Package 'simStateSpace'

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|--|
| Title Simulate Data from State Space Models |
| Version 1.0.0 |
| Description Offers an efficient and user-friendly framework for simulating data in state space models where n subjects/units is greater than one which is common in social and behavioral sciences. |
| <pre>URL https://github.com/jeksterslab/simStateSpace,</pre> |
| https://jeksterslab.github.io/simStateSpace/ |
| <pre>BugReports https://github.com/jeksterslab/simStateSpace/issues</pre> |
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| Author Ivan Jacob Agaloos Pesigan [aut, cre, cph] (https://orcid.org/0000-0003-4818-8420) |
| Maintainer Ivan Jacob Agaloos Pesigan <r.jeksterslab@gmail.com></r.jeksterslab@gmail.com> |
| R topics documented: |
| OU2SSM Sim2Matrix SimSSM0 SimSSM0Fixed SimSSM0Vary SimSSMOU 1 |

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Description

This function converts parameters from the Ornstein–Uhlenbeck model to state space model parameterization. See details for more information.

Usage

```
OU2SSM(mu, phi, sigma_sqrt, delta_t)
```

Arguments

| mu | Numeric vector. The long-term mean or equilibrium level (μ) . |
|------------|---|
| phi | Numeric matrix. The rate of mean reversion, determining how quickly the variable returns to its mean (Φ) . |
| sigma_sqrt | Numeric matrix. Cholesky decomposition of the matrix of volatility or randomness in the process (Σ) . |
| delta_t | Numeric. Time interval (δ_t) . |

Details

The state space parameters as a function of the Ornstein-Uhlenbeck model parameters are given by

$$\boldsymbol{\beta} = \exp\left(-\boldsymbol{\Phi}\boldsymbol{\Delta}_t\right)$$

$$oldsymbol{lpha} = -oldsymbol{\Phi}^{-1} \left(oldsymbol{eta} - \mathbf{I}_p
ight)$$

$$\operatorname{vec}\left(\boldsymbol{\Psi}\right) = \left\{\left[\left(-\boldsymbol{\Phi}\otimes\mathbf{I}_{p}\right) + \left(\mathbf{I}_{p}\otimes-\boldsymbol{\Phi}\right)\right]\left[\exp\left(\left[\left(-\boldsymbol{\Phi}\otimes\mathbf{I}_{p}\right) + \left(\mathbf{I}_{p}\otimes-\boldsymbol{\Phi}\right)\right]\Delta_{t}\right) - \mathbf{I}_{p\times p}\right]\operatorname{vec}\left(\boldsymbol{\Sigma}\right)\right\}$$

Author(s)

Ivan Jacob Agaloos Pesigan

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See Also

Other Simulation of State Space Models Data Functions: Sim2Matrix(), SimSSM0Fixed(), SimSSM0Vary(), SimSSM0(), SimSSM0UFixed(), SimSSM0UVary(), SimSSMVARFixed(), SimSSMVARVary(), SimSSMVAR()

Examples

```
p <- k <- 2
mu <- c(5.76, 5.18)
phi <- matrix(data = c(0.10, -0.05, -0.05, 0.10), nrow = p)
sigma_sqrt <- chol(
    matrix(data = c(2.79, 0.06, 0.06, 3.27), nrow = p)
)
delta_t <- 0.10

OU2SSM(
    mu = mu,
    phi = phi,
    sigma_sqrt = sigma_sqrt,
    delta_t = delta_t
)</pre>
```

Sim2Matrix

Simulation Output to Matrix

Description

This function converts the output of SimSSM0(), SimSSMOU(), SimSSMVAR(), SimSSMOFixed(), SimSSMOFixed(), or SimSSMVARFixed() to a matrix.

Usage

```
Sim2Matrix(x, eta = FALSE)
```

Arguments

Author(s)

Ivan Jacob Agaloos Pesigan

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See Also

Other Simulation of State Space Models Data Functions: OU2SSM(), SimSSM0Fixed(), SimSSM0Vary(), SimSSM0(), SimSSMOUFixed(), SimSSMOUVary(), SimSSMVARFixed(), SimSSMVARVary(), SimSSMVAR()

```
# prepare parameters
set.seed(42)
k <- p <- 3
I \leftarrow diag(k)
I_sqrt <- chol(I)</pre>
null\_vec \leftarrow rep(x = 0, times = k)
n <- 5
mu0 <- null_vec
sigma0_sqrt <- I_sqrt</pre>
alpha <- null_vec
beta \leftarrow diag(x = 0.50, nrow = k)
psi_sqrt <- I_sqrt</pre>
nu <- null_vec
lambda <- I
theta_sqrt <- chol(diag(x = 0.50, nrow = k))
time <- 50
burn_in <- 0</pre>
# generate data
ssm <- SimSSM0(
  mu0 = mu0,
  sigma0_sqrt = sigma0_sqrt,
  alpha = alpha,
  beta = beta,
  psi_sqrt = psi_sqrt,
  nu = nu,
  lambda = lambda,
  theta_sqrt = theta_sqrt,
  time = time,
  burn_in = burn_in
)
# list to matrix
mat <- Sim2Matrix(ssm)</pre>
str(mat)
head(mat)
# generate data
ssm <- SimSSM0Fixed(</pre>
  n = n,
  mu0 = mu0,
  sigma0_sqrt = sigma0_sqrt,
  alpha = alpha,
  beta = beta,
  psi_sqrt = psi_sqrt,
```

