Package ‘simStateSpace’

February 2, 2024

Title Simulate Data from State Space Models

Version 1.1.0.9000

Description Provides a streamlined and user-friendly framework
for simulating data in state space models,
particularly when the number of subjects/units (n) exceeds one,
a scenario commonly encountered in social and behavioral sciences.
For an introduction to state space models in social and behavioral sciences,
refer to Chow, Ho, Hamaker, and Dolan (2010) <doi:10.1080/10705511003661553>.

URL https://github.com/jeksterslab/simStateSpace,
https://jeksterslab.github.io/simStateSpace/

BugReports https://github.com/jeksterslab/simStateSpace/issues

License GPL (>= 3)

Encoding UTF-8

Roxygen list(markdown = TRUE)

Depends R (>= 3.0.0)

LinkingTo Rcpp, RcppArmadillo

Imports Rcpp

Suggests knitr, rmarkdown, testthat, Matrix

RoxygenNote 7.3.1

NeedsCompilation yes

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R topics documented:

`as.data.frame.simstatespace`

Coerce an Object of Class `simstatespace` *to a Data Frame*

Description

Coerce an Object of Class `simstatespace` to a Data Frame

Usage

```
## S3 method for class 'simstatespace'
as.data.frame(
  x,
  row.names = NULL,
  optional = FALSE,
  eta = FALSE,
  long = TRUE,
  ...
)
```

Arguments

`x`	Object of class `simstatespace`.
`row.names`	`NULL` or character vector giving the row names for the data frame. Missing values are not allowed.
`optional`	Logical. If `TRUE`, setting row names and converting column names is optional.
`eta`	Logical. If `eta = TRUE`, include `eta`. If `eta = FALSE`, exclude `eta`.
`long`	Logical. If `long = TRUE`, use long format. If `long = FALSE`, use wide format.
`...`	Additional arguments.

Author(s)

Ivan Jacob Agaloos Pesigan

Examples

```
# prepare parameters
set.seed(42)
## number of individuals
n <- 5
## time points
time <- 50
## dynamic structure
p <- 3
mu0 <- rep(x = 0, times = p)
sigma0 <- diag(p)
sigma0_l <- t(chol(sigma0))
alpha <- rep(x = 0, times = p)
beta <- 0.50 * diag(p)
psi <- diag(p)
psi_l <- t(chol(psi))
## measurement model
k <- 3
nu <- rep(x = 0, times = k)
lambda <- diag(k)
theta <- 0.50 * diag(k)
theta_l <- t(chol(theta))
## covariates
j <- 2
x <- lapply(
  X = seq_len(n),
  FUN = function(i) {
    matrix(
      data = stats::rnorm(n = time * j),
      nrow = j,
      ncol = time
    )
  }
)
gamma_eta <- diag(x = 0.10, nrow = p, ncol = j)
gamma_y <- diag(x = 0.10, nrow = k, ncol = j)

# Type 0
ssm <- SimSSMFixed(
  n = n,
  time = time,
  mu0 = mu0,
  sigma0_l = sigma0_l,
  alpha = alpha,
  beta = beta,
  psi_l = psi_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 0
)
```

```
head(as.data.frame(ssm))
head(as.data.frame(ssm, long = FALSE))

# Type 1
ssm <- SimSSMFixed(
  n = n,
  time = time,
  mu0 = mu0,
  sigma0_l = sigma0_l,
  alpha = alpha,
  beta = beta,
  psi_l = psi_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 1,
  x = x,
  gamma_eta = gamma_eta
)

head(as.data.frame(ssm))
head(as.data.frame(ssm, long = FALSE))

# Type 2
ssm <- SimSSMFixed(
  n = n,
  time = time,
  mu0 = mu0,
  sigma0_l = sigma0_l,
  alpha = alpha,
  beta = beta,
  psi_l = psi_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 2,
  x = x,
  gamma_eta = gamma_eta,
  gamma_y = gamma_y
)

head(as.data.frame(ssm))
head(as.data.frame(ssm, long = FALSE))
```

as.matrix.simstatespace

Coerce an Object of Class `simstatespace` *to a Matrix*

Description

Coerce an Object of Class `simstatespace` to a Matrix

Usage

```
## S3 method for class 'simstatespace'
as.matrix(x, eta = FALSE, long = TRUE, ...)
```

Arguments

`x`	Object of class `simstatespace`.
`eta`	Logical. If `eta = TRUE`, include `eta`. If `eta = FALSE`, exclude `eta`.
`long`	Logical. If `long = TRUE`, use long format. If `long = FALSE`, use wide format.
`...`	Additional arguments.

Author(s)

Ivan Jacob Agaloos Pesigan

Examples

```
# prepare parameters
set.seed(42)
## number of individuals
n <- 5
## time points
time <- 50
## dynamic structure
p <- 3
mu0 <- rep(x = 0, times = p)
sigma0 <- diag(p)
sigma0_l <- t(chol(sigma0))
alpha <- rep(x = 0, times = p)
beta <- 0.50 * diag(p)
psi <- diag(p)
psi_l <- t(chol(psi))
## measurement model
k <- 3
nu <- rep(x = 0, times = k)
lambda <- diag(k)
theta <- 0.50 * diag(k)
theta_l <- t(chol(theta))
## covariates
j <- 2
x <- lapply(
  X = seq_len(n),
  FUN = function(i) {
    matrix(
      data = stats::rnorm(n = time * j),
      nrow = j,
```

```
      ncol = time
    )
  }
)
gamma_eta <- diag(x = 0.10, nrow = p, ncol = j)
gamma_y <- diag(x = 0.10, nrow = k, ncol = j)

# Type 0
ssm <- SimSSMFixed(
  n = n,
  time = time,
  mu0 = mu0,
  sigma0_l = sigma0_l,
  alpha = alpha,
  beta = beta,
  psi_l = psi_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 0
)

head(as.matrix(ssm))
head(as.matrix(ssm, long = FALSE))

# Type 1
ssm <- SimSSMFixed(
  n = n,
  time = time,
  mu0 = mu0,
  sigma0_l = sigma0_l,
  alpha = alpha,
  beta = beta,
  psi_l = psi_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 1,
  x = x,
  gamma_eta = gamma_eta
)

head(as.matrix(ssm))
head(as.matrix(ssm, long = FALSE))

# Type 2
ssm <- SimSSMFixed(
  n = n,
  time = time,
  mu0 = mu0,
  sigma0_l = sigma0_l,
  alpha = alpha,
  beta = beta,
```

```
  psi_l = psi_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 2,
  x = x,
  gamma_eta = gamma_eta,
  gamma_y = gamma_y
)

head(as.matrix(ssm))
head(as.matrix(ssm, long = FALSE))
```

`LinSDE2SSM` *Convert Parameters from the Linear Stochastic Differential Equation Model to State Space Model Parameterization*

Description

This function converts parameters from the linear stochastic differential equation model to state space model parameterization.

Usage

```
LinSDE2SSM(gamma, phi, sigma_l, delta_t)
```

Arguments

`gamma`	Numeric vector. An unobserved term that is constant over time ($\boldsymbol{\gamma}$).
`phi`	Numeric matrix. The drift matrix which represents the rate of change of the solution in the absence of any random fluctuations ($\boldsymbol{\Phi}$).
`sigma_l`	Numeric matrix. Cholesky factorization (`t(chol(sigma))`) of the covariance matrix of volatility or randomness in the process $\boldsymbol{\Sigma}$.
`delta_t`	Numeric. Time interval (Δ_t).

Details

Let the linear stochastic equation model be given by

$$\mathrm{d}\boldsymbol{\eta}_{i,t} = \left(\boldsymbol{\gamma} + \boldsymbol{\Phi}\boldsymbol{\eta}_{i,t}\right)\mathrm{d}t + \boldsymbol{\Sigma}^{\frac{1}{2}}\mathrm{d}\mathbf{W}_{i,t}$$

for individual i and time t. The state space parameters as a function of the linear stochastic differential equation model parameters are given by

$$\boldsymbol{\beta} = \exp\left(\boldsymbol{\Phi}\Delta_t\right)$$

$$\boldsymbol{\alpha} = \boldsymbol{\Phi}^{-1}\left(\boldsymbol{\beta} - \mathbf{I}_p\right)\boldsymbol{\gamma}$$

$$\mathrm{vec}\left(\boldsymbol{\Psi}\right) = \left[\left(\boldsymbol{\Phi} \otimes \mathbf{I}_p\right) + \left(\mathbf{I}_p \otimes \boldsymbol{\Phi}\right)\right] \left[\exp\left(\left[\left(\boldsymbol{\Phi} \otimes \mathbf{I}_p\right) + \left(\mathbf{I}_p \otimes \boldsymbol{\Phi}\right)\right] \Delta_t\right) - \mathbf{I}_{p \times p}\right] \mathrm{vec}\left(\boldsymbol{\Sigma}\right)$$

where p is the number of latent variables and Δ_t is the time interval.

Value

Returns a list of state space parameters:

- `alpha`: Numeric vector. Vector of constant values for the dynamic model ($\boldsymbol{\alpha}$).
- `beta`: Numeric matrix. Transition matrix relating the values of the latent variables from the previous time point to the current time point. ($\boldsymbol{\beta}$).
- `psi_l`: Numeric matrix. Cholesky factorization (`t(chol(psi))`) of the process noise covariance matrix $\boldsymbol{\Psi}$.

Author(s)

Ivan Jacob Agaloos Pesigan

See Also

Other Simulation of State Space Models Data Functions: `SimSSMFixed()`, `SimSSMIVary()`, `SimSSMLinGrowth()`, `SimSSMLinGrowthIVary()`, `SimSSMLinSDEFixed()`, `SimSSMLinSDEIVary()`, `SimSSMOUFixed()`, `SimSSMOUIVary()`, `SimSSMVARFixed()`, `SimSSMVARIVary()`

Examples

```
p <- 2
gamma <- c(0.317, 0.230)
phi <- matrix(
  data = c(
   -0.10,
   0.05,
   0.05,
   -0.10
 ),
 nrow = p
)
sigma <- matrix(
  data = c(
    2.79,
    0.06,
    0.06,
    3.27
  ),
  nrow = p
)
sigma_l <- t(chol(sigma))
delta_t <- 0.10

LinSDE2SSM(
  gamma = gamma,
```

```
  phi = phi,
  sigma_l = sigma_l,
  delta_t = delta_t
)
```

---

plot.simstatespace *Plot Method for an Object of Class* simstatespace

---

Description

Plot Method for an Object of Class simstatespace

Usage

```
## S3 method for class 'simstatespace'
plot(x, id = NULL, time = NULL, eta = FALSE, type = "b", ...)
```

Arguments

x	Object of class simstatespace.
id	Numeric vector. Optional id numbers to plot. If id = NULL, plot all available data.
time	Numeric vector. Optional time points to plot. If time = NULL, plot all available data.
eta	Logical. If eta = TRUE, plot the latent variables. If eta = FALSE, plot the observed variables.
type	Character indicating the type of plotting; actually any of the types as in plot.default().
...	Additional arguments.

