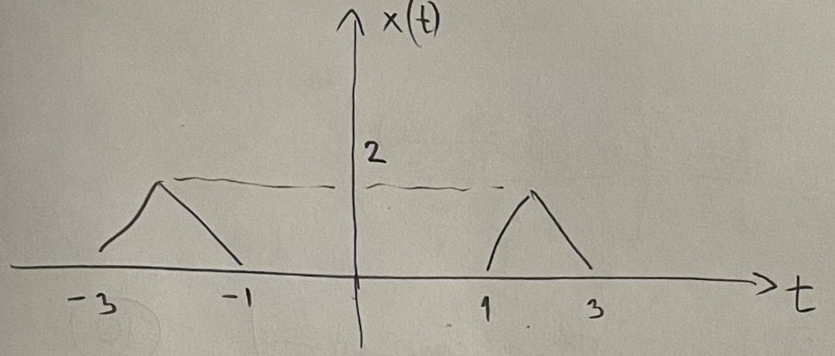
1.

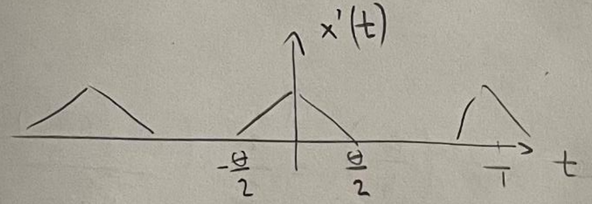


$T = 4\,ms$

$\theta = 2\,ms$

$\alpha = \frac{\theta}{T} = \frac{1}{2}$

$N = \frac{2}{\alpha} = 4$



$$x'(t) = \frac{a_0}{2} + \sum_{n=1}^{+\infty} a_n \cdot \cos(n\omega_0 t) + \sum_{n=1}^{+\infty} b_n \cdot \sin(n\omega_0 t) \quad 2$$

$b_n = 0$ 1

$$a_n = \frac{2}{T}\int_T x'(t)\cdot\cos(n\omega_0 t)\,dt = \frac{2}{T}\int_{-\frac{\theta}{2}}^{\frac{\theta}{2}} x'(t)\cdot\cos(n\omega_0 t)\,dt = \frac{4}{T}\int_0^{\frac{\theta}{2}} x'(t)\cos(n\omega_0 t)\,dt$$

1° $(0, A)$

2° $\left(\frac{\theta}{2}, 0\right)$

$$x' = \frac{-2A}{\theta}\left(t - \frac{\theta}{2}\right) = -\frac{2A}{\theta}t + A$$

$$a_n = \frac{4}{T}\int_0^{\frac{\theta}{2}}\left(-\frac{2A}{\theta}t + A\right)\cdot\cos(n\omega_0 t)\,dt = -\frac{8A}{\theta T}\cdot\int_0^{\frac{\theta}{2}} t\cdot\cos(n\omega_0 t)\,dt + \frac{4A}{T}\int_0^{\frac{\theta}{2}}\cos(n\omega_0 t)\,dt =$$

$$\left|\begin{array}{ll} u = t & dv = \cos(n\omega_0 t)\,dt \\ du = dt & v = \frac{1}{n\omega_0}\cdot\sin(n\omega_0 t) \end{array}\right|$$

$$= \frac{-8A}{\theta T}\cdot\left[\frac{t}{n\omega_0}\cdot\sin(n\omega_0 t) + \frac{1}{(n\omega_0)^2}\cdot\cos(n\omega_0 t)\right]\Bigg|_0^{\frac{\theta}{2}} + \frac{4A}{n\omega_0 \cdot T}\cdot\sin(n\omega_0 t)\Bigg|_0^{\frac{\theta}{2}} =$$

$$= \frac{-8A}{\theta T}\left[\frac{\theta}{2n\omega_0}\cdot\sin(n\omega_0 t) + \frac{1}{(n\omega_0)^2}\left(\cos\left(n\omega_0\frac{\theta}{2}\right) - 1\right)\right] + \frac{4A}{n\omega_0\cdot T}\cdot\sin\left(n\omega_0\frac{\theta}{2}\right) =$$

$$= \cancel{\frac{-4A}{n\omega_0 T}\cdot\sin(n\omega_0 t)} + \frac{8A}{\theta\cdot T\cdot(n\omega_0)^2}\left(1 - \cos\left(n\omega_0\frac{\theta}{2}\right)\right) + \cancel{\frac{4A}{n\omega_0 T}\cdot\sin\left(n\omega_0\frac{\theta}{2}\right)} =$$

