Covariance Matrix from Correlation Matrix

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Scalar Notation

The covariance between X and Y from the correlation between X and Y is given by Equation (1)

$$\sigma_{X,Y} = \sigma_X \sigma_Y \rho_{X,Y} \tag{1}$$

where $\rho_{X,Y}$ is the correlation between X and Y, σ_X is the standard deviation of X, and σ_Y is the standard deviation of Y.

Matrix Notation

The covariance matrix from the correlation matrix is given by Equation (2)

$$\boldsymbol{\Sigma} = \text{diag}\left(\boldsymbol{\Sigma}\right)^{\frac{1}{2}} \ \boldsymbol{P} \ \text{diag}\left(\boldsymbol{\Sigma}\right)^{\frac{1}{2}} \tag{2}$$

where $\boldsymbol{\Sigma}$ is the covariance matrix, $\boldsymbol{P}$ is the correlation matrix, $\text{diag}\left(\cdot\right)$ is an operator that creates a diagonal matrix from the diagonal elements of the input matrix $\boldsymbol{\Sigma}$.

Table 1: Variables

Variable	Symbol	Description
`sigmacap_i`	$\boldsymbol{\Sigma}$	covariance matrix
`rhocap_i`	$\boldsymbol{P}$	correlation matrix
`sd_i`	$\boldsymbol{\sigma}$	standard deviation vector

Examples

$$\boldsymbol{\Sigma} = \begin{pmatrix} 0.6856935 & -0.042434 & 1.2743154 & 0.5162707 \\ -0.042434 & 0.1899794 & -0.3296564 & -0.1216394 \\ 1.2743154 & -0.3296564 & 3.1162779 & 1.2956094 \\ 0.5162707 & -0.1216394 & 1.2956094 & 0.5810063 \end{pmatrix} \tag{3}$$

$$\boldsymbol{P} = \begin{pmatrix} 1 & -0.1175698 & 0.8717538 & 0.8179411 \\ -0.1175698 & 1 & -0.4284401 & -0.3661259 \\ 0.8717538 & -0.4284401 & 1 & 0.9628654 \\ 0.8179411 & -0.3661259 & 0.9628654 & 1 \end{pmatrix} \tag{4}$$

```
library(rhoMatrix)
```

```
sigmacap_i
```

```
##              Sepal.Length Sepal.Width Petal.Length Petal.Width
## Sepal.Length    0.6856935  -0.0424340    1.2743154   0.5162707
## Sepal.Width    -0.0424340   0.1899794   -0.3296564  -0.1216394
## Petal.Length    1.2743154  -0.3296564    3.1162779   1.2956094
## Petal.Width     0.5162707  -0.1216394    1.2956094   0.5810063
```

```
sd_i
```

```
## Sepal.Length  Sepal.Width Petal.Length  Petal.Width
```

```
##    0.8280661    0.4358663    1.7652982    0.7622377
```

```
rhocap_i
```

```
##              Sepal.Length Sepal.Width Petal.Length Petal.Width
## Sepal.Length    1.0000000  -0.1175698    0.8717538   0.8179411
## Sepal.Width    -0.1175698   1.0000000   -0.4284401  -0.3661259
## Petal.Length    0.8717538  -0.4284401    1.0000000   0.9628654
## Petal.Width     0.8179411  -0.3661259    0.9628654   1.0000000
```

Covariance Matrix from Correlation Matrix

```
cov_of_cor(
  rhocap_i,
  sd = sd_i
)
```

```
##              Sepal.Length Sepal.Width Petal.Length Petal.Width
## Sepal.Length    0.6856935  -0.0424340    1.2743154   0.5162707
## Sepal.Width    -0.0424340   0.1899794   -0.3296564  -0.1216394
## Petal.Length    1.2743154  -0.3296564    3.1162779   1.2956094
## Petal.Width     0.5162707  -0.1216394    1.2956094   0.5810063
```