Package 'simStateSpace'

November 16, 2023				
Title Simulate Data from State Space Models				
Version 1.0.1				
Description Provides a streamlined and user-friendly framework for simulating data in state space models, particularly when the number of subjects/units (n) exceeds one, a scenario commonly encountered in social and behavioral sciences. For an introduction to state space models in social and behavioral sciences, refer to Chow, Ho, Hamaker, and Dolan (2010) <doi:10.1080 10705511003661553="">.</doi:10.1080>				
<pre>URL https://github.com/jeksterslab/simStateSpace,</pre>				
https://jeksterslab.github.io/simStateSpace/				
<pre>BugReports https://github.com/jeksterslab/simStateSpace/issues</pre>				
License GPL (>= 3)				
Encoding UTF-8				
Roxygen list(markdown = TRUE)				
Depends R (>= $3.0.0$)				
LinkingTo Rcpp, RcppArmadillo				
Imports Rcpp				
Suggests knitr, rmarkdown, testthat, Matrix				
RoxygenNote 7.2.3				
NeedsCompilation yes				
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R topics documented:				
OU2SSM Sim2Matrix SimSSM0 SimSSM0Fixed				

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Description

This function converts parameters from the Ornstein–Uhlenbeck model to state space model parameterization. See details for more information.

Usage

```
OU2SSM(mu, phi, sigma_sqrt, delta_t)
```

Arguments

mu	Numeric vector. The long-term mean or equilibrium level (μ) .
phi	Numeric matrix. The rate of mean reversion, determining how quickly the variable returns to its mean (Φ) .
sigma_sqrt	Numeric matrix. Cholesky decomposition of the matrix of volatility or randomness in the process (Σ) .
delta_t	Numeric. Time interval (δ_t).

Details

The state space parameters as a function of the Ornstein-Uhlenbeck model parameters are given by

$$\beta = \exp(-\Phi \Delta_t)$$

$$oldsymbol{lpha} = -oldsymbol{\Phi}^{-1} \left(oldsymbol{eta} - \mathbf{I}_p
ight)$$

$$\operatorname{vec}\left(\mathbf{\Psi}\right) = \left\{ \left[\left(-\mathbf{\Phi} \otimes \mathbf{I}_{p} \right) + \left(\mathbf{I}_{p} \otimes -\mathbf{\Phi} \right) \right] \left[\exp\left(\left[\left(-\mathbf{\Phi} \otimes \mathbf{I}_{p} \right) + \left(\mathbf{I}_{p} \otimes -\mathbf{\Phi} \right) \right] \Delta_{t} \right) - \mathbf{I}_{p \times p} \right] \operatorname{vec}\left(\mathbf{\Sigma}\right) \right\}$$

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Value

Returns a list of state space parameters:

- alpha: Numeric vector. Vector of intercepts for the dynamic model (α) .
- beta: Numeric matrix. Transition matrix relating the values of the latent variables at time t 1 to those at time t (β).
- psi: Numeric matrix. The process noise covariance matrix (Ψ) .

Author(s)

Ivan Jacob Agaloos Pesigan

See Also

Other Simulation of State Space Models Data Functions: Sim2Matrix(), SimSSM0Fixed(), SimSSM0Vary(), SimSSM0(), SimSSM0UFixed(), SimSSM0UVary(), SimSSMVARFixed(), SimSSMVARVary(), SimSSMVAR()

Examples

```
p <- k <- 2
mu <- c(5.76, 5.18)
phi <- matrix(data = c(0.10, -0.05, -0.05, 0.10), nrow = p)
sigma_sqrt <- chol(
    matrix(data = c(2.79, 0.06, 0.06, 3.27), nrow = p)
)
delta_t <- 0.10

OU2SSM(
    mu = mu,
    phi = phi,
    sigma_sqrt = sigma_sqrt,
    delta_t = delta_t
)</pre>
```

Sim2Matrix

Simulation Output to Matrix

Description

This function converts the output of SimSSM0(), SimSSMOU(), SimSSMVAR(), SimSSMOFixed(), SimSSMOUFixed(), or SimSSMVARFixed() to a matrix.

Usage

```
Sim2Matrix(x, eta = FALSE)
```

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Arguments

Value

Returns a matrix of simulated data.

Author(s)

Ivan Jacob Agaloos Pesigan

See Also

```
Other Simulation of State Space Models Data Functions: OU2SSM(), SimSSM0Fixed(), SimSSM0Vary(), SimSSM0(), SimSSMOUFixed(), SimSSMOUVary(), SimSSMVARFixed(), SimSSMVARVary(), SimSSMVAR()
```

```
# prepare parameters
set.seed(42)
k <- p <- 3
I \leftarrow diag(k)
I_sqrt <- chol(I)</pre>
null\_vec \leftarrow rep(x = 0, times = k)
n <- 5
mu0 <- null_vec
sigma0_sqrt <- I_sqrt</pre>
alpha <- null_vec</pre>
beta \leftarrow diag(x = 0.50, nrow = k)
psi_sqrt <- I_sqrt</pre>
nu <- null_vec
lambda <- I
theta_sqrt <- chol(diag(x = 0.50, nrow = k))
time <- 50
burn_in <- 0
# generate data
ssm <- SimSSM0(
  mu0 = mu0,
  sigma0_sqrt = sigma0_sqrt,
  alpha = alpha,
  beta = beta,
  psi_sqrt = psi_sqrt,
  nu = nu,
  lambda = lambda,
  theta_sqrt = theta_sqrt,
  time = time,
  burn_in = burn_in
```

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```
)
# list to matrix
mat <- Sim2Matrix(ssm)</pre>
str(mat)
head(mat)
# generate data
ssm <- SimSSM0Fixed(</pre>
  n = n,
  mu0 = mu0,
  sigma0_sqrt = sigma0_sqrt,
  alpha = alpha,
  beta = beta,
  psi_sqrt = psi_sqrt,
  nu = nu,
  lambda = lambda,
  theta_sqrt = theta_sqrt,
  time = time,
  burn_in = burn_in
)
# list to matrix
mat <- Sim2Matrix(ssm)</pre>
str(mat)
head(mat)
```

SimSSM0

Simulate Data from a State Space Model (n = 1)

Description

This function simulates data from a state space model. See details for more information.

Usage

