

# Reticular Action Model (RAM) Notation Notes

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# Chapter 1

## Description

This is a collection of my personal notes on the Reticular Action Model (RAM) notation that accompanies the **ramR** package (Pesigan, 2021). You can install the released version of **ramR** from GitHub with:

```
remotes::install_github("jeksterslab/ramR")
```

These notes are based on the following resources:

- Boker and McArdle (2005)
- McArdle and McDonald (1984)
- McArdle (2005)

See GitHub Pages for the html deployment.



## Chapter 2

# Reticular Action Model (RAM) Matrix Notation

### 2.1 Full Model

**Definition 2.1.**

$$\mathbf{v} = \mathbf{A}\mathbf{v} + \mathbf{u} \quad (2.1)$$

where

- $\mathbf{v}$  and  $\mathbf{u}$  are  $t \times 1$  vectors of random variables
- $\mathbf{A}$  is a  $t \times t$  matrix of *directed* or *asymmetric* relationship from column variable  $v_j$  to row variable  $v_i$ 
  - $\mathbf{A}$  represent the regression of each of the  $t$  variables  $\mathbf{v}$  on the other  $t - 1$  variables
  - diagonal  $a_{i,i}$  is zero
  - $u_i$  represent the residual of  $v_i$
  - if all regression coefficients on other variables are zero, then the variable  $v_i$  is considered the same as its own residual  $u_i$

**Definition 2.2.**

$$\mathbf{S} = \mathbb{E} \{ \mathbf{u}\mathbf{u}' \}, \quad (2.2)$$

where

- $\mathbf{S}$  is a  $t \times t$  matrix of *undirected* or *symmetric* relationship
  - the notation  $\Omega$  is used in other sources for  $\mathbf{S}$
- $\mathbb{E}$  is the expectation operator

**Definition 2.3.**

$$\mathbf{C} = \mathbb{E} \{ \mathbf{v}\mathbf{v}' \}, \quad (2.3)$$

where

- $\mathbf{C}$  is a  $t \times t$  variance-covariance matrix
  - the notation  $\Sigma$  is used in other sources for  $\mathbf{C}$

**Definition 2.4.**

$$\mathbf{v} = \mathbf{A}\mathbf{v} + \mathbf{u}$$

can be rewritten as

$$\begin{aligned}\mathbf{v} - \mathbf{A}\mathbf{v} &= \mathbf{u} \\ \mathbf{u} &= \mathbf{v} - \mathbf{A}\mathbf{v} \\ \mathbf{u} &= (\mathbf{I} - \mathbf{A})\mathbf{v}\end{aligned}\tag{2.4}$$

assuming that  $(\mathbf{I} - \mathbf{A})$  is non-singular,

$$\mathbf{E} = (\mathbf{I} - \mathbf{A})^{-1}\tag{2.5}$$

then

$$\begin{aligned}\mathbf{v} &= (\mathbf{I} - \mathbf{A})^{-1}\mathbf{u} \\ &= \mathbf{E}\mathbf{u}.\end{aligned}\tag{2.6}$$

Using the definitions above,  $\mathbf{S}$  and  $\mathbf{C}$  are given by

$$\begin{aligned}\mathbf{S} &= (\mathbf{I} - \mathbf{A})\mathbf{C}(\mathbf{I} - \mathbf{A})^{-1} \\ &= \mathbf{E}^{-1}\mathbf{C}(\mathbf{E}^{-1})^{\top}\end{aligned}\tag{2.7}$$

$$\begin{aligned}\mathbf{C} &= (\mathbf{I} - \mathbf{A})^{-1}\mathbf{S}[(\mathbf{I} - \mathbf{A})^{-1}]^{\top} \\ &= \mathbf{E}\mathbf{S}\mathbf{E}^{\top}\end{aligned}\tag{2.8}$$

