mathbf{v} \end{aligned} \tag{2.4}$$

assuming that $(\mathbf{I} - \mathbf{A})$ is non-singular,

$$\mathbf{E} = (\mathbf{I} - \mathbf{A})^{-1} \tag{2.5}$$

then

$$\begin{aligned} \mathbf{v} &= (\mathbf{I} - \mathbf{A})^{-1} \mathbf{u} \\ &= \mathbf{E}\mathbf{u}. \end{aligned} \tag{2.6}$$

Using the definitions above, \mathbf{S} and \mathbf{C} are given by

$$\begin{aligned} \mathbf{S} &= (\mathbf{I} - \mathbf{A})\mathbf{C}(\mathbf{I} - \mathbf{A})^{-1} \\ &= \mathbf{E}^{-1}\mathbf{C}(\mathbf{E}^{-1})^{\top} \end{aligned} \tag{2.7}$$

$$\begin{aligned} \mathbf{C} &= (\mathbf{I} - \mathbf{A})^{-1}\mathbf{S}[(\mathbf{I} - \mathbf{A})^{-1}]^{\top} \\ &= \mathbf{E}\mathbf{S}\mathbf{E}^{\top} \end{aligned} \tag{2.8}$$

2.2 Given vs. Hidden Variables

Definition 2.5.

$$\mathbf{v} = \begin{bmatrix} \mathbf{g}_{p \times 1} \\ \mathbf{h}_{q \times 1} \end{bmatrix} \tag{2.9}$$

$$t = p + q \tag{2.10}$$

- \mathbf{g} may be considered observed, manifest or *given* variables
- \mathbf{h} may be considered unobserved, latent, or *hidden* variables

Definition 2.6.

$$\mathbf{F} = [\mathbf{I}_{p \times p} : \mathbf{0}_{p \times q}] \tag{2.11}$$

- the \mathbf{F} matrix acts as a *filter* to select the manifest variables out of the full set of manifest and latent variables

$$\mathbf{g} = \mathbf{F}\mathbf{v} \quad (2.12)$$

$$\begin{aligned} \mathbf{g} &= \mathbf{F}(\mathbf{I} - \mathbf{A})^{-1} \mathbf{u} \\ &= \mathbf{F}\mathbf{E}\mathbf{u} \end{aligned} \quad (2.13)$$

Definition 2.7.

$$\mathbf{M} = \mathbb{E} \{ \mathbf{g}\mathbf{g}^T \} \quad (2.14)$$

$$\begin{aligned} \mathbf{M} &= \mathbf{F}(\mathbf{I} - \mathbf{A})^{-1} \mathbf{S} [(\mathbf{I} - \mathbf{A})^{-1}]^T \mathbf{F}^T \\ &= \mathbf{F}\mathbf{E}\mathbf{S}\mathbf{E}^T \mathbf{F}^T \\ &= \mathbf{F}\mathbf{C}\mathbf{F}^T \end{aligned} \quad (2.15)$$

- when components of \mathbf{v} are permuted, the columns of \mathbf{F} can be correspondingly permuted
- the rows and columns of \mathbf{C} that are filtered out by \mathbf{F} contain useful information about the latent variable structure.

The equations above completely define RAM.

Chapter 3

Reticular Action Model (RAM) Path Diagram

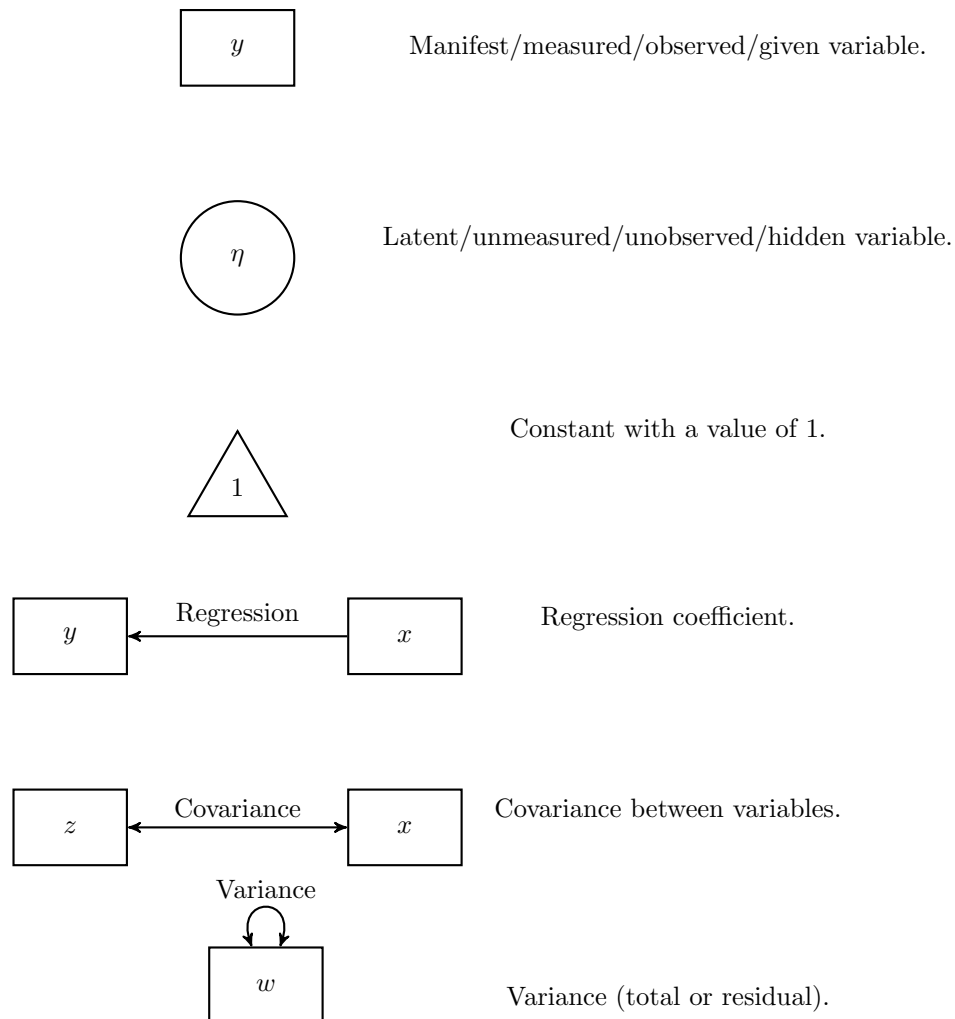
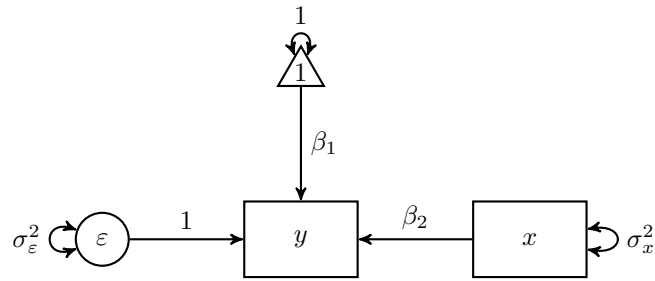
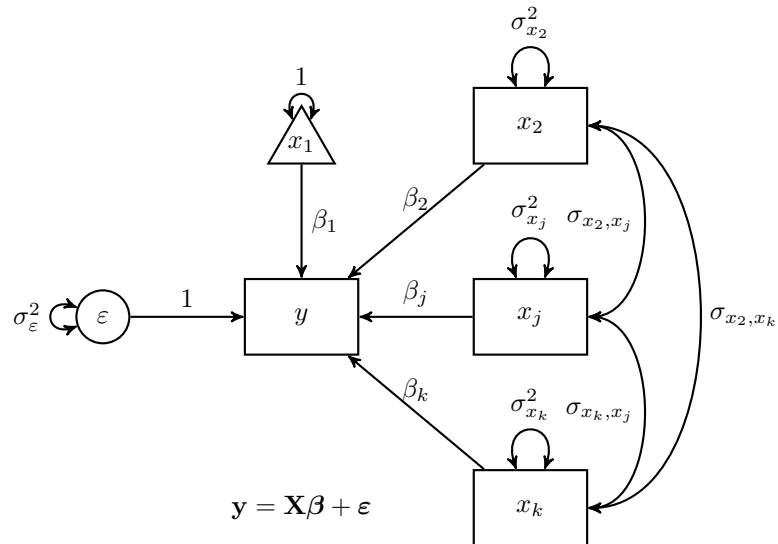