```
SimSSM0(
  mu0,
  sigma0_sqrt,
  alpha,
  beta,
  psi_sqrt,
  nu,
  lambda,
  theta_sqrt,
  time,
  burn_in
)
```

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Arguments

mu0	Numeric vector. Mean of initial latent variable values $(\mu_{\eta 0})$.
sigma0_sqrt	Numeric matrix. Cholesky decomposition of the covariance matrix of initial latent variable values $(\Sigma_{\eta 0})$.
alpha	Numeric vector. Vector of intercepts for the dynamic model (α) .
beta	Numeric matrix. Transition matrix relating the values of the latent variables at time $t-1$ to those at time $t(\beta)$.
psi_sqrt	Numeric matrix. Cholesky decomposition of the process noise covariance matrix (Ψ) .
nu	Numeric vector. Vector of intercepts for the measurement model (ν) .
lambda	Numeric matrix. Factor loading matrix linking the latent variables to the observed variables (Λ) .
theta_sqrt	Numeric matrix. Cholesky decomposition of the measurement error covariance matrix (Θ) .
time	Positive integer. Number of time points to simulate.
burn_in	Positive integer. Number of burn-in points to exclude before returning the results.

Details

The measurement model is given by

$$\mathbf{y}_{t} = \boldsymbol{\nu} + \boldsymbol{\Lambda} \boldsymbol{\eta}_{t} + \boldsymbol{arepsilon}_{t} \quad ext{with} \quad \boldsymbol{arepsilon}_{t} \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Theta}
ight)$$

where \mathbf{y}_t , $\boldsymbol{\eta}_t$, and $\boldsymbol{\varepsilon}_t$ are random variables and $\boldsymbol{\nu}$, $\boldsymbol{\Lambda}$, and $\boldsymbol{\Theta}$ are model parameters. \mathbf{y}_t is a vector of observed random variables at time t, $\boldsymbol{\eta}_t$ is a vector of latent random variables at time t, and $\boldsymbol{\varepsilon}_t$ is a vector of random measurement errors at time t, while $\boldsymbol{\nu}$ is a vector of intercept, $\boldsymbol{\Lambda}$ is a matrix of factor loadings, and $\boldsymbol{\Theta}$ is the covariance matrix of $\boldsymbol{\varepsilon}$.

The dynamic structure is given by

$$oldsymbol{\eta}_t = oldsymbol{lpha} + oldsymbol{eta} oldsymbol{\eta}_{t-1} + oldsymbol{\zeta}_t \quad ext{with} \quad oldsymbol{\zeta}_t \sim \mathcal{N}\left(\mathbf{0}, oldsymbol{\Psi}
ight)$$

where η_t , η_{t-1} , and ζ_t are random variables and α , β , and Ψ are model parameters. η_t is a vector of latent variables at time t, η_{t-1} is a vector of latent variables at time t-1, and ζ_t is a vector of dynamic noise at time t while α is a vector of intercepts, β is a matrix of autoregression and cross regression coefficients, and Ψ is the covariance matrix of ζ_t .

Value

Returns a list with the following elements:

- y: A t by k matrix of values for the manifest variables.
- eta: A t by p matrix of values for the latent variables.
- time: A vector of discrete time points from 1 to t.
- n: Number of individuals.

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Author(s)

Ivan Jacob Agaloos Pesigan

References

Chow, S.-M., Ho, M. R., Hamaker, E. L., & Dolan, C. V. (2010). Equivalence and differences between structural equation modeling and state-space modeling techniques. *Structural Equation Modeling: A Multidisciplinary Journal*, 17(2), 303–332. doi:10.1080/10705511003661553

Chow, S.-M., Losardo, D., Park, J., & Molenaar, P. C. M. (2023). Continuous-time dynamic models: Connections to structural equation models and other discrete-time models. In R. H. Hoyle (Ed.), Handbook of structural equation modeling (2nd ed.). The Guilford Press.

Shumway, R. H., & Stoffer, D. S. (2017). *Time series analysis and its applications: With R examples*. Springer International Publishing. doi:10.1007/9783319524528

See Also

Other Simulation of State Space Models Data Functions: OU2SSM(), Sim2Matrix(), SimSSM0Fixed(), SimSSM0Vary(), SimSSM0UFixed(), SimSSM0UVary(), SimSSMVARFixed(), SimSSMVARVary(), SimSSMVAR()

```
# prepare parameters
set.seed(42)
k < -p < -3
I \leftarrow diag(k)
I_sqrt <- chol(I)</pre>
null\_vec \leftarrow rep(x = 0, times = k)
mu0 <- null_vec
sigma0_sqrt <- I_sqrt</pre>
alpha <- null_vec
beta \leftarrow diag(x = 0.50, nrow = k)
psi_sqrt <- I_sqrt</pre>
nu <- null_vec
lambda <- I
theta_sqrt <- chol(diag(x = 0.50, nrow = k))
time <- 50
burn_in <- 0
# generate data
ssm <- SimSSM0(
  mu0 = mu0,
  sigma0_sqrt = sigma0_sqrt,
  alpha = alpha,
  beta = beta,
  psi_sqrt = psi_sqrt,
  nu = nu,
  lambda = lambda,
  theta_sqrt = theta_sqrt,
  time = time,
```

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```
burn_in = burn_in
)
str(ssm)
```

SimSSM0Fixed

Simulate Data using a State Space Model Parameterization for n > 1 Individuals (Fixed Parameters)

Description

This function simulates data using a state space model parameterization for n > 1 individuals. In this model, the parameters are invariant across individuals.

