

Package ‘simStateSpace’

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Title Simulate Data from State Space Models

Version 1.0.0

Description Offers an efficient and user-friendly framework
for simulating data in state space models
where n subjects/units is greater than one
which is common in social and behavioral sciences.

URL <https://github.com/jeksterslab/simStateSpace>,
<https://jeksterslab.github.io/simStateSpace/>

BugReports <https://github.com/jeksterslab/simStateSpace/issues>

License GPL (>= 3)

Encoding UTF-8

Roxygen list(markdown = TRUE)

Depends R (>= 3.0.0)

LinkingTo Rcpp, RcppArmadillo

Imports Rcpp

Suggests knitr, rmarkdown, testthat, Matrix

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NeedsCompilation yes

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R topics documented:

OU2SSM	2
Sim2Matrix	3
SimSSM0	5
SimSSM0Fixed	7
SimSSM0Vary	10
SimSSMOU	12

SimSSMOUFixed	15
SimSSMOUVary	18
SimSSMVAR	20
SimSSMVARFixed	22
SimSSMVARVary	24

Index	27
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OU2SSM	<i>Convert Parameters from the Ornstein–Uhlenbeck Model to State Space Model Parameterization</i>
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Description

This function converts parameters from the Ornstein–Uhlenbeck model to state space model parameterization. See details for more information.

Usage

```
OU2SSM(mu, phi, sigma_sqrt, delta_t)
```

Arguments

mu	Numeric vector. The long-term mean or equilibrium level (μ).
phi	Numeric matrix. The rate of mean reversion, determining how quickly the variable returns to its mean (Φ).
sigma_sqrt	Numeric matrix. Cholesky decomposition of the matrix of volatility or randomness in the process (Σ).
delta_t	Numeric. Time interval (δ_t).

Details

The state space parameters as a function of the Ornstein–Uhlenbeck model parameters are given by

$$\beta = \exp(-\Phi \Delta_t)$$

$$\alpha = -\Phi^{-1}(\beta - \mathbf{I}_p)$$

$$\text{vec}(\Psi) = \{ [(-\Phi \otimes \mathbf{I}_p) + (\mathbf{I}_p \otimes -\Phi)] [\exp([(-\Phi \otimes \mathbf{I}_p) + (\mathbf{I}_p \otimes -\Phi)] \Delta_t) - \mathbf{I}_{p \times p}] \text{vec}(\Sigma) \}$$

Author(s)

Ivan Jacob Agaloos Pesigan

See Also

Other Simulation of State Space Models Data Functions: [Sim2Matrix\(\)](#), [SimSSM0Fixed\(\)](#), [SimSSM0Vary\(\)](#), [SimSSM0\(\)](#), [SimSSMOUFixed\(\)](#), [SimSSMOUVary\(\)](#), [SimSSMOU\(\)](#), [SimSSMVARFixed\(\)](#), [SimSSMVARVary\(\)](#), [SimSSMVAR\(\)](#)

Examples

```
p <- k <- 2
mu <- c(5.76, 5.18)
phi <- matrix(data = c(0.10, -0.05, -0.05, 0.10), nrow = p)
sigma_sqrt <- chol(
  matrix(data = c(2.79, 0.06, 0.06, 3.27), nrow = p)
)
delta_t <- 0.10

OU2SSM(
  mu = mu,
  phi = phi,
  sigma_sqrt = sigma_sqrt,
  delta_t = delta_t
)
```

Sim2Matrix

Simulation Output to Matrix

Description

This function converts the output of [SimSSM0\(\)](#), [SimSSMOU\(\)](#), [SimSSMVAR\(\)](#), [SimSSM0Fixed\(\)](#), [SimSSMOUFixed\(\)](#), or [SimSSMVARFixed\(\)](#) to a matrix.

Usage

```
Sim2Matrix(x, eta = FALSE)
```

Arguments

x	R object. Output of SimSSM0() , SimSSMOU() , SimSSMVAR() , SimSSM0Fixed() , SimSSMOUFixed() , or SimSSMVARFixed() .
eta	Logical. If eta = TRUE, include eta. If eta = FALSE, exclude eta.

Author(s)

Ivan Jacob Agaloos Pesigan

See Also

Other Simulation of State Space Models Data Functions: [OU2SSM\(\)](#), [SimSSM0Fixed\(\)](#), [SimSSM0Vary\(\)](#), [SimSSM0\(\)](#), [SimSSMOUFixed\(\)](#), [SimSSMOUVary\(\)](#), [SimSSMOU\(\)](#), [SimSSMVARFixed\(\)](#), [SimSSMVARVary\(\)](#), [SimSSMVAR\(\)](#)

Examples

```
# prepare parameters
set.seed(42)
k <- p <- 3
I <- diag(k)
I_sqrt <- chol(I)
null_vec <- rep(x = 0, times = k)
n <- 5
mu0 <- null_vec
sigma0_sqrt <- I_sqrt
alpha <- null_vec
beta <- diag(x = 0.50, nrow = k)
psi_sqrt <- I_sqrt
nu <- null_vec
lambda <- I
theta_sqrt <- chol(diag(x = 0.50, nrow = k))
time <- 50
burn_in <- 0

# generate data
ssm <- SimSSM0(
  mu0 = mu0,
  sigma0_sqrt = sigma0_sqrt,
  alpha = alpha,
  beta = beta,
  psi_sqrt = psi_sqrt,
  nu = nu,
  lambda = lambda,
  theta_sqrt = theta_sqrt,
  time = time,
  burn_in = burn_in
)

# list to matrix
mat <- Sim2Matrix(ssm)
str(mat)
head(mat)

# generate data
ssm <- SimSSM0Fixed(
  n = n,
  mu0 = mu0,
  sigma0_sqrt = sigma0_sqrt,
  alpha = alpha,
  beta = beta,
  psi_sqrt = psi_sqrt,
```

```

    nu = nu,
    lambda = lambda,
    theta_sqrt = theta_sqrt,
    time = time,
    burn_in = burn_in
)

# list to matrix
mat <- Sim2Matrix(ssm)
str(mat)
head(mat)

```

SimSSM0

Simulate Data from a State Space Model (n = 1)

Description

This function simulates data from a state space model. See details for more information.