## 2.2 Observed/Manifest/Given Variables vs. Unobserved/Latent/Hidden Variables

**Definition 2.5.**

$$\mathbf{v} = \begin{bmatrix} \mathbf{g}_{p \times 1} \\ \mathbf{h}_{q \times 1} \end{bmatrix}\tag{2.9}$$

$$t = p + q\tag{2.10}$$

- $\mathbf{g}$  may be considered observed, manifest or *given* variables
- $\mathbf{h}$  may be considered unobserved, latent, or *hidden* variables

**Definition 2.6.**

$$\mathbf{F} = [\mathbf{I}_{p \times p} : \mathbf{0}_{p \times q}]\tag{2.11}$$

- the  $\mathbf{F}$  matrix acts as a *filter* to select the manifest variables out of the full set of manifest and latent variables



$$\mathbf{g} = \mathbf{F}\mathbf{v} \quad (2.12)$$

$$\begin{aligned} \mathbf{g} &= \mathbf{F}(\mathbf{I} - \mathbf{A})^{-1} \mathbf{u} \\ &= \mathbf{F}\mathbf{E}\mathbf{u} \end{aligned} \quad (2.13)$$

**Definition 2.7.**

$$\mathbf{M} = \mathbb{E} \{ \mathbf{g}\mathbf{g}^T \} \quad (2.14)$$

$$\begin{aligned} \mathbf{M} &= \mathbf{F}(\mathbf{I} - \mathbf{A})^{-1} \mathbf{S} [(\mathbf{I} - \mathbf{A})^{-1}]^T \mathbf{F}^T \\ &= \mathbf{F}\mathbf{E}\mathbf{S}\mathbf{E}^T \mathbf{F}^T \\ &= \mathbf{F}\mathbf{C}\mathbf{F}^T \end{aligned} \quad (2.15)$$

- when components of  $\mathbf{v}$  are permuted, the columns of  $\mathbf{F}$  can be correspondingly permuted
- the rows and columns of  $\mathbf{C}$  that are filtered out by  $\mathbf{F}$  contain useful information about the latent variable structure.

The equations above completely define RAM.



