Formalne metode

u softverskom inženjerstvu

05 eNKA NKA DKA

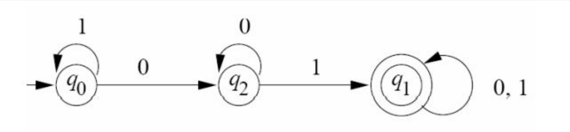
ETFBL 24-25

Dunja Vrbaški

DKA

$A = (S, \Sigma, \sigma, s_0, F)$

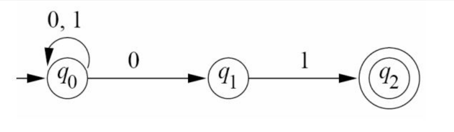
- S - skup stanja
- Σ - alfabet
- σ - funkcija prelaza, $\sigma: S \times \Sigma \rightarrow S$
- s_0 - inicijalno stanje
- F - skup ciljnih stanja



NKA

$A = (S, \Sigma, \sigma, s_0, F)$

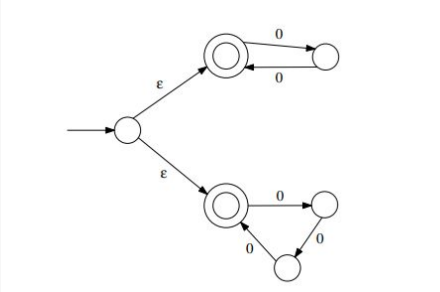
- S - skup stanja
- Σ - alfabet
- σ - funkcija prelaza, $\sigma: S \times \Sigma \rightarrow \mathscr{P}(S)$
- s_0 - inicijalno stanje
- F - skup ciljnih stanja



ε - NKA

$A = (S, \Sigma, \sigma, s_0, F)$

- S - skup stanja
- Σ - alfabet
- σ - funkcija prelaza, $\sigma: S \times (\Sigma \cup \{\varepsilon\}) \rightarrow \mathscr{P}(S)$
- s_0 - inicijalno stanje
- F - skup ciljnih stanja



DKA

Proširena funkcija prelaza σ^*

$\sigma: S \times \Sigma \to S$

$\sigma^*: S \times \Sigma^* \to S$

$\sigma^*(s, \varepsilon) = s$

$\sigma^*(s, wa) = \sigma(\sigma^*(s, w), a)$

NKA

Proširena funkcija prelaza σ^*

$\sigma: S \times \Sigma \to \mathscr{P}(S)$

$\sigma^*: S \times \Sigma^* \to \mathscr{P}(S)$

$\sigma^*(s, \varepsilon) = \{s\}$

$\sigma^*(s, wa) = \cup \{ \sigma(p, a) \mid p \in \sigma^*(s, w) \}$

NKA

Proširena funkcija prelaza σ^*

σ: S x Σ → $\mathscr{P}$(S)

σ^*: S x Σ* → $\mathscr{P}$(S)

σ^*(s, ε) = {s}

σ^*(s, wa) = ∪ { σ(p, a) | p ∈ σ^*(s, w) }

εNKA

Proširena funkcija prelaza σ^*

σ: S x Σ → $\mathscr{P}$(S)

σ^*: S x Σ* → $\mathscr{P}$(S)

σ^*(s, ε) = ε-closure({s})

σ^*(s, wa) = ε-closure(∪ { σ(p, a) | p ∈ σ^*(s, w) })

Modeli nisu zapravo prošireni

- NKA nije proširenje DKA
- ε-NKA nije proširenje NKA

Teoreme koje pokazuju:

- Za svaki NKA postoji odgovarajući DKA
- Za svaki e-NKA postoji odgovarajući NKA

Obrnute strane su “očigledne”
treba i to dokazati

e-NKA → NKA → DKA → minimizacija

Konstrukcija NKA iz ε-NKA

Za ε-NKA definisan kao $A = (S, \Sigma, \sigma, s_0, F)$
konstruiše se NKA $A_1 = (S, \Sigma, \sigma_1, s_0, F_1)$

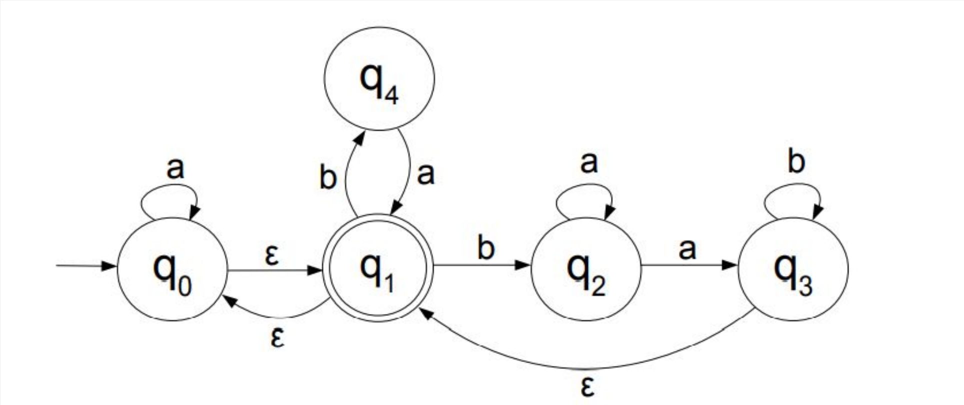
$\sigma_1(s, \varepsilon) = \varnothing$
$\sigma_1(s, a) = \sigma^*(s, a)$