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```
nu = nu,
lambda = lambda,
theta_sqrt = theta_sqrt,
time = time,
burn_in = burn_in
)

# list to matrix
mat <- Sim2Matrix(ssm)
str(mat)
head(mat)</pre>
```

SimSSM0

Simulate Data from a State Space Model (n = 1)

Description

This function simulates data from a state space model. See details for more information.

Usage

```
SimSSM0(
  mu0,
  sigma0_sqrt,
  alpha,
  beta,
  psi_sqrt,
  nu,
  lambda,
  theta_sqrt,
  time,
  burn_in
)
```

Arguments

| mu0 | Numeric vector. Mean of initial latent variable values $(\mu_{\eta 0})$. |
|-------------|---|
| sigma0_sqrt | Numeric matrix. Cholesky decomposition of the covariance matrix of initial latent variable values $(\Sigma_{\eta 0})$. |
| alpha | Numeric vector. Vector of intercepts for the dynamic model (α) . |
| beta | Numeric matrix. Transition matrix relating the values of the latent variables at time $t-1$ to those at time $t(\beta)$. |
| psi_sqrt | Numeric matrix. Cholesky decomposition of the process noise covariance matrix (Ψ) . |
| nu | Numeric vector. Vector of intercepts for the measurement model (ν) . |

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lambda Numeric matrix. Factor loading matrix linking the latent variables to the ob-

served variables (Λ) .

theta_sqrt Numeric matrix. Cholesky decomposition of the measurement error covariance

matrix (Θ) .

time Positive integer. Number of time points to simulate.

burn_in Positive integer. Number of burn-in points to exclude before returning the re-

sults.

Details

The measurement model is given by

$$\mathbf{y}_{t} = \boldsymbol{\nu} + \boldsymbol{\Lambda} \boldsymbol{\eta}_{t} + \boldsymbol{\varepsilon}_{t} \quad \mathrm{with} \quad \boldsymbol{\varepsilon}_{t} \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Theta}\right)$$

where \mathbf{y}_t , $\boldsymbol{\eta}_t$, and $\boldsymbol{\varepsilon}_t$ are random variables and $\boldsymbol{\nu}$, $\boldsymbol{\Lambda}$, and $\boldsymbol{\Theta}$ are model parameters. \mathbf{y}_t is a vector of observed random variables at time t, $\boldsymbol{\eta}_t$ is a vector of latent random variables at time t, and $\boldsymbol{\varepsilon}_t$ is a vector of random measurement errors at time t, while $\boldsymbol{\nu}$ is a vector of intercept, $\boldsymbol{\Lambda}$ is a matrix of factor loadings, and $\boldsymbol{\Theta}$ is the covariance matrix of $\boldsymbol{\varepsilon}$.

The dynamic structure is given by

$$oldsymbol{\eta}_t = oldsymbol{lpha} + oldsymbol{eta} oldsymbol{\eta}_{t-1} + oldsymbol{\zeta}_t \quad ext{with} \quad oldsymbol{\zeta}_t \sim \mathcal{N}\left(oldsymbol{0}, oldsymbol{\Psi}
ight)$$

where η_t , η_{t-1} , and ζ_t are random variables and α , β , and Ψ are model parameters. η_t is a vector of latent variables at time t, η_{t-1} is a vector of latent variables at time t-1, and ζ_t is a vector of dynamic noise at time t while α is a vector of intercepts, β is a matrix of autoregression and cross regression coefficients, and Ψ is the covariance matrix of ζ_t .

Value

Returns a list with the following elements:

- y: A t by k matrix of values for the manifest variables.
- eta: A t by p matrix of values for the latent variables.
- time: A vector of discrete time points from 1 to t.
- n: Number of individuals.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Shumway, R. H., & Stoffer, D. S. (2017). *Time series analysis and its applications: With R examples*. Springer International Publishing. doi:10.1007/9783319524528

See Also

Other Simulation of State Space Models Data Functions: OU2SSM(), Sim2Matrix(), SimSSM0Fixed(), SimSSM0Vary(), SimSSM0UFixed(), SimSSMOUVary(), SimSSMVARFixed(), SimSSMVARVary(), SimSSMVAR()

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Examples

```
# prepare parameters
set.seed(42)
k < -p < -3
I <- diag(k)</pre>
I_sqrt <- chol(I)</pre>
null_vec \leftarrow rep(x = 0, times = k)
mu0 <- null_vec
sigma0_sqrt <- I_sqrt
alpha <- null_vec</pre>
beta \leftarrow diag(x = 0.50, nrow = k)
psi_sqrt <- I_sqrt</pre>
nu <- null_vec
lambda <- I
theta_sqrt <- chol(diag(x = 0.50, nrow = k))
time <- 50
burn_in <- 0
# generate data
ssm <- SimSSM0(
  mu0 = mu0,
  sigma0_sqrt = sigma0_sqrt,
  alpha = alpha,
  beta = beta,
  psi_sqrt = psi_sqrt,
  nu = nu,
  lambda = lambda,
  theta_sqrt = theta_sqrt,
  time = time,
  burn_in = burn_in
)
str(ssm)
```

SimSSM0Fixed

Simulate Data using a State Space Model Parameterization for n > 1 Individuals (Fixed Parameters)

Description

This function simulates data using a state space model parameterization for n > 1 individuals. In this model, the parameters are invariant across individuals.

Usage