Author(s)

Ivan Jacob Agaloos Pesigan

Examples

```
# prepare parameters
set.seed(42)
## number of individuals
n <- 5
## time points
time <- 50
## dynamic structure
p <- 3
mu0 <- rep(x = 0, times = p)
sigma0 <- diag(p)
sigma0_l <- t(chol(sigma0))
```

```
alpha <- rep(x = 0, times = p)
beta <- 0.50 * diag(p)
psi <- diag(p)
psi_l <- t(chol(psi))
## measurement model
k <- 3
nu <- rep(x = 0, times = k)
lambda <- diag(k)
theta <- 0.50 * diag(k)
theta_l <- t(chol(theta))
## covariates
j <- 2
x <- lapply(
  X = seq_len(n),
  FUN = function(i) {
    matrix(
      data = stats::rnorm(n = time * j),
      nrow = j,
      ncol = time
    )
  }
)
gamma_eta <- diag(x = 0.10, nrow = p, ncol = j)
gamma_y <- diag(x = 0.10, nrow = k, ncol = j)

# Type 0
ssm <- SimSSMFixed(
  n = n,
  time = time,
  mu0 = mu0,
  sigma0_l = sigma0_l,
  alpha = alpha,
  beta = beta,
  psi_l = psi_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 0
)

plot(ssm)
plot(ssm, id = 1:3, time = 0:9)

# Type 1
ssm <- SimSSMFixed(
  n = n,
  time = time,
  mu0 = mu0,
  sigma0_l = sigma0_l,
  alpha = alpha,
  beta = beta,
  psi_l = psi_l,
  nu = nu,
```

```
  lambda = lambda,
  theta_l = theta_l,
  type = 1,
  x = x,
  gamma_eta = gamma_eta
)

plot(ssm)
plot(ssm, id = 1:3, time = 0:9)

# Type 2
ssm <- SimSSMFixed(
  n = n,
  time = time,
  mu0 = mu0,
  sigma0_l = sigma0_l,
  alpha = alpha,
  beta = beta,
  psi_l = psi_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 2,
  x = x,
  gamma_eta = gamma_eta,
  gamma_y = gamma_y
)

plot(ssm)
plot(ssm, id = 1:3, time = 0:9)
```

print.simstatespace *Print Method for an Object of Class* simstatespace

Description

Print Method for an Object of Class simstatespace

Usage

```
## S3 method for class 'simstatespace'
print(x, ...)
```

Arguments

x	Object of Class simstatespace.
...	Additional arguments.

Value

Prints simulated data in long format.

Author(s)

Ivan Jacob Agaloos Pesigan

Examples

```
# prepare parameters
set.seed(42)
## number of individuals
n <- 5
## time points
time <- 50
## dynamic structure
p <- 3
mu0 <- rep(x = 0, times = p)
sigma0 <- diag(p)
sigma0_l <- t(chol(sigma0))
alpha <- rep(x = 0, times = p)
beta <- 0.50 * diag(p)
psi <- diag(p)
psi_l <- t(chol(psi))
## measurement model
k <- 3
nu <- rep(x = 0, times = k)
lambda <- diag(k)
theta <- 0.50 * diag(k)
theta_l <- t(chol(theta))
## covariates
j <- 2
x <- lapply(
  X = seq_len(n),
  FUN = function(i) {
    matrix(
      data = stats::rnorm(n = time * j),
      nrow = j,
      ncol = time
    )
  }
)
gamma_eta <- diag(x = 0.10, nrow = p, ncol = j)
gamma_y <- diag(x = 0.10, nrow = k, ncol = j)

# Type 0
ssm <- SimSSMFixed(
  n = n,
  time = time,
  mu0 = mu0,
  sigma0_l = sigma0_l,
  alpha = alpha,
```

```
  beta = beta,
  psi_l = psi_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 0
)

print(ssm)

# Type 1
ssm <- SimSSMFixed(
  n = n,
  time = time,
  mu0 = mu0,
  sigma0_l = sigma0_l,
  alpha = alpha,
  beta = beta,
  psi_l = psi_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 1,
  x = x,
  gamma_eta = gamma_eta
)

print(ssm)

# Type 2
ssm <- SimSSMFixed(
  n = n,
  time = time,
  mu0 = mu0,
  sigma0_l = sigma0_l,
  alpha = alpha,
  beta = beta,
  psi_l = psi_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 2,
  x = x,
  gamma_eta = gamma_eta,
  gamma_y = gamma_y
)

print(ssm)
```

SimSSMFixed *Simulate Data from the State Space Model (Fixed Parameters)*

Description

This function simulates data from the state space model. In this model, the parameters are invariant cross individuals and across time.

Usage

```
SimSSMFixed(
  n,
  time,
  delta_t = 1,
  mu0,
  sigma0_l,
  alpha,
  beta,
  psi_l,
  nu,
  lambda,
  theta_l,
  type = 0,
  x = NULL,
  gamma_eta = NULL,
  gamma_y = NULL
)
```

Arguments

`n`	Positive integer. Number of individuals.
`time`	Positive integer. Number of time points.
`delta_t`	Numeric. Time interval. The default value is `1.0` with an option to use a numeric value for the discretized state space model parameterization of the linear stochastic differential equation model.
`mu0`	Numeric vector. Mean of initial latent variable values ($\boldsymbol{\mu}_{\boldsymbol{\eta}\mid 0}$).
`sigma0_l`	Numeric matrix. Cholesky factorization (`t(chol(sigma0))`) of the covariance matrix of initial latent variable values ($\boldsymbol{\Sigma}_{\boldsymbol{\eta}\mid 0}$).
`alpha`	Numeric vector. Vector of constant values for the dynamic model ($\boldsymbol{\alpha}$).
`beta`	Numeric matrix. Transition matrix relating the values of the latent variables at the previous to the current time point ($\boldsymbol{\beta}$).
`psi_l`	Numeric matrix. Cholesky factorization (`t(chol(psi))`) of the covariance matrix of the process noise ($\boldsymbol{\Psi}$).
`nu`	Numeric vector. Vector of intercept values for the measurement model ($\boldsymbol{\nu}$).
`lambda`	Numeric matrix. Factor loading matrix linking the latent variables to the observed variables ($\boldsymbol{\Lambda}$).
`theta_l`	Numeric matrix. Cholesky factorization (`t(chol(theta))`) of the covariance matrix of the measurement error ($\boldsymbol{\Theta}$).
`type`	Integer. State space model type. See Details for more information.

`x` List. Each element of the list is a matrix of covariates for each individual `i` in `n`. The number of columns in each matrix should be equal to `time`.

`gamma_eta` Numeric matrix. Matrix linking the covariates to the latent variables at current time point ($\boldsymbol{\Gamma}_{\boldsymbol{\eta}}$).

`gamma_y` Numeric matrix. Matrix linking the covariates to the observed variables at current time point ($\boldsymbol{\Gamma}_{\mathbf{y}}$).

Details

Type 0:

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \boldsymbol{\Lambda} \boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t}, \quad \mathrm{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N} \left(\mathbf{0}, \boldsymbol{\Theta} \right)$$

where $\mathbf{y}_{i,t}$, $\boldsymbol{\eta}_{i,t}$, and $\boldsymbol{\varepsilon}_{i,t}$ are random variables and $\boldsymbol{\nu}$, $\boldsymbol{\Lambda}$, and $\boldsymbol{\Theta}$ are model parameters. $\mathbf{y}_{i,t}$ is a vector of observed random variables, $\boldsymbol{\eta}_{i,t}$ is a vector of latent random variables, and $\boldsymbol{\varepsilon}_{i,t}$ is a vector of random measurement errors, at time t and individual i. $\boldsymbol{\nu}$ is a vector of intercepts, $\boldsymbol{\Lambda}$ is a matrix of factor loadings, and $\boldsymbol{\Theta}$ is the covariance matrix of $\boldsymbol{\varepsilon}$.

An alternative representation of the measurement error is given by

$$\boldsymbol{\varepsilon}_{i,t} = \boldsymbol{\Theta}^{\frac{1}{2}} \mathbf{z}_{i,t}, \quad \mathrm{with} \quad \mathbf{z}_{i,t} \sim \mathcal{N} \left(\mathbf{0}, \mathbf{I} \right)$$

where $\mathbf{z}_{i,t}$ is a vector of independent standard normal random variables and $\left(\boldsymbol{\Theta}^{\frac{1}{2}} \right) \left(\boldsymbol{\Theta}^{\frac{1}{2}} \right)^{\prime} = \boldsymbol{\Theta}$.

The dynamic structure is given by

$$\boldsymbol{\eta}_{i,t} = \boldsymbol{\alpha} + \boldsymbol{\beta} \boldsymbol{\eta}_{i,t-1} + \boldsymbol{\zeta}_{i,t}, \quad \mathrm{with} \quad \boldsymbol{\zeta}_{i,t} \sim \mathcal{N} \left(\mathbf{0}, \boldsymbol{\Psi} \right)$$

where $\boldsymbol{\eta}_{i,t}$, $\boldsymbol{\eta}_{i,t-1}$, and $\boldsymbol{\zeta}_{i,t}$ are random variables, and $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, and $\boldsymbol{\Psi}$ are model parameters. $\boldsymbol{\eta}_{i,t}$ is a vector of latent variables at time t and individual i, $\boldsymbol{\eta}_{i,t-1}$ is a vector of latent variables at time $t-1$ and individual i, and $\boldsymbol{\zeta}_{i,t}$ is a vector of dynamic noise at time t and individual i. $\boldsymbol{\alpha}$ is a vector of intercepts, $\boldsymbol{\beta}$ is a matrix of autoregression and cross regression coefficients, and $\boldsymbol{\Psi}$ is the covariance matrix of $\boldsymbol{\zeta}_{i,t}$.

An alternative representation of the dynamic noise is given by

$$\boldsymbol{\zeta}_{i,t} = \boldsymbol{\Psi}^{\frac{1}{2}} \mathbf{z}_{i,t}, \quad \mathrm{with} \quad \mathbf{z}_{i,t} \sim \mathcal{N} \left(\mathbf{0}, \mathbf{I} \right)$$

where $\left(\boldsymbol{\Psi}^{\frac{1}{2}} \right) \left(\boldsymbol{\Psi}^{\frac{1}{2}} \right)^{\prime} = \boldsymbol{\Psi}$.

Type 1:

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \boldsymbol{\Lambda} \boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t}, \quad \mathrm{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N} \left(\mathbf{0}, \boldsymbol{\Theta} \right).$$

The dynamic structure is given by

$$\boldsymbol{\eta}_{i,t} = \boldsymbol{\alpha} + \boldsymbol{\beta} \boldsymbol{\eta}_{i,t-1} + \boldsymbol{\Gamma}_{\boldsymbol{\eta}} \mathbf{x}_{i,t} + \boldsymbol{\zeta}_{i,t}, \quad \mathrm{with} \quad \boldsymbol{\zeta}_{i,t} \sim \mathcal{N} \left(\mathbf{0}, \boldsymbol{\Psi} \right)$$

where $\mathbf{x}_{i,t}$ is a vector of covariates at time t and individual i, and $\boldsymbol{\Gamma}_{\boldsymbol{\eta}}$ is the coefficient matrix linking the covariates to the latent variables.

