Technical University of Madrid

Master in Data Science

PRACTICAL APPLICATION 4

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Introduction

This practical application focuses on probabilistic graphical models, specifically Bayesian Networks, to explore and analyze the relationships among variables in a dataset. The primary aim is to construct and interpret a directed acyclic graph (DAG) that represents the probabilistic dependencies between variables and to use the model for exact and approximate inference. Bayesian Networks offer a robust and interpretable framework for understanding complex systems where variables interact probabilistically. To achieve this, different approaches for learning the structure will be explored, combining constraint-based and score-and-search methods, as well as maximum likelihood estimation and Bayesian estimation for learning the parameters. This exploration aims to identify the most effective way to uncover meaningful patterns and dependencies, providing insights into the probabilistic relationships and conditional independencies among the variables. This approach highlights the flexibility and utility of Bayesian Networks in modeling categorical data and performing probabilistic reasoning.

Problem description

The dataset used in this application is the Car Evaluation dataset, which assesses the acceptability of cars based on multiple categorical criteria. It consists of 1,728 instances. The dataset is entirely composed of discrete features, making it well-suited for Bayesian Network modeling. The dataset consists of seven features: buying (buying price), categorized as low, med, high, or vhigh (very high); maint (maintenance cost), also categorized as low, med, high, or vhigh; doors (number of doors), with values 2, 3, 4, or 5more; persons, representing seating capacity, categorized as 2, 4, or more; lug_boot (luggage boot size), categorized as small, med (medium), or big; safety, indicating safety levels as low, med, or high; and car, which reflects the overall acceptability of the vehicle with values unacc (unacceptable), acc (acceptable), good, or vgood (very good).

The objective of this application is to construct and analyze a Bayesian Network to uncover the probabilistic dependencies among these features and identify how they interact. The learned network will also be used to perform inferences, such as estimating probabilities under specific conditions or exploring the effects of certain variables on others.

Methodology

The tool that was used for this application is Python, precisely package pgmpy. Considering that the dataset has no missing values and data is uniform, no preprocessing is necessary. To explore how different learning techniques affect the structure and parameterization of the Bayesian Network, four networks were constructed using combinations of parameter learning and structure learning methods. No background knowledge was initially added. All queries were done using exact inference, unless stated otherwise.

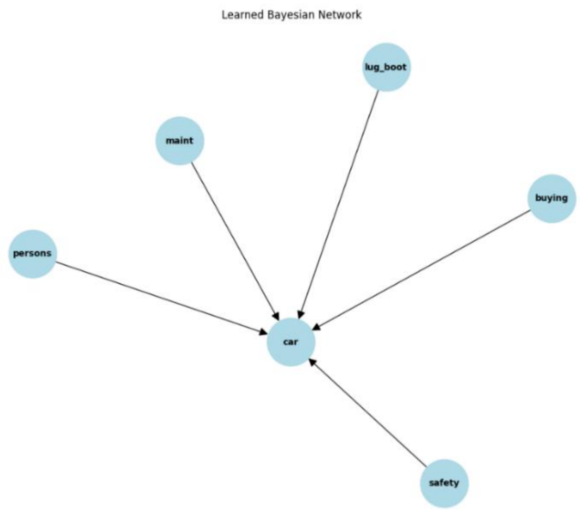
For parameter learning, Maximum Likelihood Estimation was used in two networks because the dataset contains enough observations to reliably estimate conditional probabilities directly from the data. MLE ensures that the Conditional Probability Tables closely reflect the observed frequencies in the dataset without relying on priors. In contrast, Bayesian Estimation was applied in the other two networks to incorporate a regularization effect through priors. This method is particularly useful for avoiding overfitting, especially in cases where some combinations of variables may have sparse data due to the categorical nature of the features.

For structure learning, both constraint-based and score-and-search approaches were employed to capture different perspectives on the relationships within the dataset. The PC Algorithm was selected for the constraint-based approach as it efficiently identifies conditional independencies, making it suitable for datasets with a moderate number of variables. This method is particularly advantageous for discovering dependencies in categorical variables, such as how safety and buying prices interact to influence acceptability. For the score-and-search approach, when combining MLE with score-and-search methods, the Hill Climbing algorithm paired with the K2 scoring metric is a suitable choice. The K2Score evaluates the fit of the structure by considering the data likelihood while assuming a uniform prior, focusing on simplicity and efficiency. Finally, for Bayesian Estimation and score-and-search methods, the Hill Climbing algorithm with a Bayesian Dirichlet Equivalent (BDe) score was used, as it combines Bayesian principles with an efficient search for optimal network structures.

