Package 'simStateSpace'

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Title Simulate Data from State Space Models		
Version 1.2.3.9000		
Description Provides a streamlined and user-friendly framework for simulating data in state space models, particularly when the number of subjects/units (n) exceeds one, a scenario commonly encountered in social and behavioral sciences. For an introduction to state space models in social and behavioral sciences, refer to Chow, Ho, Hamaker, and Dolan (2010) <doi:10.1080 10705511003661553="">.</doi:10.1080>		
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as.data.frame.simstatespace

Coerce an Object of Class simstatespace to a Data Frame

Description

Coerce an Object of Class simstatespace to a Data Frame

Usage

```
## S3 method for class 'simstatespace'
as.data.frame(
    x,
    row.names = NULL,
    optional = FALSE,
    eta = FALSE,
    long = TRUE,
    ...
)
```

Arguments

X	Object of class simstatespace.
row.names	NULL or character vector giving the row names for the data frame. Missing values are not allowed.
optional	Logical. If TRUE, setting row names and converting column names is optional.
eta	Logical. If eta = TRUE, include eta. If eta = FALSE, exclude eta.
long	Logical. If long = TRUE, use long format. If long = FALSE, use wide format.
	Additional arguments.

Author(s)

Ivan Jacob Agaloos Pesigan

```
# prepare parameters
set.seed(42)
## number of individuals
n <- 5
## time points
time <- 50
## dynamic structure
p <- 3
mu0 < -rep(x = 0, times = p)
sigma0 <- diag(p)</pre>
sigma0_l <- t(chol(sigma0))</pre>
alpha <- rep(x = 0, times = p)
beta <- 0.50 * diag(p)
psi <- diag(p)</pre>
psi_l <- t(chol(psi))</pre>
## measurement model
k <- 3
nu \leftarrow rep(x = 0, times = k)
lambda <- diag(k)</pre>
theta <- 0.50 * diag(k)
theta_l <- t(chol(theta))</pre>
## covariates
j <- 2
x <- lapply(
  X = seq_len(n),
  FUN = function(i) {
    matrix(
      data = stats::rnorm(n = time * j),
      nrow = j,
      ncol = time
 }
)
gamma \leftarrow diag(x = 0.10, nrow = p, ncol = j)
kappa \leftarrow diag(x = 0.10, nrow = k, ncol = j)
```

```
# Type 0
ssm <- SimSSMFixed(</pre>
 n = n,
 time = time,
 mu0 = mu0,
  sigma0_1 = sigma0_1,
  alpha = alpha,
  beta = beta,
  psi_l = psi_l,
  nu = nu,
  lambda = lambda,
  theta_1 = theta_1,
  type = 0
)
head(as.data.frame(ssm))
head(as.data.frame(ssm, long = FALSE))
# Type 1
ssm <- SimSSMFixed(</pre>
  n = n,
  time = time,
  mu0 = mu0,
  sigma0_1 = sigma0_1,
  alpha = alpha,
  beta = beta,
  psi_l = psi_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 1,
  x = x,
  gamma = gamma
head(as.data.frame(ssm))
head(as.data.frame(ssm, long = FALSE))
# Type 2
ssm <- SimSSMFixed(</pre>
 n = n,
  time = time,
 mu0 = mu0,
  sigma0_1 = sigma0_1,
  alpha = alpha,
  beta = beta,
  psi_l = psi_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 2,
  x = x,
```

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```
gamma = gamma,
kappa = kappa
)
head(as.data.frame(ssm))
head(as.data.frame(ssm, long = FALSE))
```

as.matrix.simstatespace

Coerce an Object of Class simstatespace to a Matrix

Description

Coerce an Object of Class simstatespace to a Matrix

Usage

```
## S3 method for class 'simstatespace'
as.matrix(x, eta = FALSE, long = TRUE, ...)
```

Arguments

x Object of class simstatespace.
 eta Logical. If eta = TRUE, include eta. If eta = FALSE, exclude eta.
 long Logical. If long = TRUE, use long format. If long = FALSE, use wide format.

.. Additional arguments.

Author(s)

Ivan Jacob Agaloos Pesigan

```
# prepare parameters
set.seed(42)
## number of individuals
n <- 5
## time points
time <- 50
## dynamic structure
p <- 3
mu0 <- rep(x = 0, times = p)
sigma0 <- diag(p)
sigma0_1 <- t(chol(sigma0))
alpha <- rep(x = 0, times = p)
beta <- 0.50 * diag(p)
psi <- diag(p)</pre>
```

```
psi_l <- t(chol(psi))</pre>
## measurement model
k <- 3
nu \leftarrow rep(x = 0, times = k)
lambda <- diag(k)</pre>
theta <-0.50 * diag(k)
theta_l <- t(chol(theta))</pre>
## covariates
j <- 2
x <- lapply(
  X = seq_len(n),
  FUN = function(i) {
    matrix(
      data = stats::rnorm(n = time * j),
      nrow = j,
      ncol = time
    )
  }
)
gamma \leftarrow diag(x = 0.10, nrow = p, ncol = j)
kappa \leftarrow diag(x = 0.10, nrow = k, ncol = j)
# Type 0
ssm <- SimSSMFixed(</pre>
  n = n,
  time = time,
  mu0 = mu0,
  sigma0_1 = sigma0_1,
  alpha = alpha,
  beta = beta,
  psi_l = psi_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 0
)
head(as.matrix(ssm))
head(as.matrix(ssm, long = FALSE))
# Type 1
ssm <- SimSSMFixed(</pre>
  n = n,
  time = time,
  mu0 = mu0,
  sigma0_1 = sigma0_1,
  alpha = alpha,
  beta = beta,
  psi_l = psi_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 1,
```

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```
x = x,
  gamma = gamma
)
head(as.matrix(ssm))
head(as.matrix(ssm, long = FALSE))
# Type 2
ssm <- SimSSMFixed(</pre>
  n = n,
  time = time,
  mu0 = mu0,
  sigma0_1 = sigma0_1,
  alpha = alpha,
  beta = beta,
  psi_l = psi_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 2,
  x = x,
  gamma = gamma,
  kappa = kappa
)
head(as.matrix(ssm))
head(as.matrix(ssm, long = FALSE))
```

 ${\tt coef.statespacepb}$

Estimated Parameter Method for an Object of Class statespacepb

Description

Estimated Parameter Method for an Object of Class statespacepb

Usage

```
## S3 method for class 'statespacepb'
coef(object, ...)
```

Arguments

```
object Object of Class statespacepb.
... additional arguments.
```

Value

Returns a vector of estimated parameters.

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Author(s)

Ivan Jacob Agaloos Pesigan

Description

Confidence Intervals Method for an Object of Class statespacepb

Usage

```
## S3 method for class 'statespacepb'
confint(object, parm = NULL, level = 0.95, type = "pc", ...)
```

Arguments

object	Object of Class statespacepb.
parm	a specification of which parameters are to be given confidence intervals, either a vector of numbers or a vector of names. If missing, all parameters are considered.
level	the confidence level required.
type	Charater string. Confidence interval type, that is, type = "pc" for percentile; type = "bc" for bias corrected.
	additional arguments.

Value

Returns a matrix of confidence intervals.

Author(s)

Ivan Jacob Agaloos Pesigan

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LinSDE2SSM	Convert Parameters from the Linear Stochastic Differential Equation
LINODEZOON	Convert Turameters from the Linear Stochastic Differential Equation
	Model to State Space Model Parameterization

Description

This function converts parameters from the linear stochastic differential equation model to state space model parameterization.

Usage

LinSDE2SSM(iota, phi, sigma_l, delta_t)

Arguments

iota	Numeric vector. An unobserved term that is constant over time (ι) .
phi	Numeric matrix. The drift matrix which represents the rate of change of the solution in the absence of any random fluctuations (Φ) .
sigma_l	Numeric matrix. Cholesky factorization (t(chol(sigma))) of the covariance matrix of volatility or randomness in the process (Σ) .
delta_t	Numeric. Time interval (Δ_t) .

Details

Let the linear stochastic equation model be given by

$$\mathrm{d} oldsymbol{\eta}_{i,t} = \left(oldsymbol{\iota} + oldsymbol{\Phi} oldsymbol{\eta}_{i,t}
ight) \mathrm{d} t + oldsymbol{\Sigma}^{rac{1}{2}} \mathrm{d} \mathbf{W}_{i,t}$$

for individual i and time t. The discrete-time state space model given below represents the discrete-time solution for the linear stochastic differential equation.

$$oldsymbol{\eta}_{i,t_{l_i}} = oldsymbol{lpha}_{\Delta t_{l_i}} + oldsymbol{eta}_{\Delta t_{l_i}} oldsymbol{\eta}_{i,t_{l_i-1}} + oldsymbol{\zeta}_{i,t_{l_i}}, \quad ext{with} \quad oldsymbol{\zeta}_{i,t_{l_i}} \sim \mathcal{N}\left(oldsymbol{0}, oldsymbol{\Psi}_{\Delta t_{l_i}}
ight)$$

with

$$\boldsymbol{\beta}_{\Delta t_{l_i}} = \exp{(\Delta t \boldsymbol{\Phi})},$$

$$\boldsymbol{lpha}_{\Delta t_{l_i}} = \boldsymbol{\Phi}^{-1} \left(\boldsymbol{eta} - \mathbf{I}_p \right) \boldsymbol{\iota}, \quad ext{and}$$

$$\operatorname{vec}\left(\boldsymbol{\Psi}_{\Delta t_{l_{i}}}\right) = \left[\left(\boldsymbol{\Phi} \otimes \mathbf{I}_{p}\right) + \left(\mathbf{I}_{p} \otimes \boldsymbol{\Phi}\right)\right] \left[\exp\left(\left[\left(\boldsymbol{\Phi} \otimes \mathbf{I}_{p}\right) + \left(\mathbf{I}_{p} \otimes \boldsymbol{\Phi}\right)\right] \Delta t\right) - \mathbf{I}_{p \times p}\right] \operatorname{vec}\left(\boldsymbol{\Sigma}\right)$$

where t denotes continuous-time processes that can be defined by any arbitrary time point, t_{l_i} the $l^{\rm th}$ observed measurement occassion for individual i, p the number of latent variables and Δt the time interval.

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Value

Returns a list of state space parameters:

- alpha: Numeric vector. Vector of constant values for the dynamic model (α) .
- beta: Numeric matrix. Transition matrix relating the values of the latent variables from the previous time point to the current time point. (β).
- psi_1: Numeric matrix. Cholesky factorization (t(chol(psi))) of the process noise covariance matrix Ψ.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Harvey, A. C. (1990). Forecasting, structural time series models and the Kalman filter. Cambridge University Press. doi:10.1017/cbo9781107049994

See Also

```
Other Simulation of State Space Models Data Functions: PBSSMLinSDEFixed(), PBSSMOUFixed(), PBSSMVARFixed(), SimBetaN(), SimPhiN(), SimSSMFixed(), SimSSMIVary(), SimSSMLinGrowth(), SimSSMLinGrowthIVary(), SimSSMLinSDEFixed(), SimSSMLinSDEIVary(), SimSSMOUFixed(), SimSSMOUIVary(), SimSSMVARFixed(), SimSSMVARIVary(), TestPhi(), TestStability(), TestStationarity()
```

```
p <- 2
iota <- c(0.317, 0.230)
phi <- matrix(</pre>
  data = c(
   -0.10,
   0.05,
   0.05,
   -0.10
 ),
 nrow = p
)
sigma <- matrix(
  data = c(
    2.79,
    0.06,
    0.06,
    3.27
  nrow = p
sigma_l <- t(chol(sigma))</pre>
delta_t <- 0.10
LinSDE2SSM(
```

```
iota = iota,
phi = phi,
sigma_l = sigma_l,
delta_t = delta_t
)
```

PBSSMLinSDEFixed

Parametric Bootstrap for the Linear Stochastic Differential Equation Model using a State Space Model Parameterization (Fixed Parameters)

Description

This function simulates data from a linear stochastic differential equation model using a state-space model parameterization and fits the model using the dynr package. The process is repeated R times. It assumes that the parameters remain constant across individuals and over time. At the moment, the function only supports type = \emptyset .

Usage

```
PBSSMLinSDEFixed(
  R,
  path,
  prefix,
  n,
  time,
  delta_t = 0.1,
 mu0,
  sigma0_l,
  iota,
  phi,
  sigma_1,
  nu,
  lambda,
  theta_1,
  type = 0,
  x = NULL
  gamma = NULL,
  kappa = NULL,
 mu0_fixed = FALSE,
  sigma0_fixed = FALSE,
  alpha_level = 0.05,
  optimization_flag = TRUE,
  hessian_flag = FALSE,
  verbose = FALSE,
  weight_flag = FALSE,
```

```
debug_flag = FALSE,
perturb_flag = FALSE,
xtol_rel = 1e-07,
stopval = -9999,
ftol_rel = -1,
ftol_abs = -1,
maxeval = as.integer(-1),
maxtime = -1,
ncores = NULL,
seed = NULL
```

Arguments

R	Positive integer. Number of bootstrap samples.
path	Path to a directory to store bootstrap samples and estimates.
prefix	Character string. Prefix used for the file names for the bootstrap samples and estimates.
n	Positive integer. Number of individuals.
time	Positive integer. Number of time points.
delta_t	Numeric. Time interval (Δ_t) .
mu0	Numeric vector. Mean of initial latent variable values $(\mu_{\eta 0})$.
sigma0_l	Numeric matrix. Cholesky factorization (t(chol(sigma0))) of the covariance matrix of initial latent variable values ($\Sigma_{\eta 0}$).
iota	Numeric vector. An unobserved term that is constant over time (ι) .
phi	Numeric matrix. The drift matrix which represents the rate of change of the solution in the absence of any random fluctuations (Φ) .
sigma_l	Numeric matrix. Cholesky factorization ($t(chol(sigma))$) of the covariance matrix of volatility or randomness in the process (Σ).
nu	Numeric vector. Vector of intercept values for the measurement model (ν) .
lambda	Numeric matrix. Factor loading matrix linking the latent variables to the observed variables (Λ) .
theta_l	Numeric matrix. Cholesky factorization (t(chol(theta))) of the covariance matrix of the measurement error (Θ) .
type	Integer. State space model type. See Details for more information.
X	List. Each element of the list is a matrix of covariates for each individual i in n. The number of columns in each matrix should be equal to time.
gamma	Numeric matrix. Matrix linking the covariates to the latent variables at current time point (Γ) .
kappa	Numeric matrix. Matrix linking the covariates to the observed variables at current time point (κ) .
mu0_fixed	Logical. If mu0_fixed = TRUE, fix the initial mean vector to mu0. If mu0_fixed = FALSE, mu0 is estimated.

sigma0_fixed	Logical. If sigma0_fixed = TRUE, fix the initial covariance matrix to tcrossprod(sigma0_1). If sigma0_fixed = FALSE, sigma0 is estimated.
alpha_level	Numeric vector. Significance level α .
optimization_f	lag
	a flag (TRUE/FALSE) indicating whether optimization is to be done.
hessian_flag	a flag (TRUE/FALSE) indicating whether the Hessian matrix is to be calculated.
verbose	a flag (TRUE/FALSE) indicating whether more detailed intermediate output during the estimation process should be printed
weight_flag	a flag (TRUE/FALSE) indicating whether the negative log likelihood function should be weighted by the length of the time series for each individual
debug_flag	a flag (TRUE/FALSE) indicating whether users want additional dynr output that can be used for diagnostic purposes
perturb_flag	a flag (TRUE/FLASE) indicating whether to perturb the latent states during estimation. Only useful for ensemble forecasting.
xtol_rel	Stopping criteria option for parameter optimization. See dynr::dynr.model() for more details.
stopval	Stopping criteria option for parameter optimization. See dynr::dynr.model() for more details.
ftol_rel	Stopping criteria option for parameter optimization. See dynr.model () for more details.
ftol_abs	Stopping criteria option for parameter optimization. See dynr::dynr.model() for more details.
maxeval	Stopping criteria option for parameter optimization. See dynr::dynr.model() for more details.
maxtime	Stopping criteria option for parameter optimization. See dynr::dynr.model() for more details.
ncores	Positive integer. Number of cores to use. If ncores = NULL, use a single core. Consider using multiple cores when number of bootstrap samples R is a large value.
seed	Random seed.