$$= \frac{8A}{\theta\cdot T\cdot(n\omega_0)^2}\cdot 2\cdot\sin^2\left(n\omega_0\frac{\theta}{4}\right) = \frac{16A}{\theta\cdot T}\cdot\frac{\sin^2\left(n\omega_0\frac{\theta}{4}\right)}{(n\omega_0)^2}\cdot\frac{\theta}{16}\cdot\frac{16}{\theta} =$$

$$= \frac{A\theta}{T}\cdot\left(\frac{\sin\left(n\omega_0\frac{\theta}{4}\right)}{n\omega_0\frac{\theta}{4}}\right)^2$$

$$F_n' = \frac{a_n - jb_n}{2} = \frac{a_n}{2} = \frac{A\theta}{2T}\cdot\left(\frac{\sin\left(n\cdot\frac{2\pi}{T_0}\cdot\frac{\theta}{4}\right)}{n\cdot\frac{2\pi}{T_0}\cdot\frac{\theta}{4}}\right)^2 =$$

$$= \frac{A\alpha}{2}\cdot\left(\frac{\sin\left(n\pi\frac{\alpha}{2}\right)}{n\pi\frac{\alpha}{2}}\right)^2 = \frac{A\alpha}{2}\cdot \mathrm{sinc}^2\left(n\frac{\alpha}{2}\right)$$ 10

$$\left.\begin{array}{l}\alpha = \frac{1}{2}\\ A = 2\end{array}\right\} \Rightarrow F_n' = \frac{1}{2}\cdot \mathrm{sinc}^2\left(\frac{n}{4}\right) = \frac{1}{2}\cdot\left(\frac{\sin(n\pi/4)}{n\pi/4}\right)^2$$

$$F_n = F_n'\cdot e^{-jn\omega_0 t_0} \boxed{= \frac{A\alpha}{2}\cdot \mathrm{sinc}^2\left(n\frac{\alpha}{2}\right)\cdot e^{-jn\omega_0 t_0}}$$ 5

$$\boxed{t_0 = 2ms}$$

$$|F_n'| = |F_n|$$

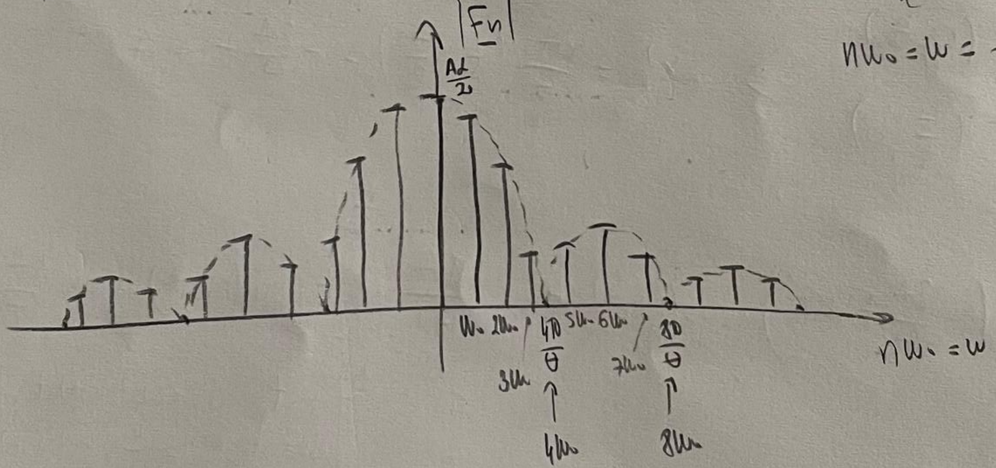
$$n\omega_0\frac{\theta}{4} = k\pi$$

$$n\omega_0 = \omega = \frac{4\pi}{\theta}\cdot k = \frac{4\pi}{T}\cdot\frac{T}{\theta}\cdot k =$$

