Package 'simStateSpace'

January 15, 2024

Title Simulate Data from State Space Models

Version 1.1.0

Description Provides a streamlined and user-friendly framework
for simulating data in state space models,
particularly when the number of subjects/units (n) exceeds one,
a scenario commonly encountered in social and behavioral sciences.
For an introduction to state space models in social and behavioral sciences,
refer to Chow, Ho, Hamaker, and Dolan (2010) <doi:10.1080/10705511003661553>.

URL https://github.com/jeksterslab/simStateSpace,
https://jeksterslab.github.io/simStateSpace/

BugReports https://github.com/jeksterslab/simStateSpace/issues

License GPL (>= 3)

Encoding UTF-8

Roxygen list(markdown = TRUE)

Depends R (>= 3.0.0)

LinkingTo Rcpp, RcppArmadillo

Imports Rcpp, graphics

Suggests knitr, rmarkdown, testthat, Matrix

RoxygenNote 7.3.0

NeedsCompilation yes

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R topics documented:

`as.data.frame.simstatespace`

Coerce an Object of Class `simstatespace` to a Data Frame

Description

Coerce an Object of Class `simstatespace` to a Data Frame

Usage

```
## S3 method for class 'simstatespace'
as.data.frame(
  x,
  row.names = NULL,
  optional = FALSE,
  eta = FALSE,
  long = TRUE,
  ...
)
```

Arguments

`x`	Object of class `simstatespace`.
`row.names`	`NULL` or character vector giving the row names for the data frame. Missing values are not allowed.
`optional`	Logical. If `TRUE`, setting row names and converting column names is optional.
`eta`	Logical. If `eta = TRUE`, include `eta`. If `eta = FALSE`, exclude `eta`.
`long`	Logical. If `long = TRUE`, use long format. If `long = FALSE`, use wide format.
`...`	Additional arguments.

Author(s)

Ivan Jacob Agaloos Pesigan

Examples

```
# prepare parameters
set.seed(42)
k <- p <- 3
iden <- diag(k)
null_vec <- rep(x = 0, times = k)
n <- 5
mu0 <- null_vec
sigma0 <- iden
alpha <- null_vec
beta <- diag(x = 0.50, nrow = k)
psi <- iden
nu <- null_vec
lambda <- iden
theta <- diag(x = 0.50, nrow = k)
time <- 50
burn_in <- 0
gamma_y <- gamma_eta <- 0.10 * diag(k)
x <- lapply(
  X = seq_len(n),
  FUN = function(i) {
    return(
      matrix(
        data = rnorm(n = k * (time + burn_in)),
        ncol = k
      )
    )
  }
)

# Type 0
ssm <- SimSSMFixed(
  n = n,
  mu0 = mu0,
  sigma0 = sigma0,
  alpha = alpha,
  beta = beta,
  psi = psi,
  nu = nu,
  lambda = lambda,
  theta = theta,
  type = 0,
  time = time,
  burn_in = burn_in
)

head(as.data.frame(ssm))
head(as.data.frame(ssm, long = FALSE))
```

as.matrix.simstatespace — *Coerce an Object of Class* `simstatespace` *to a Matrix*

Description

Coerce an Object of Class `simstatespace` to a Matrix

Usage

```
## S3 method for class 'simstatespace'
as.matrix(x, eta = FALSE, long = TRUE, ...)
```

Arguments

`x`	Object of class `simstatespace`.
`eta`	Logical. If `eta = TRUE`, include `eta`. If `eta = FALSE`, exclude `eta`.
`long`	Logical. If `long = TRUE`, use long format. If `long = FALSE`, use wide format.
`...`	Additional arguments.

Author(s)

Ivan Jacob Agaloos Pesigan

Examples

```
# prepare parameters
set.seed(42)
k <- p <- 3
iden <- diag(k)
null_vec <- rep(x = 0, times = k)
n <- 5
mu0 <- null_vec
sigma0 <- iden
alpha <- null_vec
beta <- diag(x = 0.50, nrow = k)
psi <- iden
nu <- null_vec
lambda <- iden
theta <- diag(x = 0.50, nrow = k)
time <- 50
burn_in <- 0
gamma_y <- gamma_eta <- 0.10 * diag(k)
x <- lapply(
  X = seq_len(n),
  FUN = function(i) {
```

```
    return(
      matrix(
        data = rnorm(n = k * (time + burn_in)),
        ncol = k
      )
    )
  }
)

# Type 0
ssm <- SimSSMFixed(
  n = n,
  mu0 = mu0,
  sigma0 = sigma0,
  alpha = alpha,
  beta = beta,
  psi = psi,
  nu = nu,
  lambda = lambda,
  theta = theta,
  type = 0,
  time = time,
  burn_in = burn_in
)

head(as.matrix(ssm))
head(as.matrix(ssm, long = FALSE))
```

OU2SSM *Convert Parameters from the Ornstein–Uhlenbeck Model to State Space Model Parameterization*

Description

This function converts parameters from the Ornstein–Uhlenbeck model to state space model parameterization.

Usage

```
OU2SSM(mu, phi, sigma, delta_t)
```

Arguments

mu	Numeric vector. The long-term mean or equilibrium level ($\boldsymbol{\mu}$).
phi	Numeric matrix. The rate of mean reversion, determining how quickly the variable returns to its mean ($\boldsymbol{\Phi}$).
sigma	Numeric matrix. The matrix of volatility or randomness in the process ($\boldsymbol{\Sigma}$).
delta_t	Numeric. Time interval (δ_t).

Details

The state space parameters as a function of the Ornstein–Uhlenbeck model parameters are given by

$$\boldsymbol{\beta} = \exp\left(-\boldsymbol{\Phi}\Delta_t\right)$$

$$\boldsymbol{\alpha} = -\boldsymbol{\Phi}^{-1}\left(\boldsymbol{\beta} - \mathbf{I}_p\right)$$

$$\mathrm{vec}\left(\boldsymbol{\Psi}\right) = \left\{\left[\left(-\boldsymbol{\Phi} \otimes \mathbf{I}_p\right) + \left(\mathbf{I}_p \otimes -\boldsymbol{\Phi}\right)\right]\left[\exp\left(\left[\left(-\boldsymbol{\Phi} \otimes \mathbf{I}_p\right) + \left(\mathbf{I}_p \otimes -\boldsymbol{\Phi}\right)\right]\Delta_t\right) - \mathbf{I}_{p\times p}\right]\mathrm{vec}\left(\boldsymbol{\Sigma}\right)\right\}$$

Value

Returns a list of state space parameters:

- `alpha`: Numeric vector. Vector of intercepts for the dynamic model ($\boldsymbol{\alpha}$).
- `beta`: Numeric matrix. Transition matrix relating the values of the latent variables at time `t - 1` to those at time `t` ($\boldsymbol{\beta}$).
- `psi`: Numeric matrix. The process noise covariance matrix ($\boldsymbol{\Psi}$).

Author(s)

Ivan Jacob Agaloos Pesigan

See Also

Other Simulation of State Space Models Data Functions: `SimSSMFixed()`, `SimSSMIVary()`, `SimSSMLinGrowthIVary()`, `SimSSMLinGrowth()`, `SimSSMOUFixed()`, `SimSSMOUIVary()`, `SimSSMOU()`, `SimSSMVARFixed()`, `SimSSMVARIVary()`, `SimSSMVAR()`, `SimSSM()`

Examples

```
p <- k <- 2
mu <- c(5.76, 5.18)
phi <- matrix(
  data = c(0.10, -0.05, -0.05, 0.10),
  nrow = p
)
sigma <- matrix(
  data = c(2.79, 0.06, 0.06, 3.27),
  nrow = p
)
delta_t <- 0.10

OU2SSM(
  mu = mu,
  phi = phi,
  sigma = sigma,
  delta_t = delta_t
)
```

`plot.simstatespace` *Plot Method for an Object of Class* `simstatespace`

Description

Plot Method for an Object of Class `simstatespace`

Usage

```
## S3 method for class 'simstatespace'
plot(x, id = NULL, time = NULL, eta = FALSE, type = "b", ...)
```

Arguments

`x`	Object of class `simstatespace`.
`id`	Numeric vector. Optional `id` numbers to plot. If `id = NULL`, plot all available data.
`time`	Numeric vector. Optional `time` points to plot. If `time = NULL`, plot all available data.
`eta`	Logical. If `eta = TRUE`, plot the latent variables. If `eta = FALSE`, plot the observed variables.
`type`	Character indicating the type of plotting; actually any of the types as in `plot.default()`.
`...`	Additional arguments.

Author(s)

Ivan Jacob Agaloos Pesigan

Examples

```
# prepare parameters
set.seed(42)
k <- p <- 3
iden <- diag(k)
null_vec <- rep(x = 0, times = k)
n <- 5
mu0 <- null_vec
sigma0 <- iden
alpha <- null_vec
beta <- diag(x = 0.50, nrow = k)
psi <- iden
nu <- null_vec
lambda <- iden
theta <- diag(x = 0.50, nrow = k)
time <- 50
burn_in <- 0
gamma_y <- gamma_eta <- 0.10 * diag(k)
```

```
x <- lapply(
  X = seq_len(n),
  FUN = function(i) {
    return(
      matrix(
        data = rnorm(n = k * (time + burn_in)),
        ncol = k
      )
    )
  }
)

# Type 0
ssm <- SimSSMFixed(
  n = n,
  mu0 = mu0,
  sigma0 = sigma0,
  alpha = alpha,
  beta = beta,
  psi = psi,
  nu = nu,
  lambda = lambda,
  theta = theta,
  type = 0,
  time = time,
  burn_in = burn_in
)

plot(ssm)
plot(ssm, id = 1:3, time = 1:10)
```

`print.simstatespace` *Print Method for an Object of Class* `simstatespace`

Description

Print Method for an Object of Class `simstatespace`

Usage

```
## S3 method for class 'simstatespace'
print(x, ...)
```

Arguments

`x`	Object of Class `simstatespace`.
`...`	Additional arguments.

Value

Prints simulated data in long format.

Author(s)

Ivan Jacob Agaloos Pesigan

Examples

```
# prepare parameters
set.seed(42)
k <- p <- 3
iden <- diag(k)
null_vec <- rep(x = 0, times = k)
n <- 5
mu0 <- null_vec
sigma0 <- iden
alpha <- null_vec
beta <- diag(x = 0.50, nrow = k)
psi <- iden
nu <- null_vec
lambda <- iden
theta <- diag(x = 0.50, nrow = k)
time <- 50
burn_in <- 0
gamma_y <- gamma_eta <- 0.10 * diag(k)
x <- lapply(
  X = seq_len(n),
  FUN = function(i) {
    return(
      matrix(
        data = rnorm(n = k * (time + burn_in)),
        ncol = k
      )
    )
  }
)

# Type 0
ssm <- SimSSMFixed(
  n = n,
  mu0 = mu0,
  sigma0 = sigma0,
  alpha = alpha,
  beta = beta,
  psi = psi,
  nu = nu,
  lambda = lambda,
  theta = theta,
  type = 0,
  time = time,
  burn_in = burn_in
```

```
)

print(ssm)
```

SimSSM *Simulate Data from a State Space Model (n = 1)*

Description

This function simulates data from a state space model.

