

Formalne metode

u softverskom inženjerstvu

07 RE; Gramatike

ETFBL 24-25

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$$r = b(ab)^* \mid ab^*a$$

$$r = r_1 \mid r_2$$

$$r_1 = b(ab)^*$$

$$r_2 = ab^*a$$

$$r_1 = r_3 r_4$$

$$r_3 = b$$

$$r_4 = (ab)^*$$

$$r_4 = r_5^*$$

$$r_5 = ab$$

$$r_5 = r_6 r_7$$

$$r_6 = a$$

$$r_7 = b$$

$$r_2 = r_8 r_9$$

$$r_8 = a$$

$$r_9 = b^*a$$

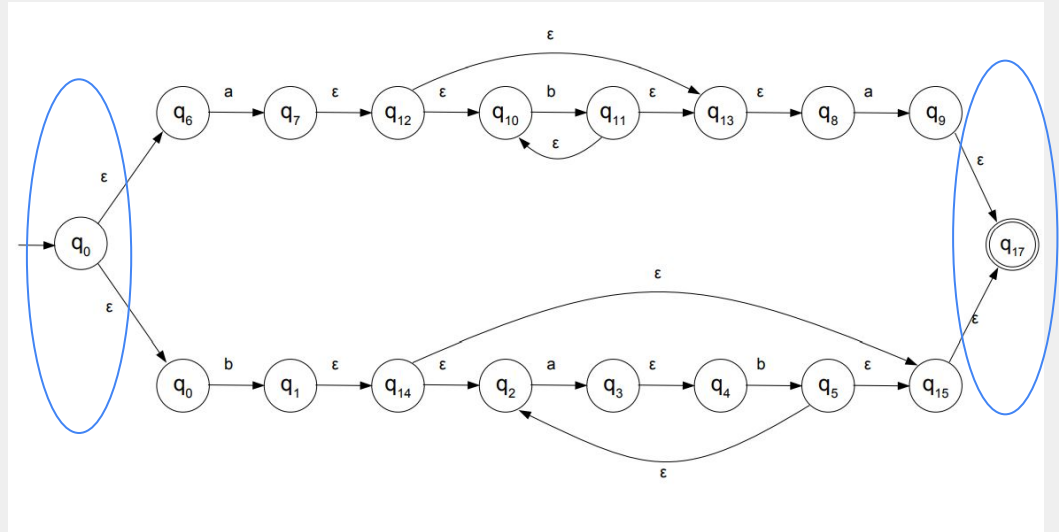
$$r_9 = r_{10} r_{11}$$

$$r_{10} = b^*$$

$$r_{11} = a$$

$$r_{10} = r_{12}^*$$

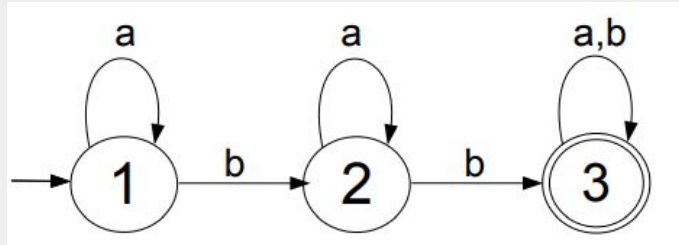
$$r_{12} = b$$



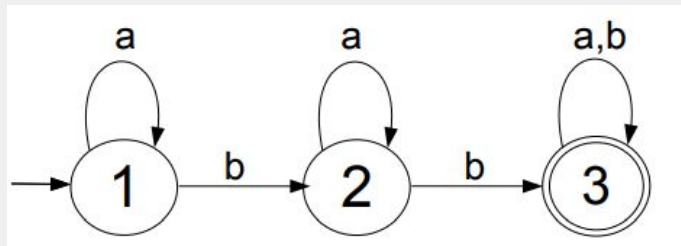
$RE \rightarrow e\text{-NKA} \rightarrow NKA \rightarrow DKA$

$RE \leftarrow e\text{-NKA} \leftarrow NKA \leftarrow DKA \quad ??$

Konstrukcija RE na osnovu automata



Regularni izraz?