```
SimSSM0Fixed(
  n,
  mu0,
```

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```
sigma0_sqrt,
alpha,
beta,
psi_sqrt,
nu,
lambda,
theta_sqrt,
time,
burn_in
```

Arguments

n Positive integer. Number of individuals.

mu0 Numeric vector. Mean of initial latent variable values $(\mu_{\eta|0})$.

sigma@_sqrt Numeric matrix. Cholesky decomposition of the covariance matrix of initial

latent variable values ($\Sigma_{\eta|0}$).

alpha Numeric vector. Vector of intercepts for the dynamic model (α) .

beta Numeric matrix. Transition matrix relating the values of the latent variables at

time t - 1 to those at time t (β) .

psi_sqrt Numeric matrix. Cholesky decomposition of the process noise covariance ma-

trix (Ψ) .

nu Numeric vector. Vector of intercepts for the measurement model (ν) .

lambda Numeric matrix. Factor loading matrix linking the latent variables to the ob-

served variables (Λ) .

theta_sqrt Numeric matrix. Cholesky decomposition of the measurement error covariance

matrix (Θ) .

time Positive integer. Number of time points to simulate.

burn_in Positive integer. Number of burn-in points to exclude before returning the re-

sults.

Details

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \boldsymbol{\Lambda} \boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t} \quad ext{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Theta}\right)$$

where $\mathbf{y}_{i,t}$, $\eta_{i,t}$, and $\varepsilon_{i,t}$ are random variables and $\boldsymbol{\nu}$, $\boldsymbol{\Lambda}$, and $\boldsymbol{\Theta}$ are model parameters. $\mathbf{y}_{i,t}$ is a vector of observed random variables at time t and individual i, $\eta_{i,t}$ is a vector of latent random variables at time t and individual i, while $\boldsymbol{\nu}$ is a vector of intercept, $\boldsymbol{\Lambda}$ is a matrix of factor loadings, and $\boldsymbol{\Theta}$ is the covariance matrix of ε .

The dynamic structure is given by

$$oldsymbol{\eta}_{i,t} = oldsymbol{lpha} + oldsymbol{eta} oldsymbol{\eta}_{i,t-1} + oldsymbol{\zeta}_{i,t} \quad ext{with} \quad oldsymbol{\zeta}_{i,t} \sim \mathcal{N}\left(oldsymbol{0}, oldsymbol{\Psi}
ight)$$

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where $\eta_{i,t}$, $\eta_{i,t-1}$, and $\zeta_{i,t}$ are random variables and α , β , and Ψ are model parameters. $\eta_{i,t}$ is a vector of latent variables at time t and individual i, $\eta_{i,t-1}$ is a vector of latent variables at time t-1 and individual i, and $\zeta_{i,t}$ is a vector of dynamic noise at time t and individual i while α is a vector of intercepts, β is a matrix of autoregression and cross regression coefficients, and Ψ is the covariance matrix of $\zeta_{i,t}$.

Value

Returns a list of length n. Each element is a list with the following elements:

- y: A t by k matrix of values for the manifest variables.
- eta: A t by p matrix of values for the latent variables.
- time: A vector of discrete time points from 1 to t.
- id: A vector of ID numbers of length t.
- n: Number of individuals.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Shumway, R. H., & Stoffer, D. S. (2017). *Time series analysis and its applications: With R examples*. Springer International Publishing. doi:10.1007/9783319524528

See Also

```
Other Simulation of State Space Models Data Functions: OU2SSM(), Sim2Matrix(), SimSSM0Vary(), SimSSM0(), SimSSMOUFixed(), SimSSMOUVary(), SimSSMOU(), SimSSMVARFixed(), SimSSMVARVary(), SimSSMVAR()
```

```
# prepare parameters
set.seed(42)
k <- p <- 3
I \leftarrow diag(k)
I_sqrt <- chol(I)</pre>
null\_vec \leftarrow rep(x = 0, times = k)
n <- 5
mu0 <- null_vec
sigma0_sqrt <- I_sqrt
alpha <- null_vec
beta \leftarrow diag(x = 0.50, nrow = k)
psi_sqrt <- I_sqrt</pre>
nu <- null_vec
lambda <- I
theta_sqrt <- chol(diag(x = 0.50, nrow = k))
time <- 50
burn_in <- 0
```

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```
# generate data
ssm <- SimSSM0Fixed(
    n = n,
    mu0 = mu0,
    sigma0_sqrt = sigma0_sqrt,
    alpha = alpha,
    beta = beta,
    psi_sqrt = psi_sqrt,
    nu = nu,
    lambda = lambda,
    theta_sqrt = theta_sqrt,
    time = time,
    burn_in = burn_in
)</pre>
```

SimSSM0Vary

Simulate Data using a State Space Model Parameterization for n > 1 Individuals (Varying Parameters)

Description

This function simulates data using a state space model parameterization for n > 1 individuals. In this model, the parameters can vary across individuals.

Usage