Usage

```

SimSSM0(
  mu0,
  sigma0_sqrt,
  alpha,
  beta,
  psi_sqrt,
  nu,
  lambda,
  theta_sqrt,
  time,
  burn_in
)

```

Arguments

mu0	Numeric vector. Mean of initial latent variable values ($\mu_{\eta 0}$).
sigma0_sqrt	Numeric matrix. Cholesky decomposition of the covariance matrix of initial latent variable values ($\Sigma_{\eta 0}$).
alpha	Numeric vector. Vector of intercepts for the dynamic model (α).
beta	Numeric matrix. Transition matrix relating the values of the latent variables at time $t - 1$ to those at time t (β).
psi_sqrt	Numeric matrix. Cholesky decomposition of the process noise covariance matrix (Ψ).
nu	Numeric vector. Vector of intercepts for the measurement model (ν).

lambda	Numeric matrix. Factor loading matrix linking the latent variables to the observed variables (Λ).
theta_sqrt	Numeric matrix. Cholesky decomposition of the measurement error covariance matrix (Θ).
time	Positive integer. Number of time points to simulate.
burn_in	Positive integer. Number of burn-in points to exclude before returning the results.

Details

The measurement model is given by

$$\mathbf{y}_t = \boldsymbol{\nu} + \Lambda \boldsymbol{\eta}_t + \boldsymbol{\varepsilon}_t \quad \text{with} \quad \boldsymbol{\varepsilon}_t \sim \mathcal{N}(\mathbf{0}, \Theta)$$

where \mathbf{y}_t , $\boldsymbol{\eta}_t$, and $\boldsymbol{\varepsilon}_t$ are random variables and $\boldsymbol{\nu}$, Λ , and Θ are model parameters. \mathbf{y}_t is a vector of observed random variables at time t , $\boldsymbol{\eta}_t$ is a vector of latent random variables at time t , and $\boldsymbol{\varepsilon}_t$ is a vector of random measurement errors at time t , while $\boldsymbol{\nu}$ is a vector of intercept, Λ is a matrix of factor loadings, and Θ is the covariance matrix of $\boldsymbol{\varepsilon}$.

The dynamic structure is given by

$$\boldsymbol{\eta}_t = \boldsymbol{\alpha} + \beta \boldsymbol{\eta}_{t-1} + \boldsymbol{\zeta}_t \quad \text{with} \quad \boldsymbol{\zeta}_t \sim \mathcal{N}(\mathbf{0}, \Psi)$$

where $\boldsymbol{\eta}_t$, $\boldsymbol{\eta}_{t-1}$, and $\boldsymbol{\zeta}_t$ are random variables and $\boldsymbol{\alpha}$, β , and Ψ are model parameters. $\boldsymbol{\eta}_t$ is a vector of latent variables at time t , $\boldsymbol{\eta}_{t-1}$ is a vector of latent variables at time $t - 1$, and $\boldsymbol{\zeta}_t$ is a vector of dynamic noise at time t while $\boldsymbol{\alpha}$ is a vector of intercepts, β is a matrix of autoregression and cross regression coefficients, and Ψ is the covariance matrix of $\boldsymbol{\zeta}_t$.

Value

Returns a list with the following elements:

- y: A t by k matrix of values for the manifest variables.
- eta: A t by p matrix of values for the latent variables.
- time: A vector of discrete time points from 1 to t.
- n: Number of individuals.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Shumway, R. H., & Stoffer, D. S. (2017). *Time series analysis and its applications: With R examples*. Springer International Publishing. doi:10.1007/9783319524528

See Also

Other Simulation of State Space Models Data Functions: [OU2SSM\(\)](#), [Sim2Matrix\(\)](#), [SimSSM0Fixed\(\)](#), [SimSSM0Vary\(\)](#), [SimSSM0UFixed\(\)](#), [SimSSM0UVary\(\)](#), [SimSSMOU\(\)](#), [SimSSMVARFixed\(\)](#), [SimSSMVARVary\(\)](#), [SimSSMVAR\(\)](#)

Examples

```

# prepare parameters
set.seed(42)
k <- p <- 3
I <- diag(k)
I_sqrt <- chol(I)
null_vec <- rep(x = 0, times = k)
mu0 <- null_vec
sigma0_sqrt <- I_sqrt
alpha <- null_vec
beta <- diag(x = 0.50, nrow = k)
psi_sqrt <- I_sqrt
nu <- null_vec
lambda <- I
theta_sqrt <- chol(diag(x = 0.50, nrow = k))
time <- 50
burn_in <- 0

# generate data
ssm <- SimSSM0(
  mu0 = mu0,
  sigma0_sqrt = sigma0_sqrt,
  alpha = alpha,
  beta = beta,
  psi_sqrt = psi_sqrt,
  nu = nu,
  lambda = lambda,
  theta_sqrt = theta_sqrt,
  time = time,
  burn_in = burn_in
)

str(ssm)

```

SimSSM0Fixed

Simulate Data using a State Space Model Parameterization for $n > 1$ Individuals (Fixed Parameters)

Description

This function simulates data using a state space model parameterization for $n > 1$ individuals. In this model, the parameters are invariant across individuals.

Usage

```

SimSSM0Fixed(
  n,
  mu0,

```

```

    sigma0_sqrt,
    alpha,
    beta,
    psi_sqrt,
    nu,
    lambda,
    theta_sqrt,
    time,
    burn_in
)

```

Arguments

n	Positive integer. Number of individuals.
mu0	Numeric vector. Mean of initial latent variable values ($\mu_{\eta 0}$).
sigma0_sqrt	Numeric matrix. Cholesky decomposition of the covariance matrix of initial latent variable values ($\Sigma_{\eta 0}$).
alpha	Numeric vector. Vector of intercepts for the dynamic model (α).
beta	Numeric matrix. Transition matrix relating the values of the latent variables at time $t - 1$ to those at time t (β).
psi_sqrt	Numeric matrix. Cholesky decomposition of the process noise covariance matrix (Ψ).
nu	Numeric vector. Vector of intercepts for the measurement model (ν).
lambda	Numeric matrix. Factor loading matrix linking the latent variables to the observed variables (Λ).
theta_sqrt	Numeric matrix. Cholesky decomposition of the measurement error covariance matrix (Θ).
time	Positive integer. Number of time points to simulate.
burn_in	Positive integer. Number of burn-in points to exclude before returning the results.