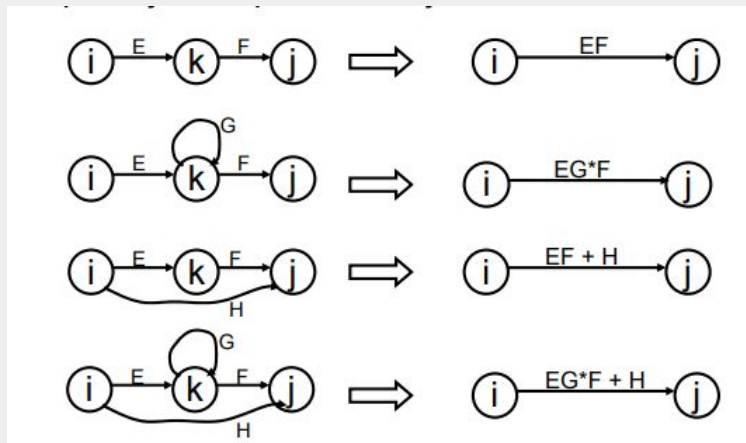


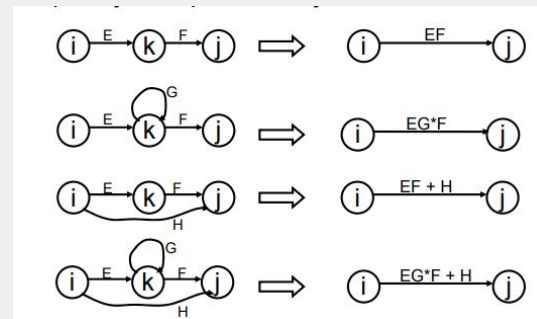
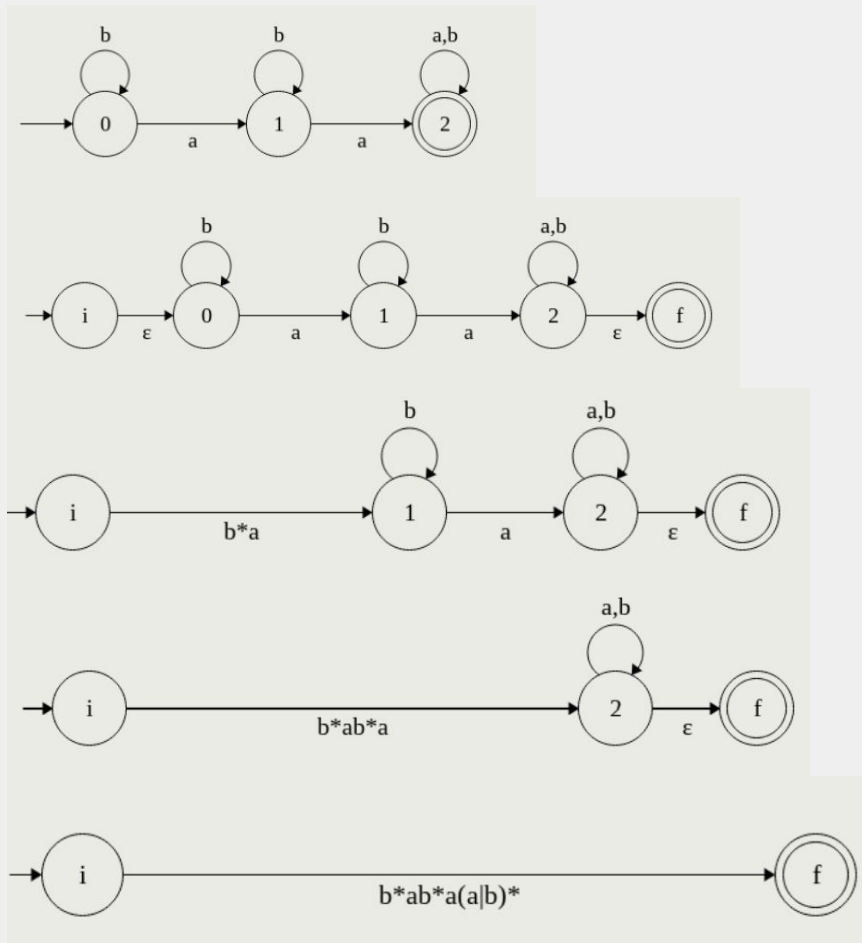
$a^*ba^*b(a|b)^*$

Složeniji automat?