$F_1 = F$ ako $\varepsilon \notin L$
$F_1 = F \cup \{s_0\}$ ako $\varepsilon \in L$ (ako ε-closure($\{s_0\}$) sadrži bar jedno stanje iz F)

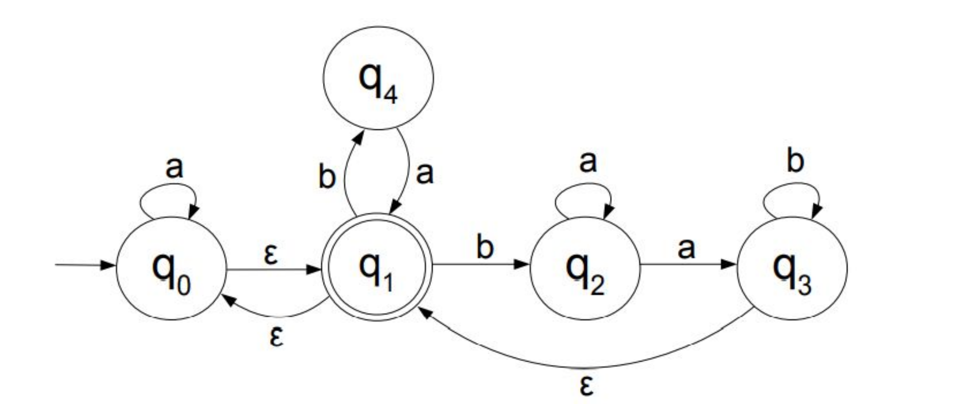
podsetnik:

$\sigma^*(s, \varepsilon) = \varepsilon\text{-closure}(\{s\})$

$\sigma^*(s, wa) = \varepsilon\text{-closure}(\bigcup \{ \sigma(p, a) \mid p \in \sigma^*(s, w) \})$



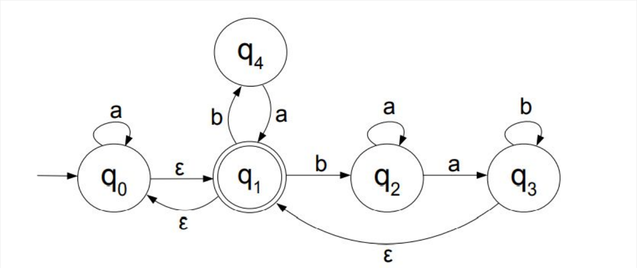
q4
a
b
a
a
b
ε
q0
q1
b
q2
a
q3
ε
ε



$S = \{q_0, q_1, q_2, q_3, q_4\}$
$\Sigma = \{a, b\}$
$s_0 = q_0$

$F_1 = ?$
Da li $\varepsilon \in F$?

$\sigma_1 = ?$



$$\begin{aligned}\delta_1(q_0, a) &= \delta^*(q_0, a)\\ &= \varepsilon\text{-closure}\left(\bigcup\left(\delta(p, a) \mid p \in \delta^*(q_0, \varepsilon)\right)\right)\\ &= \varepsilon\text{-closure}\left(\bigcup\left(\delta(p, a) \mid p \in \{q_0, q_1\}\right)\right)\\ &= \varepsilon\text{-closure}(\{q_0\} \cup \emptyset)\\ &= \varepsilon\text{-closure}(\{q_0\})\\ &= \varepsilon\text{-closure}(q_0)\\ &= \{q_0, q_1\}\end{aligned}$$

$$\begin{aligned}\delta_1(q_0, b) &= \delta^*(q_0, b)\\ &= \varepsilon\text{-closure}\left(\bigcup\left(\delta(p, b) \mid p \in \delta^*(q_0, \varepsilon)\right)\right)\\ &= \varepsilon\text{-closure}\left(\bigcup\left(\delta(p, b) \mid p \in \{q_0, q_1\}\right)\right)\\ &= \varepsilon\text{-closure}(\emptyset \cup \{q_4, q_2\})\\ &= \varepsilon\text{-closure}(\{q_4, q_2\})\\ &= \varepsilon\text{-closure}(q_4) \cup \varepsilon\text{-closure}(q_2)\\ &= \{q_4, q_2\}\end{aligned}$$

