Reticular Action Model (RAM) Notation

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Contents

Chapter 1

Description

This is a collection of my personal notes on the Reticular Action Model (RAM) notation that accompanies the `ram` package. You can install the released version of `ram` from GitHub with:

```
remotes::install_github("jeksterslab/ram")
```

See GitHub Pages for the html deployment.

Chapter 2

Reticular Action Model (RAM) Matrix Notation

The model-implied mean vector $\mu\left(\theta\right)$ as a function of Reticular Action Model (RAM) matrices is given by

$$\mu\left(\theta\right) = \mathbf{F}\left(\mathbf{I} - \mathbf{A}\right)^{-1}\mathbf{m}. \tag{2.1}$$

The `ram::mutheta()` function can be used to derive the model-implied mean vector.

The model-implied variance-covariance matrix $\Sigma\left(\theta\right)$ as a function of Reticular Action Model (RAM) matrices is given by

$$\Sigma\left(\theta\right) = \mathbf{F}\left(\mathbf{I} - \mathbf{A}\right)^{-1}\Omega\left[\left(\mathbf{I} - \mathbf{A}\right)^{-1}\right]^{\mathsf{T}}\mathbf{F}^{\mathsf{T}}. \tag{2.2}$$

The `ram::Sigmatheta()` function can be used to derive the model-implied variance-covariance matrix.

Chapter 3

Simple Regression

Let v_1, v_2, and u be random variables whose associations are given by the regression equation

$$\begin{aligned} v_1 &= m_1 + a_{1,2} v_2 + u \\ &= -3.951208 + 1.269259 \cdot v_2 + u. \end{aligned} \tag{3.1}$$

v_1 and v_2 are observed variables and u is a stochastic error term which is normally distributed around zero with constant variance across values of v_2

$$u \sim \mathcal{N}\left(m_3 = 0, \omega_{3,3} = 47.659854\right). \tag{3.2}$$

v_2 has a mean of $m_2 = 13.038328$ and a variance of $\omega_{2,2} = 7.151261$.

Below are two ways of specifying this model. The first specification includes the error term u as a latent variable. The second specification only includes the observed variables.

3.1 Specification 1 - Includes Error Term as a Latent Variable

3.1.1 Matrix Notation

$$\text{variables} = \begin{bmatrix} v_1 \\ v_2 \\ u \end{bmatrix} \tag{3.3}$$

$$\mathbf{A} = \begin{bmatrix} 0 & a_{1,2} & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1.269259 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{3.4}$$

$$\Omega = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \omega_{2,2} & 0 \\ 0 & 0 & \omega_{3,3} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 7.151261 & 0 \\ 0 & 0 & 47.659854 \end{bmatrix} \tag{3.5}$$

$$\mathbf{m} = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} -3.951208 \\ 13.038328 \\ 0 \end{bmatrix} \tag{3.6}$$

To filter the observed variables, use the following filter matrix

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}. \tag{3.7}$$

To include all variables, use the following filter matrix

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{3.8}$$

3.1.2 Expectations

$$\mathbb{E}\left(u\right) = m_3 = 0 \tag{3.9}$$

$$\mathbb{E}\left(v_2\right) = m_2 = 13.038328 \tag{3.10}$$

$$\begin{aligned} \mathbb{E}\left(v_1\right) &= \mathbb{E}\left(m_1 + a_{1,2}v_2 + u\right) \\ &= \mathbb{E}\left(m_1\right) + \mathbb{E}\left(a_{1,2}v_2\right) + \mathbb{E}\left(u\right) \\ &= m_1 + a_{1,2}\mathbb{E}\left(v_2\right) + 0 \\ &= m_1 + a_{1,2}m_2 \\ &= -3.951208 + 1.269259 \times 13.038328 \\ &= 12.5978072 \end{aligned} \tag{3.11}$$