Details

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \boldsymbol{\Lambda}\boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t} \quad \text{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Theta})$$

where $\mathbf{y}_{i,t}$, $\boldsymbol{\eta}_{i,t}$, and $\boldsymbol{\varepsilon}_{i,t}$ are random variables and $\boldsymbol{\nu}$, $\boldsymbol{\Lambda}$, and $\boldsymbol{\Theta}$ are model parameters. $\mathbf{y}_{i,t}$ is a vector of observed random variables at time t and individual i , $\boldsymbol{\eta}_{i,t}$ is a vector of latent random variables at time t and individual i , and $\boldsymbol{\varepsilon}_{i,t}$ is a vector of random measurement errors at time t and individual i , while $\boldsymbol{\nu}$ is a vector of intercept, $\boldsymbol{\Lambda}$ is a matrix of factor loadings, and $\boldsymbol{\Theta}$ is the covariance matrix of $\boldsymbol{\varepsilon}$.

The dynamic structure is given by

$$\boldsymbol{\eta}_{i,t} = \boldsymbol{\alpha} + \boldsymbol{\beta}\boldsymbol{\eta}_{i,t-1} + \boldsymbol{\zeta}_{i,t} \quad \text{with} \quad \boldsymbol{\zeta}_{i,t} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Psi})$$

where $\eta_{i,t}$, $\eta_{i,t-1}$, and $\zeta_{i,t}$ are random variables and α , β , and Ψ are model parameters. $\eta_{i,t}$ is a vector of latent variables at time t and individual i , $\eta_{i,t-1}$ is a vector of latent variables at time $t - 1$ and individual i , and $\zeta_{i,t}$ is a vector of dynamic noise at time t and individual i while α is a vector of intercepts, β is a matrix of autoregression and cross regression coefficients, and Ψ is the covariance matrix of $\zeta_{i,t}$.

Value

Returns a list of length n. Each element is a list with the following elements:

- y: A t by k matrix of values for the manifest variables.
- eta: A t by p matrix of values for the latent variables.
- time: A vector of discrete time points from 1 to t.
- id: A vector of ID numbers of length t.
- n: Number of individuals.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Shumway, R. H., & Stoffer, D. S. (2017). *Time series analysis and its applications: With R examples*. Springer International Publishing. doi:10.1007/9783319524528

See Also

Other Simulation of State Space Models Data Functions: [OU2SSM\(\)](#), [Sim2Matrix\(\)](#), [SimSSM0Vary\(\)](#), [SimSSM0\(\)](#), [SimSSMOUFixed\(\)](#), [SimSSMOUVary\(\)](#), [SimSSMOU\(\)](#), [SimSSMVARFixed\(\)](#), [SimSSMVARVary\(\)](#), [SimSSMVAR\(\)](#)

Examples

```
# prepare parameters
set.seed(42)
k <- p <- 3
I <- diag(k)
I_sqrt <- chol(I)
null_vec <- rep(x = 0, times = k)
n <- 5
mu0 <- null_vec
sigma0_sqrt <- I_sqrt
alpha <- null_vec
beta <- diag(x = 0.50, nrow = k)
psi_sqrt <- I_sqrt
nu <- null_vec
lambda <- I
theta_sqrt <- chol(diag(x = 0.50, nrow = k))
time <- 50
burn_in <- 0
```

```
# generate data
ssm <- SimSSM0Fixed(
  n = n,
  mu0 = mu0,
  sigma0_sqrt = sigma0_sqrt,
  alpha = alpha,
  beta = beta,
  psi_sqrt = psi_sqrt,
  nu = nu,
  lambda = lambda,
  theta_sqrt = theta_sqrt,
  time = time,
  burn_in = burn_in
)

str(ssm)
```

SimSSM0Vary

Simulate Data using a State Space Model Parameterization for $n > 1$ Individuals (Varying Parameters)

Description

This function simulates data using a state space model parameterization for $n > 1$ individuals. In this model, the parameters can vary across individuals.

Usage

```
SimSSM0Vary(
  n,
  mu0,
  sigma0_sqrt,
  alpha,
  beta,
  psi_sqrt,
  nu,
  lambda,
  theta_sqrt,
  time,
  burn_in
)
```

Arguments

n	Positive integer. Number of individuals.
mu0	List of numeric vectors. Mean of initial latent variable values ($\mu_{\eta 0}$).

sigma0_sqrt	List of numeric matrices. Cholesky decomposition of the covariance matrix of initial latent variable values ($\Sigma_{\eta 0}$).
alpha	List of numeric vectors. Vector of intercepts for the dynamic model (α).
beta	List of numeric matrices. Transition matrix relating the values of the latent variables at time $t - 1$ to those at time t (β).
psi_sqrt	List of numeric matrices. Cholesky decomposition of the process noise covariance matrix (Ψ).
nu	List of numeric vectors. Vector of intercepts for the measurement model (ν).
lambda	List of numeric matrices. Factor loading matrix linking the latent variables to the observed variables (Λ).
theta_sqrt	List of numeric matrices. Cholesky decomposition of the measurement error covariance matrix (Θ).
time	Positive integer. Number of time points to simulate.
burn_in	Positive integer. Number of burn-in points to exclude before returning the results.

Details

Parameters can vary across individuals by providing a list of parameter values. If the length of any of the parameters (mu0, sigma0_sqrt, alpha, beta, psi_sqrt, nu, lambda, and theta_sqrt) is less than n , the function will cycle through the available values.