To present the results, the dependencies among variables will be visualized using networkx package from Python. For performing exact inference, the variable elimination algorithm was used, and for approximate inference, likelihood weighted sampling and forward sampling.

Results

Maximum likelihood estimation and PC algorithm



Picture 1. Network learned with MLE and PC algorithm

The nodes represent features such as buying, maint, doors, persons, lug_boot, safety, and car, while the arcs indicate the dependencies between them. The node car is central to the network, receiving arrows from multiple variables, indicating that it is heavily influenced by factors like persons, lug_boot, safety, buying and maint. These relationships align with domain intuition. The absence of the 'doors' variable may result from its low variability, redundancy with other variables like lug_boot or persons, or limitations in the PC algorithm's conditional independence tests. The PC algorithm's reliance on statistical tests may have been affected by the limited sample size or low variability in certain variables, such as doors, reducing its ability to detect relationships. Overall, the relationships follow a real-life logic, the state of the car is heavily influenced by all these factors, although a few more relationships that do exist in real life between nodes like buying and maint, safety and persons etc. would make a network more complete.

The conditional probability distributions reveal key relationships among variables, with lug_boot, safety, maint, buying, and persons showing uniform distributions due to their independence, suggesting the dataset lacks detailed context to capture real-world complexity. In contrast, car depends heavily on its parent variables. The tables show that cars with only two seats are consistently classified as unacceptable, while those with higher passenger capacity and safety ratings are deemed acceptable. The dominance of car(unacc) highlights stringent evaluation criteria, mirroring real-world consumer behavior where cars are judged harshly on practicality and safety. While the network aligns with general trends, such as the prioritization of safety and capacity, the uniform distributions suggest it oversimplifies consumer preferences, missing nuances shaped by specific contexts or demographics. The network offers an insightful but simplified representation of real-world car evaluations.

The computed independencies and d-separation tests highlight the intricate relationships. Initially, these variables are independent of each other, but specific observations introduce conditional independencies. Observing safety and lug_boot makes maint independent of buying and persons, showing how observed factors dominate the influence on maint. Similarly, persons becomes independent of lug_boot when maint, safety, and buying are observed, reducing the complexity of their relationships. These patterns illustrate the mediating role of variables like safety, maint, or car, which create dependencies, such as between lug_boot and persons or buying and safety, while blocking pathways in other contexts. These results align with broader patterns of conditional independencies, emphasizing the dynamic interplay between dependencies and independencies that shape the flow of information within the network. Observed factors like safety and luggage space often dominate others like maintenance and price in real-life decisions.

The Markov blanket analysis for this network reveals that every node's blanket includes all other nodes, suggesting a densely connected structure with no apparent simplification at first glance. However, a closer look at the conditional independencies found through d-separation tests indicates that the network captures significant underlying dependencies and independencies.

Predictive reasoning results show how evidence shifts car classification from baseline probabilities, where car(unacc) dominates at 70%, and car(acc) (22.2%), car(good) (3.99%), and car(vgood) (3.76%) are much less likely. Evidence such as safety = high raises car(acc) to 35.4% and car(vgood) to 11.3%, reducing car(unacc) to 48.1%, highlighting safety’s impact on improving classification. However, when persons = more and safety = low, car is classified as car(unacc) with 100% certainty, showing that low safety negates higher capacity benefits. Balanced evidence, such as buying=low, maint=med, persons=4 and safety=med, safety, results in a 50% likelihood for both car(acc) and car(good), eliminating unacceptable outcomes. In cases with maint=low, safety=high, and persons=4 and buying=high, car is classified as car(acc) with 100% certainty, showing that premium features ensure acceptability. These results emphasize how evidence influences classifications, with safety, maintenance, and capacity playing key roles.