$$= 2\omega_0\cdot\frac{1}{\alpha}\cdot k =$$

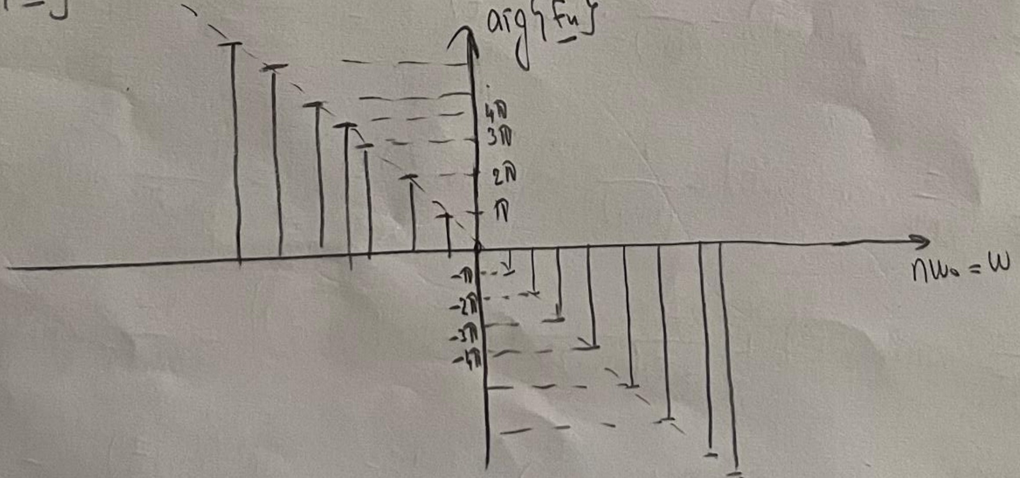
$$= 4\omega_0\cdot k$$

5



$$\arg\{F_n\} = -n\omega_0 t_0 = -2n\omega_0 = -2n\cdot\frac{2\pi}{T_0} \boxed{= -n\pi}$$

5



$$F_0 = \frac{A\alpha}{2} = \frac{1}{2}$$

$$F_1 = \frac{A\alpha}{2} \cdot \frac{\sin^2\left(\frac{\pi}{4}\right)}{\left(\frac{\pi}{4}\right)^2} = \frac{1}{\cancel{2}} \cdot \frac{\cancel{\frac{1}{2}}}{\frac{\pi^2}{\cancel{16}\,4}} = \frac{4}{\pi^2}$$

$$F_2 = \frac{A\alpha}{2} \cdot \frac{\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{\pi}{2}\right)^2} = \frac{1}{\cancel{2}} \cdot \frac{1}{\frac{\pi^2}{\cancel{4}\,2}} = \frac{2}{\pi^2}$$

$$F_3 = \frac{A\alpha}{2} \cdot \frac{\sin^2\left(\frac{3\pi}{4}\right)}{\left(\frac{3\pi}{4}\right)^2} = \frac{1}{\cancel{2}} \cdot \frac{\cancel{\frac{1}{2}}}{\frac{9\pi^2}{\cancel{16}\,4}} = \frac{4}{9\pi^2}$$

$$P' = |F_0|^2 + 2\left(|F_1|^2 + |F_2|^2 + |F_3|^2\right) =$$

$$= \frac{1}{4} + 2\left(\frac{16}{\pi^4} + \frac{4}{\pi^4} + \frac{16}{81\pi^4}\right) =$$

$$= \frac{1}{4} + 2\left(\frac{20}{\pi^4} + \frac{16}{81\pi^4}\right) = \frac{1}{4} + 2 \cdot \frac{1636}{81\pi^4} =$$

$$= \frac{1}{4} + \frac{3272}{81\pi^4} \doteq \boxed{0{,}6646\ [V^2]}$$

3

$$P' = \frac{1}{T}\int_T x'^2(t)\,dt = \frac{1}{T}\int_{-\frac{\theta}{2}}^{\frac{\theta}{2}} x'^2(t)\,dt = \frac{2}{T}\int_0^{\frac{\theta}{2}} x'^2(t)\,dt = \frac{2}{T}\int_0^{\frac{\theta}{2}} \left(-\frac{2A}{\theta}t + A\right)^2 dt =$$

$$= \frac{2}{T}\int_0^{\frac{\theta}{2}} \left(A^2 - \frac{4A^2}{\theta}t + \frac{4A^2}{\theta^2}t^2\right) dt = \frac{2}{T}\cdot\left[A^2 \cdot t\Big|_0^{\frac{\theta}{2}} - \frac{2A^2}{\theta}\cdot t^2\Big|_0^{\frac{\theta}{2}} + \frac{4A^2}{3\theta^2}\cdot t^3\Big|_0^{\frac{\theta}{2}}\right] =$$