Usage

```
SimSSM0Fixed(
    n,
    mu0,
    sigma0_sqrt,
    alpha,
    beta,
    psi_sqrt,
    nu,
    lambda,
    theta_sqrt,
    time,
    burn_in
)
```

Arguments

n	Positive integer. Number of individuals.
mu0	Numeric vector. Mean of initial latent variable values $(\mu_{\eta 0})$.
sigma0_sqrt	Numeric matrix. Cholesky decomposition of the covariance matrix of initial latent variable values ($\Sigma_{\eta 0}$).
alpha	Numeric vector. Vector of intercepts for the dynamic model (α) .
beta	Numeric matrix. Transition matrix relating the values of the latent variables at time $t-1$ to those at time $t(\beta)$.
psi_sqrt	Numeric matrix. Cholesky decomposition of the process noise covariance matrix (Ψ) .
nu	Numeric vector. Vector of intercepts for the measurement model (ν) .
lambda	Numeric matrix. Factor loading matrix linking the latent variables to the observed variables (Λ).

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theta_sqrt Numeric matrix. Cholesky decomposition of the measurement error covariance

matrix (Θ) .

time Positive integer. Number of time points to simulate.

burn_in Positive integer. Number of burn-in points to exclude before returning the re-

sults.

Details

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \boldsymbol{\Lambda} \boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t} \quad ext{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Theta}\right)$$

where $\mathbf{y}_{i,t}$, $\eta_{i,t}$, and $\varepsilon_{i,t}$ are random variables and $\boldsymbol{\nu}$, $\boldsymbol{\Lambda}$, and $\boldsymbol{\Theta}$ are model parameters. $\mathbf{y}_{i,t}$ is a vector of observed random variables at time t and individual i, $\eta_{i,t}$ is a vector of latent random variables at time t and individual i, and $\varepsilon_{i,t}$ is a vector of random measurement errors at time t and individual i, while $\boldsymbol{\nu}$ is a vector of intercept, $\boldsymbol{\Lambda}$ is a matrix of factor loadings, and $\boldsymbol{\Theta}$ is the covariance matrix of ε .

The dynamic structure is given by

$$oldsymbol{\eta}_{i.t} = oldsymbol{lpha} + oldsymbol{eta} oldsymbol{\eta}_{i.t-1} + oldsymbol{\zeta}_{i.t} \quad ext{with} \quad oldsymbol{\zeta}_{i.t} \sim \mathcal{N}\left(oldsymbol{0}, oldsymbol{\Psi}
ight)$$

where $\eta_{i,t}$, $\eta_{i,t-1}$, and $\zeta_{i,t}$ are random variables and α , β , and Ψ are model parameters. $\eta_{i,t}$ is a vector of latent variables at time t and individual i, $\eta_{i,t-1}$ is a vector of latent variables at time t-1 and individual i, and $\zeta_{i,t}$ is a vector of dynamic noise at time t and individual i while α is a vector of intercepts, β is a matrix of autoregression and cross regression coefficients, and Ψ is the covariance matrix of $\zeta_{i,t}$.

Value

Returns a list of length n. Each element is a list with the following elements:

- y: A t by k matrix of values for the manifest variables.
- eta: A t by p matrix of values for the latent variables.
- time: A vector of discrete time points from 1 to t.
- id: A vector of ID numbers of length t.
- n: Number of individuals.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Chow, S.-M., Ho, M. R., Hamaker, E. L., & Dolan, C. V. (2010). Equivalence and differences between structural equation modeling and state-space modeling techniques. *Structural Equation Modeling: A Multidisciplinary Journal*, 17(2), 303–332. doi:10.1080/10705511003661553

Chow, S.-M., Losardo, D., Park, J., & Molenaar, P. C. M. (2023). Continuous-time dynamic models: Connections to structural equation models and other discrete-time models. In R. H. Hoyle (Ed.), Handbook of structural equation modeling (2nd ed.). The Guilford Press.

Shumway, R. H., & Stoffer, D. S. (2017). *Time series analysis and its applications: With R examples*. Springer International Publishing. doi:10.1007/9783319524528

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See Also

Other Simulation of State Space Models Data Functions: OU2SSM(), Sim2Matrix(), SimSSM0Vary(), SimSSM0(), SimSSMOUFixed(), SimSSMOUVary(), SimSSMOU(), SimSSMVARFixed(), SimSSMVARVary(), SimSSMVAR()

Examples

```
# prepare parameters
set.seed(42)
k <- p <- 3
I \leftarrow diag(k)
I_sqrt <- chol(I)</pre>
null_vec <- rep(x = 0, times = k)
n <- 5
mu0 <- null_vec
sigma0_sqrt <- I_sqrt</pre>
alpha <- null_vec</pre>
beta \leftarrow diag(x = 0.50, nrow = k)
psi_sqrt <- I_sqrt</pre>
nu <- null_vec
lambda <- I
theta_sqrt <- chol(diag(x = 0.50, nrow = k))
time <- 50
burn_in <- 0
# generate data
ssm <- SimSSM0Fixed(</pre>
  n = n,
  mu0 = mu0,
  sigma0_sqrt = sigma0_sqrt,
  alpha = alpha,
  beta = beta,
  psi_sqrt = psi_sqrt,
  nu = nu,
  lambda = lambda,
  theta_sqrt = theta_sqrt,
  time = time,
  burn_in = burn_in
)
str(ssm)
```

SimSSM0Vary

Simulate Data using a State Space Model Parameterization for n > 1 Individuals (Varying Parameters)

Description

This function simulates data using a state space model parameterization for n > 1 individuals. In this model, the parameters can vary across individuals.