The results of diagnostic reasoning reveal interesting shifts in probabilities driven by evidence. With car = 'unacc', the strong increase of 14.27% in persons(2) and safety(low) highlights a strong link between unacceptable cars, low safety, and limited seating capacity. With car = 'acc', safety(high) rose by 19.79%, and persons(4) increased by 18.23%, indicating that acceptable cars are closely associated with higher safety and family suitability. With car = 'vgood', the dramatic rise of 66.67% in safety(high) to 100% and the 28.21% increase in lug_boot(big) emphasize the dominant perception of very good cars as being exceptionally safe and spacious.

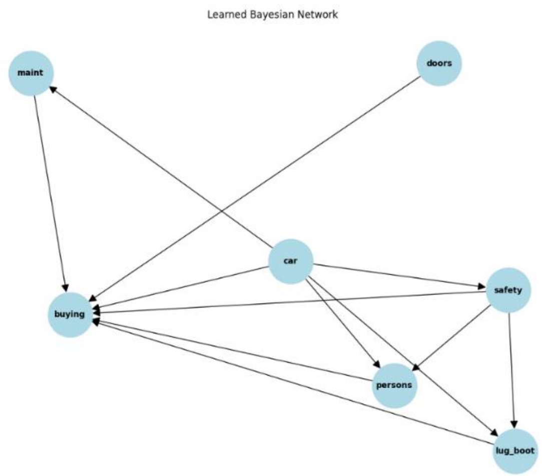
With car = 'good', buying(low) and maint(low) increased significantly by 41.67%, showing that affordability is a defining characteristic of good cars, while safety(med) rose by 23.19%, suggesting moderate safety aligns well with this category.

The intercausal reasoning results reveal unexpected insights into the relationships in the network. With car = acc, buying = low, safety(med) increased significantly by 29.59%, while safety(low) dropped to 0.00%, suggesting that low buying costs are associated with moderate to high safety levels rather than low safety, which might initially seem counterintuitive. This scenario was chosen to explore how affordability influences acceptability and interacts with safety and maintenance. With car = vgood, maint = low, safety(high) increased by 66.67% to 100.00%, and lug_boot(small) dropped entirely to 0.00%, indicating that very good cars are perceived as needing high safety and large storage when maintenance is low. This scenario was designed to test how high-quality cars align with affordability in maintenance and practical features like storage. Finally, with car = good, lug_boot = big, buying(low) rose by 41.67%, showing a strong connection between affordability and cars with large storage. This scenario was selected to analyze how practical features like storage interact with affordability and safety in defining good cars.

Using maximum a posteriori, two scenarios were queried. The first explored the most probable states of buying, maint, and safety when the car is classified as "very good," showing that such cars typically have "low" buying cost, "high" maint, and "high" safety. This balance between affordability and safety reflects key attributes valued in the dataset. The second scenario assessed the car's classification under optimized conditions (buying = low, maint = low, safety = high), resulting in car = unacc. This suggests that even ideal features often lead to "unacceptable" classifications, indicating possible biases in the dataset or disproportionate influence of other factors.

The sensitivity analysis assessed the impact of maint and lug_boot on car classifications. In the first scenario, it was tested how the probability of a car being "very good" changes with maint levels, given favorable conditions (buying = low, safety = high). The constant probability showed that maint has minimal influence. In the second scenario, the effect of lug_boot was evaluated on the probability of a car being "acceptable", given persons = 4. Uniform probabilities indicated lug_boot is not a significant factor.

Maximum likelihood estimation and Hill climbing algorithm with K2 scoring



Picture 2. Network learned using MLE and Hill climbing algorithm

The network learned using the Hill Climbing algorithm with MLE highlights car as the central node, influencing variables like safety, persons, buying, maint, and lug_boot, reflecting its critical role in car evaluations. Buying is directly shaped by maint, doors, persons, lug_boot, and safety, demonstrating how purchasing decisions are influenced by interconnected factors. The strong influence of safety on persons, lug_boot, and buying underscores its central importance, as consumers prioritize well-being and usability in their choices. Compared to the PC algorithm's sparser network, this model incorporates the doors variable and richer connections, capturing the complexity of real-life decision-making where factors like capacity, long-term affordability, and safety dynamically interact to shape outcomes. By reflecting these multifaceted relationships, the network mirrors the subtle priorities of consumers, offering a realistic and comprehensive view of the car evaluation process.