Value

Returns a list of length n . Each element is a list with the following elements:

- y: A t by k matrix of values for the manifest variables.
- eta: A t by p matrix of values for the latent variables.
- time: A vector of discrete time points from 1 to t .
- id: A vector of ID numbers of length t .
- n: Number of individuals.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Shumway, R. H., & Stoffer, D. S. (2017). *Time series analysis and its applications: With R examples*. Springer International Publishing. doi:10.1007/9783319524528

See Also

Other Simulation of State Space Models Data Functions: [OU2SSM\(\)](#), [Sim2Matrix\(\)](#), [SimSSM0Fixed\(\)](#), [SimSSM0\(\)](#), [SimSSM0UFixed\(\)](#), [SimSSM0UVary\(\)](#), [SimSSM0U\(\)](#), [SimSSMVARFixed\(\)](#), [SimSSMVARVary\(\)](#), [SimSSMVAR\(\)](#)

Examples

```

# prepare parameters
# In this example, beta varies across individuals
set.seed(42)
k <- p <- 3
iden <- diag(k)
iden_sqrt <- chol(iden)
null_vec <- rep(x = 0, times = k)
n <- 5
mu0 <- list(null_vec)
sigma0_sqrt <- list(iden_sqrt)
alpha <- list(null_vec)
beta <- list(
  diag(x = 0.1, nrow = k),
  diag(x = 0.2, nrow = k),
  diag(x = 0.3, nrow = k),
  diag(x = 0.4, nrow = k),
  diag(x = 0.5, nrow = k)
)
psi_sqrt <- list(iden_sqrt)
nu <- list(null_vec)
lambda <- list(iden)
theta_sqrt <- list(chol(diag(x = 0.50, nrow = k)))
time <- 50
burn_in <- 0

ssm <- SimSSM0Vary(
  n = n,
  mu0 = mu0,
  sigma0_sqrt = sigma0_sqrt,
  alpha = alpha,
  beta = beta,
  psi_sqrt = psi_sqrt,
  nu = nu,
  lambda = lambda,
  theta_sqrt = theta_sqrt,
  time = time,
  burn_in = burn_in
)

str(ssm)

```

SimSSMOU

Simulate Data from the Ornstein–Uhlenbeck Model using a State Space Model Parameterization ($n = 1$)

Description

This function simulates data from the Ornstein–Uhlenbeck model using a state space model parameterization. See details for more information.

Usage

```

SimSSMOU(
  mu0,
  sigma0_sqrt,
  mu,
  phi,
  sigma_sqrt,
  nu,
  lambda,
  theta_sqrt,
  delta_t,
  time,
  burn_in
)

```

Arguments

mu0	Numeric vector. Mean of initial latent variable values ($\mu_{\eta 0}$).
sigma0_sqrt	Numeric matrix. Cholesky decomposition of the covariance matrix of initial latent variable values ($\Sigma_{\eta 0}$).
mu	Numeric vector. The long-term mean or equilibrium level (μ).
phi	Numeric matrix. The rate of mean reversion, determining how quickly the variable returns to its mean (Φ).
sigma_sqrt	Numeric matrix. Cholesky decomposition of the matrix of volatility or randomness in the process (Σ).
nu	Numeric vector. Vector of intercepts for the measurement model (ν).
lambda	Numeric matrix. Factor loading matrix linking the latent variables to the observed variables (Λ).
theta_sqrt	Numeric matrix. Cholesky decomposition of the measurement error covariance matrix (Θ).
delta_t	Numeric. Time interval (δ_t).
time	Positive integer. Number of time points to simulate.
burn_in	Positive integer. Number of burn-in points to exclude before returning the results.

Details

The measurement model is given by

$$\mathbf{y}_t = \boldsymbol{\nu} + \boldsymbol{\Lambda}\boldsymbol{\eta}_t + \boldsymbol{\varepsilon}_t \quad \text{with} \quad \boldsymbol{\varepsilon}_t \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Theta})$$

where \mathbf{y}_t , $\boldsymbol{\eta}_t$, and $\boldsymbol{\varepsilon}_t$ are random variables and $\boldsymbol{\nu}$, $\boldsymbol{\Lambda}$, and $\boldsymbol{\Theta}$ are model parameters. \mathbf{y}_t is a vector of observed random variables at time t , $\boldsymbol{\eta}_t$ is a vector of latent random variables at time t , and $\boldsymbol{\varepsilon}_t$ is a vector of random measurement errors at time t , while $\boldsymbol{\nu}$ is a vector of intercept, $\boldsymbol{\Lambda}$ is a matrix of factor loadings, and $\boldsymbol{\Theta}$ is the covariance matrix of $\boldsymbol{\varepsilon}$.

The dynamic structure is given by

$$d\eta_t = \Phi (\mu - \eta_t) dt + \Sigma^{\frac{1}{2}} dW_t$$

where μ is the long-term mean or equilibrium level, Φ is the rate of mean reversion, determining how quickly the variable returns to its mean, Σ is the matrix of volatility or randomness in the process, and dW is a Wiener process or Brownian motion, which represents random fluctuations.

Value

Returns a list with the following elements:

- y: A t by k matrix of values for the manifest variables.
- eta: A t by p matrix of values for the latent variables.
- time: A vector of continuous time points of length t starting from 0 with delta_t increments.
- n: Number of individuals.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Uhlenbeck, G. E., & Ornstein, L. S. (1930). On the theory of the brownian motion. *Physical Review*, 36(5), 823–841. doi:10.1103/physrev.36.823

See Also

Other Simulation of State Space Models Data Functions: [OU2SSM\(\)](#), [Sim2Matrix\(\)](#), [SimSSM0Fixed\(\)](#), [SimSSM0Vary\(\)](#), [SimSSM0\(\)](#), [SimSSM0UFixed\(\)](#), [SimSSM0UVary\(\)](#), [SimSSMVARFixed\(\)](#), [SimSSMVARVary\(\)](#), [SimSSMVAR\(\)](#)

