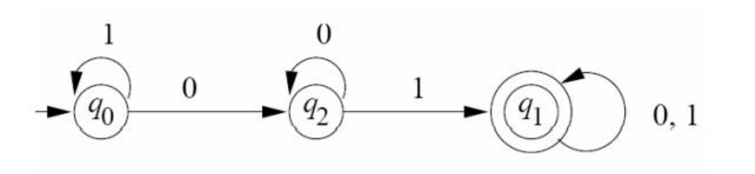
Formalne metode

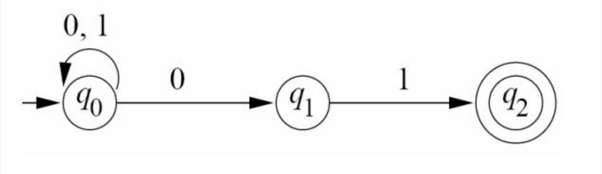
u softverskom inženjerstvu

04 Nedeterministički konačni automati

ETFBL 24-25

Dunja Vrbaški





Automat je deterministički

Jednoznačno je određena promena stanja za svaki ulazni simbol.

Automat je nedeterministički

Iz stanja q_0 se na osnovu simbola 0 može preći i u stanje q_0 i u stanje $q_{1.}$

Deterministički konačni automat

$A = (S, \Sigma, \sigma, s_0, F)$

- S - skup stanja
- Σ - alfabet
- σ - funkcija prelaza, $\sigma: S \times \Sigma \rightarrow S$
- s_0 - inicijalno stanje
- F - skup ciljnih stanja

Nedeterministički konačni automat

$A = (S, \Sigma, \sigma, s_0, F)$

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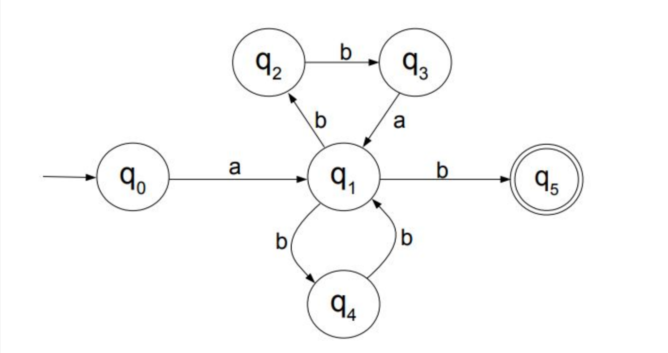
- skup svih podskupova
- prazan skup pripada ovom skupu

- iz jednog stanja, na osnovu jednog simbola možemo preći:
  - u jedno novo stanje
  - u više stanja
  - ni u jedno stanje



	a	b
$\rightarrow q_0$	$\{q_1\}$	$\emptyset$
q_1	$\emptyset$	$\{q_2, q_4, q_5\}$
q_2	$\emptyset$	$\{q_3\}$
q_3	$\{q_1\}$	$\emptyset$
q_4	$\emptyset$	$\{q_1\}$
$*q_5$	$\emptyset$	$\emptyset$

$\delta(q_0, a)=\{q_1\}$, $\delta(q_0, b)=\emptyset$

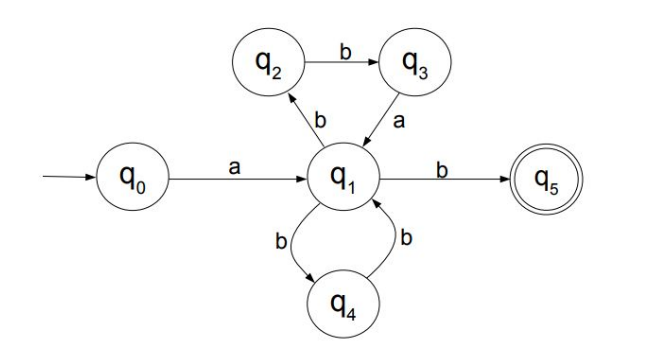
$\delta(q_1, a)=\emptyset$, $\delta(q_1, b)=\{q_2, q_4, q_5\}$