Usage

```
SimSSM(
  mu0,
  sigma0,
  alpha,
  beta,
  psi,
  nu,
  lambda,
  theta,
  gamma_y = NULL,
  gamma_eta = NULL,
  x = NULL,
  type = 0,
  time,
  burn_in = 0
)
```

Arguments

`mu0`	Numeric vector. Mean of initial latent variable values ($\boldsymbol{\mu}_{\boldsymbol{\eta}\|0}$).
`sigma0`	Numeric matrix. The covariance matrix of initial latent variable values ($\boldsymbol{\Sigma}_{\boldsymbol{\eta}\|0}$).
`alpha`	Numeric vector. Vector of intercepts for the dynamic model ($\boldsymbol{\alpha}$).
`beta`	Numeric matrix. Transition matrix relating the values of the latent variables at time `t - 1` to those at time `t` ($\boldsymbol{\beta}$).
`psi`	Numeric matrix. The process noise covariance matrix ($\boldsymbol{\Psi}$).
`nu`	Numeric vector. Vector of intercepts for the measurement model ($\boldsymbol{\nu}$).
`lambda`	Numeric matrix. Factor loading matrix linking the latent variables to the observed variables ($\boldsymbol{\Lambda}$).
`theta`	Numeric matrix. The measurement error covariance matrix ($\boldsymbol{\Theta}$).
`gamma_y`	Numeric matrix. Matrix relating the values of the covariate matrix at time `t` to the observed variables at time `t` ($\boldsymbol{\Gamma}_{\mathbf{y}}$).

gamma_eta	Numeric matrix. Matrix relating the values of the covariate matrix at time `t` to the latent variables at time `t` ($\boldsymbol{\Gamma}_{\boldsymbol{\eta}}$).
x	Numeric matrix. The matrix of observed covariates in `type = 1` or `type = 2`. The number of rows should be equal to `time + burn_in`.
type	Integer. State space model type. See Details for more information.
time	Positive integer. Number of time points to simulate.
burn_in	Positive integer. Number of burn-in points to exclude before returning the results.

Details

Type 0:
The measurement model is given by

$$\mathbf{y}_t = \boldsymbol{\nu} + \boldsymbol{\Lambda}\boldsymbol{\eta}_t + \boldsymbol{\varepsilon}_t \quad \text{with} \quad \boldsymbol{\varepsilon}_t \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Theta}\right)$$

where $\mathbf{y}_t$, $\boldsymbol{\eta}_t$, and $\boldsymbol{\varepsilon}_t$ are random variables and $\boldsymbol{\nu}$, $\boldsymbol{\Lambda}$, and $\boldsymbol{\Theta}$ are model parameters. $\mathbf{y}_t$ is a vector of observed random variables, $\boldsymbol{\eta}_t$ is a vector of latent random variables, and $\boldsymbol{\varepsilon}_t$ is a vector of random measurement errors, at time t. $\boldsymbol{\nu}$ is a vector of intercepts, $\boldsymbol{\Lambda}$ is a matrix of factor loadings, and $\boldsymbol{\Theta}$ is the covariance matrix of $\boldsymbol{\varepsilon}$.
The dynamic structure is given by

$$\boldsymbol{\eta}_t = \boldsymbol{\alpha} + \boldsymbol{\beta}\boldsymbol{\eta}_{t-1} + \boldsymbol{\zeta}_t \quad \text{with} \quad \boldsymbol{\zeta}_t \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Psi}\right)$$

where $\boldsymbol{\eta}_t$, $\boldsymbol{\eta}_{t-1}$, and $\boldsymbol{\zeta}_t$ are random variables, and $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, and $\boldsymbol{\Psi}$ are model parameters. $\boldsymbol{\eta}_t$ is a vector of latent variables at time t, $\boldsymbol{\eta}_{t-1}$ is a vector of latent variables at time $t-1$, and $\boldsymbol{\zeta}_t$ is a vector of dynamic noise at time t. $\boldsymbol{\alpha}$ is a vector of intercepts, $\boldsymbol{\beta}$ is a matrix of autoregression and cross regression coefficients, and $\boldsymbol{\Psi}$ is the covariance matrix of $\boldsymbol{\zeta}_t$.

Type 1:
The measurement model is given by

$$\mathbf{y}_t = \boldsymbol{\nu} + \boldsymbol{\Lambda}\boldsymbol{\eta}_t + \boldsymbol{\varepsilon}_t \quad \text{with} \quad \boldsymbol{\varepsilon}_t \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Theta}\right).$$

The dynamic structure is given by

$$\boldsymbol{\eta}_t = \boldsymbol{\alpha} + \boldsymbol{\beta}\boldsymbol{\eta}_{t-1} + \boldsymbol{\Gamma}_{\boldsymbol{\eta}}\mathbf{x}_t + \boldsymbol{\zeta}_t \quad \text{with} \quad \boldsymbol{\zeta}_t \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Psi}\right)$$

where $\mathbf{x}_t$ is a vector of covariates at time t, and $\boldsymbol{\Gamma}_{\boldsymbol{\eta}}$ is the coefficient matrix linking the covariates to the latent variables.

Type 2:
The measurement model is given by

$$\mathbf{y}_t = \boldsymbol{\nu} + \boldsymbol{\Lambda}\boldsymbol{\eta}_t + \boldsymbol{\Gamma}_{\mathbf{y}}\mathbf{x}_t + \boldsymbol{\varepsilon}_t \quad \text{with} \quad \boldsymbol{\varepsilon}_t \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Theta}\right)$$

where $\boldsymbol{\Gamma}_{\mathbf{y}}$ is the coefficient matrix linking the covariates to the observed variables.
The dynamic structure is given by

$$\boldsymbol{\eta}_t = \boldsymbol{\alpha} + \boldsymbol{\beta}\boldsymbol{\eta}_{t-1} + \boldsymbol{\Gamma}_{\boldsymbol{\eta}}\mathbf{x}_t + \boldsymbol{\zeta}_t \quad \text{with} \quad \boldsymbol{\zeta}_t \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Psi}\right).$$

Value

Returns an object of class `simstatespace` which is a list with the following elements:

- `call`: Function call.
- `args`: Function arguments.
- `data`: Generated data which is a list of length `n`. `data` is a list with the following elements:
  - `id`: A vector of ones of length `t`.
  - `time`: A vector of time points of length `t`.
  - `y`: A `t` by `k` matrix of values for the manifest variables.
  - `eta`: A `t` by `p` matrix of values for the latent variables.
  - `x`: A `t` by `j` matrix of values for the covariates.
- `fun`: Function used.

References

Chow, S.-M., Ho, M. R., Hamaker, E. L., & Dolan, C. V. (2010). Equivalence and differences between structural equation modeling and state-space modeling techniques. *Structural Equation Modeling: A Multidisciplinary Journal*, 17(2), 303–332. doi:10.1080/10705511003661553

See Also

Other Simulation of State Space Models Data Functions: `OU2SSM()`, `SimSSMFixed()`, `SimSSMIVary()`, `SimSSMLinGrowthIVary()`, `SimSSMLinGrowth()`, `SimSSMOUFixed()`, `SimSSMOUIVary()`, `SimSSMOU()`, `SimSSMVARFixed()`, `SimSSMVARIVary()`, `SimSSMVAR()`

Examples

```
# prepare parameters
set.seed(42)
k <- p <- 3
iden <- diag(k)
null_vec <- rep(x = 0, times = k)
mu0 <- null_vec
sigma0 <- iden
alpha <- null_vec
beta <- diag(x = 0.50, nrow = k)
psi <- iden
nu <- null_vec
lambda <- iden
theta <- diag(x = 0.50, nrow = k)
time <- 1000
burn_in <- 0
gamma_y <- gamma_eta <- 0.10 * diag(k)
x <- matrix(
  data = rnorm(n = k * (time + burn_in)),
  ncol = k
)

# Type 0
```

```
ssm <- SimSSM(
  mu0 = mu0,
  sigma0 = sigma0,
  alpha = alpha,
  beta = beta,
  psi = psi,
  nu = nu,
  lambda = lambda,
  theta = theta,
  type = 0,
  time = time,
  burn_in = burn_in
)

plot(ssm)

# Type 1
ssm <- SimSSM(
  mu0 = mu0,
  sigma0 = sigma0,
  alpha = alpha,
  beta = beta,
  psi = psi,
  nu = nu,
  lambda = lambda,
  theta = theta,
  gamma_eta = gamma_eta,
  x = x,
  type = 1,
  time = time,
  burn_in = burn_in
)

plot(ssm)

# Type 2
ssm <- SimSSM(
  mu0 = mu0,
  sigma0 = sigma0,
  alpha = alpha,
  beta = beta,
  psi = psi,
  nu = nu,
  lambda = lambda,
  theta = theta,
  gamma_y = gamma_y,
  gamma_eta = gamma_eta,
  x = x,
  type = 2,
  time = time,
  burn_in = burn_in
)
```

```
plot(ssm)
```

SimSSMFixed *Simulate Data using a State Space Model Parameterization for n > 1 Individuals (Fixed Parameters)*

Description

This function simulates data using a state space model parameterization for n > 1 individuals. In this model, the parameters are invariant across individuals.

Usage

```
SimSSMFixed(
  n,
  mu0,
  sigma0,
  alpha,
  beta,
  psi,
  nu,
  lambda,
  theta,
  gamma_y = NULL,
  gamma_eta = NULL,
  x = NULL,
  type = 0,
  time,
  burn_in = 0
)
```

Arguments

n	Positive integer. Number of individuals.
mu0	Numeric vector. Mean of initial latent variable values ($\boldsymbol{\mu}_{\boldsymbol{\eta}\mid 0}$).
sigma0	Numeric matrix. The covariance matrix of initial latent variable values ($\boldsymbol{\Sigma}_{\boldsymbol{\eta}\mid 0}$).
alpha	Numeric vector. Vector of intercepts for the dynamic model ($\boldsymbol{\alpha}$).
beta	Numeric matrix. Transition matrix relating the values of the latent variables at time t - 1 to those at time t ($\boldsymbol{\beta}$).
psi	Numeric matrix. The process noise covariance matrix ($\boldsymbol{\Psi}$).
nu	Numeric vector. Vector of intercepts for the measurement model ($\boldsymbol{\nu}$).
lambda	Numeric matrix. Factor loading matrix linking the latent variables to the observed variables ($\boldsymbol{\Lambda}$).
theta	Numeric matrix. The measurement error covariance matrix ($\boldsymbol{\Theta}$).

gamma_y	Numeric matrix. Matrix relating the values of the covariate matrix at time t to the observed variables at time t ($\boldsymbol{\Gamma}_{\mathbf{y}}$).
gamma_eta	Numeric matrix. Matrix relating the values of the covariate matrix at time t to the latent variables at time t ($\boldsymbol{\Gamma}_{\boldsymbol{\eta}}$).
x	A list of length n of numeric matrices. Each element of the list is a matrix of observed covariates in type = 1 or type = 2. The number of rows in each matrix should be equal to time + burn_in.
type	Integer. State space model type. See Details for more information.
time	Positive integer. Number of time points to simulate.
burn_in	Positive integer. Number of burn-in points to exclude before returning the results.