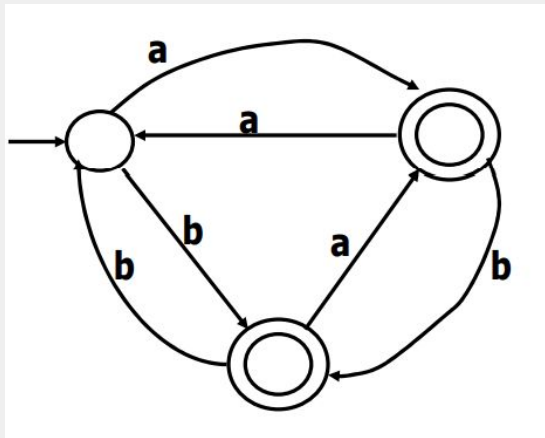
1. Dodajemo jedno novo početno stanje sa ε -prelazom do starog početnog
2. Dodajemo jedno novo završno stanje sa ε -prelazima od svih završnih
3. odaberemo stanje za eliminaciju (k)

Zamenjujemo na sledeći način:

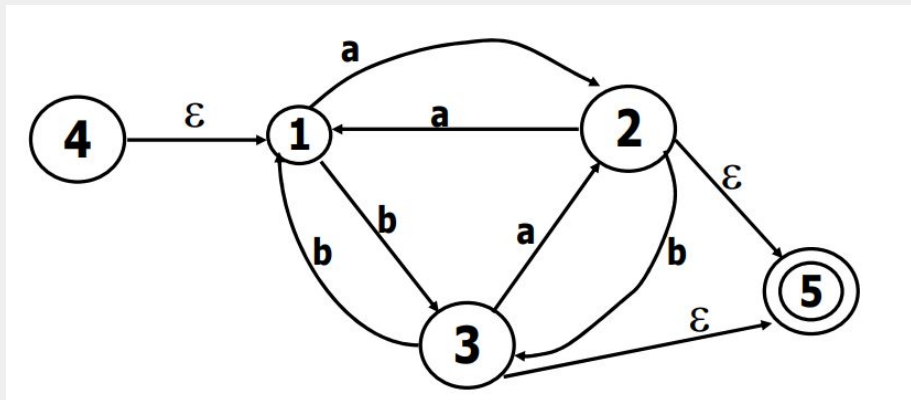
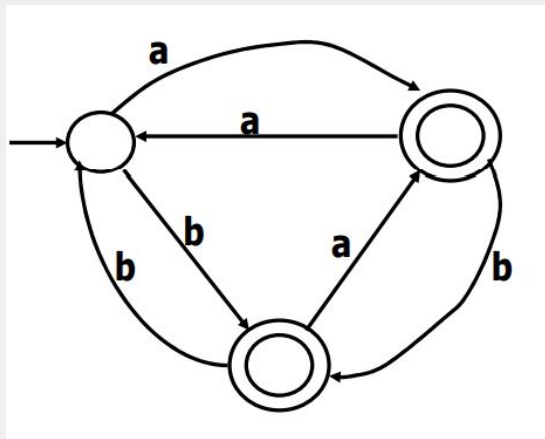




Konačan regularan izraz zavisi od izbora redosleda stanja koji se eliminišu



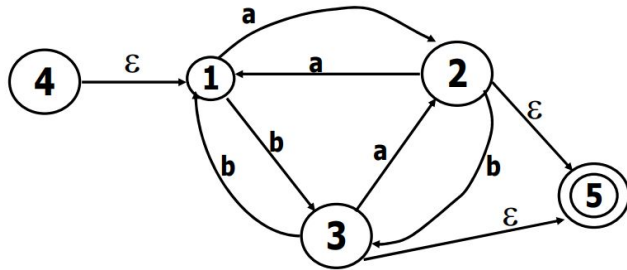
Složeniji automat?
Ideja u nastavku.