Examples

```
# prepare parameters
set.seed(42)
p <- k <- 2
I <- diag(p)
I_sqrt <- chol(I)
mu0 <- c(-3.0, 1.5)
sigma0_sqrt <- I_sqrt
mu <- c(5.76, 5.18)
phi <- matrix(data = c(0.10, -0.05, -0.05, 0.10), nrow = p)
sigma_sqrt <- chol(
  matrix(data = c(2.79, 0.06, 0.06, 3.27), nrow = p)
)
nu <- rep(x = 0, times = k)
lambda <- diag(k)
theta_sqrt <- chol(diag(x = 0.50, nrow = k))
delta_t <- 0.10
time <- 50
```

```

burn_in <- 0

# generate data
ssm <- SimSSMOU(
  mu0 = mu0,
  sigma0_sqrt = sigma0_sqrt,
  mu = mu,
  phi = phi,
  sigma_sqrt = sigma_sqrt,
  nu = nu,
  lambda = lambda,
  theta_sqrt = theta_sqrt,
  delta_t = delta_t,
  time = time,
  burn_in = burn_in
)

str(ssm)

```

SimSSMOUFixed

Simulate Data from an Ornstein–Uhlenbeck Model using a State Space Model Parameterization for $n > 1$ Individuals (Fixed Parameters)

Description

This function simulates data from an Ornstein–Uhlenbeck model using a state space model parameterization for $n > 1$ individuals. In this model, the parameters are invariant across individuals. See details for more information.

Usage

```

SimSSMOUFixed(
  n,
  mu0,
  sigma0_sqrt,
  mu,
  phi,
  sigma_sqrt,
  nu,
  lambda,
  theta_sqrt,
  delta_t,
  time,
  burn_in
)

```

Arguments

n	Positive integer. Number of individuals.
mu0	Numeric vector. Mean of initial latent variable values ($\mu_{\eta 0}$).
sigma0_sqrt	Numeric matrix. Cholesky decomposition of the covariance matrix of initial latent variable values ($\Sigma_{\eta 0}$).
mu	Numeric vector. The long-term mean or equilibrium level (μ).
phi	Numeric matrix. The rate of mean reversion, determining how quickly the variable returns to its mean (Φ).
sigma_sqrt	Numeric matrix. Cholesky decomposition of the matrix of volatility or randomness in the process (Σ).
nu	Numeric vector. Vector of intercepts for the measurement model (ν).
lambda	Numeric matrix. Factor loading matrix linking the latent variables to the observed variables (Λ).
theta_sqrt	Numeric matrix. Cholesky decomposition of the measurement error covariance matrix (Θ).
delta_t	Numeric. Time interval (δ_t).
time	Positive integer. Number of time points to simulate.
burn_in	Positive integer. Number of burn-in points to exclude before returning the results.

Details

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \boldsymbol{\Lambda}\boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t} \quad \text{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Theta})$$

where $\mathbf{y}_{i,t}$, $\boldsymbol{\eta}_{i,t}$, and $\boldsymbol{\varepsilon}_{i,t}$ are random variables and $\boldsymbol{\nu}$, $\boldsymbol{\Lambda}$, and $\boldsymbol{\Theta}$ are model parameters. $\mathbf{y}_{i,t}$ is a vector of observed random variables at time t and individual i , $\boldsymbol{\eta}_{i,t}$ is a vector of latent random variables at time t and individual i , and $\boldsymbol{\varepsilon}_{i,t}$ is a vector of random measurement errors at time t and individual i , while $\boldsymbol{\nu}$ is a vector of intercept, $\boldsymbol{\Lambda}$ is a matrix of factor loadings, and $\boldsymbol{\Theta}$ is the covariance matrix of $\boldsymbol{\varepsilon}$.

The dynamic structure is given by

$$d\boldsymbol{\eta}_{i,t} = \boldsymbol{\Phi}(\boldsymbol{\mu} - \boldsymbol{\eta}_{i,t}) dt + \boldsymbol{\Sigma}^{\frac{1}{2}} d\mathbf{W}_{i,t}$$

where $\boldsymbol{\mu}$ is the long-term mean or equilibrium level, $\boldsymbol{\Phi}$ is the rate of mean reversion, determining how quickly the variable returns to its mean, $\boldsymbol{\Sigma}$ is the matrix of volatility or randomness in the process, and $d\mathbf{W}$ is a Wiener process or Brownian motion, which represents random fluctuations.

Value

Returns a list of length n. Each element is a list with the following elements:

- y: A t by k matrix of values for the manifest variables.
- eta: A t by p matrix of values for the latent variables.
- time: A vector of continuous time points of length t starting from 0 with delta_t increments.
- id: A vector of ID numbers of length t.
- n: Number of individuals.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Uhlenbeck, G. E., & Ornstein, L. S. (1930). On the theory of the brownian motion. *Physical Review*, 36(5), 823–841. doi:[10.1103/physrev.36.823](https://doi.org/10.1103/physrev.36.823)

See Also

Other Simulation of State Space Models Data Functions: [OU2SSM\(\)](#), [Sim2Matrix\(\)](#), [SimSSM0Fixed\(\)](#), [SimSSM0Vary\(\)](#), [SimSSM0\(\)](#), [SimSSMOUVary\(\)](#), [SimSSMOU\(\)](#), [SimSSMVARFixed\(\)](#), [SimSSMVARVary\(\)](#), [SimSSMVAR\(\)](#)

Examples

```
# prepare parameters
set.seed(42)
p <- k <- 2
I <- diag(p)
I_sqrt <- chol(I)
n <- 5
mu0 <- c(-3.0, 1.5)
sigma0_sqrt <- I_sqrt
mu <- c(5.76, 5.18)
phi <- matrix(data = c(0.10, -0.05, -0.05, 0.10), nrow = p)
sigma_sqrt <- chol(
  matrix(data = c(2.79, 0.06, 0.06, 3.27), nrow = p)
)
nu <- rep(x = 0, times = k)
lambda <- diag(k)
theta_sqrt <- chol(diag(x = 0.50, nrow = k))
delta_t <- 0.10
time <- 50
burn_in <- 0

# generate data
ssm <- SimSSMOUFixed(
  n = n,
  mu0 = mu0,
  sigma0_sqrt = sigma0_sqrt,
  mu = mu,
  phi = phi,
  sigma_sqrt = sigma_sqrt,
  nu = nu,
  lambda = lambda,
  theta_sqrt = theta_sqrt,
  delta_t = delta_t,
  time = time,
  burn_in = burn_in
)
```

```
str(ssm)
```

SimSSMOUVary

Simulate Data from an Ornstein–Uhlenbeck Model using a State Space Model Parameterization for $n > 1$ Individuals (Varying Parameters)

Description

This function simulates data from an Ornstein–Uhlenbeck model using a state space model parameterization for $n > 1$ individuals. In this model, the parameters can vary across individuals.