Details

Type 0:
The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \boldsymbol{\Lambda} \boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t} \quad \text{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N} \left(\mathbf{0}, \boldsymbol{\Theta} \right)$$

where $\mathbf{y}_{i,t}$, $\boldsymbol{\eta}_{i,t}$, and $\boldsymbol{\varepsilon}_{i,t}$ are random variables and $\boldsymbol{\nu}$, $\boldsymbol{\Lambda}$, and $\boldsymbol{\Theta}$ are model parameters. $\mathbf{y}_{i,t}$ is a vector of observed random variables, $\boldsymbol{\eta}_{i,t}$ is a vector of latent random variables, and $\boldsymbol{\varepsilon}_{i,t}$ is a vector of random measurement errors, at time t and individual i. $\boldsymbol{\nu}$ is a vector of intercepts, $\boldsymbol{\Lambda}$ is a matrix of factor loadings, and $\boldsymbol{\Theta}$ is the covariance matrix of $\boldsymbol{\varepsilon}$.
The dynamic structure is given by

$$\boldsymbol{\eta}_{i,t} = \boldsymbol{\alpha} + \boldsymbol{\beta} \boldsymbol{\eta}_{i,t-1} + \boldsymbol{\zeta}_{i,t} \quad \text{with} \quad \boldsymbol{\zeta}_{i,t} \sim \mathcal{N} \left(\mathbf{0}, \boldsymbol{\Psi} \right)$$

where $\boldsymbol{\eta}_{i,t}$, $\boldsymbol{\eta}_{i,t-1}$, and $\boldsymbol{\zeta}_{i,t}$ are random variables, and $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, and $\boldsymbol{\Psi}$ are model parameters. $\boldsymbol{\eta}_{i,t}$ is a vector of latent variables at time t and individual i, $\boldsymbol{\eta}_{i,t-1}$ is a vector of latent variables at time $t-1$ and individual i, and $\boldsymbol{\zeta}_{i,t}$ is a vector of dynamic noise at time t and individual i. $\boldsymbol{\alpha}$ is a vector of intercepts, $\boldsymbol{\beta}$ is a matrix of autoregression and cross regression coefficients, and $\boldsymbol{\Psi}$ is the covariance matrix of $\boldsymbol{\zeta}_{i,t}$.

Type 1:
The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \boldsymbol{\Lambda} \boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t} \quad \text{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N} \left(\mathbf{0}, \boldsymbol{\Theta} \right).$$

The dynamic structure is given by

$$\boldsymbol{\eta}_{i,t} = \boldsymbol{\alpha} + \boldsymbol{\beta} \boldsymbol{\eta}_{i,t-1} + \boldsymbol{\Gamma}_{\boldsymbol{\eta}} \mathbf{x}_{i,t} + \boldsymbol{\zeta}_{i,t} \quad \text{with} \quad \boldsymbol{\zeta}_{i,t} \sim \mathcal{N} \left(\mathbf{0}, \boldsymbol{\Psi} \right)$$

where $\mathbf{x}_{i,t}$ is a vector of covariates at time t and individual i, and $\boldsymbol{\Gamma}_{\boldsymbol{\eta}}$ is the coefficient matrix linking the covariates to the latent variables.

Type 2:
The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \boldsymbol{\Lambda} \boldsymbol{\eta}_{i,t} + \boldsymbol{\Gamma}_{\mathbf{y}} \mathbf{x}_{i,t} + \boldsymbol{\varepsilon}_{i,t} \quad \text{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N} \left(\mathbf{0}, \boldsymbol{\Theta} \right)$$

where $\boldsymbol{\Gamma}_{\mathbf{y}}$ is the coefficient matrix linking the covariates to the observed variables.
The dynamic structure is given by

$$\boldsymbol{\eta}_{i,t} = \boldsymbol{\alpha} + \boldsymbol{\beta} \boldsymbol{\eta}_{i,t-1} + \boldsymbol{\Gamma}_{\boldsymbol{\eta}} \mathbf{x}_{i,t} + \boldsymbol{\zeta}_{i,t} \quad \text{with} \quad \boldsymbol{\zeta}_{i,t} \sim \mathcal{N} \left(\mathbf{0}, \boldsymbol{\Psi} \right).$$

Value

Returns an object of class `simstatespace` which is a list with the following elements:

- `call`: Function call.
- `args`: Function arguments.
- `data`: Generated data which is a list of length `n`. Each element of `data` is a list with the following elements:
  - `id`: A vector of ID numbers of length `t`.
  - `time`: A vector time points of length `t`.
  - `y`: A `t` by `k` matrix of values for the manifest variables.
  - `eta`: A `t` by `p` matrix of values for the latent variables.
  - `x`: A `t` by `j` matrix of values for the covariates.
- `fun`: Function used.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Chow, S.-M., Ho, M. R., Hamaker, E. L., & Dolan, C. V. (2010). Equivalence and differences between structural equation modeling and state-space modeling techniques. *Structural Equation Modeling: A Multidisciplinary Journal*, 17(2), 303–332. doi:10.1080/10705511003661553

See Also

Other Simulation of State Space Models Data Functions: `OU2SSM()`, `SimSSMIVary()`, `SimSSMLinGrowthIVary()`, `SimSSMLinGrowth()`, `SimSSMOUFixed()`, `SimSSMOUIVary()`, `SimSSMOU()`, `SimSSMVARFixed()`, `SimSSMVARIVary()`, `SimSSMVAR()`, `SimSSM()`

Examples

```
# prepare parameters
set.seed(42)
k <- p <- 3
iden <- diag(k)
null_vec <- rep(x = 0, times = k)
n <- 5
mu0 <- null_vec
sigma0 <- iden
alpha <- null_vec
beta <- diag(x = 0.50, nrow = k)
psi <- iden
nu <- null_vec
lambda <- iden
theta <- diag(x = 0.50, nrow = k)
time <- 50
burn_in <- 0
gamma_y <- gamma_eta <- 0.10 * diag(k)
```

```
x <- lapply(
  X = seq_len(n),
  FUN = function(i) {
    return(
      matrix(
        data = rnorm(n = k * (time + burn_in)),
        ncol = k
      )
    )
  }
)

# Type 0
ssm <- SimSSMFixed(
  n = n,
  mu0 = mu0,
  sigma0 = sigma0,
  alpha = alpha,
  beta = beta,
  psi = psi,
  nu = nu,
  lambda = lambda,
  theta = theta,
  type = 0,
  time = time,
  burn_in = burn_in
)

plot(ssm)

# Type 1
ssm <- SimSSMFixed(
  n = n,
  mu0 = mu0,
  sigma0 = sigma0,
  alpha = alpha,
  beta = beta,
  psi = psi,
  nu = nu,
  lambda = lambda,
  theta = theta,
  gamma_eta = gamma_eta,
  x = x,
  type = 1,
  time = time,
  burn_in = burn_in
)

plot(ssm)

# Type 2
ssm <- SimSSMFixed(
  n = n,
```

```
  mu0 = mu0,
  sigma0 = sigma0,
  alpha = alpha,
  beta = beta,
  psi = psi,
  nu = nu,
  lambda = lambda,
  theta = theta,
  gamma_y = gamma_y,
  gamma_eta = gamma_eta,
  x = x,
  type = 2,
  time = time,
  burn_in = burn_in
)

plot(ssm)
```

`SimSSMIVary` *Simulate Data using a State Space Model Parameterization for n > 1 Individuals (Individual-Varying Parameters)*

Description

This function simulates data using a state space model parameterization for `n > 1` individuals. In this model, the parameters can vary across individuals.

Usage

```
SimSSMIVary(
  n,
  mu0,
  sigma0,
  alpha,
  beta,
  psi,
  nu,
  lambda,
  theta,
  gamma_y = NULL,
  gamma_eta = NULL,
  x = NULL,
  type = 0,
  time,
  burn_in = 0
)
```

Arguments

- `n` — Positive integer. Number of individuals.
- `mu0` — List of numeric vectors. Each element of the list is the mean of initial latent variable values ($\boldsymbol{\mu}_{\boldsymbol{\eta}|0}$).
- `sigma0` — List of numeric matrices. Each element of the list is the covariance matrix of initial latent variable values ($\boldsymbol{\Sigma}_{\boldsymbol{\eta}|0}$).
- `alpha` — List of numeric vectors. Each element of the list is the vector of intercepts for the dynamic model ($\boldsymbol{\alpha}$).
- `beta` — List of numeric matrices. Each element of the list is the transition matrix relating the values of the latent variables at time `t - 1` to those at time `t` ($\boldsymbol{\beta}$).
- `psi` — List of numeric matrices. Each element of the list is the process noise covariance matrix ($\boldsymbol{\Psi}$).
- `nu` — List of numeric vectors. Each element of the list is the vector of intercepts for the measurement model ($\boldsymbol{\nu}$).
- `lambda` — List of numeric matrices. Each element of the list is the factor loading matrix linking the latent variables to the observed variables ($\boldsymbol{\Lambda}$).
- `theta` — List of numeric matrices. Each element of the list is the measurement error covariance matrix ($\boldsymbol{\Theta}$).
- `gamma_y` — Numeric matrix. Matrix relating the values of the covariate matrix at time `t` to the observed variables at time `t` ($\boldsymbol{\Gamma}_{\mathbf{y}}$).
- `gamma_eta` — Numeric matrix. Matrix relating the values of the covariate matrix at time `t` to the latent variables at time `t` ($\boldsymbol{\Gamma}_{\boldsymbol{\eta}}$).
- `x` — A list of length `n` of numeric matrices. Each element of the list is a matrix of observed covariates in `type = 1` or `type = 2`. The number of rows in each matrix should be equal to `time + burn_in`.
- `type` — Integer. State space model type. See Details for more information.
- `time` — Positive integer. Number of time points to simulate.
- `burn_in` — Positive integer. Number of burn-in points to exclude before returning the results.

Details

Parameters can vary across individuals by providing a list of parameter values. If the length of any of the parameters (`mu0`, `sigma0`, `alpha`, `beta`, `psi`, `nu`, `lambda`, `theta`, `gamma_y`, or `gamma_eta`) is less the `n`, the function will cycle through the available values.

Value

Returns an object of class `simstatespace` which is a list with the following elements:

- `call`: Function call.
- `args`: Function arguments.
- `data`: Generated data which is a list of length `n`. Each element of `data` is a list with the following elements:

- `id`: A vector of ID numbers of length `t`.
- `time`: A vector time points of length `t`.
- `y`: A `t` by `k` matrix of values for the manifest variables.
- `eta`: A `t` by `p` matrix of values for the latent variables.
- `x`: A `t` by `j` matrix of values for the covariates.