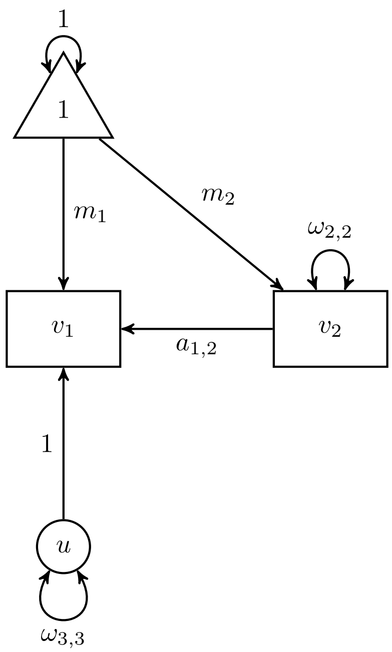

Figure 3.1: The Simple Linear Regression Model (with u)

$$
\begin{aligned}
\mathbb{E}\left(\begin{bmatrix} v_1 \\ v_2 \\ u \end{bmatrix}\right) &= \begin{bmatrix} m_1 + a_{1,2} m_2 \\ m_2 \\ m_3 \end{bmatrix} \\
&= \begin{bmatrix} 12.5978072 \\ 13.038328 \\ 0 \end{bmatrix}
\end{aligned} \tag{3.12}
$$

$$
\begin{aligned}
\operatorname{Cov}(u, u) &= \operatorname{Var}(u) \\
&= \omega_{3,3} \\
&= 47.659854
\end{aligned} \tag{3.13}
$$

$$\begin{aligned}
\operatorname{Cov}(v_1, u) &= \operatorname{Cov}\left(a_{1,2}v_2 + u, u\right) \\
&= \operatorname{Cov}\left(a_{1,2}v_2, u\right) + \operatorname{Cov}(u, u) \\
&= a_{1,2}^2 \operatorname{Cov}(v_2, u) + \operatorname{Var}(u) \\
&= a_{1,2}^2 \cdot 0 + \omega_{3,3} \\
&= 0 + \omega_{3,3} \\
&= \omega_{3,3} \\
&= 47.659854
\end{aligned} \tag{3.14}$$

$$\operatorname{Cov}(v_2, u) = 0 \tag{3.15}$$

$$\begin{aligned}
\operatorname{Cov}(v_1, v_1) &= \operatorname{Cov}\left(a_{1,2}v_2 + u, a_{1,2}v_2 + u\right) \\
&= \operatorname{Cov}\left(a_{1,2}v_2, a_{1,2}v_2\right) + \operatorname{Cov}\left(a_{1,2}v_2, u\right) + \operatorname{Cov}\left(a_{1,2}v_2, u\right) + \operatorname{Cov}(u, u) \\
&= a_{1,2}^2 \operatorname{Cov}(v_2, v_2) + a_{1,2} \operatorname{Cov}(v_2, u) + a_{1,2} \operatorname{Cov}(v_2, u) + \operatorname{Var}(u) \\
&= a_{1,2}^2 \operatorname{Var}(v_2) + a_{1,2} \cdot 0 + a_{1,2} \cdot 0 + \omega_{3,3} \\
&= a_{1,2}^2 \operatorname{Var}(v_2) + 0 + 0 + \omega_{3,3} \\
&= a_{1,2}^2 \omega_{2,2} + \omega_{3,3} \\
&= 1.269259^2 \times 7.151261 + 47.659854 \\
&= 59.1806671
\end{aligned} \tag{3.16}$$

$$\begin{aligned}
\operatorname{Cov}(v_2, v_1) &= \operatorname{Cov}\left(v_2, a_{1,2}v_2 + u\right) \\
&= \operatorname{Cov}\left(v_2, a_{1,2}v_2\right) + \operatorname{Cov}(v_2, u) \\
&= a_{1,2} \operatorname{Cov}(v_2, v_2) + 0 \\
&= a_{1,2} \operatorname{Var}(v_2) \\
&= a_{1,2}\omega_{2,2} \\
&= 1.269259 \times 7.151261 \\
&= 9.0768024
\end{aligned} \tag{3.17}$$

$$\begin{aligned}
\operatorname{Cov}(v_2, v_2) &= \operatorname{Var}(v_2) \\
&= \omega_{2,2} \\
&= 7.151261
\end{aligned} \tag{3.18}$$