Details

Type 0:

The measurement model is given by

$$\mathbf{y}_{i,t} = \mathbf{\nu} + \mathbf{\Lambda} oldsymbol{\eta}_{i,t} + oldsymbol{arepsilon}_{i,t}, \quad ext{with} \quad oldsymbol{arepsilon}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, oldsymbol{\Theta}
ight)$$

where $\mathbf{y}_{i,t}$, $\boldsymbol{\eta}_{i,t}$, and $\boldsymbol{\varepsilon}_{i,t}$ are random variables and $\boldsymbol{\nu}$, $\boldsymbol{\Lambda}$, and $\boldsymbol{\Theta}$ are model parameters. $\mathbf{y}_{i,t}$ represents a vector of observed random variables, $\boldsymbol{\eta}_{i,t}$ a vector of latent random variables, and $\boldsymbol{\varepsilon}_{i,t}$ a vector of random measurement errors, at time t and individual i. $\boldsymbol{\nu}$ denotes a vector of intercepts, $\boldsymbol{\Lambda}$ a matrix of factor loadings, and $\boldsymbol{\Theta}$ the covariance matrix of $\boldsymbol{\varepsilon}$.

An alternative representation of the measurement error is given by

$$oldsymbol{arepsilon}_{i,t} = oldsymbol{\Theta}^{rac{1}{2}} \mathbf{z}_{i,t}, \quad ext{with} \quad \mathbf{z}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \mathbf{I}
ight)$$

where $\mathbf{z}_{i,t}$ is a vector of independent standard normal random variables and $\left(\mathbf{\Theta}^{\frac{1}{2}}\right)\left(\mathbf{\Theta}^{\frac{1}{2}}\right)' = \mathbf{\Theta}$. The dynamic structure is given by

$$\mathrm{d}\boldsymbol{\eta}_{i,t} = \left(\boldsymbol{\iota} + \boldsymbol{\Phi}\boldsymbol{\eta}_{i,t}\right) \mathrm{d}t + \boldsymbol{\Sigma}^{\frac{1}{2}} \mathrm{d}\mathbf{W}_{i,t}$$

where ι is a term which is unobserved and constant over time, Φ is the drift matrix which represents the rate of change of the solution in the absence of any random fluctuations, Σ is the matrix of volatility or randomness in the process, and $\mathrm{d}W$ is a Wiener process or Brownian motion, which represents random fluctuations.

Type 1:

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \boldsymbol{\Lambda} \boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t}, \quad ext{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Theta}\right).$$

The dynamic structure is given by

$$d\boldsymbol{\eta}_{i:t} = (\boldsymbol{\iota} + \boldsymbol{\Phi} \boldsymbol{\eta}_{i:t}) dt + \boldsymbol{\Gamma} \mathbf{x}_{i:t} + \boldsymbol{\Sigma}^{\frac{1}{2}} d\mathbf{W}_{i:t}$$

where $\mathbf{x}_{i,t}$ represents a vector of covariates at time t and individual i, and Γ the coefficient matrix linking the covariates to the latent variables.

Type 2:

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \boldsymbol{\Lambda} \boldsymbol{\eta}_{i,t} + \kappa \mathbf{x}_{i,t} + \boldsymbol{\varepsilon}_{i,t}, \quad \text{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Theta}\right)$$

where κ represents the coefficient matrix linking the covariates to the observed variables.

The dynamic structure is given by

$$d\boldsymbol{\eta}_{i,t} = (\boldsymbol{\iota} + \boldsymbol{\Phi} \boldsymbol{\eta}_{i,t}) dt + \Gamma \mathbf{x}_{i,t} + \boldsymbol{\Sigma}^{\frac{1}{2}} d\mathbf{W}_{i,t}.$$

State Space Parameterization:

The state space parameters as a function of the linear stochastic differential equation model parameters are given by

$$\boldsymbol{\beta}_{\Delta t_{l.}} = \exp\left(\Delta t \boldsymbol{\Phi}\right)$$

$$oldsymbol{lpha}_{\Delta t_{l_i}} = oldsymbol{\Phi}^{-1} \left(oldsymbol{eta} - \mathbf{I}_p
ight) oldsymbol{\iota}$$

$$\operatorname{vec}\left(\mathbf{\Psi}_{\Delta t_{l_{i}}}\right) = \left[\left(\mathbf{\Phi} \otimes \mathbf{I}_{p}\right) + \left(\mathbf{I}_{p} \otimes \mathbf{\Phi}\right)\right] \left[\exp\left(\left[\left(\mathbf{\Phi} \otimes \mathbf{I}_{p}\right) + \left(\mathbf{I}_{p} \otimes \mathbf{\Phi}\right)\right] \Delta t\right) - \mathbf{I}_{p \times p}\right] \operatorname{vec}\left(\mathbf{\Sigma}\right)$$

where p is the number of latent variables and Δt is the time interval.

Value

Returns an object of class statespacepb which is a list with the following elements:

call Function call.

args Function arguments.

thetahatstar Sampling distribution of $\hat{\theta}$.

vcov Sampling variance-covariance matrix of $\hat{\theta}$.

est Vector of estimated $\hat{\theta}$.

fun Function used ("PBSSMLinSDEFixed").

Author(s)

Ivan Jacob Agaloos Pesigan

References

Chow, S.-M., Ho, M. R., Hamaker, E. L., & Dolan, C. V. (2010). Equivalence and differences between structural equation modeling and state-space modeling techniques. *Structural Equation Modeling: A Multidisciplinary Journal*, 17(2), 303–332. doi:10.1080/10705511003661553

Chow, S.-M., Losardo, D., Park, J., & Molenaar, P. C. M. (2023). Continuous-time dynamic models: Connections to structural equation models and other discrete-time models. In R. H. Hoyle (Ed.), Handbook of structural equation modeling (2nd ed.). The Guilford Press.

Harvey, A. C. (1990). Forecasting, structural time series models and the Kalman filter. Cambridge University Press. doi:10.1017/cbo9781107049994

See Also

Other Simulation of State Space Models Data Functions: LinSDE2SSM(), PBSSMOUFixed(), PBSSMVARFixed(), SimBetaN(), SimPhiN(), SimSSMFixed(), SimSSMIVary(), SimSSMLinGrowth(), SimSSMLinGrowthIVary(), SimSSMLinSDEFixed(), SimSSMLinSDEIVary(), SimSSMOUFixed(), SimSSMOUIVary(), SimSSMVARFixed(), SimSSMVARIVary(), TestPhi(), TestStability(), TestStationarity()

```
## Not run:
# prepare parameters
## number of individuals
n <- 5
## time points
time <- 50
delta_t <- 0.10
## dynamic structure
p <- 2
mu0 < -c(-3.0, 1.5)
sigma0 <- 0.001 * diag(p)
sigma0_l <- t(chol(sigma0))</pre>
iota <- c(0.317, 0.230)
phi <- matrix(</pre>
  data = c(
    -0.10,
    0.05,
    0.05,
    -0.10
  ),
  nrow = p
sigma <- matrix(</pre>
  data = c(
    2.79,
    0.06,
    0.06,
```

```
3.27
  ),
  nrow = p
)
sigma_l <- t(chol(sigma))</pre>
## measurement model
k <- 2
nu \leftarrow rep(x = 0, times = k)
lambda <- diag(k)</pre>
theta <-0.001 * diag(k)
theta_l <- t(chol(theta))</pre>
pb <- PBSSMLinSDEFixed(</pre>
  R = 10L
  path = getwd(),
  prefix = "lse",
  n = n,
  time = time,
  delta_t = delta_t,
  mu0 = mu0,
  sigma0_1 = sigma0_1,
  iota = iota,
  phi = phi,
  sigma_l = sigma_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 0,
  ncores = parallel::detectCores() - 1,
  seed = 42
)
print(pb)
summary(pb)
vcov(pb)
coef(pb)
confint(pb)
## End(Not run)
```

PBSSMOUFixed

Parametric Bootstrap for the Ornstein-Uhlenbeck Model using a State Space Model Parameterization (Fixed Parameters)

Description

This function simulates data from a Ornstein–Uhlenbeck (OU) model using a state-space model parameterization and fits the model using the dynr package. The process is repeated R times. It assumes that the parameters remain constant across individuals and over time. At the moment, the function only supports type = \emptyset .

Usage

```
PBSSMOUFixed(
 R,
  path,
  prefix,
 n,
  time,
  delta_t = 0.1,
 mu0,
 sigma0_l,
 mu,
 phi,
  sigma_l,
  nu,
  lambda,
  theta_l,
  type = 0,
  x = NULL,
  gamma = NULL,
  kappa = NULL,
 mu0_fixed = FALSE,
  sigma0_fixed = FALSE,
  alpha_level = 0.05,
  optimization_flag = TRUE,
  hessian_flag = FALSE,
  verbose = FALSE,
 weight_flag = FALSE,
  debug_flag = FALSE,
 perturb_flag = FALSE,
  xtol_rel = 1e-07,
  stopval = -9999,
  ftol_rel = -1,
  ftol_abs = -1,
 maxeval = as.integer(-1),
 maxtime = -1,
 ncores = NULL,
  seed = NULL
)
```

Arguments

R	Positive integer. Number of bootstrap samples.
path	Path to a directory to store bootstrap samples and estimates.
prefix	Character string. Prefix used for the file names for the bootstrap samples and estimates.
n	Positive integer. Number of individuals.
time	Positive integer. Number of time points.

delta_t	Numeric. Time interval (Δ_t) .
mu0	Numeric vector. Mean of initial latent variable values $(\mu_{\eta 0})$.
sigma0_l	Numeric matrix. Cholesky factorization (t(chol(sigma0))) of the covariance matrix of initial latent variable values ($\Sigma_{\eta 0}$).
mu	Numeric vector. The long-term mean or equilibrium level (μ) .
phi	Numeric matrix. The drift matrix which represents the rate of change of the solution in the absence of any random fluctuations (Φ). It also represents the rate of mean reversion, determining how quickly the variable returns to its mean.
sigma_l	Numeric matrix. Cholesky factorization (t(chol(sigma))) of the covariance matrix of volatility or randomness in the process (Σ) .
nu	Numeric vector. Vector of intercept values for the measurement model (ν) .
lambda	Numeric matrix. Factor loading matrix linking the latent variables to the observed variables (Λ) .
theta_l	Numeric matrix. Cholesky factorization ($t(chol(theta))$) of the covariance matrix of the measurement error (Θ).
type	Integer. State space model type. See Details for more information.
X	List. Each element of the list is a matrix of covariates for each individual i in n. The number of columns in each matrix should be equal to time.
gamma	Numeric matrix. Matrix linking the covariates to the latent variables at current time point (Γ) .
kappa	Numeric matrix. Matrix linking the covariates to the observed variables at current time point (κ).
mu0_fixed	Logical. If mu0_fixed = TRUE, fix the initial mean vector to mu0. If mu0_fixed = FALSE, mu0 is estimated.
sigma0_fixed	Logical. If sigma0_fixed = TRUE, fix the initial covariance matrix to tcrossprod(sigma0_1). If sigma0_fixed = FALSE, sigma0 is estimated.
alpha_level	Numeric vector. Significance level α .
optimization_f	
	a flag (TRUE/FALSE) indicating whether optimization is to be done.
hessian_flag	a flag (TRUE/FALSE) indicating whether the Hessian matrix is to be calculated.
verbose	a flag (TRUE/FALSE) indicating whether more detailed intermediate output during the estimation process should be printed
weight_flag	a flag (TRUE/FALSE) indicating whether the negative log likelihood function should be weighted by the length of the time series for each individual
debug_flag	a flag (TRUE/FALSE) indicating whether users want additional dynr output that can be used for diagnostic purposes
perturb_flag	a flag (TRUE/FLASE) indicating whether to perturb the latent states during estimation. Only useful for ensemble forecasting.
xtol_rel	Stopping criteria option for parameter optimization. See dynr.model () for more details.
stopval	Stopping criteria option for parameter optimization. See dynr.model () for more details.

ftol_rel	Stopping criteria option for parameter optimization. See dynr::dynr.model() for more details.
ftol_abs	Stopping criteria option for parameter optimization. See dynr::dynr.model() for more details.
maxeval	Stopping criteria option for parameter optimization. See dynr::dynr.model() for more details.
maxtime	Stopping criteria option for parameter optimization. See dynr::dynr.model() for more details.
ncores	Positive integer. Number of cores to use. If ncores = NULL, use a single core. Consider using multiple cores when number of bootstrap samples R is a large value.
seed	Random seed.

Details

Type 0:

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \boldsymbol{\Lambda} \boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t}, \quad ext{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Theta}\right)$$

where $\mathbf{y}_{i,t}$, $\eta_{i,t}$, and $\varepsilon_{i,t}$ are random variables and ν , Λ , and Θ are model parameters. $\mathbf{y}_{i,t}$ represents a vector of observed random variables, $\eta_{i,t}$ a vector of latent random variables, and $\varepsilon_{i,t}$ a vector of random measurement errors, at time t and individual t. ν denotes a vector of intercepts, Λ a matrix of factor loadings, and Θ the covariance matrix of ε .

An alternative representation of the measurement error is given by

$$\boldsymbol{\varepsilon}_{i,t} = \boldsymbol{\Theta}^{\frac{1}{2}} \mathbf{z}_{i,t}, \quad \text{with} \quad \mathbf{z}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \mathbf{I}\right)$$

where $\mathbf{z}_{i,t}$ is a vector of independent standard normal random variables and $\left(\Theta^{\frac{1}{2}}\right)\left(\Theta^{\frac{1}{2}}\right)' = \Theta$. The dynamic structure is given by

$$\mathrm{d}oldsymbol{\eta}_{i,t} = oldsymbol{\Phi}\left(oldsymbol{\eta}_{i,t} - oldsymbol{\mu}
ight)\mathrm{d}t + oldsymbol{\Sigma}^{rac{1}{2}}\mathrm{d}\mathbf{W}_{i,t}$$

where μ is the long-term mean or equilibrium level, Φ is the rate of mean reversion, determining how quickly the variable returns to its mean, Σ is the matrix of volatility or randomness in the process, and dW is a Wiener process or Brownian motion, which represents random fluctuations.

Type 1:

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \boldsymbol{\Lambda} \boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t}, \quad ext{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Theta}\right).$$

The dynamic structure is given by

$$\mathrm{d} oldsymbol{\eta}_{i,t} = oldsymbol{\Phi} \left(oldsymbol{\eta}_{i,t} - oldsymbol{\mu}
ight) \mathrm{d} t + oldsymbol{\Gamma} \mathbf{x}_{i,t} + oldsymbol{\Sigma}^{rac{1}{2}} \mathrm{d} \mathbf{W}_{i,t}$$

where $\mathbf{x}_{i,t}$ represents a vector of covariates at time t and individual i, and Γ the coefficient matrix linking the covariates to the latent variables.

Type 2:

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \boldsymbol{\Lambda} \boldsymbol{\eta}_{i,t} + \kappa \mathbf{x}_{i,t} + \boldsymbol{\varepsilon}_{i,t}, \quad ext{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Theta}\right)$$

where κ represents the coefficient matrix linking the covariates to the observed variables. The dynamic structure is given by

$$d\boldsymbol{\eta}_{i,t} = \boldsymbol{\Phi} \left(\boldsymbol{\eta}_{i,t} - \boldsymbol{\mu} \right) dt + \boldsymbol{\Gamma} \mathbf{x}_{i,t} + \boldsymbol{\Sigma}^{\frac{1}{2}} d\mathbf{W}_{i,t}.$$

The OU model as a linear stochastic differential equation model:

The OU model is a first-order linear stochastic differential equation model in the form of

$$\mathrm{d} oldsymbol{\eta}_{i,t} = \left(oldsymbol{\iota} + oldsymbol{\Phi} oldsymbol{\eta}_{i,t}
ight) \mathrm{d} t + oldsymbol{\Sigma}^{rac{1}{2}} \mathrm{d} \mathbf{W}_{i,t}$$

where $\mu = -\Phi^{-1}\iota$ and, equivalently $\iota = -\Phi\mu$.

Value

Returns an object of class statespacepb which is a list with the following elements:

call Function call.

args Function arguments.

thetahatstar Sampling distribution of $\hat{\theta}$.

vcov Sampling variance-covariance matrix of $\hat{\theta}$.

est Vector of estimated $\hat{\theta}$.

fun Function used ("PBSSMOUFixed").

Author(s)

Ivan Jacob Agaloos Pesigan

References

Chow, S.-M., Ho, M. R., Hamaker, E. L., & Dolan, C. V. (2010). Equivalence and differences between structural equation modeling and state-space modeling techniques. *Structural Equation Modeling: A Multidisciplinary Journal*, 17(2), 303–332. doi:10.1080/10705511003661553

Chow, S.-M., Losardo, D., Park, J., & Molenaar, P. C. M. (2023). Continuous-time dynamic models: Connections to structural equation models and other discrete-time models. In R. H. Hoyle (Ed.), Handbook of structural equation modeling (2nd ed.). The Guilford Press.