Figure 3.1: Path Diagram Elements



$$y = \alpha + \beta x + \varepsilon$$

Figure 3.2: Two-Variable Regression Model



$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

Figure 3.3: k -Variable Regression Model

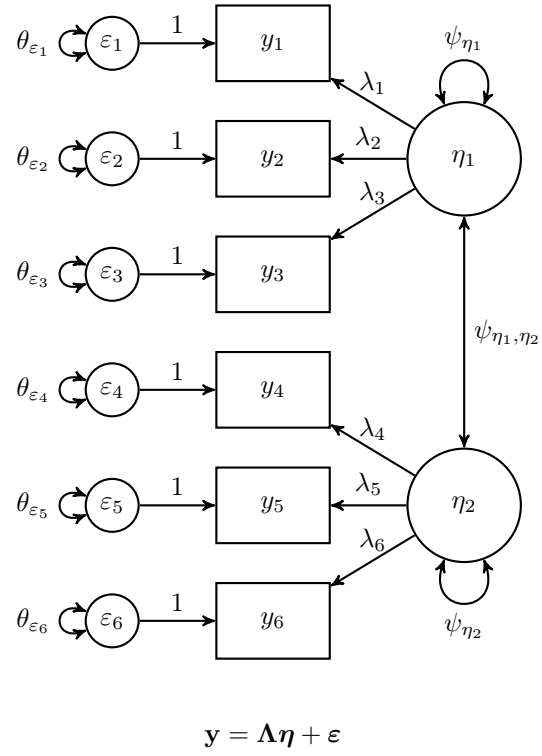


Figure 3.4: Two-Factor Confirmatory Factor Analysis Model

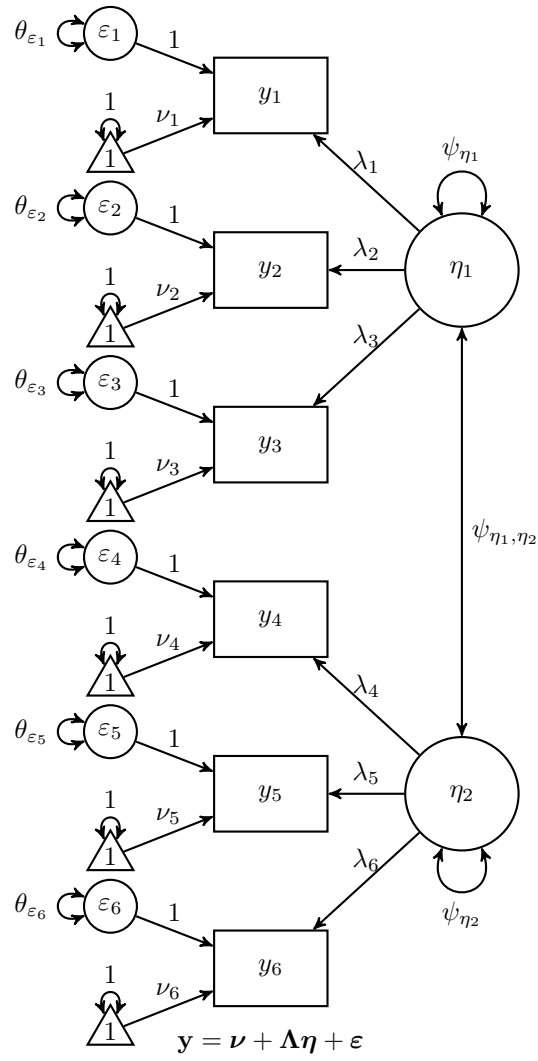
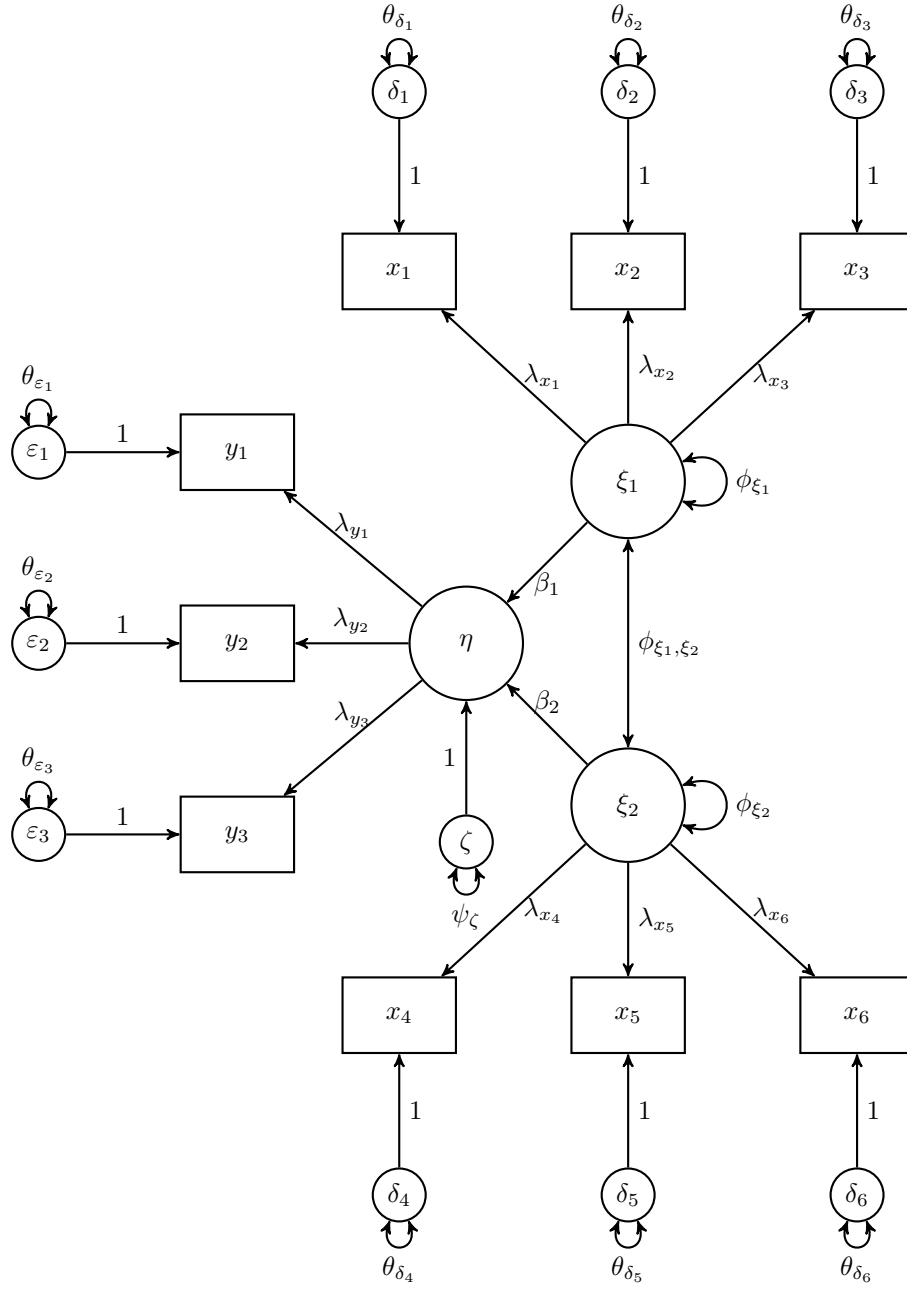


Figure 3.5: Two-Factor Confirmatory Factor Analysis Model with Mean Structure



$$\eta = \mathbf{B}\eta + \mathbf{\Gamma}\xi + \zeta, \mathbf{y} = \mathbf{\Lambda}_y\eta + \varepsilon, \mathbf{x} = \mathbf{\Lambda}_x\xi + \delta$$

Figure 3.6: Path Model with Latent Variables

Chapter 4

Student's t -test

In this section, the Student's t -test is presented as a structural equation model using the RAM notation. Let y be a continuous dependent variable, x be a dichotomous independent variable ($x = \{0, 1\}$), and ε be the stochastic error term with mean 0 and constant variance of σ_ε^2 across the values of x . The associations of the variables are given by

$$y = \alpha + \beta x + \varepsilon$$

where

- α is the expected value of y when $x = 0$
- β is the unit change in y for unit change in x
- $\alpha + \beta$ is the expected value of y when $x = 1$

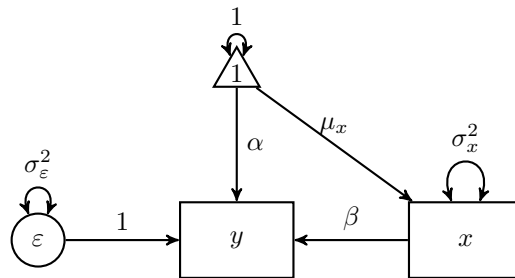


Figure 4.1: Student's t -test

4.1 Symbolic

Let $\{y, x, \varepsilon\}$ be the variables of interest.

$$\mathbf{A} = \begin{pmatrix} 0 & \beta & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{S} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \sigma_x^2 & 0 \\ 0 & 0 & \sigma_\varepsilon^2 \end{pmatrix}$$

$$\mathbf{C} = (\mathbf{I} - \mathbf{A})^{-1} \mathbf{S} [(\mathbf{I} - \mathbf{A})^{-1}]^\top$$

$$= \mathbf{E} \mathbf{S} \mathbf{E}^\top$$

$$\begin{aligned} &= \begin{pmatrix} 1 & \beta & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \sigma_x^2 & 0 \\ 0 & 0 & \sigma_\varepsilon^2 \end{pmatrix} \begin{pmatrix} 1 & \beta & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}^\top \\ &= \begin{pmatrix} \sigma_x^2 \beta^2 + \sigma_\varepsilon^2 & \beta \sigma_x^2 & \sigma_\varepsilon^2 \\ \sigma_x^2 \beta & \sigma_x^2 & 0 \\ \sigma_\varepsilon^2 & 0 & \sigma_\varepsilon^2 \end{pmatrix} \end{aligned}$$

$$\mathbf{F} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\mathbf{M} = \mathbf{F} (\mathbf{I} - \mathbf{A})^{-1} \mathbf{S} [(\mathbf{I} - \mathbf{A})^{-1}]^\top \mathbf{F}^\top$$

$$= \mathbf{F} \mathbf{E} \mathbf{S} \mathbf{E}^\top \mathbf{F}^\top$$

$$= \mathbf{F} \mathbf{C} \mathbf{F}^\top$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \sigma_x^2 \beta^2 + \sigma_\varepsilon^2 & \beta \sigma_x^2 & \sigma_\varepsilon^2 \\ \sigma_x^2 \beta & \sigma_x^2 & 0 \\ \sigma_\varepsilon^2 & 0 & \sigma_\varepsilon^2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}^\top$$

$$= \begin{pmatrix} \sigma_x^2 \beta^2 + \sigma_\varepsilon^2 & \beta \sigma_x^2 \\ \sigma_x^2 \beta & \sigma_x^2 \end{pmatrix}$$

$$\mathbf{v} = (\mathbf{I} - \mathbf{A})^{-1} \mathbf{u}$$

$$= \left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & \beta & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right]^{-1} \begin{pmatrix} \alpha \\ \mu_x \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} \alpha + \beta \mu_x \\ \mu_x \\ 0 \end{pmatrix}$$

$$\begin{aligned}
\mathbf{u} &= (\mathbf{I} - \mathbf{A}) \mathbf{v} \\
&= \left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & \beta & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right] \begin{pmatrix} \alpha + \beta\mu_x \\ \mu_x \\ 0 \end{pmatrix} \\
&= \begin{pmatrix} \alpha \\ \mu_x \\ 0 \end{pmatrix}
\end{aligned}$$

$$\begin{aligned}
\mathbf{g} &= \mathbf{F} (\mathbf{I} - \mathbf{A})^{-1} \mathbf{u} \\
&= \left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & \beta & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right]^{-1} \begin{pmatrix} \alpha \\ \mu_x \\ 0 \end{pmatrix} \\
&= \begin{pmatrix} \alpha + \beta\mu_x \\ \mu_x \\ 0 \end{pmatrix}
\end{aligned}$$

4.1.1 Using the ramR Package

A

```
##   y   x       e
## y "0" "beta" "1"
## x "0" "0"    "0"
## e "0" "0"    "0"
```

S

```
##   y   x       e
## y "0" "0"      "0"
## x "0" "sigma[x]^2" "0"
## e "0" "0"      "sigma[varepsilon]^2"
```

u

```
##   u
## y "alpha"
## x "mu[x]"
## e "0"
```

filter

```
##   y x e
## y 1 0 0
## x 0 1 0
```

The covariance expectations can be symbolically derived using the `ramR::C_sym()` function.

```
ramR::C_sym(A, S)
```

```
## {{sigma[x]^2*beta^2+sigma[varepsilon]^2,          beta*sigma[x]^2,
## {          sigma[x]^2*beta,                      sigma[x]^2,
## {          sigma[varepsilon]^2,                      0,
```

$$\mathbf{C} = \begin{pmatrix} \sigma_x^2 \beta^2 + \sigma_\varepsilon^2 & \beta \sigma_x^2 & \sigma_\varepsilon^2 \\ \sigma_x^2 \beta & \sigma_x^2 & 0 \\ \sigma_\varepsilon^2 & 0 & \sigma_\varepsilon^2 \end{pmatrix}$$

The covariance expectations for the observed variables can be symbolically derived using the `ramR::M_sym()` function.

```
ramR::M_sym(A, S, filter)
```

```
## {{sigma[x]^2*beta^2+sigma[varepsilon]^2,          beta*sigma[x]^2},
## {          sigma[x]^2*beta,                      sigma[x]^2}}
```

$$\mathbf{M} = \begin{pmatrix} \sigma_x^2 \beta^2 + \sigma_\varepsilon^2 & \beta \sigma_x^2 \\ \sigma_x^2 \beta & \sigma_x^2 \end{pmatrix}$$

The mean expectations can be symbolically derived using the `ramR::v_sym()` function.

```
ramR::v_sym(A, u)
```

```
## {{alpha+beta*mu[x]},
## {          mu[x]},
## {          0}}
```

$$\mathbf{v} = \begin{pmatrix} \alpha + \beta \mu_x \\ \mu_x \\ 0 \end{pmatrix}$$

The mean expectations for the observed variables can be symbolically derived using the `ramR::g_sym()` function.

```
ramR::g_sym(A, u, filter)
```

```
## {{alpha+beta*mu[x]},
## {          mu[x]}}
```

$$\mathbf{g} = \begin{pmatrix} \alpha + \beta \mu_x \\ \mu_x \end{pmatrix}$$

4.2 Numerical Example

Let \mathbf{df} be a random sample from a population with the following parameters

