# Package 'simStateSpace'

November 16, 2023
Title Simulate Data from State Space Models
Version 1.0.1
Description Provides a streamlined and user-friendly framework for simulating data in state space models, particularly when the number of subjects/units (n) exceeds one, a scenario commonly encountered in social and behavioral sciences. For an introduction to state space models in social and behavioral sciences, refer to Chow, Ho, Hamaker, and Dolan (2010) <doi:10.1080 10705511003661553="">.</doi:10.1080>
<pre>URL https://github.com/jeksterslab/simStateSpace,</pre>
https://jeksterslab.github.io/simStateSpace/
<pre>BugReports https://github.com/jeksterslab/simStateSpace/issues</pre>
License GPL (>= 3)
Encoding UTF-8
<b>Roxygen</b> list(markdown = TRUE)
<b>Depends</b> R (>= 3.0.0)
LinkingTo Rcpp, RcppArmadillo
Imports Rcpp
Suggests knitr, rmarkdown, testthat, Matrix
RoxygenNote 7.2.3
NeedsCompilation yes
Author Ivan Jacob Agaloos Pesigan [aut, cre, cph] ( <a href="https://orcid.org/0000-0003-4818-8420">https://orcid.org/0000-0003-4818-8420</a> )
Maintainer Ivan Jacob Agaloos Pesigan <r.jeksterslab@gmail.com></r.jeksterslab@gmail.com>
R topics documented:
OU2SSM

2 OU2SSM

0U2S	SM	Conv Space			,		Or	ns	tei	n–	Uk	lei	nbe	eck	t I	Мо	de	l i	to	Sta	ıte
Index																					27
	SimSSMVARVary		 	 	•	 ٠	 •	•		•	•		•	•			•		•	٠	. 24
	SimSSMVARFixe																				
	SimSSMVAR																				
	SimSSMOUVary																				
	SimSSMOUFixed																				
	SimSSMOU																				
	SimSSM0Vary .		 	 																	. 10

# Description

This function converts parameters from the Ornstein–Uhlenbeck model to state space model parameterization. See details for more information.

# Usage

```
OU2SSM(mu, phi, sigma_sqrt, delta_t)
```

# Arguments

mu	Numeric vector. The long-term mean or equilibrium level $(\mu)$ .
phi	Numeric matrix. The rate of mean reversion, determining how quickly the variable returns to its mean $(\Phi)$ .
sigma_sqrt	Numeric matrix. Cholesky decomposition of the matrix of volatility or randomness in the process ( $\Sigma$ ).
delta_t	Numeric. Time interval $(\delta_t)$ .

# **Details**

The state space parameters as a function of the Ornstein-Uhlenbeck model parameters are given by

$$\boldsymbol{\beta} = \exp\left(-\boldsymbol{\Phi}\boldsymbol{\Delta}_t\right)$$

$$oldsymbol{lpha} = -oldsymbol{\Phi}^{-1} \left( oldsymbol{eta} - \mathbf{I}_p 
ight)$$

$$\operatorname{vec}\left(\mathbf{\Psi}\right) = \left\{ \left[ \left( -\mathbf{\Phi} \otimes \mathbf{I}_{p} \right) + \left( \mathbf{I}_{p} \otimes -\mathbf{\Phi} \right) \right] \left[ \exp\left( \left[ \left( -\mathbf{\Phi} \otimes \mathbf{I}_{p} \right) + \left( \mathbf{I}_{p} \otimes -\mathbf{\Phi} \right) \right] \Delta_{t} \right) - \mathbf{I}_{p \times p} \right] \operatorname{vec}\left(\mathbf{\Sigma}\right) \right\}$$

Sim2Matrix 3

# Value

Returns a list of state space parameters:

- alpha: Numeric vector. Vector of intercepts for the dynamic model  $(\alpha)$ .
- beta: Numeric matrix. Transition matrix relating the values of the latent variables at time t 1 to those at time t (β).
- psi: Numeric matrix. The process noise covariance matrix  $(\Psi)$ .

# Author(s)

Ivan Jacob Agaloos Pesigan

# See Also

Other Simulation of State Space Models Data Functions: Sim2Matrix(), SimSSM0Fixed(), SimSSM0Vary(), SimSSM0(), SimSSM0UFixed(), SimSSM0UVary(), SimSSMVARFixed(), SimSSMVARVary(), SimSSMVAR()

# **Examples**

```
p <- k <- 2
mu <- c(5.76, 5.18)
phi <- matrix(data = c(0.10, -0.05, -0.05, 0.10), nrow = p)
sigma_sqrt <- chol(
    matrix(data = c(2.79, 0.06, 0.06, 3.27), nrow = p)
)
delta_t <- 0.10

OU2SSM(
    mu = mu,
    phi = phi,
    sigma_sqrt = sigma_sqrt,
    delta_t = delta_t
)</pre>
```

Sim2Matrix

Simulation Output to Matrix

# Description

This function converts the output of SimSSMO(), SimSSMOU(), SimSSMVAR(), SimSSMOFixed(), SimSSMOUFixed(), or SimSSMVARFixed() to a matrix.

# Usage

```
Sim2Matrix(x, eta = FALSE)
```

4 Sim2Matrix

# Arguments

#### Value

Returns a matrix of simulated data.

# Author(s)

Ivan Jacob Agaloos Pesigan

# See Also

```
Other Simulation of State Space Models Data Functions: OU2SSM(), SimSSM0Fixed(), SimSSM0Vary(), SimSSM0(), SimSSMOUFixed(), SimSSMOUVary(), SimSSMVARFixed(), SimSSMVARVary(), SimSSMVAR()
```

```
# prepare parameters
set.seed(42)
k <- p <- 3
I \leftarrow diag(k)
I_sqrt <- chol(I)</pre>
null\_vec \leftarrow rep(x = 0, times = k)
n <- 5
mu0 <- null_vec
sigma0_sqrt <- I_sqrt</pre>
alpha <- null_vec</pre>
beta \leftarrow diag(x = 0.50, nrow = k)
psi_sqrt <- I_sqrt</pre>
nu <- null_vec
lambda <- I
theta_sqrt <- chol(diag(x = 0.50, nrow = k))
time <- 50
burn_in <- 0</pre>
# generate data
ssm <- SimSSM0(
  mu0 = mu0,
  sigma0_sqrt = sigma0_sqrt,
  alpha = alpha,
  beta = beta,
  psi_sqrt = psi_sqrt,
  nu = nu,
  lambda = lambda,
  theta_sqrt = theta_sqrt,
  time = time,
  burn_in = burn_in
```