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Usage

```
SimSSM0Vary(
    n,
    mu0,
    sigma0_sqrt,
    alpha,
    beta,
    psi_sqrt,
    nu,
    lambda,
    theta_sqrt,
    time,
    burn_in
)
```

Arguments

n	Positive integer. Number of individuals.
mu0	List of numeric vectors. Mean of initial latent variable values $(\mu_{\eta 0})$.
sigma0_sqrt	List of numeric matrices. Cholesky decomposition of the covariance matrix of initial latent variable values $(\Sigma_{\eta 0})$.
alpha	List of numeric vectors. Vector of intercepts for the dynamic model (α) .
beta	List of numeric matrices. Transition matrix relating the values of the latent variables at time $t-1$ to those at time $t(\beta)$.
psi_sqrt	List of numeric matrices. Cholesky decomposition of the process noise covariance matrix (Ψ) .
nu	List of numeric vectors. Vector of intercepts for the measurement model (ν) .
lambda	List of numeric matrices. Factor loading matrix linking the latent variables to the observed variables (Λ) .
theta_sqrt	List of numeric matrices. Cholesky decomposition of the measurement error covariance matrix (Θ) .
time	Positive integer. Number of time points to simulate.
burn_in	Positive integer. Number of burn-in points to exclude before returning the results.

Details

Parameters can vary across individuals by providing a list of parameter values. If the length of any of the parameters (mu0, sigma0_sqrt, alpha, beta, psi_sqrt, nu, lambda, and theta_sqrt) is less the n, the function will cycle through the available values.

Value

Returns a list of length n. Each element is a list with the following elements:

• y: A t by k matrix of values for the manifest variables.

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- eta: A t by p matrix of values for the latent variables.
- time: A vector of discrete time points from 1 to t.
- id: A vector of ID numbers of length t.
- n: Number of individuals.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Chow, S.-M., Ho, M. R., Hamaker, E. L., & Dolan, C. V. (2010). Equivalence and differences between structural equation modeling and state-space modeling techniques. *Structural Equation Modeling: A Multidisciplinary Journal*, 17(2), 303–332. doi:10.1080/10705511003661553

Chow, S.-M., Losardo, D., Park, J., & Molenaar, P. C. M. (2023). Continuous-time dynamic models: Connections to structural equation models and other discrete-time models. In R. H. Hoyle (Ed.), Handbook of structural equation modeling (2nd ed.). The Guilford Press.

Shumway, R. H., & Stoffer, D. S. (2017). *Time series analysis and its applications: With R examples*. Springer International Publishing. doi:10.1007/9783319524528

See Also

Other Simulation of State Space Models Data Functions: OU2SSM(), Sim2Matrix(), SimSSM0Fixed(), SimSSM0(), SimSSMOUFixed(), SimSSMOUVary(), SimSSMOU(), SimSSMVARFixed(), SimSSMVARVary(), SimSSMVAR()

```
# prepare parameters
# In this example, beta varies across individuals
set.seed(42)
k < -p < -3
iden <- diag(k)
iden_sqrt <- chol(iden)</pre>
null\_vec \leftarrow rep(x = 0, times = k)
n <- 5
mu0 <- list(null_vec)</pre>
sigma0_sqrt <- list(iden_sqrt)</pre>
alpha <- list(null_vec)</pre>
beta <- list(
  diag(x = 0.1, nrow = k),
  diag(x = 0.2, nrow = k),
  diag(x = 0.3, nrow = k),
  diag(x = 0.4, nrow = k),
  diag(x = 0.5, nrow = k)
psi_sqrt <- list(iden_sqrt)</pre>
nu <- list(null_vec)</pre>
lambda <- list(iden)</pre>
theta_sqrt <- list(chol(diag(x = 0.50, nrow = k)))</pre>
```

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```
time <- 50
burn_in <- 0</pre>
ssm <- SimSSM0Vary(
  n = n,
  mu0 = mu0,
  sigma0_sqrt = sigma0_sqrt,
  alpha = alpha,
  beta = beta,
  psi_sqrt = psi_sqrt,
  nu = nu,
  lambda = lambda,
  theta_sqrt = theta_sqrt,
  time = time,
  burn_in = burn_in
)
str(ssm)
```

SimSSMOU

Simulate Data from the Ornstein-Uhlenbeck Model using a State Space Model Parameterization (n = 1)

Description

This function simulates data from the Ornstein–Uhlenbeck model using a state space model parameterization. See details for more information.

Usage

```
SimSSMOU(
   mu0,
   sigma0_sqrt,
   mu,
   phi,
   sigma_sqrt,
   nu,
   lambda,
   theta_sqrt,
   delta_t,
   time,
   burn_in
)
```

Arguments

mu0

Numeric vector. Mean of initial latent variable values $(\mu_{\eta|0})$.

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sigma0_sqrt	Numeric matrix. Cholesky decomposition of the covariance matrix of initial latent variable values ($\Sigma_{\eta 0}$).
mu	Numeric vector. The long-term mean or equilibrium level (μ) .
phi	Numeric matrix. The rate of mean reversion, determining how quickly the variable returns to its mean (Φ) .
sigma_sqrt	Numeric matrix. Cholesky decomposition of the matrix of volatility or randomness in the process (Σ) .
nu	Numeric vector. Vector of intercepts for the measurement model (ν) .
lambda	Numeric matrix. Factor loading matrix linking the latent variables to the observed variables (Λ).
theta_sqrt	Numeric matrix. Cholesky decomposition of the measurement error covariance matrix (Θ) .
delta_t	Numeric. Time interval (δ_t).
time	Positive integer. Number of time points to simulate.
burn_in	Positive integer. Number of burn-in points to exclude before returning the results.

Details

The measurement model is given by

$$\mathbf{y}_{t} = \boldsymbol{\nu} + \boldsymbol{\Lambda} \boldsymbol{\eta}_{t} + \boldsymbol{\varepsilon}_{t} \quad \text{with} \quad \boldsymbol{\varepsilon}_{t} \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Theta}\right)$$

where y_t , η_t , and ε_t are random variables and ν , Λ , and Θ are model parameters. y_t is a vector of observed random variables at time t, η_t is a vector of latent random variables at time t, and ε_t is a vector of random measurement errors at time t, while ν is a vector of intercept, Λ is a matrix of factor loadings, and Θ is the covariance matrix of ε .

The dynamic structure is given by

$$\mathrm{d}\boldsymbol{\eta}_t = \boldsymbol{\Phi}\left(\boldsymbol{\mu} - \boldsymbol{\eta}_t\right) \mathrm{d}t + \boldsymbol{\Sigma}^{\frac{1}{2}} \mathrm{d}\mathbf{W}_t$$

where μ is the long-term mean or equilibrium level, Φ is the rate of mean reversion, determining how quickly the variable returns to its mean, Σ is the matrix of volatility or randomness in the process, and dW is a Wiener process or Brownian motion, which represents random fluctuations.

Value

Returns a list with the following elements:

- y: A t by k matrix of values for the manifest variables.
- eta: A t by p matrix of values for the latent variables.
- time: A vector of continuous time points of length t starting from 0 with delta_t increments.
- n: Number of individuals.

Author(s)

Ivan Jacob Agaloos Pesigan

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References

Chow, S.-M., Losardo, D., Park, J., & Molenaar, P. C. M. (2023). Continuous-time dynamic models: Connections to structural equation models and other discrete-time models. In R. H. Hoyle (Ed.), Handbook of structural equation modeling (2nd ed.). The Guilford Press.

Uhlenbeck, G. E., & Ornstein, L. S. (1930). On the theory of the brownian motion. *Physical Review*, 36(5), 823–841. doi:10.1103/physrev.36.823

See Also

Other Simulation of State Space Models Data Functions: OU2SSM(), Sim2Matrix(), SimSSM0Fixed(), SimSSM0Vary(), SimSSM0UFixed(), SimSSMOUVary(), SimSSMVARFixed(), SimSSMVARVary(), SimSSMVAR()