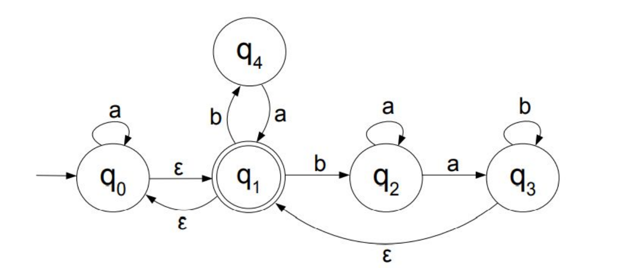
$$\begin{aligned}\delta_1(q_1, a) &= \delta^*(q_1, a)\\ &= \varepsilon\text{-closure}\left(\bigcup\left(\delta(p, a) \mid p \in \delta^*(q_1, \varepsilon)\right)\right)\\ &= \varepsilon\text{-closure}\left(\bigcup\left(\delta(p, a) \mid p \in \{q_1, q_0\}\right)\right)\\ &= \varepsilon\text{-closure}(\emptyset \cup \{q_0\})\\ &= \varepsilon\text{-closure}(\{q_0\})\\ &= \varepsilon\text{-closure}(q_0)\\ &= \{q_0, q_1\}\end{aligned}$$

$$\begin{aligned}\delta_1(q_1, b) &= \delta^*(q_1, b)\\ &= \varepsilon\text{-closure}\left(\bigcup\left(\delta(p, b) \mid p \in \delta^*(q_1, \varepsilon)\right)\right)\\ &= \varepsilon\text{-closure}\left(\bigcup\left(\delta(p, b) \mid p \in \{q_1, q_0\}\right)\right)\\ &= \varepsilon\text{-closure}(\{q_4, q_2\} \cup \emptyset)\\ &= \varepsilon\text{-closure}(\{q_4, q_2\})\\ &= \varepsilon\text{-closure}(q_4) \cup \varepsilon\text{-closure}(q_2)\\ &= \{q_4, q_2\}\end{aligned}$$

podsetnik:

$\sigma^*(s, \varepsilon) = \varepsilon\text{-closure}(\{s\})$

$\sigma^*(s, wa) = \varepsilon\text{-closure}(\cup \{ \sigma(p, a) \mid p \in \sigma^*(s, w) \})$



$$
\begin{aligned}
\delta_1(q_2, a) &= \delta^*(q_2, a)\\
&= \varepsilon\text{-closure}\left(\bigcup(\delta(p, a) \mid p \in \delta^*(q_2, \varepsilon))\right)\\
&= \varepsilon\text{-closure}\left(\bigcup(\delta(p, a) \mid p \in \{q_2\})\right)\\
&= \varepsilon\text{-closure}(\{q_2, q_3\})\\
&= \varepsilon\text{-closure}(\{q_2\}) \cup \varepsilon\text{-closure}(\{q_3\})\\
&= \varepsilon\text{-closure}(q_2) \cup \varepsilon\text{-closure}(q_3)\\
&= \{q_2\} \cup \{q_3, q_1, q_0\} = \{q_0, q_1, q_2, q_3\}
\end{aligned}
$$

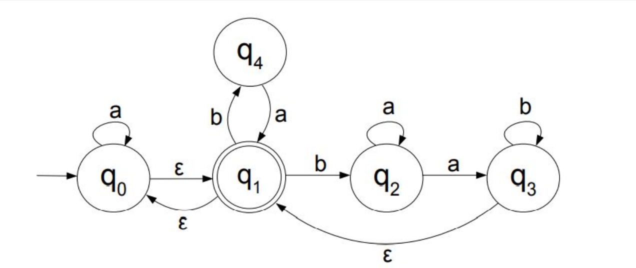
$$
\begin{aligned}
\delta_1(q_3, a) &= \delta^*(q_3, a)\\
&= \varepsilon\text{-closure}\left(\bigcup(\delta(p, a) \mid p \in \delta^*(q_3, \varepsilon))\right)\\
&= \varepsilon\text{-closure}\left(\bigcup(\delta(p, a) \mid p \in \{q_3, q_1, q_0\})\right)\\
&= \varepsilon\text{-closure}(\emptyset \cup \emptyset \cup \{q_0\})\\
&= \varepsilon\text{-closure}(\{q_0\})\\
&= \varepsilon\text{-closure}(q_0)\\
&= \{q_0, q_1\}
\end{aligned}
$$