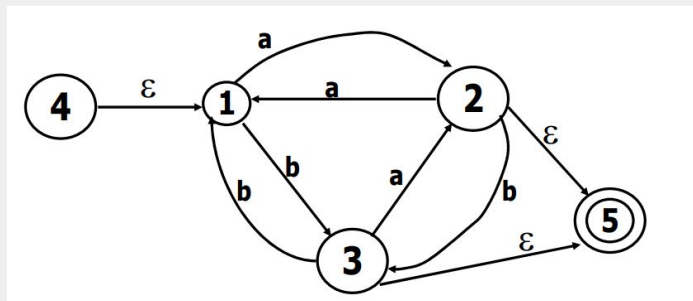
Numerišu se stanja

Dodaju se novo početno i završno i numerišu se sa n-1 i n

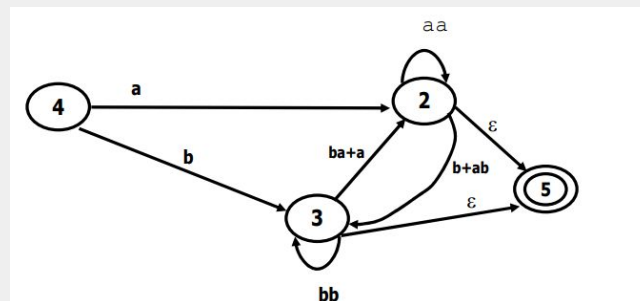
Eliminacija stanja 1



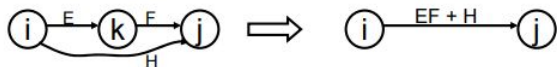
| | |
|-------|-------------|
| 2-1-2 | 2-(aa)-2 |
| 2-1-3 | 2-(b+ab)-3 |
| 2-1-4 | \emptyset |
| 2-1-5 | \emptyset |
| 3-1-2 | 3-(ba+a)-2 |
| 3-1-3 | 3-(bb)-3 |
| 3-1-4 | \emptyset |
| 3-1-5 | \emptyset |
| 4-1-2 | 4-(a)-2 |
| 4-1-3 | 4-(b)-3 |
| 4-1-4 | \emptyset |
| 4-1-5 | \emptyset |



2-1-2 2-(aa)-2
 2-1-3 2-(b+ab)-3
 2-1-4 \emptyset
 2-1-5 \emptyset
 3-1-2 3-(ba+a)-2
 3-1-3 3-(bb)-3
 3-1-4 \emptyset
 3-1-5 \emptyset
 4-1-2 4-(a)-2
 4-1-3 4-(b)-3
 4-1-4 \emptyset
 4-1-5 \emptyset



Može se nastaviti proces za novodobijeni automat.
Eliminiše se naredno stanje (2).



Od i do j uvek imamo:

Ili se direktno prelazi iz i u j bez prolaska kroz stanje k .

Ili se prolazi kroz k

- prvo ide prelaz do k
pa eventualno petlja u k
pa nastavak do j .

Uopštavanje:

Zamislamo da ima više prelaza od i do k , k do k , i od k do j , da nisu u pitanju nužno direktni prelazi.

Ili se prelazi iz i u j bez prolaska kroz stanje k .

Ili se prolazi kroz k

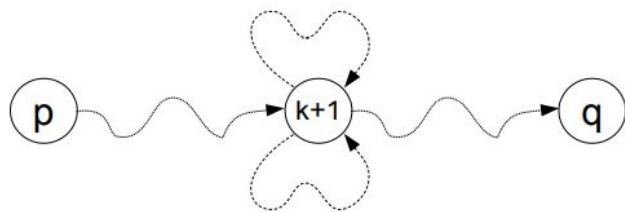
- prvo idu prelazi do k
pa eventualno "petlja/ciklus" u k
pa prelazi do j .