$\delta(q_2, a)=\emptyset$, $\delta(q_2, b)=\{q_3\}$

$\delta(q_3, a)=\{q_1\}$, $\delta(q_3, b)=\emptyset$

$\delta(q_4, a)=\emptyset$, $\delta(q_4, b)=\{q_1\}$

$\delta(q_5, a)=\emptyset$, $\delta(q_5, b)=\emptyset$



	a	b
$\rightarrow q_0$	$\{q_1\}$	$\emptyset$
q_1	$\emptyset$	$\{q_2, q_4, q_5\}$
q_2	$\emptyset$	$\{q_3\}$
q_3	$\{q_1\}$	$\emptyset$
q_4	$\emptyset$	$\{q_1\}$
$*q_5$	$\emptyset$	$\emptyset$

$\delta(q_0, a)=\{q_1\},\ \delta(q_0, b)=\emptyset$

$\delta(q_1, a)=\emptyset,\ \delta(q_1, b)=\{q_2, q_4, q_5\}$

$\delta(q_2, a)=\emptyset,\ \delta(q_2, b)=\{q_3\}$

$\delta(q_3, a)=\{q_1\},\ \delta(q_3, b)=\emptyset$

$\delta(q_4, a)=\emptyset,\ \delta(q_4, b)=\{q_1\}$

$\delta(q_5, a)=\emptyset,\ \delta(q_5, b)=\emptyset$

abbabbb

q0 → q1 → q2 → q3 → q1 → q4 → q1 → q5

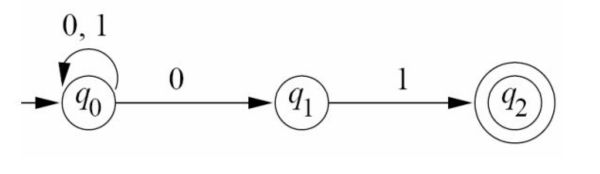
q0 → q1 → q2 → q3 → q1 → q2 → q3

q0 → q1 → q4 → q1

Zašto bismo imali nedeterministički automat?

- NKA može biti intuitivniji
- NKA može biti jednostavniji po strukturi
- → NKA može biti jednostavniji za kreiranje i programiranje

DKA - eng. DFA (deterministic finite automaton/automata)
NKA - eng. NFA (deterministic finite automaton/automata)



Kako bi izgledao deterministički automat?

Da li svaki NKA ima odgovarajući DKA?

Deterministički konačni automat

Proširena funkcija prelaza σ^*

σ: S x Σ → S

σ^*: S x Σ* → S

$\sigma^*(s, \varepsilon) = s$

$\sigma^*(s, wa) = \sigma(\sigma^*(s, w), a)$

Nedeterministički konačni automat

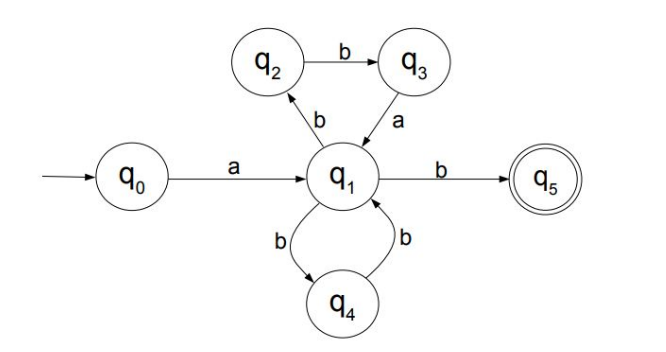
Proširena funkcija prelaza σ^*

σ: S x Σ → $\mathscr{P}$(S)

σ^*: S x Σ* → $\mathscr{P}$(S)