Harvey, A. C. (1990). Forecasting, structural time series models and the Kalman filter. Cambridge University Press. doi:10.1017/cbo9781107049994

Oravecz, Z., Tuerlinckx, F., & Vandekerckhove, J. (2011). A hierarchical latent stochastic differential equation model for affective dynamics. Psychological Methods, 16 (4), 468–490. doi:10.1037/a0024375

Uhlenbeck, G. E., & Ornstein, L. S. (1930). On the theory of the brownian motion. Physical Review, 36 (5), 823–841. doi:10.1103/physrev.36.823

See Also

```
Other Simulation of State Space Models Data Functions: LinSDE2SSM(), PBSSMLinSDEFixed(), PBSSMVARFixed(), SimBetaN(), SimPhiN(), SimSSMFixed(), SimSSMIVary(), SimSSMLinGrowth(), SimSSMLinGrowthIVary(), SimSSMLinSDEFixed(), SimSSMLinSDEIVary(), SimSSMOUFixed(), SimSSMOUIVary(), SimSSMVARFixed(), SimSSMVARIVary(), TestPhi(), TestStability(), TestStationarity()
```

```
## Not run:
# prepare parameters
## number of individuals
n <- 5
## time points
time <- 50
delta_t <- 0.10
## dynamic structure
p <- 2
mu0 <- c(-3.0, 1.5)
sigma0 <- 0.001 * diag(p)
sigma0_l <- t(chol(sigma0))</pre>
mu < -c(5.76, 5.18)
phi <- matrix(</pre>
  data = c(
    -0.10,
    0.05,
    0.05,
    -0.10
  ),
  nrow = p
)
sigma <- matrix(</pre>
  data = c(
    2.79,
    0.06,
    0.06,
    3.27
  ),
  nrow = p
)
sigma_l <- t(chol(sigma))</pre>
## measurement model
nu \leftarrow rep(x = 0, times = k)
lambda <- diag(k)</pre>
theta <-0.001 * diag(k)
theta_l <- t(chol(theta))</pre>
pb <- PBSSMOUFixed(</pre>
  R = 10L,
  path = getwd(),
  prefix = "ou",
  n = n,
```

```
time = time,
 delta_t = delta_t,
 mu0 = mu0,
 sigma0_1 = sigma0_1,
 mu = mu,
 phi = phi,
 sigma_l = sigma_l,
 nu = nu,
 lambda = lambda,
 theta_1 = theta_1,
 type = 0,
 ncores = parallel::detectCores() - 1,
 seed = 42
print(pb)
summary(pb)
vcov(pb)
coef(pb)
confint(pb)
## End(Not run)
```

PBSSMVARFixed

Parametric Bootstrap for the Vector Autoregressive Model (Fixed Parameters)

Description

This function simulates data from a vector autoregressive model using a state-space model parameterization and fits the model using the dynr package. The process is repeated R times. It assumes that the parameters remain constant across individuals and over time. At the moment, the function only supports type = \emptyset .

Usage

```
PBSSMVARFixed(
    R,
    path,
    prefix,
    n,
    time,
    mu0,
    sigma0_l,
    alpha,
    beta,
    psi_l,
    type = 0,
    x = NULL,
```

```
gamma = NULL,
 mu0_fixed = FALSE,
 sigma0_fixed = FALSE,
 alpha_level = 0.05,
 optimization_flag = TRUE,
 hessian_flag = FALSE,
 verbose = FALSE,
 weight_flag = FALSE,
 debug_flag = FALSE,
 perturb_flag = FALSE,
 xtol_rel = 1e-07,
  stopval = -9999,
  ftol_rel = -1,
  ftol_abs = -1,
 maxeval = as.integer(-1),
 maxtime = -1,
 ncores = NULL,
 seed = NULL
)
```

Arguments R

path	Path to a directory to store bootstrap samples and estimates.
prefix	Character string. Prefix used for the file names for the bootstrap samples and estimates.
n	Positive integer. Number of individuals.
time	Positive integer. Number of time points.
mu0	Numeric vector. Mean of initial latent variable values $(\mu_{\eta 0})$.
sigma0_l	Numeric matrix. Cholesky factorization (t(chol(sigma0))) of the covariance matrix of initial latent variable values ($\Sigma_{\eta 0}$).
alpha	Numeric vector. Vector of constant values for the dynamic model (α) .
beta	Numeric matrix. Transition matrix relating the values of the latent variables at the previous to the current time point (β) .
psi_l	Numeric matrix. Cholesky factorization $(t(chol(psi)))$ of the covariance matrix of the process noise (Ψ) .
type	Integer. State space model type. See Details for more information.
X	List. Each element of the list is a matrix of covariates for each individual i in n. The number of columns in each matrix should be equal to time.
gamma	Numeric matrix. Matrix linking the covariates to the latent variables at current time point (Γ).
mu0_fixed	Logical. If mu0_fixed = TRUE, fix the initial mean vector to mu0. If mu0_fixed = FALSE, mu0 is estimated.
sigma0_fixed	Logical. If sigma0_fixed = TRUE, fix the initial covariance matrix to tcrossprod(sigma0_1). If sigma0_fixed = FALSE, sigma0 is estimated.

Positive integer. Number of bootstrap samples.

alpha_level	Numeric vector. Significance level α .
optimization_flag	
	a flag (TRUE/FALSE) indicating whether optimization is to be done.
hessian_flag	a flag (TRUE/FALSE) indicating whether the Hessian matrix is to be calculated.
verbose	a flag (TRUE/FALSE) indicating whether more detailed intermediate output during the estimation process should be printed
weight_flag	a flag (TRUE/FALSE) indicating whether the negative log likelihood function should be weighted by the length of the time series for each individual
debug_flag	a flag (TRUE/FALSE) indicating whether users want additional dynr output that can be used for diagnostic purposes
perturb_flag	a flag (TRUE/FLASE) indicating whether to perturb the latent states during estimation. Only useful for ensemble forecasting.
xtol_rel	Stopping criteria option for parameter optimization. See dynr.model () for more details.
stopval	Stopping criteria option for parameter optimization. See dynr.model () for more details.
ftol_rel	Stopping criteria option for parameter optimization. See dynr::dynr.model() for more details.
ftol_abs	Stopping criteria option for parameter optimization. See dynr::dynr.model() for more details.
maxeval	Stopping criteria option for parameter optimization. See dynr::dynr.model() for more details.
maxtime	Stopping criteria option for parameter optimization. See dynr::dynr.model() for more details.
ncores	Positive integer. Number of cores to use. If ncores = NULL, use a single core. Consider using multiple cores when number of bootstrap samples R is a large value.
seed	Random seed.

Details

Type 0:

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\eta}_{i,t}$$

where $\mathbf{y}_{i,t}$ represents a vector of observed variables and $\eta_{i,t}$ a vector of latent variables for individual i and time t. Since the observed and latent variables are equal, we only generate data from the dynamic structure.

The dynamic structure is given by

$$oldsymbol{\eta}_{i,t} = oldsymbol{lpha} + oldsymbol{eta} oldsymbol{\eta}_{i,t-1} + oldsymbol{\zeta}_{i,t}, \quad ext{with} \quad oldsymbol{\zeta}_{i,t} \sim \mathcal{N}\left(oldsymbol{0}, oldsymbol{\Psi}
ight)$$

where $\eta_{i,t}$, $\eta_{i,t-1}$, and $\zeta_{i,t}$ are random variables, and α , β , and Ψ are model parameters. Here, $\eta_{i,t}$ is a vector of latent variables at time t and individual i, $\eta_{i,t-1}$ represents a vector of latent variables at time t-1 and individual i, and $\zeta_{i,t}$ represents a vector of dynamic noise at time t and

individual i. α denotes a vector of intercepts, β a matrix of autoregression and cross regression coefficients, and Ψ the covariance matrix of $\zeta_{i,t}$.

An alternative representation of the dynamic noise is given by

$$oldsymbol{\zeta}_{i.t} = oldsymbol{\Psi}^{rac{1}{2}} oldsymbol{\mathbf{z}}_{i.t}, \quad ext{with} \quad oldsymbol{\mathbf{z}}_{i.t} \sim \mathcal{N}\left(oldsymbol{0}, oldsymbol{\mathbf{I}}
ight)$$

where
$$\left(\Psi^{rac{1}{2}}
ight)\left(\Psi^{rac{1}{2}}
ight)'=\Psi.$$

Type 1:

The measurement model is given by

$$\mathbf{y}_{i,t} = oldsymbol{\eta}_{i,t}.$$

The dynamic structure is given by

$$oldsymbol{\eta}_{i,t} = oldsymbol{lpha} + oldsymbol{eta} oldsymbol{\eta}_{i,t-1} + oldsymbol{\Gamma} \mathbf{x}_{i,t} + oldsymbol{\zeta}_{i,t}, \quad ext{with} \quad oldsymbol{\zeta}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, oldsymbol{\Psi}
ight)$$

where $\mathbf{x}_{i,t}$ represents a vector of covariates at time t and individual i, and Γ the coefficient matrix linking the covariates to the latent variables.

Value

Returns an object of class statespacepb which is a list with the following elements:

call Function call.

args Function arguments.

thetahatstar Sampling distribution of $\hat{\theta}$.

vcov Sampling variance-covariance matrix of $\hat{\theta}$.

est Vector of estimated $\hat{\theta}$.

fun Function used ("PBSSMVARFixed").

Author(s)

Ivan Jacob Agaloos Pesigan

References

Chow, S.-M., Ho, M. R., Hamaker, E. L., & Dolan, C. V. (2010). Equivalence and differences between structural equation modeling and state-space modeling techniques. *Structural Equation Modeling: A Multidisciplinary Journal*, 17(2), 303–332. doi:10.1080/10705511003661553

See Also

Other Simulation of State Space Models Data Functions: LinSDE2SSM(), PBSSMLinSDEFixed(), PBSSMOUFixed(), SimBetaN(), SimPhiN(), SimSSMFixed(), SimSSMIVary(), SimSSMLinGrowth(), SimSSMLinGrowthIVary(), SimSSMLinSDEFixed(), SimSSMLinSDEIVary(), SimSSMOUFixed(), SimSSMOUIVary(), SimSSMVARFixed(), SimSSMVARIVary(), TestPhi(), TestStability(), TestStationarity()

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Examples

```
## Not run:
# prepare parameters
## number of individuals
n <- 5
## time points
time <- 50
## dynamic structure
p <- 3
mu0 < -rep(x = 0, times = p)
sigma0 <- 0.001 * diag(p)
sigma0_l <- t(chol(sigma0))</pre>
alpha <- rep(x = 0, times = p)
beta <- 0.50 * diag(p)
psi <- 0.001 * diag(p)
psi_l <- t(chol(psi))</pre>
pb <- PBSSMVARFixed(</pre>
  R = 10L
  path = getwd(),
  prefix = "var",
  n = n,
  time = time,
  mu0 = mu0,
  sigma0_1 = sigma0_1,
  alpha = alpha,
  beta = beta,
  psi_l = psi_l,
  type = 0,
  ncores = parallel::detectCores() - 1,
  seed = 42
print(pb)
summary(pb)
vcov(pb)
coef(pb)
confint(pb)
## End(Not run)
```

plot.simstatespace

Plot Method for an Object of Class simstatespace

Description

Plot Method for an Object of Class simstatespace

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Usage

```
## S3 method for class 'simstatespace'
plot(x, id = NULL, time = NULL, eta = FALSE, type = "b", ...)
```

Arguments

x	Object of class simstatespace.
id	Numeric vector. Optional id numbers to plot. If id = NULL, plot all available data.
time	Numeric vector. Optional time points to plot. If time = NULL, plot all available data.
eta	Logical. If eta = TRUE, plot the latent variables. If eta = FALSE, plot the observed variables.
type	Character indicating the type of plotting; actually any of the types as in plot.default().
	Additional arguments.

Author(s)

Ivan Jacob Agaloos Pesigan

```
# prepare parameters
set.seed(42)
## number of individuals
n <- 5
## time points
time <- 50
## dynamic structure
p <- 3
mu0 < -rep(x = 0, times = p)
sigma0 <- diag(p)</pre>
sigma0_l <- t(chol(sigma0))</pre>
alpha <- rep(x = 0, times = p)
beta <- 0.50 * diag(p)
psi <- diag(p)</pre>
psi_l <- t(chol(psi))</pre>
## measurement model
k <- 3
nu \leftarrow rep(x = 0, times = k)
lambda <- diag(k)</pre>
theta <-0.50 * diag(k)
theta_l <- t(chol(theta))</pre>
## covariates
j <- 2
x <- lapply(
  X = seq_len(n),
  FUN = function(i) {
    matrix(
```

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```
data = stats::rnorm(n = time * j),
      nrow = j,
      ncol = time
   )
 }
)
gamma \leftarrow diag(x = 0.10, nrow = p, ncol = j)
kappa <- diag(x = 0.10, nrow = k, ncol = j)
# Type 0
ssm <- SimSSMFixed(</pre>
 n = n,
  time = time,
 mu0 = mu0,
  sigma0_1 = sigma0_1,
  alpha = alpha,
  beta = beta,
  psi_l = psi_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 0
)
plot(ssm)
plot(ssm, id = 1:3, time = 0:9)
# Type 1
ssm <- SimSSMFixed(</pre>
 n = n,
 time = time,
  mu0 = mu0,
  sigma0_1 = sigma0_1,
  alpha = alpha,
  beta = beta,
  psi_l = psi_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 1,
  x = x,
  gamma = gamma
plot(ssm)
plot(ssm, id = 1:3, time = 0:9)
# Type 2
ssm <- SimSSMFixed(</pre>
 n = n,
 time = time,
  mu0 = mu0,
  sigma0_1 = sigma0_1,
```

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```
alpha = alpha,
beta = beta,
psi_l = psi_l,
nu = nu,
lambda = lambda,
theta_l = theta_l,
type = 2,
x = x,
gamma = gamma,
kappa = kappa
)

plot(ssm)
plot(ssm, id = 1:3, time = 0:9)
```

print.simstatespace

Print Method for an Object of Class simstatespace

Description

Print Method for an Object of Class simstatespace

Usage

```
## S3 method for class 'simstatespace' print(x, ...)
```

Arguments

x Object of Class simstatespace.

.. Additional arguments.

Value

Prints simulated data in long format.

Author(s)

Ivan Jacob Agaloos Pesigan

```
# prepare parameters
set.seed(42)
## number of individuals
n <- 5
## time points
time <- 50</pre>
```

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```
## dynamic structure
p <- 3
mu0 \leftarrow rep(x = 0, times = p)
sigma0 <- diag(p)</pre>
sigma0_l <- t(chol(sigma0))</pre>
alpha <- rep(x = 0, times = p)
beta <- 0.50 * diag(p)
psi <- diag(p)</pre>
psi_l <- t(chol(psi))</pre>
## measurement model
k <- 3
nu \leftarrow rep(x = 0, times = k)
lambda <- diag(k)</pre>
theta <-0.50 * diag(k)
theta_l \leftarrow t(chol(theta))
## covariates
j <- 2
x <- lapply(
  X = seq_len(n),
  FUN = function(i) {
    matrix(
      data = stats::rnorm(n = time * j),
      nrow = j,
      ncol = time
  }
gamma \leftarrow diag(x = 0.10, nrow = p, ncol = j)
kappa \leftarrow diag(x = 0.10, nrow = k, ncol = j)
# Type 0
ssm <- SimSSMFixed(</pre>
  n = n,
  time = time,
  mu0 = mu0,
  sigma0_1 = sigma0_1,
  alpha = alpha,
  beta = beta,
  psi_l = psi_l,
  nu = nu,
  lambda = lambda,
  theta_1 = theta_1,
  type = 0
)
print(ssm)
# Type 1
ssm <- SimSSMFixed(</pre>
  n = n,
  time = time,
  mu0 = mu0,
  sigma0_1 = sigma0_1,
```

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```
alpha = alpha,
  beta = beta,
  psi_l = psi_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 1,
  x = x,
  gamma = gamma
)
print(ssm)
# Type 2
ssm <- SimSSMFixed(</pre>
  n = n,
  time = time,
  mu0 = mu0,
  sigma0_1 = sigma0_1,
  alpha = alpha,
  beta = beta,
  psi_l = psi_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 2,
  x = x,
  gamma = gamma,
  kappa = kappa
)
print(ssm)
```

print.statespacepb

Print Method for an Object of Class statespacepb

Description

Print Method for an Object of Class statespacepb

Usage

```
## S3 method for class 'statespacepb'
print(x, alpha = NULL, type = "pc", digits = 4, ...)
```

Arguments

Х

Object of Class statespacepb.