## Chapter 3

# Reticular Action Model (RAM) Path Diagram

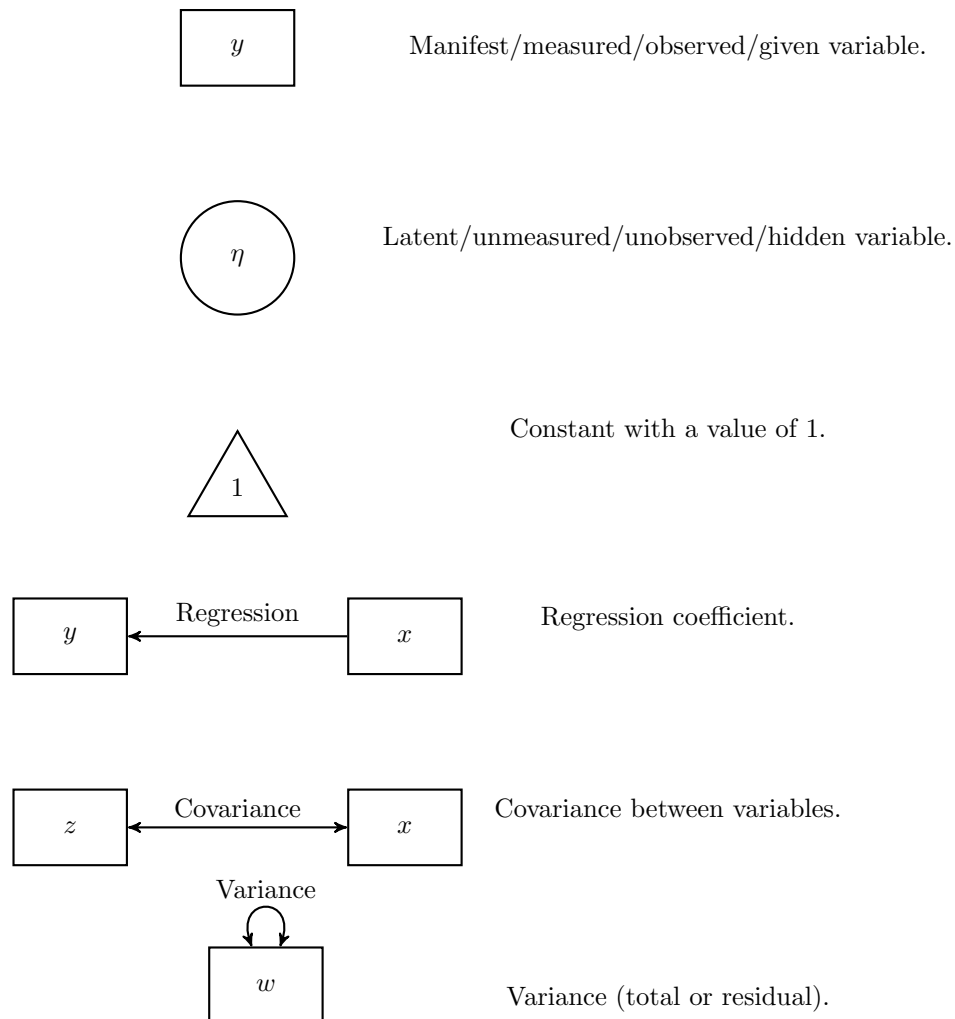
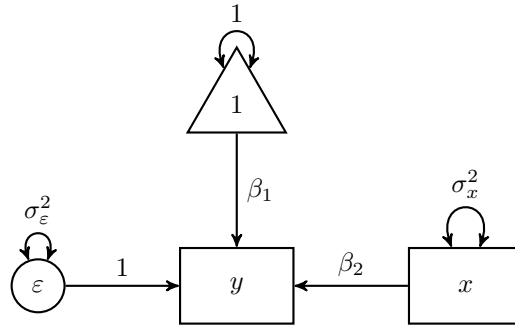
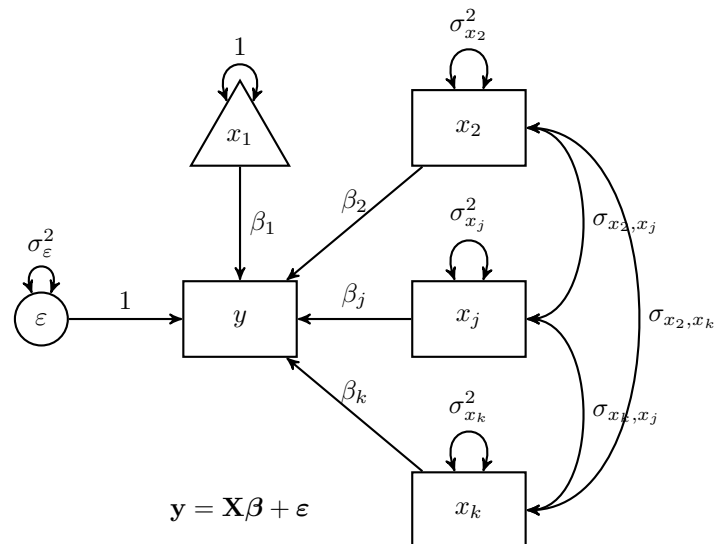