```
SimSSM0Vary(
    n,
    mu0,
    sigma0_sqrt,
    alpha,
    beta,
    psi_sqrt,
    nu,
    lambda,
    theta_sqrt,
    time,
    burn_in
)
```

Arguments

n Positive integer. Number of individuals.

mu0 List of numeric vectors. Mean of initial latent variable values $(\mu_{\eta|0})$.

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| sigma0_sqrt | List of numeric matrices. Cholesky decomposition of the covariance matrix of initial latent variable values $(\Sigma_{\eta 0})$. |
|-------------|---|
| alpha | List of numeric vectors. Vector of intercepts for the dynamic model (α) . |
| beta | List of numeric matrices. Transition matrix relating the values of the latent variables at time $t-1$ to those at time $t(\beta)$. |
| psi_sqrt | List of numeric matrices. Cholesky decomposition of the process noise covariance matrix (Ψ) . |
| nu | List of numeric vectors. Vector of intercepts for the measurement model (ν) . |
| lambda | List of numeric matrices. Factor loading matrix linking the latent variables to the observed variables (Λ) . |
| theta_sqrt | List of numeric matrices. Cholesky decomposition of the measurement error covariance matrix (Θ) . |
| time | Positive integer. Number of time points to simulate. |
| burn_in | Positive integer. Number of burn-in points to exclude before returning the results. |

Details

Parameters can vary across individuals by providing a list of parameter values. If the length of any of the parameters (mu0, sigma0_sqrt, alpha, beta, psi_sqrt, nu, lambda, and theta_sqrt) is less the n, the function will cycle through the available values.

Value

Returns a list of length n. Each element is a list with the following elements:

- y: A t by k matrix of values for the manifest variables.
- eta: A t by p matrix of values for the latent variables.
- time: A vector of discrete time points from 1 to t.
- id: A vector of ID numbers of length t.
- n: Number of individuals.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Shumway, R. H., & Stoffer, D. S. (2017). *Time series analysis and its applications: With R examples*. Springer International Publishing. doi:10.1007/9783319524528

See Also

Other Simulation of State Space Models Data Functions: OU2SSM(), Sim2Matrix(), SimSSM0Fixed(), SimSSM0(), SimSSMOUFixed(), SimSSMOUVary(), SimSSMOU(), SimSSMVARFixed(), SimSSMVARVary(), SimSSMVAR()

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Examples

```
# prepare parameters
# In this example, beta varies across individuals
set.seed(42)
k <- p <- 3
iden <- diag(k)
iden_sqrt <- chol(iden)</pre>
null_vec <- rep(x = 0, times = k)
n <- 5
mu0 <- list(null_vec)</pre>
sigma0_sqrt <- list(iden_sqrt)</pre>
alpha <- list(null_vec)</pre>
beta <- list(</pre>
  diag(x = 0.1, nrow = k),
  diag(x = 0.2, nrow = k),
  diag(x = 0.3, nrow = k),
  diag(x = 0.4, nrow = k),
  diag(x = 0.5, nrow = k)
)
psi_sqrt <- list(iden_sqrt)</pre>
nu <- list(null_vec)</pre>
lambda <- list(iden)</pre>
theta_sqrt <- list(chol(diag(x = 0.50, nrow = k)))</pre>
time <- 50
burn_in <- 0</pre>
ssm <- SimSSM0Vary(</pre>
  n = n,
  mu0 = mu0,
  sigma0_sqrt = sigma0_sqrt,
  alpha = alpha,
  beta = beta,
  psi_sqrt = psi_sqrt,
  nu = nu,
  lambda = lambda,
  theta_sqrt = theta_sqrt,
  time = time,
  burn_in = burn_in
)
str(ssm)
```

SimSSMOU

Simulate Data from the Ornstein-Uhlenbeck Model using a State Space Model Parameterization (n = 1)

Description

This function simulates data from the Ornstein–Uhlenbeck model using a state space model parameterization. See details for more information.

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Usage

```
SimSSMOU(
   mu0,
   sigma0_sqrt,
   mu,
   phi,
   sigma_sqrt,
   nu,
   lambda,
   theta_sqrt,
   delta_t,
   time,
   burn_in
)
```

Arguments

| mu0 | Numeric vector. Mean of initial latent variable values $(\mu_{\eta 0})$. |
|-------------|---|
| sigma0_sqrt | Numeric matrix. Cholesky decomposition of the covariance matrix of initial latent variable values $(\Sigma_{\eta 0})$. |
| mu | Numeric vector. The long-term mean or equilibrium level (μ) . |
| phi | Numeric matrix. The rate of mean reversion, determining how quickly the variable returns to its mean (Φ) . |
| sigma_sqrt | Numeric matrix. Cholesky decomposition of the matrix of volatility or randomness in the process (Σ) . |
| nu | Numeric vector. Vector of intercepts for the measurement model (ν) . |
| lambda | Numeric matrix. Factor loading matrix linking the latent variables to the observed variables (Λ) . |
| theta_sqrt | Numeric matrix. Cholesky decomposition of the measurement error covariance matrix (Θ) . |
| delta_t | Numeric. Time interval (δ_t). |
| time | Positive integer. Number of time points to simulate. |
| burn_in | Positive integer. Number of burn-in points to exclude before returning the results. |

Details

The measurement model is given by

$$\mathbf{y}_{t} = oldsymbol{
u} + oldsymbol{\Lambda} oldsymbol{\eta}_{t} + oldsymbol{arepsilon}_{t} \quad ext{with} \quad oldsymbol{arepsilon}_{t} \sim \mathcal{N}\left(\mathbf{0}, oldsymbol{\Theta}
ight)$$

where \mathbf{y}_t , $\boldsymbol{\eta}_t$, and $\boldsymbol{\varepsilon}_t$ are random variables and $\boldsymbol{\nu}$, $\boldsymbol{\Lambda}$, and $\boldsymbol{\Theta}$ are model parameters. \mathbf{y}_t is a vector of observed random variables at time t, $\boldsymbol{\eta}_t$ is a vector of latent random variables at time t, and $\boldsymbol{\varepsilon}_t$ is a vector of random measurement errors at time t, while $\boldsymbol{\nu}$ is a vector of intercept, $\boldsymbol{\Lambda}$ is a matrix of factor loadings, and $\boldsymbol{\Theta}$ is the covariance matrix of $\boldsymbol{\varepsilon}$.

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The dynamic structure is given by

$$\mathrm{d}\boldsymbol{\eta}_t = \boldsymbol{\Phi} \left(\boldsymbol{\mu} - \boldsymbol{\eta}_t \right) \mathrm{d}t + \boldsymbol{\Sigma}^{\frac{1}{2}} \mathrm{d}\mathbf{W}_t$$

where μ is the long-term mean or equilibrium level, Φ is the rate of mean reversion, determining how quickly the variable returns to its mean, Σ is the matrix of volatility or randomness in the process, and dW is a Wiener process or Brownian motion, which represents random fluctuations.

Value

Returns a list with the following elements:

- y: A t by k matrix of values for the manifest variables.
- eta: A t by p matrix of values for the latent variables.
- time: A vector of continuous time points of length t starting from 0 with delta_t increments.
- n: Number of individuals.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Uhlenbeck, G. E., & Ornstein, L. S. (1930). On the theory of the brownian motion. *Physical Review*, 36(5), 823–841. doi:10.1103/physrev.36.823

See Also

Other Simulation of State Space Models Data Functions: OU2SSM(), Sim2Matrix(), SimSSM0Fixed(), SimSSM0Vary(), SimSSM0Vary(), SimSSM0UFixed(), SimSSM0UVary(), SimSSMVARFixed(), SimSSMVARVary(), SimSSMVAR()