From the conditional probability distributions it is visible that the variable maint strongly influences other aspects of the network, with significant shifts in probabilities, confirming its central role in shaping connected variables. Safety shows clear differences in its influence: high safety leads to higher probabilities of positive states across multiple dimensions, while low safety strongly correlates with less desirable outcomes. This suggests that safety acts as a critical factor, with its variations significantly shaping other probabilities. The variables persons and lug_boot are evenly distributed in some contexts but show distinct probabilities based on combinations with other variables. For example, higher passenger capacity and larger luggage size align with more favorable conditions, reflecting their practical importance in the system.

The computed independencies and d-separation scenarios reveal a comprehensive view of how variables interact. Variables like persons and doors are independent in most cases, even when conditioned on safety or lug_boot, reflecting their limited direct interaction. Similarly, safety and lug_boot are independent of doors across various contexts, and maint and doors show independence unless conditioned on overarching factors like car. These patterns highlight the modular nature of the network, where structural and maintenance-related attributes are largely unrelated. However, the d-separation scenarios uncover subtle dependencies, such as lug_boot and persons being d-connected when conditioned on safety, reflecting real-life scenarios where passenger capacity and luggage space jointly influence safety perceptions. Likewise, maint and safety are connected through lug_boot, illustrating how practical considerations like luggage capacity can link maintenance costs and safety concerns through wear and performance. These insights mirror real-world decision-making processes that balances distinct evaluations with meaningful interdependencies.

The Markov blanket analysis highlights the unique role of maint as a variable tightly connected to all others, emphasizing its critical function in balancing safety, practicality, and cost—key elements in car evaluations. Similarly, the consistent presence of car across all blankets underscores its central role as a mediator. Notably, the inclusion of doors and persons in multiple blankets reflects their foundational influence on the network, suggesting these variables serve as essential indicators of practicality and usability in real-life decision-making.

The results of predictive reasoning reveal that when car=acc, the probability of safety(high) increases 19.79%, while safety(low) drops entirely to 0%. This aligns with real-life expectations, where cars deemed acceptable often prioritize higher safety standards. When car=unacc and safety=med, the probability of persons(2) rises sharply for 20.45%, while persons(4) and persons(more) decrease, indicating that cars classified as unacceptable are strongly associated with limited seating capacity. This reflects how practical features like seating often correlate with overall car evaluations in reality. When car=good and maint=med, the probability of buying(low) increases dramatically for 61.17%, with all other categories dropping to 4.70%. This suggests that good cars with moderate maintenance costs are overwhelmingly perceived as affordable, aligning with real-world preferences where consumers associate quality with reasonable costs.

The results of diagnostic reasoning reveal that when lug_boot = small, the probability of car(unacc) increases by 8.10%, while car(vgood) drops entirely to 0%. This scenario illustrates how limited luggage space impacts acceptability, as small storage capacity can significantly diminish a car's appeal for families or travelers who require practicality.

The intercausal reasoning results reveal significant insights. When lug_boot = big and car = unacc, the probability of safety(low) increases sharply by 18.84%, while safety(high) and safety(med) both drop equally by 9.42%. This scenario examines the trade-offs between practicality (large luggage space) and safety in unacceptable cars. The result highlights that consumers often associate poor safety with cars emphasizing practical features but failing in essential safety standards. This reflects real-life tendencies where practical designs may not compensate for inadequate safety, particularly in lower-quality vehicles.

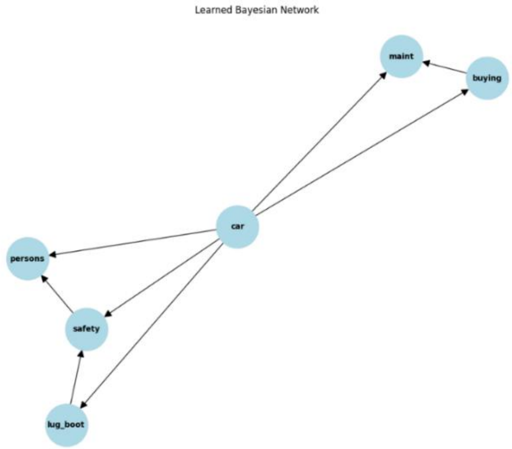
One of the most interesting results for a MAP query is for car='unacc' with lug_boot='small', where the most probable configuration includes persons='2' and doors='2'. This scenario explores how practical constraints like seating capacity and accessibility influence rejection. It reflects real-life situations where cars with limited usability, such as minimal seating and restricted access, fail to meet consumer needs, especially for families or groups. Similarly, for maint='vhigh' and safety='high', the most probable features are lug_boot='small' and buying='low'. This scenario examines how high maintenance expenses are offset by affordability and compact practicality, highlighting the trade-offs consumers face between long-term costs and niche design priorities, such as compact cars for urban use.