Figure 3.1: Path Diagram Elements



$$y = \alpha + \beta x + \varepsilon$$

Figure 3.2: Two-Variable Regression Model



$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

Figure 3.3:  $k$ -Variable Regression Model

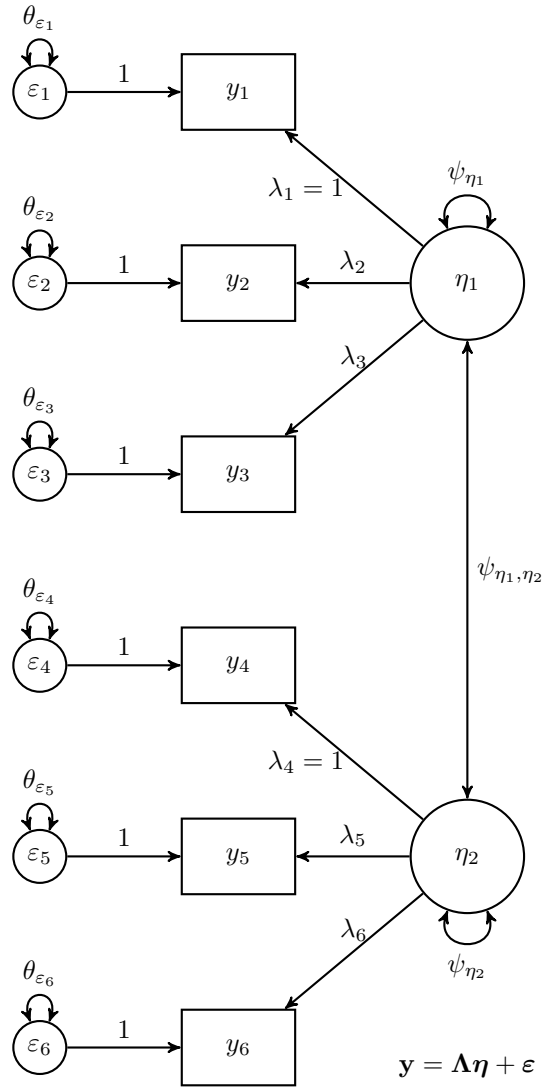


Figure 3.4: Two-Factor Confirmatory Factor Analysis Model



# Chapter 4

## Student's $t$ -test

In this section, the Student's  $t$ -test is presented as a structural equation model using the RAM notation. Let  $y$  be a continuous dependent variable,  $x$  be a dichotomous independent variable ( $x = \{0, 1\}$ ), and  $\varepsilon$  be the stochastic error term with mean 0 and constant variance of  $\sigma_\varepsilon^2$  across the values of  $x$ . The associations of the variables are given by

$$y = \alpha + \beta x + \varepsilon$$

where

- $\alpha$  is the expected value of  $y$  when  $x = 0$
- $\beta$  is the unit change in  $y$  for unit change in  $x$
- $\alpha + \beta$  is the expected value of  $y$  when  $x = 1$

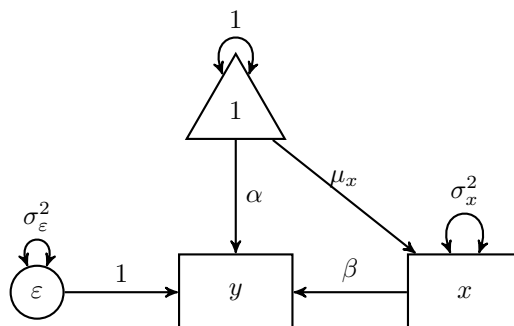


Figure 4.1: Student's  $t$ -test

### 4.1 Symbolic

Let  $\{y, x, \varepsilon\}$  be the variables of interest.

$$\mathbf{A} = \begin{pmatrix} 0 & \beta & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{S} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \sigma_x^2 & 0 \\ 0 & 0 & \sigma_\varepsilon^2 \end{pmatrix}$$

$$\mathbf{C} = (\mathbf{I} - \mathbf{A})^{-1} \mathbf{S} [(\mathbf{I} - \mathbf{A})^{-1}]^\top$$

$$= \mathbf{E} \mathbf{S} \mathbf{E}^\top$$

$$\begin{aligned} &= \begin{pmatrix} 1 & \beta & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \sigma_x^2 & 0 \\ 0 & 0 & \sigma_\varepsilon^2 \end{pmatrix} \begin{pmatrix} 1 & \beta & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}^\top \\ &= \begin{pmatrix} \sigma_x^2 \beta^2 + \sigma_\varepsilon^2 & \beta \sigma_x^2 & \sigma_\varepsilon^2 \\ \sigma_x^2 \beta & \sigma_x^2 & 0 \\ \sigma_\varepsilon^2 & 0 & \sigma_\varepsilon^2 \end{pmatrix} \end{aligned}$$