SimSSM0 5

```
)
# list to matrix
mat <- Sim2Matrix(ssm)</pre>
str(mat)
head(mat)
# generate data
ssm <- SimSSM0Fixed(</pre>
  n = n,
  mu0 = mu0,
  sigma0_sqrt = sigma0_sqrt,
  alpha = alpha,
  beta = beta,
  psi_sqrt = psi_sqrt,
  nu = nu,
  lambda = lambda,
  theta_sqrt = theta_sqrt,
  time = time,
  burn_in = burn_in
)
# list to matrix
mat <- Sim2Matrix(ssm)</pre>
str(mat)
head(mat)
```

SimSSM0

Simulate Data from a State Space Model (n = 1)

# Description

This function simulates data from a state space model. See details for more information.

# Usage

```
SimSSM0(
  mu0,
  sigma0_sqrt,
  alpha,
  beta,
  psi_sqrt,
  nu,
  lambda,
  theta_sqrt,
  time,
  burn_in
)
```

6 SimSSM0

# Arguments

mu0	Numeric vector. Mean of initial latent variable values $(\mu_{\eta 0})$ .
sigma0_sqrt	Numeric matrix. Cholesky decomposition of the covariance matrix of initial latent variable values $(\Sigma_{\eta 0})$ .
alpha	Numeric vector. Vector of intercepts for the dynamic model $(\alpha)$ .
beta	Numeric matrix. Transition matrix relating the values of the latent variables at time $t-1$ to those at time $t(\beta)$ .
psi_sqrt	Numeric matrix. Cholesky decomposition of the process noise covariance matrix $(\Psi)$ .
nu	Numeric vector. Vector of intercepts for the measurement model $(\nu)$ .
lambda	Numeric matrix. Factor loading matrix linking the latent variables to the observed variables $(\Lambda)$ .
theta_sqrt	Numeric matrix. Cholesky decomposition of the measurement error covariance matrix $(\Theta)$ .
time	Positive integer. Number of time points to simulate.
burn_in	Positive integer. Number of burn-in points to exclude before returning the results.

# **Details**

The measurement model is given by

$$\mathbf{y}_{t} = \boldsymbol{\nu} + \boldsymbol{\Lambda} \boldsymbol{\eta}_{t} + \boldsymbol{arepsilon}_{t} \quad ext{with} \quad \boldsymbol{arepsilon}_{t} \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Theta}
ight)$$

where  $\mathbf{y}_t$ ,  $\boldsymbol{\eta}_t$ , and  $\boldsymbol{\varepsilon}_t$  are random variables and  $\boldsymbol{\nu}$ ,  $\boldsymbol{\Lambda}$ , and  $\boldsymbol{\Theta}$  are model parameters.  $\mathbf{y}_t$  is a vector of observed random variables at time t,  $\boldsymbol{\eta}_t$  is a vector of latent random variables at time t, and  $\boldsymbol{\varepsilon}_t$  is a vector of random measurement errors at time t, while  $\boldsymbol{\nu}$  is a vector of intercept,  $\boldsymbol{\Lambda}$  is a matrix of factor loadings, and  $\boldsymbol{\Theta}$  is the covariance matrix of  $\boldsymbol{\varepsilon}$ .

The dynamic structure is given by

$$oldsymbol{\eta}_t = oldsymbol{lpha} + oldsymbol{eta} oldsymbol{\eta}_{t-1} + oldsymbol{\zeta}_t \quad ext{with} \quad oldsymbol{\zeta}_t \sim \mathcal{N}\left( oldsymbol{0}, oldsymbol{\Psi} 
ight)$$

where  $\eta_t$ ,  $\eta_{t-1}$ , and  $\zeta_t$  are random variables and  $\alpha$ ,  $\beta$ , and  $\Psi$  are model parameters.  $\eta_t$  is a vector of latent variables at time t,  $\eta_{t-1}$  is a vector of latent variables at time t-1, and  $\zeta_t$  is a vector of dynamic noise at time t while  $\alpha$  is a vector of intercepts,  $\beta$  is a matrix of autoregression and cross regression coefficients, and  $\Psi$  is the covariance matrix of  $\zeta_t$ .

# Value

Returns a list with the following elements:

- y: A t by k matrix of values for the manifest variables.
- eta: A t by p matrix of values for the latent variables.
- time: A vector of discrete time points from 1 to t.
- n: Number of individuals.

SimSSM0 7

# Author(s)

Ivan Jacob Agaloos Pesigan

# References

Shumway, R. H., & Stoffer, D. S. (2017). *Time series analysis and its applications: With R examples*. Springer International Publishing. doi:10.1007/9783319524528

# See Also

```
Other Simulation of State Space Models Data Functions: OU2SSM(), Sim2Matrix(), SimSSM0Fixed(), SimSSM0Vary(), SimSSM0UFixed(), SimSSMOUVary(), SimSSMVARFixed(), SimSSMVARVary(), SimSSMVAR()
```

```
# prepare parameters
set.seed(42)
k <- p <- 3
I <- diag(k)</pre>
I_sqrt <- chol(I)</pre>
null\_vec \leftarrow rep(x = 0, times = k)
mu0 <- null_vec
sigma0_sqrt <- I_sqrt</pre>
alpha <- null_vec</pre>
beta \leftarrow diag(x = 0.50, nrow = k)
psi_sqrt <- I_sqrt</pre>
nu <- null_vec
lambda <- I
theta_sqrt <- chol(diag(x = 0.50, nrow = k))
time <- 50
burn_in <- 0
# generate data
ssm <- SimSSM0(
  mu0 = mu0,
  sigma0_sqrt = sigma0_sqrt,
  alpha = alpha,
  beta = beta,
  psi_sqrt = psi_sqrt,
  nu = nu,
  lambda = lambda,
  theta_sqrt = theta_sqrt,
  time = time,
  burn_in = burn_in
str(ssm)
```

8 SimSSM0Fixed

SimSSMØFixed	Simulate Data using a State Space Model Parameterization for $n > 1$ Individuals (Fixed Parameters)

# Description

This function simulates data using a state space model parameterization for n > 1 individuals. In this model, the parameters are invariant across individuals.