The sensitivity analysis highlights key dependencies, showcasing meaningful real-life trade-offs captured through approximate inference. When maint='med', larger luggage boots are more likely for medium and high buying prices, while very high buying costs favor compact designs. Approximate inference shows slight variations in probabilities but aligns with real-life preferences for balancing storage and cost in mid-range vehicles. Another finding is the link between doors and persons for safety=high and buying-low, where two-door cars are strongly associated with two seats. Approximate inference captures this pattern, reflecting real-world trends where safe, affordable cars often prioritize compactness. Lastly, for maint=low and doors=5more, larger seating capacities align with bigger luggage spaces, as revealed by approximate inference, emphasizing practicality for family-oriented designs.

Bayesian estimation and PC algorithm

The networks learned using MLE with PC and the Bayesian estimation with PC are identical because the PC algorithm focuses solely on structure learning through conditional independence tests, unaffected by the parameter estimation method. The PC algorithm identifies edges in the network based on statistical dependencies, defining the structure independently of how probabilities are assigned. Since parameter estimation, whether through MLE or Bayesian methods, only affects the conditional probability distributions and not the structure itself, the resulting networks are structurally the same. Therefore, further analysis of the Bayesian estimation with PC is unnecessary as it would yield no new structural insights.

Bayesian estimation and Hill climbing algorithm with BD score



Picture 3. Network learned using Bayesian estimation and Hill climbing algorithm

The network reveals that car is serving as the central node influencing the rest of the nodes. This highlights that the car's classification directly shapes perceptions of safety, its suitability for passengers, purchasing decisions, and maintenance requirements. The variable buying is influenced by car, reflecting how purchasing decisions are primarily driven by the car's quality. Safety also plays a pivotal role, impacting persons while being influenced by car and lug_boot, indicating that the classification of the car determines its perceived safety and suitability for different passenger capacities. The outgoing edges from car to variables like lug_boot and maint further emphasize how car quality impacts practical aspects such as luggage space and maintenance costs. Compared to the network learned with PC and Bayesian Estimation, this network, created using Hill Climbing with BD score, captures a richer and more dynamic structure, better reflecting real-world decision-making processes by including more subtle relationships between variables.

The network's conditional probability disctributions reveal signicifant relationships between variables. Buying price and maintenance cost show strong dependencies with other nodes, indicating their central role in shaping the network. For instance, higher buying prices increase the likelihood of unfavorable maintenance and safety combinations, while lower prices balance these probabilities. The maintenance variable similarly influences outcomes across multiple nodes, with very high maintenance skewing probabilities toward less desirable scenarios. In contrast, variables like lug_boot size and number of persons exhibit more stable, independent distributions, suggesting they have minimal impact on the broader network. Safety, however, shows a clear influence: higher safety levels correlate with more favorable configurations across nodes, while lower safety leads to riskier combinations.

The get_independencies method shows that persons is independent of safety and lug_boot and becomes conditionally independent of buying and maintenance once car is known, suggesting that car quality summarizes key features. Similarly, lug_boot shows conditional independence from safety and persons, indicating minimal direct influence between these variables. However, d-separation tests highlight persistent dependencies that align with real-life decision-making. Maintenance cost and buying price remain dependent even after accounting for other variables, reflecting how buyers consider both upfront costs and long-term expenses together. Safety remains linked to maintenance, suggesting concerns about reliability. Interestingly, buying price becomes independent of lug_boot size when car and safety are known, confirming that luggage space is a secondary concern once key features are addressed. Yet, lug_boot size remains linked to safety, reflecting the perception that larger vehicles are safer. The network effectively captures real-world decision-making patterns by emphasizing core features like price, safety, and maintenance, while secondary features like luggage space and seating capacity play a smaller role.

The Markov Blanket results further confirm the car variable's central role in the network, directly influencing buying, maint, persons, lug_boot, and safety. The Markov blanket for persons includes safety and lug_boot, indicating that seating capacity interacts with practical features like storage and perceived safety, both of which are influenced by car quality. Similarly, lug_boot size and safety share overlapping blankets, emphasizing a connection between storage capacity and vehicle safety, which reflects real-world perceptions that larger vehicles are considered safer.