$$\mathbf{F} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\mathbf{M} = \mathbf{F} (\mathbf{I} - \mathbf{A})^{-1} \mathbf{S} [(\mathbf{I} - \mathbf{A})^{-1}]^\top \mathbf{F}^\top$$

$$= \mathbf{F} \mathbf{E} \mathbf{S} \mathbf{E}^\top \mathbf{F}^\top$$

$$= \mathbf{F} \mathbf{C} \mathbf{F}^\top$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \sigma_x^2 \beta^2 + \sigma_\varepsilon^2 & \beta \sigma_x^2 & \sigma_\varepsilon^2 \\ \sigma_x^2 \beta & \sigma_x^2 & 0 \\ \sigma_\varepsilon^2 & 0 & \sigma_\varepsilon^2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}^\top$$

$$= \begin{pmatrix} \sigma_x^2 \beta^2 + \sigma_\varepsilon^2 & \beta \sigma_x^2 \\ \sigma_x^2 \beta & \sigma_x^2 \end{pmatrix}$$

$$\mathbf{v} = (\mathbf{I} - \mathbf{A})^{-1} \mathbf{u}$$

$$= \left[ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & \beta & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right]^{-1} \begin{pmatrix} \alpha \\ \mu_x \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} \alpha + \beta \mu_x \\ \mu_x \\ 0 \end{pmatrix}$$



$$\begin{aligned}
\mathbf{u} &= (\mathbf{I} - \mathbf{A}) \mathbf{v} \\
&= \left[ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & \beta & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right] \begin{pmatrix} \alpha + \beta\mu_x \\ \mu_x \\ 0 \end{pmatrix} \\
&= \begin{pmatrix} \alpha \\ \mu_x \\ 0 \end{pmatrix}
\end{aligned}$$

$$\begin{aligned}
\mathbf{g} &= \mathbf{F} (\mathbf{I} - \mathbf{A})^{-1} \mathbf{u} \\
&= \left[ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & \beta & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right]^{-1} \begin{pmatrix} \alpha \\ \mu_x \\ 0 \end{pmatrix} \\
&= \begin{pmatrix} \alpha + \beta\mu_x \\ \mu_x \\ 0 \end{pmatrix}
\end{aligned}$$

#### 4.1.1 Using the ramR Package

A

```
##      [,1] [,2] [,3]
## [1,] "0"  "beta" "1"
## [2,] "0"  "0"    "0"
## [3,] "0"  "0"    "0"
```

S

```
##      [,1] [,2]      [,3]
## [1,] "0"  "0"      "0"
## [2,] "0"  "sigma[x]^2" "0"
## [3,] "0"  "0"      "sigma[varepsilon]^2"
```

u

```
##      [,1]
## [1,] "alpha"
## [2,] "mu[x]"
## [3,] "0"
```

filter

```
##      [,1] [,2] [,3]
## [1,] 1    0    0
## [2,] 0    1    0
```

The covariance expectations can be symbolically derived using the `ramR::C_sym()` function.

```
ramR::C_sym(A, S)
```

```
## {{sigma[x]^2*beta^2+sigma[varepsilon]^2,          beta*sigma[x]^2,          si
## {          sigma[x]^2*beta,          sigma[x]^2,
## {          sigma[varepsilon]^2,          0,          si
```

$$\mathbf{C} = \begin{pmatrix} \sigma_x^2\beta^2 + \sigma_\varepsilon^2 & \beta\sigma_x^2 & \sigma_\varepsilon^2 \\ \sigma_x^2\beta & \sigma_x^2 & 0 \\ \sigma_\varepsilon^2 & 0 & \sigma_\varepsilon^2 \end{pmatrix}$$

The covariance expectations for the observed variables can be symbolically derived using the `ramR::M_sym()` function.

```
ramR::M_sym(A, S, filter)
```

```
## {{sigma[x]^2*beta^2+sigma[varepsilon]^2,          beta*sigma[x]^2},
## {          sigma[x]^2*beta,          sigma[x]^2}}
```

$$\mathbf{M} = \begin{pmatrix} \sigma_x^2\beta^2 + \sigma_\varepsilon^2 & \beta\sigma_x^2 \\ \sigma_x^2\beta & \sigma_x^2 \end{pmatrix}$$