```
# prepare parameters
set.seed(42)
p <- k <- 2
I <- diag(p)</pre>
I_sqrt <- chol(I)</pre>
mu0 < -c(-3.0, 1.5)
sigma0_sqrt <- I_sqrt
mu < -c(5.76, 5.18)
phi <- matrix(data = c(0.10, -0.05, -0.05, 0.10), nrow = p)
sigma_sqrt <- chol(</pre>
  matrix(data = c(2.79, 0.06, 0.06, 3.27), nrow = p)
nu < -rep(x = 0, times = k)
lambda <- diag(k)</pre>
theta_sqrt <- chol(diag(x = 0.50, nrow = k))
delta_t <- 0.10
time <- 50
```

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```
burn_in <- 0
# generate data
ssm <- SimSSMOU(
 mu0 = mu0,
 sigma0_sqrt = sigma0_sqrt,
 mu = mu,
 phi = phi,
 sigma_sqrt = sigma_sqrt,
 nu = nu,
 lambda = lambda,
 theta_sqrt = theta_sqrt,
 delta_t = delta_t,
 time = time,
 burn_in = burn_in
)
str(ssm)
```

SimSSMOUFixed

Simulate Data from an Ornstein-Uhlenbeck Model using a State Space Model Parameterization for n > 1 Individuals (Fixed Parameters)

Description

This function simulates data from an Ornstein–Uhlenbeck model using a state space model parameterization for n > 1 individuals. In this model, the parameters are invariant across individuals. See details for more information.

Usage

```
SimSSMOUFixed(
    n,
    mu0,
    sigma0_sqrt,
    mu,
    phi,
    sigma_sqrt,
    nu,
    lambda,
    theta_sqrt,
    delta_t,
    time,
    burn_in
)
```

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Arguments

Positive integer. Number of individuals. n Numeric vector. Mean of initial latent variable values $(\mu_{\eta|0})$. mu0 sigma0_sqrt Numeric matrix. Cholesky decomposition of the covariance matrix of initial latent variable values $(\Sigma_{\eta|0})$. mu Numeric vector. The long-term mean or equilibrium level (μ) . Numeric matrix. The rate of mean reversion, determining how quickly the variphi able returns to its mean (Φ) . Numeric matrix. Cholesky decomposition of the matrix of volatility or randomsigma_sqrt ness in the process (Σ) . nu Numeric vector. Vector of intercepts for the measurement model (ν) . Numeric matrix. Factor loading matrix linking the latent variables to the oblambda served variables (Λ) . theta_sqrt

Numeric matrix. Cholesky decomposition of the measurement error covariance matrix (Θ) .

delta_t Numeric. Time interval (δ_t).

Positive integer. Number of time points to simulate. time

burn in Positive integer. Number of burn-in points to exclude before returning the re-

sults.

Details

The measurement model is given by

$$\mathbf{y}_{i,t} = \boldsymbol{\nu} + \boldsymbol{\Lambda} \boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t} \quad \text{with} \quad \boldsymbol{\varepsilon}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Theta}\right)$$

where $y_{i,t}$, $\eta_{i,t}$, and $\varepsilon_{i,t}$ are random variables and ν , Λ , and Θ are model parameters. $y_{i,t}$ is a vector of observed random variables at time t and individual i, $\eta_{i,t}$ is a vector of latent random variables at time t and individual i, and $\varepsilon_{i,t}$ is a vector of random measurement errors at time t and individual i, while ν is a vector of intercept, Λ is a matrix of factor loadings, and Θ is the covariance matrix of ε .