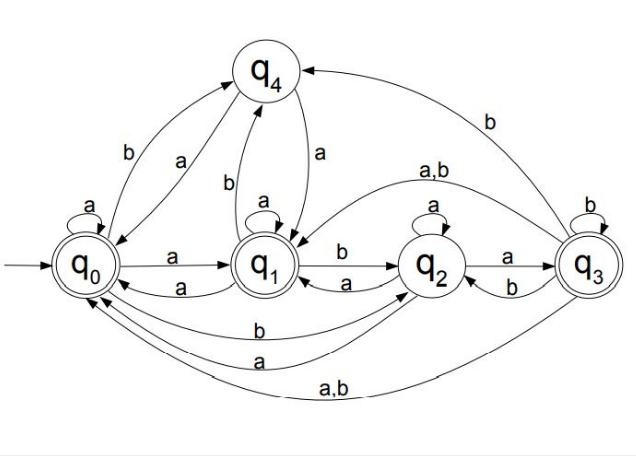
$$
\begin{aligned}
\delta_1(q_3, b) &= \delta^*(q_3, b)\\
&= \varepsilon\text{-closure}\left(\bigcup(\delta(p, b) \mid p \in \delta^*(q_3, \varepsilon))\right)\\
&= \varepsilon\text{-closure}\left(\bigcup(\delta(p, b) \mid p \in \{q_3, q_1, q_0\})\right)\\
&= \varepsilon\text{-closure}(\{q_3\} \cup \{q_2, q_4\} \cup \emptyset)\\
&= \varepsilon\text{-closure}(\{q_3, q_2, q_4\})\\
&= \varepsilon\text{-closure}(q_3) \cup \varepsilon\text{-closure}(q_2) \cup \varepsilon\text{-closure}(q_4)\\
&= \{q_3, q_1, q_0\} \cup \{q_2\} \cup \{q_4\} = \{q_0, q_1, q_2, q_3, q_4\}
\end{aligned}
$$

$$
\begin{aligned}
\delta_1(q_2, b) &= \delta^*(q_2, b)\\
&= \varepsilon\text{-closure}\left(\bigcup(\delta(p, b) \mid p \in \delta^*(q_2, \varepsilon))\right)\\
&= \varepsilon\text{-closure}\left(\bigcup(\delta(p, b) \mid p \in \{q_2\})\right)\\
&= \varepsilon\text{-closure}(\emptyset)\\
&= \emptyset
\end{aligned}
$$

$$
\begin{aligned}
\delta_1(q_4, a) &= \delta^*(q_4, a)\\
&= \varepsilon\text{-closure}\left(\bigcup(\delta(p, a) \mid p \in \delta^*(q_4, \varepsilon))\right)\\
&= \varepsilon\text{-closure}\left(\bigcup(\delta(p, a) \mid p \in \{q_4\})\right)\\
&= \varepsilon\text{-closure}(\{q_1\})\\
&= \varepsilon\text{-closure}(q_1)\\
&= \{q_0, q_1\}
\end{aligned}
$$

$$
\begin{aligned}
\delta_1(q_4, b) &= \delta^*(q_4, b)\\
&= \varepsilon\text{-closure}\left(\bigcup(\delta(p, b) \mid p \in \delta^*(q_4, \varepsilon))\right)\\
&= \varepsilon\text{-closure}\left(\bigcup(\delta(p, b) \mid p \in \{q_4\})\right)\\
&= \varepsilon\text{-closure}(\emptyset)\\
&= \emptyset
\end{aligned}
$$





	a	b
→*q_0	$\{q_0, q_1\}$	$\{q_2, q_4\}$
*q_1	$\{q_0, q_1\}$	$\{q_2, q_4\}$
q_2	$\{q_0, q_1, q_2, q_3\}$	Ø
*q_3	$\{q_0, q_1\}$	$\{q_0, q_1, q_2, q_3, q_4\}$
q_4	$\{q_0, q_1\}$	Ø

Konstrukcija DKA iz NKA

Za NKA definisan kao $A = (S, \Sigma, \sigma, s_0, F)$
konstruiše se DKA $A_1 = (S_1, \Sigma, \sigma_1, s_{01}, F_1)$

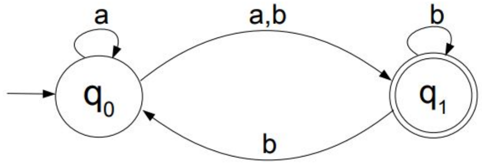
$S_1 = \mathscr{P}(S)$ (skup svih podskupova)

$s_{01} = \{s_0\}$

$\sigma_1(s, a) = \bigcup \{\sigma(p, a) \mid p \in s\}$ (s je skup)

$F_1 = \{p \in S_1 \mid p \cap F \neq \varnothing\}$ (ako skup p sadrži bar jedno završno stanje iz F)

	a	b
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_1\}$
$^*q_1$	$\varnothing$	$\{q_0, q_1\}$

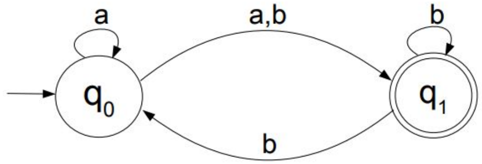


NKA - za jednu reč imamo potencijalno više putanja
Ideja za DKA - paralelan prolaz

→ partitivan skup, pokrivamo sve moguće kombinacije

obratiti pažnju: potencijalan broj stanja novog automata je 2^n

	a	b
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_1\}$
$*q_1$	$\varnothing$	$\{q_0, q_1\}$