# Usage

```
SimSSM0Fixed(
    n,
    mu0,
    sigma0_sqrt,
    alpha,
    beta,
    psi_sqrt,
    nu,
    lambda,
    theta_sqrt,
    time,
    burn_in
)
```

# Arguments

n	Positive integer. Number of individuals.
mu0	Numeric vector. Mean of initial latent variable values $(\mu_{\eta 0})$ .
sigma0_sqrt	Numeric matrix. Cholesky decomposition of the covariance matrix of initial latent variable values $(\Sigma_{\eta 0})$ .
alpha	Numeric vector. Vector of intercepts for the dynamic model $(\alpha)$ .
beta	Numeric matrix. Transition matrix relating the values of the latent variables at time $t-1$ to those at time $t(\beta)$ .
psi_sqrt	Numeric matrix. Cholesky decomposition of the process noise covariance matrix $(\Psi)$ .
nu	Numeric vector. Vector of intercepts for the measurement model $(\nu)$ .
lambda	Numeric matrix. Factor loading matrix linking the latent variables to the observed variables $(\Lambda)$ .
theta_sqrt	Numeric matrix. Cholesky decomposition of the measurement error covariance matrix $(\Theta)$ .
time	Positive integer. Number of time points to simulate.
burn_in	Positive integer. Number of burn-in points to exclude before returning the results.

SimSSM0Fixed 9

#### **Details**

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \boldsymbol{\Lambda} \boldsymbol{\eta}_{i:t} + \boldsymbol{\varepsilon}_{i:t} \quad \text{with} \quad \boldsymbol{\varepsilon}_{i:t} \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Theta}\right)$$

where  $\mathbf{y}_{i,t}$ ,  $\boldsymbol{\eta}_{i,t}$ , and  $\boldsymbol{\varepsilon}_{i,t}$  are random variables and  $\boldsymbol{\nu}$ ,  $\boldsymbol{\Lambda}$ , and  $\boldsymbol{\Theta}$  are model parameters.  $\mathbf{y}_{i,t}$  is a vector of observed random variables at time t and individual i,  $\boldsymbol{\eta}_{i,t}$  is a vector of latent random variables at time t and individual i, and  $\boldsymbol{\varepsilon}_{i,t}$  is a vector of random measurement errors at time t and individual i, while  $\boldsymbol{\nu}$  is a vector of intercept,  $\boldsymbol{\Lambda}$  is a matrix of factor loadings, and  $\boldsymbol{\Theta}$  is the covariance matrix of  $\boldsymbol{\varepsilon}$ .

The dynamic structure is given by

$$oldsymbol{\eta}_{i,t} = oldsymbol{lpha} + oldsymbol{eta} oldsymbol{\eta}_{i,t-1} + oldsymbol{\zeta}_{i,t} \quad ext{with} \quad oldsymbol{\zeta}_{i,t} \sim \mathcal{N}\left(oldsymbol{0}, oldsymbol{\Psi}
ight)$$

where  $\eta_{i,t}$ ,  $\eta_{i,t-1}$ , and  $\zeta_{i,t}$  are random variables and  $\alpha$ ,  $\beta$ , and  $\Psi$  are model parameters.  $\eta_{i,t}$  is a vector of latent variables at time t and individual i,  $\eta_{i,t-1}$  is a vector of latent variables at time t-1 and individual i, and  $\zeta_{i,t}$  is a vector of dynamic noise at time t and individual i while  $\alpha$  is a vector of intercepts,  $\beta$  is a matrix of autoregression and cross regression coefficients, and  $\Psi$  is the covariance matrix of  $\zeta_{i,t}$ .

#### Value

Returns a list of length n. Each element is a list with the following elements:

- y: A t by k matrix of values for the manifest variables.
- eta: A t by p matrix of values for the latent variables.
- time: A vector of discrete time points from 1 to t.
- id: A vector of ID numbers of length t.
- n: Number of individuals.

# Author(s)

Ivan Jacob Agaloos Pesigan

#### References

Shumway, R. H., & Stoffer, D. S. (2017). *Time series analysis and its applications: With R examples*. Springer International Publishing. doi:10.1007/9783319524528

# See Also

Other Simulation of State Space Models Data Functions: OU2SSM(), Sim2Matrix(), SimSSM0Vary(), SimSSM0(), SimSSMOUFixed(), SimSSMOUVary(), SimSSMOU(), SimSSMVARFixed(), SimSSMVARVary(), SimSSMVAR()

10 SimSSM0Vary

# **Examples**

```
# prepare parameters
set.seed(42)
k < -p < -3
I <- diag(k)</pre>
I_sqrt <- chol(I)</pre>
null_vec \leftarrow rep(x = 0, times = k)
n <- 5
mu0 <- null_vec
sigma0_sqrt <- I_sqrt</pre>
alpha <- null_vec</pre>
beta \leftarrow diag(x = 0.50, nrow = k)
psi_sqrt <- I_sqrt</pre>
nu <- null_vec
lambda <- I
theta_sqrt <- chol(diag(x = 0.50, nrow = k))
time <- 50
burn_in <- 0
# generate data
ssm <- SimSSM0Fixed(
  n = n,
  mu0 = mu0,
  sigma0_sqrt = sigma0_sqrt,
  alpha = alpha,
  beta = beta,
  psi_sqrt = psi_sqrt,
  nu = nu,
  lambda = lambda,
  theta_sqrt = theta_sqrt,
  time = time,
  burn_in = burn_in
)
str(ssm)
```

SimSSM0Vary

Simulate Data using a State Space Model Parameterization for n > 1 Individuals (Varying Parameters)

# **Description**

This function simulates data using a state space model parameterization for n > 1 individuals. In this model, the parameters can vary across individuals.