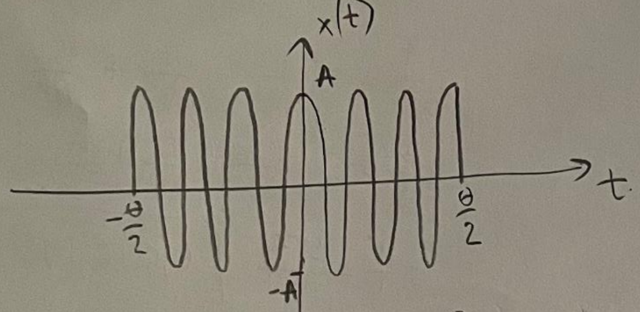
$$= \frac{2}{T}\cdot\left[\cancel{\frac{A^2\theta}{2}} - \cancel{\frac{A^2\theta}{2}} + \frac{A^2\theta}{6}\right] = \frac{A^2\theta}{3T} = \boxed{\frac{A^2\alpha}{3}} = \boxed{\frac{2}{3} = 0{,}667\ [V^2]}$$

3

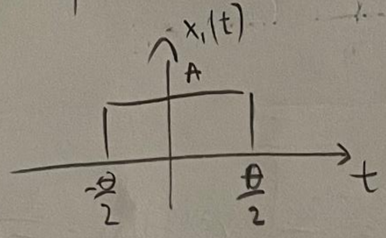
$$\eta = \frac{P'}{P} = \frac{0{,}6646}{0{,}667} \doteq \boxed{0{,}996 = 99{,}6\ \%}$$

1

2. a)



$x(t) =$



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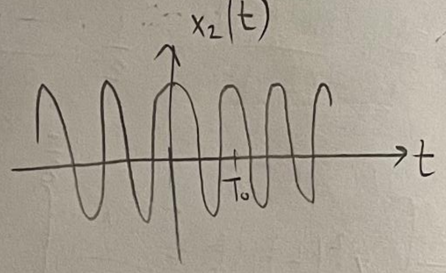


$$X_{1n} = \frac{A\theta}{T} \cdot \frac{\sin\left(n\omega_0 \frac{\theta}{2}\right)}{n\omega_0 \frac{\theta}{2}}$$

$$X_1(j\omega) = \lim_{T\to\infty} T \cdot X_{n_1} = A\theta \cdot \frac{\sin\left(\omega \frac{\theta}{2}\right)}{\omega \frac{\theta}{2}} \longrightarrow$$

Нуле: $\omega \frac{\theta}{2} = k\pi$

$\boxed{\omega = \frac{2\pi}{\theta} \cdot k}$

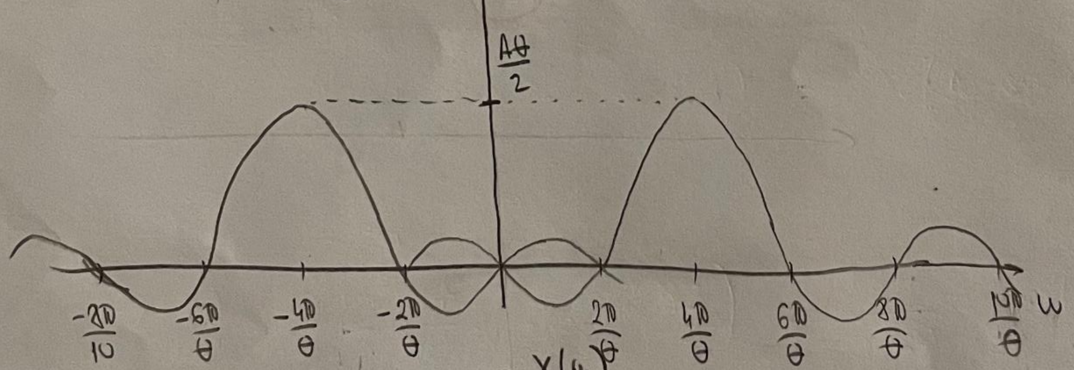
$$x(t) = x_1(t) \cdot \cos(\omega_0 t) = x_1(t) \cdot \frac{1}{2}\left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right)$$