$S_1 = \mathscr{P}(S) = \{\varnothing, \{q_0\}, \{q_1\}, \{q_0, q_1\}\}$

$\sigma_1(s, a) = \bigcup \{\sigma(p, a) \mid p \in s\}$

$\delta_1(\varnothing, a) = \varnothing$

$\delta_1(\varnothing, b) = \varnothing$

$\delta_1(\{q_0\}, a) = \delta(q_0, a) = \{q_0, q_1\}$

$\delta_1(\{q_0\}, b) = \delta(q_0, b) = \{q_1\}$

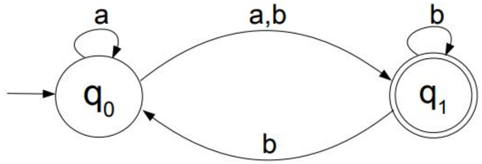
$\delta_1(\{q_1\}, a) = \delta(q_1, a) = \varnothing$

$\delta_1(\{q_1\}, b) = \delta(q_1, b) = \{q_0, q_1\}$

$\delta_1(\{q_0, q_1\}, a) = \delta(q_0, a) \bigcup \delta(q_1, a) = \{q_0, q_1\} \bigcup \varnothing = \{q_0, q_1\}$

$\delta_1(\{q_0, q_1\}, b) = \delta(q_0, b) \bigcup \delta(q_1, b) = \{q_1\} \bigcup \{q_0, q_1\} = \{q_0, q_1\}$

	a	b
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_1\}$
$*q_1$	$\varnothing$	$\{q_0, q_1\}$



$S_1 = \mathscr{P}(S) = \{\varnothing, \{q_0\}, \{q_1\}, \{q_0, q_1\}\}$

$p_0 \quad p_1 \quad p_2 \quad p_3$

$\sigma_1(s, a) = \bigcup \{\sigma(p, a) \mid p \in s\}$

$F1 = \{\{q_1\}, \{q_0, q_1\}\}$ $\qquad s_{01} = \{q_0\}$

$p_2 \quad p_3 \qquad p_1$

$\delta_1(\varnothing, a) = \varnothing$

$\delta_1(\varnothing, b) = \varnothing$

$\delta_1(\{q_0\}, a) = \delta(q_0, a) = \{q_0, q_1\}$

$\delta_1(\{q_0\}, b) = \delta(q_0, b) = \{q_1\}$

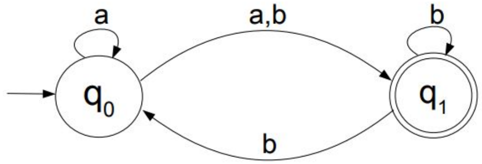
$\delta_1(\{q_1\}, a) = \delta(q_1, a) = \varnothing$

$\delta_1(\{q_1\}, b) = \delta(q_1, b) = \{q_0, q_1\}$

$\delta_1(\{q_0, q_1\}, a) = \delta(q_0, a) \bigcup \delta(q_1, a) = \{q_0, q_1\} \bigcup \varnothing = \{q_0, q_1\}$

$\delta_1(\{q_0, q_1\}, b) = \delta(q_0, b) \bigcup \delta(q_1, b) = \{q_1\} \bigcup \{q_0, q_1\} = \{q_0, q_1\}$

	a	b
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_1\}$
$*q_1$	$\varnothing$	$\{q_0, q_1\}$



$S_1 = \mathscr{P}(S) = \{\varnothing, \{q_0\}, \{q_1\}, \{q_0, q_1\}\}$

$p_0 \quad p_1 \quad p_2 \quad p_3$

$\sigma_1(s, a) = \bigcup \{\sigma(p, a) \mid p \in s\}$

$F1 = \{\{q_1\}, \{q_0, q_1\}\}$ $\qquad s_{01} = \{q_0\}$

$p_2 \quad p_3 \qquad p_1$

$\delta_1(\varnothing, a) = \varnothing$

$\delta_1(\varnothing, b) = \varnothing$

$\delta_1(\{q_0\}, a) = \delta(q_0, a) = \{q_0, q_1\}$

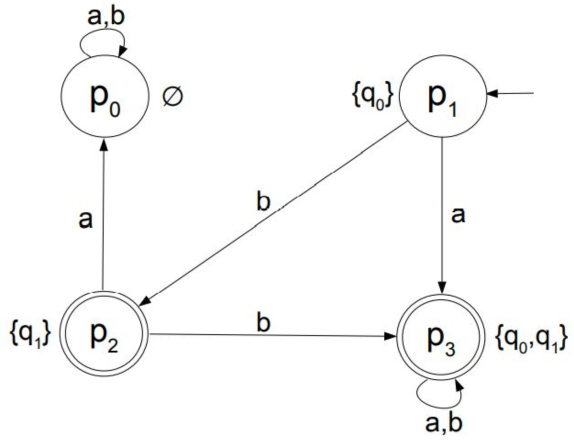
$\delta_1(\{q_0\}, b) = \delta(q_0, b) = \{q_1\}$