- `fun`: Function used.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Chow, S.-M., Ho, M. R., Hamaker, E. L., & Dolan, C. V. (2010). Equivalence and differences between structural equation modeling and state-space modeling techniques. *Structural Equation Modeling: A Multidisciplinary Journal*, 17(2), 303–332. doi:10.1080/10705511003661553

See Also

Other Simulation of State Space Models Data Functions: `OU2SSM()`, `SimSSMFixed()`, `SimSSMLinGrowthIVary()`, `SimSSMLinGrowth()`, `SimSSMOUFixed()`, `SimSSMOUIVary()`, `SimSSMOU()`, `SimSSMVARFixed()`, `SimSSMVARIVary()`, `SimSSMVAR()`, `SimSSM()`

Examples

```
# prepare parameters
# In this example, beta varies across individuals
set.seed(42)
k <- p <- 3
iden <- diag(k)
null_vec <- rep(x = 0, times = k)
n <- 5
mu0 <- list(null_vec)
sigma0 <- list(iden)
alpha <- list(null_vec)
beta <- list(
  diag(x = 0.1, nrow = k),
  diag(x = 0.2, nrow = k),
  diag(x = 0.3, nrow = k),
  diag(x = 0.4, nrow = k),
  diag(x = 0.5, nrow = k)
)
psi <- list(iden)
nu <- list(null_vec)
lambda <- list(iden)
theta <- list(diag(x = 0.50, nrow = k))
time <- 50
burn_in <- 0
gamma_y <- gamma_eta <- list(0.10 * diag(k))
x <- lapply(
  X = seq_len(n),
```

```
    FUN = function(i) {
      return(
        matrix(
          data = rnorm(n = k * (time + burn_in)),
          ncol = k
        )
      )
    }
  )

  # Type 0
  ssm <- SimSSMIVary(
    n = n,
    mu0 = mu0,
    sigma0 = sigma0,
    alpha = alpha,
    beta = beta,
    psi = psi,
    nu = nu,
    lambda = lambda,
    theta = theta,
    type = 0,
    time = time,
    burn_in = burn_in
  )

  plot(ssm)

  # Type 1
  ssm <- SimSSMIVary(
    n = n,
    mu0 = mu0,
    sigma0 = sigma0,
    alpha = alpha,
    beta = beta,
    psi = psi,
    nu = nu,
    lambda = lambda,
    theta = theta,
    gamma_eta = gamma_eta,
    x = x,
    type = 1,
    time = time,
    burn_in = burn_in
  )

  plot(ssm)

  # Type 2
  ssm <- SimSSMIVary(
    n = n,
    mu0 = mu0,
    sigma0 = sigma0,
```

```
  alpha = alpha,
  beta = beta,
  psi = psi,
  nu = nu,
  lambda = lambda,
  theta = theta,
  gamma_y = gamma_y,
  gamma_eta = gamma_eta,
  x = x,
  type = 2,
  time = time,
  burn_in = burn_in
)

plot(ssm)
```

SimSSMLinGrowth *Simulate Data from a Linear Growth Curve Model*

Description

This function simulates data from a linear growth curve model for `n > 1` individuals.

Usage

```
SimSSMLinGrowth(
  n,
  mu0,
  sigma0,
  theta,
  gamma_y = NULL,
  gamma_eta = NULL,
  x = NULL,
  type = 0,
  time
)
```

Arguments

n	Positive integer. Number of individuals.
mu0	Numeric vector. A vector of length two. The first element is the mean of the intercept, and the second element is the mean of the slope.
sigma0	Numeric matrix. The covariance matrix of the intercept and the slope.
theta	Numeric. The common measurement error variance.
gamma_y	Numeric matrix. Matrix relating the values of the covariate matrix at time `t` to `y` at time `t` ($\boldsymbol{\Gamma}_{\mathbf{y}}$).

gamma_eta	Numeric matrix. Matrix relating the values of the covariate matrix at time t to the latent variables (intercept and slope) at time t ($\boldsymbol{\Gamma}_{\boldsymbol{\eta}}$).
x	A list of length n of numeric matrices. Each element of the list is a matrix of observed covariates in type = 1 or type = 2. The number of rows in each matrix should be equal to time.
type	Integer. State space model type. See Details for more information.
time	Positive integer. Number of time points to simulate.

Details

Type 0:

The measurement model is given by

$$y_{i,t} = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} \eta 0_{i,t} \\ \eta 1_{i,t} \end{pmatrix} + \boldsymbol{\varepsilon}_{i,t} \quad \text{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}(0, \theta)$$

where $y_{i,t}$, $\eta 0_{i,t}$, $\eta 1_{i,t}$, and $\boldsymbol{\varepsilon}_{i,t}$ are random variables and θ is a model parameter. $y_{i,t}$ is a vector of observed random variables at time t and individual i, $\eta 0_{i,t}$ and $\eta 1_{i,t}$ form a vector of latent random variables at time t and individual i, and $\boldsymbol{\varepsilon}_{i,t}$ is a vector of random measurement errors at time t and individual i. θ is the variance of $\boldsymbol{\varepsilon}$.

The dynamic structure is given by

$$\begin{pmatrix} \eta 0_{i,t} \\ \eta 1_{i,t} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \eta 0_{i,t-1} \\ \eta 1_{i,t-1} \end{pmatrix}.$$

The mean vector and covariance matrix of the intercept and slope are captured in the mean vector and covariance matrix of the initial condition given by

$$\boldsymbol{\mu}_{\boldsymbol{\eta} \mid 0} = \begin{pmatrix} \mu_{\eta_0} \\ \mu_{\eta_1} \end{pmatrix} \quad \text{and,}$$

$$\boldsymbol{\Sigma}_{\boldsymbol{\eta} \mid 0} = \begin{pmatrix} \sigma^2_{\eta_0} & \sigma_{\eta_0, \eta_1} \\ \sigma_{\eta_1, \eta_0} & \sigma^2_{\eta_1} \end{pmatrix}.$$

Type 1:

The measurement model is given by

$$y_{i,t} = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} \eta 0_{i,t} \\ \eta 1_{i,t} \end{pmatrix} + \boldsymbol{\varepsilon}_{i,t} \quad \text{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}(0, \theta).$$

The dynamic structure is given by

$$\begin{pmatrix} \eta 0_{i,t} \\ \eta 1_{i,t} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \eta 0_{i,t-1} \\ \eta 1_{i,t-1} \end{pmatrix} + \boldsymbol{\Gamma}_{\boldsymbol{\eta}} \mathbf{x}_{i,t}$$

where $\mathbf{x}_{i,t}$ is a vector of covariates at time t and individual i, and $\boldsymbol{\Gamma}_{\boldsymbol{\eta}}$ is the coefficient matrix linking the covariates to the latent variables.

Type 2:

The measurement model is given by

$$y_{i,t} = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} \eta_{0_{i,t}} \\ \eta_{1_{i,t}} \end{pmatrix} + \boldsymbol{\Gamma}_{\mathbf{y}} \mathbf{x}_{i,t} + \boldsymbol{\varepsilon}_{i,t} \quad \text{with} \quad \varepsilon_{i,t} \sim \mathcal{N}(0, \theta)$$

where $\boldsymbol{\Gamma}_{\mathbf{y}}$ is the coefficient matrix linking the covariates to the observed variables.

The dynamic structure is given by

$$\begin{pmatrix} \eta_{0_{i,t}} \\ \eta_{1_{i,t}} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \eta_{0_{i,t-1}} \\ \eta_{1_{i,t-1}} \end{pmatrix} + \boldsymbol{\Gamma}_{\boldsymbol{\eta}} \mathbf{x}_{i,t}.$$

Value

Returns an object of class `simstatespace` which is a list with the following elements:

- `call`: Function call.
- `args`: Function arguments.
- `data`: Generated data which is a list of length `n`. Each element of `data` is a list with the following elements:
  - `id`: A vector of ID numbers of length `t`.
  - `time`: A vector time points of length `t`.
  - `y`: A `t` by `k` matrix of values for the manifest variables.
  - `eta`: A `t` by `p` matrix of values for the latent variables.
  - `x`: A `t` by `j` matrix of values for the covariates.
- `fun`: Function used.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Chow, S.-M., Ho, M. R., Hamaker, E. L., & Dolan, C. V. (2010). Equivalence and differences between structural equation modeling and state-space modeling techniques. *Structural Equation Modeling: A Multidisciplinary Journal*, 17(2), 303–332. doi:10.1080/10705511003661553

See Also

Other Simulation of State Space Models Data Functions: `OU2SSM()`, `SimSSMFixed()`, `SimSSMIVary()`, `SimSSMLinGrowthIVary()`, `SimSSMOUFixed()`, `SimSSMOUIVary()`, `SimSSMOU()`, `SimSSMVARFixed()`, `SimSSMVARIVary()`, `SimSSMVAR()`, `SimSSM()`

Examples

```
# prepare parameters
set.seed(42)
n <- 10
mu0 <- c(0.615, 1.006)
sigma0 <- matrix(
  data = c(
    1.932,
    0.618,
    0.618,
    0.587
  ),
  nrow = 2
)
theta <- 0.6
time <- 10
gamma_y <- matrix(data = 0.10, nrow = 1, ncol = 2)
gamma_eta <- matrix(data = 0.10, nrow = 2, ncol = 2)
x <- lapply(
  X = seq_len(n),
  FUN = function(i) {
    return(
      matrix(
        data = rnorm(n = 2 * time),
        ncol = 2
      )
    )
  }
)

# Type 0
ssm <- SimSSMLinGrowth(
  n = n,
  mu0 = mu0,
  sigma0 = sigma0,
  theta = theta,
  type = 0,
  time = time
)

plot(ssm)

# Type 1
ssm <- SimSSMLinGrowth(
  n = n,
  mu0 = mu0,
  sigma0 = sigma0,
  theta = theta,
  gamma_eta = gamma_eta,
  x = x,
  type = 1,
  time = time
```

```
)

plot(ssm)

# Type 2
ssm <- SimSSMLinGrowth(
  n = n,
  mu0 = mu0,
  sigma0 = sigma0,
  theta = theta,
  gamma_y = gamma_y,
  gamma_eta = gamma_eta,
  x = x,
  type = 2,
  time = time
)

plot(ssm)
```

SimSSMLinGrowthIVary *Simulate Data from a Linear Growth Curve Model (Individual-Varying Parameters)*

Description

This function simulates data from a linear growth curve model for n > 1 individuals. In this model, the parameters can vary across individuals.