The dynamic structure is given by

$$\mathrm{d} oldsymbol{\eta}_{i,t} = oldsymbol{\Phi} \left(oldsymbol{\mu} - oldsymbol{\eta}_{i,t}
ight) \mathrm{d} t + oldsymbol{\Sigma}^{rac{1}{2}} \mathrm{d} \mathbf{W}_{i,t}$$

where μ is the long-term mean or equilibrium level, Φ is the rate of mean reversion, determining how quickly the variable returns to its mean, Σ is the matrix of volatility or randomness in the process, and dW is a Wiener process or Brownian motion, which represents random fluctuations.

Value

Returns a list of length n. Each element is a list with the following elements:

- y: A t by k matrix of values for the manifest variables.
- eta: A t by p matrix of values for the latent variables.
- time: A vector of continuous time points of length t starting from 0 with delta_t increments.
- id: A vector of ID numbers of length t.
- n: Number of individuals.

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Author(s)

Ivan Jacob Agaloos Pesigan

References

Uhlenbeck, G. E., & Ornstein, L. S. (1930). On the theory of the brownian motion. *Physical Review*, 36(5), 823–841. doi:10.1103/physrev.36.823

See Also

Other Simulation of State Space Models Data Functions: OU2SSM(), Sim2Matrix(), SimSSM0Fixed(), SimSSM0Vary(), SimSSM0Vary(), SimSSM0U(), SimSSMVARFixed(), SimSSMVARVary(), SimSSMVAR()

```
# prepare parameters
set.seed(42)
p < -k < -2
I <- diag(p)</pre>
I_sqrt <- chol(I)</pre>
n <- 5
mu0 < -c(-3.0, 1.5)
sigma0_sqrt <- I_sqrt</pre>
mu <- c(5.76, 5.18)
phi <- matrix(data = c(0.10, -0.05, -0.05, 0.10), nrow = p)
sigma_sqrt <- chol(</pre>
  matrix(data = c(2.79, 0.06, 0.06, 3.27), nrow = p)
)
nu \leftarrow rep(x = 0, times = k)
lambda <- diag(k)</pre>
theta_sqrt <- chol(diag(x = 0.50, nrow = k))
delta_t <- 0.10
time <- 50
burn_in <- 0</pre>
# generate data
ssm <- SimSSMOUFixed(</pre>
  n = n,
  mu0 = mu0,
  sigma0_sqrt = sigma0_sqrt,
  mu = mu,
  phi = phi,
  sigma_sqrt = sigma_sqrt,
  nu = nu,
  lambda = lambda,
  theta_sqrt = theta_sqrt,
  delta_t = delta_t,
  time = time,
  burn_in = burn_in
)
```

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str(ssm)

SimSSMOUVary

Simulate Data from an Ornstein-Uhlenbeck Model using a State Space Model Parameterization for n > 1 Individuals (Varying Parameters)

Description

This function simulates data from an Ornstein–Uhlenbeck model using a state space model parameterization for n > 1 individuals. In this model, the parameters can vary across individuals.

Usage

```
SimSSMOUVary(
    n,
    mu0,
    sigma0_sqrt,
    mu,
    phi,
    sigma_sqrt,
    nu,
    lambda,
    theta_sqrt,
    delta_t,
    time,
    burn_in
)
```

Arguments

| n | Positive integer. Number of individuals. |
|-------------|---|
| mu0 | Numeric vector. Mean of initial latent variable values $(\mu_{\eta 0})$. |
| sigma0_sqrt | Numeric matrix. Cholesky decomposition of the covariance matrix of initial latent variable values ($\Sigma_{\eta 0}$). |
| mu | List of numeric vectors. The long-term mean or equilibrium level (μ) . |
| phi | List of numeric matrices. The rate of mean reversion, determining how quickly the variable returns to its mean (Φ) . |
| sigma_sqrt | List of numeric matrices. Cholesky decomposition of the matrix of volatility or randomness in the process (Σ) . |
| nu | Numeric vector. Vector of intercepts for the measurement model (ν) . |
| lambda | Numeric matrix. Factor loading matrix linking the latent variables to the observed variables (Λ). |
| theta_sqrt | Numeric matrix. Cholesky decomposition of the measurement error covariance matrix (Θ) . |

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| delta_t | Numeric. Time interval (δ_t) . |
|---------|--|
| time | Positive integer. Number of time points to simulate. |
| burn_in | Positive integer. Number of burn-in points to exclude before returning the re- |

Details

Parameters can vary across individuals by providing a list of parameter values. If the length of any of the parameters (mu0, sigma0_sqrt, mu, phi, sigma_sqrt, nu, lambda, theta_sqrt) is less the n, the function will cycle through the available values.

Value

Returns a list of length n. Each element is a list with the following elements:

- y: A t by k matrix of values for the manifest variables.
- eta: A t by p matrix of values for the latent variables.
- time: A vector of discrete time points from 1 to t.
- id: A vector of ID numbers of length t.

sults.