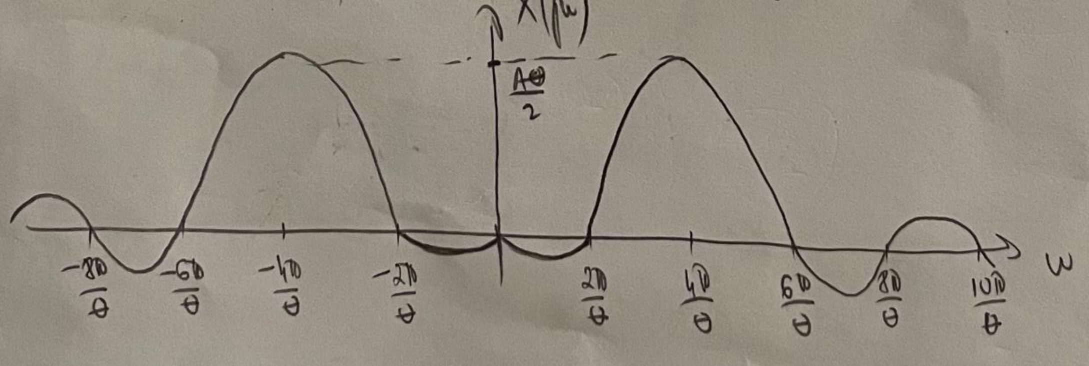
$$= \frac{1}{2} x_1(t) e^{j\omega_0 t} + \frac{1}{2} x_1(t) e^{-j\omega_0 t}$$

5

$$X(j\omega) = \frac{1}{2} X_1(\omega - \omega_0) + \frac{1}{2} X_1(\omega + \omega_0)$$

i) $\omega_0 = \frac{4\pi}{\theta}$





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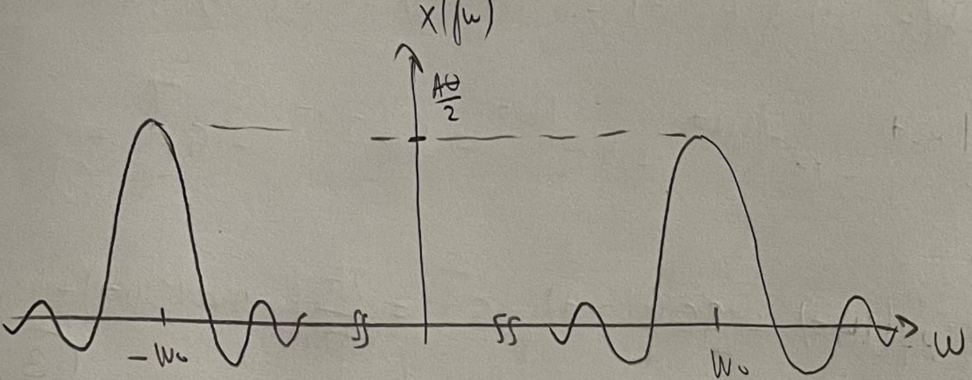
ii) $\omega_0 >> \frac{2\pi}{\theta}$



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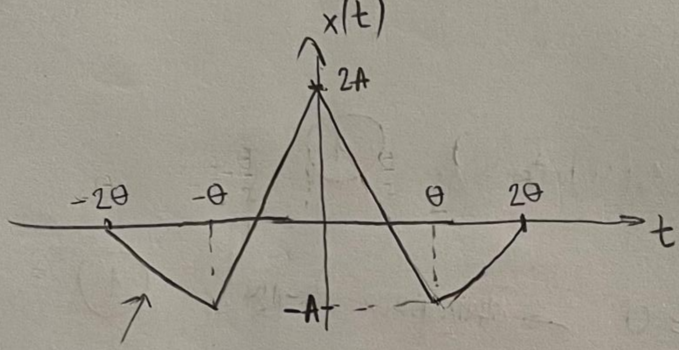
d)