Usage

```
SimSSMLinGrowthIVary(
  n,
  mu0,
  sigma0,
  theta,
  gamma_y = NULL,
  gamma_eta = NULL,
  x = NULL,
  type = 0,
  time
)
```

Arguments

n	Positive integer. Number of individuals.
mu0	A list of numeric vectors. Each element of the list is a vector of length two. The first element is the mean of the intercept, and the second element is the mean of the slope.

sigma0	A list of numeric matrices. Each element of the list is the covariance matrix of the intercept and the slope.
theta	A list numeric values. Each element of the list is the common measurement error variance.
gamma_y	Numeric matrix. Matrix relating the values of the covariate matrix at time t to y at time t ($\mathbf{\Gamma}_{\mathbf{y}}$).
gamma_eta	Numeric matrix. Matrix relating the values of the covariate matrix at time t to the latent variables (intercept and slope) at time t ($\mathbf{\Gamma}_{\boldsymbol{\eta}}$).
x	A list of length n of numeric matrices. Each element of the list is a matrix of observed covariates in type = 1 or type = 2. The number of rows in each matrix should be equal to time.
type	Integer. State space model type. See Details for more information.
time	Positive integer. Number of time points to simulate.

Details

Parameters can vary across individuals by providing a list of parameter values. If the length of any of the parameters (mu0, sigma0, mu, theta, gamma_y, or gamma_eta) is less the n, the function will cycle through the available values.

Value

Returns an object of class simstatespace which is a list with the following elements:

- call: Function call.
- args: Function arguments.
- data: Generated data which is a list of length n. Each element of data is a list with the following elements:
  - id: A vector of ID numbers of length t.
  - time: A vector time points of length t.
  - y: A t by k matrix of values for the manifest variables.
  - eta: A t by p matrix of values for the latent variables.
  - x: A t by j matrix of values for the covariates.
- fun: Function used.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Chow, S.-M., Ho, M. R., Hamaker, E. L., & Dolan, C. V. (2010). Equivalence and differences between structural equation modeling and state-space modeling techniques. *Structural Equation Modeling: A Multidisciplinary Journal*, 17(2), 303–332. doi:10.1080/10705511003661553

See Also

Other Simulation of State Space Models Data Functions: OU2SSM(), SimSSMFixed(), SimSSMIVary(), SimSSMLinGrowth(), SimSSMOUFixed(), SimSSMOUIVary(), SimSSMOU(), SimSSMVARFixed(), SimSSMVARIVary(), SimSSMVAR(), SimSSM()

Examples

```
# prepare parameters
# In this example, the mean vector of the intercept and slope vary.
# Specifically,
# there are two sets of values representing two latent classes.
set.seed(42)
n <- 10
mu0_1 <- c(0.615, 1.006) # lower starting point, higher growth
mu0_2 <- c(1.000, 0.500) # higher starting point, lower growth
mu0 <- list(mu0_1, mu0_2)
sigma0 <- list(
  matrix(
    data = c(
      1.932,
      0.618,
      0.618,
      0.587
    ),
    nrow = 2
  )
)
theta <- list(0.6)
time <- 10
gamma_y <- list(matrix(data = 0.10, nrow = 1, ncol = 2))
gamma_eta <- list(matrix(data = 0.10, nrow = 2, ncol = 2))
x <- lapply(
  X = seq_len(n),
  FUN = function(i) {
    return(
      matrix(
        data = rnorm(n = 2 * time),
        ncol = 2
      )
    )
  }
)

# Type 0
ssm <- SimSSMLinGrowthIVary(
  n = n,
  mu0 = mu0,
  sigma0 = sigma0,
  theta = theta,
  type = 0,
  time = time
)
```

```
plot(ssm)

# Type 1
ssm <- SimSSMLinGrowthIVary(
  n = n,
  mu0 = mu0,
  sigma0 = sigma0,
  theta = theta,
  gamma_eta = gamma_eta,
  x = x,
  type = 1,
  time = time
)

plot(ssm)

# Type 2
ssm <- SimSSMLinGrowthIVary(
  n = n,
  mu0 = mu0,
  sigma0 = sigma0,
  theta = theta,
  gamma_y = gamma_y,
  gamma_eta = gamma_eta,
  x = x,
  type = 2,
  time = time
)

plot(ssm)
```

SimSSMOU *Simulate Data from the Ornstein–Uhlenbeck Model using a State Space Model Parameterization (n = 1)*

Description

This function simulates data from the Ornstein–Uhlenbeck model using a state space model parameterization.

Usage

```
SimSSMOU(
  mu0,
  sigma0,
  mu,
  phi,
  sigma,
```

```
  nu,
  lambda,
  theta,
  gamma_y = NULL,
  gamma_eta = NULL,
  x = NULL,
  type = 0,
  delta_t,
  time,
  burn_in = 0
)
```

Arguments

mu0	Numeric vector. Mean of initial latent variable values ($\boldsymbol{\mu}_{\boldsymbol{\eta}\|0}$).
sigma0	Numeric matrix. The covariance matrix of initial latent variable values ($\boldsymbol{\Sigma}_{\boldsymbol{\eta}\|0}$).
mu	Numeric vector. The long-term mean or equilibrium level ($\boldsymbol{\mu}$).
phi	Numeric matrix. The rate of mean reversion, determining how quickly the variable returns to its mean ($\boldsymbol{\Phi}$).
sigma	Numeric matrix. The matrix of volatility or randomness in the process ($\boldsymbol{\Sigma}$).
nu	Numeric vector. Vector of intercepts for the measurement model ($\boldsymbol{\nu}$).
lambda	Numeric matrix. Factor loading matrix linking the latent variables to the observed variables ($\boldsymbol{\Lambda}$).
theta	Numeric matrix. The measurement error covariance matrix ($\boldsymbol{\Theta}$).
gamma_y	Numeric matrix. Matrix relating the values of the covariate matrix at time `t` to the observed variables at time `t` ($\boldsymbol{\Gamma}_{\mathbf{y}}$).
gamma_eta	Numeric matrix. Matrix relating the values of the covariate matrix at time `t` to the latent variables at time `t` ($\boldsymbol{\Gamma}_{\boldsymbol{\eta}}$).
x	Numeric matrix. The matrix of observed covariates in `type = 1` or `type = 2`. The number of rows should be equal to `time + burn_in`.
type	Integer. State space model type. See Details for more information.
delta_t	Numeric. Time interval (δ_t).
time	Positive integer. Number of time points to simulate.
burn_in	Positive integer. Number of burn-in points to exclude before returning the results.

Details

Type 0:

The measurement model is given by

$$\mathbf{y}_t = \boldsymbol{\nu} + \boldsymbol{\Lambda}\boldsymbol{\eta}_t + \boldsymbol{\varepsilon}_t \quad \text{with} \quad \boldsymbol{\varepsilon}_t \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Theta}\right)$$

where $\mathbf{y}_t$, $\boldsymbol{\eta}_t$, and $\boldsymbol{\varepsilon}_t$ are random variables and $\boldsymbol{\nu}$, $\boldsymbol{\Lambda}$, and $\boldsymbol{\Theta}$ are model parameters. $\mathbf{y}_t$ is a vector of observed random variables, $\boldsymbol{\eta}_t$ is a vector of latent random variables, and $\boldsymbol{\varepsilon}_t$ is a vector of random measurement errors, at time t. $\boldsymbol{\nu}$ is a vector of intercepts, $\boldsymbol{\Lambda}$ is a matrix of factor loadings, and $\boldsymbol{\Theta}$ is the covariance matrix of $\boldsymbol{\varepsilon}$.

The dynamic structure is given by

$$\mathrm{d}\boldsymbol{\eta}_t = \boldsymbol{\Phi} \left(\boldsymbol{\mu} - \boldsymbol{\eta}_t \right) \mathrm{d}t + \boldsymbol{\Sigma}^{\frac{1}{2}} \mathrm{d}\mathbf{W}_t$$

where $\boldsymbol{\mu}$ is the long-term mean or equilibrium level, $\boldsymbol{\Phi}$ is the rate of mean reversion, determining how quickly the variable returns to its mean, $\boldsymbol{\Sigma}$ is the matrix of volatility or randomness in the process, and $\mathrm{d}\boldsymbol{W}$ is a Wiener process or Brownian motion, which represents random fluctuations.

Type 1:

The measurement model is given by

$$\mathbf{y}_t = \boldsymbol{\nu} + \boldsymbol{\Lambda} \boldsymbol{\eta}_t + \boldsymbol{\varepsilon}_t \quad \mathrm{with} \quad \boldsymbol{\varepsilon}_t \sim \mathcal{N} \left(\mathbf{0}, \boldsymbol{\Theta} \right).$$

The dynamic structure is given by

$$\mathrm{d}\boldsymbol{\eta}_t = \boldsymbol{\Phi} \left(\boldsymbol{\mu} - \boldsymbol{\eta}_t \right) \mathrm{d}t + \boldsymbol{\Gamma}_{\boldsymbol{\eta}} \mathbf{x}_t + \boldsymbol{\Sigma}^{\frac{1}{2}} \mathrm{d}\mathbf{W}_t$$

where $\mathbf{x}_t$ is a vector of covariates at time t, and $\boldsymbol{\Gamma}_{\boldsymbol{\eta}}$ is the coefficient matrix linking the covariates to the latent variables.

Type 2:

The measurement model is given by

$$\mathbf{y}_t = \boldsymbol{\nu} + \boldsymbol{\Lambda} \boldsymbol{\eta}_t + \boldsymbol{\Gamma}_{\mathbf{y}} \mathbf{x}_t + \boldsymbol{\varepsilon}_t \quad \mathrm{with} \quad \boldsymbol{\varepsilon}_t \sim \mathcal{N} \left(\mathbf{0}, \boldsymbol{\Theta} \right)$$

where $\boldsymbol{\Gamma}_{\mathbf{y}}$ is the coefficient matrix linking the covariates to the observed variables.

The dynamic structure is given by

$$\mathrm{d}\boldsymbol{\eta}_t = \boldsymbol{\Phi} \left(\boldsymbol{\mu} - \boldsymbol{\eta}_t \right) \mathrm{d}t + \boldsymbol{\Gamma}_{\boldsymbol{\eta}} \mathbf{x}_t + \boldsymbol{\Sigma}^{\frac{1}{2}} \mathrm{d}\mathbf{W}_t.$$

Value

Returns an object of class `simstatespace` which is a list with the following elements:

- `call`: Function call.
- `args`: Function arguments.
- `data`: Generated data which is a list of length `n`. `data` is a list with the following elements:
  - `id`: A vector of ones of length `t`.
  - `time`: A vector of time points of length `t`.
  - `y`: A `t` by `k` matrix of values for the manifest variables.
  - `eta`: A `t` by `p` matrix of values for the latent variables.
  - `x`: A `t` by `j` matrix of values for the covariates.
- `fun`: Function used.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Chow, S.-M., Losardo, D., Park, J., & Molenaar, P. C. M. (2023). Continuous-time dynamic models: Connections to structural equation models and other discrete-time models. In R. H. Hoyle (Ed.), Handbook of structural equation modeling (2nd ed.). The Guilford Press.