$\sigma^*(s, \varepsilon) = \{s\}$

$\sigma^*(s, wa) = \cup \{ \sigma(p, a) \mid p \in \sigma^*(s, w) \}$



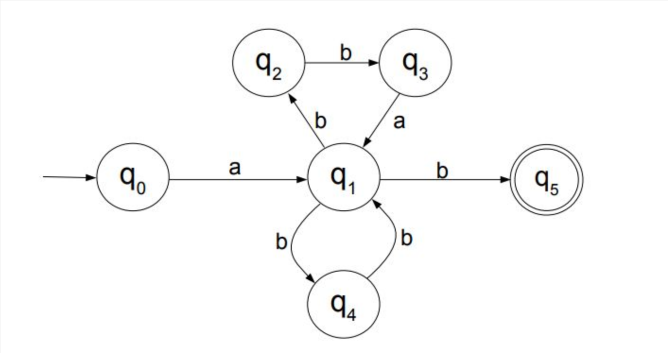
abbab

$$
\begin{aligned}
\delta^*(q_0, \varepsilon) &= \{q_0\} \\
\delta^*(q_0, a) &= \bigcup \{\delta(q, a) \mid q \in \delta^*(q_0, \varepsilon)\} \\
&= \bigcup \{\delta(q, a) \mid q \in \{q_0\}\} = \{q_1\} \\
\delta^*(q_0, ab) &= \bigcup \{\delta(q, b) \mid q \in \delta^*(q_0, \varepsilon a)\} \\
&= \bigcup \{\delta(q, b) \mid q \in \{q_1\}\} = \{q_2, q_4, q_5\} \\
\delta^*(q_0, abb) &= \bigcup \{\delta(q, b) \mid q \in \delta^*(q_0, \varepsilon ab)\} \\
&= \bigcup \{\delta(q, b) \mid q \in \{q_2, q_4, q_5\}\} = \{q_3, q_1\} \cup \emptyset \\
\delta^*(q_0, abba) &= \bigcup \{\delta(q, a) \mid q \in \delta^*(q_0, \varepsilon abb)\} \\
&= \bigcup \{\delta(q, a) \mid q \in \{q_1, q_3\}\} = \{q_1\} \cup \emptyset \\
\delta^*(q_0, abbab) &= \bigcup \{\delta(q, b) \mid q \in \delta^*(q_0, \varepsilon abba)\} \\
&= \bigcup \{\delta(q, b) \mid q \in \{q_1\}\} = \{q_2, q_4, q_5\}
\end{aligned}
$$

NKA prihvata reč ako je presek skupova

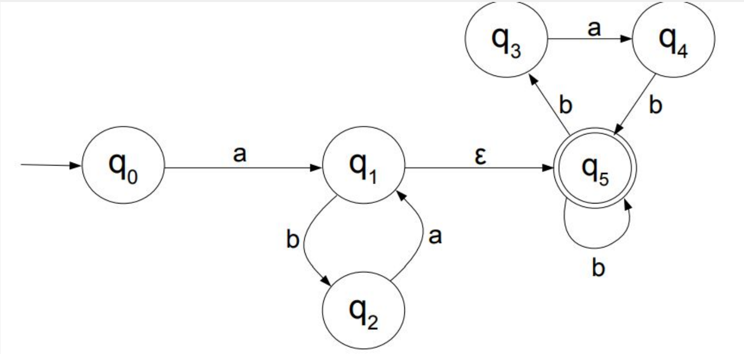
- završnih stanja i
- stanja dobijenih σ*

neprazan



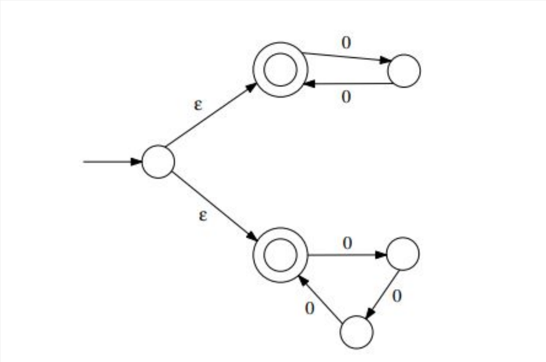
$$\delta^*(q_0, abbab) = \bigcup\{\delta(q, b) \mid q \in \delta^*(q_0, \varepsilon abba)\}$$
$$= \bigcup\{\delta(q, b) \mid q \in \{q_1\}\} = \{q_2, q_4, q_5\}$$