Usage

```
SimSSMOUVary(
  n,
  mu0,
  sigma0_sqrt,
  mu,
  phi,
  sigma_sqrt,
  nu,
  lambda,
  theta_sqrt,
  delta_t,
  time,
  burn_in
)
```

Arguments

n	Positive integer. Number of individuals.
mu0	Numeric vector. Mean of initial latent variable values ($\mu_{\eta 0}$).
sigma0_sqrt	Numeric matrix. Cholesky decomposition of the covariance matrix of initial latent variable values ($\Sigma_{\eta 0}$).
mu	List of numeric vectors. The long-term mean or equilibrium level (μ).
phi	List of numeric matrices. The rate of mean reversion, determining how quickly the variable returns to its mean (Φ).
sigma_sqrt	List of numeric matrices. Cholesky decomposition of the matrix of volatility or randomness in the process (Σ).
nu	Numeric vector. Vector of intercepts for the measurement model (ν).
lambda	Numeric matrix. Factor loading matrix linking the latent variables to the observed variables (Λ).
theta_sqrt	Numeric matrix. Cholesky decomposition of the measurement error covariance matrix (Θ).

delta_t	Numeric. Time interval (δ_t).
time	Positive integer. Number of time points to simulate.
burn_in	Positive integer. Number of burn-in points to exclude before returning the results.

Details

Parameters can vary across individuals by providing a list of parameter values. If the length of any of the parameters (`mu0`, `sigma0_sqrt`, `mu`, `phi`, `sigma_sqrt`, `nu`, `lambda`, `theta_sqrt`) is less than `n`, the function will cycle through the available values.

Value

Returns a list of length `n`. Each element is a list with the following elements:

- `y`: A `t` by `k` matrix of values for the manifest variables.
- `eta`: A `t` by `p` matrix of values for the latent variables.
- `time`: A vector of discrete time points from 1 to `t`.
- `id`: A vector of ID numbers of length `t`.
- `n`: Number of individuals.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Uhlenbeck, G. E., & Ornstein, L. S. (1930). On the theory of the brownian motion. *Physical Review*, 36(5), 823–841. doi:[10.1103/physrev.36.823](https://doi.org/10.1103/physrev.36.823)

See Also

Other Simulation of State Space Models Data Functions: [OU2SSM\(\)](#), [Sim2Matrix\(\)](#), [SimSSM0Fixed\(\)](#), [SimSSM0Vary\(\)](#), [SimSSM0\(\)](#), [SimSSMOUFixed\(\)](#), [SimSSMOU\(\)](#), [SimSSMVARFixed\(\)](#), [SimSSMVARVary\(\)](#), [SimSSMVAR\(\)](#)

Examples

```
# prepare parameters
# In this example, phi varies across individuals
set.seed(42)
p <- k <- 2
iden <- diag(p)
iden_sqrt <- chol(iden)
n <- 5
mu0 <- list(c(-3.0, 1.5))
sigma0_sqrt <- list(iden_sqrt)
mu <- list(c(5.76, 5.18))
phi <- list(
```

```

as.matrix(Matrix::expm(diag(x = -0.1, nrow = k))),
as.matrix(Matrix::expm(diag(x = -0.2, nrow = k))),
as.matrix(Matrix::expm(diag(x = -0.3, nrow = k))),
as.matrix(Matrix::expm(diag(x = -0.4, nrow = k))),
as.matrix(Matrix::expm(diag(x = -0.5, nrow = k)))
)
sigma_sqrt <- list(
  chol(
    matrix(data = c(2.79, 0.06, 0.06, 3.27), nrow = p)
  )
)
nu <- list(rep(x = 0, times = k))
lambda <- list(diag(k))
theta_sqrt <- list(chol(diag(x = 0.50, nrow = k)))
delta_t <- 0.10
time <- 50
burn_in <- 0

ssm <- SimSSMOUVary(
  n = n,
  mu0 = mu0,
  sigma0_sqrt = sigma0_sqrt,
  mu = mu,
  phi = phi,
  sigma_sqrt = sigma_sqrt,
  nu = nu,
  lambda = lambda,
  theta_sqrt = theta_sqrt,
  delta_t = delta_t,
  time = time,
  burn_in = burn_in
)

str(ssm)

```

SimSSMVAR

Simulate Data from the Vector Autoregressive Model using a State Space Model Parameterization ($n = 1$)

Description

This function simulates data from the vector autoregressive model using a state space model parameterization. See details for more information.

Usage

```
SimSSMVAR(mu0, sigma0_sqrt, alpha, beta, psi_sqrt, time, burn_in)
```

Arguments

mu0	Numeric vector. Mean of initial latent variable values ($\mu_{\eta 0}$).
sigma0_sqrt	Numeric matrix. Cholesky decomposition of the covariance matrix of initial latent variable values ($\Sigma_{\eta 0}$).
alpha	Numeric vector. Vector of intercepts for the dynamic model (α).
beta	Numeric matrix. Transition matrix relating the values of the latent variables at time $t - 1$ to those at time t (β).
psi_sqrt	Numeric matrix. Cholesky decomposition of the process noise covariance matrix (Ψ).
time	Positive integer. Number of time points to simulate.
burn_in	Positive integer. Number of burn-in points to exclude before returning the results.

