sigmaMatrix: Covariance Matrix

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Definition

Let X and Y be random variables.

$$\begin{aligned}\operatorname{Cov}(X, Y) &= \mathbb{E}\left[(X - \mathbb{E}[X])(Y - \mathbb{E}[Y])\right] \\ &= \mathbb{E}[XY] - \mathbb{E}[X]\,\mathbb{E}[Y]\end{aligned} \tag{1}$$

Consistent Estimator of the Covariance Matrix

$$\hat{\sigma}_{X,Y} = n^{-1} \sum_{i=1}^{n} \left[(X_i - \hat{\mu}_X)(Y_i - \hat{\mu}_Y)\right] \tag{2}$$

$$\widehat{\boldsymbol{\Sigma}} = n^{-1} \sum_{i=1}^{n} \left[(\boldsymbol{x}_i - \hat{\boldsymbol{\mu}})(\boldsymbol{x}_i - \hat{\boldsymbol{\mu}})'\right] \tag{3}$$

Unbiased Estimator of the Covariance Matrix

$$\hat{\sigma}_{X,Y} = (n-1)^{-1} \sum_{i=1}^{n} \left[(X_i - \hat{\mu}_X)(Y_i - \hat{\mu}_Y)\right] \tag{4}$$

$$\widehat{\boldsymbol{\Sigma}} = (n-1)^{-1} \sum_{i=1}^{n} \left[(\boldsymbol{x}_i - \hat{\boldsymbol{\mu}})(\boldsymbol{x}_i - \hat{\boldsymbol{\mu}})'\right] \tag{5}$$

Examples

Consistent Estimate of the Covariance Matrix

```
covc(iris[, 1:4])
```

```
##              Sepal.Length Sepal.Width Petal.Length Petal.Width
## Sepal.Length   0.68112222 -0.04215111    1.2658200   0.5128289
## Sepal.Width   -0.04215111  0.18871289   -0.3274587  -0.1208284
## Petal.Length   1.26582000 -0.32745867    3.0955027   1.2869720
## Petal.Width    0.51282889 -0.12082844    1.2869720   0.5771329
```

```
n <- dim(iris)[1]
cov(iris[, 1:4]) * (n - 1) / n
```

```
##              Sepal.Length Sepal.Width Petal.Length Petal.Width
## Sepal.Length   0.68112222 -0.04215111    1.2658200   0.5128289
## Sepal.Width   -0.04215111  0.18871289   -0.3274587  -0.1208284
## Petal.Length   1.26582000 -0.32745867    3.0955027   1.2869720
## Petal.Width    0.51282889 -0.12082844    1.2869720   0.5771329
```

Unbiased Estimate of the Covariance Matrix

```
cov(iris[, 1:4])
```

```
##              Sepal.Length Sepal.Width Petal.Length Petal.Width
## Sepal.Length    0.6856935  -0.0424340    1.2743154   0.5162707
## Sepal.Width    -0.0424340   0.1899794   -0.3296564  -0.1216394
## Petal.Length    1.2743154  -0.3296564    3.1162779   1.2956094
```

```
## Petal.Width     0.5162707  -0.1216394    1.2956094   0.5810063
```