Uhlenbeck, G. E., & Ornstein, L. S. (1930). On the theory of the brownian motion. *Physical Review*, *36*(5), 823–841. doi:10.1103/physrev.36.823

See Also

Other Simulation of State Space Models Data Functions: `OU2SSM()`, `SimSSMFixed()`, `SimSSMIVary()`, `SimSSMLinGrowthIVary()`, `SimSSMLinGrowth()`, `SimSSMOUFixed()`, `SimSSMOUIVary()`, `SimSSMVARFixed()`, `SimSSMVARIVary()`, `SimSSMVAR()`, `SimSSM()`

Examples

```
# prepare parameters
set.seed(42)
p <- k <- 2
iden <- diag(p)
mu0 <- c(-3.0, 1.5)
sigma0 <- iden
mu <- c(5.76, 5.18)
phi <- matrix(data = c(0.10, -0.05, -0.05, 0.10), nrow = p)
sigma <- matrix(
  data = c(2.79, 0.06, 0.06, 3.27),
  nrow = p
)
nu <- rep(x = 0, times = k)
lambda <- diag(k)
theta <- diag(x = 0.50, nrow = k)
delta_t <- 0.10
time <- 1000
burn_in <- 0
gamma_y <- gamma_eta <- 0.10 * diag(k)
x <- matrix(
  data = rnorm(n = k * (time + burn_in)),
  ncol = k
)

# Type 0
ssm <- SimSSMOU(
  mu0 = mu0,
  sigma0 = sigma0,
  mu = mu,
  phi = phi,
  sigma = sigma,
  nu = nu,
  lambda = lambda,
  theta = theta,
  type = 0,
  delta_t = delta_t,
```

```
  time = time,
  burn_in = burn_in
)

plot(ssm)

# Type 1
ssm <- SimSSMOU(
  mu0 = mu0,
  sigma0 = sigma0,
  mu = mu,
  phi = phi,
  sigma = sigma,
  nu = nu,
  lambda = lambda,
  theta = theta,
  gamma_eta = gamma_eta,
  x = x,
  type = 1,
  delta_t = delta_t,
  time = time,
  burn_in = burn_in
)

plot(ssm)

# Type 2
ssm <- SimSSMOU(
  mu0 = mu0,
  sigma0 = sigma0,
  mu = mu,
  phi = phi,
  sigma = sigma,
  nu = nu,
  lambda = lambda,
  theta = theta,
  gamma_y = gamma_y,
  gamma_eta = gamma_eta,
  x = x,
  type = 2,
  delta_t = delta_t,
  time = time,
  burn_in = burn_in
)

plot(ssm)
```

SimSSMOUFixed — *Simulate Data from an Ornstein–Uhlenbeck Model using a State Space Model Parameterization for $n > 1$ Individuals (Fixed Parameters)*

Description

This function simulates data from an Ornstein–Uhlenbeck model using a state space model parameterization for `n > 1` individuals. In this model, the parameters are invariant across individuals.

Usage

```
SimSSMOUFixed(
  n,
  mu0,
  sigma0,
  mu,
  phi,
  sigma,
  nu,
  lambda,
  theta,
  gamma_y = NULL,
  gamma_eta = NULL,
  x = NULL,
  type = 0,
  delta_t,
  time,
  burn_in = 0
)
```

Arguments

n	Positive integer. Number of individuals.
mu0	Numeric vector. Mean of initial latent variable values ($\boldsymbol{\mu}_{\boldsymbol{\eta} \mid 0}$).
sigma0	Numeric matrix. The covariance matrix of initial latent variable values ($\boldsymbol{\Sigma}_{\boldsymbol{\eta} \mid 0}$).
mu	Numeric vector. The long-term mean or equilibrium level ($\boldsymbol{\mu}$).
phi	Numeric matrix. The rate of mean reversion, determining how quickly the variable returns to its mean ($\boldsymbol{\Phi}$).
sigma	Numeric matrix. The matrix of volatility or randomness in the process ($\boldsymbol{\Sigma}$).
nu	Numeric vector. Vector of intercepts for the measurement model ($\boldsymbol{\nu}$).
lambda	Numeric matrix. Factor loading matrix linking the latent variables to the observed variables ($\boldsymbol{\Lambda}$).
theta	Numeric matrix. The measurement error covariance matrix ($\boldsymbol{\Theta}$).
gamma_y	Numeric matrix. Matrix relating the values of the covariate matrix at time `t` to the observed variables at time `t` ($\boldsymbol{\Gamma}_{\mathbf{y}}$).
gamma_eta	Numeric matrix. Matrix relating the values of the covariate matrix at time `t` to the latent variables at time `t` ($\boldsymbol{\Gamma}_{\boldsymbol{\eta}}$).
x	Numeric matrix. The matrix of observed covariates in `type = 1` or `type = 2`. The number of rows should be equal to `time + burn_in`.
type	Integer. State space model type. See Details for more information.

`delta_t` Numeric. Time interval (δ_t).

`time` Positive integer. Number of time points to simulate.

`burn_in` Positive integer. Number of burn-in points to exclude before returning the results.

Details

Type 0:

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \boldsymbol{\Lambda} \boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t} \quad \text{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N} \left(\mathbf{0}, \boldsymbol{\Theta} \right)$$

where $\mathbf{y}_{i,t}$, $\boldsymbol{\eta}_{i,t}$, and $\boldsymbol{\varepsilon}_{i,t}$ are random variables and $\boldsymbol{\nu}$, $\boldsymbol{\Lambda}$, and $\boldsymbol{\Theta}$ are model parameters. $\mathbf{y}_{i,t}$ is a vector of observed random variables, $\boldsymbol{\eta}_{i,t}$ is a vector of latent random variables, and $\boldsymbol{\varepsilon}_{i,t}$ is a vector of random measurement errors, at time t and individual i. $\boldsymbol{\nu}$ is a vector of intercepts, $\boldsymbol{\Lambda}$ is a matrix of factor loadings, and $\boldsymbol{\Theta}$ is the covariance matrix of $\boldsymbol{\varepsilon}$.

The dynamic structure is given by

$$\mathrm{d} \boldsymbol{\eta}_{i,t} = \boldsymbol{\Phi} \left(\boldsymbol{\mu} - \boldsymbol{\eta}_{i,t} \right) \mathrm{d}t + \boldsymbol{\Sigma}^{\frac{1}{2}} \mathrm{d} \mathbf{W}_{i,t}$$

where $\boldsymbol{\mu}$ is the long-term mean or equilibrium level, $\boldsymbol{\Phi}$ is the rate of mean reversion, determining how quickly the variable returns to its mean, $\boldsymbol{\Sigma}$ is the matrix of volatility or randomness in the process, and $\mathrm{d}\boldsymbol{W}$ is a Wiener process or Brownian motion, which represents random fluctuations.

Type 1:

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \boldsymbol{\Lambda} \boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t} \quad \text{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N} \left(\mathbf{0}, \boldsymbol{\Theta} \right).$$

The dynamic structure is given by

$$\mathrm{d} \boldsymbol{\eta}_{i,t} = \boldsymbol{\Phi} \left(\boldsymbol{\mu} - \boldsymbol{\eta}_{i,t} \right) \mathrm{d}t + \boldsymbol{\Gamma}_{\boldsymbol{\eta}} \mathbf{x}_{i,t} + \boldsymbol{\Sigma}^{\frac{1}{2}} \mathrm{d} \mathbf{W}_{i,t}$$

where $\mathbf{x}_{i,t}$ is a vector of covariates at time t and individual i, and $\boldsymbol{\Gamma}_{\boldsymbol{\eta}}$ is the coefficient matrix linking the covariates to the latent variables.

Type 2:

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \boldsymbol{\Lambda} \boldsymbol{\eta}_{i,t} + \boldsymbol{\Gamma}_{\mathbf{y}} \mathbf{x}_{i,t} + \boldsymbol{\varepsilon}_{i,t} \quad \text{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N} \left(\mathbf{0}, \boldsymbol{\Theta} \right)$$

where $\boldsymbol{\Gamma}_{\mathbf{y}}$ is the coefficient matrix linking the covariates to the observed variables.

The dynamic structure is given by

$$\mathrm{d} \boldsymbol{\eta}_{i,t} = \boldsymbol{\Phi} \left(\boldsymbol{\mu} - \boldsymbol{\eta}_{i,t} \right) \mathrm{d}t + \boldsymbol{\Gamma}_{\boldsymbol{\eta}} \mathbf{x}_{i,t} + \boldsymbol{\Sigma}^{\frac{1}{2}} \mathrm{d} \mathbf{W}_{i,t}.$$

Value

Returns an object of class simstatespace which is a list with the following elements:

- call: Function call.
- args: Function arguments.
- data: Generated data which is a list of length n. Each element of data is a list with the following elements:
  - id: A vector of ID numbers of length t.
  - time: A vector time points of length t.
  - y: A t by k matrix of values for the manifest variables.
  - eta: A t by p matrix of values for the latent variables.
  - x: A t by j matrix of values for the covariates.
- fun: Function used.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Chow, S.-M., Losardo, D., Park, J., & Molenaar, P. C. M. (2023). Continuous-time dynamic models: Connections to structural equation models and other discrete-time models. In R. H. Hoyle (Ed.), Handbook of structural equation modeling (2nd ed.). The Guilford Press.

Uhlenbeck, G. E., & Ornstein, L. S. (1930). On the theory of the brownian motion. *Physical Review*, *36*(5), 823–841. doi:10.1103/physrev.36.823

See Also

Other Simulation of State Space Models Data Functions: OU2SSM(), SimSSMFixed(), SimSSMIVary(), SimSSMLinGrowthIVary(), SimSSMLinGrowth(), SimSSMOUIVary(), SimSSMOU(), SimSSMVARFixed(), SimSSMVARIVary(), SimSSMVAR(), SimSSM()

Examples

```
# prepare parameters
set.seed(42)
p <- k <- 2
iden <- diag(p)
n <- 5
mu0 <- c(-3.0, 1.5)
sigma0 <- iden
mu <- c(5.76, 5.18)
phi <- matrix(data = c(0.10, -0.05, -0.05, 0.10), nrow = p)
sigma <- matrix(
  data = c(2.79, 0.06, 0.06, 3.27),
  nrow = p
)
nu <- rep(x = 0, times = k)
```

```
lambda <- diag(k)
theta <- diag(x = 0.50, nrow = k)
delta_t <- 0.10
time <- 50
burn_in <- 0
gamma_y <- gamma_eta <- 0.10 * diag(k)
x <- lapply(
  X = seq_len(n),
  FUN = function(i) {
    return(
      matrix(
        data = rnorm(n = k * (time + burn_in)),
        ncol = k
      )
    )
  }
)

# Type 0
ssm <- SimSSMOUFixed(
  n = n,
  mu0 = mu0,
  sigma0 = sigma0,
  mu = mu,
  phi = phi,
  sigma = sigma,
  nu = nu,
  lambda = lambda,
  theta = theta,
  type = 0,
  delta_t = delta_t,
  time = time,
  burn_in = burn_in
)

plot(ssm)

# Type 1
ssm <- SimSSMOUFixed(
  n = n,
  mu0 = mu0,
  sigma0 = sigma0,
  mu = mu,
  phi = phi,
  sigma = sigma,
  nu = nu,
  lambda = lambda,
  theta = theta,
  gamma_eta = gamma_eta,
  x = x,
  type = 1,
  delta_t = delta_t,
  time = time,
```

```
  burn_in = burn_in
)

plot(ssm)

# Type 2
ssm <- SimSSMOUFixed(
  n = n,
  mu0 = mu0,
  sigma0 = sigma0,
  mu = mu,
  phi = phi,
  sigma = sigma,
  nu = nu,
  lambda = lambda,
  theta = theta,
  gamma_y = gamma_y,
  gamma_eta = gamma_eta,
  x = x,
  type = 2,
  delta_t = delta_t,
  time = time,
  burn_in = burn_in
)

plot(ssm)
```

SimSSMOUIVary *Simulate Data from an Ornstein–Uhlenbeck Model using a State Space Model Parameterization for n > 1 Individuals (Individual-Varying Parameters)*

Description

This function simulates data from an Ornstein–Uhlenbeck model using a state space model parameterization for n > 1 individuals. In this model, the parameters can vary across individuals.