1° $(-2\theta, 0)$
$(-\theta, -A)$

$$x_1 = \frac{-A}{\theta}(t+2\theta) = -\frac{A}{\theta}t - 2A$$

2° $(-\theta, -A)$
$(0, 2A)$

$$x_1 + A = \frac{3A}{\theta}(t+\theta) = \frac{3A}{\theta}t + 2A$$

3° $(0, 2A)$
$(\theta, -A)$

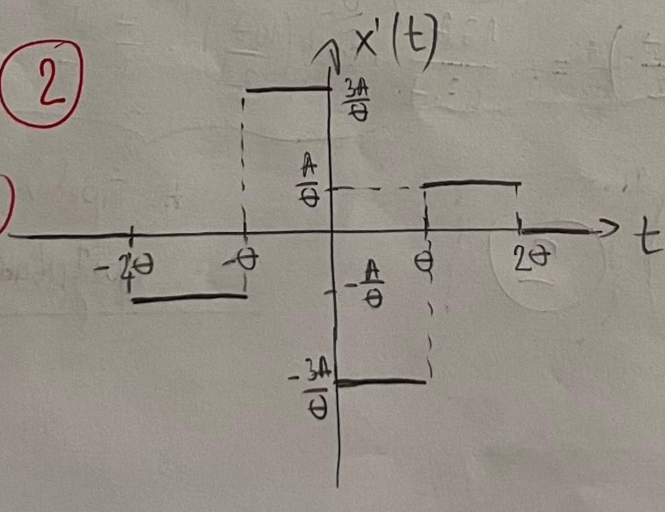
$$x_1 + A = \frac{-3A}{\theta}(t-\theta) = -\frac{3A}{\theta}t + 2A$$

4° $(\theta, -A)$
$(2\theta, 0)$

$$x_1 = \frac{-A}{-\theta}(t - \cancel{2\theta}) = \frac{A}{\theta}t - 2A$$

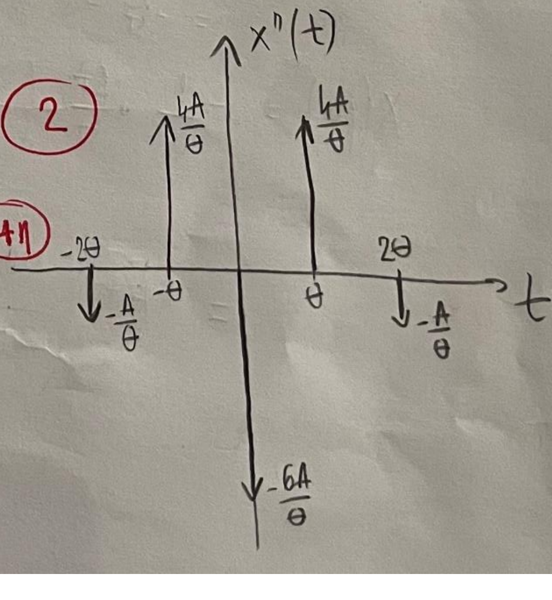
2

+1



2

+1



$$x''(t) = x_2(t) = -\frac{A}{\theta}\cdot\delta(t+2\theta) + \frac{4A}{\theta}\delta(t+\theta) - \frac{6A}{\theta}\cdot\delta(t) + \frac{4A}{\theta}\cdot\delta(t-\theta) - \frac{A}{\theta}\cdot\delta(t-2\theta)$$

$$= \frac{A}{\theta}\left[-\delta(t+2\theta) - \delta(t-2\theta) + 4\delta(t+\theta) + 4\delta(t-\theta) - 6\delta(t)\right]$$

$$X_2(j\omega) = \frac{A}{\theta}\cdot\left[-e^{j\omega 2\theta} - e^{-j\omega 2\theta} + 4e^{j\omega\theta} + 4e^{-j\omega\theta} - 6\right] =$$

$$= \frac{A}{\theta}\left[-2\cos(2\omega\theta) + 8\cos(\omega\theta) - 6\right] = \frac{2A}{\theta}\left(-\cos(2\omega\theta) + 4\cos(\omega\theta) - 3\right) \quad 3$$

$$X_2(0) = 0 \quad 1$$

$$X_1(j\omega) = \frac{1}{j\omega}\cdot X_2(j\omega) \quad 1$$

$$X(j\omega) = \frac{1}{j\omega}\cdot X_1(j\omega) + \pi\cdot X_1(0)\cdot\delta(\omega) \quad 1$$

$$X_1(0) = \int_{-\infty}^{\infty} x_1(t)\,dt = 0 \rightarrow \text{нечетна ф-ја!} \quad 1$$

$$\Rightarrow X(j\omega) = \frac{1}{(j\omega)^2}\cdot X_2(j\omega) = -\frac{1}{\omega^2}\cdot\frac{2A}{\theta}\left(-\cos(2\omega\theta) + 4\cos(\omega\theta) - 3\right) = \quad 2$$

$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1$$

$$-\cos 2x + 4\cos x - 3 = 1 - 2\cos^2 x + 4\cos x - 3 = -2\cos^2 x + 4\cos x - 2 =$$

$$= -2(\cos^2 x - 2\cos x + 1) = -2\cdot(1-\cos x)^2 = -2\cdot\left(2\cdot\sin^2\frac{x}{2}\right)^2 =$$

