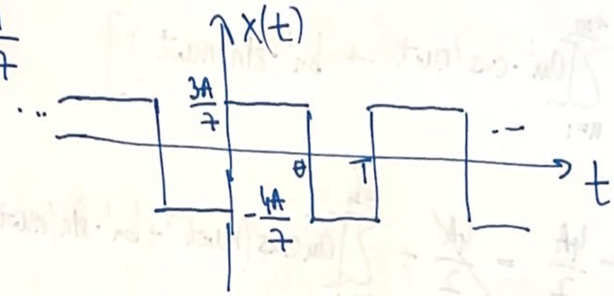
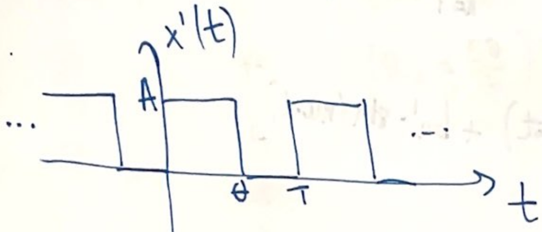
1. $\alpha = \frac{4}{7}$





$$x(t) = x'(t) - \frac{4A}{7}$$

$$X_n' = \frac{1}{T}\int_T x'(t)\cdot e^{-jn\omega_0 t}\,dt = \frac{1}{T}\int_0^\theta A\cdot e^{-jn\omega_0 t}\,dt = \frac{-A}{T}\cdot\frac{1}{jn\omega_0}\cdot e^{-jn\omega_0 t}\Big|_0^\theta$$

$$= \frac{-A}{T}\cdot\frac{1}{jn\omega_0}\left(e^{-jn\omega_0\theta} - 1\right) = \frac{A}{T}\cdot\frac{1}{jn\omega_0}\left(1 - e^{-jn\omega_0\theta}\right) =$$

$$= \frac{A}{T}\cdot\frac{1}{jn\omega_0}\cdot e^{-jn\omega_0\frac{\theta}{2}}\left(e^{jn\omega_0\frac{\theta}{2}} - e^{-jn\omega_0\frac{\theta}{2}}\right)\Big|\cdot\frac{2}{2} =$$

$$= \frac{2A}{n\omega_0 T}\cdot\sin\left(n\omega_0\frac{\theta}{2}\right)\cdot e^{-jn\omega_0\frac{\theta}{2}}\ \Big|\cdot\frac{\frac{\theta}{2}}{\frac{\theta}{2}} =$$

$$\boxed{= \frac{A\theta}{T}\cdot\frac{\sin\left(n\omega_0\frac{\theta}{2}\right)}{n\omega_0\frac{\theta}{2}}\cdot e^{-jn\omega_0\frac{\theta}{2}}}$$

$$= A\alpha\cdot\frac{\sin(n\pi\alpha)}{n\pi\alpha}\cdot e^{-jn\pi\alpha}\ \boxed{= A\alpha\cdot\mathrm{sinc}(n\alpha)\cdot e^{-jn\pi\alpha}}$$

$$\left(X_n' = \frac{a_n' - jb_n'}{2}\right)\quad a_n' = 2\mathrm{Re}\{X_n'\} = \frac{2A\theta}{T}\cdot\frac{\sin\left(n\omega_0\frac{\theta}{2}\right)}{n\omega_0\frac{\theta}{2}}\cdot\cos\left(n\omega_0\frac{\theta}{2}\right)$$

$$a_0' = \frac{2A\theta}{T} = 2A\alpha = \frac{8A}{7}$$

$$\boxed{\frac{a_0'}{2} = \frac{4A}{7}}$$

$$x'(t) = \frac{a_0'}{2} + \sum_{n=1}^{+\infty}\left[a_n'\cdot\cos(n\omega_0 t) + b_n'\cdot\sin(n\omega_0 t)\right]$$

$$x(t) = x'(t) - \frac{4A}{7} = \cancel{\frac{4A}{7}} + \sum_{n=1}^{+\infty}\left[a_n'\cos(n\omega_0 t) + b_n'\cdot\sin(n\omega_0 t)\right] - \cancel{\frac{4A}{7}}$$

$$= \sum_{n=1}^{+\infty}\left[a_n'\cdot\cos(n\omega_0 t) + b_n'\cdot\sin(n\omega_0 t)\right]$$