Type 2:
The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \boldsymbol{\Lambda}\boldsymbol{\eta}_{i,t} + \boldsymbol{\Gamma}_{\mathbf{y}}\mathbf{x}_{i,t} + \boldsymbol{\varepsilon}_{i,t}, \quad \text{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Theta}\right)$$

where $\boldsymbol{\Gamma}_{\mathbf{y}}$ is the coefficient matrix linking the covariates to the observed variables.
The dynamic structure is given by

$$\boldsymbol{\eta}_{i,t} = \boldsymbol{\alpha} + \boldsymbol{\beta}\boldsymbol{\eta}_{i,t-1} + \boldsymbol{\Gamma}_{\boldsymbol{\eta}}\mathbf{x}_{i,t} + \boldsymbol{\zeta}_{i,t}, \quad \text{with} \quad \boldsymbol{\zeta}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Psi}\right).$$

Value

Returns an object of class `simstatespace` which is a list with the following elements:

- `call`: Function call.
- `args`: Function arguments.
- `data`: Generated data which is a list of length `n`. Each element of `data` is a list with the following elements:
  - `id`: A vector of ID numbers with length `t`, where `t` is the value of the function argument `time`.
  - `time`: A vector time points of length `t`.
  - `y`: A `t` by `k` matrix of values for the manifest variables.
  - `eta`: A `t` by `p` matrix of values for the latent variables.
  - `x`: A `t` by `j` matrix of values for the covariates (when covariates are included).
- `fun`: Function used.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Chow, S.-M., Ho, M. R., Hamaker, E. L., & Dolan, C. V. (2010). Equivalence and differences between structural equation modeling and state-space modeling techniques. *Structural Equation Modeling: A Multidisciplinary Journal*, 17(2), 303–332. doi:10.1080/10705511003661553

See Also

Other Simulation of State Space Models Data Functions: `LinSDE2SSM()`, `SimSSMIVary()`, `SimSSMLinGrowth()`, `SimSSMLinGrowthIVary()`, `SimSSMLinSDEFixed()`, `SimSSMLinSDEIVary()`, `SimSSMOUFixed()`, `SimSSMOUIVary()`, `SimSSMVARFixed()`, `SimSSMVARIVary()`

Examples

```
# prepare parameters
set.seed(42)
## number of individuals
n <- 5
## time points
```

```
time <- 50
## dynamic structure
p <- 3
mu0 <- rep(x = 0, times = p)
sigma0 <- diag(p)
sigma0_l <- t(chol(sigma0))
alpha <- rep(x = 0, times = p)
beta <- 0.50 * diag(p)
psi <- diag(p)
psi_l <- t(chol(psi))
## measurement model
k <- 3
nu <- rep(x = 0, times = k)
lambda <- diag(k)
theta <- 0.50 * diag(k)
theta_l <- t(chol(theta))
## covariates
j <- 2
x <- lapply(
  X = seq_len(n),
  FUN = function(i) {
    matrix(
      data = stats::rnorm(n = time * j),
      nrow = j,
      ncol = time
    )
  }
)
gamma_eta <- diag(x = 0.10, nrow = p, ncol = j)
gamma_y <- diag(x = 0.10, nrow = k, ncol = j)

# Type 0
ssm <- SimSSMFixed(
  n = n,
  time = time,
  mu0 = mu0,
  sigma0_l = sigma0_l,
  alpha = alpha,
  beta = beta,
  psi_l = psi_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 0
)

plot(ssm)

# Type 1
ssm <- SimSSMFixed(
  n = n,
  time = time,
  mu0 = mu0,
```

```
  sigma0_l = sigma0_l,
  alpha = alpha,
  beta = beta,
  psi_l = psi_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 1,
  x = x,
  gamma_eta = gamma_eta
)

plot(ssm)

# Type 2
ssm <- SimSSMFixed(
  n = n,
  time = time,
  mu0 = mu0,
  sigma0_l = sigma0_l,
  alpha = alpha,
  beta = beta,
  psi_l = psi_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 2,
  x = x,
  gamma_eta = gamma_eta,
  gamma_y = gamma_y
)

plot(ssm)
```

SimSSMIVary *Simulate Data from the State Space Model (Individual-Varying Parameters)*

Description

This function simulates data from the state space model. In this model, the parameters can vary across individuals.

Usage

```
SimSSMIVary(
  n,
  time,
  delta_t = 1,
```

```
  mu0,
  sigma0_l,
  alpha,
  beta,
  psi_l,
  nu,
  lambda,
  theta_l,
  type = 0,
  x = NULL,
  gamma_eta = NULL,
  gamma_y = NULL
)
```

Arguments

n	Positive integer. Number of individuals.
time	Positive integer. Number of time points.
delta_t	Numeric. Time interval. The default value is 1.0 with an option to use a numeric value for the discretized state space model parameterization of the linear stochastic differential equation model.
mu0	List of numeric vectors. Each element of the list is the mean of initial latent variable values ($\boldsymbol{\mu}_{\boldsymbol{\eta}\mid 0}$).
sigma0_l	List of numeric matrices. Each element of the list is the Cholesky factorization (t(chol(sigma0))) of the covariance matrix of initial latent variable values ($\boldsymbol{\Sigma}_{\boldsymbol{\eta}\mid 0}$).
alpha	List of numeric vectors. Each element of the list is the vector of constant values for the dynamic model ($\boldsymbol{\alpha}$).
beta	List of numeric matrices. Each element of the list is the transition matrix relating the values of the latent variables at the previous to the current time point ($\boldsymbol{\beta}$).
psi_l	List of numeric matrices. Each element of the list is the Cholesky factorization (t(chol(psi))) of the covariance matrix of the process noise ($\boldsymbol{\Psi}$).
nu	List of numeric vectors. Each element of the list is the vector of intercept values for the measurement model ($\boldsymbol{\nu}$).
lambda	List of numeric matrices. Each element of the list is the factor loading matrix linking the latent variables to the observed variables ($\boldsymbol{\Lambda}$).
theta_l	List of numeric matrices. Each element of the list is the Cholesky factorization (t(chol(theta))) of the covariance matrix of the measurement error ($\boldsymbol{\Theta}$).
type	Integer. State space model type. See Details for more information.
x	List. Each element of the list is a matrix of covariates for each individual i in n. The number of columns in each matrix should be equal to time.
gamma_eta	List of numeric matrices. Each element of the list is the matrix linking the covariates to the latent variables at current time point ($\boldsymbol{\Gamma}_{\boldsymbol{\eta}}$).
gamma_y	List of numeric matrices. Each element of the list is the matrix linking the covariates to the observed variables at current time point ($\boldsymbol{\Gamma}_{\mathbf{y}}$).

Details

Parameters can vary across individuals by providing a list of parameter values. If the length of any of the parameters (`mu0`, `sigma0_l`, `alpha`, `beta`, `psi_l`, `nu`, `lambda`, `theta_l`, `gamma_eta`, or `gamma_y`) is less the `n`, the function will cycle through the available values.

Value

Returns an object of class `simstatespace` which is a list with the following elements:

- `call`: Function call.
- `args`: Function arguments.
- `data`: Generated data which is a list of length `n`. Each element of `data` is a list with the following elements:
  - `id`: A vector of ID numbers with length `t`, where `t` is the value of the function argument `time`.
  - `time`: A vector time points of length `t`.
  - `y`: A `t` by `k` matrix of values for the manifest variables.
  - `eta`: A `t` by `p` matrix of values for the latent variables.
  - `x`: A `t` by `j` matrix of values for the covariates (when covariates are included).
- `fun`: Function used.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Chow, S.-M., Ho, M. R., Hamaker, E. L., & Dolan, C. V. (2010). Equivalence and differences between structural equation modeling and state-space modeling techniques. *Structural Equation Modeling: A Multidisciplinary Journal*, 17(2), 303–332. doi:10.1080/10705511003661553

See Also

Other Simulation of State Space Models Data Functions: `LinSDE2SSM()`, `SimSSMFixed()`, `SimSSMLinGrowth()`, `SimSSMLinGrowthIVary()`, `SimSSMLinSDEFixed()`, `SimSSMLinSDEIVary()`, `SimSSMOUFixed()`, `SimSSMOUIVary()`, `SimSSMVARFixed()`, `SimSSMVARIVary()`

Examples

```
# prepare parameters
# In this example, beta varies across individuals.
set.seed(42)
## number of individuals
n <- 5
## time points
time <- 50
## dynamic structure
p <- 3
```

```
mu0 <- list(
  rep(x = 0, times = p)
)
sigma0 <- diag(p)
sigma0_l <- list(
  t(chol(sigma0))
)
alpha <- list(
  rep(x = 0, times = p)
)
beta <- list(
  0.1 * diag(p),
  0.2 * diag(p),
  0.3 * diag(p),
  0.4 * diag(p),
  0.5 * diag(p)
)
psi <- diag(p)
psi_l <- list(
  t(chol(psi))
)
## measurement model
k <- 3
nu <- list(
  rep(x = 0, times = k)
)
lambda <- list(
  diag(k)
)
theta <- 0.50 * diag(k)
theta_l <- list(
  t(chol(theta))
)
## covariates
j <- 2
x <- lapply(
  X = seq_len(n),
  FUN = function(i) {
    matrix(
      data = stats::rnorm(n = time * j),
      nrow = j,
      ncol = time
    )
  }
)
gamma_eta <- list(
  diag(x = 0.10, nrow = p, ncol = j)
)
gamma_y <- list(
  diag(x = 0.10, nrow = k, ncol = j)
)

# Type 0
```

```
ssm <- SimSSMIVary(
  n = n,
  time = time,
  mu0 = mu0,
  sigma0_l = sigma0_l,
  alpha = alpha,
  beta = beta,
  psi_l = psi_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 0
)

plot(ssm)

# Type 1
ssm <- SimSSMIVary(
  n = n,
  time = time,
  mu0 = mu0,
  sigma0_l = sigma0_l,
  alpha = alpha,
  beta = beta,
  psi_l = psi_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 1,
  x = x,
  gamma_eta = gamma_eta
)

plot(ssm)

# Type 2
ssm <- SimSSMIVary(
  n = n,
  time = time,
  mu0 = mu0,
  sigma0_l = sigma0_l,
  alpha = alpha,
  beta = beta,
  psi_l = psi_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 2,
  x = x,
  gamma_eta = gamma_eta,
  gamma_y = gamma_y
)
```

```
plot(ssm)
```

`SimSSMLinGrowth` *Simulate Data from the Linear Growth Curve Model*

Description

This function simulates data from the linear growth curve model.

Usage

```
SimSSMLinGrowth(
  n,
  time,
  mu0,
  sigma0_l,
  theta_l,
  type = 0,
  x = NULL,
  gamma_eta = NULL,
  gamma_y = NULL
)
```

Arguments

`n`	Positive integer. Number of individuals.
`time`	Positive integer. Number of time points.
`mu0`	Numeric vector. A vector of length two. The first element is the mean of the intercept, and the second element is the mean of the slope.
`sigma0_l`	Numeric matrix. Cholesky factorization (`t(chol(sigma0))`) of the covariance matrix of the intercept and the slope.
`theta_l`	Numeric. Square root of the common measurement error variance.
`type`	Integer. State space model type. See Details for more information.
`x`	List. Each element of the list is a matrix of covariates for each individual `i` in `n`. The number of columns in each matrix should be equal to `time`.
`gamma_eta`	Numeric matrix. Matrix linking the covariates to the latent variables at current time point ($\boldsymbol{\Gamma}_{\boldsymbol{\eta}}$).
`gamma_y`	Numeric matrix. Matrix linking the covariates to the observed variables at current time point ($\boldsymbol{\Gamma}_{\mathbf{y}}$).