Parameter	$x = 0$	$x = 1$
Sample Size	500	500
$\mathbb{E}(y \mid x)$	0	1
$\text{Var}(y \mid x)$	1	1

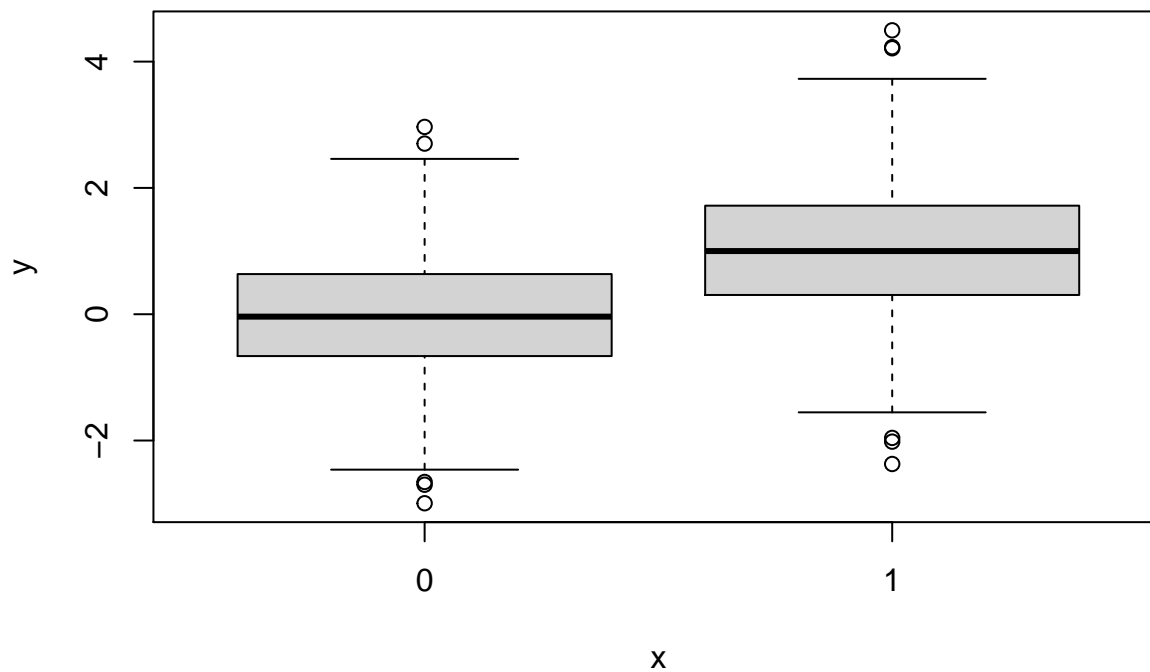
Parameter	Description	Value
α	$\mathbb{E}(y \mid x = 0)$	0
β	$\mathbb{E}(y \mid x = 1) - \mathbb{E}(y \mid x = 0)$	1

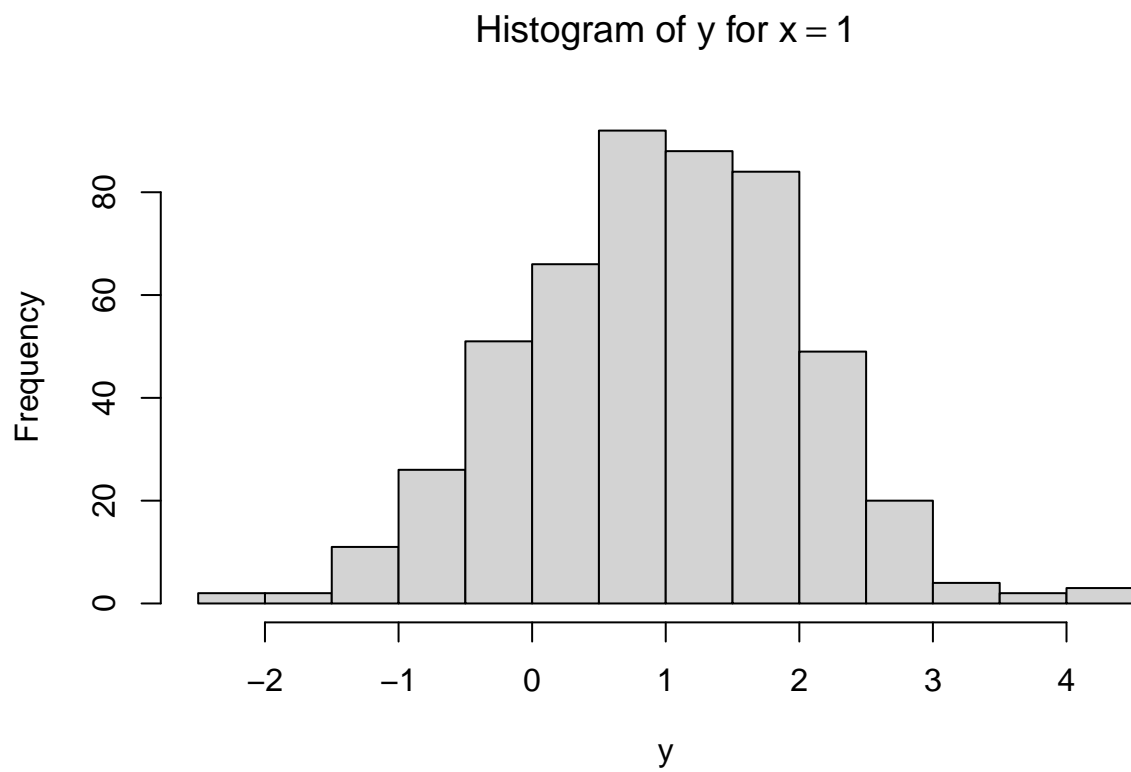
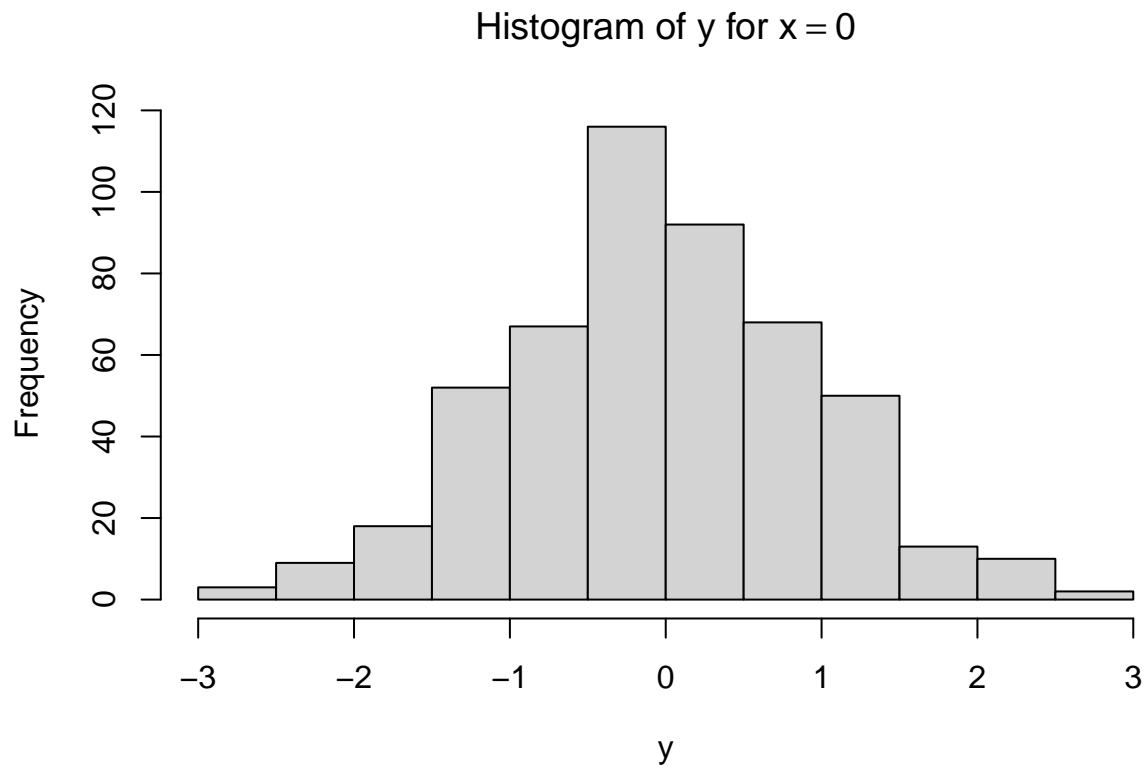
```
head(df)
```

```
##           y x
## 1  1.3709584 0
## 2 -0.5646982 0
## 3  0.3631284 0
## 4  0.6328626 0
## 5  0.4042683 0
## 6 -0.1061245 0
```

```
summary(df)
```

```
##           y           x
## Min.   :-2.9931  Min.   :0.0
## 1st Qu.: -0.2770  1st Qu.:0.0
## Median :  0.4503  Median :0.5
## Mean    :  0.4742  Mean    :0.5
## 3rd Qu.:  1.2492  3rd Qu.:1.0
## Max.    :  4.4953  Max.    :1.0
```





4.2.1 t -test

```
t.test(y ~ x, data = df)
```

```
##
## Welch Two Sample t-test
##
## data: y by x
## t = -15.897, df = 994.36, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
## -1.1329278 -0.8839594
## sample estimates:
## mean in group 0 mean in group 1
## -0.03004622 0.97839737
```

4.2.2 Linear Regression

```
summary(lm(y ~ x, data = df))
```

```
##
## Call:
## lm(formula = y ~ x, data = df)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -3.3501 -0.6517  0.0086  0.6858  3.5169
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.03005    0.04486   -0.67   0.503
## x            1.00844    0.06344   15.90 <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.003 on 998 degrees of freedom
## Multiple R-squared:  0.2021, Adjusted R-squared:  0.2013
## F-statistic: 252.7 on 1 and 998 DF, p-value: < 2.2e-16
```

4.2.3 Structural Equation Modeling

4.2.3.1 lavaan (Rosseel, 2012)

```
model <- "
  y ~ x
"
fit <- lavaan::sem(
  model,
  data = df,
  meanstructure = TRUE,
```

```

fixed.x = FALSE
)
lavaan::summary(fit)

## lavaan 0.6-7 ended normally after 12 iterations
##
##      Estimator                      ML
##      Optimization method          NLMINB
##      Number of free parameters      5
##
##      Number of observations          1000
##
## Model Test User Model:
##
##      Test statistic                  0.000
##      Degrees of freedom              0
##
## Parameter Estimates:
##
##      Standard errors                Standard
##      Information                    Expected
##      Information saturated (h1) model Structured
##
## Regressions:
##              Estimate Std.Err z-value P(>|z|)
## y ~
## x              1.008   0.063  15.913   0.000
##
## Intercepts:
##              Estimate Std.Err z-value P(>|z|)
## .y            -0.030   0.045  -0.671   0.503
## x              0.500   0.016  31.623   0.000
##
## Variances:
##              Estimate Std.Err z-value P(>|z|)
## .y              1.004   0.045  22.361   0.000
## x              0.250   0.011  22.361   0.000

```

4.2.3.2 OpenMx (Boker et al., 2020)

RAM matrices can be used to specify models in `OpenMx`. Note, however, that the `u` vector in the RAM notation is `M` in the `OpenMx` notation.

```

mxData <- OpenMx::mxData(
  observed = df,
  type = "raw"
)
mxA <- OpenMx::mxMatrix(
  type = "Full",
  nrow = 3,
  ncol = 3,
  free = c(