DKA

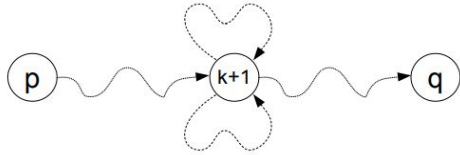
Numerišemo stanja

$L(p, q, k)$ - svi prelazi od p do q ali samo prolaskom kroz stanja $\leq k$

$$L = L(p, k+1, k) L(k+1, k+1, k) * L(k+1, q, k)$$



$$L=L(p, k+1, k)L(k+1, k+1, k)*L(k+1, q, k)$$

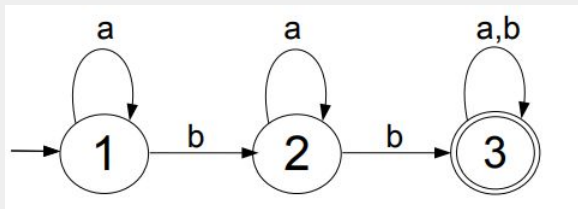


Rekurzivna formula

$$r(p, q, k+1) = r(p, q, k) \mid r(p, k+1, k)(r(k+1, k+1, k))^*r(k+1, q, k)$$

$r(p, q, 0)$ - direktni prelazi

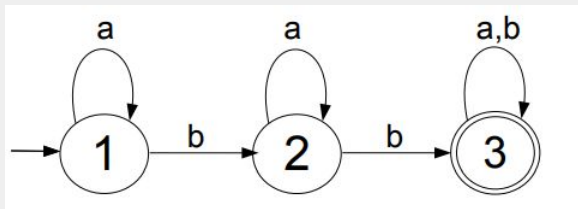
$r(s_0, f, n)$ - konačni rezultat (f - finalno stanje, n - broj stanja)



1 - početno

3 - završno

$r(1, 3, 3) = ?$



$$r(p, q, k+1) = r(p, q, k) \mid r(p, k+1, k) r(k+1, k+1, k) * r(k+1, q, k)$$

$$r(1, 3, 3) = r(1, 3, 2) \mid r(1, 3, 2) r(3, 3, 2) * r(3, 3, 2)$$

$$r(1, 3, 2) = r(1, 3, 1) \mid r(1, 2, 1) r(2, 2, 1) * r(2, 3, 1)$$

$$r(1, 3, 1) = r(1, 3, 0) \mid r(1, 1, 0) r(1, 1, 0) * r(1, 3, 0)$$

$$r(1, 2, 1) = r(1, 2, 0) \mid r(1, 1, 0) r(1, 1, 0) * r(1, 2, 0)$$

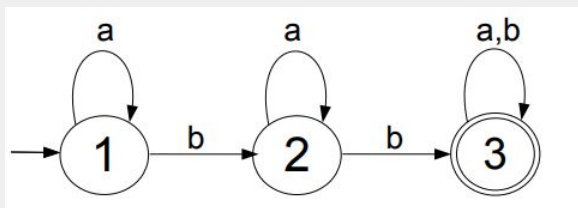
$$r(2, 2, 1) = r(2, 2, 0) \mid r(2, 1, 0) r(1, 1, 0) * r(1, 2, 0)$$

$$r(2, 3, 1) = r(2, 3, 0) \mid r(2, 1, 0) r(1, 1, 0) * r(1, 3, 0)$$

$$r(3, 3, 2) = r(3, 3, 1) \mid r(3, 2, 0) r(2, 2, 1) * r(2, 3, 1)$$

$$r(3, 3, 1) = r(3, 3, 0) \mid r(3, 1, 0) r(1, 1, 0) * r(1, 3, 0)$$

| p | r(p,1,0) | r(p,2,0) | r(p,3,0) |
|---|-------------------|-------------------|--------------------------|
| 1 | $\epsilon \mid a$ | b | \emptyset |
| 2 | \emptyset | $\epsilon \mid a$ | b |
| 3 | \emptyset | \emptyset | $\epsilon \mid a \mid b$ |