Details

Type 0:

The measurement model is given by

$$Y_{i,t} = \left(\begin{array}{cc} 1 & 0 \end{array} \right) \left(\begin{array}{c} \eta_{0_{i,t}} \\ \eta_{1_{i,t}} \end{array} \right) + \boldsymbol{\varepsilon}_{i,t}, \quad \text{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}\left(0, \theta\right)$$

where $Y_{i,t}$, $\eta_{0_{i,t}}$, $\eta_{1_{i,t}}$, and $\boldsymbol{\varepsilon}_{i,t}$ are random variables and θ is a model parameter. $Y_{i,t}$ is the observed random variable at time t and individual i, $\eta_{0_{i,t}}$ and $\eta_{1_{i,t}}$ form a vector of latent random variables at time t and individual i, and $\boldsymbol{\varepsilon}_{i,t}$ is a vector of random measurement errors at time t and individual i. θ is the variance of $\boldsymbol{\varepsilon}$.

The dynamic structure is given by

$$\left(\begin{array}{c} \eta_{0_{i,t}} \\ \eta_{1_{i,t}} \end{array} \right) = \left(\begin{array}{cc} 1 & 1 \\ 0 & 1 \end{array} \right) \left(\begin{array}{c} \eta_{0_{i,t-1}} \\ \eta_{1_{i,t-1}} \end{array} \right).$$

The mean vector and covariance matrix of the intercept and slope are captured in the mean vector and covariance matrix of the initial condition given by

$$\boldsymbol{\mu}_{\boldsymbol{\eta} \mid 0} = \left(\begin{array}{c} \mu_{\eta_{0}} \\ \mu_{\eta_{1}} \end{array} \right) \quad \text{and,}$$

$$\boldsymbol{\Sigma}_{\boldsymbol{\eta} \mid 0} = \left(\begin{array}{cc} \sigma^{2}_{\eta_{0}} & \sigma_{\eta_{0}, \eta_{1}} \\ \sigma_{\eta_{1}, \eta_{0}} & \sigma^{2}_{\eta_{1}} \end{array} \right).$$

Type 1:

The measurement model is given by

$$Y_{i,t} = \left(\begin{array}{cc} 1 & 0 \end{array} \right) \left(\begin{array}{c} \eta_{0_{i,t}} \\ \eta_{1_{i,t}} \end{array} \right) + \boldsymbol{\varepsilon}_{i,t}, \quad \text{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}\left(0, \theta\right).$$

The dynamic structure is given by

$$\left(\begin{array}{c} \eta_{0_{i,t}} \\ \eta_{1_{i,t}} \end{array} \right) = \left(\begin{array}{cc} 1 & 1 \\ 0 & 1 \end{array} \right) \left(\begin{array}{c} \eta_{0_{i,t-1}} \\ \eta_{1_{i,t-1}} \end{array} \right) + \boldsymbol{\Gamma}_{\boldsymbol{\eta}} \mathbf{x}_{i,t}$$

where $\mathbf{x}_{i,t}$ is a vector of covariates at time t and individual i, and $\boldsymbol{\Gamma}_{\boldsymbol{\eta}}$ is the coefficient matrix linking the covariates to the latent variables.

Type 2:

The measurement model is given by

$$Y_{i,t} = \left(\begin{array}{cc} 1 & 0 \end{array} \right) \left(\begin{array}{c} \eta_{0_{i,t}} \\ \eta_{1_{i,t}} \end{array} \right) + \boldsymbol{\Gamma}_{\mathbf{y}} \mathbf{x}_{i,t} + \boldsymbol{\varepsilon}_{i,t}, \quad \text{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}\left(0, \theta\right)$$

where $\boldsymbol{\Gamma}_{\mathbf{y}}$ is the coefficient matrix linking the covariates to the observed variables.

The dynamic structure is given by

$$\left(\begin{array}{c} \eta_{0_{i,t}} \\ \eta_{1_{i,t}} \end{array} \right) = \left(\begin{array}{cc} 1 & 1 \\ 0 & 1 \end{array} \right) \left(\begin{array}{c} \eta_{0_{i,t-1}} \\ \eta_{1_{i,t-1}} \end{array} \right) + \boldsymbol{\Gamma}_{\boldsymbol{\eta}} \mathbf{x}_{i,t}.$$

Value

Returns an object of class `simstatespace` which is a list with the following elements:

- `call`: Function call.
- `args`: Function arguments.
- `data`: Generated data which is a list of length `n`. Each element of `data` is a list with the following elements:
  - `id`: A vector of ID numbers with length `t`, where `t` is the value of the function argument `time`.
  - `time`: A vector time points of length `t`.
  - `y`: A `t` by `k` matrix of values for the manifest variables.
  - `eta`: A `t` by `p` matrix of values for the latent variables.
  - `x`: A `t` by `j` matrix of values for the covariates (when covariates are included).
- `fun`: Function used.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Chow, S.-M., Ho, M. R., Hamaker, E. L., & Dolan, C. V. (2010). Equivalence and differences between structural equation modeling and state-space modeling techniques. *Structural Equation Modeling: A Multidisciplinary Journal*, 17(2), 303–332. doi:10.1080/10705511003661553

See Also

Other Simulation of State Space Models Data Functions: `LinSDE2SSM()`, `SimSSMFixed()`, `SimSSMIVary()`, `SimSSMLinGrowthIVary()`, `SimSSMLinSDEFixed()`, `SimSSMLinSDEIVary()`, `SimSSMOUFixed()`, `SimSSMOUIVary()`, `SimSSMVARFixed()`, `SimSSMVARIVary()`

Examples

```
# prepare parameters
set.seed(42)
## number of individuals
n <- 5
## time points
time <- 50
## dynamic structure
p <- 2
mu0 <- c(0.615, 1.006)
sigma0 <- matrix(
  data = c(
    1.932,
    0.618,
    0.618,
    0.587
  ),
```

```
  nrow = p
)
sigma0_l <- t(chol(sigma0))
## measurement model
k <- 1
theta <- 0.50
theta_l <- sqrt(theta)
## covariates
j <- 2
x <- lapply(
  X = seq_len(n),
  FUN = function(i) {
    return(
      matrix(
        data = rnorm(n = j * time),
        nrow = j
      )
    )
  }
)
gamma_eta <- diag(x = 0.10, nrow = p, ncol = j)
gamma_y <- diag(x = 0.10, nrow = k, ncol = j)

# Type 0
ssm <- SimSSMLinGrowth(
  n = n,
  time = time,
  mu0 = mu0,
  sigma0_l = sigma0_l,
  theta_l = theta_l,
  type = 0
)

plot(ssm)

# Type 1
ssm <- SimSSMLinGrowth(
  n = n,
  time = time,
  mu0 = mu0,
  sigma0_l = sigma0_l,
  theta_l = theta_l,
  type = 1,
  x = x,
  gamma_eta = gamma_eta
)

plot(ssm)

# Type 2
ssm <- SimSSMLinGrowth(
  n = n,
  time = time,
```

```
  mu0 = mu0,
  sigma0_l = sigma0_l,
  theta_l = theta_l,
  type = 2,
  x = x,
  gamma_eta = gamma_eta,
  gamma_y = gamma_y
)

plot(ssm)
```

SimSSMLinGrowthIVary *Simulate Data from the Linear Growth Curve Model (Individual-Varying Parameters)*

Description

This function simulates data from the linear growth curve model. In this model, the parameters can vary across individuals.

Usage

```
SimSSMLinGrowthIVary(
  n,
  time,
  mu0,
  sigma0_l,
  theta_l,
  type = 0,
  x = NULL,
  gamma_eta = NULL,
  gamma_y = NULL
)
```

Arguments

n	Positive integer. Number of individuals.
time	Positive integer. Number of time points.
mu0	A list of numeric vectors. Each element of the list is a vector of length two. The first element is the mean of the intercept, and the second element is the mean of the slope.
sigma0_l	A list of numeric matrices. Each element of the list is the Cholesky factorization (`t(chol(sigma0))`) of the covariance matrix of the intercept and the slope.
theta_l	A list numeric values. Each element of the list is the square root of the common measurement error variance.
type	Integer. State space model type. See Details for more information.

x	List. Each element of the list is a matrix of covariates for each individual i in n. The number of columns in each matrix should be equal to time.
gamma_eta	List of numeric matrices. Each element of the list is the matrix linking the covariates to the latent variables at current time point ($\boldsymbol{\Gamma}_{\boldsymbol{\eta}}$).
gamma_y	List of numeric matrices. Each element of the list is the matrix linking the covariates to the observed variables at current time point ($\boldsymbol{\Gamma}_{\mathbf{y}}$).

Details

Parameters can vary across individuals by providing a list of parameter values. If the length of any of the parameters (mu0, sigma0, mu, theta_l, gamma_eta, or gamma_y) is less the n, the function will cycle through the available values.

Value

Returns an object of class simstatespace which is a list with the following elements:

- call: Function call.
- args: Function arguments.
- data: Generated data which is a list of length n. Each element of data is a list with the following elements:
  - id: A vector of ID numbers with length t, where t is the value of the function argument time.
  - time: A vector time points of length t.
  - y: A t by k matrix of values for the manifest variables.
  - eta: A t by p matrix of values for the latent variables.
  - x: A t by j matrix of values for the covariates (when covariates are included).
- fun: Function used.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Chow, S.-M., Ho, M. R., Hamaker, E. L., & Dolan, C. V. (2010). Equivalence and differences between structural equation modeling and state-space modeling techniques. *Structural Equation Modeling: A Multidisciplinary Journal*, 17(2), 303–332. doi:10.1080/10705511003661553

See Also

Other Simulation of State Space Models Data Functions: LinSDE2SSM(), SimSSMFixed(), SimSSMIVary(), SimSSMLinGrowth(), SimSSMLinSDEFixed(), SimSSMLinSDEIVary(), SimSSMOUFixed(), SimSSMOUIVary(), SimSSMVARFixed(), SimSSMVARIVary()

Other Simulation of State Space Models Data Functions: LinSDE2SSM(), SimSSMFixed(), SimSSMIVary(), SimSSMLinGrowth(), SimSSMLinSDEFixed(), SimSSMLinSDEIVary(), SimSSMOUFixed(), SimSSMOUIVary(), SimSSMVARFixed(), SimSSMVARIVary()

Examples

```
# prepare parameters
# In this example, the mean vector of the intercept and slope vary.
# Specifically,
# there are two sets of values representing two latent classes.
set.seed(42)
## number of individuals
n <- 10
## time points
time <- 50
## dynamic structure
p <- 2
mu0_1 <- c(0.615, 1.006) # lower starting point, higher growth
mu0_2 <- c(1.000, 0.500) # higher starting point, lower growth
mu0 <- list(mu0_1, mu0_2)
sigma0 <- matrix(
  data = c(
    1.932,
    0.618,
    0.618,
    0.587
  ),
  nrow = p
)
sigma0_l <- list(t(chol(sigma0)))
## measurement model
k <- 1
theta <- 0.50
theta_l <- list(sqrt(theta))
## covariates
j <- 2
x <- lapply(
  X = seq_len(n),
  FUN = function(i) {
    matrix(
      data = stats::rnorm(n = time * j),
      nrow = j,
      ncol = time
    )
  }
)
gamma_eta <- list(
  diag(x = 0.10, nrow = p, ncol = j)
)
gamma_y <- list(
  diag(x = 0.10, nrow = k, ncol = j)
)

# Type 0
ssm <- SimSSMLinGrowthIVary(
  n = n,
  time = time,
```

```
  mu0 = mu0,
  sigma0_l = sigma0_l,
  theta_l = theta_l,
  type = 0
)

plot(ssm)

# Type 1
ssm <- SimSSMLinGrowthIVary(
  n = n,
  time = time,
  mu0 = mu0,
  sigma0_l = sigma0_l,
  theta_l = theta_l,
  type = 1,
  x = x,
  gamma_eta = gamma_eta
)

plot(ssm)

# Type 2
ssm <- SimSSMLinGrowthIVary(
  n = n,
  time = time,
  mu0 = mu0,
  sigma0_l = sigma0_l,
  theta_l = theta_l,
  type = 2,
  x = x,
  gamma_eta = gamma_eta,
  gamma_y = gamma_y
)

plot(ssm)
```

`SimSSMLinSDEFixed` *Simulate Data from the Linear Stochastic Differential Equation Model using a State Space Model Parameterization (Fixed Parameters)*

Description

This function simulates data from the linear stochastic differential equation model using a state space model parameterization. In this model, the parameters are invariant across individuals and across time.