```
# prepare parameters
set.seed(42)
p <- k <- 2
I <- diag(p)</pre>
I_sqrt <- chol(I)</pre>
mu0 < -c(-3.0, 1.5)
sigma0_sqrt <- I_sqrt</pre>
mu < -c(5.76, 5.18)
phi <- matrix(data = c(0.10, -0.05, -0.05, 0.10), nrow = p)
sigma_sqrt <- chol(</pre>
  matrix(data = c(2.79, 0.06, 0.06, 3.27), nrow = p)
)
nu \leftarrow rep(x = 0, times = k)
lambda <- diag(k)</pre>
theta_sqrt <- chol(diag(x = 0.50, nrow = k))
delta_t <- 0.10
time <- 50
burn_in <- 0
# generate data
ssm <- SimSSMOU(
  mu0 = mu0,
  sigma0_sqrt = sigma0_sqrt,
  mu = mu,
  phi = phi,
  sigma_sqrt = sigma_sqrt,
  nu = nu,
  lambda = lambda,
  theta_sqrt = theta_sqrt,
  delta_t = delta_t,
  time = time,
  burn_in = burn_in
str(ssm)
```

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SimSSMOUFixed	Simulate Data from an Ornstein-Uhlenbeck Model using a State Space
	Model Parameterization for $n > 1$ Individuals (Fixed Parameters)

Description

This function simulates data from an Ornstein–Uhlenbeck model using a state space model parameterization for n > 1 individuals. In this model, the parameters are invariant across individuals. See details for more information.

Usage

```
SimSSMOUFixed(
    n,
    mu0,
    sigma0_sqrt,
    mu,
    phi,
    sigma_sqrt,
    nu,
    lambda,
    theta_sqrt,
    delta_t,
    time,
    burn_in
)
```

Arguments

n	Positive integer. Number of individuals.
mu0	Numeric vector. Mean of initial latent variable values $(\mu_{\eta 0})$.
sigma0_sqrt	Numeric matrix. Cholesky decomposition of the covariance matrix of initial latent variable values $(\Sigma_{\eta 0})$.
mu	Numeric vector. The long-term mean or equilibrium level (μ) .
phi	Numeric matrix. The rate of mean reversion, determining how quickly the variable returns to its mean (Φ) .
sigma_sqrt	Numeric matrix. Cholesky decomposition of the matrix of volatility or randomness in the process (Σ).
nu	Numeric vector. Vector of intercepts for the measurement model (ν) .
lambda	Numeric matrix. Factor loading matrix linking the latent variables to the observed variables ($\pmb{\Lambda}$).
theta_sqrt	Numeric matrix. Cholesky decomposition of the measurement error covariance matrix $(\boldsymbol{\Theta}).$
delta_t	Numeric. Time interval (δ_t) .

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time Positive integer. Number of time points to simulate.

burn_in Positive integer. Number of burn-in points to exclude before returning the re-

sults.

Details

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \boldsymbol{\Lambda} \boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t} \quad ext{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Theta}\right)$$

where $\mathbf{y}_{i,t}$, $\boldsymbol{\eta}_{i,t}$, and $\boldsymbol{\varepsilon}_{i,t}$ are random variables and $\boldsymbol{\nu}$, $\boldsymbol{\Lambda}$, and $\boldsymbol{\Theta}$ are model parameters. $\mathbf{y}_{i,t}$ is a vector of observed random variables at time t and individual i, $\boldsymbol{\eta}_{i,t}$ is a vector of latent random variables at time t and individual i, and $\boldsymbol{\varepsilon}_{i,t}$ is a vector of random measurement errors at time t and individual i, while $\boldsymbol{\nu}$ is a vector of intercept, $\boldsymbol{\Lambda}$ is a matrix of factor loadings, and $\boldsymbol{\Theta}$ is the covariance matrix of $\boldsymbol{\varepsilon}$.

The dynamic structure is given by

$$d\boldsymbol{\eta}_{i:t} = \boldsymbol{\Phi} \left(\boldsymbol{\mu} - \boldsymbol{\eta}_{i:t} \right) dt + \boldsymbol{\Sigma}^{\frac{1}{2}} d\mathbf{W}_{i,t}$$

where μ is the long-term mean or equilibrium level, Φ is the rate of mean reversion, determining how quickly the variable returns to its mean, Σ is the matrix of volatility or randomness in the process, and $\mathrm{d}W$ is a Wiener process or Brownian motion, which represents random fluctuations.

Value

Returns a list of length n. Each element is a list with the following elements:

- y: A t by k matrix of values for the manifest variables.
- eta: A t by p matrix of values for the latent variables.
- time: A vector of continuous time points of length t starting from 0 with delta_t increments.
- id: A vector of ID numbers of length t.
- n: Number of individuals.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Chow, S.-M., Losardo, D., Park, J., & Molenaar, P. C. M. (2023). Continuous-time dynamic models: Connections to structural equation models and other discrete-time models. In R. H. Hoyle (Ed.), Handbook of structural equation modeling (2nd ed.). The Guilford Press.

Uhlenbeck, G. E., & Ornstein, L. S. (1930). On the theory of the brownian motion. *Physical Review*, *36*(5), 823–841. doi:10.1103/physrev.36.823

See Also

Other Simulation of State Space Models Data Functions: OU2SSM(), Sim2Matrix(), SimSSM0Fixed(), SimSSM0Vary(), SimSSM0Vary(), SimSSM0U(), SimSSMVARFixed(), SimSSMVARVary(), SimSSMVAR()

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Examples