$$r(p, q, k+1) = r(p, q, k) \mid r(p, k+1, k) r(k+1, k+1, k) * r(k+1, q, k)$$

$$r(1, 3, 3) = r(1, 3, 2) \mid r(1, 3, 2) r(3, 3, 2) * r(3, 3, 2)$$

$$r(1, 3, 2) = r(1, 3, 1) \mid r(1, 2, 1) r(2, 2, 1) * r(2, 3, 1)$$

$$r(1, 3, 1) = r(1, 3, 0) \mid r(1, 1, 0) r(1, 1, 0) * r(1, 3, 0)$$

$$r(1, 2, 1) = r(1, 2, 0) \mid r(1, 1, 0) r(1, 1, 0) * r(1, 2, 0)$$

$$r(2, 2, 1) = r(2, 2, 0) \mid r(2, 1, 0) r(1, 1, 0) * r(1, 2, 0)$$

$$r(2, 3, 1) = r(2, 3, 0) \mid r(2, 1, 0) r(1, 1, 0) * r(1, 3, 0)$$

$$r(3, 3, 2) = r(3, 3, 1) \mid r(3, 2, 0) r(2, 2, 1) * r(2, 3, 1)$$

$$r(3, 3, 1) = r(3, 3, 0) \mid r(3, 1, 0) r(1, 1, 0) * r(1, 3, 0)$$

| p | r(p,1,0) | r(p,2,0) | r(p,3,0) |
|---|-------------------|-------------------|--------------------------|
| 1 | $\epsilon \mid a$ | b | \emptyset |
| 2 | \emptyset | $\epsilon \mid a$ | b |
| 3 | \emptyset | \emptyset | $\epsilon \mid a \mid b$ |

$$r(3, 3, 1) = (\epsilon \mid a \mid b) \mid \emptyset (\epsilon \mid a) * \emptyset = \epsilon \mid a \mid b$$

$$r(2, 2, 1) = \epsilon \mid a \mid \emptyset (\epsilon \mid a) * b = \epsilon \mid a$$

$$r(2, 3, 1) = b \mid \emptyset (\epsilon \mid a) * \emptyset = b$$

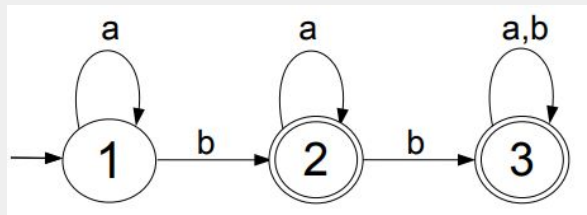
$$r(1, 2, 1) = b \mid (\epsilon \mid a) (\epsilon \mid a) * b = b \mid a * b = (\epsilon \mid a^*) b = a^* b$$

$$r(3, 3, 2) = \epsilon \mid a \mid b \mid \emptyset a * b = \epsilon \mid a \mid b$$

$$r(1, 3, 1) = \emptyset \mid (\epsilon \mid a) (\epsilon \mid a) * \emptyset = \emptyset$$

$$r(1, 3, 2) = \emptyset \mid a * b (\epsilon \mid a) * b = a * b (\epsilon \mid a) * b = a * b a * b$$

$$r(1, 3, 3) = a * b a * b \mid a * b a * b (\epsilon \mid a \mid b) * (\epsilon \mid a \mid b) \\ = a * b a * b \mid (a \mid b)^*$$



Sada je i stanje 2 konačno

$$r=r(1,2,3) \mid r(1,3,3)$$

$$r=r(1,2,3) \mid (a^*ba^*b \mid (a \mid b)^*)$$

$$r(p, q, k+1) = r(p, q, k) \mid r(p, k+1, k) r(k+1, k+1, k)^* r(k+1, q, k)$$

$$r(1, 2, 3) = r(1, 2, 2) \mid r(1, 3, 2) r(3, 3, 2)^* r(3, 2, 2)$$

$$r(1, 2, 2) = r(1, 2, 1) \mid r(1, 2, 1) r(2, 2, 1)^* r(2, 2, 1)$$

$$r(3, 2, 2) = r(3, 2, 1) \mid r(3, 2, 1) r(2, 2, 1)^* r(2, 2, 1)$$

$$r(1, 2, 1) = a^*b, \quad r(2, 2, 1) = \varepsilon \mid a$$

$$r(1, 2, 2) = a^*b \mid (a^*b)(\varepsilon \mid a)^*(\varepsilon \mid a) = a^*b \mid a^*b(\varepsilon \mid a)^* = a^*b \mid a^*ba^*$$

$$r(3, 2, 2) = \emptyset \mid \emptyset(\varepsilon \mid a)^*(\varepsilon \mid a) = \emptyset$$

$$r(1, 2, 3) = a^*b \mid a^*ba^* \mid a^*ba^*b(\varepsilon \mid a \mid b)^*\emptyset = a^*b \mid a^*ba^*$$

$$r = a^*b \mid a^*ba^* \mid a^*ba^*b \mid a^*ba^*b(\varepsilon \mid a \mid b)^*(\varepsilon \mid a \mid b)$$