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alpha	Numeric vector. Significance level α . If alpha = NULL, use the argument alpha used in x.
type	Charater string. Confidence interval type, that is, type = "pc" for percentile; type = "bc" for bias corrected.
digits	Digits to print.
	additional arguments.

Value

Prints a matrix of estimates, standard errors, number of bootstrap replications, and confidence intervals.

Author(s)

Ivan Jacob Agaloos Pesigan

SimBetaN	Simulate Transition Matrices from the Multivariate Normal Distribution

Description

This function simulates random transition matrices from the multivariate normal distribution. The function ensures that the generated transition matrices are stationary using TestStationarity().

Usage

```
SimBetaN(n, beta, vcov_beta_vec_l)
```

Arguments

n Positive integer. Number of replications. beta Numeric matrix. The transition matrix (β) . vcov_beta_vec_1 Numeric matrix. Cholesky factorization (t(chol(vcov_beta_vec))) of the

Numeric matrix. Cholesky factorization (t(chol(vcov_beta_vec))) of the sampling variance-covariance matrix $vec(\beta)$.

Author(s)

Ivan Jacob Agaloos Pesigan

See Also

```
Other Simulation of State Space Models Data Functions: LinSDE2SSM(), PBSSMLinSDEFixed(), PBSSMOUFixed(), PBSSMVARFixed(), SimPhiN(), SimSSMFixed(), SimSSMIVary(), SimSSMLinGrowth(), SimSSMLinGrowthIVary(), SimSSMLinSDEFixed(), SimSSMLinSDEIVary(), SimSSMOUFixed(), SimSSMOUIVary(), SimSSMVARFixed(), SimSSMVARIVary(), TestPhi(), TestStability(), TestStationarity()
```

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Examples

```
n <- 10
beta <- matrix(
    data = c(
        0.7, 0.5, -0.1,
        0.0, 0.6, 0.4,
        0, 0, 0.5
    ),
    nrow = 3
)
vcov_beta_vec_l <- t(chol(0.001 * diag(9)))
SimBetaN(n = n, beta = beta, vcov_beta_vec_l = vcov_beta_vec_l)</pre>
```

SimPhiN

Simulate Random Drift Matrices from the Multivariate Normal Distribution

Description

This function simulates random drift matrices from the multivariate normal distribution. The function ensures that the generated drift matrices are stable using TestPhi().

Usage

```
SimPhiN(n, phi, vcov_phi_vec_1)
```

Arguments

```
n Positive integer. Number of replications.

phi Numeric matrix. The drift matrix (\Phi).

vcov_phi_vec_1 Numeric matrix. Cholesky factorization (t(chol(vcov_phi_vec))) of the sampling variance-covariance matrix vec (\Phi).
```

Author(s)

Ivan Jacob Agaloos Pesigan

See Also

```
Other Simulation of State Space Models Data Functions: LinSDE2SSM(), PBSSMLinSDEFixed(), PBSSMOUFixed(), PBSSMVARFixed(), SimBetaN(), SimSSMFixed(), SimSSMIVary(), SimSSMLinGrowth(), SimSSMLinGrowthIVary(), SimSSMLinSDEFixed(), SimSSMLinSDEIVary(), SimSSMOUFixed(), SimSSMOUIVary(), SimSSMVARFixed(), SimSSMVARIVary(), TestPhi(), TestStability(), TestStationarity()
```

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Examples

```
n <- 10
phi <- matrix(
    data = c(
        -0.357, 0.771, -0.450,
        0.0, -0.511, 0.729,
        0, 0, -0.693
    ),
    nrow = 3
)
vcov_phi_vec_l <- t(chol(0.001 * diag(9)))
SimPhiN(n = n, phi = phi, vcov_phi_vec_l = vcov_phi_vec_l)</pre>
```

SimSSMFixed

Simulate Data from a State Space Model (Fixed Parameters)

Description

This function simulates data using a state space model. It assumes that the parameters remain constant across individuals and over time.

Usage

```
SimSSMFixed(
  n,
  time,
  delta_t = 1,
 mu0,
  sigma0_1,
  alpha,
 beta,
  psi_l,
  nu,
  lambda,
  theta_1,
  type = 0,
  x = NULL,
 gamma = NULL,
 kappa = NULL
)
```

Arguments

n Positive integer. Number of individuals.time Positive integer. Number of time points.

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delta_t	Numeric. Time interval. The default value is 1.0 with an option to use a numeric value for the discretized state space model parameterization of the linear stochastic differential equation model.
mu0	Numeric vector. Mean of initial latent variable values $(\mu_{\eta 0})$.
sigma0_l	Numeric matrix. Cholesky factorization (t(chol(sigma0))) of the covariance matrix of initial latent variable values ($\Sigma_{\eta 0}$).
alpha	Numeric vector. Vector of constant values for the dynamic model (α) .
beta	Numeric matrix. Transition matrix relating the values of the latent variables at the previous to the current time point (β) .
psi_l	Numeric matrix. Cholesky factorization (t(chol(psi))) of the covariance matrix of the process noise (Ψ).
nu	Numeric vector. Vector of intercept values for the measurement model (ν) .
lambda	Numeric matrix. Factor loading matrix linking the latent variables to the observed variables (Λ) .
theta_l	Numeric matrix. Cholesky factorization (t(chol(theta))) of the covariance matrix of the measurement error (Θ) .
type	Integer. State space model type. See Details for more information.
X	List. Each element of the list is a matrix of covariates for each individual i in n. The number of columns in each matrix should be equal to time.
gamma	Numeric matrix. Matrix linking the covariates to the latent variables at current time point (Γ) .
kappa	Numeric matrix. Matrix linking the covariates to the observed variables at current time point (κ) .

Details

Type 0:

The measurement model is given by

$$\mathbf{y}_{i,t} = \mathbf{\nu} + \mathbf{\Lambda} \boldsymbol{\eta}_{i,t} + \boldsymbol{arepsilon}_{i,t}, \quad ext{with} \quad \boldsymbol{arepsilon}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \mathbf{\Theta}
ight)$$

where $\mathbf{y}_{i,t}$, $\eta_{i,t}$, and $\varepsilon_{i,t}$ are random variables and ν , Λ , and Θ are model parameters. $\mathbf{y}_{i,t}$ represents a vector of observed random variables, $\eta_{i,t}$ a vector of latent random variables, and $\varepsilon_{i,t}$ a vector of random measurement errors, at time t and individual t. ν denotes a vector of intercepts, Λ a matrix of factor loadings, and Θ the covariance matrix of ε .

An alternative representation of the measurement error is given by

$$\boldsymbol{\varepsilon}_{i,t} = \boldsymbol{\Theta}^{\frac{1}{2}} \mathbf{z}_{i,t}, \quad \text{with} \quad \mathbf{z}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \mathbf{I}\right)$$

where $\mathbf{z}_{i,t}$ is a vector of independent standard normal random variables and $\left(\Theta^{\frac{1}{2}}\right)\left(\Theta^{\frac{1}{2}}\right)' = \Theta$. The dynamic structure is given by

$$oldsymbol{\eta}_{i,t} = oldsymbol{lpha} + oldsymbol{eta} oldsymbol{\eta}_{i,t-1} + oldsymbol{\zeta}_{i,t}, \quad ext{with} \quad oldsymbol{\zeta}_{i,t} \sim \mathcal{N}\left(oldsymbol{0}, oldsymbol{\Psi}
ight)$$

where $\eta_{i,t}$, $\eta_{i,t-1}$, and $\zeta_{i,t}$ are random variables, and α , β , and Ψ are model parameters. Here, $\eta_{i,t}$ is a vector of latent variables at time t and individual i, $\eta_{i,t-1}$ represents a vector of latent

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variables at time t-1 and individual i, and $\zeta_{i,t}$ represents a vector of dynamic noise at time t and individual i. α denotes a vector of intercepts, β a matrix of autoregression and cross regression coefficients, and Ψ the covariance matrix of $\zeta_{i,t}$.

An alternative representation of the dynamic noise is given by

$$oldsymbol{\zeta}_{i.t} = oldsymbol{\Psi}^{rac{1}{2}} oldsymbol{\mathbf{z}}_{i.t}, \quad ext{with} \quad oldsymbol{\mathbf{z}}_{i.t} \sim \mathcal{N}\left(oldsymbol{0}, oldsymbol{\mathbf{I}}
ight)$$

where
$$\left(\Psi^{rac{1}{2}}
ight)\left(\Psi^{rac{1}{2}}
ight)'=\Psi.$$

Type 1:

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \boldsymbol{\Lambda} \boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t}, \quad ext{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Theta}\right).$$

The dynamic structure is given by

$$oldsymbol{\eta}_{i,t} = oldsymbol{lpha} + oldsymbol{eta} oldsymbol{\eta}_{i,t-1} + oldsymbol{\Gamma} \mathbf{x}_{i,t} + oldsymbol{\zeta}_{i,t}, \quad ext{with} \quad oldsymbol{\zeta}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, oldsymbol{\Psi}
ight)$$

where $\mathbf{x}_{i,t}$ represents a vector of covariates at time t and individual i, and Γ the coefficient matrix linking the covariates to the latent variables.

Type 2:

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \boldsymbol{\Lambda} \boldsymbol{\eta}_{i,t} + \kappa \mathbf{x}_{i,t} + \boldsymbol{\varepsilon}_{i,t}, \quad ext{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Theta}\right)$$

where κ represents the coefficient matrix linking the covariates to the observed variables.

The dynamic structure is given by

$$\eta_{i,t} = \alpha + \beta \eta_{i,t-1} + \Gamma \mathbf{x}_{i,t} + \zeta_{i,t}, \text{ with } \zeta_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \mathbf{\Psi}\right).$$

Value

Returns an object of class simstatespace which is a list with the following elements:

- call: Function call.
- args: Function arguments.
- data: Generated data which is a list of length n. Each element of data is a list with the following elements:
 - id: A vector of ID numbers with length 1, where 1 is the value of the function argument time.
 - time: A vector time points of length 1.
 - y: A 1 by k matrix of values for the manifest variables.
 - eta: A 1 by p matrix of values for the latent variables.
 - x: A 1 by j matrix of values for the covariates (when covariates are included).
- fun: Function used.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Chow, S.-M., Ho, M. R., Hamaker, E. L., & Dolan, C. V. (2010). Equivalence and differences between structural equation modeling and state-space modeling techniques. *Structural Equation Modeling: A Multidisciplinary Journal*, 17(2), 303–332. doi:10.1080/10705511003661553

See Also

```
Other Simulation of State Space Models Data Functions: LinSDE2SSM(), PBSSMLinSDEFixed(), PBSSMOUFixed(), PBSSMVARFixed(), SimBetaN(), SimPhiN(), SimSSMIVary(), SimSSMLinGrowth(), SimSSMLinGrowthIVary(), SimSSMLinSDEFixed(), SimSSMLinSDEIVary(), SimSSMOUFixed(), SimSSMOUIVary(), SimSSMVARFixed(), SimSSMVARIVary(), TestPhi(), TestStability(), TestStationarity()
```

```
# prepare parameters
set.seed(42)
## number of individuals
n <- 5
## time points
time <- 50
## dynamic structure
p <- 3
mu0 < -rep(x = 0, times = p)
sigma0 < -0.001 * diag(p)
sigma0_l <- t(chol(sigma0))</pre>
alpha <- rep(x = 0, times = p)
beta <- 0.50 * diag(p)
psi <- 0.001 * diag(p)
psi_l <- t(chol(psi))</pre>
## measurement model
k <- 3
nu \leftarrow rep(x = 0, times = k)
lambda <- diag(k)</pre>
theta <- 0.001 * diag(k)
theta_l <- t(chol(theta))</pre>
## covariates
j <- 2
x <- lapply(
  X = seq_len(n),
  FUN = function(i) {
    matrix(
      data = stats::rnorm(n = time * j),
      nrow = j,
      ncol = time
    )
 }
)
```

```
gamma \leftarrow diag(x = 0.10, nrow = p, ncol = j)
kappa \leftarrow diag(x = 0.10, nrow = k, ncol = j)
# Type 0
ssm <- SimSSMFixed(</pre>
 n = n,
 time = time,
  mu0 = mu0,
  sigma0_1 = sigma0_1,
  alpha = alpha,
  beta = beta,
  psi_l = psi_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 0
)
plot(ssm)
# Type 1
ssm <- SimSSMFixed(</pre>
  n = n,
  time = time,
  mu0 = mu0,
  sigma0_1 = sigma0_1,
  alpha = alpha,
  beta = beta,
  psi_1 = psi_1,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 1,
  x = x,
  gamma = gamma
)
plot(ssm)
# Type 2
ssm <- SimSSMFixed(</pre>
 n = n,
  time = time,
  mu0 = mu0,
  sigma0_1 = sigma0_1,
  alpha = alpha,
  beta = beta,
  psi_l = psi_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 2,
  x = x,
```

```
gamma = gamma,
kappa = kappa
)
plot(ssm)
```

SimSSMIVary

Simulate Data from a State Space Model (Individual-Varying Parameters)

Description

This function simulates data using a state space model. It assumes that the parameters can vary across individuals.

Usage

```
SimSSMIVary(
  n,
  time,
 delta_t = 1,
 mu0,
  sigma0_1,
  alpha,
 beta,
 psi_l,
 nu,
 lambda,
  theta_l,
  type = 0,
  x = NULL,
  gamma = NULL,
 kappa = NULL
)
```

Arguments

n	Positive integer. Number of individuals.
time	Positive integer. Number of time points.
delta_t	Numeric. Time interval. The default value is 1.0 with an option to use a numeric value for the discretized state space model parameterization of the linear stochastic differential equation model.
mu0	List of numeric vectors. Each element of the list is the mean of initial latent variable values $(\mu_{\eta 0})$.

sigma0_l	List of numeric matrices. Each element of the list is the Cholesky factorization (t(chol(sigma0))) of the covariance matrix of initial latent variable values $(\Sigma_{\eta 0})$.
alpha	List of numeric vectors. Each element of the list is the vector of constant values for the dynamic model (α) .
beta	List of numeric matrices. Each element of the list is the transition matrix relating the values of the latent variables at the previous to the current time point (β) .
psi_l	List of numeric matrices. Each element of the list is the Cholesky factorization $(t(chol(psi)))$ of the covariance matrix of the process noise (Ψ) .
nu	List of numeric vectors. Each element of the list is the vector of intercept values for the measurement model (ν).
lambda	List of numeric matrices. Each element of the list is the factor loading matrix linking the latent variables to the observed variables (Λ) .
theta_l	List of numeric matrices. Each element of the list is the Cholesky factorization $(t(chol(theta)))$ of the covariance matrix of the measurement error (Θ) .
type	Integer. State space model type. See Details in SimSSMFixed() for more information.
Х	List. Each element of the list is a matrix of covariates for each individual i in n. The number of columns in each matrix should be equal to time.
gamma	List of numeric matrices. Each element of the list is the matrix linking the covariates to the latent variables at current time point (Γ) .
kappa	List of numeric matrices. Each element of the list is the matrix linking the covariates to the observed variables at current time point (κ) .

Details

Parameters can vary across individuals by providing a list of parameter values. If the length of any of the parameters (mu0, sigma0_1, alpha, beta, psi_1, nu, lambda, theta_1, gamma, or kappa) is less the n, the function will cycle through the available values.