```



```

      F, T, F,
      F, F, F,
      F, F, F
    ),
    values = c(
      0, 0.20, 1,
      0, 0, 0,
      0, 0, 0
    ),
    labels = c(
      NA, "beta", NA,
      NA, NA, NA,
      NA, NA, NA
    ),
    byrow = TRUE,
    name = "mxA"
  )
mxS <- OpenMx::mxMatrix(
  type = "Symm",
  nrow = 3,
  ncol = 3,
  free = c(
    F, F, F,
    F, T, F,
    F, F, T
  ),
  values = c(
    0, 0, 0,
    0, 0.20, 0,
    0, 0, 0.20
  ),
  labels = c(
    NA, NA, NA,
    NA, "sigma2x", NA,
    NA, NA, "sigma2e"
  ),
  byrow = TRUE,
  name = "mxS"
)
mxM <- OpenMx::mxMatrix(
  type = "Full",
  nrow = 1,
  ncol = 3,
  free = c(
    T, T, F
  ),
  values = c(
    0.20,
    0.20,
    0
  ),
  labels = c(
    "alpha",

```

```

    "mux",
    NA
  ),
  byrow = TRUE,
  name = "mxM"
)
mxF <- OpenMx::mxMatrix(
  type = "Full",
  nrow = 2,
  ncol = 3,
  free = FALSE,
  values = c(
    1, 0, 0,
    0, 1, 0
  ),
  byrow = TRUE,
  name = "mxF"
)
expRAM <- OpenMx::mxExpectationRAM(
  A = "mxA",
  S = "mxS",
  F = "mxF",
  M = "mxM",
  dimnames = c(
    "y",
    "x",
    "e"
  )
)
objML <- OpenMx::mxFitFunctionML()
mxMod <- OpenMx::mxModel(
  name = "Student's t test",
  data = mxData,
  matrices = list(
    mxA,
    mxS,
    mxF,
    mxM
  ),
  expectation = expRAM,
  fitfunction = objML
)
fit <- OpenMx::mxRun(mxMod)

```

```
## Running Student's t test with 5 parameters
```

```
summary(fit)
```

```
## Summary of Student's t test
```

```
##
```

```
## free parameters:
```

```
##      name matrix row col      Estimate Std.Error A
```

```
## 1    beta    mxA    1    2    1.00844356 0.06337369
## 2 sigma2x    mxS    2    2    0.25000000 0.01118034
## 3 sigma2e    mxS    3    3    1.00402596 0.04490152
## 4    alpha    mxM    1    y   -0.03004621 0.04481202
## 5     mux     mxM    1    x    0.49999999 0.01581140
##
## Model Statistics:
##           | Parameters | Degrees of Freedom | Fit (-2lnL units)
##      Model:              5              1995              4293.478
##      Saturated:          5              1995              NA
##      Independence:       4              1996              NA
## Number of observations/statistics: 1000/2000
##
## Information Criteria:
##           | df Penalty | Parameters Penalty | Sample-Size Adjusted
##      AIC:      303.4776              4303.478              4303.538
##      BIC:     -9487.4941              4328.016              4312.136
##      CFI: NA
##      TLI: 1    (also known as NNFI)
##      RMSEA: 0   [95% CI (NA, NA)]
##      Prob(RMSEA <= 0.05): NA
## To get additional fit indices, see help(mxRefModels)
## timestamp: 2021-01-24 17:35:20
## Wall clock time: 0.03866959 secs
## optimizer:  SLSQP
## OpenMx version number: 2.18.1
## Need help?  See help(mxSummary)
```

4.2.4 Using the ramR Package

A

```
##      y      x e
## y 0 1.008444 1
## x 0 0.000000 0
## e 0 0.000000 0
```

S

```
##      y      x      e
## y 0 0.0000000 0.000000
## x 0 0.2502503 0.000000
## e 0 0.0000000 1.006038
```

u

```
##      u
## y -0.03004622
## x  0.50000000
## e  0.00000000
```

```
filter
```

```
##   y x e
## y 1 0 0
## x 0 1 0
```

The covariance expectations can be numerically derived using the `ramR::C_num()` function.

```
ramR::C_num(A, S)
```

```
##           y           x           e
## y 1.2605321 0.2523633 1.006038
## x 0.2523633 0.2502503 0.000000
## e 1.0060380 0.0000000 1.006038
```

The covariance expectations for the observed variables can be numerically derived using the `ramR::M_num()` function.

```
ramR::M_num(A, S, filter)
```

```
##           y           x
## y 1.2605321 0.2523633
## x 0.2523633 0.2502503
```

The mean expectations can be numerically derived using the `ramR::v_num()` function.

```
ramR::v_num(A, u)
```

```
##           v
## y 0.4741756
## x 0.5000000
## e 0.0000000
```

The mean expectations for the observed variables can be numerically derived using the `ramR::v_num()` function.

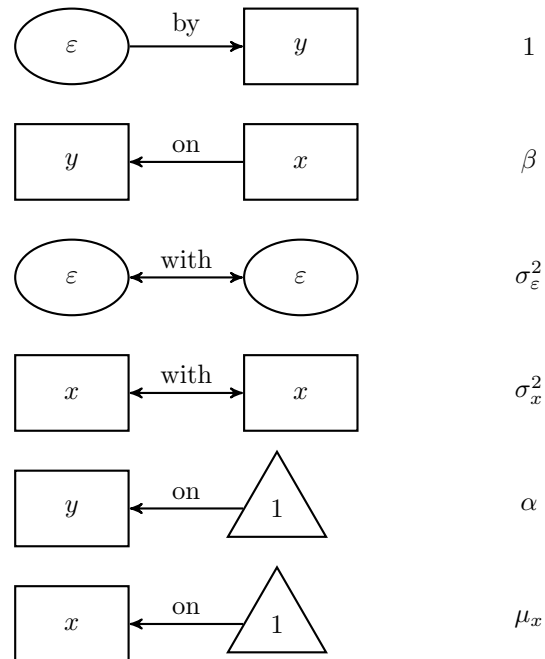
```
ramR::g_num(A, u, filter)
```

```
##           g
## y 0.4741756
## x 0.5000000
```

4.3 Equations to RAM

The `ramR` package has a utility function to convert structural equations to RAM notation. The Student's *t*-test can be expressed in the following equations

```
eq <- "
# VARIABLE1 OPERATION VARIABLE2 LABEL
e      by      y      1
y      on      x      beta
e      with    e      sigma[varepsilon]^2
x      with    x      sigma[x]^2
y      on      1      alpha
x      on      1      mu[x]
"
```

Figure 4.2: Student's *t*-test's Structural Equations

The error term is treated as a latent variable and defined with the operation **by**. Its value is constrained to 1. The regression of y on x is defined by operation **on**. It is labeled as **beta**. The variance of x and the error variance are defined using the operation **with**. These are labeled **sigma[x]^2** and **sigma[varepsilon]^2** respectively. The intercept and the mean of x are defined using the operation **on** 1. These are labeled **alpha** and **mu[x]** respectively.

The `ramR::eq2ram` converts the equations to RAM notation.

```
ramR::eq2ram(eq)
```

```
## $eq
##   var1  op var2      label
## 1    e  by   y       1
## 2    y  on   x     beta
## 3    e with  e sigma[varepsilon]^2
## 4    x with  x   sigma[x]^2
## 5    y  on   1     alpha
## 6    x  on   1    mu[x]
##
```

```
## $variables
## [1] "y" "x" "e"
##
## $A
##   y   x   e
## y "0" "beta" "1"
## x "0" "0"    "0"
## e "0" "0"    "0"
##
## $S
##   y   x   e
## y "0" "0"   "0"
## x "0" "sigma[x]^2" "0"
## e "0" "0"   "sigma[varepsilon]^2"
##
## $filter
##   y x e
## y 1 0 0
## x 0 1 0
##
## $u
##   u
## y "alpha"
## x "mu[x]"
## e "0"
```

4.4 Equations to Expectations

The `ramR` package has a utility function to convert structural equations to expectations both symbolically and numerically.

```
eq <- "
# VARIABLE1 OPERATION VARIABLE2 LABEL
e          by          y          1
y          on          x          beta
e          with        e          sigma[varepsilon]^2
x          with        x          sigma[x]^2
y          on          1          alpha
x          on          1          mu[x]
"
```

```
ramR::eq2exp_sym(eq)
```

```
## $variables
## [1] "y" "x" "e"
##
## $A
## {{ 0, beta, 1},
## { 0, 0, 0},
## { 0, 0, 0}}
##
## $S
```

```
## {{
##      0,          0,          0},
##      {          0,      sigma[x]^2,      0},
##      {          0,          0, sigma[varepsilon]^2}}
##
## $u
## {{alpha},
##      {mu[x]},
##      {      0}}
##
## $filter
## {{1, 0, 0},
##      {0, 1, 0}}
##
## $v
## {{alpha+beta*mu[x]},
##      {      mu[x]},
##      {      0}}
##
## $g
## {{alpha+beta*mu[x]},
##      {      mu[x]}}
##
## $C
## {{sigma[x]^2*beta^2+sigma[varepsilon]^2,      beta*sigma[x]^2,
##      {      sigma[x]^2*beta,      sigma[x]^2,
##      {      sigma[varepsilon]^2,      0,
##
## $M
## {{sigma[x]^2*beta^2+sigma[varepsilon]^2,      beta*sigma[x]^2,
##      {      sigma[x]^2*beta,      sigma[x]^2}}
```

```
eq <- "
# VARIABLE1 OPERATION VARIABLE2 VALUE
e          by          y          1.00
y          on          x          1.00
e          with        e          1.00
x          with        x          0.25
y          on          1          0.00
x          on          1          0.50
"
```

```
ramR::eq2exp_num(eq)
```

```
## $variables
## [1] "y" "x" "e"
##
## $A
##      y x e
## y 0 1 1
## x 0 0 0
## e 0 0 0
##
## $S
```

```

##      y      x e
## y 0 0.00 0
## x 0 0.25 0
## e 0 0.00 1
##
## $u
##      u
## y 0.0
## x 0.5
## e 0.0
##
## $filter
##      y x e
## y 1 0 0
## x 0 1 0
##
## $v
##      v
## y 0.5
## x 0.5
## e 0.0
##
## $g
##      g
## y 0.5
## x 0.5
##
## $C
##      y      x e
## y 1.25 0.25 1
## x 0.25 0.25 0
## e 1.00 0.00 1
##
## $M
##      y      x
## y 1.25 0.25
## x 0.25 0.25

```


Chapter 5

One-Way Analysis of Variance

In this section, one-way analysis of variance is presented as a structural equation model using the RAM notation. Let y be a continuous dependent variable, x be a categorical independent variable with three levels ($x = \{0, 1, 2\}$). The dependent variable x can be dummy coded as

x	x_1	x_2
$x = 0$	0	0
$x = 1$	1	0
$x = 2$	0	1

ε be the stochastic error term with mean 0 and constant variance of σ_ε^2 across the values of the regressors. The associations of the variables are given by

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \varepsilon$$

where

- β_0 is the expected value of y when $x = 0$
- β_1 is the unit change in y for unit change in x_1 while x_2 is constant
- β_2 is the unit change in y for unit change in x_2 while x_1 is constant
- $\beta_0 + \beta_1$ is the expected value of y when $x = 1$
- $\beta_0 + \beta_2$ is the expected value of y when $x = 2$

5.1 Symbolic

Let $\{y, x_1, x_2, \varepsilon\}$ be the variables of interest.

$$\mathbf{A} = \begin{pmatrix} 0 & \beta_1 & \beta_2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{S} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \sigma_{x_1}^2 & 0 & 0 \\ 0 & 0 & \sigma_{x_2}^2 & 0 \\ 0 & 0 & 0 & \sigma_\varepsilon^2 \end{pmatrix}$$