Usage

```
SimSSMLinSDEFixed(
  n,
  time,
  delta_t = 1,
  mu0,
  sigma0_l,
  gamma,
  phi,
  sigma_l,
  nu,
  lambda,
  theta_l,
  type = 0,
  x = NULL,
  gamma_eta = NULL,
  gamma_y = NULL
)
```

Arguments

n	Positive integer. Number of individuals.
time	Positive integer. Number of time points.
delta_t	Numeric. Time interval (Δ_t).
mu0	Numeric vector. Mean of initial latent variable values ($\boldsymbol{\mu}_{\boldsymbol{\eta} \mid 0}$).
sigma0_l	Numeric matrix. Cholesky factorization (`t(chol(sigma0))`) of the covariance matrix of initial latent variable values ($\boldsymbol{\Sigma}_{\boldsymbol{\eta} \mid 0}$).
gamma	Numeric vector. An unobserved term that is constant over time (γ).
phi	Numeric matrix. The drift matrix which represents the rate of change of the solution in the absence of any random fluctuations ($\boldsymbol{\Phi}$).
sigma_l	Numeric matrix. Cholesky factorization (`t(chol(sigma))`) of the covariance matrix of volatility or randomness in the process $\boldsymbol{\Sigma}$.
nu	Numeric vector. Vector of intercept values for the measurement model ($\boldsymbol{\nu}$).
lambda	Numeric matrix. Factor loading matrix linking the latent variables to the observed variables ($\boldsymbol{\Lambda}$).
theta_l	Numeric matrix. Cholesky factorization (`t(chol(theta))`) of the covariance matrix of the measurement error ($\boldsymbol{\Theta}$).
type	Integer. State space model type. See Details for more information.
x	List. Each element of the list is a matrix of covariates for each individual `i` in `n`. The number of columns in each matrix should be equal to `time`.
gamma_eta	Numeric matrix. Matrix linking the covariates to the latent variables at current time point ($\boldsymbol{\Gamma}_{\boldsymbol{\eta}}$).
gamma_y	Numeric matrix. Matrix linking the covariates to the observed variables at current time point ($\boldsymbol{\Gamma}_{\mathbf{y}}$).

Details

Type 0:

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \boldsymbol{\Lambda}\boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t}, \quad \text{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Theta}\right)$$

where $\mathbf{y}_{i,t}$, $\boldsymbol{\eta}_{i,t}$, and $\boldsymbol{\varepsilon}_{i,t}$ are random variables and $\boldsymbol{\nu}$, $\boldsymbol{\Lambda}$, and $\boldsymbol{\Theta}$ are model parameters. $\mathbf{y}_{i,t}$ is a vector of observed random variables, $\boldsymbol{\eta}_{i,t}$ is a vector of latent random variables, and $\boldsymbol{\varepsilon}_{i,t}$ is a vector of random measurement errors, at time t and individual i. $\boldsymbol{\nu}$ is a vector of intercepts, $\boldsymbol{\Lambda}$ is a matrix of factor loadings, and $\boldsymbol{\Theta}$ is the covariance matrix of $\boldsymbol{\varepsilon}$.

An alternative representation of the measurement error is given by

$$\boldsymbol{\varepsilon}_{i,t} = \boldsymbol{\Theta}^{\frac{1}{2}}\mathbf{z}_{i,t}, \quad \text{with} \quad \mathbf{z}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \mathbf{I}\right)$$

where $\mathbf{z}_{i,t}$ is a vector of independent standard normal random variables and $\left(\boldsymbol{\Theta}^{\frac{1}{2}}\right)\left(\boldsymbol{\Theta}^{\frac{1}{2}}\right)' = \boldsymbol{\Theta}$.

The dynamic structure is given by

$$\mathrm{d}\boldsymbol{\eta}_{i,t} = \left(\boldsymbol{\gamma} + \boldsymbol{\Phi}\boldsymbol{\eta}_{i,t}\right)\mathrm{d}t + \boldsymbol{\Sigma}^{\frac{1}{2}}\mathrm{d}\mathbf{W}_{i,t}$$

where $\boldsymbol{\gamma}$ is a term which is unobserved and constant over time, $\boldsymbol{\Phi}$ is the drift matrix which represents the rate of change of the solution in the absence of any random fluctuations, $\boldsymbol{\Sigma}$ is the matrix of volatility or randomness in the process, and $\mathrm{d}\boldsymbol{W}$ is a Wiener process or Brownian motion, which represents random fluctuations.

Type 1:

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \boldsymbol{\Lambda}\boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t}, \quad \text{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Theta}\right).$$

The dynamic structure is given by

$$\mathrm{d}\boldsymbol{\eta}_{i,t} = \left(\boldsymbol{\gamma} + \boldsymbol{\Phi}\boldsymbol{\eta}_{i,t}\right)\mathrm{d}t + \boldsymbol{\Gamma}_{\boldsymbol{\eta}}\mathbf{x}_{i,t} + \boldsymbol{\Sigma}^{\frac{1}{2}}\mathrm{d}\mathbf{W}_{i,t}$$

where $\mathbf{x}_{i,t}$ is a vector of covariates at time t and individual i, and $\boldsymbol{\Gamma}_{\boldsymbol{\eta}}$ is the coefficient matrix linking the covariates to the latent variables.

Type 2:

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \boldsymbol{\Lambda}\boldsymbol{\eta}_{i,t} + \boldsymbol{\Gamma}_{\mathbf{y}}\mathbf{x}_{i,t} + \boldsymbol{\varepsilon}_{i,t}, \quad \text{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Theta}\right)$$

where $\boldsymbol{\Gamma}_{\mathbf{y}}$ is the coefficient matrix linking the covariates to the observed variables.

The dynamic structure is given by

$$\mathrm{d}\boldsymbol{\eta}_{i,t} = \left(\boldsymbol{\gamma} + \boldsymbol{\Phi}\boldsymbol{\eta}_{i,t}\right)\mathrm{d}t + \boldsymbol{\Gamma}_{\boldsymbol{\eta}}\mathbf{x}_{i,t} + \boldsymbol{\Sigma}^{\frac{1}{2}}\mathrm{d}\mathbf{W}_{i,t}.$$

Value

Returns an object of class `simstatespace` which is a list with the following elements:

- `call`: Function call.
- `args`: Function arguments.
- `data`: Generated data which is a list of length `n`. Each element of `data` is a list with the following elements:
  - `id`: A vector of ID numbers with length `t`, where `t` is the value of the function argument `time`.
  - `time`: A vector time points of length `t`.
  - `y`: A `t` by `k` matrix of values for the manifest variables.
  - `eta`: A `t` by `p` matrix of values for the latent variables.
  - `x`: A `t` by `j` matrix of values for the covariates (when covariates are included).
- `fun`: Function used.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Chow, S.-M., Ho, M. R., Hamaker, E. L., & Dolan, C. V. (2010). Equivalence and differences between structural equation modeling and state-space modeling techniques. *Structural Equation Modeling: A Multidisciplinary Journal*, 17(2), 303–332. doi:10.1080/10705511003661553

See Also

Other Simulation of State Space Models Data Functions: `LinSDE2SSM()`, `SimSSMFixed()`, `SimSSMIVary()`, `SimSSMLinGrowth()`, `SimSSMLinGrowthIVary()`, `SimSSMLinSDEIVary()`, `SimSSMOUFixed()`, `SimSSMOUIVary()`, `SimSSMVARFixed()`, `SimSSMVARIVary()`

Examples

```
# prepare parameters
set.seed(42)
## number of individuals
n <- 5
## time points
time <- 50
delta_t <- 0.10
## dynamic structure
p <- 2
mu0 <- c(-3.0, 1.5)
sigma0 <- diag(p)
sigma0_l <- t(chol(sigma0))
gamma <- c(0.317, 0.230)
phi <- matrix(
  data = c(
    -0.10,
```

```
    0.05,
    0.05,
    -0.10
  ),
  nrow = p
)
sigma <- matrix(
  data = c(
    2.79,
    0.06,
    0.06,
    3.27
  ),
  nrow = p
)
sigma_l <- t(chol(sigma))
## measurement model
k <- 2
nu <- rep(x = 0, times = k)
lambda <- diag(k)
theta <- 0.50 * diag(k)
theta_l <- t(chol(theta))
## covariates
j <- 2
x <- lapply(
  X = seq_len(n),
  FUN = function(i) {
    matrix(
      data = stats::rnorm(n = time * j),
      nrow = j,
      ncol = time
    )
  }
)
gamma_eta <- diag(x = 0.10, nrow = p, ncol = j)
gamma_y <- diag(x = 0.10, nrow = k, ncol = j)

# Type 0
ssm <- SimSSMLinSDEFixed(
  n = n,
  time = time,
  delta_t = delta_t,
  mu0 = mu0,
  sigma0_l = sigma0_l,
  gamma = gamma,
  phi = phi,
  sigma_l = sigma_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 0
)
```

```
plot(ssm)

# Type 1
ssm <- SimSSMLinSDEFixed(
  n = n,
  time = time,
  delta_t = delta_t,
  mu0 = mu0,
  sigma0_l = sigma0_l,
  gamma = gamma,
  phi = phi,
  sigma_l = sigma_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 1,
  x = x,
  gamma_eta = gamma_eta
)

plot(ssm)

# Type 2
ssm <- SimSSMLinSDEFixed(
  n = n,
  time = time,
  delta_t = delta_t,
  mu0 = mu0,
  sigma0_l = sigma0_l,
  gamma = gamma,
  phi = phi,
  sigma_l = sigma_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 2,
  x = x,
  gamma_eta = gamma_eta,
  gamma_y = gamma_y
)

plot(ssm)
```

SimSSMLinSDEIVary — *Simulate Data from the Linear Stochastic Differential Equation Model using a State Space Model Parameterization (Individual-Varying Parameters)*

Description

This function simulates data from the linear stochastic differential equation model using a state space model parameterization. In this model, the parameters can vary across individuals.