Value

Returns an object of class simstatespace which is a list with the following elements:

- call: Function call.
- args: Function arguments.
- data: Generated data which is a list of length n. Each element of data is a list with the following elements:
 - id: A vector of ID numbers with length 1, where 1 is the value of the function argument time.
 - time: A vector time points of length 1.
 - y: A 1 by k matrix of values for the manifest variables.
 - eta: A 1 by p matrix of values for the latent variables.
 - x: A 1 by j matrix of values for the covariates (when covariates are included).
- fun: Function used.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Chow, S.-M., Ho, M. R., Hamaker, E. L., & Dolan, C. V. (2010). Equivalence and differences between structural equation modeling and state-space modeling techniques. *Structural Equation Modeling: A Multidisciplinary Journal*, 17(2), 303–332. doi:10.1080/10705511003661553

See Also

```
Other Simulation of State Space Models Data Functions: LinSDE2SSM(), PBSSMLinSDEFixed(), PBSSMOUFixed(), PBSSMVARFixed(), SimBetaN(), SimPhiN(), SimSSMFixed(), SimSSMLinGrowth(), SimSSMLinGrowthIVary(), SimSSMLinSDEFixed(), SimSSMLinSDEIVary(), SimSSMOUFixed(), SimSSMOUIVary(), SimSSMVARFixed(), SimSSMVARIVary(), TestPhi(), TestStability(), TestStationarity()
```

```
# prepare parameters
# In this example, beta varies across individuals.
set.seed(42)
## number of individuals
n <- 5
## time points
time <- 50
## dynamic structure
p < -3
mu0 <- list(
  rep(x = 0, times = p)
sigma0 <- 0.001 * diag(p)
sigma0_l <- list(</pre>
  t(chol(sigma0))
alpha <- list(
  rep(x = 0, times = p)
beta <- list(
  0.1 * diag(p),
  0.2 * diag(p),
  0.3 * diag(p),
  0.4 * diag(p),
  0.5 * diag(p)
psi <- 0.001 * diag(p)
psi_l <- list(</pre>
  t(chol(psi))
## measurement model
k <- 3
nu <- list(
```

```
rep(x = 0, times = k)
lambda <- list(</pre>
  diag(k)
)
theta <- 0.001 * diag(k)
theta_l <- list(</pre>
 t(chol(theta))
)
## covariates
j <- 2
x <- lapply(
 X = seq_len(n),
 FUN = function(i) {
    matrix(
      data = stats::rnorm(n = time * j),
      nrow = j,
      ncol = time
  }
)
gamma <- list(</pre>
 diag(x = 0.10, nrow = p, ncol = j)
kappa <- list(</pre>
  diag(x = 0.10, nrow = k, ncol = j)
# Type 0
ssm <- SimSSMIVary(</pre>
 n = n,
 time = time,
  mu0 = mu0,
  sigma0_1 = sigma0_1,
  alpha = alpha,
  beta = beta,
  psi_1 = psi_1,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 0
)
plot(ssm)
# Type 1
ssm <- SimSSMIVary(</pre>
 n = n,
  time = time,
  mu0 = mu0,
  sigma0_1 = sigma0_1,
  alpha = alpha,
  beta = beta,
```

```
psi_l = psi_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 1,
  x = x,
  gamma = gamma
plot(ssm)
# Type 2
ssm <- SimSSMIVary(</pre>
  n = n,
  time = time,
  mu0 = mu0,
  sigma0_1 = sigma0_1,
  alpha = alpha,
  beta = beta,
  psi_l = psi_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 2,
  x = x,
  gamma = gamma,
  kappa = kappa
)
plot(ssm)
```

 ${\tt SimSSMLinGrowth}$

Simulate Data from the Linear Growth Curve Model

Description

This function simulates data from the linear growth curve model.

Usage

```
SimSSMLinGrowth(
    n,
    time,
    mu0,
    sigma0_l,
    theta_l,
    type = 0,
    x = NULL,
```

```
gamma = NULL,
kappa = NULL
)
```

Arguments

Positive integer. Number of individuals. n time Positive integer. Number of time points. Numeric vector. A vector of length two. The first element is the mean of the mu0 intercept, and the second element is the mean of the slope. sigma0_l Numeric matrix. Cholesky factorization (t(chol(sigma0))) of the covariance matrix of the intercept and the slope. theta_1 Numeric. Square root of the common measurement error variance. type Integer. State space model type. See Details for more information. List. Each element of the list is a matrix of covariates for each individual i in n. The number of columns in each matrix should be equal to time. Numeric matrix. Matrix linking the covariates to the latent variables at current gamma time point (Γ) . kappa Numeric matrix. Matrix linking the covariates to the observed variables at current time point (κ) .

Details

Type 0:

The measurement model is given by

$$Y_{i,t} = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} \eta_{0_{i,t}} \\ \eta_{1_{i,t}} \end{pmatrix} + \boldsymbol{\varepsilon}_{i,t}, \quad ext{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}\left(0, heta
ight)$$

where $Y_{i,t}$, $\eta_{0_{i,t}}$, $\eta_{1_{i,t}}$, and $\varepsilon_{i,t}$ are random variables and θ is a model parameter. $Y_{i,t}$ is the observed random variable at time t and individual i, $\eta_{0_{i,t}}$ (intercept) and $\eta_{1_{i,t}}$ (slope) form a vector of latent random variables at time t and individual i, and $\varepsilon_{i,t}$ a vector of random measurement errors at time t and individual i. θ is the variance of ε .

The dynamic structure is given by

$$\left(\begin{array}{c}\eta_{0_{i,t}}\\\eta_{1_{i,t}}\end{array}\right)=\left(\begin{array}{cc}1&1\\0&1\end{array}\right)\left(\begin{array}{c}\eta_{0_{i,t-1}}\\\eta_{1_{i,t-1}}\end{array}\right).$$

The mean vector and covariance matrix of the intercept and slope are captured in the mean vector and covariance matrix of the initial condition given by

$$\mu_{\eta|0} = \begin{pmatrix} \mu_{\eta_0} \\ \mu_{\eta_1} \end{pmatrix}$$
 and,

$$\boldsymbol{\Sigma_{\eta|0}} = \left(\begin{array}{cc} \sigma_{\eta_0}^2 & \sigma_{\eta_0,\eta_1} \\ \sigma_{\eta_1,\eta_0} & \sigma_{\eta_1}^2 \end{array} \right).$$

Type 1:

The measurement model is given by

$$Y_{i,t} = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} \eta_{0_{i,t}} \\ \eta_{1_{i,t}} \end{pmatrix} + \boldsymbol{\varepsilon}_{i,t}, \quad \text{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}\left(0, \theta\right).$$

The dynamic structure is given by

$$\left(\begin{array}{c} \eta_{0_{i,t}} \\ \eta_{1_{i,t}} \end{array}\right) = \left(\begin{array}{cc} 1 & 1 \\ 0 & 1 \end{array}\right) \left(\begin{array}{c} \eta_{0_{i,t-1}} \\ \eta_{1_{i,t-1}} \end{array}\right) + \mathbf{\Gamma} \mathbf{x}_{i,t}$$

where $\mathbf{x}_{i,t}$ represents a vector of covariates at time t and individual i, and Γ the coefficient matrix linking the covariates to the latent variables.

Type 2:

The measurement model is given by

$$Y_{i,t} = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} \eta_{0_{i,t}} \\ \eta_{1_{i,t}} \end{pmatrix} + \kappa \mathbf{x}_{i,t} + \boldsymbol{\varepsilon}_{i,t}, \quad \text{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}\left(0, \theta\right)$$

where κ represents the coefficient matrix linking the covariates to the observed variables.

The dynamic structure is given by

$$\begin{pmatrix} \eta_{0_{i,t}} \\ \eta_{1_{i,t}} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \eta_{0_{i,t-1}} \\ \eta_{1_{i,t-1}} \end{pmatrix} + \mathbf{\Gamma} \mathbf{x}_{i,t}.$$

Value

Returns an object of class simstatespace which is a list with the following elements:

- call: Function call.
- args: Function arguments.
- data: Generated data which is a list of length n. Each element of data is a list with the following elements:
 - id: A vector of ID numbers with length 1, where 1 is the value of the function argument
 - time: A vector time points of length 1.
 - y: A 1 by k matrix of values for the manifest variables.
 - eta: A 1 by p matrix of values for the latent variables.
 - x: A 1 by j matrix of values for the covariates (when covariates are included).
- fun: Function used.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Chow, S.-M., Ho, M. R., Hamaker, E. L., & Dolan, C. V. (2010). Equivalence and differences between structural equation modeling and state-space modeling techniques. *Structural Equation Modeling: A Multidisciplinary Journal*, 17(2), 303–332. doi:10.1080/10705511003661553

See Also

```
Other Simulation of State Space Models Data Functions: LinSDE2SSM(), PBSSMLinSDEFixed(), PBSSMOUFixed(), PBSSMVARFixed(), SimBetaN(), SimPhiN(), SimSSMFixed(), SimSSMIVary(), SimSSMLinGrowthIVary(), SimSSMLinSDEFixed(), SimSSMLinSDEIVary(), SimSSMOUFixed(), SimSSMOUIVary(), SimSSMVARFixed(), SimSSMVARIVary(), TestPhi(), TestStability(), TestStationarity()
```

```
# prepare parameters
set.seed(42)
## number of individuals
n <- 5
## time points
time <-5
## dynamic structure
p <- 2
mu0 <- c(0.615, 1.006)
sigma0 <- matrix(</pre>
  data = c(
    1.932,
    0.618,
    0.618,
    0.587
  ),
  nrow = p
)
sigma0_l <- t(chol(sigma0))</pre>
## measurement model
k <- 1
theta <- 0.50
theta_l <- sqrt(theta)</pre>
## covariates
j <- 2
x <- lapply(
  X = seq_len(n),
  FUN = function(i) {
    return(
      matrix(
        data = rnorm(n = j * time),
        nrow = j
      )
    )
  }
gamma \leftarrow diag(x = 0.10, nrow = p, ncol = j)
kappa \leftarrow diag(x = 0.10, nrow = k, ncol = j)
# Type 0
ssm <- SimSSMLinGrowth(</pre>
 n = n,
  time = time,
  mu0 = mu0,
```

```
sigma0_1 = sigma0_1,
  theta_1 = theta_1,
  type = 0
)
plot(ssm)
# Type 1
ssm <- SimSSMLinGrowth(</pre>
 n = n,
  time = time,
  mu0 = mu0,
  sigma0_1 = sigma0_1,
  theta_l = theta_l,
  type = 1,
  x = x,
  gamma = gamma
)
plot(ssm)
# Type 2
ssm <- SimSSMLinGrowth(</pre>
  n = n,
  time = time,
  mu0 = mu0,
  sigma0_1 = sigma0_1,
  theta_1 = theta_1,
  type = 2,
  x = x,
  gamma = gamma,
  kappa = kappa
)
plot(ssm)
```

SimSSMLinGrowthIVary Simulate Data from the Linear Growth Curve Model (Individual-Varying Parameters)

Description

This function simulates data from the linear growth curve model. It assumes that the parameters can vary across individuals.

Usage

```
SimSSMLinGrowthIVary(
   n,
```

```
time,
mu0,
sigma0_1,
theta_1,
type = 0,
x = NULL,
gamma = NULL,
kappa = NULL
)
```

Arguments

n	Positive integer. Number of individuals.
time	Positive integer. Number of time points.
mu0	A list of numeric vectors. Each element of the list is a vector of length two. The first element is the mean of the intercept, and the second element is the mean of the slope.
sigma0_l	A list of numeric matrices. Each element of the list is the Cholesky factorization (t(chol(sigma0))) of the covariance matrix of the intercept and the slope.
theta_l	A list numeric values. Each element of the list is the square root of the common measurement error variance.
type	Integer. State space model type. See Details in SimSSMLinGrowth() for more information.
X	List. Each element of the list is a matrix of covariates for each individual i in n. The number of columns in each matrix should be equal to time.
gamma	List of numeric matrices. Each element of the list is the matrix linking the covariates to the latent variables at current time point (Γ) .
kappa	List of numeric matrices. Each element of the list is the matrix linking the covariates to the observed variables at current time point (κ) .

Details

Parameters can vary across individuals by providing a list of parameter values. If the length of any of the parameters (mu0, sigma0, mu, theta_1, gamma, or kappa) is less the n, the function will cycle through the available values.

Value

Returns an object of class simstatespace which is a list with the following elements:

- call: Function call.
- args: Function arguments.
- data: Generated data which is a list of length n. Each element of data is a list with the following elements:
 - id: A vector of ID numbers with length 1, where 1 is the value of the function argument time.

- time: A vector time points of length 1.
- y: A 1 by k matrix of values for the manifest variables.
- eta: A 1 by p matrix of values for the latent variables.
- x: A 1 by j matrix of values for the covariates (when covariates are included).
- fun: Function used.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Chow, S.-M., Ho, M. R., Hamaker, E. L., & Dolan, C. V. (2010). Equivalence and differences between structural equation modeling and state-space modeling techniques. *Structural Equation Modeling: A Multidisciplinary Journal*, 17(2), 303–332. doi:10.1080/10705511003661553

See Also

```
Other Simulation of State Space Models Data Functions: LinSDE2SSM(), PBSSMLinSDEFixed(), PBSSMOUFixed(), PBSSMVARFixed(), SimBetaN(), SimPhiN(), SimSSMFixed(), SimSSMIVary(), SimSSMLinGrowth(), SimSSMLinSDEFixed(), SimSSMLinSDEIVary(), SimSSMOUFixed(), SimSSMOUIVary(), SimSSMVARFixed(), SimSSMVARIVary(), TestPhi(), TestStability(), TestStationarity()

Other Simulation of State Space Models Data Functions: LinSDE2SSM(), PBSSMLinSDEFixed(), PBSSMOUFixed(), SimSSMVARFixed(), SimSSMIVary(), SimSSMLinGrowth(), SimSSMLinSDEFixed(), SimSSMLinSDEIVary(), SimSSMOUFixed(), SimSSMOUIVary(), SimSSMVARFixed(), SimSSMVARIVary(), TestPhi(), TestStability(), TestStationarity()
```

```
# prepare parameters
# In this example, the mean vector of the intercept and slope vary.
# Specifically,
# there are two sets of values representing two latent classes.
set.seed(42)
## number of individuals
n <- 10
## time points
time <-5
## dynamic structure
p <- 2
mu0_1 \leftarrow c(0.615, 1.006) # lower starting point, higher growth
mu0_2 \leftarrow c(1.000, 0.500) # higher starting point, lower growth
mu0 <- list(mu0_1, mu0_2)</pre>
sigma0 <- matrix(</pre>
  data = c(
    1.932,
    0.618,
    0.618,
    0.587
  ),
```

```
nrow = p
)
sigma0_l <- list(t(chol(sigma0)))</pre>
## measurement model
k <- 1
theta <- 0.50
theta_l <- list(sqrt(theta))</pre>
## covariates
j <- 2
x <- lapply(
 X = seq_len(n),
 FUN = function(i) {
    matrix(
      data = stats::rnorm(n = time * j),
      nrow = j,
      ncol = time
    )
  }
)
gamma <- list(</pre>
 diag(x = 0.10, nrow = p, ncol = j)
kappa <- list(</pre>
  diag(x = 0.10, nrow = k, ncol = j)
# Type 0
ssm <- SimSSMLinGrowthIVary(</pre>
 n = n,
 time = time,
 mu0 = mu0,
  sigma0_1 = sigma0_1,
  theta_1 = theta_1,
  type = 0
)
plot(ssm)
# Type 1
ssm <- SimSSMLinGrowthIVary(</pre>
 n = n,
  time = time,
 mu0 = mu0,
  sigma0_1 = sigma0_1,
  theta_l = theta_l,
  type = 1,
 x = x,
  gamma = gamma
)
plot(ssm)
# Type 2
```

```
ssm <- SimSSMLinGrowthIVary(
  n = n,
  time = time,
  mu0 = mu0,
  sigma0_1 = sigma0_1,
  theta_1 = theta_1,
  type = 2,
  x = x,
  gamma = gamma,
  kappa = kappa
)
plot(ssm)</pre>
```

SimSSMLinSDEFixed

Simulate Data from the Linear Stochastic Differential Equation Model using a State Space Model Parameterization (Fixed Parameters)

Description

This function simulates data from the linear stochastic differential equation model using a state space model parameterization. It assumes that the parameters remain constant across individuals and over time.

Usage

```
SimSSMLinSDEFixed(
  n,
  time,
 delta_t = 1,
 mu0,
  sigma0_1,
  iota,
 phi,
  sigma_l,
 nu,
  lambda,
  theta_1,
  type = 0,
 x = NULL
 gamma = NULL,
 kappa = NULL
)
```

Arguments

Positive integer. Number of individuals.

Positive integer. Number of time points.
Numeric. Time interval (Δ_t) .
Numeric vector. Mean of initial latent variable values $(\mu_{\eta 0})$.
Numeric matrix. Cholesky factorization (t(chol(sigma0))) of the covariance matrix of initial latent variable values ($\Sigma_{\eta 0}$).
Numeric vector. An unobserved term that is constant over time (ι) .
Numeric matrix. The drift matrix which represents the rate of change of the solution in the absence of any random fluctuations (Φ) .
Numeric matrix. Cholesky factorization (t(chol(sigma))) of the covariance matrix of volatility or randomness in the process (Σ) .
Numeric vector. Vector of intercept values for the measurement model (ν) .
Numeric matrix. Factor loading matrix linking the latent variables to the observed variables ($\pmb{\Lambda}$).
Numeric matrix. Cholesky factorization (t(chol(theta))) of the covariance matrix of the measurement error (Θ) .
Integer. State space model type. See Details for more information.
List. Each element of the list is a matrix of covariates for each individual i in n . The number of columns in each matrix should be equal to time.
Numeric matrix. Matrix linking the covariates to the latent variables at current time point $(\Gamma).$
Numeric matrix. Matrix linking the covariates to the observed variables at current time point (κ) .

