Untitled

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6. An email message can travel through one of three server routes. The percentage of errors in each of the servers and the percentage of messages that travel through each route are shown in the following table. Assume that the servers are independent.

(a) What is the probability of receiving an email containing an error?
(b) What is the probability that a message will arrive without error?
(c) If a message arrives without error, what is the probability that it was sent through server 1?

```
Server_Routes <- c("Server 1", "Server 2", "Server 3")
Percentage_of_Messages <- c(40, 25, 35)
Percentage_of_Errors <- c(1, 2, 1.5)
Messages_with_Errors <- (Percentage_of_Messages / 100) *
  (Percentage_of_Errors / 100) * 100
Messages_without_Errors <- (Percentage_of_Messages / 100) *
  (1 - (Percentage_of_Errors / 100)) * 100
Total_Messages_with_Errors <- sum(Messages_with_Errors)
Total_Messages_without_Errors <- sum(Messages_without_Errors)

df <- data.frame(Server_Routes, Percentage_of_Messages, Percentage_of_Errors,
                 Messages_with_Errors, Messages_without_Errors)
final.df <- rbind(df, c("Total", NA, NA, Total_Messages_with_Errors,
                        Total_Messages_without_Errors))
print(final.df)
```

```
##   Server_Routes Percentage_of_Messages Percentage_of_Errors
## 1      Server 1                    40                   1
## 2      Server 2                    25                   2
## 3      Server 3                    35                 1.5
## 4         Total                  <NA>                <NA>
##   Messages_with_Errors Messages_without_Errors
## 1                  0.4                   39.6
## 2                  0.5                   24.5
## 3                0.525                 34.475
## 4                1.425                 98.575
```

```
cat(sprintf("The probability of receiving an email containing an error is %.5f",
            Total_Messages_with_Errors/100))
```

```
## The probability of receiving an email containing an error is 0.01425
```

```r
cat(sprintf("\nThe probability that a message will arrive without error is %.5f",
            Total_Messages_without_Errors/100))
```

```
##
## The probability that a message will arrive without error is 0.98575
```

```r
cat(sprintf("\nThe probability that a message without error was sent through
            server 1 is %.3f", Messages_without_Errors[1] / Total_Messages_without_Errors))
```

```
##
## The probability that a message without error was sent through
##             server 1 is 0.402
```

9. A software company surveyed managers to determine the probability that they would buy a new graphics package that includes three-dimensional graphics. About 20% of office managers were certain that they would not buy the package, 70% claimed that they would buy, and the others were undecided. Of those who said that they would not buy the package, only 10% said that they were interested in upgrading their computer hardware. Of those interested in buying the graphics package, 40% were also interested in upgrading their computer hardware. Of the undecided, 20% were interested in upgrading their computer hardware.

Let A denote the intention of not buying, B the intention of buying, C the undecided, and G the intention of upgrading the computer hardware. (a) Calculate the probability that a manager chosen at random will not upgrade the computer hardware (P(complement of G)).

(b) Explain what is meant by the posterior probability of B given G, P(B given G).

(c) Construct a tree diagram and use it to calculate the following probabilities: P(G), P(B given G), P(B given complement of G), P(C given G), P(C given complement of G).

```r
prob_A <- 0.2
prob_B <- 0.7
prob_C <- 1 - prob_A - prob_B

prob_G_given_A <- 0.1
prob_G_given_B <- 0.4
prob_G_given_C <- 0.2

prob_complement_G_given_A <- 1 - prob_G_given_A
prob_complement_G_given_B <- 1 - prob_G_given_B
prob_complement_G_given_C <- 1 - prob_G_given_C

total_prob <- ((prob_G_given_A) * prob_A) + ((prob_G_given_B) * prob_B) +
  ((prob_G_given_C) * prob_C)
post_B <- ((prob_G_given_B) * prob_B) / total_prob
post_B_complement_G <- ((prob_complement_G_given_B) * prob_B) / (1 - total_prob)
post_C <- ((prob_G_given_C) * prob_C) / total_prob
post_C_complement_G <- ((prob_complement_G_given_C) * prob_C) / (1 - total_prob)

cat(sprintf("The probability that a manager will not upgrade the computer
            hardware is: %.2f", 1 - total_prob))
```

```
## The probability that a manager will not upgrade the computer
##             hardware is: 0.68
```