# Usage

```
SimSSM0Vary(
```

SimSSM0Vary 11

```
n,
mu0,
sigma0_sqrt,
alpha,
beta,
psi_sqrt,
nu,
lambda,
theta_sqrt,
time,
burn_in
)
```

# **Arguments**

n	Positive integer. Number of individuals.
mu0	List of numeric vectors. Mean of initial latent variable values $(\mu_{\eta 0})$ .
sigma0_sqrt	List of numeric matrices. Cholesky decomposition of the covariance matrix of initial latent variable values ( $\Sigma_{\eta 0}$ ).
alpha	List of numeric vectors. Vector of intercepts for the dynamic model $(\alpha)$ .
beta	List of numeric matrices. Transition matrix relating the values of the latent variables at time $t-1$ to those at time $t(\beta)$ .
psi_sqrt	List of numeric matrices. Cholesky decomposition of the process noise covariance matrix $(\Psi).$
nu	List of numeric vectors. Vector of intercepts for the measurement model $(\nu)$ .
lambda	List of numeric matrices. Factor loading matrix linking the latent variables to the observed variables $(\Lambda)$ .
theta_sqrt	List of numeric matrices. Cholesky decomposition of the measurement error covariance matrix $(\boldsymbol{\Theta}).$
time	Positive integer. Number of time points to simulate.
burn_in	Positive integer. Number of burn-in points to exclude before returning the results.

# **Details**

Parameters can vary across individuals by providing a list of parameter values. If the length of any of the parameters (mu0, sigma0\_sqrt, alpha, beta, psi\_sqrt, nu, lambda, and theta\_sqrt) is less the n, the function will cycle through the available values.

#### Value

Returns a list of length n. Each element is a list with the following elements:

- y: A t by k matrix of values for the manifest variables.
- eta: A t by p matrix of values for the latent variables.
- time: A vector of discrete time points from 1 to t.

12 SimSSM0Vary

- id: A vector of ID numbers of length t.
- n: Number of individuals.

# Author(s)

Ivan Jacob Agaloos Pesigan

#### References

Shumway, R. H., & Stoffer, D. S. (2017). *Time series analysis and its applications: With R examples*. Springer International Publishing. doi:10.1007/9783319524528

#### See Also

```
Other Simulation of State Space Models Data Functions: OU2SSM(), Sim2Matrix(), SimSSM0Fixed(), SimSSM0(), SimSSMOUFixed(), SimSSMOUVary(), SimSSMVARFixed(), SimSSMVARVary(), SimSSMVAR()
```

```
# prepare parameters
# In this example, beta varies across individuals
set.seed(42)
k <- p <- 3
iden <- diag(k)</pre>
iden_sqrt <- chol(iden)</pre>
null_vec <- rep(x = 0, times = k)
mu0 <- list(null_vec)</pre>
sigma0_sqrt <- list(iden_sqrt)</pre>
alpha <- list(null_vec)</pre>
beta <- list(</pre>
  diag(x = 0.1, nrow = k),
  diag(x = 0.2, nrow = k),
  diag(x = 0.3, nrow = k),
  diag(x = 0.4, nrow = k),
  diag(x = 0.5, nrow = k)
)
psi_sqrt <- list(iden_sqrt)</pre>
nu <- list(null_vec)</pre>
lambda <- list(iden)</pre>
theta_sqrt <- list(chol(diag(x = 0.50, nrow = k)))
time <- 50
burn_in <- 0
ssm <- SimSSM0Vary(</pre>
  n = n,
  mu0 = mu0,
  sigma0_sqrt = sigma0_sqrt,
  alpha = alpha,
  beta = beta,
  psi_sqrt = psi_sqrt,
```

SimSSMOU 13

```
nu = nu,
lambda = lambda,
theta_sqrt = theta_sqrt,
time = time,
burn_in = burn_in
)
str(ssm)
```

 ${\tt SimSSMOU}$ 

Simulate Data from the Ornstein-Uhlenbeck Model using a State Space Model Parameterization (n = 1)

# Description

This function simulates data from the Ornstein–Uhlenbeck model using a state space model parameterization. See details for more information.

# Usage

```
SimSSMOU(
   mu0,
   sigma0_sqrt,
   mu,
   phi,
   sigma_sqrt,
   nu,
   lambda,
   theta_sqrt,
   delta_t,
   time,
   burn_in
)
```

# Arguments

mu0	Numeric vector. Mean of initial latent variable values $(\mu_{\eta 0})$ .
sigma0_sqrt	Numeric matrix. Cholesky decomposition of the covariance matrix of initial latent variable values $(\Sigma_{\eta 0})$ .
mu	Numeric vector. The long-term mean or equilibrium level $(\mu)$ .
phi	Numeric matrix. The rate of mean reversion, determining how quickly the variable returns to its mean $(\Phi)$ .
sigma_sqrt	Numeric matrix. Cholesky decomposition of the matrix of volatility or randomness in the process $(\Sigma)$ .
nu	Numeric vector. Vector of intercepts for the measurement model $(\nu)$ .

14 SimSSMOU

Numeric matrix. Factor loading matrix linking the latent variables to the observed variables ( $\Lambda$ ). Numeric matrix. Cholesky decomposition of the measurement error covariance matrix ( $\Theta$ ). delta\_t Numeric. Time interval ( $\delta_t$ ). time Positive integer. Number of time points to simulate. Positive integer. Number of burn-in points to exclude before returning the results.

# **Details**

The measurement model is given by

$$\mathbf{y}_{t} = \boldsymbol{\nu} + \boldsymbol{\Lambda} \boldsymbol{\eta}_{t} + \boldsymbol{\varepsilon}_{t} \quad \text{with} \quad \boldsymbol{\varepsilon}_{t} \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Theta}\right)$$

where  $y_t$ ,  $\eta_t$ , and  $\varepsilon_t$  are random variables and  $\nu$ ,  $\Lambda$ , and  $\Theta$  are model parameters.  $y_t$  is a vector of observed random variables at time t,  $\eta_t$  is a vector of latent random variables at time t, and  $\varepsilon_t$  is a vector of random measurement errors at time t, while  $\nu$  is a vector of intercept,  $\Lambda$  is a matrix of factor loadings, and  $\Theta$  is the covariance matrix of  $\varepsilon$ .