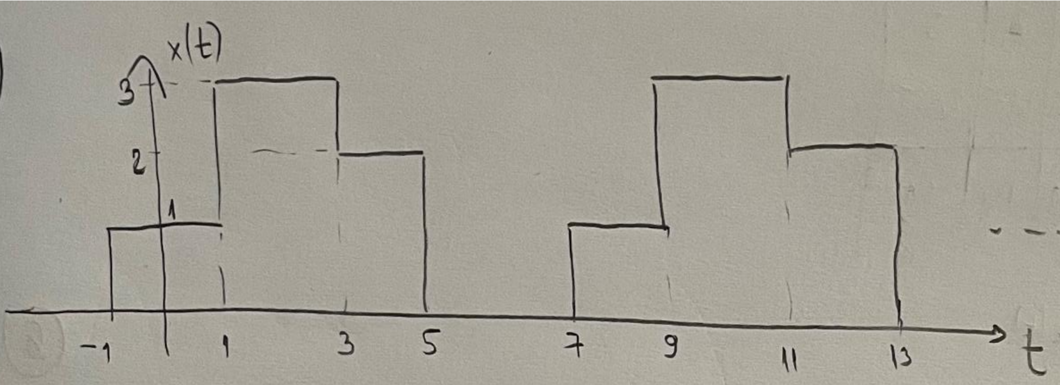
$$= -8\cdot\sin^4\frac{x}{2}$$

$$X(j\omega) = -\frac{1}{\omega^2}\cdot\frac{2A}{\theta}\cdot\left(-8\cdot\sin^4\frac{\omega\theta}{2}\right) = \underbrace{\frac{16A}{\omega^2\cdot\theta}\cdot\sin^4\left(\frac{\omega\theta}{2}\right)\cdot\frac{\theta}{4}\cdot\frac{4}{\theta}}_{\text{Не треба средување}} =$$

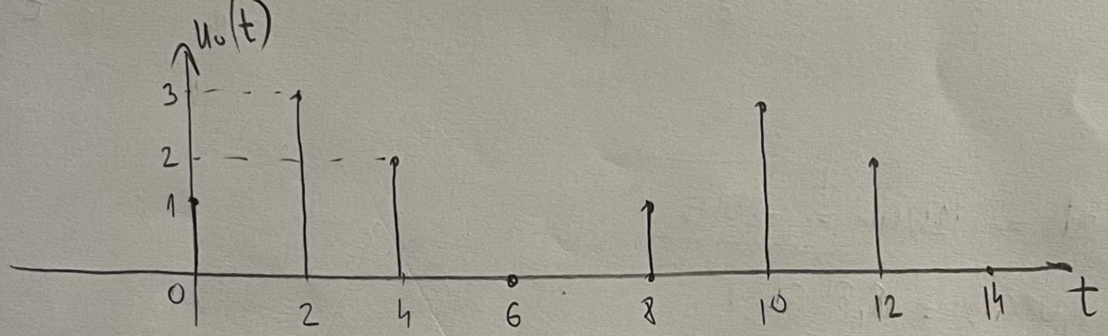
$$= 4A\theta\cdot\left(\frac{\sin^2\left(\frac{\omega\theta}{2}\right)}{\frac{\omega\theta}{2}}\right)^2 \quad 2$$

Не треба средување

3. a)



$T_s = 2s$



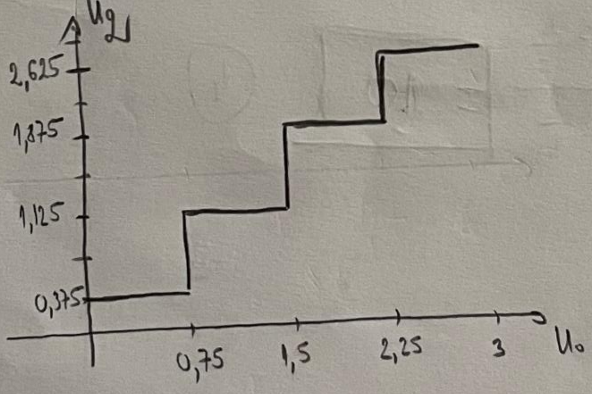
6

$\Delta = 0{,}75V$

1

$$\Delta = \frac{X_{max} - X_{min}}{q} = \frac{3-0}{q} = \frac{3}{q} \Rightarrow q = \frac{3}{0{,}75} \boxed{= 4} = 2^n$$