• n: Number of individuals.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Uhlenbeck, G. E., & Ornstein, L. S. (1930). On the theory of the brownian motion. *Physical Review*, 36(5), 823–841. doi:10.1103/physrev.36.823

See Also

```
Other Simulation of State Space Models Data Functions: OU2SSM(), Sim2Matrix(), SimSSM0Fixed(), SimSSM0Vary(), SimSSM0(), SimSSM0UFixed(), SimSSM0U(), SimSSMVARFixed(), SimSSMVARVary(), SimSSMVAR()
```

```
# prepare parameters
# In this example, phi varies across individuals
set.seed(42)
p <- k <- 2
iden <- diag(p)
iden_sqrt <- chol(iden)
n <- 5
mu0 <- list(c(-3.0, 1.5))
sigma0_sqrt <- list(iden_sqrt)
mu <- list(c(5.76, 5.18))
phi <- list(</pre>
```

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```
as.matrix(Matrix::expm(diag(x = -0.1, nrow = k))),
  as.matrix(Matrix::expm(diag(x = -0.2, nrow = k))),
  as.matrix(Matrix::expm(diag(x = -0.3, nrow = k))),
  as.matrix(Matrix::expm(diag(x = -0.4, nrow = k))),
  as.matrix(Matrix::expm(diag(x = -0.5, nrow = k)))
)
sigma_sqrt <- list(</pre>
  chol(
    matrix(data = c(2.79, 0.06, 0.06, 3.27), nrow = p)
)
nu \leftarrow list(rep(x = 0, times = k))
lambda <- list(diag(k))</pre>
theta_sqrt <- list(chol(diag(x = 0.50, nrow = k)))
delta_t <- 0.10
time <- 50
burn_in <- 0
ssm <- SimSSMOUVary(</pre>
  n = n,
  mu0 = mu0,
  sigma0_sqrt = sigma0_sqrt,
  mu = mu,
  phi = phi,
  sigma_sqrt = sigma_sqrt,
  nu = nu,
  lambda = lambda,
  theta_sqrt = theta_sqrt,
  delta_t = delta_t,
  time = time,
  burn_in = burn_in
)
str(ssm)
```

SimSSMVAR

Simulate Data from the Vector Autoregressive Model using a State Space Model Parameterization (n = 1)

Description

This function simulates data from the vector autoregressive model using a state space model parameterization. See details for more information.

Usage

```
SimSSMVAR(mu0, sigma0_sqrt, alpha, beta, psi_sqrt, time, burn_in)
```

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Arguments

| mu0 | Numeric vector. Mean of initial latent variable values $(\mu_{\eta 0})$. |
|-------------|---|
| sigma0_sqrt | Numeric matrix. Cholesky decomposition of the covariance matrix of initial latent variable values $(\Sigma_{\eta 0})$. |
| alpha | Numeric vector. Vector of intercepts for the dynamic model (α) . |
| beta | Numeric matrix. Transition matrix relating the values of the latent variables at time $t-1$ to those at time $t(\beta)$. |
| psi_sqrt | Numeric matrix. Cholesky decomposition of the process noise covariance matrix (Ψ) . |
| time | Positive integer. Number of time points to simulate. |
| burn_in | Positive integer. Number of burn-in points to exclude before returning the results. |

Details

The measurement model is given by

$$\mathbf{y}_t = \boldsymbol{\eta}_t$$
.

The dynamic structure is given by

$$oldsymbol{\eta}_t = oldsymbol{lpha} + oldsymbol{eta} oldsymbol{\eta}_{t-1} + oldsymbol{\zeta}_t \quad ext{with} \quad oldsymbol{\zeta}_t \sim \mathcal{N}\left(oldsymbol{0}, oldsymbol{\Psi}
ight)$$

where η_t, η_{t-1} , and ζ_t are random variables and α , β , and Ψ are model parameters. η_t is a vector of latent variables at time t, η_{t-1} is a vector of latent variables at t-1, and ζ_t is a vector of dynamic noise at time t while α is a vector of intercepts, β is a matrix of autoregression and cross regression coefficients, and Ψ is the covariance matrix of ζ_t .

Value

Returns a list with the following elements:

- y: A t by k matrix of values for the manifest variables.
- eta: A t by p matrix of values for the latent variables.
- time: A vector of discrete time points from 1 to t.
- n: Number of individuals.

References

Shumway, R. H., & Stoffer, D. S. (2017). *Time series analysis and its applications: With R examples*. Springer International Publishing. doi:10.1007/9783319524528

See Also

Other Simulation of State Space Models Data Functions: OU2SSM(), Sim2Matrix(), SimSSM0Fixed(), SimSSM0Vary(), SimSSM0U(), SimSSM0UVary(), SimSSM0U(), SimSSMVARFixed(), SimSSMVARVary()

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Examples

```
# prepare parameters
set.seed(42)
k < -3
I \leftarrow diag(k)
I_sqrt <- chol(I)</pre>
null_vec \leftarrow rep(x = 0, times = k)
mu0 <- null_vec
sigma0_sqrt <- I_sqrt
alpha <- null_vec</pre>
beta \leftarrow diag(x = 0.5, nrow = k)
psi_sqrt <- I_sqrt</pre>
time <- 50
burn_in <- 0
# generate data
ssm <- SimSSMVAR(
  mu0 = mu0,
  sigma0_sqrt = sigma0_sqrt,
  alpha = alpha,
  beta = beta,
  psi_sqrt = psi_sqrt,
  time = time,
  burn_in = burn_in
)
str(ssm)
```

SimSSMVARFixed

Simulate Data from a Vector Autoregressive Model using a State Space Model Parameterization for n > 1 Individuals (Fixed Parameters)

Description

This function simulates data from a vector autoregressive model using a state space model parameterization for n > 1 individuals. In this model, the parameters are invariant across individuals.

Usage