```
# prepare parameters
set.seed(42)
p <- k <- 2
I <- diag(p)</pre>
I_sqrt <- chol(I)</pre>
n <- 5
mu0 < -c(-3.0, 1.5)
sigma0_sqrt <- I_sqrt</pre>
mu <- c(5.76, 5.18)
phi <- matrix(data = c(0.10, -0.05, -0.05, 0.10), nrow = p)
sigma_sqrt <- chol(</pre>
  matrix(data = c(2.79, 0.06, 0.06, 3.27), nrow = p)
)
nu \leftarrow rep(x = 0, times = k)
lambda <- diag(k)</pre>
theta_sqrt <- chol(diag(x = 0.50, nrow = k))
delta_t <- 0.10
time <- 50
burn_in <- 0
# generate data
ssm <- SimSSMOUFixed(</pre>
  n = n,
  mu0 = mu0,
  sigma0_sqrt = sigma0_sqrt,
  mu = mu,
  phi = phi,
  sigma_sqrt = sigma_sqrt,
  nu = nu,
  lambda = lambda,
  theta_sqrt = theta_sqrt,
  delta_t = delta_t,
  time = time,
  burn_in = burn_in
)
str(ssm)
```

SimSSMOUVary

Simulate Data from an Ornstein-Uhlenbeck Model using a State Space Model Parameterization for n > 1 Individuals (Varying Parameters)

Description

This function simulates data from an Ornstein–Uhlenbeck model using a state space model parameterization for n > 1 individuals. In this model, the parameters can vary across individuals.

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Usage

```
SimSSMOUVary(
    n,
    mu0,
    sigma0_sqrt,
    mu,
    phi,
    sigma_sqrt,
    nu,
    lambda,
    theta_sqrt,
    delta_t,
    time,
    burn_in
)
```

Arguments

n	Positive integer. Number of individuals.
mu0	Numeric vector. Mean of initial latent variable values $(\mu_{\eta 0})$.
sigma0_sqrt	Numeric matrix. Cholesky decomposition of the covariance matrix of initial latent variable values $(\Sigma_{\eta 0})$.
mu	List of numeric vectors. The long-term mean or equilibrium level (μ) .
phi	List of numeric matrices. The rate of mean reversion, determining how quickly the variable returns to its mean (Φ) .
sigma_sqrt	List of numeric matrices. Cholesky decomposition of the matrix of volatility or randomness in the process (Σ) .
nu	Numeric vector. Vector of intercepts for the measurement model (ν) .
lambda	Numeric matrix. Factor loading matrix linking the latent variables to the observed variables (Λ) .
theta_sqrt	Numeric matrix. Cholesky decomposition of the measurement error covariance matrix (Θ) .
delta_t	Numeric. Time interval (δ_t) .
time	Positive integer. Number of time points to simulate.
burn_in	Positive integer. Number of burn-in points to exclude before returning the results.

Details

Parameters can vary across individuals by providing a list of parameter values. If the length of any of the parameters (mu0, sigma0_sqrt, mu, phi, sigma_sqrt, nu, lambda, theta_sqrt) is less the n, the function will cycle through the available values.

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Value

Returns a list of length n. Each element is a list with the following elements:

- y: A t by k matrix of values for the manifest variables.
- eta: A t by p matrix of values for the latent variables.
- time: A vector of discrete time points from 1 to t.
- id: A vector of ID numbers of length t.
- n: Number of individuals.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Chow, S.-M., Losardo, D., Park, J., & Molenaar, P. C. M. (2023). Continuous-time dynamic models: Connections to structural equation models and other discrete-time models. In R. H. Hoyle (Ed.), Handbook of structural equation modeling (2nd ed.). The Guilford Press.

Uhlenbeck, G. E., & Ornstein, L. S. (1930). On the theory of the brownian motion. *Physical Review*, 36(5), 823–841. doi:10.1103/physrev.36.823

See Also

Other Simulation of State Space Models Data Functions: OU2SSM(), Sim2Matrix(), SimSSM0Fixed(), SimSSM0Vary(), SimSSM0(), SimSSM0UFixed(), SimSSM0U(), SimSSMVARFixed(), SimSSMVARVary(), SimSSMVAR()