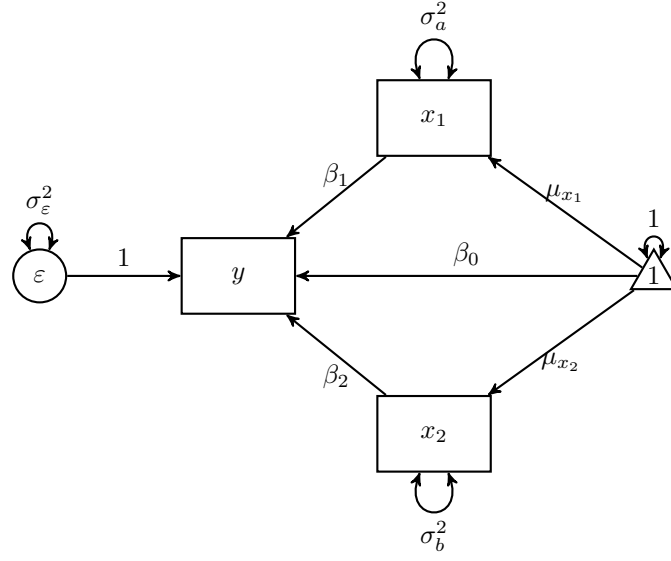


Figure 5.1: One-Way Analysis of Variance

$$\begin{aligned}
 \mathbf{C} &= (\mathbf{I} - \mathbf{A})^{-1} \mathbf{S} [(\mathbf{I} - \mathbf{A})^{-1}]^\top \\
 &= \mathbf{E} \mathbf{S} \mathbf{E}^\top \\
 &= \begin{pmatrix} 1 & \beta_1 & \beta_2 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \sigma_{x_1}^2 & 0 & 0 \\ 0 & 0 & \sigma_{x_2}^2 & 0 \\ 0 & 0 & 0 & \sigma_\varepsilon^2 \end{pmatrix} \begin{pmatrix} 1 & \beta_1 & \beta_2 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}^\top \\
 &= \begin{pmatrix} \sigma_{x_1}^2 \beta_1^2 + \sigma_{x_2}^2 \beta_2^2 + \sigma_\varepsilon^2 & \beta_1 \sigma_{x_1}^2 & \beta_2 \sigma_{x_2}^2 & \sigma_\varepsilon^2 \\ \sigma_{x_1}^2 \beta_1 & \sigma_{x_1}^2 & 0 & 0 \\ \sigma_{x_2}^2 \beta_2 & 0 & \sigma_{x_2}^2 & 0 \\ \sigma_\varepsilon^2 & 0 & 0 & \sigma_\varepsilon^2 \end{pmatrix}
 \end{aligned}$$

$$\mathbf{F} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{aligned}
\mathbf{M} &= \mathbf{F}(\mathbf{I} - \mathbf{A})^{-1} \mathbf{S} \left[(\mathbf{I} - \mathbf{A})^{-1} \right]^\top \mathbf{F}^\top \\
&= \mathbf{F} \mathbf{E} \mathbf{S} \mathbf{E}^\top \mathbf{F}^\top \\
&= \mathbf{F} \mathbf{C} \mathbf{F}^\top \\
&= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \sigma_{x_1}^2 \beta_1^2 + \sigma_{x_2}^2 \beta_2^2 + \sigma_\varepsilon^2 & \beta_1 \sigma_{x_1}^2 & \beta_2 \sigma_{x_2}^2 & \sigma_\varepsilon^2 \\ \sigma_{x_1}^2 \beta_1 & \sigma_{x_1}^2 & 0 & 0 \\ \sigma_{x_2}^2 \beta_2 & 0 & \sigma_{x_2}^2 & 0 \\ \sigma_\varepsilon^2 & 0 & 0 & \sigma_\varepsilon^2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}^\top \\
&= \begin{pmatrix} \sigma_{x_1}^2 \beta_1^2 + \sigma_{x_2}^2 \beta_2^2 + \sigma_\varepsilon^2 & \beta_1 \sigma_{x_1}^2 & \beta_2 \sigma_{x_2}^2 \\ \sigma_{x_1}^2 \beta_1 & \sigma_{x_1}^2 & 0 \\ \sigma_{x_2}^2 \beta_2 & 0 & \sigma_{x_2}^2 \end{pmatrix}
\end{aligned}$$

$$\begin{aligned}
\mathbf{v} &= (\mathbf{I} - \mathbf{A})^{-1} \mathbf{u} \\
&= \left[\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & \beta_1 & \beta_2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right]^{-1} \begin{pmatrix} \beta_0 \\ \mu_{x_1} \\ \mu_{x_2} \\ 0 \end{pmatrix} \\
&= \begin{pmatrix} \beta_0 + \beta_1 \mu_{x_1} + \beta_2 \mu_{x_2} \\ \mu_{x_1} \\ \mu_{x_2} \\ 0 \end{pmatrix}
\end{aligned}$$

$$\begin{aligned}
\mathbf{u} &= (\mathbf{I} - \mathbf{A}) \mathbf{v} \\
&= \left[\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & \beta_1 & \beta_2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right] \begin{pmatrix} \beta_0 + \beta_1 \mu_{x_1} + \beta_2 \mu_{x_2} \\ \mu_{x_1} \\ \mu_{x_2} \\ 0 \end{pmatrix} \\
&= \begin{pmatrix} \beta_0 \\ \mu_{x_1} \\ \mu_{x_2} \\ 0 \end{pmatrix}
\end{aligned}$$

$$\begin{aligned}
\mathbf{g} &= \mathbf{F}(\mathbf{I} - \mathbf{A})^{-1} \mathbf{u} \\
&= \left[\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & \beta_1 & \beta_2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right]^{-1} \begin{pmatrix} \beta_0 \\ \mu_{x_1} \\ \mu_{x_2} \\ 0 \end{pmatrix} \\
&= \begin{pmatrix} \beta_0 + \beta_1 \mu_{x_1} + \beta_2 \mu_{x_2} \\ \mu_{x_1} \\ \mu_{x_2} \end{pmatrix}
\end{aligned}$$

5.1.1 Using the ramR Package

A

```
##      y      x1      x2      e
## y  "0" "beta[1]" "beta[2]" "1"
## x1 "0" "0"      "0"      "0"
## x2 "0" "0"      "0"      "0"
## e  "0" "0"      "0"      "0"
```

S

```
##      y      x1      x2      e
## y  "0" "0"      "0"      "0"
## x1 "0" "sigma[x[1]]^2" "0"      "0"
## x2 "0" "0"      "sigma[x[2]]^2" "0"
## e  "0" "0"      "0"      "sigma[varepsilon]^2"
```

u

```
##      u
## y  "beta[0]"
## x1 "mu[x[1]]"
## x2 "mu[x[2]]"
## e  "0"
```

filter

```
##      y x1 x2 e
## y  1  0  0  0
## x1 0  1  0  0
## x2 0  0  1  0
```

The covariance expectations can be symbolically derived using the `ramR::C_sym()` function.

```
ramR::C_sym(A, S)
```

```
## {{sigma[x[1]]^2*beta[1]^2+sigma[x[2]]^2*beta[2]^2+sigma[varepsilon]^2,
## {                                     sigma[x[1]]^2*beta[1],
## {                                     sigma[x[2]]^2*beta[2],
## {                                     sigma[varepsilon]^2,
```

$$\mathbf{C} = \begin{pmatrix} \sigma_{x_1}^2 \beta_1^2 + \sigma_{x_2}^2 \beta_2^2 + \sigma_\varepsilon^2 & \beta_1 \sigma_{x_1}^2 & \beta_2 \sigma_{x_2}^2 & \sigma_\varepsilon^2 \\ \sigma_{x_1}^2 \beta_1 & \sigma_{x_1}^2 & 0 & 0 \\ \sigma_{x_2}^2 \beta_2 & 0 & \sigma_{x_2}^2 & 0 \\ \sigma_\varepsilon^2 & 0 & 0 & \sigma_\varepsilon^2 \end{pmatrix}$$

The covariance expectations for the observed variables can be symbolically derived using the `ramR::M_sym()` function.

```
ramR::M_sym(A, S, filter)
```

```
## {{sigma[x[1]]^2*beta[1]^2+sigma[x[2]]^2*beta[2]^2+sigma[varepsilon]^2,
## {                                     sigma[x[1]]^2*beta[1],
## {                                     sigma[x[2]]^2*beta[2],
```

$$\mathbf{M} = \begin{pmatrix} \sigma_{x_1}^2 \beta_1^2 + \sigma_{x_2}^2 \beta_2^2 + \sigma_\varepsilon^2 & \beta_1 \sigma_{x_1}^2 & \beta_2 \sigma_{x_2}^2 \\ \sigma_{x_1}^2 \beta_1 & \sigma_{x_1}^2 & 0 \\ \sigma_{x_2}^2 \beta_2 & 0 & \sigma_{x_2}^2 \end{pmatrix}$$

The mean expectations can be symbolically derived using the `ramR::v_sym()` function.

```
ramR::v_sym(A, u)
```

```
## {beta[0]+beta[1]*mu[x[1]]+beta[2]*mu[x[2]]},
## {                                     mu[x[1]]},
## {                                     mu[x[2]]},
## {                                     0}}
```

$$\mathbf{v} = \begin{pmatrix} \beta_0 + \beta_1 \mu_{x_1} + \beta_2 \mu_{x_2} \\ \mu_{x_1} \\ \mu_{x_2} \\ 0 \end{pmatrix}$$

The mean expectations for the observed variables can be symbolically derived using the `ramR::g_sym()` function.

```
ramR::g_sym(A, u, filter)
```

```
## {beta[0]+beta[1]*mu[x[1]]+beta[2]*mu[x[2]]},
## {                                     mu[x[1]]},
## {                                     mu[x[2]]}}
```

$$\mathbf{g} = \begin{pmatrix} \beta_0 + \beta_1 \mu_{x_1} + \beta_2 \mu_{x_2} \\ \mu_{x_1} \\ \mu_{x_2} \end{pmatrix}$$

5.2 Numerical Example

Let \mathbf{df} be a random sample from a population with the following parameters

Parameter	$x = 0$	$x = 1$	$x = 2$
Sample Size	500	500	500
$\mathbb{E}(y x)$	0	2	1
$\text{Var}(y x)$	1	1	1