The dynamic structure is given by

$$\mathrm{d}\boldsymbol{\eta}_t = \boldsymbol{\Phi} \left( \boldsymbol{\mu} - \boldsymbol{\eta}_t \right) \mathrm{d}t + \boldsymbol{\Sigma}^{\frac{1}{2}} \mathrm{d}\mathbf{W}_t$$

where  $\mu$  is the long-term mean or equilibrium level,  $\Phi$  is the rate of mean reversion, determining how quickly the variable returns to its mean,  $\Sigma$  is the matrix of volatility or randomness in the process, and dW is a Wiener process or Brownian motion, which represents random fluctuations.

#### Value

Returns a list with the following elements:

- y: A t by k matrix of values for the manifest variables.
- eta: A t by p matrix of values for the latent variables.
- time: A vector of continuous time points of length t starting from 0 with delta\_t increments.
- n: Number of individuals.

#### Author(s)

Ivan Jacob Agaloos Pesigan

# References

Uhlenbeck, G. E., & Ornstein, L. S. (1930). On the theory of the brownian motion. *Physical Review*, 36(5), 823–841. doi:10.1103/physrev.36.823

#### See Also

Other Simulation of State Space Models Data Functions: OU2SSM(), Sim2Matrix(), SimSSM0Fixed(), SimSSM0Vary(), SimSSM0UFixed(), SimSSMOUVary(), SimSSMVARFixed(), SimSSMVARVary(), SimSSMVAR()

SimSSMOUFixed 15

# **Examples**

```
# prepare parameters
set.seed(42)
p <- k <- 2
I <- diag(p)</pre>
I_sqrt <- chol(I)</pre>
mu0 < -c(-3.0, 1.5)
sigma0\_sqrt \leftarrow I\_sqrt
mu < -c(5.76, 5.18)
phi <- matrix(data = c(0.10, -0.05, -0.05, 0.10), nrow = p)
sigma_sqrt <- chol(</pre>
  matrix(data = c(2.79, 0.06, 0.06, 3.27), nrow = p)
)
nu \leftarrow rep(x = 0, times = k)
lambda <- diag(k)</pre>
theta_sqrt <- chol(diag(x = 0.50, nrow = k))
delta_t <- 0.10
time <- 50
burn_in <- 0
# generate data
ssm <- SimSSMOU(
  mu0 = mu0,
  sigma0_sqrt = sigma0_sqrt,
  mu = mu,
  phi = phi,
  sigma_sqrt = sigma_sqrt,
  nu = nu,
  lambda = lambda,
  theta_sqrt = theta_sqrt,
  delta_t = delta_t,
  time = time,
  burn_in = burn_in
)
str(ssm)
```

SimSSMOUFixed

Simulate Data from an Ornstein-Uhlenbeck Model using a State Space Model Parameterization for n > 1 Individuals (Fixed Parameters)

# Description

This function simulates data from an Ornstein–Uhlenbeck model using a state space model parameterization for n > 1 individuals. In this model, the parameters are invariant across individuals. See details for more information.

16 SimSSMOUFixed

#### Usage

```
SimSSMOUFixed(
    n,
    mu0,
    sigma0_sqrt,
    mu,
    phi,
    sigma_sqrt,
    nu,
    lambda,
    theta_sqrt,
    delta_t,
    time,
    burn_in
)
```

# **Arguments**

s.
9

mu0 Numeric vector. Mean of initial latent variable values  $(\mu_{n|0})$ .

sigma@\_sqrt Numeric matrix. Cholesky decomposition of the covariance matrix of initial

latent variable values  $(\Sigma_{\eta|0})$ .

mu Numeric vector. The long-term mean or equilibrium level  $(\mu)$ .

phi Numeric matrix. The rate of mean reversion, determining how quickly the vari-

able returns to its mean  $(\Phi)$ .

sigma\_sqrt Numeric matrix. Cholesky decomposition of the matrix of volatility or random-

ness in the process  $(\Sigma)$ .

nu Numeric vector. Vector of intercepts for the measurement model  $(\nu)$ .

lambda Numeric matrix. Factor loading matrix linking the latent variables to the ob-

served variables  $(\Lambda)$ .

theta\_sqrt Numeric matrix. Cholesky decomposition of the measurement error covariance

matrix  $(\Theta)$ .

delta\_t Numeric. Time interval  $(\delta_t)$ .

time Positive integer. Number of time points to simulate.

burn\_in Positive integer. Number of burn-in points to exclude before returning the re-

sults.

#### **Details**

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \boldsymbol{\Lambda} \boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t} \quad ext{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Theta}\right)$$

where  $\mathbf{y}_{i,t}$ ,  $\boldsymbol{\eta}_{i,t}$ , and  $\boldsymbol{\varepsilon}_{i,t}$  are random variables and  $\boldsymbol{\nu}$ ,  $\boldsymbol{\Lambda}$ , and  $\boldsymbol{\Theta}$  are model parameters.  $\mathbf{y}_{i,t}$  is a vector of observed random variables at time t and individual i,  $\boldsymbol{\eta}_{i,t}$  is a vector of latent random

SimSSMOUFixed 17

variables at time t and individual i, and  $\varepsilon_{i,t}$  is a vector of random measurement errors at time t and individual i, while  $\nu$  is a vector of intercept,  $\Lambda$  is a matrix of factor loadings, and  $\Theta$  is the covariance matrix of  $\varepsilon$ .

The dynamic structure is given by

$$\mathrm{d}\boldsymbol{\eta}_{i,t} = \boldsymbol{\Phi} \left( \boldsymbol{\mu} - \boldsymbol{\eta}_{i,t} \right) \mathrm{d}t + \boldsymbol{\Sigma}^{\frac{1}{2}} \mathrm{d}\mathbf{W}_{i,t}$$

where  $\mu$  is the long-term mean or equilibrium level,  $\Phi$  is the rate of mean reversion, determining how quickly the variable returns to its mean,  $\Sigma$  is the matrix of volatility or randomness in the process, and  $\mathrm{d}W$  is a Wiener process or Brownian motion, which represents random fluctuations.

#### Value

Returns a list of length n. Each element is a list with the following elements:

- y: A t by k matrix of values for the manifest variables.
- eta: A t by p matrix of values for the latent variables.
- time: A vector of continuous time points of length t starting from 0 with delta\_t increments.
- id: A vector of ID numbers of length t.
- n: Number of individuals.