Details

Type 0:

The measurement model is given by

$$\mathbf{y}_{i,t} = oldsymbol{
u} + oldsymbol{\Lambda} oldsymbol{\eta}_{i,t} + oldsymbol{arepsilon}_{i,t}, \quad ext{with} \quad oldsymbol{arepsilon}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, oldsymbol{\Theta}
ight)$$

where $\mathbf{y}_{i,t}$, $\eta_{i,t}$, and $\varepsilon_{i,t}$ are random variables and $\boldsymbol{\nu}$, $\boldsymbol{\Lambda}$, and $\boldsymbol{\Theta}$ are model parameters. $\mathbf{y}_{i,t}$ represents a vector of observed random variables, $\eta_{i,t}$ a vector of latent random variables, and $\varepsilon_{i,t}$ a vector of random measurement errors, at time t and individual i. $\boldsymbol{\nu}$ denotes a vector of intercepts, $\boldsymbol{\Lambda}$ a matrix of factor loadings, and $\boldsymbol{\Theta}$ the covariance matrix of ε .

An alternative representation of the measurement error is given by

$$oldsymbol{arepsilon}_{i,t} = oldsymbol{\Theta}^{rac{1}{2}} \mathbf{z}_{i,t}, \quad ext{with} \quad \mathbf{z}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \mathbf{I}
ight)$$

where $\mathbf{z}_{i,t}$ is a vector of independent standard normal random variables and $\left(\Theta^{\frac{1}{2}}\right)\left(\Theta^{\frac{1}{2}}\right)' = \Theta$. The dynamic structure is given by

$$\mathrm{d}oldsymbol{\eta}_{i,t} = \left(oldsymbol{\iota} + oldsymbol{\Phi}oldsymbol{\eta}_{i,t}
ight)\mathrm{d}t + oldsymbol{\Sigma}^{rac{1}{2}}\mathrm{d}\mathbf{W}_{i,t}$$

where ι is a term which is unobserved and constant over time, Φ is the drift matrix which represents the rate of change of the solution in the absence of any random fluctuations, Σ is the matrix of volatility or randomness in the process, and $\mathrm{d}W$ is a Wiener process or Brownian motion, which represents random fluctuations.

Type 1:

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \boldsymbol{\Lambda} \boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t}, \quad \text{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Theta}\right).$$

The dynamic structure is given by

$$\mathrm{d}\boldsymbol{\eta}_{i,t} = \left(\boldsymbol{\iota} + \boldsymbol{\Phi}\boldsymbol{\eta}_{i,t}\right) \mathrm{d}t + \Gamma \mathbf{x}_{i,t} + \boldsymbol{\Sigma}^{\frac{1}{2}} \mathrm{d}\mathbf{W}_{i,t}$$

where $\mathbf{x}_{i,t}$ represents a vector of covariates at time t and individual i, and Γ the coefficient matrix linking the covariates to the latent variables.

Type 2:

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \boldsymbol{\Lambda} \boldsymbol{\eta}_{i,t} + \boldsymbol{\kappa} \mathbf{x}_{i,t} + \boldsymbol{\varepsilon}_{i,t}, \quad ext{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Theta}\right)$$

where κ represents the coefficient matrix linking the covariates to the observed variables.

The dynamic structure is given by

$$d\boldsymbol{\eta}_{i,t} = \left(\boldsymbol{\iota} + \boldsymbol{\Phi} \boldsymbol{\eta}_{i,t}\right) dt + \boldsymbol{\Gamma} \mathbf{x}_{i,t} + \boldsymbol{\Sigma}^{\frac{1}{2}} d\mathbf{W}_{i,t}.$$

State Space Parameterization:

The state space parameters as a function of the linear stochastic differential equation model parameters are given by

$$\boldsymbol{\beta}_{\Delta t_{li}} = \exp\left(\Delta t \boldsymbol{\Phi}\right)$$

$$oldsymbol{lpha}_{\Delta t_{lz}} = oldsymbol{\Phi}^{-1} \left(oldsymbol{eta} - \mathbf{I}_{p}
ight) oldsymbol{\iota}$$

$$\operatorname{vec}\left(\mathbf{\Psi}_{\Delta t_{l_{i}}}\right) = \left[\left(\mathbf{\Phi} \otimes \mathbf{I}_{p}\right) + \left(\mathbf{I}_{p} \otimes \mathbf{\Phi}\right)\right] \left[\exp\left(\left[\left(\mathbf{\Phi} \otimes \mathbf{I}_{p}\right) + \left(\mathbf{I}_{p} \otimes \mathbf{\Phi}\right)\right] \Delta t\right) - \mathbf{I}_{p \times p}\right] \operatorname{vec}\left(\mathbf{\Sigma}\right)$$

where p is the number of latent variables and Δt is the time interval.

Value

Returns an object of class simstatespace which is a list with the following elements:

- call: Function call.
- args: Function arguments.
- data: Generated data which is a list of length n. Each element of data is a list with the following elements:
 - id: A vector of ID numbers with length 1, where 1 is the value of the function argument time.
 - time: A vector time points of length 1.
 - y: A 1 by k matrix of values for the manifest variables.
 - eta: A 1 by p matrix of values for the latent variables.
 - x: A 1 by j matrix of values for the covariates (when covariates are included).
- fun: Function used.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Chow, S.-M., Ho, M. R., Hamaker, E. L., & Dolan, C. V. (2010). Equivalence and differences between structural equation modeling and state-space modeling techniques. *Structural Equation Modeling: A Multidisciplinary Journal*, 17(2), 303–332. doi:10.1080/10705511003661553

Chow, S.-M., Losardo, D., Park, J., & Molenaar, P. C. M. (2023). Continuous-time dynamic models: Connections to structural equation models and other discrete-time models. In R. H. Hoyle (Ed.), Handbook of structural equation modeling (2nd ed.). The Guilford Press.

Harvey, A. C. (1990). Forecasting, structural time series models and the Kalman filter. Cambridge University Press. doi:10.1017/cbo9781107049994

See Also

```
Other Simulation of State Space Models Data Functions: LinSDE2SSM(), PBSSMLinSDEFixed(), PBSSMOUFixed(), PBSSMVARFixed(), SimBetaN(), SimPhiN(), SimSSMFixed(), SimSSMIVary(), SimSSMLinGrowth(), SimSSMLinGrowthIVary(), SimSSMLinSDEIVary(), SimSSMOUFixed(), SimSSMOUFixed(), SimSSMVARFixed(), SimSSMVARIVary(), TestPhi(), TestStability(), TestStationarity()
```

```
# prepare parameters
set.seed(42)
## number of individuals
n <- 5
## time points
time <- 50
delta_t <- 0.10
## dynamic structure
p <- 2
mu0 < -c(-3.0, 1.5)
sigma0 <- 0.001 * diag(p)
sigma0_l <- t(chol(sigma0))</pre>
iota <- c(0.317, 0.230)
phi <- matrix(</pre>
  data = c(
    -0.10,
    0.05,
    0.05,
    -0.10
  ),
  nrow = p
sigma <- matrix(</pre>
  data = c(
    2.79,
    0.06,
    0.06,
```

```
3.27
 ),
  nrow = p
)
sigma_l <- t(chol(sigma))</pre>
## measurement model
k <- 2
nu \leftarrow rep(x = 0, times = k)
lambda <- diag(k)</pre>
theta <- 0.001 * diag(k)
theta_l <- t(chol(theta))</pre>
## covariates
j <- 2
x <- lapply(
  X = seq_len(n),
  FUN = function(i) {
    matrix(
      data = stats::rnorm(n = time * j),
      nrow = j,
      ncol = time
    )
  }
)
gamma \leftarrow diag(x = 0.10, nrow = p, ncol = j)
kappa <- diag(x = 0.10, nrow = k, ncol = j)
# Type 0
ssm <- SimSSMLinSDEFixed(</pre>
  n = n,
  time = time,
  delta_t = delta_t,
  mu0 = mu0,
  sigma0_1 = sigma0_1,
  iota = iota,
  phi = phi,
  sigma_l = sigma_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 0
plot(ssm)
# Type 1
ssm <- SimSSMLinSDEFixed(</pre>
  n = n,
  time = time,
  delta_t = delta_t,
  mu0 = mu0,
  sigma0_1 = sigma0_1,
  iota = iota,
  phi = phi,
```

```
sigma_l = sigma_l,
  nu = nu,
  lambda = lambda,
  theta_1 = theta_1,
  type = 1,
  x = x,
  gamma = gamma
plot(ssm)
# Type 2
ssm <- SimSSMLinSDEFixed(</pre>
  n = n,
  time = time,
  delta_t = delta_t,
  mu0 = mu0,
  sigma0_1 = sigma0_1,
  iota = iota,
  phi = phi,
  sigma_l = sigma_l,
  nu = nu,
  lambda = lambda,
  theta_1 = theta_1,
  type = 2,
  x = x,
  gamma = gamma,
  kappa = kappa
plot(ssm)
```

SimSSMLinSDEIVary

Simulate Data from the Linear Stochastic Differential Equation Model using a State Space Model Parameterization (Individual-Varying Parameters)

Description

This function simulates data from the linear stochastic differential equation model using a state space model parameterization. It assumes that the parameters can vary across individuals.

Usage

```
SimSSMLinSDEIVary(
  n,
  time,
  delta_t = 1,
  mu0,
```

SimSSMLinSDEIVary

```
sigma0_1,
iota,
phi,
sigma_1,
nu,
lambda,
theta_1,
type = 0,
x = NULL,
gamma = NULL,
kappa = NULL
)
```

Arguments

n	Positive integer. Number of individuals.
time	Positive integer. Number of time points.
delta_t	Numeric. Time interval. The default value is 1.0 with an option to use a numeric value for the discretized state space model parameterization of the linear stochastic differential equation model.
mu0	List of numeric vectors. Each element of the list is the mean of initial latent variable values $(\mu_{\eta 0})$.
sigma0_l	List of numeric matrices. Each element of the list is the Cholesky factorization (t(chol(sigma0))) of the covariance matrix of initial latent variable values $(\Sigma_{\eta 0})$.
iota	List of numeric vectors. Each element of the list is an unobserved term that is constant over time (ι) .
phi	List of numeric matrix. Each element of the list is the drift matrix which represents the rate of change of the solution in the absence of any random fluctuations (Φ) .
sigma_l	List of numeric matrix. Each element of the list is the Cholesky factorization (t(chol(sigma))) of the covariance matrix of volatility or randomness in the process Σ .
nu	List of numeric vectors. Each element of the list is the vector of intercept values for the measurement model (ν) .
lambda	List of numeric matrices. Each element of the list is the factor loading matrix linking the latent variables to the observed variables (Λ) .
theta_l	List of numeric matrices. Each element of the list is the Cholesky factorization $(t(chol(theta)))$ of the covariance matrix of the measurement error (Θ) .
type	Integer. State space model type. See Details in SimSSMLinSDEFixed() for more information.
х	List. Each element of the list is a matrix of covariates for each individual i in n. The number of columns in each matrix should be equal to time.
gamma	List of numeric matrices. Each element of the list is the matrix linking the covariates to the latent variables at current time point (Γ) .

kappa

List of numeric matrices. Each element of the list is the matrix linking the covariates to the observed variables at current time point (κ) .

Details

Parameters can vary across individuals by providing a list of parameter values. If the length of any of the parameters (mu0, sigma0_1, iota, phi, sigma_1, nu, lambda, theta_1, gamma, or kappa) is less the n, the function will cycle through the available values.

Value

Returns an object of class simstatespace which is a list with the following elements:

- · call: Function call.
- args: Function arguments.
- data: Generated data which is a list of length n. Each element of data is a list with the following elements:
 - id: A vector of ID numbers with length 1, where 1 is the value of the function argument time.
 - time: A vector time points of length 1.
 - y: A 1 by k matrix of values for the manifest variables.
 - eta: A 1 by p matrix of values for the latent variables.
 - x: A 1 by j matrix of values for the covariates (when covariates are included).
- fun: Function used.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Chow, S.-M., Ho, M. R., Hamaker, E. L., & Dolan, C. V. (2010). Equivalence and differences between structural equation modeling and state-space modeling techniques. *Structural Equation Modeling: A Multidisciplinary Journal*, 17(2), 303–332. doi:10.1080/10705511003661553

Chow, S.-M., Losardo, D., Park, J., & Molenaar, P. C. M. (2023). Continuous-time dynamic models: Connections to structural equation models and other discrete-time models. In R. H. Hoyle (Ed.), Handbook of structural equation modeling (2nd ed.). The Guilford Press.

Harvey, A. C. (1990). Forecasting, structural time series models and the Kalman filter. Cambridge University Press. doi:10.1017/cbo9781107049994

See Also

```
Other Simulation of State Space Models Data Functions: LinSDE2SSM(), PBSSMLinSDEFixed(), PBSSMOUFixed(), PBSSMVARFixed(), SimBetaN(), SimPhiN(), SimSSMFixed(), SimSSMIVary(), SimSSMLinGrowth(), SimSSMLinGrowthIVary(), SimSSMLinSDEFixed(), SimSSMOUFixed(), SimSSMOUIVary(), SimSSMVARFixed(), SimSSMVARIVary(), TestPhi(), TestStability(), TestStationarity()
```

```
# prepare parameters
# In this example, phi varies across individuals.
set.seed(42)
## number of individuals
n <- 5
## time points
time <- 50
delta_t <- 0.10
## dynamic structure
p <- 2
mu0 <- list(
  c(-3.0, 1.5)
sigma0 <- 0.001 * diag(p)
sigma0_1 \leftarrow list(
  t(chol(sigma0))
iota <- list(</pre>
  c(0.317, 0.230)
phi <- list(</pre>
 -0.1 * diag(p),
  -0.2 * diag(p),
  -0.3 * diag(p),
  -0.4 * diag(p),
  -0.5 * diag(p)
sigma <- matrix(</pre>
  data = c(
    2.79,
    0.06,
    0.06,
    3.27
  ),
  nrow = p
)
sigma_l <- list(</pre>
  t(chol(sigma))
)
## measurement model
k <- 2
nu <- list(
  rep(x = 0, times = k)
lambda <- list(</pre>
  diag(k)
theta <- 0.001 * diag(k)
theta_l <- list(</pre>
  t(chol(theta))
)
```

```
## covariates
j <- 2
x <- lapply(
 X = seq_len(n),
 FUN = function(i) {
    matrix(
      data = stats::rnorm(n = time * j),
      nrow = j,
      ncol = time
    )
  }
)
gamma <- list(</pre>
  diag(x = 0.10, nrow = p, ncol = j)
kappa <- list(</pre>
  diag(x = 0.10, nrow = k, ncol = j)
# Type 0
ssm <- SimSSMLinSDEIVary(</pre>
 n = n,
  time = time,
  delta_t = delta_t,
  mu0 = mu0,
  sigma0_1 = sigma0_1,
  iota = iota,
  phi = phi,
  sigma_l = sigma_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 0
)
plot(ssm)
# Type 1
ssm <- SimSSMLinSDEIVary(</pre>
 n = n,
  time = time,
  delta_t = delta_t,
  mu0 = mu0,
  sigma0_1 = sigma0_1,
  iota = iota,
  phi = phi,
  sigma_l = sigma_l,
  nu = nu,
  lambda = lambda,
  theta_1 = theta_1,
  type = 1,
  x = x,
  gamma = gamma
```

```
)
plot(ssm)
# Type 2
ssm <- SimSSMLinSDEIVary(</pre>
  n = n,
  time = time,
  delta_t = delta_t,
  mu0 = mu0,
  sigma0_1 = sigma0_1,
  iota = iota,
  phi = phi,
  sigma_l = sigma_l,
  nu = nu,
  lambda = lambda,
  theta_1 = theta_1,
  type = 2,
  x = x,
  gamma = gamma,
  kappa = kappa
)
plot(ssm)
```

SimSSMOUFixed

Simulate Data from the Ornstein-Uhlenbeck Model using a State Space Model Parameterization (Fixed Parameters)

Description

This function simulates data from the Ornstein–Uhlenbeck (OU) model using a state space model parameterization. It assumes that the parameters remain constant across individuals and over time.