```
# prepare parameters
# In this example, phi varies across individuals
set.seed(42)
p < -k < -2
iden <- diag(p)</pre>
iden_sqrt <- chol(iden)</pre>
n <- 5
mu0 <- list(c(-3.0, 1.5))
sigma0_sqrt <- list(iden_sqrt)</pre>
mu \leftarrow list(c(5.76, 5.18))
phi <- list(</pre>
  as.matrix(Matrix::expm(diag(x = -0.1, nrow = k))),
  as.matrix(Matrix::expm(diag(x = -0.2, nrow = k))),
  as.matrix(Matrix::expm(diag(x = -0.3, nrow = k))),
  as.matrix(Matrix::expm(diag(x = -0.4, nrow = k))),
  as.matrix(Matrix::expm(diag(x = -0.5, nrow = k)))
sigma_sqrt <- list(</pre>
  chol(
    matrix(data = c(2.79, 0.06, 0.06, 3.27), nrow = p)
```

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```
)
)
nu \leftarrow list(rep(x = 0, times = k))
lambda <- list(diag(k))</pre>
theta_sqrt <- list(chol(diag(x = 0.50, nrow = k)))</pre>
delta_t <- 0.10
time <- 50
burn_in <- 0</pre>
ssm <- SimSSMOUVary(</pre>
  n = n,
  mu0 = mu0,
  sigma0_sqrt = sigma0_sqrt,
  mu = mu,
  phi = phi,
  sigma_sqrt = sigma_sqrt,
  nu = nu,
  lambda = lambda,
  theta_sqrt = theta_sqrt,
  delta_t = delta_t,
  time = time,
  burn_in = burn_in
str(ssm)
```

 ${\tt SimSSMVAR}$

Simulate Data from the Vector Autoregressive Model using a State Space Model Parameterization (n = 1)

Description

This function simulates data from the vector autoregressive model using a state space model parameterization. See details for more information.

Usage

```
SimSSMVAR(mu0, sigma0_sqrt, alpha, beta, psi_sqrt, time, burn_in)
```

Arguments

mu0	Numeric vector. Mean of initial latent variable values $(\mu_{\eta 0})$.
sigma0_sqrt	Numeric matrix. Cholesky decomposition of the covariance matrix of initial latent variable values ($\Sigma_{\eta 0}$).
alpha	Numeric vector. Vector of intercepts for the dynamic model (α) .
beta	Numeric matrix. Transition matrix relating the values of the latent variables at time $t-1$ to those at time $t(\beta)$.

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psi_sqrt Numeric matrix. Cholesky decomposition of the process noise covariance matrix (Ψ).

time Positive integer. Number of time points to simulate.

burn_in Positive integer. Number of burn-in points to exclude before returning the re-

sults.

Details

The measurement model is given by

$$\mathbf{y}_t = \boldsymbol{\eta}_t$$
.

The dynamic structure is given by

$$\eta_t = \alpha + \beta \eta_{t-1} + \zeta_t$$
 with $\zeta_t \sim \mathcal{N}\left(0, \Psi\right)$

where η_t, η_{t-1} , and ζ_t are random variables and α , β , and Ψ are model parameters. η_t is a vector of latent variables at time t, η_{t-1} is a vector of latent variables at t-1, and ζ_t is a vector of dynamic noise at time t while α is a vector of intercepts, β is a matrix of autoregression and cross regression coefficients, and Ψ is the covariance matrix of ζ_t .

Value

Returns a list with the following elements:

- y: A t by k matrix of values for the manifest variables.
- eta: A t by p matrix of values for the latent variables.
- time: A vector of discrete time points from 1 to t.
- n: Number of individuals.

References

Chow, S.-M., Ho, M. R., Hamaker, E. L., & Dolan, C. V. (2010). Equivalence and differences between structural equation modeling and state-space modeling techniques. *Structural Equation Modeling: A Multidisciplinary Journal*, 17(2), 303–332. doi:10.1080/10705511003661553

Chow, S.-M., Losardo, D., Park, J., & Molenaar, P. C. M. (2023). Continuous-time dynamic models: Connections to structural equation models and other discrete-time models. In R. H. Hoyle (Ed.), Handbook of structural equation modeling (2nd ed.). The Guilford Press.

Shumway, R. H., & Stoffer, D. S. (2017). *Time series analysis and its applications: With R examples*. Springer International Publishing. doi:10.1007/9783319524528

See Also

Other Simulation of State Space Models Data Functions: OU2SSM(), Sim2Matrix(), SimSSM0Fixed(), SimSSM0Vary(), SimSSM0(), SimSSM0UFixed(), SimSSM0UVary(), SimSSM0U(), SimSSMVARFixed(), SimSSMVARVary()

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Examples

```
# prepare parameters
set.seed(42)
k < -3
I \leftarrow diag(k)
I_sqrt <- chol(I)</pre>
null_vec \leftarrow rep(x = 0, times = k)
mu0 <- null_vec
sigma0_sqrt <- I_sqrt
alpha <- null_vec</pre>
beta \leftarrow diag(x = 0.5, nrow = k)
psi_sqrt <- I_sqrt</pre>
time <- 50
burn_in <- 0
# generate data
ssm <- SimSSMVAR(</pre>
  mu0 = mu0,
  sigma0_sqrt = sigma0_sqrt,
  alpha = alpha,
  beta = beta,
  psi_sqrt = psi_sqrt,
  time = time,
  burn_in = burn_in
)
str(ssm)
```

SimSSMVARFixed

Simulate Data from a Vector Autoregressive Model using a State Space Model Parameterization for n > 1 Individuals (Fixed Parameters)

Description

This function simulates data from a vector autoregressive model using a state space model parameterization for n > 1 individuals. In this model, the parameters are invariant across individuals.

Usage

```
SimSSMVARFixed(n, mu0, sigma0_sqrt, alpha, beta, psi_sqrt, time, burn_in)
```

Arguments

n Positive integer. Number of individuals.

mu0 Numeric vector. Mean of initial latent variable values $(\mu_{\eta|0})$.

sigma0_sqrt Numeric matrix. Cholesky decomposition of the covariance matrix of initial

latent variable values $(\Sigma_{\eta|0})$.

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alpha	Numeric vector. Vector of intercepts for the dynamic model (α) .
beta	Numeric matrix. Transition matrix relating the values of the latent variables at time $t-1$ to those at time $t(\beta)$.
psi_sqrt	Numeric matrix. Cholesky decomposition of the process noise covariance matrix (Ψ) .
time	Positive integer. Number of time points to simulate.
burn_in	Positive integer. Number of burn-in points to exclude before returning the results.