Table 3.1: $\boldsymbol{\mu}\left(\boldsymbol{\theta}\right)$

	μ
v_1	12.59781
v_2	13.03833
u	0.00000

$$\begin{aligned}
\mathrm{Cov}\left(\begin{bmatrix} v_1 \\ v_2 \\ u \end{bmatrix}\right) &= \begin{bmatrix} a_{1,2}^2\omega_{2,2} + \omega_{3,3} & a_{1,2}\omega_{2,2} & \omega_{3,3} \\ a_{1,2}\omega_{2,2} & \omega_{2,2} & 0 \\ \omega_{3,3} & 0 & \omega_{3,3} \end{bmatrix} \\
&= \begin{bmatrix} 59.1806671 & 9.0768024 & 47.659854 \\ 9.0768024 & 7.151261 & 0 \\ 47.659854 & 0 & 47.659854 \end{bmatrix}
\end{aligned} \tag{3.19}$$

3.1.2.1 Using the `ram()` Package

```
knitr::kable(
  ram::mutheta(
    m,
    A = A,
    filter = filter
  ),
  col.names = "$\\boldsymbol{\\mu}$",
  caption = "$\\boldsymbol{\\mu} \\left( \\boldsymbol{\\theta} \\right)$",
  escape = FALSE
)
```

```
knitr::kable(
  ram::Sigmatheta(
    A = A,
    Omega = Omega,
    filter = filter
  ),
  caption = "$\\boldsymbol{\\Sigma} \\left( \\boldsymbol{\\theta} \\right)$",
  escape = FALSE
)
```

Table 3.2: $\Sigma(\theta)$

	v_1	v_2	u
v_1	59.180667	9.076802	47.65985
v_2	9.076802	7.151261	0.00000
u	47.659854	0.000000	47.65985

3.2 Specification 2 - Observed Variables

3.2.1 Matrix Notation

$$\text{variables} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{3.20}$$

$$\mathbf{A} = \begin{bmatrix} 0 & a_{1,2} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1.269259 \\ 0 & 0 \end{bmatrix} \tag{3.21}$$

$$\Omega = \begin{bmatrix} \omega_{1,1} & 0 \\ 0 & \omega_{2,2} \end{bmatrix} = \begin{bmatrix} 47.659854 & 0 \\ 0 & 7.151261 \end{bmatrix} \tag{3.22}$$

$$\mathbf{m} = \begin{bmatrix} m_1 \\ m_2 \end{bmatrix} = \begin{bmatrix} -3.951208 \\ 13.038328 \end{bmatrix} \tag{3.23}$$

$$\mathbf{F} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{3.24}$$

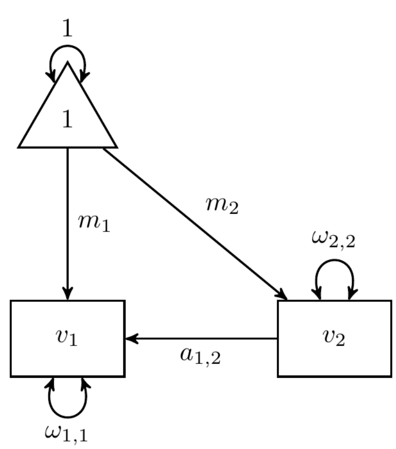

Figure 3.2: The Simple Linear Regression Model (without u)

Table 3.3: $\boldsymbol{\mu}\left(\boldsymbol{\theta}\right)$

	$\boldsymbol{\mu}$
v_1	12.59781
v_2	13.03833

Table 3.4: $\boldsymbol{\Sigma}\left(\boldsymbol{\theta}\right)$

	v_1	v_2
v_1	59.180667	9.076802
v_2	9.076802	7.151261

3.2.1.1 Using the `ram()` Package

```
knitr::kable(
  ram::mutheta(
    m,
    A = A,
    filter = filter
  ),
  col.names = "$\\boldsymbol{\\mu}$",
  caption = "$\\boldsymbol{\\mu} \\left( \\boldsymbol{\\theta} \\right)$",
  escape = FALSE
)
```

```
knitr::kable(
  ram::Sigmatheta(
    A = A,
    Omega = Omega,
    filter = filter
  ),
  caption = "$\\boldsymbol{\\Sigma} \\left( \\boldsymbol{\\theta} \\right)$",
  escape = FALSE
)
```