Usage

```
SimSSMOUFixed(
    n,
    time,
    delta_t = 1,
    mu0,
    sigma0_l,
    mu,
    phi,
    sigma_l,
    nu,
    lambda,
    theta_l,
```

```
type = 0,
x = NULL,
gamma = NULL,
kappa = NULL
```

Arguments

n Positive integer. Number of individuals.time Positive integer. Number of time points.

delta_t Numeric. Time interval (Δ_t) .

mu0 Numeric vector. Mean of initial latent variable values $(\mu_{n|0})$.

sigma0_l Numeric matrix. Cholesky factorization (t(chol(sigma0))) of the covariance

matrix of initial latent variable values $(\Sigma_{n|0})$.

mu Numeric vector. The long-term mean or equilibrium level (μ) .

phi Numeric matrix. The drift matrix which represents the rate of change of the

solution in the absence of any random fluctuations (Φ) . It also represents the rate of mean reversion, determining how quickly the variable returns to its mean.

sigma_1 Numeric matrix. Cholesky factorization (t(chol(sigma))) of the covariance

matrix of volatility or randomness in the process (Σ) .

nu Numeric vector. Vector of intercept values for the measurement model (ν) .

lambda Numeric matrix. Factor loading matrix linking the latent variables to the ob-

served variables (Λ) .

theta_1 Numeric matrix. Cholesky factorization (t(chol(theta))) of the covariance

matrix of the measurement error (Θ) .

type Integer. State space model type. See Details for more information.

x List. Each element of the list is a matrix of covariates for each individual i in n.

The number of columns in each matrix should be equal to time.

gamma Numeric matrix. Matrix linking the covariates to the latent variables at current

time point (Γ) .

kappa Numeric matrix. Matrix linking the covariates to the observed variables at cur-

rent time point (κ) .

Details

Type 0:

The measurement model is given by

$$\mathbf{y}_{i,t} = oldsymbol{
u} + oldsymbol{\Lambda} oldsymbol{\eta}_{i,t} + oldsymbol{arepsilon}_{i,t}, \quad ext{with} \quad oldsymbol{arepsilon}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, oldsymbol{\Theta}
ight)$$

where $\mathbf{y}_{i,t}$, $\eta_{i,t}$, and $\varepsilon_{i,t}$ are random variables and $\boldsymbol{\nu}$, $\boldsymbol{\Lambda}$, and $\boldsymbol{\Theta}$ are model parameters. $\mathbf{y}_{i,t}$ represents a vector of observed random variables, $\eta_{i,t}$ a vector of latent random variables, and $\varepsilon_{i,t}$ a vector of random measurement errors, at time t and individual i. $\boldsymbol{\nu}$ denotes a vector of intercepts, $\boldsymbol{\Lambda}$ a matrix of factor loadings, and $\boldsymbol{\Theta}$ the covariance matrix of ε .

An alternative representation of the measurement error is given by

$$\boldsymbol{\varepsilon}_{i.t} = \boldsymbol{\Theta}^{\frac{1}{2}} \mathbf{z}_{i.t}, \quad \text{with} \quad \mathbf{z}_{i.t} \sim \mathcal{N}\left(\mathbf{0}, \mathbf{I}\right)$$

where $\mathbf{z}_{i,t}$ is a vector of independent standard normal random variables and $\left(\Theta^{\frac{1}{2}}\right)\left(\Theta^{\frac{1}{2}}\right)' = \Theta$. The dynamic structure is given by

$$\mathrm{d}\boldsymbol{\eta}_{i,t} = \boldsymbol{\Phi} \left(\boldsymbol{\eta}_{i,t} - \boldsymbol{\mu} \right) \mathrm{d}t + \boldsymbol{\Sigma}^{\frac{1}{2}} \mathrm{d}\mathbf{W}_{i,t}$$

where μ is the long-term mean or equilibrium level, Φ is the rate of mean reversion, determining how quickly the variable returns to its mean, Σ is the matrix of volatility or randomness in the process, and dW is a Wiener process or Brownian motion, which represents random fluctuations.

Type 1:

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \boldsymbol{\Lambda} \boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t}, \quad \text{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Theta}\right).$$

The dynamic structure is given by

$$\mathrm{d} \boldsymbol{\eta}_{i,t} = \boldsymbol{\Phi} \left(\boldsymbol{\eta}_{i,t} - \boldsymbol{\mu} \right) \mathrm{d} t + \boldsymbol{\Gamma} \mathbf{x}_{i,t} + \boldsymbol{\Sigma}^{\frac{1}{2}} \mathrm{d} \mathbf{W}_{i,t}$$

where $\mathbf{x}_{i,t}$ represents a vector of covariates at time t and individual i, and Γ the coefficient matrix linking the covariates to the latent variables.

Type 2:

The measurement model is given by

$$\mathbf{y}_{i,t} = \mathbf{\nu} + \mathbf{\Lambda} \boldsymbol{\eta}_{i,t} + \kappa \mathbf{x}_{i,t} + \mathbf{arepsilon}_{i,t}, \quad ext{with} \quad \mathbf{arepsilon}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \mathbf{\Theta}
ight)$$

where κ represents the coefficient matrix linking the covariates to the observed variables.

The dynamic structure is given by

$$d\boldsymbol{\eta}_{i:t} = \boldsymbol{\Phi} \left(\boldsymbol{\eta}_{i:t} - \boldsymbol{\mu} \right) dt + \boldsymbol{\Gamma} \mathbf{x}_{i:t} + \boldsymbol{\Sigma}^{\frac{1}{2}} d\mathbf{W}_{i:t}.$$

The OU model as a linear stochastic differential equation model:

The OU model is a first-order linear stochastic differential equation model in the form of

$$\mathrm{d} oldsymbol{\eta}_{i,t} = \left(oldsymbol{\iota} + oldsymbol{\Phi} oldsymbol{\eta}_{i,t}
ight) \mathrm{d} t + oldsymbol{\Sigma}^{rac{1}{2}} \mathrm{d} \mathbf{W}_{i,t}$$

where $\mu = -\Phi^{-1}\iota$ and, equivalently $\iota = -\Phi\mu$.

Value

Returns an object of class simstatespace which is a list with the following elements:

- call: Function call.
- args: Function arguments.
- data: Generated data which is a list of length n. Each element of data is a list with the following elements:

- id: A vector of ID numbers with length 1, where 1 is the value of the function argument time.
- time: A vector time points of length 1.
- y: A 1 by k matrix of values for the manifest variables.
- eta: A 1 by p matrix of values for the latent variables.
- x: A 1 by j matrix of values for the covariates (when covariates are included).
- fun: Function used.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Chow, S.-M., Ho, M. R., Hamaker, E. L., & Dolan, C. V. (2010). Equivalence and differences between structural equation modeling and state-space modeling techniques. *Structural Equation Modeling: A Multidisciplinary Journal*, 17(2), 303–332. doi:10.1080/10705511003661553

Chow, S.-M., Losardo, D., Park, J., & Molenaar, P. C. M. (2023). Continuous-time dynamic models: Connections to structural equation models and other discrete-time models. In R. H. Hoyle (Ed.), Handbook of structural equation modeling (2nd ed.). The Guilford Press.

Harvey, A. C. (1990). Forecasting, structural time series models and the Kalman filter. Cambridge University Press. doi:10.1017/cbo9781107049994

Oravecz, Z., Tuerlinckx, F., & Vandekerckhove, J. (2011). A hierarchical latent stochastic differential equation model for affective dynamics. Psychological Methods, 16 (4), 468–490. doi:10.1037/a0024375

Uhlenbeck, G. E., & Ornstein, L. S. (1930). On the theory of the brownian motion. Physical Review, 36 (5), 823–841. doi:10.1103/physrev.36.823

See Also

```
Other Simulation of State Space Models Data Functions: LinSDE2SSM(), PBSSMLinSDEFixed(), PBSSMOUFixed(), PBSSMVARFixed(), SimBetaN(), SimPhiN(), SimSSMFixed(), SimSSMIVary(), SimSSMLinGrowth(), SimSSMLinGrowthIVary(), SimSSMLinSDEFixed(), SimSSMLinSDEIVary(), SimSSMOUIVary(), SimSSMVARFixed(), SimSSMVARIVary(), TestPhi(), TestStability(), TestStationarity()
```

```
# prepare parameters
set.seed(42)
## number of individuals
n <- 5
## time points
time <- 50
delta_t <- 0.10
## dynamic structure
p <- 2
mu0 <- c(-3.0, 1.5)
sigma0 <- 0.001 * diag(p)</pre>
```

```
sigma0_l <- t(chol(sigma0))</pre>
mu < -c(5.76, 5.18)
phi <- matrix(</pre>
  data = c(
    -0.10,
    0.05,
    0.05,
    -0.10
  ),
  nrow = p
sigma <- matrix(</pre>
  data = c(
    2.79,
    0.06,
    0.06,
    3.27
  ),
  nrow = p
sigma_l <- t(chol(sigma))</pre>
## measurement model
k <- 2
nu \leftarrow rep(x = 0, times = k)
lambda <- diag(k)</pre>
theta <- 0.001 * diag(k)
theta_l <- t(chol(theta))</pre>
## covariates
j <- 2
x <- lapply(
  X = seq_len(n),
  FUN = function(i) {
    matrix(
      data = stats::rnorm(n = time * j),
      nrow = j,
      ncol = time
    )
  }
gamma \leftarrow diag(x = 0.10, nrow = p, ncol = j)
kappa \leftarrow diag(x = 0.10, nrow = k, ncol = j)
# Type 0
ssm <- SimSSMOUFixed(</pre>
  n = n,
  time = time,
  delta_t = delta_t,
  mu0 = mu0,
  sigma0_1 = sigma0_1,
  mu = mu,
  phi = phi,
  sigma_l = sigma_l,
  nu = nu,
```

```
lambda = lambda,
  theta_1 = theta_1,
  type = 0
)
plot(ssm)
# Type 1
ssm <- SimSSMOUFixed(</pre>
 n = n,
  time = time,
  delta_t = delta_t,
  mu0 = mu0,
  sigma0_1 = sigma0_1,
  mu = mu,
  phi = phi,
  sigma_l = sigma_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 1,
  x = x,
  gamma = gamma
plot(ssm)
# Type 2
ssm <- SimSSMOUFixed(</pre>
  n = n,
  time = time,
  delta_t = delta_t,
  mu0 = mu0,
  sigma0_1 = sigma0_1,
  mu = mu,
  phi = phi,
  sigma_l = sigma_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 2,
  x = x,
  gamma = gamma,
  kappa = kappa
)
plot(ssm)
```

SimSSMOUIVary

Simulate Data from the Ornstein-Uhlenbeck Model using a State Space Model Parameterization (Individual-Varying Parameters)

Description

This function simulates data from the Ornstein–Uhlenbeck model using a state space model parameterization. It assumes that the parameters can vary across individuals.

Usage

```
SimSSMOUIVary(
  n,
  time,
 delta_t = 1,
 mu0,
  sigma0_1,
 mu,
 phi,
 sigma_l,
  nu,
  lambda,
  theta_1,
  type = 0,
  x = NULL
 gamma = NULL,
 kappa = NULL
)
```

Arguments

n	Positive integer. Number of individuals.
time	Positive integer. Number of time points.
delta_t	Numeric. Time interval. The default value is 1.0 with an option to use a numeric value for the discretized state space model parameterization of the linear stochastic differential equation model.
mu0	List of numeric vectors. Each element of the list is the mean of initial latent variable values $(\mu_{\eta 0})$.
sigma0_l	List of numeric matrices. Each element of the list is the Cholesky factorization (t(chol(sigma0))) of the covariance matrix of initial latent variable values $(\Sigma_{\eta 0})$.
mu	List of numeric vectors. Each element of the list is the long-term mean or equilibrium level (μ) .
phi	List of numeric matrix. Each element of the list is the drift matrix which represents the rate of change of the solution in the absence of any random fluctuations (Φ) . It also represents the rate of mean reversion, determining how quickly the variable returns to its mean.

sigma_l	List of numeric matrix. Each element of the list is the Cholesky factorization $(t(chol(sigma)))$ of the covariance matrix of volatility or randomness in the process Σ .
nu	List of numeric vectors. Each element of the list is the vector of intercept values for the measurement model (ν) .
lambda	List of numeric matrices. Each element of the list is the factor loading matrix linking the latent variables to the observed variables (Λ) .
theta_l	List of numeric matrices. Each element of the list is the Cholesky factorization $(t(chol(theta)))$ of the covariance matrix of the measurement error (Θ) .
type	Integer. State space model type. See Details in SimSSMOUFixed() for more information.
X	List. Each element of the list is a matrix of covariates for each individual i in n. The number of columns in each matrix should be equal to time.
gamma	List of numeric matrices. Each element of the list is the matrix linking the covariates to the latent variables at current time point (Γ) .
kappa	List of numeric matrices. Each element of the list is the matrix linking the covariates to the observed variables at current time point (κ) .

Details

Parameters can vary across individuals by providing a list of parameter values. If the length of any of the parameters (mu0, sigma0_1, mu, phi, sigma_1, nu, lambda, theta_1, gamma, or kappa) is less the n, the function will cycle through the available values.

Value

Returns an object of class simstatespace which is a list with the following elements:

- call: Function call.
- args: Function arguments.
- data: Generated data which is a list of length n. Each element of data is a list with the following elements:
 - id: A vector of ID numbers with length 1, where 1 is the value of the function argument time.
 - time: A vector time points of length 1.
 - y: A 1 by k matrix of values for the manifest variables.
 - eta: A 1 by p matrix of values for the latent variables.
 - x: A 1 by j matrix of values for the covariates (when covariates are included).
- fun: Function used.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Chow, S.-M., Ho, M. R., Hamaker, E. L., & Dolan, C. V. (2010). Equivalence and differences between structural equation modeling and state-space modeling techniques. *Structural Equation Modeling: A Multidisciplinary Journal*, 17(2), 303–332. doi:10.1080/10705511003661553

Chow, S.-M., Losardo, D., Park, J., & Molenaar, P. C. M. (2023). Continuous-time dynamic models: Connections to structural equation models and other discrete-time models. In R. H. Hoyle (Ed.), Handbook of structural equation modeling (2nd ed.). The Guilford Press.

Harvey, A. C. (1990). Forecasting, structural time series models and the Kalman filter. Cambridge University Press. doi:10.1017/cbo9781107049994

Oravecz, Z., Tuerlinckx, F., & Vandekerckhove, J. (2011). A hierarchical latent stochastic differential equation model for affective dynamics. Psychological Methods, 16 (4), 468–490. doi:10.1037/a0024375

Uhlenbeck, G. E., & Ornstein, L. S. (1930). On the theory of the brownian motion. Physical Review, 36 (5), 823–841. doi:10.1103/physrev.36.823

See Also

```
Other Simulation of State Space Models Data Functions: LinSDE2SSM(), PBSSMLinSDEFixed(), PBSSMOUFixed(), PBSSMVARFixed(), SimBetaN(), SimPhiN(), SimSSMFixed(), SimSSMIVary(), SimSSMLinGrowth(), SimSSMLinGrowthIVary(), SimSSMLinSDEFixed(), SimSSMLinSDEIVary(), SimSSMOUFixed(), SimSSMVARFixed(), SimSSMVARFixed(), TestPhi(), TestStability(), TestStationarity()
```

```
# prepare parameters
# In this example, phi varies across individuals.
set.seed(42)
## number of individuals
n <- 5
## time points
time <- 50
delta_t <- 0.10
## dynamic structure
p <- 2
mu0 <- list(
  c(-3.0, 1.5)
sigma0 <- 0.001 * diag(p)
sigma0_l <- list(</pre>
  t(chol(sigma0))
mu <- list(</pre>
  c(5.76, 5.18)
phi <- list(</pre>
  -0.1 * diag(p),
  -0.2 * diag(p),
  -0.3 * diag(p),
  -0.4 * diag(p),
```

```
-0.5 * diag(p)
sigma <- matrix(</pre>
  data = c(
    2.79,
    0.06,
    0.06,
    3.27
  ),
  nrow = p
)
sigma_l <- list(</pre>
  t(chol(sigma))
## measurement model
k <- 2
nu <- list(
  rep(x = 0, times = k)
lambda <- list(</pre>
  diag(k)
theta <- 0.001 * diag(k)
theta_1 \leftarrow list(
  t(chol(theta))
## covariates
j <- 2
x <- lapply(
 X = seq_len(n),
  FUN = function(i) {
    matrix(
      data = stats::rnorm(n = time * j),
      nrow = j,
      ncol = time
    )
  }
)
gamma <- list(</pre>
  diag(x = 0.10, nrow = p, ncol = j)
kappa <- list(</pre>
  diag(x = 0.10, nrow = k, ncol = j)
# Type 0
ssm <- SimSSMOUIVary(</pre>
  n = n,
  time = time,
  delta_t = delta_t,
  mu0 = mu0,
  sigma0_1 = sigma0_1,
  mu = mu,
```

```
phi = phi,
  sigma_l = sigma_l,
  nu = nu,
  lambda = lambda,
  theta_1 = theta_1,
  type = 0
)
plot(ssm)
# Type 1
ssm <- SimSSMOUIVary(</pre>
 n = n,
  time = time,
  delta_t = delta_t,
 mu0 = mu0,
  sigma0_1 = sigma0_1,
  mu = mu,
  phi = phi,
  sigma_l = sigma_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 1,
  x = x,
  gamma = gamma
plot(ssm)
# Type 2
ssm <- SimSSMOUIVary(</pre>
 n = n,
  time = time,
  delta_t = delta_t,
  mu0 = mu0,
  sigma0_1 = sigma0_1,
  mu = mu,
  phi = phi,
  sigma_l = sigma_l,
  nu = nu,
  lambda = lambda,
  theta_l = theta_l,
  type = 2,
  x = x,
  gamma = gamma,
  kappa = kappa
)
plot(ssm)
```

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SimSSMVARFixed	Simulate Data from the Vector Autoregressive Model (Fixed Parameters)

Description

This function simulates data from the vector autoregressive model using a state space model parameterization. It assumes that the parameters remain constant across individuals and over time.