Usage

```
SimSSMLinSDEIVary(
  n,
  time,
  delta_t = 1,
  mu0,
  sigma0_l,
  gamma,
  phi,
  sigma_l,
  nu,
  lambda,
  theta_l,
  type = 0,
  x = NULL,
  gamma_eta = NULL,
  gamma_y = NULL
)
```

Arguments

n	Positive integer. Number of individuals.
time	Positive integer. Number of time points.
delta_t	Numeric. Time interval. The default value is `1.0` with an option to use a numeric value for the discretized state space model parameterization of the linear stochastic differential equation model.
mu0	List of numeric vectors. Each element of the list is the mean of initial latent variable values ($\boldsymbol{\mu}_{\boldsymbol{\eta}\|0}$).
sigma0_l	List of numeric matrices. Each element of the list is the Cholesky factorization (`t(chol(sigma0))`) of the covariance matrix of initial latent variable values ($\boldsymbol{\Sigma}_{\boldsymbol{\eta}\|0}$).
gamma	List of numeric vectors. Each element of the list is an unobserved term that is constant over time (γ).
phi	List of numeric matrix. Each element of the list is the drift matrix which represents the rate of change of the solution in the absence of any random fluctuations ($\boldsymbol{\Phi}$).
sigma_l	List of numeric matrix. Each element of the list is the Cholesky factorization (`t(chol(sigma))`) of the covariance matrix of volatility or randomness in the process $\boldsymbol{\Sigma}$.
nu	List of numeric vectors. Each element of the list is the vector of intercept values for the measurement model ($\boldsymbol{\nu}$).

lambda	List of numeric matrices. Each element of the list is the factor loading matrix linking the latent variables to the observed variables ($\boldsymbol{\Lambda}$).
theta_l	List of numeric matrices. Each element of the list is the Cholesky factorization (t(chol(theta))) of the covariance matrix of the measurement error ($\boldsymbol{\Theta}$).
type	Integer. State space model type. See Details for more information.
x	List. Each element of the list is a matrix of covariates for each individual i in n. The number of columns in each matrix should be equal to time.
gamma_eta	List of numeric matrices. Each element of the list is the matrix linking the covariates to the latent variables at current time point ($\boldsymbol{\Gamma}_{\boldsymbol{\eta}}$).
gamma_y	List of numeric matrices. Each element of the list is the matrix linking the covariates to the observed variables at current time point ($\boldsymbol{\Gamma}_{\mathbf{y}}$).

Details

Parameters can vary across individuals by providing a list of parameter values. If the length of any of the parameters (mu0, sigma0_l, gamma, phi, sigma_l, nu, lambda, theta_l, gamma_eta, or gamma_y) is less the n, the function will cycle through the available values.

Value

Returns an object of class simstatespace which is a list with the following elements:

- call: Function call.
- args: Function arguments.
- data: Generated data which is a list of length n. Each element of data is a list with the following elements:
  - id: A vector of ID numbers with length t, where t is the value of the function argument time.
  - time: A vector time points of length t.
  - y: A t by k matrix of values for the manifest variables.
  - eta: A t by p matrix of values for the latent variables.
  - x: A t by j matrix of values for the covariates (when covariates are included).
- fun: Function used.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Chow, S.-M., Ho, M. R., Hamaker, E. L., & Dolan, C. V. (2010). Equivalence and differences between structural equation modeling and state-space modeling techniques. *Structural Equation Modeling: A Multidisciplinary Journal*, 17(2), 303–332. doi:10.1080/10705511003661553

See Also

Other Simulation of State Space Models Data Functions: `LinSDE2SSM()`, `SimSSMFixed()`, `SimSSMIVary()`, `SimSSMLinGrowth()`, `SimSSMLinGrowthIVary()`, `SimSSMLinSDEFixed()`, `SimSSMOUFixed()`, `SimSSMOUIVary()`, `SimSSMVARFixed()`, `SimSSMVARIVary()`

Examples

```
# prepare parameters
# In this example, phi varies across individuals.
set.seed(42)
## number of individuals
n <- 5
## time points
time <- 50
delta_t <- 0.10
## dynamic structure
p <- 2
mu0 <- list(
  c(-3.0, 1.5)
)
sigma0 <- diag(p)
sigma0_l <- list(
  t(chol(sigma0))
)
gamma <- list(
  c(0.317, 0.230)
)
phi <- list(
  -0.1 * diag(p),
  -0.2 * diag(p),
  -0.3 * diag(p),
  -0.4 * diag(p),
  -0.5 * diag(p)
)
sigma <- matrix(
  data = c(
    2.79,
    0.06,
    0.06,
    3.27
  ),
  nrow = p
)
sigma_l <- list(
  t(chol(sigma))
)
## measurement model
k <- 2
nu <- list(
  rep(x = 0, times = k)
)
lambda <- list(
```

```
  diag(k)
)
theta <- 0.50 * diag(k)
theta_l <- list(
  t(chol(theta))
)
## covariates
j <- 2
x <- lapply(
  X = seq_len(n),
  FUN = function(i) {
    matrix(
      data = stats::rnorm(n = time * j),
      nrow = j,
      ncol = time
    )
  }
)
gamma_eta <- list(
  diag(x = 0.10, nrow = p, ncol = j)
)
gamma_y <- list(
  diag(x = 0.10, nrow = k, ncol = j)
)

# Type 0
ssm <- SimSSMLinSDEIVary(
  n = n,
  time = time,
  delta_t = delta_t,
  mu0 = mu0,
  sigma0_l = sigma0_l,
  gamma = gamma,
  phi = phi,
  sigma_l = sigma_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 0
)

plot(ssm)

# Type 1
ssm <- SimSSMLinSDEIVary(
  n = n,
  time = time,
  delta_t = delta_t,
  mu0 = mu0,
  sigma0_l = sigma0_l,
  gamma = gamma,
  phi = phi,
  sigma_l = sigma_l,
```

```
    nu = nu,
    lambda = lambda,
    theta_l = theta_l,
    type = 1,
    x = x,
    gamma_eta = gamma_eta
  )

  plot(ssm)

  # Type 2
  ssm <- SimSSMLinSDEIVary(
    n = n,
    time = time,
    delta_t = delta_t,
    mu0 = mu0,
    sigma0_l = sigma0_l,
    gamma = gamma,
    phi = phi,
    sigma_l = sigma_l,
    nu = nu,
    lambda = lambda,
    theta_l = theta_l,
    type = 2,
    x = x,
    gamma_eta = gamma_eta,
    gamma_y = gamma_y
  )

  plot(ssm)
```

SimSSMOUFixed — *Simulate Data from the Ornstein–Uhlenbeck Model using a State Space Model Parameterization (Fixed Parameters)*

Description

This function simulates data from the Ornstein–Uhlenbeck model using a state space model parameterization. In this model, the parameters are invariant across individuals and across time.

Usage

```
SimSSMOUFixed(
  n,
  time,
  delta_t = 1,
  mu0,
  sigma0_l,
```

```
  mu,
  phi,
  sigma_l,
  nu,
  lambda,
  theta_l,
  type = 0,
  x = NULL,
  gamma_eta = NULL,
  gamma_y = NULL
)
```

Arguments

n	Positive integer. Number of individuals.
time	Positive integer. Number of time points.
delta_t	Numeric. Time interval (Δ_t).
mu0	Numeric vector. Mean of initial latent variable values ($\boldsymbol{\mu}_{\boldsymbol{\eta}\|0}$).
sigma0_l	Numeric matrix. Cholesky factorization (`t(chol(sigma0))`) of the covariance matrix of initial latent variable values ($\boldsymbol{\Sigma}_{\boldsymbol{\eta}\|0}$).
mu	Numeric vector. The long-term mean or equilibrium level ($\boldsymbol{\mu}$).
phi	Numeric matrix. The drift matrix which represents the rate of change of the solution in the absence of any random fluctuations ($\boldsymbol{\Phi}$). The negative value of `phi` is the rate of mean reversion, determining how quickly the variable returns to its mean ($-\boldsymbol{\Phi}$).
sigma_l	Numeric matrix. Cholesky factorization (`t(chol(sigma))`) of the covariance matrix of volatility or randomness in the process $\boldsymbol{\Sigma}$.
nu	Numeric vector. Vector of intercept values for the measurement model ($\boldsymbol{\nu}$).
lambda	Numeric matrix. Factor loading matrix linking the latent variables to the observed variables ($\boldsymbol{\Lambda}$).
theta_l	Numeric matrix. Cholesky factorization (`t(chol(theta))`) of the covariance matrix of the measurement error ($\boldsymbol{\Theta}$).
type	Integer. State space model type. See Details for more information.
x	List. Each element of the list is a matrix of covariates for each individual `i` in `n`. The number of columns in each matrix should be equal to `time`.
gamma_eta	Numeric matrix. Matrix linking the covariates to the latent variables at current time point ($\boldsymbol{\Gamma}_{\boldsymbol{\eta}}$).
gamma_y	Numeric matrix. Matrix linking the covariates to the observed variables at current time point ($\boldsymbol{\Gamma}_{\mathbf{y}}$).

Details

Type 0:

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \boldsymbol{\Lambda}\boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t}, \quad \text{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Theta}\right)$$

where $\mathbf{y}_{i,t}$, $\boldsymbol{\eta}_{i,t}$, and $\boldsymbol{\varepsilon}_{i,t}$ are random variables and $\boldsymbol{\nu}$, $\boldsymbol{\Lambda}$, and $\boldsymbol{\Theta}$ are model parameters. $\mathbf{y}_{i,t}$ is a vector of observed random variables, $\boldsymbol{\eta}_{i,t}$ is a vector of latent random variables, and $\boldsymbol{\varepsilon}_{i,t}$ is a vector of random measurement errors, at time t and individual i. $\boldsymbol{\nu}$ is a vector of intercepts, $\boldsymbol{\Lambda}$ is a matrix of factor loadings, and $\boldsymbol{\Theta}$ is the covariance matrix of $\boldsymbol{\varepsilon}$.

An alternative representation of the measurement error is given by

$$\boldsymbol{\varepsilon}_{i,t} = \boldsymbol{\Theta}^{\frac{1}{2}}\mathbf{z}_{i,t}, \quad \text{with} \quad \mathbf{z}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \mathbf{I}\right)$$

where $\mathbf{z}_{i,t}$ is a vector of independent standard normal random variables and $\left(\boldsymbol{\Theta}^{\frac{1}{2}}\right)\left(\boldsymbol{\Theta}^{\frac{1}{2}}\right)' = \boldsymbol{\Theta}$.

The dynamic structure is given by

$$\mathrm{d}\boldsymbol{\eta}_{i,t} = -\boldsymbol{\Phi}\left(\boldsymbol{\mu} - \boldsymbol{\eta}_{i,t}\right)\mathrm{d}t + \boldsymbol{\Sigma}^{\frac{1}{2}}\mathrm{d}\mathbf{W}_{i,t}$$

where $\boldsymbol{\mu}$ is the long-term mean or equilibrium level, $-\boldsymbol{\Phi}$ is the rate of mean reversion, determining how quickly the variable returns to its mean, $\boldsymbol{\Sigma}$ is the matrix of volatility or randomness in the process, and $\mathrm{d}\boldsymbol{W}$ is a Wiener process or Brownian motion, which represents random fluctuations.

Type 1:

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \boldsymbol{\Lambda}\boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t}, \quad \text{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Theta}\right).$$

The dynamic structure is given by

$$\mathrm{d}\boldsymbol{\eta}_{i,t} = -\boldsymbol{\Phi}\left(\boldsymbol{\mu} - \boldsymbol{\eta}_{i,t}\right)\mathrm{d}t + \boldsymbol{\Gamma}_{\boldsymbol{\eta}}\mathbf{x}_{i,t} + \boldsymbol{\Sigma}^{\frac{1}{2}}\mathrm{d}\mathbf{W}_{i,t}$$

where $\mathbf{x}_{i,t}$ is a vector of covariates at time t and individual i, and $\boldsymbol{\Gamma}_{\boldsymbol{\eta}}$ is the coefficient matrix linking the covariates to the latent variables.