Using predictive reasoning, with lug_boot = big and car = good, the probability of persons(more) increases by 20.44%, while persons(2) drops by 27.79%. This scenario highlights the connection between storage capacity and seating capacity, indicating that larger luggage space leads to expectations of higher passenger capacity, aligning with perceptions of family-friendly cars. With lug_boot = small, the probability of persons(2) increases by 4.57%, while persons(4) and persons(more) decrease. This result shows that small luggage space is associated with lower seating capacity, reflecting practical expectations that compact cars accommodate fewer passengers.

The diagnostic reasoning results highlight key insights into consumer perceptions. With persons = more, the probability of the car being acceptable increases by 10.04%, while unacceptable drops by 14.08%, indicating that larger seating capacity is linked to more practical cars. When safety = low, the probabilities for lug_boot size remain unchanged, showing that storage capacity is perceived independently of safety. Lastly, with high maintenance costs, the probabilities for buying price remain unchanged, suggesting that purchase price and upkeep costs are evaluated separately, and affordable cars aren't necessarily cheaper to maintain, which is a bit counter intuitive to real-life situations. These findings reflect how practical features and costs are viewed as distinct considerations in real-world car assessments.

The intercausal reasoning results reveal that when maint=high and the car=vgood, the probability of buying(low) increases by 73.24%, while the probabilities of buying(high), buying(med), and buying(vhigh) each drop by 24.21%. This scenario was chosen to explore the interaction between maintenance costs and car quality on purchase price perceptions, showing that premium cars with higher upkeep are perceived as more affordable at the point of purchase, reflecting real-world assumptions about long-term value. When persons=4 and car=good, the probabilities of safety(high) and safety(med) both increase by 16.48%, while safety(low) drops by 32.95%. This indicates that practical, family-oriented cars are expected to have higher safety ratings, aligning with consumer expectations that family-friendly vehicles should prioritize safety.

The MAP query results give insights into how practical features like safety, storage capacity, and maintenance costs shape consumer perceptions. When safety=high and lug_boot=small, the most likely values for buying and maint are both low, suggesting that safe, compact cars are perceived as affordable to buy and maintain. This reflects a real-world expectation that practical vehicles are typically more cost-effective. When car=acc and buying=high, the most likely values are safety = high and persons = 4, indicating that family-friendly cars are expected to prioritize safety and practicality. Lastly, when maint=high and safety=low, the most likely values are persons = 2 and car = unacc, suggesting that unsafe, expensive maintenance cars are perceived as impractical and undesirable, especially for families.

The sensitivity analysis, conducted using approximate inference, reveals how changes in key features impact consumer perceptions. With maint = high, the car is 100% unacceptable when safety is low, but with high safety ratings, the probability of the car being acceptable increases by 38.84%, showing that safety can offset high upkeep costs. Using approximate inference allowed for the analysis of complex dependencies that may not be fully captured with exact methods. With safety = high, the probability of buying(low) is highest at 34.65% for vehicles with big luggage space, compared to 28.13% for those with small luggage space, indicating that safe, spacious cars are perceived as more affordable. Lastly, with lug_boot = big, maint=low reduce the probability of the car being unacceptable by 14.28% and increase the likelihood of it being good or very good, reflecting that lower upkeep costs improve perceptions of car quality.

Discussion

The comparison between the Bayesian Networks shows how parameter and structure learning choices directly impact reasoning and accuracy. The **MLE with PC Algorithm** network produced a sparser structure, focusing on essential relationships, but it missed several important dependencies due to its reliance on conditional independence tests. This limited its ability to reflect real-world car evaluations where multiple factors interact simultaneously. For example, consumers consider both **safety** and **maint** when assessing a car's practicality, but the PC algorithm often missed such combined influences. The MLE with Hill Climbing algorithm, on the other hand, captured a richer structure with more dependencies that align closely with real-world consumer behavior. However, the increased number of dependencies in the MLE + Hill Climbing network also raises concerns about potential overfitting. Since MLE is purely data-driven and does not incorporate prior knowledge, there is a risk that the network might capture rare patterns specific to the dataset rather than generalizable relationships. This risk is mitigated in the Bayesian estimation with BD scoring network, which incorporates priors to handle sparse data and prevent overfitting.