Usage

```
SimSSMVARFixed(
    n,
    time,
    mu0,
    sigma0_l,
    alpha,
    beta,
    psi_l,
    type = 0,
    x = NULL,
    gamma = NULL
)
```

Arguments

n	Positive integer. Number of individuals.
time	Positive integer. Number of time points.
mu0	Numeric vector. Mean of initial latent variable values $(\mu_{\eta 0})$.
sigma0_l	Numeric matrix. Cholesky factorization (t(chol(sigma0))) of the covariance matrix of initial latent variable values ($\Sigma_{\eta 0}$).
alpha	Numeric vector. Vector of constant values for the dynamic model (α) .
beta	Numeric matrix. Transition matrix relating the values of the latent variables at the previous to the current time point (β) .
psi_l	Numeric matrix. Cholesky factorization (t(chol(psi))) of the covariance matrix of the process noise (Ψ).
type	Integer. State space model type. See Details for more information.
Х	List. Each element of the list is a matrix of covariates for each individual i in n. The number of columns in each matrix should be equal to time.
gamma	Numeric matrix. Matrix linking the covariates to the latent variables at current time point (Γ).

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Details

Type 0:

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\eta}_{i,t}$$

where $\mathbf{y}_{i,t}$ represents a vector of observed variables and $\boldsymbol{\eta}_{i,t}$ a vector of latent variables for individual i and time t. Since the observed and latent variables are equal, we only generate data from the dynamic structure.

The dynamic structure is given by

$$oldsymbol{\eta}_{i,t} = oldsymbol{lpha} + oldsymbol{eta} oldsymbol{\eta}_{i,t-1} + oldsymbol{\zeta}_{i,t}, \quad ext{with} \quad oldsymbol{\zeta}_{i,t} \sim \mathcal{N}\left(oldsymbol{0}, oldsymbol{\Psi}
ight)$$

where $\eta_{i,t}$, $\eta_{i,t-1}$, and $\zeta_{i,t}$ are random variables, and α , β , and Ψ are model parameters. Here, $\eta_{i,t}$ is a vector of latent variables at time t and individual i, $\eta_{i,t-1}$ represents a vector of latent variables at time t-1 and individual i, and $\zeta_{i,t}$ represents a vector of dynamic noise at time t and individual i. α denotes a vector of intercepts, β a matrix of autoregression and cross regression coefficients, and Ψ the covariance matrix of $\zeta_{i,t}$.

An alternative representation of the dynamic noise is given by

$$oldsymbol{\zeta}_{i,t} = oldsymbol{\Psi}^{rac{1}{2}} oldsymbol{\mathbf{z}}_{i,t}, \quad ext{with} \quad oldsymbol{\mathbf{z}}_{i,t} \sim \mathcal{N}\left(oldsymbol{0}, oldsymbol{\mathbf{I}}
ight)$$

where
$$\left(\mathbf{\Psi}^{rac{1}{2}}
ight) \left(\mathbf{\Psi}^{rac{1}{2}}
ight)' = \mathbf{\Psi}.$$

Type 1:

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\eta}_{i,t}.$$

The dynamic structure is given by

$$oldsymbol{\eta}_{i,t} = oldsymbol{lpha} + oldsymbol{eta} oldsymbol{\eta}_{i,t-1} + oldsymbol{\Gamma} \mathbf{x}_{i,t} + oldsymbol{\zeta}_{i,t}, \quad ext{with} \quad oldsymbol{\zeta}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, oldsymbol{\Psi}
ight)$$

where $\mathbf{x}_{i,t}$ represents a vector of covariates at time t and individual i, and Γ the coefficient matrix linking the covariates to the latent variables.

Value

Returns an object of class simstatespace which is a list with the following elements:

- call: Function call.
- args: Function arguments.
- data: Generated data which is a list of length n. Each element of data is a list with the following elements:
 - id: A vector of ID numbers with length 1, where 1 is the value of the function argument time.
 - time: A vector time points of length 1.
 - y: A 1 by k matrix of values for the manifest variables.
 - eta: A 1 by p matrix of values for the latent variables.
 - x: A 1 by j matrix of values for the covariates (when covariates are included).
- fun: Function used.

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Author(s)

Ivan Jacob Agaloos Pesigan

References

Chow, S.-M., Ho, M. R., Hamaker, E. L., & Dolan, C. V. (2010). Equivalence and differences between structural equation modeling and state-space modeling techniques. *Structural Equation Modeling: A Multidisciplinary Journal*, 17(2), 303–332. doi:10.1080/10705511003661553

See Also

```
Other Simulation of State Space Models Data Functions: LinSDE2SSM(), PBSSMLinSDEFixed(), PBSSMOUFixed(), PBSSMVARFixed(), SimBetaN(), SimPhiN(), SimSSMFixed(), SimSSMIVary(), SimSSMLinGrowth(), SimSSMLinGrowthIVary(), SimSSMLinSDEFixed(), SimSSMLinSDEIVary(), SimSSMOUFixed(), S
```

Examples

```
# prepare parameters
set.seed(42)
## number of individuals
n <- 5
## time points
time <- 50
## dynamic structure
p <- 3
mu0 < -rep(x = 0, times = p)
sigma0 < -0.001 * diag(p)
sigma0_l <- t(chol(sigma0))</pre>
alpha <- rep(x = 0, times = p)
beta <- 0.50 * diag(p)
psi <- 0.001 * diag(p)
psi_l <- t(chol(psi))</pre>
## covariates
j <- 2
x <- lapply(
  X = seq_len(n),
  FUN = function(i) {
      data = stats::rnorm(n = time * j),
      nrow = j,
      ncol = time
    )
  }
gamma \leftarrow diag(x = 0.10, nrow = p, ncol = j)
# Type 0
ssm <- SimSSMVARFixed(</pre>
 n = n,
 time = time,
```

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```
mu0 = mu0,
  sigma0_1 = sigma0_1,
  alpha = alpha,
  beta = beta,
  psi_1 = psi_1,
  type = 0
)
plot(ssm)
# Type 1
ssm <- SimSSMVARFixed(</pre>
  n = n,
  time = time,
  mu0 = mu0,
  sigma0_1 = sigma0_1,
  alpha = alpha,
  beta = beta,
  psi_l = psi_l,
  type = 1,
  x = x,
  gamma = gamma
)
plot(ssm)
```

SimSSMVARIVary

Simulate Data from the Vector Autoregressive Model (Individual-Varying Parameters)

Description

This function simulates data from the vector autoregressive model using a state space model parameterization. It assumes that the parameters can vary across individuals.

Usage

```
SimSSMVARIVary(
    n,
    time,
    mu0,
    sigma0_l,
    alpha,
    beta,
    psi_l,
    type = 0,
    x = NULL,
    gamma = NULL
)
```

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Arguments

n	Positive integer. Number of individuals.
time	Positive integer. Number of time points.
mu0	List of numeric vectors. Each element of the list is the mean of initial latent variable values $(\mu_{\eta 0})$.
sigma0_l	List of numeric matrices. Each element of the list is the Cholesky factorization (t(chol(sigma0))) of the covariance matrix of initial latent variable values $(\Sigma_{\eta 0})$.
alpha	List of numeric vectors. Each element of the list is the vector of constant values for the dynamic model (α) .
beta	List of numeric matrices. Each element of the list is the transition matrix relating the values of the latent variables at the previous to the current time point (β) .
psi_l	List of numeric matrices. Each element of the list is the Cholesky factorization $(t(chol(psi)))$ of the covariance matrix of the process noise (Ψ) .
type	Integer. State space model type. See Details in SimSSMVARFixed() for more information.
X	List. Each element of the list is a matrix of covariates for each individual i in n. The number of columns in each matrix should be equal to time.
gamma	List of numeric matrices. Each element of the list is the matrix linking the covariates to the latent variables at current time point (Γ) .

Details

Parameters can vary across individuals by providing a list of parameter values. If the length of any of the parameters (mu0, sigma0_1, alpha, beta, psi_1, gamma, or kappa) is less the n, the function will cycle through the available values.

Value

Returns an object of class simstatespace which is a list with the following elements:

- call: Function call.
- args: Function arguments.
- data: Generated data which is a list of length n. Each element of data is a list with the following elements:
 - id: A vector of ID numbers with length 1, where 1 is the value of the function argument time.
 - time: A vector time points of length 1.
 - y: A 1 by k matrix of values for the manifest variables.
 - eta: A 1 by p matrix of values for the latent variables.
 - x: A 1 by j matrix of values for the covariates (when covariates are included).
- fun: Function used.

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Author(s)

Ivan Jacob Agaloos Pesigan

References

Chow, S.-M., Ho, M. R., Hamaker, E. L., & Dolan, C. V. (2010). Equivalence and differences between structural equation modeling and state-space modeling techniques. *Structural Equation Modeling: A Multidisciplinary Journal*, 17(2), 303–332. doi:10.1080/10705511003661553

See Also

```
Other Simulation of State Space Models Data Functions: LinSDE2SSM(), PBSSMLinSDEFixed(), PBSSMOUFixed(), PBSSMVARFixed(), SimBetaN(), SimPhiN(), SimSSMFixed(), SimSSMIVary(), SimSSMLinGrowth(), SimSSMLinGrowthIVary(), SimSSMLinSDEFixed(), SimSSMLinSDEIVary(), SimSSMOUFixed(), SimSSMOUFixed(), SimSSMOUFixed(), TestStationarity()
```

Examples

```
# prepare parameters
# In this example, beta varies across individuals.
set.seed(42)
## number of individuals
n <- 5
## time points
time <- 50
## dynamic structure
p < -3
mu0 <- list(
  rep(x = 0, times = p)
sigma0 <- 0.001 * diag(p)
sigma0_l <- list(</pre>
  t(chol(sigma0))
alpha <- list(
  rep(x = 0, times = p)
beta <- list(
  0.1 * diag(p),
  0.2 * diag(p),
  0.3 * diag(p),
  0.4 * diag(p),
  0.5 * diag(p)
psi <- 0.001 * diag(p)
psi_l <- list(</pre>
  t(chol(psi))
## covariates
j <- 2
x <- lapply(
```

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```
X = seq_len(n),
  FUN = function(i) {
    matrix(
      data = stats::rnorm(n = time * j),
      nrow = j,
      ncol = time
  }
)
gamma <- list(</pre>
  diag(x = 0.10, nrow = p, ncol = j)
# Type 0
ssm <- SimSSMVARIVary(</pre>
  n = n,
  time = time,
 mu0 = mu0,
  sigma0_1 = sigma0_1,
  alpha = alpha,
  beta = beta,
  psi_l = psi_l,
  type = 0
)
plot(ssm)
# Type 1
ssm <- SimSSMVARIVary(</pre>
 n = n,
  time = time,
  mu0 = mu0,
  sigma0_1 = sigma0_1,
  alpha = alpha,
  beta = beta,
  psi_l = psi_l,
  type = 1,
  x = x,
  gamma = gamma
plot(ssm)
```

 $summary. statespacepb \quad \textit{Summary Method for an Object of Class} \text{ statespacepb}$

Description

Summary Method for an Object of Class statespacepb

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Usage

```
## S3 method for class 'statespacepb'
summary(object, alpha = NULL, type = "pc", digits = 4, ...)
```

Arguments

object	Object of Class statespacepb.
alpha	Numeric vector. Significance level α . If alpha = NULL, use the argument alpha used in object.
type	Charater string. Confidence interval type, that is, type = "pc" for percentile; type = "bc" for bias corrected.
digits	Digits to print.
	additional arguments.

Value

Returns a matrix of estimates, standard errors, number of bootstrap replications, and confidence intervals.

Author(s)

Ivan Jacob Agaloos Pesigan

TestPhi	Test the Drift Matrix

Description

Both have to be true for the function to return TRUE.

- Test that the real part of all eigenvalues of Φ are less than zero.
- Test that the diagonal values of Φ are between 0 to negative inifinity.

Usage

```
TestPhi(phi)
```

Arguments

phi Numeric matrix. The drift matrix (Φ) .

Author(s)

Ivan Jacob Agaloos Pesigan

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See Also

Other Simulation of State Space Models Data Functions: LinSDE2SSM(), PBSSMLinSDEFixed(), PBSSMOUFixed(), PBSSMVARFixed(), SimBetaN(), SimPhiN(), SimSSMFixed(), SimSSMIVary(), SimSSMLinGrowth(), SimSSMLinGrowthIVary(), SimSSMLinSDEFixed(), SimSSMLinSDEIVary(), SimSSMOUFixed(), SimSSMOUIVary(), SimSSMVARFixed(), SimSSMVARIVary(), TestStability(), TestStationarity()

Examples

```
phi <- matrix(
  data = c(
    -0.357, 0.771, -0.450,
    0.0, -0.511, 0.729,
    0, 0, -0.693
  ),
  nrow = 3
)
TestPhi(phi = phi)</pre>
```

TestStability

Test Stability

Description

The function computes the eigenvalues of the input matrix x. It checks if the real part of all eigenvalues is negative. If all eigenvalues have negative real parts, the system is considered stable.

Usage

TestStability(x)

Arguments

Х

Numeric matrix.

Author(s)

Ivan Jacob Agaloos Pesigan

See Also

```
Other Simulation of State Space Models Data Functions: LinSDE2SSM(), PBSSMLinSDEFixed(), PBSSMOUFixed(), PBSSMVARFixed(), SimBetaN(), SimPhiN(), SimSSMFixed(), SimSSMIVary(), SimSSMLinGrowth(), SimSSMLinGrowthIVary(), SimSSMLinSDEFixed(), SimSSMLinSDEIVary(), SimSSMOUFixed(), S
```

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Examples

```
x <- matrix(
  data = c(
    -0.357, 0.771, -0.450,
    0.0, -0.511, 0.729,
    0, 0, -0.693
  ),
  nrow = 3
)
TestStability(x)</pre>
```

TestStationarity

Test Stationarity

Description

The function computes the eigenvalues of the input matrix x. It checks if all eigenvalues have moduli less than 1. If all eigenvalues have moduli less than 1, the system is considered stationary.

Usage

TestStationarity(x)

Arguments

Х

Numeric matrix.

Author(s)

Ivan Jacob Agaloos Pesigan

See Also

```
Other Simulation of State Space Models Data Functions: LinSDE2SSM(), PBSSMLinSDEFixed(), PBSSMOUFixed(), PBSSMVARFixed(), SimBetaN(), SimPhiN(), SimSSMFixed(), SimSSMIVary(), SimSSMLinGrowth(), SimSSMLinGrowthIVary(), SimSSMLinSDEFixed(), SimSSMLinSDEIVary(), SimSSMOUFixed(), SimSSMOUIVary(), SimSSMVARFixed(), SimSSMVARIVary(), TestPhi(), TestStability()
```

Examples

```
x <- matrix(
  data = c(0.5, 0.3, 0.2, 0.4),
  nrow = 2
)
TestStationarity(x)

x <- matrix(
  data = c(0.9, -0.5, 0.8, 0.7),</pre>
```

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```
nrow = 2
)
TestStationarity(x)
```

vcov.statespacepb

Sampling Variance-Covariance Matrix Method for an Object of Class statespacepb

Description

Sampling Variance-Covariance Matrix Method for an Object of Class statespacepb

Usage

```
## S3 method for class 'statespacepb'
vcov(object, ...)
```

Arguments

object Object of Class statespacepb.
... additional arguments.

Value

Returns the variance-covariance matrix of estimates.

Author(s)

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