Type 2:

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \boldsymbol{\Lambda}\boldsymbol{\eta}_{i,t} + \boldsymbol{\Gamma}_{\mathbf{y}}\mathbf{x}_{i,t} + \boldsymbol{\varepsilon}_{i,t}, \quad \text{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Theta}\right)$$

where $\boldsymbol{\Gamma}_{\mathbf{y}}$ is the coefficient matrix linking the covariates to the observed variables.

The dynamic structure is given by

$$\mathrm{d}\boldsymbol{\eta}_{i,t} = -\boldsymbol{\Phi}\left(\boldsymbol{\mu} - \boldsymbol{\eta}_{i,t}\right)\mathrm{d}t + \boldsymbol{\Gamma}_{\boldsymbol{\eta}}\mathbf{x}_{i,t} + \boldsymbol{\Sigma}^{\frac{1}{2}}\mathrm{d}\mathbf{W}_{i,t}.$$

Value

Returns an object of class `simstatespace` which is a list with the following elements:

- `call`: Function call.
- `args`: Function arguments.

- `data`: Generated data which is a list of length `n`. Each element of `data` is a list with the following elements:
  - `id`: A vector of ID numbers with length `t`, where `t` is the value of the function argument `time`.
  - `time`: A vector time points of length `t`.
  - `y`: A `t` by `k` matrix of values for the manifest variables.
  - `eta`: A `t` by `p` matrix of values for the latent variables.
  - `x`: A `t` by `j` matrix of values for the covariates (when covariates are included).
- `fun`: Function used.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Chow, S.-M., Ho, M. R., Hamaker, E. L., & Dolan, C. V. (2010). Equivalence and differences between structural equation modeling and state-space modeling techniques. *Structural Equation Modeling: A Multidisciplinary Journal*, 17(2), 303–332. doi:10.1080/10705511003661553

See Also

Other Simulation of State Space Models Data Functions: `LinSDE2SSM()`, `SimSSMFixed()`, `SimSSMIVary()`, `SimSSMLinGrowth()`, `SimSSMLinGrowthIVary()`, `SimSSMLinSDEFixed()`, `SimSSMLinSDEIVary()`, `SimSSMOUIVary()`, `SimSSMVARFixed()`, `SimSSMVARIVary()`

Examples

```
# prepare parameters
set.seed(42)
## number of individuals
n <- 5
## time points
time <- 50
delta_t <- 0.10
## dynamic structure
p <- 2
mu0 <- c(-3.0, 1.5)
sigma0 <- diag(p)
sigma0_l <- t(chol(sigma0))
mu <- c(5.76, 5.18)
phi <- matrix(
  data = c(
    -0.10,
    0.05,
    0.05,
    -0.10
  ),
  nrow = p
)
```

```
sigma <- matrix(
  data = c(
    2.79,
    0.06,
    0.06,
    3.27
  ),
  nrow = p
)
sigma_l <- t(chol(sigma))
## measurement model
k <- 2
nu <- rep(x = 0, times = k)
lambda <- diag(k)
theta <- 0.50 * diag(k)
theta_l <- t(chol(theta))
## covariates
j <- 2
x <- lapply(
  X = seq_len(n),
  FUN = function(i) {
    matrix(
      data = stats::rnorm(n = time * j),
      nrow = j,
      ncol = time
    )
  }
)
gamma_eta <- diag(x = 0.10, nrow = p, ncol = j)
gamma_y <- diag(x = 0.10, nrow = k, ncol = j)

# Type 0
ssm <- SimSSMOUFixed(
  n = n,
  time = time,
  delta_t = delta_t,
  mu0 = mu0,
  sigma0_l = sigma0_l,
  mu = mu,
  phi = phi,
  sigma_l = sigma_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 0
)

plot(ssm)

# Type 1
ssm <- SimSSMOUFixed(
  n = n,
  time = time,
```

```
  delta_t = delta_t,
  mu0 = mu0,
  sigma0_l = sigma0_l,
  mu = mu,
  phi = phi,
  sigma_l = sigma_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 1,
  x = x,
  gamma_eta = gamma_eta
)

plot(ssm)

# Type 2
ssm <- SimSSMOUFixed(
  n = n,
  time = time,
  delta_t = delta_t,
  mu0 = mu0,
  sigma0_l = sigma0_l,
  mu = mu,
  phi = phi,
  sigma_l = sigma_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 2,
  x = x,
  gamma_eta = gamma_eta,
  gamma_y = gamma_y
)

plot(ssm)
```

SimSSMOUIVary *Simulate Data from the Ornstein–Uhlenbeck Model using a State Space Model Parameterization (Individual-Varying Parameters)*

Description

This function simulates data from the Ornstein–Uhlenbeck model using a state space model parameterization. In this model, the parameters can vary across individuals.

Usage

```
SimSSMOUIVary(
```

```
  n,
  time,
  delta_t = 1,
  mu0,
  sigma0_l,
  mu,
  phi,
  sigma_l,
  nu,
  lambda,
  theta_l,
  type = 0,
  x = NULL,
  gamma_eta = NULL,
  gamma_y = NULL
)
```

Arguments

n	Positive integer. Number of individuals.
time	Positive integer. Number of time points.
delta_t	Numeric. Time interval. The default value is `1.0` with an option to use a numeric value for the discretized state space model parameterization of the linear stochastic differential equation model.
mu0	List of numeric vectors. Each element of the list is the mean of initial latent variable values ($\boldsymbol{\mu}_{\boldsymbol{\eta}\mid 0}$).
sigma0_l	List of numeric matrices. Each element of the list is the Cholesky factorization (`t(chol(sigma0))`) of the covariance matrix of initial latent variable values ($\boldsymbol{\Sigma}_{\boldsymbol{\eta}\mid 0}$).
mu	List of numeric vectors. Each element of the list is the long-term mean or equilibrium level ($\boldsymbol{\mu}$).
phi	List of numeric matrix. Each element of the list is the drift matrix which represents the rate of change of the solution in the absence of any random fluctuations ($\boldsymbol{\Phi}$). The negative value of `phi` is the rate of mean reversion, determining how quickly the variable returns to its mean ($-\boldsymbol{\Phi}$).
sigma_l	List of numeric matrix. Each element of the list is the Cholesky factorization (`t(chol(sigma))`) of the covariance matrix of volatility or randomness in the process $\boldsymbol{\Sigma}$.
nu	List of numeric vectors. Each element of the list is the vector of intercept values for the measurement model ($\boldsymbol{\nu}$).
lambda	List of numeric matrices. Each element of the list is the factor loading matrix linking the latent variables to the observed variables ($\boldsymbol{\Lambda}$).
theta_l	List of numeric matrices. Each element of the list is the Cholesky factorization (`t(chol(theta))`) of the covariance matrix of the measurement error ($\boldsymbol{\Theta}$).
type	Integer. State space model type. See Details for more information.

`x`	List. Each element of the list is a matrix of covariates for each individual `i` in `n`. The number of columns in each matrix should be equal to `time`.
`gamma_eta`	List of numeric matrices. Each element of the list is the matrix linking the covariates to the latent variables at current time point ($\boldsymbol{\Gamma}_{\boldsymbol{\eta}}$).
`gamma_y`	List of numeric matrices. Each element of the list is the matrix linking the covariates to the observed variables at current time point ($\boldsymbol{\Gamma}_{\mathbf{y}}$).

Details

Parameters can vary across individuals by providing a list of parameter values. If the length of any of the parameters (`mu0`, `sigma0_l`, `mu`, `phi`, `sigma_l`, `nu`, `lambda`, `theta_l`, `gamma_eta`, or `gamma_y`) is less the `n`, the function will cycle through the available values.

Value

Returns an object of class `simstatespace` which is a list with the following elements:

- `call`: Function call.
- `args`: Function arguments.
- `data`: Generated data which is a list of length `n`. Each element of `data` is a list with the following elements:
  - `id`: A vector of ID numbers with length `t`, where `t` is the value of the function argument `time`.
  - `time`: A vector time points of length `t`.
  - `y`: A `t` by `k` matrix of values for the manifest variables.
  - `eta`: A `t` by `p` matrix of values for the latent variables.
  - `x`: A `t` by `j` matrix of values for the covariates (when covariates are included).
- `fun`: Function used.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Chow, S.-M., Ho, M. R., Hamaker, E. L., & Dolan, C. V. (2010). Equivalence and differences between structural equation modeling and state-space modeling techniques. *Structural Equation Modeling: A Multidisciplinary Journal*, 17(2), 303–332. doi:10.1080/10705511003661553

See Also

Other Simulation of State Space Models Data Functions: `LinSDE2SSM()`, `SimSSMFixed()`, `SimSSMIVary()`, `SimSSMLinGrowth()`, `SimSSMLinGrowthIVary()`, `SimSSMLinSDEFixed()`, `SimSSMLinSDEIVary()`, `SimSSMOUFixed()`, `SimSSMVARFixed()`, `SimSSMVARIVary()`

Examples

```
# prepare parameters
# In this example, phi varies across individuals.
set.seed(42)
## number of individuals
n <- 5
## time points
time <- 50
delta_t <- 0.10
## dynamic structure
p <- 2
mu0 <- list(
  c(-3.0, 1.5)
)
sigma0 <- diag(p)
sigma0_l <- list(
  t(chol(sigma0))
)
mu <- list(
  c(5.76, 5.18)
)
phi <- list(
  -0.1 * diag(p),
  -0.2 * diag(p),
  -0.3 * diag(p),
  -0.4 * diag(p),
  -0.5 * diag(p)
)
sigma <- matrix(
  data = c(
    2.79,
    0.06,
    0.06,
    3.27
  ),
  nrow = p
)
sigma_l <- list(
  t(chol(sigma))
)
## measurement model
k <- 2
nu <- list(
  rep(x = 0, times = k)
)
lambda <- list(
  diag(k)
)
theta <- 0.50 * diag(k)
theta_l <- list(
  t(chol(theta))
)
```

```
## covariates
j <- 2
x <- lapply(
  X = seq_len(n),
  FUN = function(i) {
    matrix(
      data = stats::rnorm(n = time * j),
      nrow = j,
      ncol = time
    )
  }
)
gamma_eta <- list(
  diag(x = 0.10, nrow = p, ncol = j)
)
gamma_y <- list(
  diag(x = 0.10, nrow = k, ncol = j)
)

# Type 0
ssm <- SimSSMOUIVary(
  n = n,
  time = time,
  delta_t = delta_t,
  mu0 = mu0,
  sigma0_l = sigma0_l,
  mu = mu,
  phi = phi,
  sigma_l = sigma_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 0
)

plot(ssm)

# Type 1
ssm <- SimSSMOUIVary(
  n = n,
  time = time,
  delta_t = delta_t,
  mu0 = mu0,
  sigma0_l = sigma0_l,
  mu = mu,
  phi = phi,
  sigma_l = sigma_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 1,
  x = x,
  gamma_eta = gamma_eta
```

```
)

plot(ssm)

# Type 2
ssm <- SimSSMOUIVary(
  n = n,
  time = time,
  delta_t = delta_t,
  mu0 = mu0,
  sigma0_l = sigma0_l,
  mu = mu,
  phi = phi,
  sigma_l = sigma_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 2,
  x = x,
  gamma_eta = gamma_eta,
  gamma_y = gamma_y
)

plot(ssm)
```

`SimSSMVARFixed` *Simulate Data from the Vector Autoregressive Model (Fixed Parameters)*

Description

This function simulates data from the vector autoregressive model using a state space model parameterization. In this model, the parameters are invariant cross individuals and across time.