# Author(s)

Ivan Jacob Agaloos Pesigan

#### References

Uhlenbeck, G. E., & Ornstein, L. S. (1930). On the theory of the brownian motion. *Physical Review*, 36(5), 823–841. doi:10.1103/physrev.36.823

# See Also

Other Simulation of State Space Models Data Functions: OU2SSM(), Sim2Matrix(), SimSSM0Fixed(), SimSSM0Vary(), SimSSM0Vary(), SimSSM0U(), SimSSMVARFixed(), SimSSMVARVary(), SimSSMVAR()

```
# prepare parameters
set.seed(42)
p <- k <- 2
I <- diag(p)
I_sqrt <- chol(I)
n <- 5
mu0 <- c(-3.0, 1.5)
sigma0_sqrt <- I_sqrt
mu <- c(5.76, 5.18)
phi <- matrix(data = c(0.10, -0.05, -0.05, 0.10), nrow = p)
sigma_sqrt <- chol(
    matrix(data = c(2.79, 0.06, 0.06, 3.27), nrow = p)</pre>
```

18 SimSSMOUVary

```
nu \leftarrow rep(x = 0, times = k)
lambda <- diag(k)</pre>
theta_sqrt <- chol(diag(x = 0.50, nrow = k))
delta_t <- 0.10
time <- 50
burn_in <- 0
# generate data
ssm <- SimSSMOUFixed(</pre>
  n = n,
  mu0 = mu0,
  sigma0_sqrt = sigma0_sqrt,
  mu = mu,
  phi = phi,
  sigma_sqrt = sigma_sqrt,
  nu = nu,
  lambda = lambda,
  theta_sqrt = theta_sqrt,
  delta_t = delta_t,
  time = time,
  burn_in = burn_in
)
str(ssm)
```

SimSSMOUVary

Simulate Data from an Ornstein-Uhlenbeck Model using a State Space Model Parameterization for n > 1 Individuals (Varying Parameters)

# Description

This function simulates data from an Ornstein–Uhlenbeck model using a state space model parameterization for n > 1 individuals. In this model, the parameters can vary across individuals.

# Usage

```
SimSSMOUVary(
    n,
    mu0,
    sigma0_sqrt,
    mu,
    phi,
    sigma_sqrt,
    nu,
    lambda,
    theta_sqrt,
    delta_t,
```

SimSSMOUVary 19

```
time,
burn_in
)
```

#### **Arguments**

n Positive integer. Number of individuals. mu0 Numeric vector. Mean of initial latent variable values  $(\mu_{n|0})$ . Numeric matrix. Cholesky decomposition of the covariance matrix of initial sigma0\_sqrt latent variable values  $(\Sigma_{\eta|0})$ . List of numeric vectors. The long-term mean or equilibrium level  $(\mu)$ . mu List of numeric matrices. The rate of mean reversion, determining how quickly phi the variable returns to its mean  $(\Phi)$ . List of numeric matrices. Cholesky decomposition of the matrix of volatility or sigma\_sqrt randomness in the process ( $\Sigma$ ). Numeric vector. Vector of intercepts for the measurement model  $(\nu)$ . ทน lambda Numeric matrix. Factor loading matrix linking the latent variables to the observed variables  $(\Lambda)$ . Numeric matrix. Cholesky decomposition of the measurement error covariance theta\_sqrt matrix  $(\Theta)$ . delta\_t Numeric. Time interval ( $\delta_t$ ). time Positive integer. Number of time points to simulate.

# **Details**

burn\_in

Parameters can vary across individuals by providing a list of parameter values. If the length of any of the parameters (mu0, sigma0\_sqrt, mu, phi, sigma\_sqrt, nu, lambda, theta\_sqrt) is less the n, the function will cycle through the available values.

Positive integer. Number of burn-in points to exclude before returning the re-

#### Value

Returns a list of length n. Each element is a list with the following elements:

- y: A t by k matrix of values for the manifest variables.
- eta: A t by p matrix of values for the latent variables.
- time: A vector of discrete time points from 1 to t.
- id: A vector of ID numbers of length t.

sults.

• n: Number of individuals.

# Author(s)

Ivan Jacob Agaloos Pesigan

20 SimSSMOUVary

# References

Uhlenbeck, G. E., & Ornstein, L. S. (1930). On the theory of the brownian motion. *Physical Review*, *36*(5), 823–841. doi:10.1103/physrev.36.823

#### See Also

Other Simulation of State Space Models Data Functions: OU2SSM(), Sim2Matrix(), SimSSM0Fixed(), SimSSM0Vary(), SimSSM0(), SimSSM0UFixed(), SimSSM0U(), SimSSMVARFixed(), SimSSMVARVary(), SimSSMVAR()

```
# prepare parameters
# In this example, phi varies across individuals
set.seed(42)
p < -k < -2
iden <- diag(p)</pre>
iden_sqrt <- chol(iden)</pre>
n <- 5
mu0 <- list(c(-3.0, 1.5))
sigma0_sqrt <- list(iden_sqrt)</pre>
mu \leftarrow list(c(5.76, 5.18))
phi <- list(</pre>
  as.matrix(Matrix::expm(diag(x = -0.1, nrow = k))),
  as.matrix(Matrix::expm(diag(x = -0.2, nrow = k))),
  as.matrix(Matrix::expm(diag(x = -0.3, nrow = k))),
  as.matrix(Matrix::expm(diag(x = -0.4, nrow = k))),
  as.matrix(Matrix::expm(diag(x = -0.5, nrow = k)))
)
sigma_sqrt <- list(</pre>
  chol(
    matrix(data = c(2.79, 0.06, 0.06, 3.27), nrow = p)
nu \leftarrow list(rep(x = 0, times = k))
lambda <- list(diag(k))</pre>
theta_sqrt <- list(chol(diag(x = 0.50, nrow = k)))</pre>
delta_t <- 0.10
time <- 50
burn_in <- 0
ssm <- SimSSMOUVary(</pre>
  n = n,
  mu0 = mu0,
  sigma0_sqrt = sigma0_sqrt,
  mu = mu,
  phi = phi,
  sigma_sqrt = sigma_sqrt,
  nu = nu,
  lambda = lambda,
  theta_sqrt = theta_sqrt,
  delta_t = delta_t,
```

SimSSMVAR 21

```
time = time,
burn_in = burn_in
)
str(ssm)
```

SimSSMVAR

Simulate Data from the Vector Autoregressive Model using a State Space Model Parameterization (n = 1)

# **Description**

This function simulates data from the vector autoregressive model using a state space model parameterization. See details for more information.

