Treća laboratorijska vježba – Priprema – Jelena Matijaš 1102/23

1. Zadatak
    a. Pravougaoni signal amplitude A i trajanja od $-\Theta/2$ do $\Theta/2$

① a)
$$s(t) = \begin{cases} A, & t \in \left[-\frac{\Theta}{2}, \frac{\Theta}{2}\right] \\ 0, & t < -\frac{\Theta}{2} \wedge t > \frac{\Theta}{2} \end{cases}$$

$$F_n = \frac{1}{T}\int_T s(t)\, e^{-jn\omega_0 t}\, dt = \frac{1}{T}\int_{-\frac{\Theta}{2}}^{\frac{\Theta}{2}} A\, e^{-jn\omega_0 t}\, dt$$

$$= \frac{A}{T}\int_{-\frac{\Theta}{2}}^{\frac{\Theta}{2}} e^{-jn\omega_0 t}\, dt = \frac{A}{T}\left.\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\right|_{-\frac{\Theta}{2}}^{\frac{\Theta}{2}}$$

$$= \frac{Aj}{Tn\omega_0}\cdot\left(e^{-jn\omega_0\frac{\Theta}{2}} - e^{jn\omega_0\frac{\Theta}{2}}\right)$$

$$= \frac{Aj}{Tn\omega_0}\left(\cos\left(n\omega_0\frac{\Theta}{2}\right) - j\sin\left(n\omega_0\frac{\Theta}{2}\right) - \cos\left(n\omega_0\frac{\Theta}{2}\right) - j\sin\left(n\omega_0\frac{\Theta}{2}\right)\right)$$

$$= \frac{Aj}{Tn\omega_0}\cdot\left(-2j\sin\left(n\omega_0\frac{\Theta}{2}\right)\right) = \frac{2A\sin\left(n\omega_0\frac{\Theta}{2}\right)}{Tn\omega_0}$$

$$= \frac{\cancel{2}A\sin\left(n\omega_0\frac{\Theta}{2}\right)}{Tn\omega_0}\cdot\frac{\frac{\Theta}{\cancel{2}}}{\frac{\Theta}{2}} = \frac{A\Theta}{T}\cdot\frac{\sin\left(n\omega_0\frac{\Theta}{2}\right)}{n\omega_0\frac{\Theta}{2}}$$

$$F_n = A\alpha\,\sin(n\alpha)$$

$$F_{n_0} = \frac{1}{T}\int_T s(t)\, dt = \frac{1}{T}A\Theta$$

b. Trougaoni signal amplitude A i trajanja od -Θ do Θ

① b) $s(t) = \begin{cases} A\left(1 - \frac{|t|}{\Theta}\right), & |t| \le \Theta \\ 0, & \Theta < |t| < \frac{T}{2} \end{cases}$

$$F_n = \frac{1}{T}\int_T s(t)\, e^{-jn\omega_0 t}\, dt = \frac{1}{T}\int_T s(t)\cos(n\omega_0 t)\, dt$$

$$-j\underbrace{\frac{1}{T}\int_T s(t)\sin(n\omega_0 t)\, dt}_{=0}$$

$$\Rightarrow F_n = \frac{1}{T}\int_T s(t)\cos(n\omega_0 t)\, dt = \frac{1}{T}\int_{-\Theta}^{\Theta} A\left(1 - \frac{|t|}{\Theta}\right)\cdot \cos(n\omega_0 t)\, dt$$

$$= \frac{2}{T}\int_0^{\Theta} A\left(1 - \frac{t}{\Theta}\right)\cos(n\omega_0 t)\, dt$$

$$= \frac{2A}{T}\left[\frac{\sin(n\omega_0 t)}{n\omega_0}\Big|_0^{\Theta} - \frac{1}{\Theta}\left(t\,\frac{\sin(n\omega_0 t)}{n\omega_0}\Big|_0^{\Theta} + \frac{\cos(n\omega_0 t)}{(n\omega_0)^2}\Big|_0^{\Theta}\right)\right]$$

$$= \frac{2A}{T}\left[\frac{\sin(n\omega_0\Theta)}{n\omega_0} - \frac{1}{\Theta}\cdot\Theta\cdot\frac{\sin(n\omega_0\Theta)}{n\omega_0} - \frac{1}{\Theta}\cdot\frac{\cos(n\omega_0\Theta) - 1}{(n\omega_0)^2}\right]$$

$$= \frac{2A}{\Theta T}\cdot\frac{1 - \cos(n\omega_0\Theta)}{(n\omega_0)^2} = \frac{2A}{\Theta T}\cdot\frac{2\sin^2\left(\frac{n\omega_0\Theta}{2}\right)\cdot\frac{\Theta^2}{4}}{(n\omega_0)^2\,\frac{\Theta^2}{4}}$$

$$= \frac{\Theta A}{T}\left(\frac{\sin\left(\frac{n\omega_0\Theta}{2}\right)}{\frac{n\omega_0\Theta}{2}}\right)^2 = \frac{A\Theta}{T}\,\mathrm{sinc}^2(n\alpha)$$