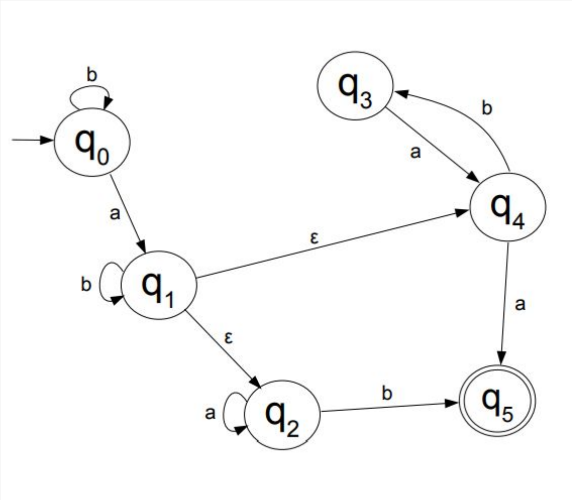
$\delta_1(\{q_1\}, a) = \delta(q_1, a) = \varnothing$

$\delta_1(\{q_1\}, b) = \delta(q_1, b) = \{q_0, q_1\}$

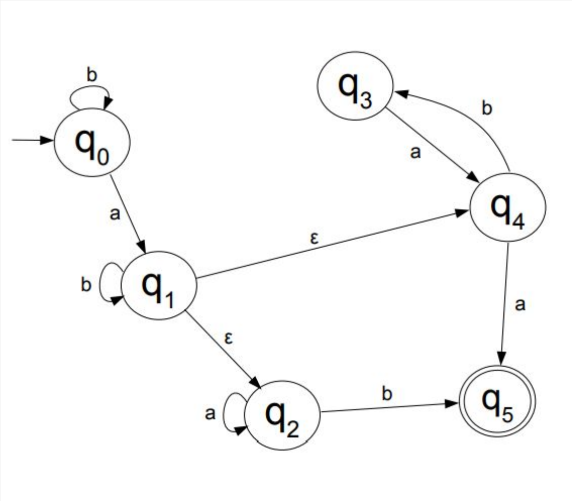
$\delta_1(\{q_0, q_1\}, a) = \delta(q_0, a) \bigcup \delta(q_1, a) = \{q_0, q_1\} \bigcup \varnothing = \{q_0, q_1\}$

$\delta_1(\{q_0, q_1\}, b) = \delta(q_0, b) \bigcup \delta(q_1, b) = \{q_1\} \bigcup \{q_0, q_1\} = \{q_0, q_1\}$