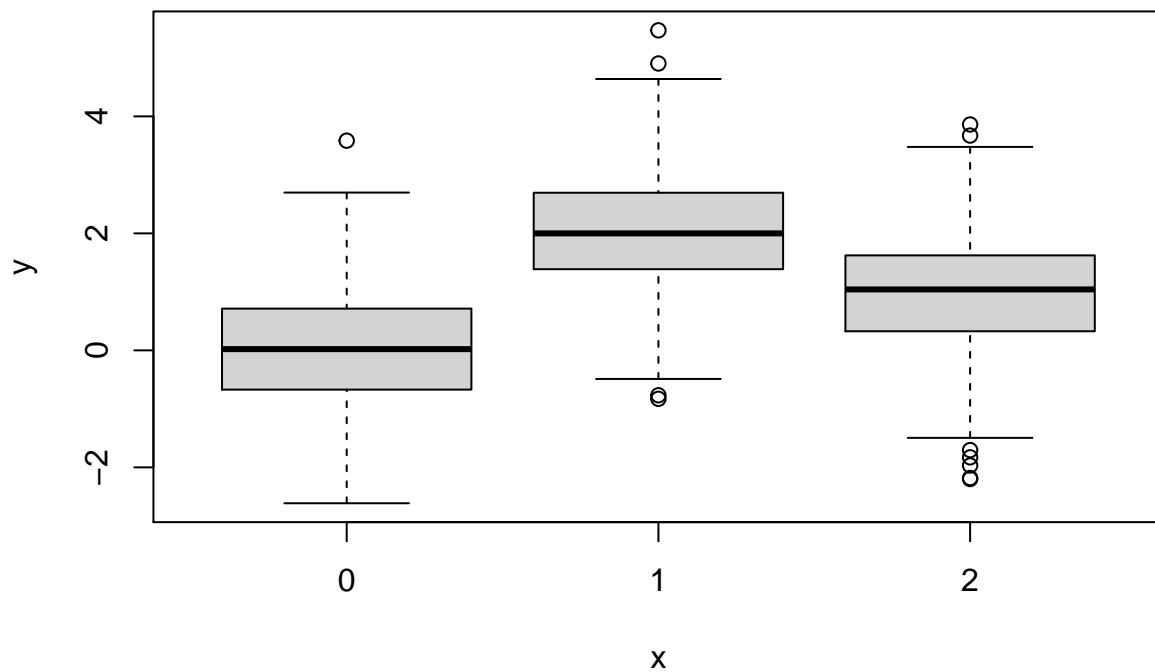
Parameter	Description	Value
β_0	$\mathbb{E}(y x = 0)$	0
β_1	$\mathbb{E}(y x = 1) - \mathbb{E}(y x = 0)$	2
β_2	$\mathbb{E}(y x = 2) - \mathbb{E}(y x = 0)$	1

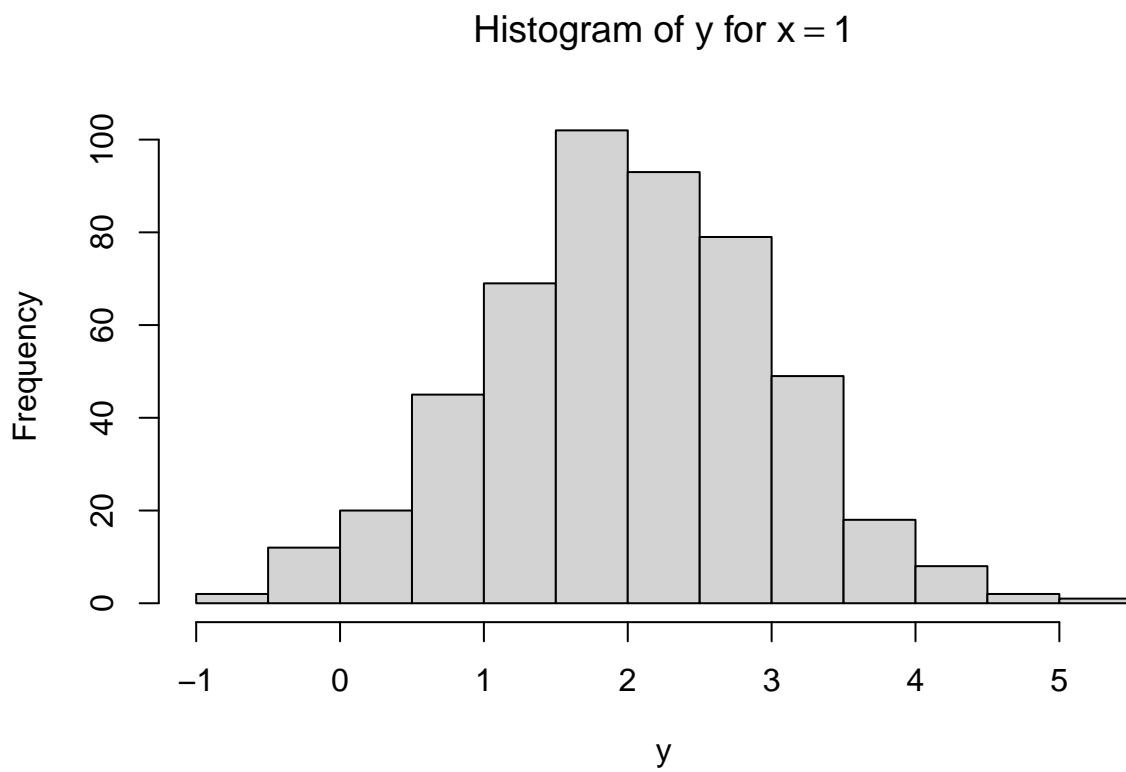
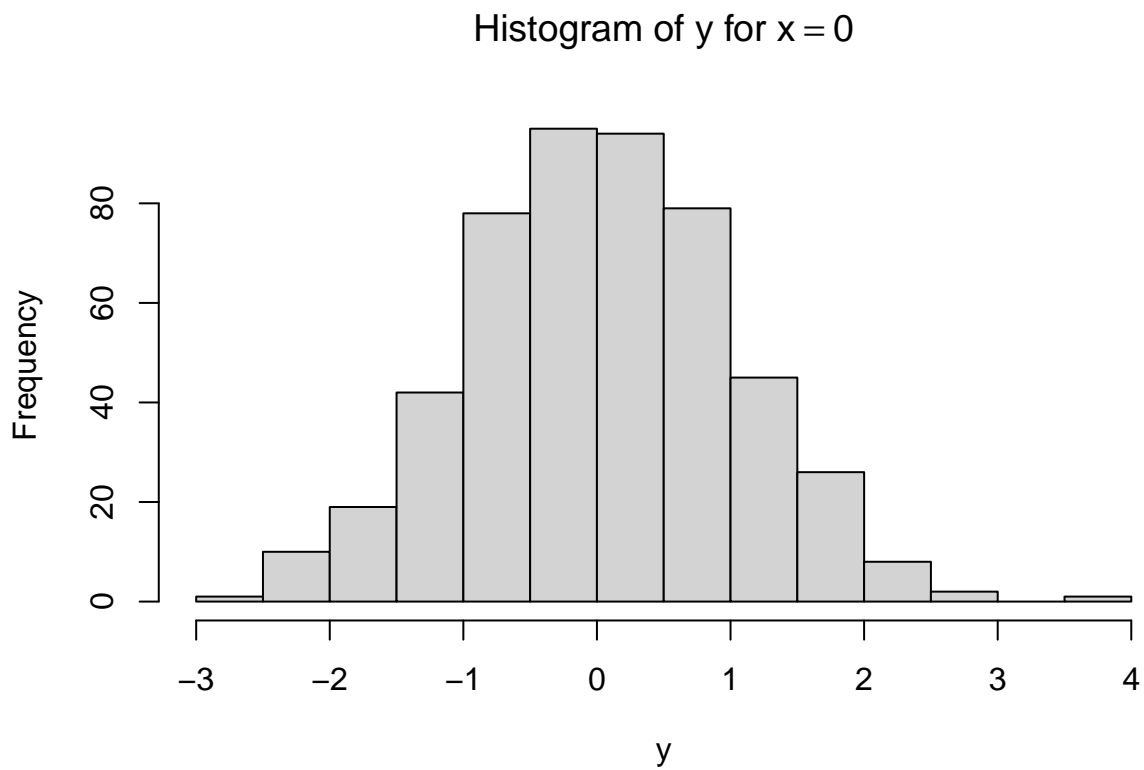
```
head(df)
```

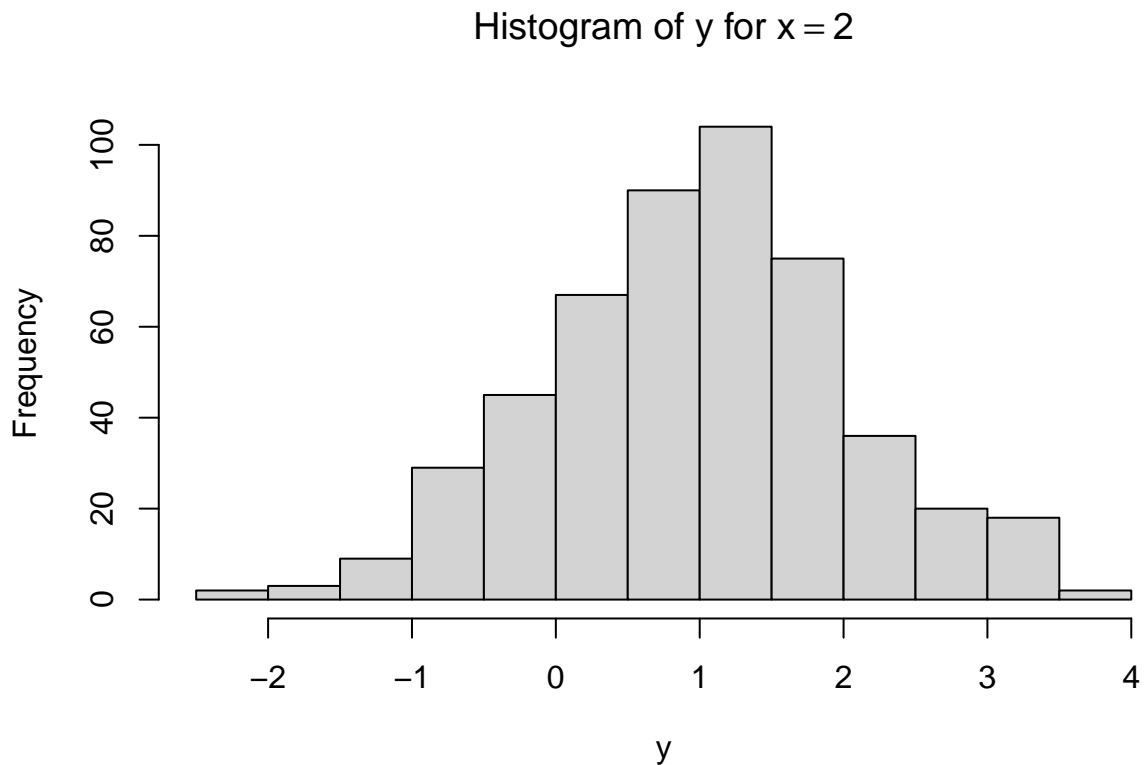
```
##           y x
## 1 -0.6013830 0
## 2 -0.1358161 0
## 3 -0.9872728 0
## 4  0.8319250 0
## 5 -0.7950595 0
## 6  0.3404646 0
```

```
summary(df)
```

```
##           y           x
## Min.      :-2.61364    0:500
## 1st Qu.:  0.08094    1:500
## Median :  1.02617    2:500
## Mean      : 1.00814
## 3rd Qu.:  1.90112
## Max.      :  5.47091
```







5.2.1 One-Way Analysis of Variance

Make sure that x is of class `factor` for `lm` and `aov` to treat it as a categorical variable.

```
str(df)
```

```
## 'data.frame': 1500 obs. of 2 variables:
## $ y: num -0.601 -0.136 -0.987 0.832 -0.795 ...
## $ x: Factor w/ 3 levels "0","1","2": 1 1 1 1 1 1 1 1 1 1 ...
```

```
summary(aov(y ~ x, data = df))
```

```
##           Df Sum Sq Mean Sq F value Pr(>F)
## x           2  983.8   491.9   471.4 <2e-16 ***
## Residuals 1497 1562.2     1.0
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

5.2.2 Linear Regression

```
summary(lm(y ~ x, data = df))
```

```
##
## Call:
```



```
## lm(formula = y ~ x, data = df)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -3.1792 -0.6469  0.0021  0.6751  3.5538
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.03083    0.04569   0.675    0.5
## x1          1.98309    0.06461  30.694 <2e-16 ***
## x2          0.94884    0.06461  14.686 <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.022 on 1497 degrees of freedom
## Multiple R-squared:  0.3864, Adjusted R-squared:  0.3856
## F-statistic: 471.4 on 2 and 1497 DF,  p-value: < 2.2e-16
```

5.2.3 Structural Equation Modeling

We have to dummy code the data set first before fitting the model. The `model.matrix` function which is used to create a design matrix can be used to dummy code `x`. Make sure that `x` is a **factor**. The first column of the design matrix is a matrix of ones. Since we do not need this column, we can replace this column with the values of `y`. Make sure to name rename the first column as `lavaan` relies on the column names.