ε-NKA

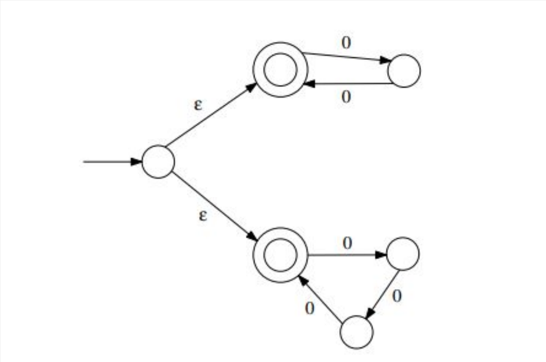


Postoje prelazi za ε.
Prelazi se vrše i bez ulaznih simbola.

Zašto bismo imali ε-NKA, gde ih koristimo?



Koji jezik prepoznaje ovaj automat?



$A = \{0^k : k \equiv 0 \bmod 2$ ILI $k \equiv 0 \bmod 3\}$

$A = A_1 \cup A_2$

$A_1 = \{0^k : k \equiv 0 \bmod 2\}$
$A_2 = \{0^k : k \equiv 0 \bmod 3\}$

ε - NKA
Nedeterministički konačni automat sa praznim (epsilon) prelazima

$A = (S, \Sigma, \sigma, s_0, F)$

- S - skup stanja
- Σ - alfabet
- σ - funkcija prelaza, $\sigma: S \times (\Sigma \cup \{\varepsilon\}) \rightarrow \mathscr{P}(S)$
- s_0 - inicijalno stanje
- F - skup ciljnih stanja

DKA

$A = (S, \Sigma, \sigma, s_0, F)$

- S - skup stanja
- Σ - alfabet
- σ - funkcija prelaza, σ: $S \times \Sigma \rightarrow S$
- s_0 - inicijalno stanje
- F - skup ciljnih stanja

NKA

$A = (S, \Sigma, \sigma, s_0, F)$

- S - skup stanja
- Σ - alfabet
- σ - funkcija prelaza, σ: $S \times \Sigma \rightarrow \mathscr{P}(S)$
- s_0 - inicijalno stanje
- F - skup ciljnih stanja

ε - NKA

$A = (S, \Sigma, \sigma, s_0, F)$

- S - skup stanja
- Σ - alfabet
- σ - funkcija prelaza, σ: $S \times (\Sigma \cup \{\varepsilon\}) \rightarrow \mathscr{P}(S)$
- s_0 - inicijalno stanje
- F - skup ciljnih stanja