$$\Rightarrow \boxed{n = 2}$$



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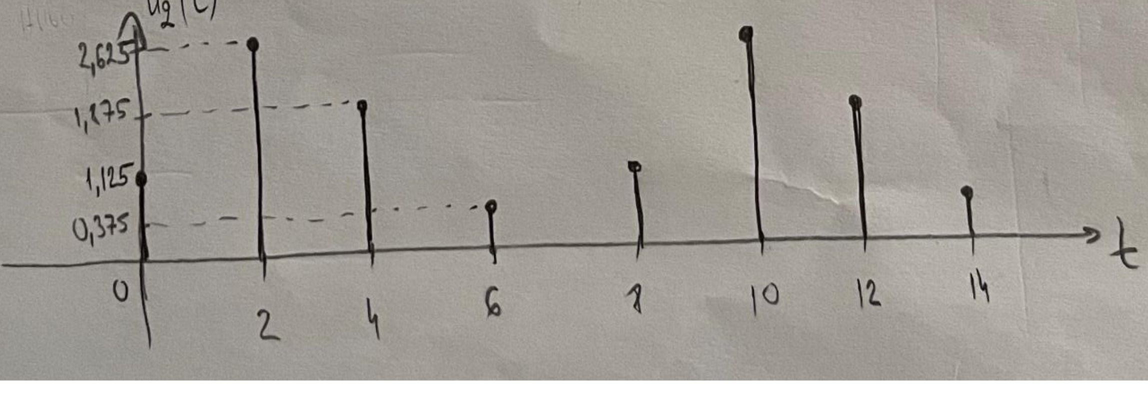
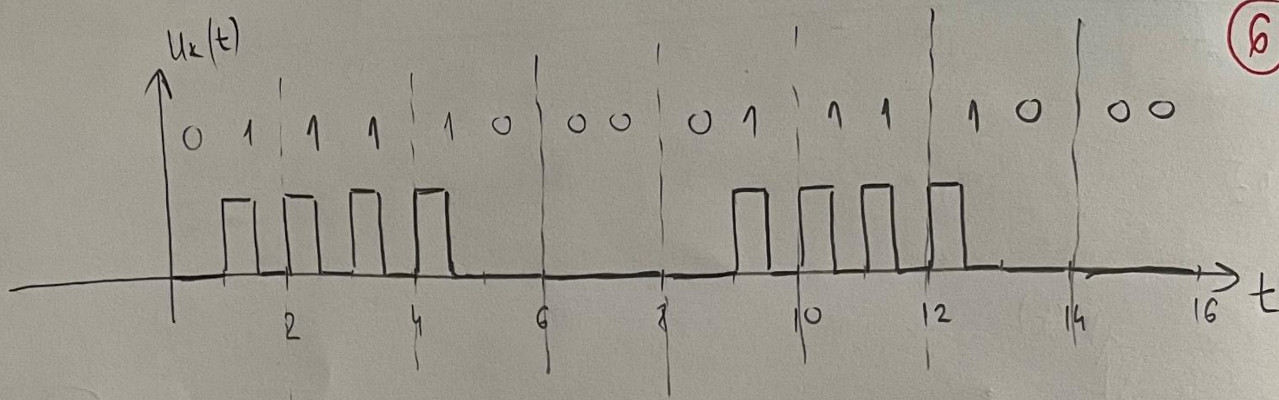


6

Ниво	0	1	2	3
ампл.	0,375	1,125	1,875	2,625
кодна ријеч	00	01	10	11



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b) $t = 2h = 120\,min = \boxed{7200\,s}$

$d = 1{,}08\,GB = \boxed{8{,}64\,Gb}$

$$\vartheta_b' = \frac{d}{t} = \frac{8{,}64\cdot 10^9\,b}{7{,}2\cdot 10^3\,s} \boxed{= 1{,}2\,\frac{Mb}{s}}$$ 2

$$\vartheta_b' = 1{,}25\,\vartheta_b \Rightarrow \vartheta_b = \frac{\vartheta_b'}{1{,}25} = \boxed{0{,}96\,\frac{Mb}{s}}$$ 4

$f_0 = 48\cdot 10^3\,Hz$

$$n = \frac{\vartheta_b}{2f_0} = \frac{960\cdot 10^3\,\frac{b}{s}}{2\cdot 48\cdot 10^3\,\frac{1}{s}} \boxed{= 10}$$ 4