Usage

```
SimSSMVARFixed(
  n,
  time,
  mu0,
  sigma0_l,
  alpha,
  beta,
  psi_l,
  type = 0,
  x = NULL,
  gamma_eta = NULL
)
```

Arguments

n	Positive integer. Number of individuals.
time	Positive integer. Number of time points.
mu0	Numeric vector. Mean of initial latent variable values ($\boldsymbol{\mu}_{\boldsymbol{\eta}\mid 0}$).
sigma0_l	Numeric matrix. Cholesky factorization (`t(chol(sigma0))`) of the covariance matrix of initial latent variable values ($\boldsymbol{\Sigma}_{\boldsymbol{\eta}\mid 0}$).
alpha	Numeric vector. Vector of constant values for the dynamic model ($\boldsymbol{\alpha}$).
beta	Numeric matrix. Transition matrix relating the values of the latent variables at the previous to the current time point ($\boldsymbol{\beta}$).
psi_l	Numeric matrix. Cholesky factorization (`t(chol(psi))`) of the covariance matrix of the process noise ($\boldsymbol{\Psi}$).
type	Integer. State space model type. See Details for more information.
x	List. Each element of the list is a matrix of covariates for each individual `i` in `n`. The number of columns in each matrix should be equal to `time`.
gamma_eta	Numeric matrix. Matrix linking the covariates to the latent variables at current time point ($\boldsymbol{\Gamma}_{\boldsymbol{\eta}}$).

Details

Type 0:

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\eta}_{i,t}.$$

The dynamic structure is given by

$$\boldsymbol{\eta}_{i,t} = \boldsymbol{\alpha} + \boldsymbol{\beta}\boldsymbol{\eta}_{i,t-1} + \boldsymbol{\zeta}_{i,t}, \quad \text{with} \quad \boldsymbol{\zeta}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Psi}\right)$$

where $\boldsymbol{\eta}_{i,t}$, $\boldsymbol{\eta}_{i,t-1}$, and $\boldsymbol{\zeta}_{i,t}$ are random variables, and $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, and $\boldsymbol{\Psi}$ are model parameters. $\boldsymbol{\eta}_{i,t}$ is a vector of latent variables at time t and individual i, $\boldsymbol{\eta}_{i,t-1}$ is a vector of latent variables at time $t-1$ and individual i, and $\boldsymbol{\zeta}_{i,t}$ is a vector of dynamic noise at time t and individual i. $\boldsymbol{\alpha}$ is a vector of intercepts, $\boldsymbol{\beta}$ is a matrix of autoregression and cross regression coefficients, and $\boldsymbol{\Psi}$ is the covariance matrix of $\boldsymbol{\zeta}_{i,t}$.

An alternative representation of the dynamic noise is given by

$$\boldsymbol{\zeta}_{i,t} = \boldsymbol{\Psi}^{\frac{1}{2}}\mathbf{z}_{i,t}, \quad \text{with} \quad \mathbf{z}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \mathbf{I}\right)$$

where $\mathbf{z}_{i,t}$ is a vector of independent standard normal random variables and $\left(\boldsymbol{\Psi}^{\frac{1}{2}}\right)\left(\boldsymbol{\Psi}^{\frac{1}{2}}\right)' = \boldsymbol{\Psi}$.

Type 1:

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\eta}_{i,t}.$$

The dynamic structure is given by

$$\boldsymbol{\eta}_{i,t} = \boldsymbol{\alpha} + \boldsymbol{\beta}\boldsymbol{\eta}_{i,t-1} + \boldsymbol{\Gamma}_{\boldsymbol{\eta}}\mathbf{x}_{i,t} + \boldsymbol{\zeta}_{i,t}, \quad \text{with} \quad \boldsymbol{\zeta}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Psi}\right)$$

where $\mathbf{x}_{i,t}$ is a vector of covariates at time t and individual i, and $\boldsymbol{\Gamma}_{\boldsymbol{\eta}}$ is the coefficient matrix linking the covariates to the latent variables.

Value

Returns an object of class simstatespace which is a list with the following elements:

- call: Function call.
- args: Function arguments.
- data: Generated data which is a list of length n. Each element of data is a list with the following elements:
  - id: A vector of ID numbers with length t, where t is the value of the function argument time.
  - time: A vector time points of length t.
  - y: A t by k matrix of values for the manifest variables.
  - eta: A t by p matrix of values for the latent variables.
  - x: A t by j matrix of values for the covariates (when covariates are included).
- fun: Function used.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Chow, S.-M., Ho, M. R., Hamaker, E. L., & Dolan, C. V. (2010). Equivalence and differences between structural equation modeling and state-space modeling techniques. *Structural Equation Modeling: A Multidisciplinary Journal*, 17(2), 303–332. doi:10.1080/10705511003661553

See Also

Other Simulation of State Space Models Data Functions: LinSDE2SSM(), SimSSMFixed(), SimSSMIVary(), SimSSMLinGrowth(), SimSSMLinGrowthIVary(), SimSSMLinSDEFixed(), SimSSMLinSDEIVary(), SimSSMOUFixed(), SimSSMOUIVary(), SimSSMVARIVary()

Examples

```
# prepare parameters
set.seed(42)
## number of individuals
n <- 5
## time points
time <- 50
## dynamic structure
p <- 3
mu0 <- rep(x = 0, times = p)
sigma0 <- diag(p)
sigma0_l <- t(chol(sigma0))
alpha <- rep(x = 0, times = p)
beta <- 0.50 * diag(p)
psi <- diag(p)
psi_l <- t(chol(psi))
## covariates
```

```
j <- 2
x <- lapply(
  X = seq_len(n),
  FUN = function(i) {
    matrix(
      data = stats::rnorm(n = time * j),
      nrow = j,
      ncol = time
    )
  }
)
gamma_eta <- diag(x = 0.10, nrow = p, ncol = j)

# Type 0
ssm <- SimSSMVARFixed(
  n = n,
  time = time,
  mu0 = mu0,
  sigma0_l = sigma0_l,
  alpha = alpha,
  beta = beta,
  psi_l = psi_l,
  type = 0
)

plot(ssm)

# Type 1
ssm <- SimSSMVARFixed(
  n = n,
  time = time,
  mu0 = mu0,
  sigma0_l = sigma0_l,
  alpha = alpha,
  beta = beta,
  psi_l = psi_l,
  type = 1,
  x = x,
  gamma_eta = gamma_eta
)

plot(ssm)
```

`SimSSMVARIVary` *Simulate Data from the Vector Autoregressive Model (Individual-Varying Parameters)*

Description

This function simulates data from the vector autoregressive model using a state space model parameterization. In this model, the parameters can vary across individuals.

Usage

```
SimSSMVARIVary(
  n,
  time,
  mu0,
  sigma0_l,
  alpha,
  beta,
  psi_l,
  type = 0,
  x = NULL,
  gamma_eta = NULL
)
```

Arguments

`n`	Positive integer. Number of individuals.
`time`	Positive integer. Number of time points.
`mu0`	List of numeric vectors. Each element of the list is the mean of initial latent variable values ($\boldsymbol{\mu}_{\boldsymbol{\eta}\|0}$).
`sigma0_l`	List of numeric matrices. Each element of the list is the Cholesky factorization (`t(chol(sigma0))`) of the covariance matrix of initial latent variable values ($\boldsymbol{\Sigma}_{\boldsymbol{\eta}\|0}$).
`alpha`	List of numeric vectors. Each element of the list is the vector of constant values for the dynamic model ($\boldsymbol{\alpha}$).
`beta`	List of numeric matrices. Each element of the list is the transition matrix relating the values of the latent variables at the previous to the current time point ($\boldsymbol{\beta}$).
`psi_l`	List of numeric matrices. Each element of the list is the Cholesky factorization (`t(chol(psi))`) of the covariance matrix of the process noise ($\boldsymbol{\Psi}$).
`type`	Integer. State space model type. See Details for more information.
`x`	List. Each element of the list is a matrix of covariates for each individual `i` in `n`. The number of columns in each matrix should be equal to `time`.
`gamma_eta`	List of numeric matrices. Each element of the list is the matrix linking the covariates to the latent variables at current time point ($\boldsymbol{\Gamma}_{\boldsymbol{\eta}}$).

Details

Parameters can vary across individuals by providing a list of parameter values. If the length of any of the parameters (`mu0`, `sigma0_l`, `alpha`, `beta`, `psi_l`, `gamma_eta`, or `gamma_y`) is less the `n`, the function will cycle through the available values.

Value

Returns an object of class `simstatespace` which is a list with the following elements:

- `call`: Function call.
- `args`: Function arguments.
- `data`: Generated data which is a list of length `n`. Each element of `data` is a list with the following elements:
  - `id`: A vector of ID numbers with length `t`, where `t` is the value of the function argument `time`.
  - `time`: A vector time points of length `t`.
  - `y`: A `t` by `k` matrix of values for the manifest variables.
  - `eta`: A `t` by `p` matrix of values for the latent variables.
  - `x`: A `t` by `j` matrix of values for the covariates (when covariates are included).
- `fun`: Function used.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Chow, S.-M., Ho, M. R., Hamaker, E. L., & Dolan, C. V. (2010). Equivalence and differences between structural equation modeling and state-space modeling techniques. *Structural Equation Modeling: A Multidisciplinary Journal*, 17(2), 303–332. doi:10.1080/10705511003661553

See Also

Other Simulation of State Space Models Data Functions: `LinSDE2SSM()`, `SimSSMFixed()`, `SimSSMIVary()`, `SimSSMLinGrowth()`, `SimSSMLinGrowthIVary()`, `SimSSMLinSDEFixed()`, `SimSSMLinSDEIVary()`, `SimSSMOUFixed()`, `SimSSMOUIVary()`, `SimSSMVARFixed()`

Examples

```
# prepare parameters
# In this example, beta varies across individuals.
set.seed(42)
## number of individuals
n <- 5
## time points
time <- 50
## dynamic structure
p <- 3
mu0 <- list(
  rep(x = 0, times = p)
)
sigma0 <- diag(p)
sigma0_l <- list(
  t(chol(sigma0))
)
```

```
alpha <- list(
  rep(x = 0, times = p)
)
beta <- list(
  0.1 * diag(p),
  0.2 * diag(p),
  0.3 * diag(p),
  0.4 * diag(p),
  0.5 * diag(p)
)
psi <- diag(p)
psi_l <- list(
  t(chol(psi))
)
## covariates
j <- 2
x <- lapply(
  X = seq_len(n),
  FUN = function(i) {
    matrix(
      data = stats::rnorm(n = time * j),
      nrow = j,
      ncol = time
    )
  }
)
gamma_eta <- list(
  diag(x = 0.10, nrow = p, ncol = j)
)

# Type 0
ssm <- SimSSMVARIVary(
  n = n,
  time = time,
  mu0 = mu0,
  sigma0_l = sigma0_l,
  alpha = alpha,
  beta = beta,
  psi_l = psi_l,
  type = 0
)

plot(ssm)

# Type 1
ssm <- SimSSMVARIVary(
  n = n,
  time = time,
  mu0 = mu0,
  sigma0_l = sigma0_l,
  alpha = alpha,
  beta = beta,
  psi_l = psi_l,
```

```
  type = 1,
  x = x,
  gamma_eta = gamma_eta
)

plot(ssm)
```

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