NKA

Proširena funkcija prelaza σ^*

$\sigma: S \times \Sigma \to \mathscr{P}(S)$

$\sigma^*: S \times \Sigma^* \to \mathscr{P}(S)$

$\sigma^*(s, \varepsilon) = \{s\}$

$\sigma^*(s, wa) = \cup \{ \sigma(p, a) \mid p \in \sigma^*(s, w) \}$

εNKA

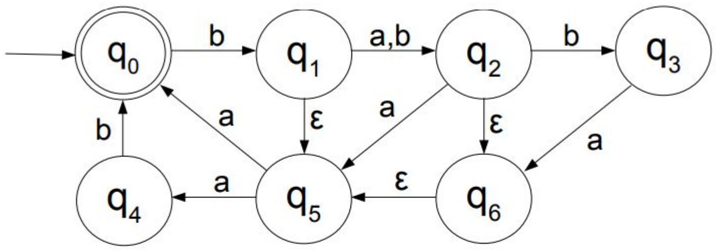
Proširena funkcija prelaza σ^*

$\sigma: S \times \Sigma \to \mathscr{P}(S)$

$\sigma^*: S \times \Sigma^* \to \mathscr{P}(S)$

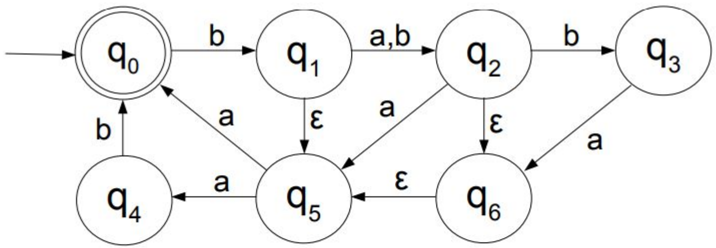
Skup stanja do kojih se dolazi od početnog, promenom stanja uključujući i ε-prelaze.

Kako ovo formalno definisati, zapisati?



$$\delta^*(q_0, b) = \{q_1, q_5\}$$

$$\delta^*(q_1, a) = \{q_0, q_2, q_4, q_5, q_6\}$$



$$\delta^*(q_0, b) = \{q_1, q_5\}$$

$$\delta^*(q_1, a) = \{q_0, q_2, q_4, q_5, q_6\}$$

$$\varepsilon\text{-closure}(q_0) = \{q_0\}$$

$$\varepsilon\text{-closure}(q_1) = \{q_1, q_5\}$$

$$\varepsilon\text{-closure}(q_2) = \{q_2, q_5, q_6\}$$

ε-closure - samo stanje + stanja do kojih se stiže ε-prelazima

$$\varepsilon-\text{closure}(P) = \bigcup_{q \in P} \left(\varepsilon-\text{closure}(q)\right)$$

ε - NKA

Proširena funkcija prelaza σ^*

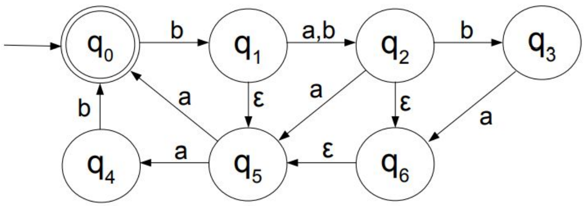
σ: S x (Σ U {ε}) → $\mathscr{P}$(S)

σ^*: S x Σ* → $\mathscr{P}$(S)

$\sigma^*(s, \varepsilon) = \varepsilon\text{-closure}(\{s\})$

$\sigma^*(s, wa) = \varepsilon\text{-closure}(\bigcup \{ \sigma(p, a) \mid p \in \sigma^*(s, w) \})$

- primjer



$$
\begin{aligned}
\delta^*(q_0, ba) &= \varepsilon\text{-closure}\left(\bigcup \left(\delta(q, a) \mid q \in \delta^*(q_0, b) = \{q_1, q_5\}\right)\right) \\
&= \varepsilon\text{-closure}\left(\delta(q_1, a) \cup \delta(q_5, a)\right) \\
&= \varepsilon\text{-closure}\left(\{q_2\} \cup \{q_0, q_4\}\right) \\
&= \varepsilon\text{-closure}\left(\{q_2, q_0, q_4\}\right) \\
&= \varepsilon\text{-closure}(q_2) \cup \varepsilon\text{-closure}(q_0) \cup \varepsilon\text{-closure}(q_4) \\
&= \{q_2, q_6, q_5\} \cup \{q_0\} \cup \{q_4\} \\
&= \{q_2, q_6, q_5, q_0, q_4\}
\end{aligned}
$$

Modeli nisu zapravo prošireni

- NKA nije proširenje DKA
- ε-NKA nije proširenje NKA

Teoreme koje pokazuju:

- Za svaki NKA postoji odgovarajući DKA
- Za svaki e-NKA postoji odgovarajući NKA

Obrnute strane su “očigledne”
treba i to dokazati

e-NKA → NKA → DKA → minimizacija