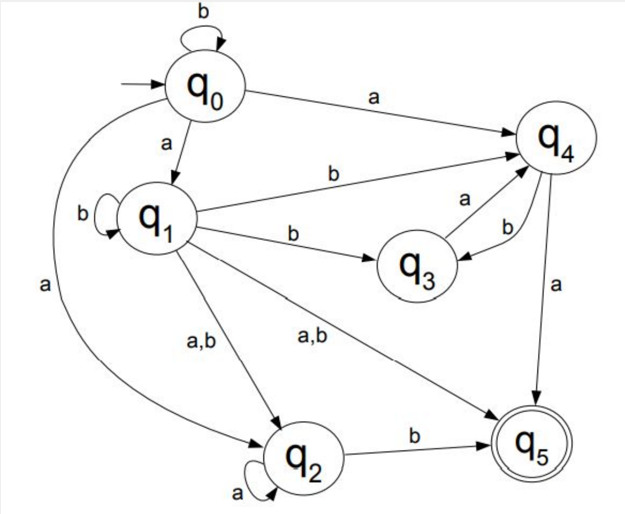




e-NKA → NKA → DKA

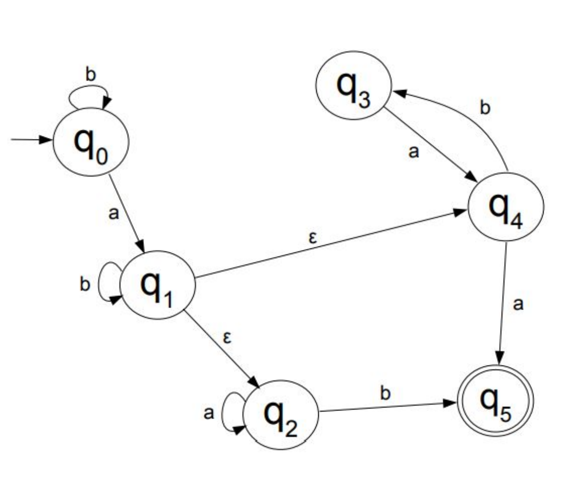


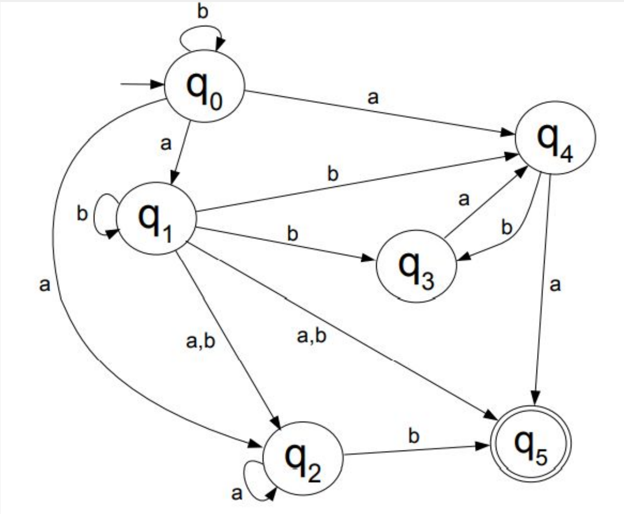
b
q0
q3
b
a
q4
a
ε
b
q1
a
ε
a
q2
b
q5



b
q0
a
a
q4
b
b
q1
a
b
b
q3
a
a
a,b
a,b
q2
b
q5
a

e-NKA → NKA → DKA

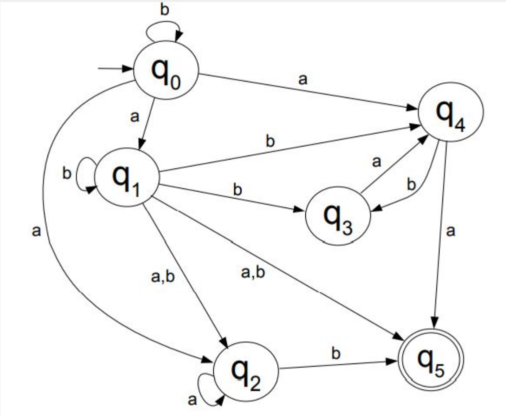




e-NKA → NKA → DKA

e-NKA → NKA → DKA ?

Problem: broj stanja novog automata je 2^6
Rešenje: ne uključivati sve podskupove, samo neophodne, iterativno



$\delta_1(\mathbf{\{q_0\}},a)=\delta(q_0,a)=\mathbf{\{q_1,q_2,q_4\}}$, $\delta_1(\{q_0\},b)=\delta(q_0,b)=\{q_0\}$

$\delta_1(\{q_1,q_2,q_4\},a)=\delta(q_1,a)\cup\delta(q_2,a)\cup\delta(q_4,a)=\{q_2,q_5\}\cup\{q_2\}\cup\{q_5\}=\mathbf{\{q_2,q_5\}}$

$\delta_1(\{q_1,q_2,q_4\},b)=\delta(q_1,b)\cup\delta(q_2,b)\cup\delta(q_4,b)=\{q_1,q_2,q_5,q_3,q_4\}\cup\{q_5\}\cup\{q_3\}=\mathbf{\{q_1,q_2,q_3,q_4,q_5\}}$

$\delta_1(\{q_2,q_5\},a)=\delta(q_2,a)\cup\delta(q_5,a)=\{q_2\}\cup\varnothing=\mathbf{\{q_2\}}$

$\delta_1(\{q_2,q_5\},b)=\delta(q_2,b)\cup\delta(q_5,b)=\{q_5\}\cup\varnothing=\mathbf{\{q_5\}}$

$\delta_1(\{q_1,q_2,q_3,q_4,q_5\},a)=\delta(q_1,a)\cup\delta(q_2,a)\cup\delta(q_3,a)\cup\delta(q_4,a)\cup\delta(q_5,a)=\{q_2,q_5\}\cup\{q_2\}\cup\{q_4\}\cup\{q_5\}\cup\varnothing=\mathbf{\{q_2,q_4,q_5\}}$

$\delta_1(\{q_1,q_2,q_3,q_4,q_5\},b)=\delta(q_1,b)\cup\delta(q_2,b)\cup\delta(q_3,b)\cup\delta(q_4,b)\cup\delta(q_5,b)=\{q_1,q_2,q_3,q_4,q_5\}\cup\{q_5\}\cup\varnothing\cup\{q_3\}\cup\varnothing=\{q_1,q_2,q_3,q_4,q_5\}$

$\delta_1(\{q_2\},a)=\delta(q_2,a)=\{q_2\}$, $\delta_1(\{q_2\},b)=\delta(q_2,b)=\{q_5\}$, $\delta_1(\{q_5\},a)=\varnothing$, $\delta_1(\{q_5\},b)=\varnothing$

$\delta_1(\{q_2,q_4,q_5\},a)=\delta(q_2,a)\cup\delta(q_4,a)\cup\delta(q_5,a)=\{q_2\}\cup\{q_5\}\cup\varnothing=\{q_2,q_5\}$