The mean expectations can be symbolically derived using the `ramR::v_sym()` function.

```
ramR::v_sym(A, u)
```

```
## {{alpha+beta*mu[x]},
## {          mu[x]},
## {          0}}
```

$$\mathbf{v} = \begin{pmatrix} \alpha + \beta\mu_x \\ \mu_x \\ 0 \end{pmatrix}$$

The mean expectations for the observed variables can be symbolically derived using the `ramR::g_sym()` function.

```
ramR::g_sym(A, u, filter)
```

```
## {{alpha+beta*mu[x]},
## {          mu[x]}}
```

$$\mathbf{g} = \begin{pmatrix} \alpha + \beta\mu_x \\ \mu_x \end{pmatrix}$$

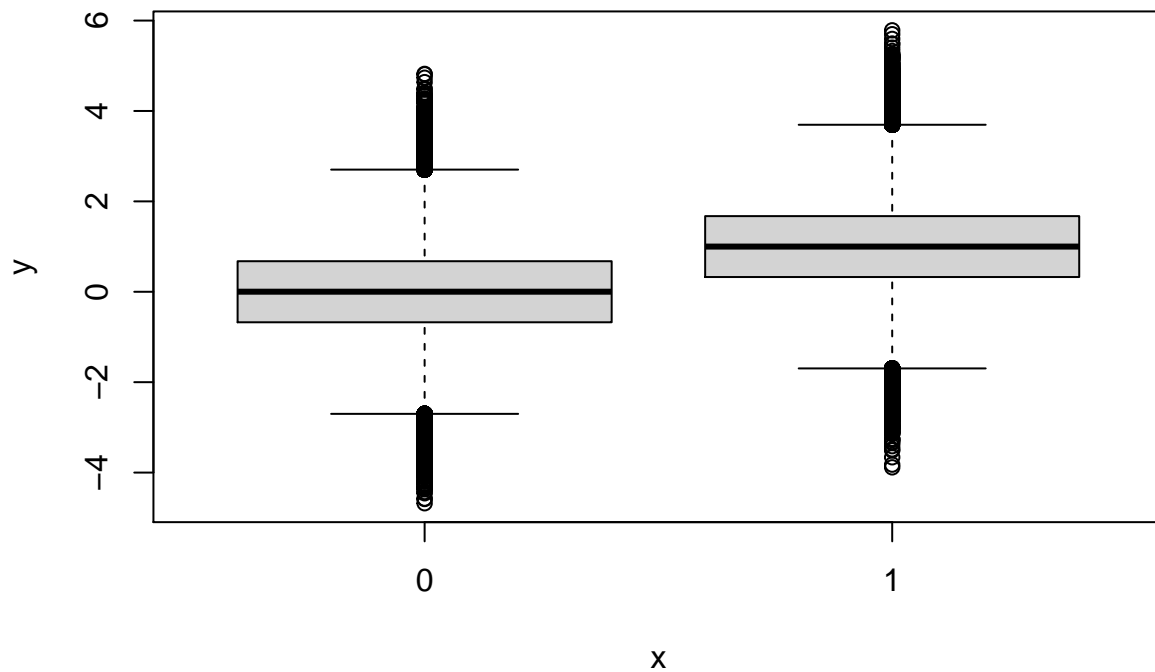
## 4.2 Numerical Example

```
head(df)
```