# Usage

```
SimSSMVAR(mu0, sigma0_sqrt, alpha, beta, psi_sqrt, time, burn_in)
```

# **Arguments**

mu0	Numeric vector. Mean of initial latent variable values $(\mu_{\eta 0})$ .
sigma0_sqrt	Numeric matrix. Cholesky decomposition of the covariance matrix of initial latent variable values $(\Sigma_{\eta 0})$ .
alpha	Numeric vector. Vector of intercepts for the dynamic model $(\alpha)$ .
beta	Numeric matrix. Transition matrix relating the values of the latent variables at time $t-1$ to those at time $t(\beta)$ .
psi_sqrt	Numeric matrix. Cholesky decomposition of the process noise covariance matrix $(\Psi)$ .
time	Positive integer. Number of time points to simulate.
burn_in	Positive integer. Number of burn-in points to exclude before returning the results.

# **Details**

The measurement model is given by

$$\mathbf{y}_t = \boldsymbol{\eta}_t$$
.

The dynamic structure is given by

$$oldsymbol{\eta}_t = oldsymbol{lpha} + oldsymbol{eta} oldsymbol{\eta}_{t-1} + oldsymbol{\zeta}_t \quad ext{with} \quad oldsymbol{\zeta}_t \sim \mathcal{N}\left(oldsymbol{0}, oldsymbol{\Psi}
ight)$$

where  $\eta_t$ ,  $\eta_{t-1}$ , and  $\zeta_t$  are random variables and  $\alpha$ ,  $\beta$ , and  $\Psi$  are model parameters.  $\eta_t$  is a vector of latent variables at time t,  $\eta_{t-1}$  is a vector of latent variables at t-1, and  $\zeta_t$  is a vector of dynamic noise at time t while  $\alpha$  is a vector of intercepts,  $\beta$  is a matrix of autoregression and cross regression coefficients, and  $\Psi$  is the covariance matrix of  $\zeta_t$ .

22 SimSSMVAR

# Value

Returns a list with the following elements:

- y: A t by k matrix of values for the manifest variables.
- eta: A t by p matrix of values for the latent variables.
- time: A vector of discrete time points from 1 to t.
- n: Number of individuals.

#### References

Shumway, R. H., & Stoffer, D. S. (2017). *Time series analysis and its applications: With R examples*. Springer International Publishing. doi:10.1007/9783319524528

# See Also

Other Simulation of State Space Models Data Functions: OU2SSM(), Sim2Matrix(), SimSSM0Fixed(), SimSSM0Vary(), SimSSM0(), SimSSM0UFixed(), SimSSM0UVary(), SimSSM0U(), SimSSMVARFixed(), SimSSMVARVary()

```
# prepare parameters
set.seed(42)
k <- 3
I <- diag(k)</pre>
I_sqrt <- chol(I)</pre>
null_vec <- rep(x = 0, times = k)
mu0 <- null_vec
sigma0\_sqrt \leftarrow I\_sqrt
alpha <- null_vec</pre>
beta <- diag(x = 0.5, nrow = k)
psi_sqrt <- I_sqrt</pre>
time <- 50
burn_in <- 0</pre>
# generate data
ssm <- SimSSMVAR(</pre>
  mu0 = mu0,
  sigma0_sqrt = sigma0_sqrt,
  alpha = alpha,
  beta = beta,
  psi_sqrt = psi_sqrt,
  time = time,
  burn_in = burn_in
str(ssm)
```

SimSSMVARFixed 23

SimSSMVARFixed	Simulate Data from a Vector Autoregressive Model using a State Space Model Parameterization for $n > 1$ Individuals (Fixed Parameters)
	•

# **Description**

This function simulates data from a vector autoregressive model using a state space model parameterization for n > 1 individuals. In this model, the parameters are invariant across individuals.

# Usage

```
SimSSMVARFixed(n, mu0, sigma0_sqrt, alpha, beta, psi_sqrt, time, burn_in)
```

# **Arguments**

n	Positive integer. Number of individuals.
mu0	Numeric vector. Mean of initial latent variable values $(\mu_{\eta 0})$ .
sigma0_sqrt	Numeric matrix. Cholesky decomposition of the covariance matrix of initial latent variable values $(\Sigma_{\eta 0})$ .
alpha	Numeric vector. Vector of intercepts for the dynamic model $(\alpha)$ .
beta	Numeric matrix. Transition matrix relating the values of the latent variables at time $t-1$ to those at time $t(\beta)$ .
psi_sqrt	Numeric matrix. Cholesky decomposition of the process noise covariance matrix $(\Psi)$ .
time	Positive integer. Number of time points to simulate.
burn_in	Positive integer. Number of burn-in points to exclude before returning the results.

# Value

Returns a list of length n. Each element is a list with the following elements:

- y: A t by k matrix of values for the manifest variables.
- eta: A t by p matrix of values for the latent variables.
- time: A vector of discrete time points from 1 to t.
- id: A vector of ID numbers of length t.
- n: Number of individuals.

# Author(s)

Ivan Jacob Agaloos Pesigan

#### References

Shumway, R. H., & Stoffer, D. S. (2017). *Time series analysis and its applications: With R examples*. Springer International Publishing. doi:10.1007/9783319524528

24 SimSSMVARVary

# See Also

Other Simulation of State Space Models Data Functions: OU2SSM(), Sim2Matrix(), SimSSM0Fixed(), SimSSM0Vary(), SimSSM0UFixed(), SimSSMOUVary(), SimSSMOU(), SimSSMVARVary(), SimSSMVAR()

# **Examples**

```
# prepare parameters
set.seed(42)
k <- 3
iden <- diag(k)</pre>
iden_sqrt <- chol(iden)</pre>
null\_vec \leftarrow rep(x = 0, times = k)
n <- 5
mu0 <- null_vec
sigma0\_sqrt \leftarrow iden\_sqrt
alpha <- null_vec</pre>
beta \leftarrow diag(x = 0.5, nrow = k)
psi_sqrt <- iden_sqrt</pre>
time <- 50
burn_in <- 0
ssm <- SimSSMVARFixed(</pre>
  n = n,
 mu0 = mu0,
  sigma0_sqrt = sigma0_sqrt,
  alpha = alpha,
  beta = beta,
  psi_sqrt = psi_sqrt,
  time = time,
  burn_in = burn_in
)
str(ssm)
```

 ${\tt SimSSMVARVary}$ 

Simulate Data from a Vector Autoregressive Model using a State Space Model Parameterization for n > 1 Individuals (Varying Parameters)

# **Description**

This function simulates data from a vector autoregressive model using a state space model parameterization for n > 1 individuals. In this model, the parameters can vary across individuals.