Usage

```
SimSSMOUIVary(
  n,
  mu0,
  sigma0,
  mu,
  phi,
  sigma,
  nu,
  lambda,
```

```
  theta,
  gamma_y = NULL,
  gamma_eta = NULL,
  x = NULL,
  type = 0,
  delta_t,
  time,
  burn_in = 0
)
```

Arguments

n Positive integer. Number of individuals.

mu0 Numeric vector. Mean of initial latent variable values ($\boldsymbol{\mu}_{\boldsymbol{\eta}|0}$).

sigma0 Numeric matrix. The covariance matrix of initial latent variable values ($\boldsymbol{\Sigma}_{\boldsymbol{\eta}|0}$).

mu List of numeric vectors. Each element of the list is the long-term mean or equilibrium level ($\boldsymbol{\mu}$).

phi List of numeric matrices. Each element of the list is the rate of mean reversion, determining how quickly the variable returns to its mean ($\boldsymbol{\Phi}$).

sigma List of numeric matrices. Each element of the list is the matrix of volatility or randomness in the process ($\boldsymbol{\Sigma}$).

nu Numeric vector. Vector of intercepts for the measurement model ($\boldsymbol{\nu}$).

lambda Numeric matrix. Factor loading matrix linking the latent variables to the observed variables ($\boldsymbol{\Lambda}$).

theta Numeric matrix. The measurement error covariance matrix ($\boldsymbol{\Theta}$).

gamma_y Numeric matrix. Matrix relating the values of the covariate matrix at time t to the observed variables at time t ($\boldsymbol{\Gamma}_{\mathbf{y}}$).

gamma_eta Numeric matrix. Matrix relating the values of the covariate matrix at time t to the latent variables at time t ($\boldsymbol{\Gamma}_{\boldsymbol{\eta}}$).

x Numeric matrix. The matrix of observed covariates in type = 1 or type = 2. The number of rows should be equal to time + burn_in.

type Integer. State space model type. See Details for more information.

delta_t Numeric. Time interval (δ_t).

time Positive integer. Number of time points to simulate.

burn_in Positive integer. Number of burn-in points to exclude before returning the results.

Details

Parameters can vary across individuals by providing a list of parameter values. If the length of any of the parameters (mu0, sigma0, mu, phi, sigma, nu, lambda, theta, gamma_y, or gamma_eta) is less the n, the function will cycle through the available values.

Value

Returns an object of class `simstatespace` which is a list with the following elements:

- `call`: Function call.
- `args`: Function arguments.
- `data`: Generated data which is a list of length `n`. Each element of `data` is a list with the following elements:
  - `id`: A vector of ID numbers of length `t`.
  - `time`: A vector time points of length `t`.
  - `y`: A `t` by `k` matrix of values for the manifest variables.
  - `eta`: A `t` by `p` matrix of values for the latent variables.
  - `x`: A `t` by `j` matrix of values for the covariates.
- `fun`: Function used.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Chow, S.-M., Losardo, D., Park, J., & Molenaar, P. C. M. (2023). Continuous-time dynamic models: Connections to structural equation models and other discrete-time models. In R. H. Hoyle (Ed.), Handbook of structural equation modeling (2nd ed.). The Guilford Press.

Uhlenbeck, G. E., & Ornstein, L. S. (1930). On the theory of the brownian motion. *Physical Review*, *36*(5), 823–841. doi:10.1103/physrev.36.823

See Also

Other Simulation of State Space Models Data Functions: `OU2SSM()`, `SimSSMFixed()`, `SimSSMIVary()`, `SimSSMLinGrowthIVary()`, `SimSSMLinGrowth()`, `SimSSMOUFixed()`, `SimSSMOU()`, `SimSSMVARFixed()`, `SimSSMVARIVary()`, `SimSSMVAR()`, `SimSSM()`

Examples

```
# prepare parameters
# In this example, phi varies across individuals
set.seed(42)
p <- k <- 2
iden <- diag(p)
n <- 5
mu0 <- list(c(-3.0, 1.5))
sigma0 <- list(iden)
mu <- list(c(5.76, 5.18))
phi <- list(
  as.matrix(Matrix::expm(diag(x = -0.1, nrow = k))),
  as.matrix(Matrix::expm(diag(x = -0.2, nrow = k))),
  as.matrix(Matrix::expm(diag(x = -0.3, nrow = k))),
  as.matrix(Matrix::expm(diag(x = -0.4, nrow = k))),
```

```
  as.matrix(Matrix::expm(diag(x = -0.5, nrow = k)))
)
sigma <- list(
  matrix(data = c(2.79, 0.06, 0.06, 3.27), nrow = p)
)
nu <- list(rep(x = 0, times = k))
lambda <- list(diag(k))
theta <- list(diag(x = 0.50, nrow = k))
delta_t <- 0.10
time <- 50
burn_in <- 0
gamma_y <- gamma_eta <- list(0.10 * diag(k))
x <- lapply(
  X = seq_len(n),
  FUN = function(i) {
    return(
      matrix(
        data = rnorm(n = k * (time + burn_in)),
        ncol = k
      )
    )
  }
)

# Type 0
ssm <- SimSSMOUIVary(
  n = n,
  mu0 = mu0,
  sigma0 = sigma0,
  mu = mu,
  phi = phi,
  sigma = sigma,
  nu = nu,
  lambda = lambda,
  theta = theta,
  type = 0,
  delta_t = delta_t,
  time = time,
  burn_in = burn_in
)

plot(ssm)

# Type 1
ssm <- SimSSMOUIVary(
  n = n,
  mu0 = mu0,
  sigma0 = sigma0,
  mu = mu,
  phi = phi,
  sigma = sigma,
  nu = nu,
  lambda = lambda,
```

```
  theta = theta,
  gamma_eta = gamma_eta,
  x = x,
  type = 1,
  delta_t = delta_t,
  time = time,
  burn_in = burn_in
)

plot(ssm)

# Type 2
ssm <- SimSSMOUIVary(
  n = n,
  mu0 = mu0,
  sigma0 = sigma0,
  mu = mu,
  phi = phi,
  sigma = sigma,
  nu = nu,
  lambda = lambda,
  theta = theta,
  gamma_y = gamma_y,
  gamma_eta = gamma_eta,
  x = x,
  type = 2,
  delta_t = delta_t,
  time = time,
  burn_in = burn_in
)

plot(ssm)
```

SimSSMVAR *Simulate Data from the Vector Autoregressive Model using a State Space Model Parameterization (n = 1)*

Description

This function simulates data from the vector autoregressive model using a state space model parameterization.

Usage

```
SimSSMVAR(
  mu0,
  sigma0,
  alpha,
  beta,
```

```
  psi,
  gamma_eta = NULL,
  x = NULL,
  time = 0,
  burn_in = 0
)
```

Arguments

mu0	Numeric vector. Mean of initial latent variable values ($\boldsymbol{\mu}_{\boldsymbol{\eta}\mid 0}$).
sigma0	Numeric matrix. The covariance matrix of initial latent variable values ($\boldsymbol{\Sigma}_{\boldsymbol{\eta}\mid 0}$).
alpha	Numeric vector. Vector of intercepts for the dynamic model ($\boldsymbol{\alpha}$).
beta	Numeric matrix. Transition matrix relating the values of the latent variables at time `t - 1` to those at time `t` ($\boldsymbol{\beta}$).
psi	Numeric matrix. The process noise covariance matrix ($\boldsymbol{\Psi}$).
gamma_eta	Numeric matrix. Matrix relating the values of the covariate matrix at time `t` to the latent variables at time `t` ($\boldsymbol{\Gamma}_{\boldsymbol{\eta}}$).
x	Numeric matrix. The matrix of observed covariates in `type = 1` or `type = 2`. The number of rows should be equal to `time + burn_in`.
time	Positive integer. Number of time points to simulate.
burn_in	Positive integer. Number of burn-in points to exclude before returning the results.

Details

The measurement model is given by

$$\mathbf{y}_t = \boldsymbol{\eta}_t .$$

The dynamic structure is given by

$$\boldsymbol{\eta}_t = \boldsymbol{\alpha} + \boldsymbol{\beta} \boldsymbol{\eta}_{t - 1} + \boldsymbol{\zeta}_t \quad \mathrm{with} \quad \boldsymbol{\zeta}_t \sim \mathcal{N} \left(\mathbf{0}, \boldsymbol{\Psi} \right)$$

where $\boldsymbol{\eta}_t$, $\boldsymbol{\eta}_{t - 1}$, and $\boldsymbol{\zeta}_t$ are random variables, and $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, and $\boldsymbol{\Psi}$ are model parameters. $\boldsymbol{\eta}_t$ is a vector of latent variables at time t, $\boldsymbol{\eta}_{t - 1}$ is a vector of latent variables at time $t - 1$, and $\boldsymbol{\zeta}_t$ is a vector of dynamic noise at time t. $\boldsymbol{\alpha}$ is a vector of intercepts, $\boldsymbol{\beta}$ is a matrix of autoregression and cross regression coefficients, and $\boldsymbol{\Psi}$ is the covariance matrix of $\boldsymbol{\zeta}_t$.

Note that when `gamma_eta` and `x` are not `NULL`, the dynamic structure is given by

$$\boldsymbol{\eta}_t = \boldsymbol{\alpha} + \boldsymbol{\beta} \boldsymbol{\eta}_{t - 1} + \boldsymbol{\Gamma}_{\boldsymbol{\eta}} \mathbf{x}_t + \boldsymbol{\zeta}_t \quad \mathrm{with} \quad \boldsymbol{\zeta}_t \sim \mathcal{N} \left(\mathbf{0}, \boldsymbol{\Psi} \right)$$

where $\mathbf{x}_t$ is a vector of covariates at time t, and $\boldsymbol{\Gamma}_{\boldsymbol{\eta}}$ is the coefficient matrix linking the covariates to the latent variables.