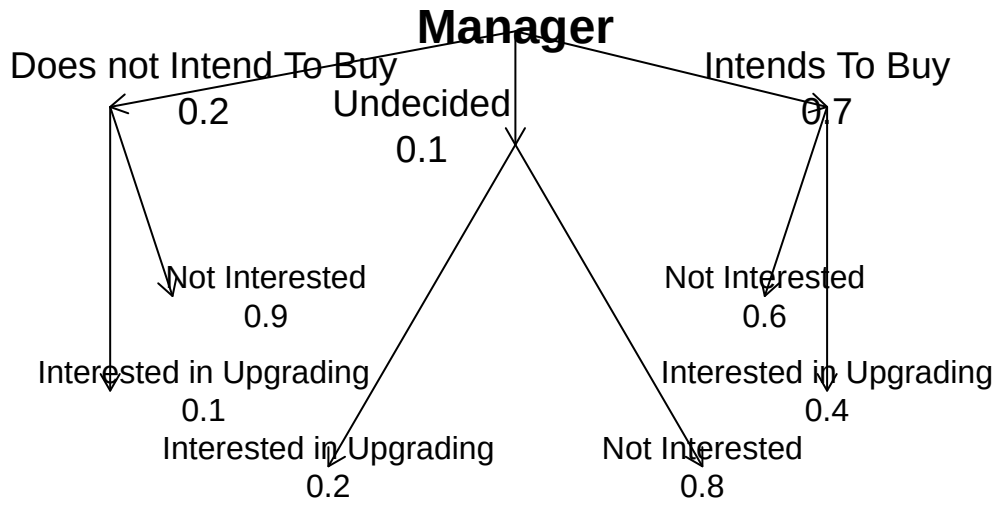
```
print("The posterior probability of B given G is the probability of a manager
      having the intention of buying after it has been determined that they
      have intentions of upgrading.")
```

```
## [1] "The posterior probability of B given G is the probability of a manager \n      having the inten
```

```
cat(sprintf("The posterior probability of B given G is: %.3f", post_B))
```

```
## The posterior probability of B given G is: 0.875
```

```
plot_tree <- function() {
  plot(1, 1, type = "n", xlab = "", ylab = "", xlim = c(0, 30), ylim = c(0, 25), axes = FALSE)

  text(15, 24, "Manager", cex = 1.5, font = 2)

  text(5, 21, "Does not Intend To Buy\n0.2", cex = 1.2)
  arrows(15, 24, 2, 20, length = 0.1)
  text(5, 5, "Interested in Upgrading\n0.1", cex = 1)
  arrows(2, 20, 2, 5, length = 0.1)
  text(7, 10, "Not Interested\n0.9", cex = 1)
  arrows(2, 20, 4, 10, length = 0.1)


  text(25, 21, "Intends To Buy\n0.7", cex = 1.2)
  arrows(15, 24, 25, 20, length = 0.1)
  text(25, 5, "Interested in Upgrading\n0.4", cex = 1)
  arrows(25, 20, 25, 5, length = 0.1)
  text(23, 10, "Not Interested\n0.6", cex = 1)
  arrows(25, 20, 23, 10, length = 0.1)

  text(12, 19, "Undecided\n0.1", cex = 1.2)
  arrows(15, 24, 15, 18, length = 0.1)
  text(9, 1, "Interested in Upgrading\n0.2", cex = 1)
  arrows(15, 18, 9, 1, length = 0.1)
  text(21, 1, "Not Interested\n0.8", cex = 1)
  arrows(15, 18, 21, 1, length = 0.1)
}

plot_tree()
```




```
cat(sprintf("The probability of event G is: %.3f", total_prob))
```

```
## The probability of event G is: 0.320
```

```
cat(sprintf("The posterior probability of B given G is: %.3f", post_B))
```

```
## The posterior probability of B given G is: 0.875
```

```
cat(sprintf("The posterior probability of B given complement of G is: %.3f",
            post_B_complement_G))
```

```
## The posterior probability of B given complement of G is: 0.618
```

```
cat(sprintf("The posterior probability of C given G is: %.3f", post_C))
```

```
## The posterior probability of C given G is: 0.063
```

```
cat(sprintf("The posterior probability of C given complement of G is: %.3f",
            post_C_complement_G))
```

```
## The posterior probability of C given complement of G is: 0.118
```

13. A malicious spyware can infect a computer system though the Internet or through email. The spyware comes through the Internet 70% of the time and 30% of the time, it gets in through email. If it enters via the Internet the anti-virus detector will detect it with probability 0.6, and via email, it is detected with probability 0.8.

(a) What is the probability that this spyware infects the system?

(b) If the spyware is detected, what is the probability that it came through the Internet?

```
Entered_Via <- c("Internet", "Email")
Percentage_of_Spyware <- c(0.7, 0.3)
Anti_Virus_Detect_Probability <- c(0.6, 0.8)
Virus_Detected <- Percentage_of_Spyware * Anti_Virus_Detect_Probability
Virus_Infect <- Percentage_of_Spyware * (1-Anti_Virus_Detect_Probability)
Total_Virus_Detected <- sum(Virus_Detected)
Total_Virus_Infect <- sum(Virus_Infect)

df <- data.frame(Entered_Via, Percentage_of_Spyware * 100,
                 Anti_Virus_Detect_Probability, Virus_Detected, Virus_Infect)
final.df <- rbind(df, c("Total", NA, NA, Total_Virus_Detected, Total_Virus_Infect))
print(final.df)
```

```
##   Entered_Via Percentage_of_Spyware...100 Anti_Virus_Detect_Probability
## 1    Internet                          70                            0.6
## 2       Email                          30                            0.8
## 3       Total                        <NA>                          <NA>
##   Virus_Detected Virus_Infect
## 1           0.42         0.28
## 2           0.24         0.06
## 3           0.66         0.34
```

```
cat(sprintf("The probability of spyware infecting the system is %.2f",
            Total_Virus_Infect))
```

```
## The probability of spyware infecting the system is 0.34
```

```
cat(sprintf("\nIf the spyware is detected, the probability it came through the
            internet is %.2f", Virus_Infect[1]/Total_Virus_Infect))
```

```
##
## If the spyware is detected, the probability it came through the
##             internet is 0.82
```