Details

The measurement model is given by

$$\mathbf{y}_t = \boldsymbol{\eta}_t.$$

The dynamic structure is given by

$$\boldsymbol{\eta}_t = \boldsymbol{\alpha} + \boldsymbol{\beta}\boldsymbol{\eta}_{t-1} + \boldsymbol{\zeta}_t \quad \text{with} \quad \boldsymbol{\zeta}_t \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Psi})$$

where $\boldsymbol{\eta}_t$, $\boldsymbol{\eta}_{t-1}$, and $\boldsymbol{\zeta}_t$ are random variables and $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, and $\boldsymbol{\Psi}$ are model parameters. $\boldsymbol{\eta}_t$ is a vector of latent variables at time t , $\boldsymbol{\eta}_{t-1}$ is a vector of latent variables at $t - 1$, and $\boldsymbol{\zeta}_t$ is a vector of dynamic noise at time t while $\boldsymbol{\alpha}$ is a vector of intercepts, $\boldsymbol{\beta}$ is a matrix of autoregression and cross regression coefficients, and $\boldsymbol{\Psi}$ is the covariance matrix of $\boldsymbol{\zeta}_t$.

Value

Returns a list with the following elements:

- y: A t by k matrix of values for the manifest variables.
- eta: A t by p matrix of values for the latent variables.
- time: A vector of discrete time points from 1 to t .
- n: Number of individuals.

References

Shumway, R. H., & Stoffer, D. S. (2017). *Time series analysis and its applications: With R examples*. Springer International Publishing. doi:10.1007/9783319524528

See Also

Other Simulation of State Space Models Data Functions: [OU2SSM\(\)](#), [Sim2Matrix\(\)](#), [SimSSM0Fixed\(\)](#), [SimSSM0Vary\(\)](#), [SimSSM0\(\)](#), [SimSSM0UFixed\(\)](#), [SimSSM0UVary\(\)](#), [SimSSM0U\(\)](#), [SimSSMVARFixed\(\)](#), [SimSSMVARVary\(\)](#)

Examples

```

# prepare parameters
set.seed(42)
k <- 3
I <- diag(k)
I_sqrt <- chol(I)
null_vec <- rep(x = 0, times = k)
mu0 <- null_vec
sigma0_sqrt <- I_sqrt
alpha <- null_vec
beta <- diag(x = 0.5, nrow = k)
psi_sqrt <- I_sqrt
time <- 50
burn_in <- 0

# generate data
ssm <- SimSSMVAR(
  mu0 = mu0,
  sigma0_sqrt = sigma0_sqrt,
  alpha = alpha,
  beta = beta,
  psi_sqrt = psi_sqrt,
  time = time,
  burn_in = burn_in
)

str(ssm)

```

SimSSMVARFixed

Simulate Data from a Vector Autoregressive Model using a State Space Model Parameterization for $n > 1$ Individuals (Fixed Parameters)

Description

This function simulates data from a vector autoregressive model using a state space model parameterization for $n > 1$ individuals. In this model, the parameters are invariant across individuals.

Usage

```
SimSSMVARFixed(n, mu0, sigma0_sqrt, alpha, beta, psi_sqrt, time, burn_in)
```

Arguments

n	Positive integer. Number of individuals.
mu0	Numeric vector. Mean of initial latent variable values ($\mu_{\eta 0}$).
sigma0_sqrt	Numeric matrix. Cholesky decomposition of the covariance matrix of initial latent variable values ($\Sigma_{\eta 0}$).

alpha	Numeric vector. Vector of intercepts for the dynamic model (α).
beta	Numeric matrix. Transition matrix relating the values of the latent variables at time $t - 1$ to those at time t (β).
psi_sqrt	Numeric matrix. Cholesky decomposition of the process noise covariance matrix (Ψ).
time	Positive integer. Number of time points to simulate.
burn_in	Positive integer. Number of burn-in points to exclude before returning the results.

Value

Returns a list of length n . Each element is a list with the following elements:

- y : A t by k matrix of values for the manifest variables.
- η : A t by p matrix of values for the latent variables.
- $time$: A vector of discrete time points from 1 to t .
- id : A vector of ID numbers of length t .
- n : Number of individuals.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Shumway, R. H., & Stoffer, D. S. (2017). *Time series analysis and its applications: With R examples*. Springer International Publishing. doi:10.1007/9783319524528

See Also

Other Simulation of State Space Models Data Functions: [OU2SSM\(\)](#), [Sim2Matrix\(\)](#), [SimSSM0Fixed\(\)](#), [SimSSM0Vary\(\)](#), [SimSSM0\(\)](#), [SimSSMOUFixed\(\)](#), [SimSSMOUVary\(\)](#), [SimSSMOU\(\)](#), [SimSSMVARVary\(\)](#), [SimSSMVAR\(\)](#)

Examples

```
# prepare parameters
set.seed(42)
k <- 3
iden <- diag(k)
iden_sqrt <- chol(iden)
null_vec <- rep(x = 0, times = k)
n <- 5
mu0 <- null_vec
sigma0_sqrt <- iden_sqrt
alpha <- null_vec
beta <- diag(x = 0.5, nrow = k)
psi_sqrt <- iden_sqrt
time <- 50
```

```

burn_in <- 0

ssm <- SimSSMVARFixed(
  n = n,
  mu0 = mu0,
  sigma0_sqrt = sigma0_sqrt,
  alpha = alpha,
  beta = beta,
  psi_sqrt = psi_sqrt,
  time = time,
  burn_in = burn_in
)

str(ssm)

```

SimSSMVARVary	<i>Simulate Data from a Vector Autoregressive Model using a State Space Model Parameterization for $n > 1$ Individuals (Varying Parameters)</i>
---------------	---

Description

This function simulates data from a vector autoregressive model using a state space model parameterization for $n > 1$ individuals. In this model, the parameters can vary across individuals.