# Usage

```
SimSSMVARVary(n, mu0, sigma0_sqrt, alpha, beta, psi_sqrt, time, burn_in)
```

SimSSMVARVary 25

# **Arguments**

n	Positive integer. Number of individuals.
mu0	List of numeric vectors. Mean of initial latent variable values $(\mu_{\eta 0})$ .
sigma0_sqrt	List of numeric matrices. Cholesky decomposition of the covariance matrix of initial latent variable values ( $\Sigma_{\eta 0}$ ).
alpha	List of numeric vectors. Vector of intercepts for the dynamic model $(\alpha)$ .
beta	List of numeric matrices. Transition matrix relating the values of the latent variables at time $t-1$ to those at time $t(\beta)$ .
psi_sqrt	List of numeric matrices. Cholesky decomposition of the process noise covariance matrix $(\boldsymbol{\Psi}).$
time	Positive integer. Number of time points to simulate.
burn_in	Positive integer. Number of burn-in points to exclude before returning the results.

#### **Details**

Parameters can vary across individuals by providing a list of parameter values. If the length of any of the parameters (mu0, sigma0\_sqrt, alpha, beta, and psi\_sqrt) is less the n, the function will cycle through the available values.

# Value

Returns a list of length n. Each element is a list with the following elements:

- y: A t by k matrix of values for the manifest variables.
- eta: A t by p matrix of values for the latent variables.
- time: A vector of discrete time points from 1 to t.
- id: A vector of ID numbers of length t.
- n: Number of individuals.

# Author(s)

Ivan Jacob Agaloos Pesigan

# References

Shumway, R. H., & Stoffer, D. S. (2017). *Time series analysis and its applications: With R examples*. Springer International Publishing. doi:10.1007/9783319524528

# See Also

Other Simulation of State Space Models Data Functions: OU2SSM(), Sim2Matrix(), SimSSM0Fixed(), SimSSM0Vary(), SimSSM0U(), SimSSM0UVary(), SimSSM0U(), SimSSMVARFixed(), SimSSMVAR()

26 SimSSMVARVary

```
# prepare parameters
# In this example, beta varies across individuals
set.seed(42)
k <- 3
iden <- diag(k)</pre>
iden_sqrt <- chol(iden)</pre>
null_vec <- rep(x = 0, times = k)
n <- 5
mu0 <- list(null_vec)</pre>
sigma0_sqrt <- list(iden_sqrt)</pre>
alpha <- list(null_vec)</pre>
beta <- list(</pre>
  diag(x = 0.1, nrow = k),
  diag(x = 0.2, nrow = k),
  diag(x = 0.3, nrow = k),
  diag(x = 0.4, nrow = k),
  diag(x = 0.5, nrow = k)
)
psi_sqrt <- list(iden_sqrt)</pre>
time <- 50
burn_in <- 0</pre>
ssm <- SimSSMVARVary(</pre>
  n = n,
 mu0 = mu0,
  sigma0_sqrt = sigma0_sqrt,
  alpha = alpha,
  beta = beta,
  psi_sqrt = psi_sqrt,
  time = time,
  burn_in = burn_in
)
str(ssm)
```

# **Index**

* Simulation of State Space Models Data Functions	Sim2Matrix, 3, 3, 7, 9, 12, 14, 17, 19, 22, 23, 25
0U2SSM, 2	SimSSM0, 3, 4, 5, 9, 12, 14, 17, 19, 22, 23, 25
Sim2Matrix, 3	SimSSMO(), 3
SimSSM0, 5	SimSSM0Fixed, 3, 4, 7, 7, 12, 14, 17, 19, 22,
SimSSM0Fixed, 7	23, 25
SimSSM0Vary, 10	SimSSM0Fixed(), 3
SimSSMOU, 13	SimSSM0Vary, 3, 4, 7, 9, 10, 14, 17, 19, 22, 23,
SimSSMOUFixed, 15	25
SimSSMOUVary, 18	SimSSMOU, 3, 4, 7, 9, 12, 13, 17, 19, 22, 23, 25
SimSSMVAR, 21	SimSSMOU(), 3
SimSSMVARFixed, 22	SimSSMOUFixed, 3, 4, 7, 9, 12, 14, 15, 19, 22,
SimSSMVARVary, 24	23, 25
* misc	SimSSMOUFixed(), 3
Sim2Matrix, 3	SimSSMOUVary, 3, 4, 7, 9, 12, 14, 17, 18, 22,
* simStateSpace	23, 25
OU2SSM, 2	SimSSMVAR, 3, 4, 7, 9, 12, 14, 17, 19, 20, 23, 25
Sim2Matrix, 3	SimSSMVAR(), 3
SimSSM0,5	SimSSMVARFixed, 3, 4, 7, 9, 12, 14, 17, 19, 22,
SimSSM0Fixed, 7	22, 25
SimSSM0Vary, 10	SimSSMVARFixed(), 3
SimSSMOU, 13	SimSSMVARVary, 3, 4, 7, 9, 12, 14, 17, 19, 22,
SimSSMOUFixed, 15	23, 24
SimSSMOUVary, 18	
SimSSMVAR, 21	
SimSSMVARFixed, 22	
SimSSMVARVary, 24	
* sim	
OU2SSM, 2	
SimSSM0, 5	
SimSSM0Fixed, 7	
SimSSM0Vary, 10	
SimSSMOU, 13	
SimSSMOUFixed, 15	
SimSSMOUVary, 18	
SimSSMVAR, 21	
SimSSMVARFixed, 22	
SimSSMVARVary, 24	
OU2SSM, 2, 4, 7, 9, 12, 14, 17, 19, 22, 23, 25	