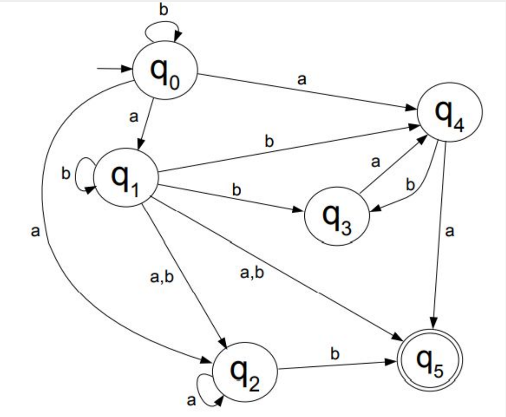
$\delta_1(\{q_2,q_4,q_5\},b)=\delta(q_2,b)\cup\delta(q_4,b)\cup\delta(q_5,b)=\{q_5\}\cup\{q_3\}\cup\varnothing=\mathbf{\{q_3,q_5\}}$

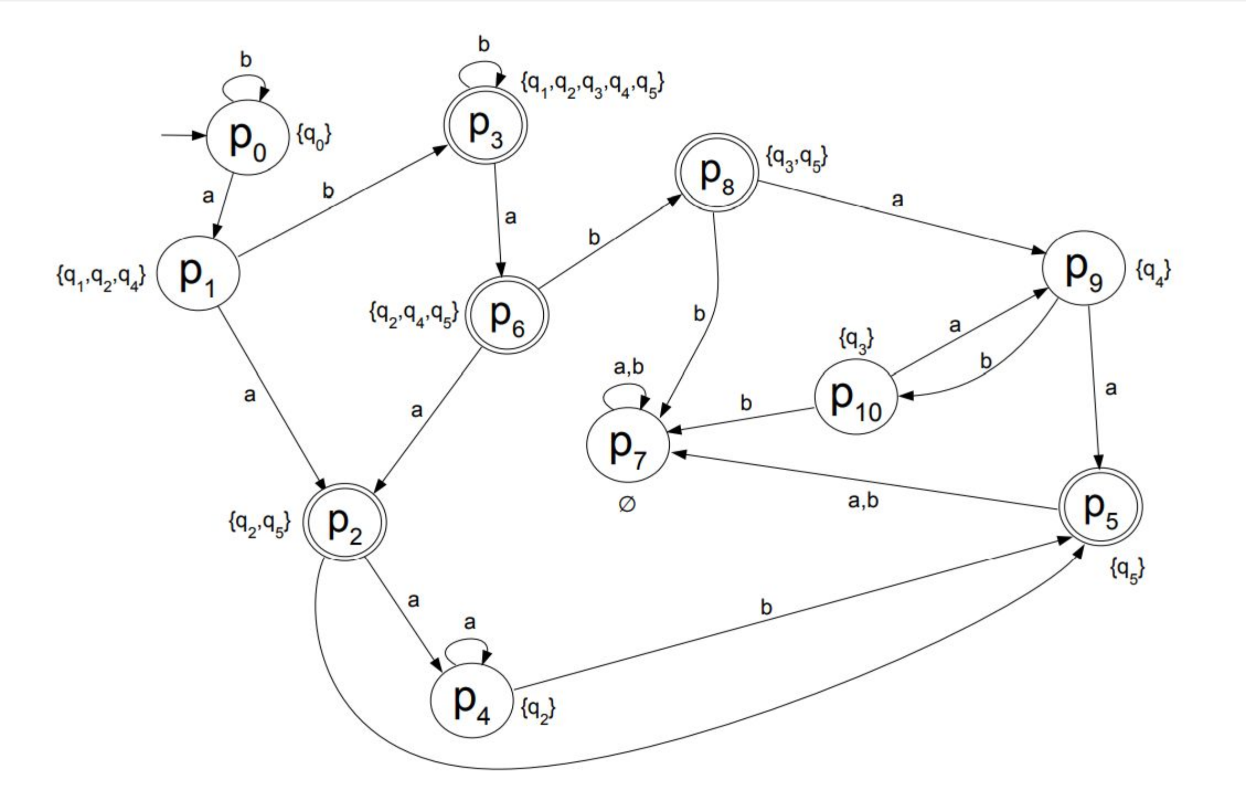
$\delta_1(\varnothing,a)=\varnothing$, $\delta_1(\varnothing,b)=\varnothing$

$\delta_1(\{q_3,q_5\},a)=\{q_4\}\cup\varnothing=\mathbf{\{q_4\}}$, $\delta_1(\{q_3,q_5\},b)=\delta(q_3,b)\cup\delta(q_5,b)=\varnothing$

$\delta_1(\{q_4\},a)=\delta(q_4,a)=\{q_5\}$, $\delta_1(\{q_4\},b)=\delta(q_4,b)=\mathbf{\{q_3\}}$,

$\delta_1(\{q_3\},a)=\delta(q_3,a)=\{q_4\}$, $\delta_1(\{q_3\},b)=\delta(q_3,b)=\varnothing$


q0
q1
q2
q3
q4
q5
a
b
a,b


p0 {q0}
p1 {q1,q2,q4}
p2 {q2,q5}
p3 {q1,q2,q3,q4,q5}
p4 {q2}
p5 {q5}
p6 {q2,q4,q5}
p7 ∅
p8 {q3,q5}
p9 {q4}
p10 {q3}
a
b
a,b