Usage

```
SimSSMVARVary(n, mu0, sigma0_sqrt, alpha, beta, psi_sqrt, time, burn_in)
```

Arguments

n	Positive integer. Number of individuals.
mu0	List of numeric vectors. Mean of initial latent variable values ($\mu_{\eta 0}$).
sigma0_sqrt	List of numeric matrices. Cholesky decomposition of the covariance matrix of initial latent variable values ($\Sigma_{\eta 0}$).
alpha	List of numeric vectors. Vector of intercepts for the dynamic model (α).
beta	List of numeric matrices. Transition matrix relating the values of the latent variables at time $t - 1$ to those at time t (β).
psi_sqrt	List of numeric matrices. Cholesky decomposition of the process noise covariance matrix (Ψ).
time	Positive integer. Number of time points to simulate.
burn_in	Positive integer. Number of burn-in points to exclude before returning the results.

Details

Parameters can vary across individuals by providing a list of parameter values. If the length of any of the parameters (μ_0 , σ_0_{sqrt} , α , β , and ψ_{sqrt}) is less than n , the function will cycle through the available values.

Value

Returns a list of length n . Each element is a list with the following elements:

- y : A t by k matrix of values for the manifest variables.
- η : A t by p matrix of values for the latent variables.
- time : A vector of discrete time points from 1 to t .
- id : A vector of ID numbers of length t .
- n : Number of individuals.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Shumway, R. H., & Stoffer, D. S. (2017). *Time series analysis and its applications: With R examples*. Springer International Publishing. doi:[10.1007/9783319524528](https://doi.org/10.1007/9783319524528)

See Also

Other Simulation of State Space Models Data Functions: [OU2SSM\(\)](#), [Sim2Matrix\(\)](#), [SimSSM0Fixed\(\)](#), [SimSSM0Vary\(\)](#), [SimSSM0\(\)](#), [SimSSM0UFixed\(\)](#), [SimSSM0UVary\(\)](#), [SimSSM0U\(\)](#), [SimSSMVARFixed\(\)](#), [SimSSMVAR\(\)](#)

Examples

```
# prepare parameters
# In this example, beta varies across individuals
set.seed(42)
k <- 3
iden <- diag(k)
iden_sqrt <- chol(iden)
null_vec <- rep(x = 0, times = k)
n <- 5
mu0 <- list(null_vec)
sigma0_sqrt <- list(iden_sqrt)
alpha <- list(null_vec)
beta <- list(
  diag(x = 0.1, nrow = k),
  diag(x = 0.2, nrow = k),
  diag(x = 0.3, nrow = k),
  diag(x = 0.4, nrow = k),
  diag(x = 0.5, nrow = k)
)
```

```
psi_sqrt <- list(iden_sqrt)
time <- 50
burn_in <- 0

ssm <- SimSSMVARVary(
  n = n,
  mu0 = mu0,
  sigma0_sqrt = sigma0_sqrt,
  alpha = alpha,
  beta = beta,
  psi_sqrt = psi_sqrt,
  time = time,
  burn_in = burn_in
)

str(ssm)
```

Index

* Simulation of State Space Models Data

Functions

OU2SSM, [2](#)
Sim2Matrix, [3](#)
SimSSM0, [5](#)
SimSSM0Fixed, [7](#)
SimSSM0Vary, [10](#)
SimSSMOU, [12](#)
SimSSMOUFixed, [15](#)
SimSSMOUVary, [18](#)
SimSSMVAR, [20](#)
SimSSMVARFixed, [22](#)
SimSSMVARVary, [24](#)

* misc

Sim2Matrix, [3](#)

* simStateSpace

OU2SSM, [2](#)
Sim2Matrix, [3](#)
SimSSM0, [5](#)
SimSSM0Fixed, [7](#)
SimSSM0Vary, [10](#)
SimSSMOU, [12](#)
SimSSMOUFixed, [15](#)
SimSSMOUVary, [18](#)
SimSSMVAR, [20](#)
SimSSMVARFixed, [22](#)
SimSSMVARVary, [24](#)

* sim

OU2SSM, [2](#)
SimSSM0, [5](#)
SimSSM0Fixed, [7](#)
SimSSM0Vary, [10](#)
SimSSMOU, [12](#)
SimSSMOUFixed, [15](#)
SimSSMOUVary, [18](#)
SimSSMVAR, [20](#)
SimSSMVARFixed, [22](#)
SimSSMVARVary, [24](#)

Sim2Matrix, [2](#), [3](#), [6](#), [9](#), [11](#), [14](#), [17](#), [19](#), [21](#), [23](#),
[25](#)
SimSSM0, [2](#), [3](#), [5](#), [9](#), [11](#), [14](#), [17](#), [19](#), [21](#), [23](#), [25](#)
SimSSM0(), [3](#)
SimSSM0Fixed, [2](#), [3](#), [6](#), [7](#), [11](#), [14](#), [17](#), [19](#), [21](#),
[23](#), [25](#)
SimSSM0Fixed(), [3](#)
SimSSM0Vary, [2](#), [3](#), [6](#), [9](#), [10](#), [14](#), [17](#), [19](#), [21](#), [23](#),
[25](#)
SimSSMOU, [2](#), [3](#), [6](#), [9](#), [11](#), [12](#), [17](#), [19](#), [21](#), [23](#), [25](#)
SimSSMOU(), [3](#)
SimSSMOUFixed, [2](#), [3](#), [6](#), [9](#), [11](#), [14](#), [15](#), [19](#), [21](#),
[23](#), [25](#)
SimSSMOUFixed(), [3](#)
SimSSMOUVary, [2](#), [3](#), [6](#), [9](#), [11](#), [14](#), [17](#), [18](#), [21](#),
[23](#), [25](#)
SimSSMVAR, [2](#), [3](#), [6](#), [9](#), [11](#), [14](#), [17](#), [19](#), [20](#), [23](#), [25](#)
SimSSMVAR(), [3](#)
SimSSMVARFixed, [2](#), [3](#), [6](#), [9](#), [11](#), [14](#), [17](#), [19](#), [21](#),
[22](#), [25](#)
SimSSMVARFixed(), [3](#)
SimSSMVARVary, [2](#), [3](#), [6](#), [9](#), [11](#), [14](#), [17](#), [19](#), [21](#),
[23](#), [24](#)

OU2SSM, [2](#), [3](#), [6](#), [9](#), [11](#), [14](#), [17](#), [19](#), [21](#), [23](#), [25](#)