```
SimSSMVARFixed(n, mu0, sigma0_sqrt, alpha, beta, psi_sqrt, time, burn_in)
```

Arguments

n Positive integer. Number of individuals.

mu0 Numeric vector. Mean of initial latent variable values $(\mu_{\eta|0})$.

sigma0_sqrt Numeric matrix. Cholesky decomposition of the covariance matrix of initial

latent variable values $(\Sigma_{\eta|0})$.

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| alpha | Numeric vector. Vector of intercepts for the dynamic model (α) . |
|----------|---|
| beta | Numeric matrix. Transition matrix relating the values of the latent variables at time $t-1$ to those at time $t(\beta)$. |
| psi_sqrt | Numeric matrix. Cholesky decomposition of the process noise covariance matrix (Ψ) . |
| time | Positive integer. Number of time points to simulate. |
| burn_in | Positive integer. Number of burn-in points to exclude before returning the results. |

Value

Returns a list of length n. Each element is a list with the following elements:

- y: A t by k matrix of values for the manifest variables.
- eta: A t by p matrix of values for the latent variables.
- time: A vector of discrete time points from 1 to t.
- id: A vector of ID numbers of length t.
- n: Number of individuals.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Shumway, R. H., & Stoffer, D. S. (2017). *Time series analysis and its applications: With R examples*. Springer International Publishing. doi:10.1007/9783319524528

See Also

```
Other Simulation of State Space Models Data Functions: OU2SSM(), Sim2Matrix(), SimSSM0Fixed(), SimSSM0Vary(), SimSSM0(), SimSSM0UFixed(), SimSSM0UVary(), SimSSMOU(), SimSSMVARVary(), SimSSMVAR()
```

```
# prepare parameters
set.seed(42)
k <- 3
iden <- diag(k)
iden_sqrt <- chol(iden)
null_vec <- rep(x = 0, times = k)
n <- 5
mu0 <- null_vec
sigma0_sqrt <- iden_sqrt
alpha <- null_vec
beta <- diag(x = 0.5, nrow = k)
psi_sqrt <- iden_sqrt
time <- 50</pre>
```

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```
burn_in <- 0

ssm <- SimSSMVARFixed(
    n = n,
    mu0 = mu0,
    sigma0_sqrt = sigma0_sqrt,
    alpha = alpha,
    beta = beta,
    psi_sqrt = psi_sqrt,
    time = time,
    burn_in = burn_in
)

str(ssm)</pre>
```

SimSSMVARVary

Simulate Data from a Vector Autoregressive Model using a State Space Model Parameterization for n > 1 Individuals (Varying Parameters)

Description

This function simulates data from a vector autoregressive model using a state space model parameterization for n > 1 individuals. In this model, the parameters can vary across individuals.

Usage

```
SimSSMVARVary(n, mu0, sigma0_sqrt, alpha, beta, psi_sqrt, time, burn_in)
```

Arguments

| n | Positive integer. Number of individuals. |
|-------------|---|
| mu0 | List of numeric vectors. Mean of initial latent variable values $(\mu_{\eta 0})$. |
| sigma0_sqrt | List of numeric matrices. Cholesky decomposition of the covariance matrix of initial latent variable values $(\Sigma_{\eta 0})$. |
| alpha | List of numeric vectors. Vector of intercepts for the dynamic model (α) . |
| beta | List of numeric matrices. Transition matrix relating the values of the latent variables at time $t-1$ to those at time $t(\beta)$. |
| psi_sqrt | List of numeric matrices. Cholesky decomposition of the process noise covariance matrix (Ψ) . |
| time | Positive integer. Number of time points to simulate. |
| burn_in | Positive integer. Number of burn-in points to exclude before returning the results. |

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Details

Parameters can vary across individuals by providing a list of parameter values. If the length of any of the parameters (mu0, sigma0_sqrt, alpha, beta, and psi_sqrt) is less the n, the function will cycle through the available values.

Value

Returns a list of length n. Each element is a list with the following elements:

- y: A t by k matrix of values for the manifest variables.
- eta: A t by p matrix of values for the latent variables.
- time: A vector of discrete time points from 1 to t.
- id: A vector of ID numbers of length t.
- n: Number of individuals.

Author(s)

Ivan Jacob Agaloos Pesigan

References

Shumway, R. H., & Stoffer, D. S. (2017). *Time series analysis and its applications: With R examples*. Springer International Publishing. doi:10.1007/9783319524528

See Also

```
Other Simulation of State Space Models Data Functions: OU2SSM(), Sim2Matrix(), SimSSM0Fixed(), SimSSM0Vary(), SimSSM0U(), SimSSMOUFixed(), SimSSMOUVary(), SimSSMOU(), SimSSMVARFixed(), SimSSMVAR()
```

```
# prepare parameters
# In this example, beta varies across individuals
set.seed(42)
k <- 3
iden <- diag(k)
iden_sqrt <- chol(iden)</pre>
null\_vec \leftarrow rep(x = 0, times = k)
n <- 5
mu0 <- list(null_vec)</pre>
sigma0_sqrt <- list(iden_sqrt)</pre>
alpha <- list(null_vec)</pre>
beta <- list(
  diag(x = 0.1, nrow = k),
  diag(x = 0.2, nrow = k),
  diag(x = 0.3, nrow = k),
  diag(x = 0.4, nrow = k),
  diag(x = 0.5, nrow = k)
)
```

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```
psi_sqrt <- list(iden_sqrt)
time <- 50
burn_in <- 0

ssm <- SimSSMVARVary(
    n = n,
    mu0 = mu0,
    sigma0_sqrt = sigma0_sqrt,
    alpha = alpha,
    beta = beta,
    psi_sqrt = psi_sqrt,
    time = time,
    burn_in = burn_in
)

str(ssm)</pre>
```

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