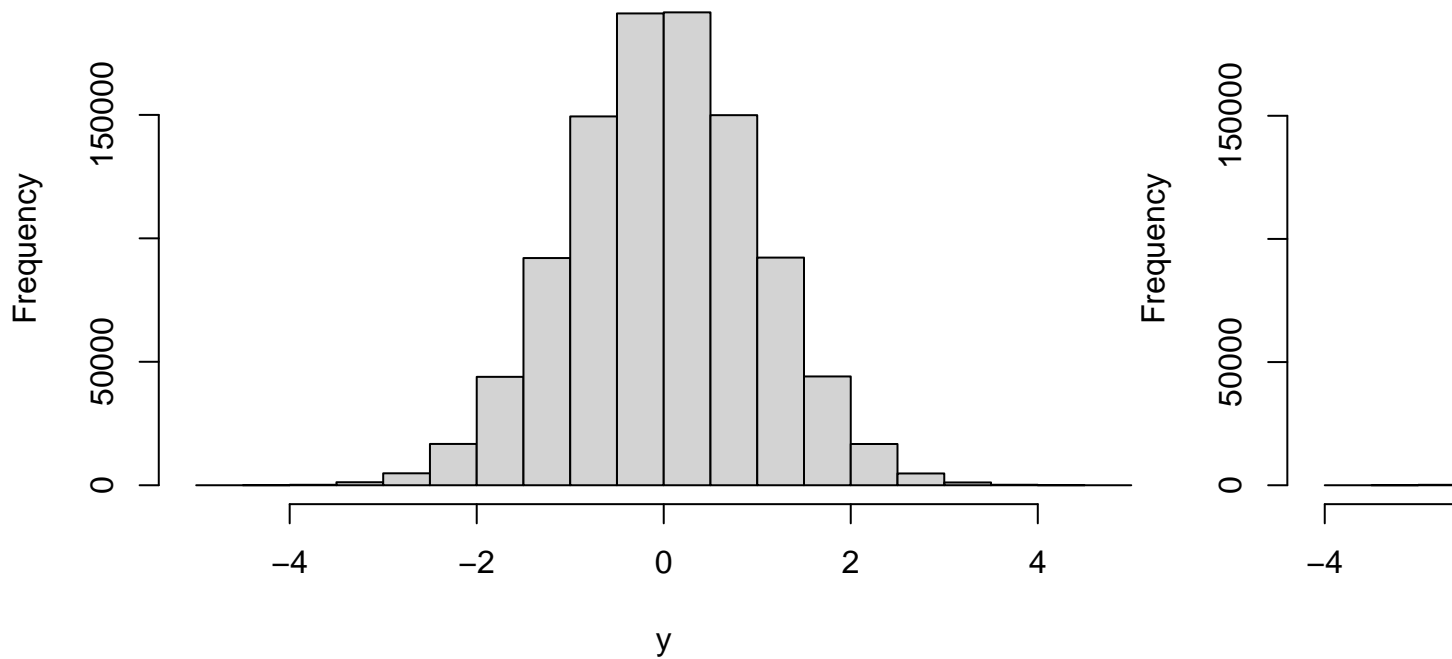
```
##           y x
## 1  1.3709584 0
## 2 -0.5646982 0
## 3  0.3631284 0
## 4  0.6328626 0
## 5  0.4042683 0
## 6 -0.1061245 0
```

```
summary(df)
```

```
##           y           x
## Min.      :-4.6785  Min.   :0.0
## 1st Qu.: -0.2622  1st Qu.:0.0
## Median :  0.5013  Median :0.5
## Mean   :  0.5000  Mean   :0.5
## 3rd Qu.:  1.2618  3rd Qu.:1.0
## Max.    :  5.7839  Max.   :1.0
```



### Histogram of y for x = 0.



#### 4.2.1 t-test

```
t.test(y ~ x, data = df)
```

```
##
## Welch Two Sample t-test
##
## data: y by x
## t = -706.06, df = 2e+06, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
## -1.0016565 -0.9961108
## sample estimates:
## mean in group 0 mean in group 1
## 0.0005737398 0.9994574009
```

#### 4.2.2 Linear Regression

```
summary(lm(y ~ x, data = df))
```

```
##
## Call:
## lm(formula = y ~ x, data = df)
##
```

```
## Residuals:
##      Min       1Q   Median       3Q      Max
## -4.8838 -0.6745  0.0005   0.6749  4.8195
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.0005737  0.0010004   0.574   0.566
## x            0.9988837  0.0014147 706.057 <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1 on 1999998 degrees of freedom
## Multiple R-squared:  0.1995, Adjusted R-squared:  0.1995
## F-statistic: 4.985e+05 on 1 and 1999998 DF,  p-value: < 2.2e-16
```