The reasoning results across all networks reveal notable differences in their ability to reflect real-world car evaluation processes. In predictive reasoning, the PC Algorithm network showed limited adjustments to evidence, often maintaining high probabilities for unacceptable cars despite positive influences like safety = high. The K2 Score network adjusted probabilities more dynamically but tended to overemphasize specific categories, such as acceptable, while neglecting others. In contrast, the BD Score network provided more balanced and realistic adjustments across categories, better reflecting buyer preferences for safer and more practical cars, despite have some contra intuitive results. Diagnostic reasoning showed that the BD Score network considered multiple contributing factors like low maintenance and small luggage space when identifying reasons for unacceptable cars, which mirrors how consumers evaluate vehicles comprehensively. The K2 Score network focused more narrowly on low safety, simplifying the diagnostic process but missing broader considerations. Intercausal reasoning highlighted that the BD Score network captured important trade-offs, such as how improvements in maintenance costs could offset higher buying prices to enhance a car’s perceived quality, whereas the PC Algorithm network struggled with these interactions. Sensitivity analysis further confirmed the BD Score network's stability in adjusting probabilities, especially when balancing safety and maintenance costs, reflecting real-world consumer behavior where safety can mitigate higher costs. The BD Score network consistently provided the most realistic and comprehensive reasoning across scenarios, aligning with practical decision-making processes in car evaluations. The MLE and Hill Climbing network, while more dynamic and reflective of real-world interactions, showed less stable adjustments in certain scenarios, which could indicate a sensitivity to specific patterns in the dataset.

Conclusion

This practical application demonstrated the strengths and limitations of different Bayesian Network learning methods in modeling car evaluation data. The networks varied in their ability to reflect real-world decision-making processes, with the **BD Score-based network** consistently providing the most realistic and comprehensive reasoning across predictive, diagnostic, and intercausal scenarios. This network's balanced handling of trade-offs between factors such as **safety**, **maintenance costs**, and **storage capacity** closely mirrors how consumers assess cars in practice. In contrast, the **PC Algorithm** and **K2 Score networks** displayed more simplistic reasoning patterns, often focusing heavily on single factors and missing broader interactions. The findings underscore the importance of selecting appropriate parameter and structure learning techniques to capture the complex relationships present in real-world datasets. To further enhance these models, future work could explore the integration of domain-specific background knowledge to guide structure learning and improve the network's interpretability. Adding expert knowledge can help address gaps in the dataset and uncover hidden dependencies that purely data-driven approaches might overlook. Overall, Bayesian Networks proved to be valuable tools for uncovering hidden dependencies and performing probabilistic reasoning, offering insights that can inform practical applications in industries such as automotive marketing, product design, and risk assessment.

References

1. Koller, D., & Friedman, N. (2009). *Probabilistic Graphical Models: Principles and Techniques*. MIT Press.
2. Darwiche, A. (2009). *Modeling and Reasoning with Bayesian Networks*. Cambridge University Press.
3. Kjaerulff, U., & Madsen, A. L. (2008). *Bayesian Networks and Influence Diagrams: A Guide to Construction and Analysis*. Springer.
4. Cooper, G. F., & Herskovits, E. (1992). A Bayesian Method for the Induction of Probabilistic Networks from Data. *Machine Learning,* 9(4), 309-347.
5. Pearl, J. (1988). *Probabilistic Reasoning in Intelligent Systems: Networks of Plausible Inference*. Morgan Kaufmann.
6. Heckerman, D., Geiger, D., & Chickering, D. M. (1995). Learning Bayesian Networks: The Combination of Knowledge and Statistical Data. *Machine Learning,* 20(3), 197-243.
7. Spirtes, P., Glymour, C., & Scheines, R. (1993). *Causation, Prediction, and Search*. MIT Press.
8. Lauritzen, S. L., & Spiegelhalter, D. J. (1988). Local Computations with Probabilities on Graphical Structures and their Application to Expert Systems. *Journal of the Royal Statistical Society: Series B (Methodological),* 50(2), 157-194.
9. Car Evaluation dataset *UCI Machine Learning Repository*. Retrieved from https://archive.ics.uci.edu/dataset/19/car+evaluation