```
df_dummy <- model.matrix(y ~ x, data = df)
df_dummy[, 1] <- df$y
colnames(df_dummy)[1] <- "y"
head(df_dummy)
```

```
##           y x1 x2
## 1 -0.6013830  0  0
## 2 -0.1358161  0  0
## 3 -0.9872728  0  0
## 4  0.8319250  0  0
## 5 -0.7950595  0  0
## 6  0.3404646  0  0
```

5.2.3.1 lavaan (Rosseel, 2012)

```
model <- "
  y ~ x1 + x2
"
fit <- lavaan::sem(
  model,
  data = df_dummy,
  meanstructure = TRUE,
  fixed.x = FALSE
)
lavaan::summary(fit)
```

```
## lavaan 0.6-7 ended normally after 22 iterations
##
##   Estimator                      ML
##   Optimization method          NLMINB
##   Number of free parameters      9
##
##   Number of observations          1500
##
## Model Test User Model:
##
##   Test statistic                  0.000
##   Degrees of freedom              0
##
## Parameter Estimates:
##
##   Standard errors                Standard
##   Information                    Expected
##   Information saturated (h1) model Structured
##
## Regressions:
##           Estimate Std.Err z-value P(>|z|)
##   y ~
##     x1           1.983   0.065  30.725   0.000
##     x2           0.949   0.065  14.701   0.000
##
## Covariances:
##           Estimate Std.Err z-value P(>|z|)
##   x1 ~~
##     x2          -0.111   0.006 -17.321   0.000
##
## Intercepts:
##           Estimate Std.Err z-value P(>|z|)
##   .y           0.031   0.046   0.676   0.499
##   x1           0.333   0.012  27.386   0.000
##   x2           0.333   0.012  27.386   0.000
##
## Variances:
##           Estimate Std.Err z-value P(>|z|)
##   .y           1.041   0.038  27.386   0.000
##   x1           0.222   0.008  27.386   0.000
##   x2           0.222   0.008  27.386   0.000
```

5.2.3.2 OpenMx (Boker et al., 2020)

RAM matrices can be used to specify models in `OpenMx`. Note, however, that the `u` vector in the RAM notation is `M` in the `OpenMx` notation.

```
mxData <- OpenMx::mxData(
  observed = df_dummy,
  type = "raw"
)
mxA <- OpenMx::mxMatrix(
  type = "Full",
```

```

nrow = 4,
ncol = 4,
free = c(
  F, T, T, F,
  F, F, F, F,
  F, F, F, F,
  F, F, F, F
),
values = c(
  0, 0.20, 0.20, 1,
  0, 0, 0, 0,
  0, 0, 0, 0,
  0, 0, 0, 0
),
labels = c(
  NA, "beta1", "beta2", NA,
  NA, NA, NA, NA,
  NA, NA, NA, NA,
  NA, NA, NA, NA
),
byrow = TRUE,
name = "mxA"
)
mxS <- OpenMx::mxMatrix(
  type = "Symm",
  nrow = 4,
  ncol = 4,
  free = c(
    F, F, F, F,
    F, T, F, F,
    F, F, T, F,
    F, F, F, T
  ),
  values = c(
    0,    0,    0,    0,
    0, 0.20,    0,    0,
    0,    0, 0.20,    0,
    0,    0,    0, 0.20
  ),
  labels = c(
    NA, NA, NA, NA,
    NA, "sigma2x1", NA, NA,
    NA, NA, "sigma2x2", NA,
    NA, NA, NA, "sigma2e"
  ),
  byrow = TRUE,
  name = "mxS"
)
mxM <- OpenMx::mxMatrix(
  type = "Full",
  nrow = 1,
  ncol = 4,
  free = c(

```

```

    T, T, T, F
  ),
  values = c(
    0.20,
    0.20,
    0.20,
    0
  ),
  labels = c(
    "beta0",
    "mux1",
    "mux2",
    NA
  ),
  byrow = TRUE,
  name = "mxM"
)
mxF <- OpenMx::mxMatrix(
  type = "Full",
  nrow = 3,
  ncol = 4,
  free = FALSE,
  values = c(
    1, 0, 0, 0,
    0, 1, 0, 0,
    0, 0, 1, 0
  ),
  byrow = TRUE,
  name = "mxF"
)
expRAM <- OpenMx::mxExpectationRAM(
  A = "mxA",
  S = "mxS",
  F = "mxF",
  M = "mxM",
  dimnames = c(
    "y",
    "x1",
    "x2",
    "e"
  )
)
)
objML <- OpenMx::mxFitFunctionML()
mxMod <- OpenMx::mxModel(
  name = "One Way Analysis of Variance",
  data = mxData,
  matrices = list(
    mxA,
    mxS,
    mxF,
    mxM
  ),
  expectation = expRAM,

```

```
fitfunction = objML
)
fit <- OpenMx::mxRun(mxMod)
```

```
## Running One Way Analysis of Variance with 8 parameters
```

```
summary(fit)
```

```
## Summary of One Way Analysis of Variance
##
## free parameters:
##      name matrix row col  Estimate  Std.Error A
## 1  beta1    mxA   1   2 1.98308662 0.064543779
## 2  beta2    mxA   1   3 0.94883814 0.064543143
## 3 sigma2x1  mxS   2   2 0.22222230 0.008114416
## 4 sigma2x2  mxS   3   3 0.22222238 0.008114420
## 5 sigma2e   mxS   4   4 1.04147460 0.038029458
## 6  beta0    mxM   1   y 0.03083127 0.045639092
## 7  mux1     mxM   1  x1 0.33333343 0.012171613
## 8  mux2     mxM   1  x2 0.33333344 0.012171612
##
## Model Statistics:
##      | Parameters | Degrees of Freedom | Fit (-2lnL units)
##      Model:      8              4492              8319.17
##      Saturated:   9              4491              NA
##      Independence: 6              4494              NA
## Number of observations/statistics: 1500/4500
##
## Information Criteria:
##      | df Penalty | Parameters Penalty | Sample-Size Adjusted
## AIC:   -664.8302      8335.170      8335.266
## BIC:   -24531.8162     8377.676      8352.262
## To get additional fit indices, see help(mxRefModels)
## timestamp: 2021-01-24 17:35:22
## Wall clock time: 0.03303933 secs
## optimizer: SLSQP
## OpenMx version number: 2.18.1
## Need help? See help(mxSummary)
```

5.2.4 Using the ramR Package

```
A
```

```
##      y      x1      x2 e
## y  0 2.008444 0.9885797 1
## x1 0 0.000000 0.0000000 0
## x2 0 0.000000 0.0000000 0
## e  0 0.000000 0.0000000 0
```

```
S
```

```
##      y      x1      x2      e
## y  0 0.0000000 0.0000000 0.0000000
## x1 0 0.2223705 0.0000000 0.0000000
## x2 0 0.0000000 0.2223705 0.0000000
## e  0 0.0000000 0.0000000 0.9823083
```

```
u
```

```
##      u
## y  0.3333333
## x1 0.3333333
## x2 0.3333333
## e  0.0000000
```

```
filter
```

```
##      y x1 x2 e
## y  1  0  0  0
## x1 0  1  0  0
## x2 0  0  1  0
```

The covariance expectations can be numerically derived using the `ramR::C_num()` function.

```
ramR::C_num(A, S)
```

```
##      y      x1      x2      e
## y  2.0966368 0.4466185 0.2198309 0.9823083
## x1 0.4466185 0.2223705 0.0000000 0.0000000
## x2 0.2198309 0.0000000 0.2223705 0.0000000
## e  0.9823083 0.0000000 0.0000000 0.9823083
```

The covariance expectations for the observed variables can be numerically derived using the `ramR::M_num()` function.

```
ramR::M_num(A, S, filter)
```

```
##      y      x1      x2
## y  2.0966368 0.4466185 0.2198309
## x1 0.4466185 0.2223705 0.0000000
## x2 0.2198309 0.0000000 0.2223705
```

The mean expectations can be numerically derived using the `ramR::v_num()` function.

```
ramR::v_num(A, u)
```

```
##      v
## y  1.3323411
## x1 0.3333333
## x2 0.3333333
## e  0.0000000
```

The mean expectations for the observed variables can be numerically derived using the `ramR::v_num()` function.

```
ramR::g_num(A, u, filter)
```

```
##           g
## y  1.3323411
## x1 0.3333333
## x2 0.3333333
```

5.3 Equations to RAM

The `ramR` package has a utility function to convert structural equations to RAM notation. One-way analysis of variance with three levels can be expressed in the following equations

```
eq <- "
# VARIABLE1 OPERATION VARIABLE2 LABEL
e          by      y          1
y          on      x1         beta[1]
y          on      x2         beta[2]
e          with    e          sigma[varepsilon]^2
x1         with    x1         sigma[x[1]]^2
x2         with    x2         sigma[x[2]]^2
y          on      1          beta[0]
x1         on      1          mu[x[1]]
x2         on      1          mu[x[2]]
"
```

The error term is treated as a latent variable and defined with the operation `by`. Its value is constrained to 1. The regression of y on x_1 and x_2 is defined by operation `on`. The coefficients are labeled as `beta[1]` and `beta[2]` respectively. The variance of x_1 , x_2 and the error variance are defined using the operation `with`. These are labeled `sigma[x[1]]^2`, `sigma[x[2]]^2`, and `sigma[varepsilon]^2` respectively. The intercept and the mean of x_1 and x_2 are defined using the operation `on` 1. These are labeled `beta[0]`, `mu[x[1]]`, and `mu[x[2]]` respectively.

The `ramR::eq2ram` converts the equations to RAM notation.