### 4.2.3 Structural Equation Modeling

```
model <- "
  y ~ x
  y ~ 1
  x ~ 1
"
fit <- lavaan::sem(model, data = df)
lavaan::summary(fit)
```

```
## lavaan 0.6-7 ended normally after 15 iterations
##
##      Estimator                      ML
##      Optimization method          NLMINB
##      Number of free parameters      5
##
##      Number of observations          2000000
##
## Model Test User Model:
##
##      Test statistic                  0.000
##      Degrees of freedom              0
##
## Parameter Estimates:
##
##      Standard errors                Standard
##      Information                    Expected
##      Information saturated (h1) model Structured
##
## Regressions:
##              Estimate Std.Err z-value P(>|z|)
## y ~
## x              0.999   0.001 706.057   0.000
##
## Intercepts:
##              Estimate Std.Err z-value P(>|z|)
## .y              0.001   0.001   0.574   0.566
```

```
##      x              0.500    0.000 1414.214    0.000
##
## Variances:
##              Estimate Std.Err z-value P(>|z|)
##      .y              1.001    0.001 1000.000    0.000
##      x              0.250    0.000 1000.000    0.000
```

label	parameter
$\alpha$	0
$\beta$	1
$\sigma^2_x$	0.25
$\sigma^2_{\epsilon}$	0.25
$\mu_x$	0.5

#### 4.2.4 Using the ramR Package

A

```
##      y      x e
## y 0 0.9988837 1
## x 0 0.0000000 0
## e 0 0.0000000 0
```

S

```
##      y      x      e
## y 0 0.0000000 0.0000000
## x 0 0.2500001 0.0000000
## e 0 0.0000000 0.2494423
```

u

```
##              [,1]
## y 0.0005737398
## x 0.5000000000
## e 0.0000000000
```

filter

```
##      y x e
## y 1 0 0
## x 0 1 0
```

The covariance expectations can be numerically derived using the `ramR::C_num()` function.

```
ramR::C_num(A, S)
```

```
##      y      x      e
## y 0.4988845 0.2497210 0.2494423
## x 0.2497210 0.2500001 0.0000000
## e 0.2494423 0.0000000 0.2494423
```

The covariance expectations for the observed variables can be numerically derived using the `ramR::M_num()` function.

```
ramR::M_num(A, S, filter)
```

```
##           y           x
## y 0.4988845 0.2497210
## x 0.2497210 0.2500001
```

The mean expectations can be numerically derived using the `ramR::v_num()` function.

```
ramR::v_num(A, u)
```

```
##           v
## y 0.5000156
## x 0.5000000
## e 0.0000000
```

The mean expectations for the observed variables can be numerically derived using the `ramR::v_num()` function.

```
ramR::g_num(A, u, filter)
```

```
##           g
## y 0.5000156
## x 0.5000000
```





# Bibliography

- Boker, S. M. and McArdle, J. J. (2005). Path analysis and path diagrams. In Everitt, B. S. and Howell, D. C., editors, *Encyclopedia of Statistics in Behavioral Science*, pages 1529–1531. John Wiley & Sons, Ltd, Chichester, UK.
- McArdle, J. J. (2005). The development of the RAM rules for latent variable structural equation modeling. In Maydeu-Olivares, A. and McArdle, J. J., editors, *Contemporary psychometrics: A festschrift for Rodrick P. McDonald*, Multivariate applications book series, pages 225–273. Lawrence Erlbaum Associates, Mahwah, NJ.
- McArdle, J. J. and McDonald, R. P. (1984). Some algebraic properties of the reticular action model for moment structures. *British Journal of Mathematical and Statistical Psychology*, 37(2):234–251.
- Pesigan, I. J. A. (2021). *ramR: Reticular Action Model (RAM) Notation*. R package version 0.9.0.