Value

Returns an object of class `simstatespace` which is a list with the following elements:

- `call`: Function call.
- `args`: Function arguments.
- `data`: Generated data which is a list of length `n`. `data` is a list with the following elements:
  - `id`: A vector of ones of length `t`.
  - `time`: A vector of time points of length `t`.
  - `y`: A `t` by `k` matrix of values for the manifest variables.
  - `eta`: A `t` by `p` matrix of values for the latent variables.
  - `x`: A `t` by `j` matrix of values for the covariates.
- `fun`: Function used.

References

Chow, S.-M., Ho, M. R., Hamaker, E. L., & Dolan, C. V. (2010). Equivalence and differences between structural equation modeling and state-space modeling techniques. *Structural Equation Modeling: A Multidisciplinary Journal*, 17(2), 303–332. doi:10.1080/10705511003661553

See Also

Other Simulation of State Space Models Data Functions: `OU2SSM()`, `SimSSMFixed()`, `SimSSMIVary()`, `SimSSMLinGrowthIVary()`, `SimSSMLinGrowth()`, `SimSSMOUFixed()`, `SimSSMOUIVary()`, `SimSSMOU()`, `SimSSMVARFixed()`, `SimSSMVARIVary()`, `SimSSM()`

Examples

```
# prepare parameters
set.seed(42)
k <- 3
iden <- diag(k)
null_vec <- rep(x = 0, times = k)
mu0 <- null_vec
sigma0 <- iden
alpha <- null_vec
beta <- diag(x = 0.5, nrow = k)
psi <- iden
time <- 1000
burn_in <- 0
gamma_eta <- 0.10 * diag(k)
x <- matrix(
  data = rnorm(n = k * (time + burn_in)),
  ncol = k
)

# No covariates
ssm <- SimSSMVAR(
  mu0 = mu0,
  sigma0 = sigma0,
```

```
  alpha = alpha,
  beta = beta,
  psi = psi,
  time = time,
  burn_in = burn_in
)

plot(ssm)

# With covariates
ssm <- SimSSMVAR(
  mu0 = mu0,
  sigma0 = sigma0,
  alpha = alpha,
  beta = beta,
  psi = psi,
  gamma_eta = gamma_eta,
  x = x,
  time = time,
  burn_in = burn_in
)

plot(ssm)
```

`SimSSMVARFixed` *Simulate Data from a Vector Autoregressive Model using a State Space Model Parameterization for n > 1 Individuals (Fixed Parameters)*

Description

This function simulates data from a vector autoregressive model using a state space model parameterization for n > 1 individuals. In this model, the parameters are invariant across individuals.

Usage

```
SimSSMVARFixed(
  n,
  mu0,
  sigma0,
  alpha,
  beta,
  psi,
  gamma_eta = NULL,
  x = NULL,
  time = 0,
  burn_in = 0
)
```

Arguments

`n`	Positive integer. Number of individuals.
`mu0`	Numeric vector. Mean of initial latent variable values ($\boldsymbol{\mu}_{\boldsymbol{\eta}\mid 0}$).
`sigma0`	Numeric matrix. The covariance matrix of initial latent variable values ($\boldsymbol{\Sigma}_{\boldsymbol{\eta}\mid 0}$).
`alpha`	Numeric vector. Vector of intercepts for the dynamic model ($\boldsymbol{\alpha}$).
`beta`	Numeric matrix. Transition matrix relating the values of the latent variables at time `t - 1` to those at time `t` ($\boldsymbol{\beta}$).
`psi`	Numeric matrix. The process noise covariance matrix ($\boldsymbol{\Psi}$).
`gamma_eta`	Numeric matrix. Matrix relating the values of the covariate matrix at time `t` to the latent variables at time `t` ($\boldsymbol{\Gamma}_{\boldsymbol{\eta}}$).
`x`	A list of length `n` of numeric matrices. Each element of the list is a matrix of observed covariates in `type = 1` or `type = 2`. The number of rows in each matrix should be equal to `time + burn_in`.
`time`	Positive integer. Number of time points to simulate.
`burn_in`	Positive integer. Number of burn-in points to exclude before returning the results.

Value

Returns an object of class `simstatespace` which is a list with the following elements:

- `call`: Function call.
- `args`: Function arguments.
- `data`: Generated data which is a list of length `n`. Each element of `data` is a list with the following elements:
  - `id`: A vector of ID numbers of length `t`.
  - `time`: A vector time points of length `t`.
  - `y`: A `t` by `k` matrix of values for the manifest variables.
  - `eta`: A `t` by `p` matrix of values for the latent variables.
  - `x`: A `t` by `j` matrix of values for the covariates.
- `fun`: Function used.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Chow, S.-M., Ho, M. R., Hamaker, E. L., & Dolan, C. V. (2010). Equivalence and differences between structural equation modeling and state-space modeling techniques. *Structural Equation Modeling: A Multidisciplinary Journal*, 17(2), 303–332. doi:10.1080/10705511003661553

See Also

Other Simulation of State Space Models Data Functions: OU2SSM(), SimSSMFixed(), SimSSMIVary(), SimSSMLinGrowthIVary(), SimSSMLinGrowth(), SimSSMOUFixed(), SimSSMOUIVary(), SimSSMOU(), SimSSMVARIVary(), SimSSMVAR(), SimSSM()

Examples

```
# prepare parameters
set.seed(42)
k <- 3
iden <- diag(k)
null_vec <- rep(x = 0, times = k)
n <- 5
mu0 <- null_vec
sigma0 <- iden
alpha <- null_vec
beta <- diag(x = 0.5, nrow = k)
psi <- iden
time <- 50
burn_in <- 0
gamma_eta <- 0.10 * diag(k)
x <- lapply(
  X = seq_len(n),
  FUN = function(i) {
    return(
      matrix(
        data = rnorm(n = k * (time + burn_in)),
        ncol = k
      )
    )
  }
)

# No covariates
ssm <- SimSSMVARFixed(
  n = n,
  mu0 = mu0,
  sigma0 = sigma0,
  alpha = alpha,
  beta = beta,
  psi = psi,
  time = time,
  burn_in = burn_in
)

plot(ssm)

# With covariates
ssm <- SimSSMVARFixed(
  n = n,
  mu0 = mu0,
  sigma0 = sigma0,
```

```
  alpha = alpha,
  beta = beta,
  psi = psi,
  gamma_eta = gamma_eta,
  x = x,
  time = time,
  burn_in = burn_in
)

plot(ssm)
```

SimSSMVARIVary — *Simulate Data from a Vector Autoregressive Model using a State Space Model Parameterization for n > 1 Individuals (Individual-Varying Parameters)*

Description

This function simulates data from a vector autoregressive model using a state space model parameterization for n > 1 individuals. In this model, the parameters can vary across individuals.

Usage

```
SimSSMVARIVary(
  n,
  mu0,
  sigma0,
  alpha,
  beta,
  psi,
  gamma_eta = NULL,
  x = NULL,
  time = 0,
  burn_in = 0
)
```

Arguments

n	Positive integer. Number of individuals.
mu0	List of numeric vectors. Each element of the list is the mean of initial latent variable values ($\boldsymbol{\mu}_{\boldsymbol{\eta}\|0}$).
sigma0	List of numeric matrices. Each element of the list is the covariance matrix of initial latent variable values ($\boldsymbol{\Sigma}_{\boldsymbol{\eta}\|0}$).
alpha	List of numeric vectors. Each element of the list is the vector of intercepts for the dynamic model ($\boldsymbol{\alpha}$).

beta	List of numeric matrices. Each element of the list is the transition matrix relating the values of the latent variables at time t - 1 to those at time t ($\boldsymbol{\beta}$).
psi	List of numeric matrices. Each element of the list is the process noise covariance matrix ($\boldsymbol{\Psi}$).
gamma_eta	Numeric matrix. Matrix relating the values of the covariate matrix at time t to the latent variables at time t ($\boldsymbol{\Gamma}_{\boldsymbol{\eta}}$).
x	A list of length n of numeric matrices. Each element of the list is a matrix of observed covariates in type = 1 or type = 2. The number of rows in each matrix should be equal to time + burn_in.
time	Positive integer. Number of time points to simulate.
burn_in	Positive integer. Number of burn-in points to exclude before returning the results.

Details

Parameters can vary across individuals by providing a list of parameter values. If the length of any of the parameters (mu0, sigma0, alpha, beta, psi, or gamma_eta) is less the n, the function will cycle through the available values.

Value

Returns an object of class simstatespace which is a list with the following elements:

- call: Function call.
- args: Function arguments.
- data: Generated data which is a list of length n. Each element of data is a list with the following elements:
  - id: A vector of ID numbers of length t.
  - time: A vector time points of length t.
  - y: A t by k matrix of values for the manifest variables.
  - eta: A t by p matrix of values for the latent variables.
  - x: A t by j matrix of values for the covariates.
- fun: Function used.

Author(s)

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References

Chow, S.-M., Ho, M. R., Hamaker, E. L., & Dolan, C. V. (2010). Equivalence and differences between structural equation modeling and state-space modeling techniques. *Structural Equation Modeling: A Multidisciplinary Journal*, 17(2), 303–332. doi:10.1080/10705511003661553

See Also

Other Simulation of State Space Models Data Functions: OU2SSM(), SimSSMFixed(), SimSSMIVary(), SimSSMLinGrowthIVary(), SimSSMLinGrowth(), SimSSMOUFixed(), SimSSMOUIVary(), SimSSMOU(), SimSSMVARFixed(), SimSSMVAR(), SimSSM()

Examples

```
# prepare parameters
# In this example, beta varies across individuals
set.seed(42)
k <- 3
iden <- diag(k)
null_vec <- rep(x = 0, times = k)
n <- 5
mu0 <- list(null_vec)
sigma0 <- list(iden)
alpha <- list(null_vec)
beta <- list(
  diag(x = 0.1, nrow = k),
  diag(x = 0.2, nrow = k),
  diag(x = 0.3, nrow = k),
  diag(x = 0.4, nrow = k),
  diag(x = 0.5, nrow = k)
)
psi <- list(iden)
time <- 50
burn_in <- 0
gamma_eta <- list(0.10 * diag(k))
x <- lapply(
  X = seq_len(n),
  FUN = function(i) {
    return(
      matrix(
        data = rnorm(n = k * (time + burn_in)),
        ncol = k
      )
    )
  }
)

# No covariates
ssm <- SimSSMVARIVary(
  n = n,
  mu0 = mu0,
  sigma0 = sigma0,
  alpha = alpha,
  beta = beta,
  psi = psi,
  time = time,
  burn_in = burn_in
)
```

```
plot(ssm)

# With covariates
ssm <- SimSSMVARIVary(
  n = n,
  mu0 = mu0,
  sigma0 = sigma0,
  alpha = alpha,
  beta = beta,
  psi = psi,
  gamma_eta = gamma_eta,
  x = x,
  time = time,
  burn_in = burn_in
)

plot(ssm)
```

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