```
ramR::eq2ram(eq)
```

```
## $eq
##   var1  op var2          label
## 1    e  by   y           1
## 2    y  on  x1         beta[1]
## 3    y  on  x2         beta[2]
## 4    e with e sigma[varepsilon]^2
## 5   x1 with x1   sigma[x[1]]^2
## 6   x2 with x2   sigma[x[2]]^2
## 7    y  on   1         beta[0]
## 8   x1  on   1        mu[x[1]]
## 9   x2  on   1        mu[x[2]]
##
```

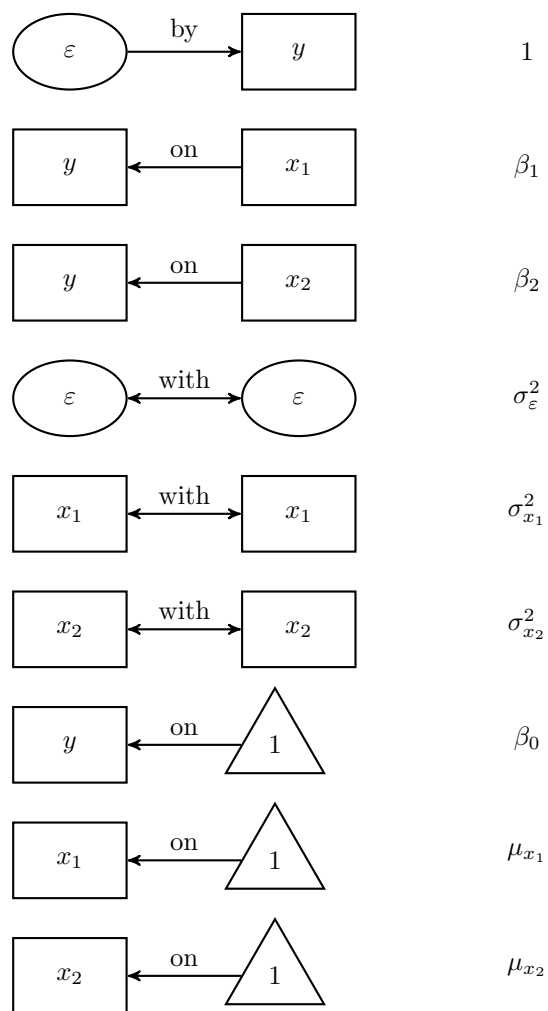


Figure 5.2: One-Way Analysis of Variance's Structural Equations


```
## $variables
## [1] "y" "x1" "x2" "e"
##
## $A
##   y   x1      x2      e
## y  "0" "beta[1]" "beta[2]" "1"
## x1 "0" "0"      "0"      "0"
## x2 "0" "0"      "0"      "0"
## e  "0" "0"      "0"      "0"
##
## $S
##   y   x1      x2      e
## y  "0" "0"      "0"      "0"
## x1 "0" "sigma[x[1]]^2" "0"      "0"
## x2 "0" "0"      "sigma[x[2]]^2" "0"
## e  "0" "0"      "0"      "sigma[varepsilon]^2"
##
## $filter
##   y x1 x2 e
## y  1  0  0  0
## x1 0  1  0  0
## x2 0  0  1  0
##
## $u
##   u
## y  "beta[0]"
## x1 "mu[x[1]]"
## x2 "mu[x[2]]"
## e  "0"
```

5.4 Equations to Expectations

The `ramR` package has a utility function to convert structural equations to expectations both symbolically and numerically.

```
eq <- "
# VARIABLE1 OPERATION VARIABLE2 LABEL
e          by          y          1
y          on          x1         beta[1]
y          on          x2         beta[2]
e          with        e          sigma[varepsilon]^2
x1         with        x1         sigma[x[1]]^2
x2         with        x2         sigma[x[2]]^2
y          on          1          beta[0]
x1         on          1          mu[x[1]]
x2         on          1          mu[x[2]]
"
```

```
ramR::eq2exp_sym(eq)
```

```
## $variables
## [1] "y" "x1" "x2" "e"
```

```

##
## $A
## {{      0, beta[1], beta[2],      1},
## {      0,      0,      0,      0},
## {      0,      0,      0,      0},
## {      0,      0,      0,      0}}
##
## $S
## {{              0,              0,              0,              0},
## {              0,      sigma[x[1]]^2,              0,              0},
## {              0,              0,      sigma[x[2]]^2,              0},
## {              0,              0,              0, sigma[varepsilon]^2}}
##
## $u
## {{ beta[0]},
## {mu[x[1]]},
## {mu[x[2]]},
## {      0}}
##
## $filter
## {{1, 0, 0, 0},
## {0, 1, 0, 0},
## {0, 0, 1, 0}}
##
## $v
## {{beta[0]+beta[1]*mu[x[1]]+beta[2]*mu[x[2]]},
## {      mu[x[1]]},
## {      mu[x[2]]},
## {      0}}
##
## $g
## {{beta[0]+beta[1]*mu[x[1]]+beta[2]*mu[x[2]]},
## {      mu[x[1]]},
## {      mu[x[2]]}}
##
## $C
## {{sigma[x[1]]^2*beta[1]^2+sigma[x[2]]^2*beta[2]^2+sigma[varepsilon]^2,
## {      sigma[x[1]]^2*beta[1],
## {      sigma[x[2]]^2*beta[2],
## {      sigma[varepsilon]^2,
##
## $M
## {{sigma[x[1]]^2*beta[1]^2+sigma[x[2]]^2*beta[2]^2+sigma[varepsilon]^2,
## {      sigma[x[1]]^2*beta[1],
## {      sigma[x[2]]^2*beta[2],

```

```

eq <- "
# VARIABLE1 OPERATION VARIABLE2 LABEL
e      by      y      1
y      on      x1      2
y      on      x2      1
e      with     e      1
x1     with    x1      0.2222222222
x2     with    x2      0.2222222222

```

```

y          on          1          0
x1         on          1          0.3333333333
x2         on          1          0.3333333333
"

```

```
ramR::eq2exp_num(eq)
```

```

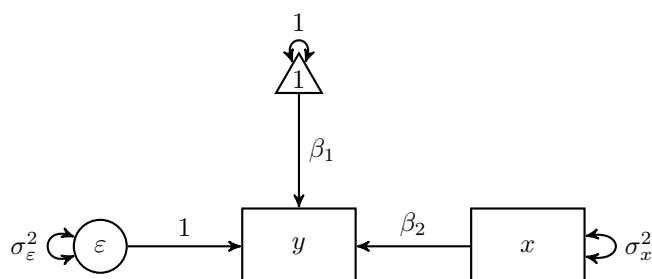
## $variables
## [1] "y" "x1" "x2" "e"
##
## $A
##      y x1 x2 e
## y  0  2  1  1
## x1 0  0  0  0
## x2 0  0  0  0
## e  0  0  0  0
##
## $S
##      y          x1          x2 e
## y  0 0.0000000 0.0000000 0
## x1 0 0.2222222 0.0000000 0
## x2 0 0.0000000 0.2222222 0
## e  0 0.0000000 0.0000000 1
##
## $u
##          u
## y  0.0000000
## x1 0.3333333
## x2 0.3333333
## e  0.0000000
##
## $filter
##      y x1 x2 e
## y  1  0  0  0
## x1 0  1  0  0
## x2 0  0  1  0
##
## $v
##          v
## y  1.0000000
## x1 0.3333333
## x2 0.3333333
## e  0.0000000
##
## $g
##          g
## y  1.0000000
## x1 0.3333333
## x2 0.3333333
##
## $C
##          y          x1          x2 e
## y  2.1111111 0.4444444 0.2222222 1

```

```
## x1 0.4444444 0.2222222 0.0000000 0
## x2 0.2222222 0.0000000 0.2222222 0
## e 1.0000000 0.0000000 0.0000000 1
##
## $M
##          y          x1          x2
## y  2.1111111 0.4444444 0.2222222
## x1 0.4444444 0.2222222 0.0000000
## x2 0.2222222 0.0000000 0.2222222
```

Chapter 6

Two-Variable Regression Model



$$y = \alpha + \beta x + \varepsilon$$

Figure 6.1: Two-Variable Regression Model

Chapter 7

k -Variable Regression Model

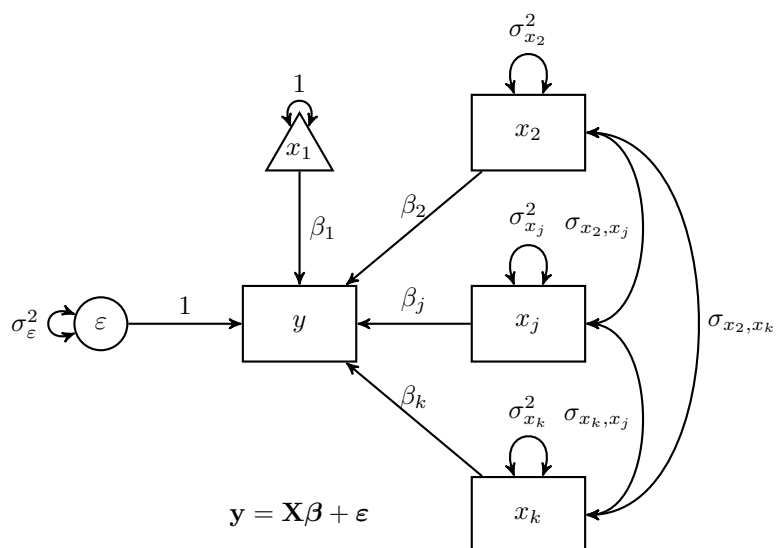


Figure 7.1: k -Variable Regression Model

Chapter 8

The Simple Mediation Model

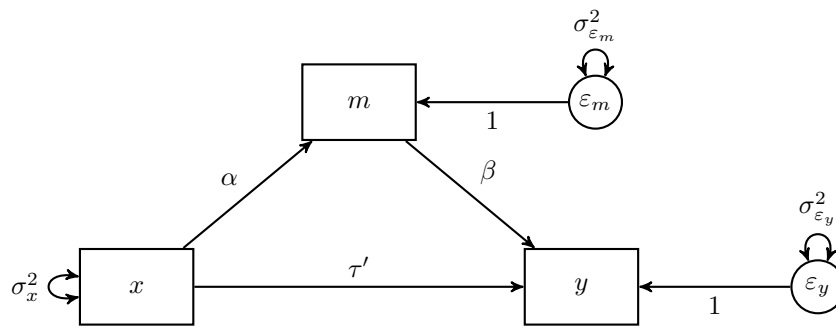


Figure 8.1: The Simple Mediation Model

Chapter 9

The Standardized Simple Mediation Model

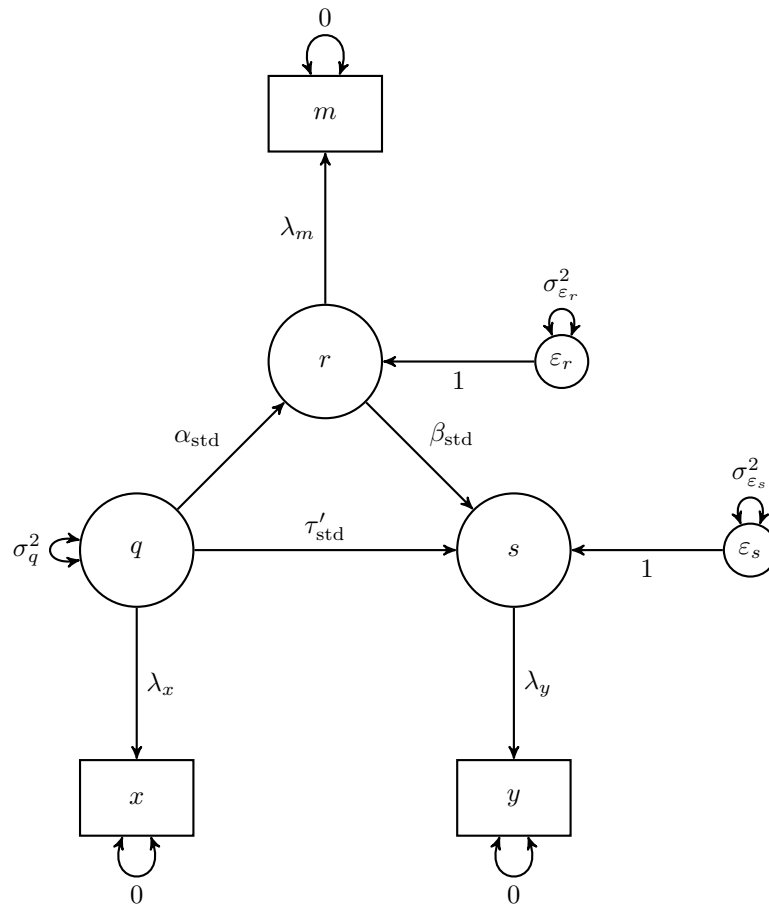


Figure 9.1: The Standardized Simple Mediation Model

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