Value

Returns a list of length n. Each element is a list with the following elements:

- y: A t by k matrix of values for the manifest variables.
- eta: A t by p matrix of values for the latent variables.
- time: A vector of discrete time points from 1 to t.
- id: A vector of ID numbers of length t.
- n: Number of individuals.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Chow, S.-M., Ho, M. R., Hamaker, E. L., & Dolan, C. V. (2010). Equivalence and differences between structural equation modeling and state-space modeling techniques. *Structural Equation Modeling: A Multidisciplinary Journal*, 17(2), 303–332. doi:10.1080/10705511003661553

Chow, S.-M., Losardo, D., Park, J., & Molenaar, P. C. M. (2023). Continuous-time dynamic models: Connections to structural equation models and other discrete-time models. In R. H. Hoyle (Ed.), Handbook of structural equation modeling (2nd ed.). The Guilford Press.

Shumway, R. H., & Stoffer, D. S. (2017). *Time series analysis and its applications: With R examples*. Springer International Publishing. doi:10.1007/9783319524528

See Also

Other Simulation of State Space Models Data Functions: OU2SSM(), Sim2Matrix(), SimSSM0Fixed(), SimSSM0Vary(), SimSSM0(), SimSSM0UFixed(), SimSSM0UVary(), SimSSMOU(), SimSSMVARVary(), SimSSMVAR()

```
# prepare parameters
set.seed(42)
k <- 3
iden <- diag(k)
iden_sqrt <- chol(iden)</pre>
```

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```
null\_vec \leftarrow rep(x = 0, times = k)
n <- 5
mu0 <- null_vec
sigma0_sqrt <- iden_sqrt</pre>
alpha <- null_vec
beta <- diag(x = 0.5, nrow = k)
psi_sqrt <- iden_sqrt</pre>
time <- 50
burn_in <- 0</pre>
ssm <- SimSSMVARFixed(</pre>
  n = n,
  mu0 = mu0,
  sigma0_sqrt = sigma0_sqrt,
  alpha = alpha,
  beta = beta,
  psi_sqrt = psi_sqrt,
  time = time,
  burn_in = burn_in
str(ssm)
```

SimSSMVARVary

Simulate Data from a Vector Autoregressive Model using a State Space Model Parameterization for n > 1 Individuals (Varying Parameters)

Description

This function simulates data from a vector autoregressive model using a state space model parameterization for n > 1 individuals. In this model, the parameters can vary across individuals.

Usage

```
SimSSMVARVary(n, mu0, sigma0_sqrt, alpha, beta, psi_sqrt, time, burn_in)
```

Arguments

n	Positive integer. Number of individuals.
mu0	List of numeric vectors. Mean of initial latent variable values $(\mu_{\eta 0})$.
sigma0_sqrt	List of numeric matrices. Cholesky decomposition of the covariance matrix of initial latent variable values $(\Sigma_{\eta 0})$.
alpha	List of numeric vectors. Vector of intercepts for the dynamic model (α) .
beta	List of numeric matrices. Transition matrix relating the values of the latent variables at time $t-1$ to those at time $t(\beta)$.
psi_sqrt	List of numeric matrices. Cholesky decomposition of the process noise covariance matrix (Ψ) .

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time Positive integer. Number of time points to simulate.

burn_in Positive integer. Number of burn-in points to exclude before returning the results.

Details

Parameters can vary across individuals by providing a list of parameter values. If the length of any of the parameters (mu0, sigma0_sqrt, alpha, beta, and psi_sqrt) is less the n, the function will cycle through the available values.

Value

Returns a list of length n. Each element is a list with the following elements:

- y: A t by k matrix of values for the manifest variables.
- eta: A t by p matrix of values for the latent variables.
- time: A vector of discrete time points from 1 to t.
- id: A vector of ID numbers of length t.
- n: Number of individuals.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Chow, S.-M., Ho, M. R., Hamaker, E. L., & Dolan, C. V. (2010). Equivalence and differences between structural equation modeling and state-space modeling techniques. *Structural Equation Modeling: A Multidisciplinary Journal*, 17(2), 303–332. doi:10.1080/10705511003661553

Chow, S.-M., Losardo, D., Park, J., & Molenaar, P. C. M. (2023). Continuous-time dynamic models: Connections to structural equation models and other discrete-time models. In R. H. Hoyle (Ed.), Handbook of structural equation modeling (2nd ed.). The Guilford Press.

Shumway, R. H., & Stoffer, D. S. (2017). *Time series analysis and its applications: With R examples*. Springer International Publishing. doi:10.1007/9783319524528

See Also

Other Simulation of State Space Models Data Functions: OU2SSM(), Sim2Matrix(), SimSSM0Fixed(), SimSSM0Vary(), SimSSM0U(), SimSSMOUFixed(), SimSSMOUVary(), SimSSMOU(), SimSSMVARFixed(), SimSSMVAR()

```
# prepare parameters
# In this example, beta varies across individuals
set.seed(42)
k <- 3
iden <- diag(k)</pre>
```

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```
iden_sqrt <- chol(iden)</pre>
null_vec <- rep(x = 0, times = k)
n <- 5
mu0 <- list(null_vec)</pre>
sigma0_sqrt <- list(iden_sqrt)</pre>
alpha <- list(null_vec)</pre>
beta <- list(</pre>
  diag(x = 0.1, nrow = k),
  diag(x = 0.2, nrow = k),
  diag(x = 0.3, nrow = k),
  diag(x = 0.4, nrow = k),
  diag(x = 0.5, nrow = k)
psi_sqrt <- list(iden_sqrt)</pre>
time <- 50
burn_in <- 0</pre>
ssm <- SimSSMVARVary(</pre>
  n = n,
  mu0 = mu0,
  sigma0_sqrt = sigma0_sqrt,
  alpha = alpha,
  beta = beta,
  psi_sqrt = psi_sqrt,
  time = time,
  burn_in = burn_in
str(ssm)
```

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