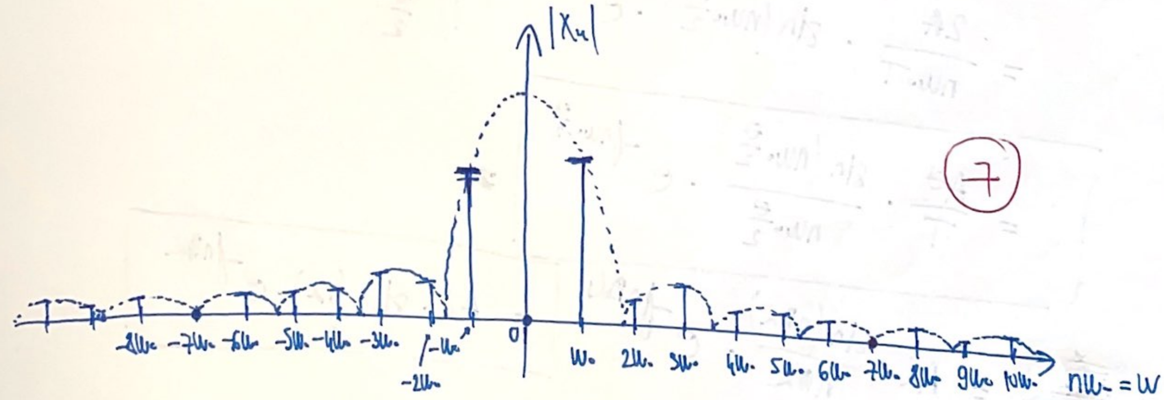
$\Rightarrow$ X_n исти као и X_n', само без DC компоненте!

$$\boxed{X_n = \frac{A\theta}{T}\cdot\frac{\sin\left(n\omega_0\frac{\theta}{2}\right)}{n\omega_0\frac{\theta}{2}}\cdot e^{-jn\omega_0\frac{\theta}{2}}}$$

10

$$n\omega_0\frac{\theta}{2} = k\pi$$

$$n\omega_0 = \omega = \frac{2\pi}{\theta}\cdot k = \frac{2\pi}{T}\cdot\frac{T}{\theta}\cdot k = \omega_0\cdot\frac{7}{4}\cdot k = \boxed{1{,}75\,\omega_0\cdot k}$$



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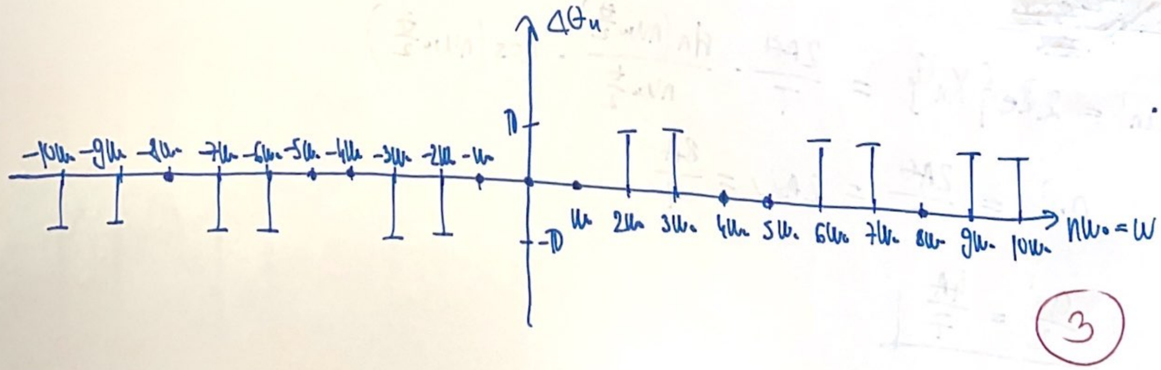

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$$\theta_n = -n\omega_0\frac{\theta}{2} + \Delta\theta_n = -n\cdot\frac{2\pi}{T}\cdot\frac{\theta}{\cancel{2}} + \Delta\theta_n = -n\pi\alpha + \Delta\theta_n$$

$$\theta_0 = 0 + 0 = 0$$

$$\theta_1 = -\frac{4\pi}{7} + 0 = -\frac{4\pi}{7} \qquad \left(\theta_{-1} = \frac{4\pi}{7}\right)$$

$$\theta_2 = -\frac{8\pi}{7} + \pi = -\frac{\pi}{7} \qquad \left(\theta_{-2} = \frac{\pi}{7}\right)$$

$$\theta_3 = -\frac{12\pi}{7} + \pi = -\frac{5\pi}{7} \qquad \left(\theta_{-3} = \frac{5\pi}{7}\right)$$

$$\vdots$$

$$\theta_4 = -\frac{16\pi}{7} + 0 = -\frac{16\pi}{7}$$

$$\theta_5 = -\frac{20\pi}{7} + 0 = -\frac{20\pi}{7}$$

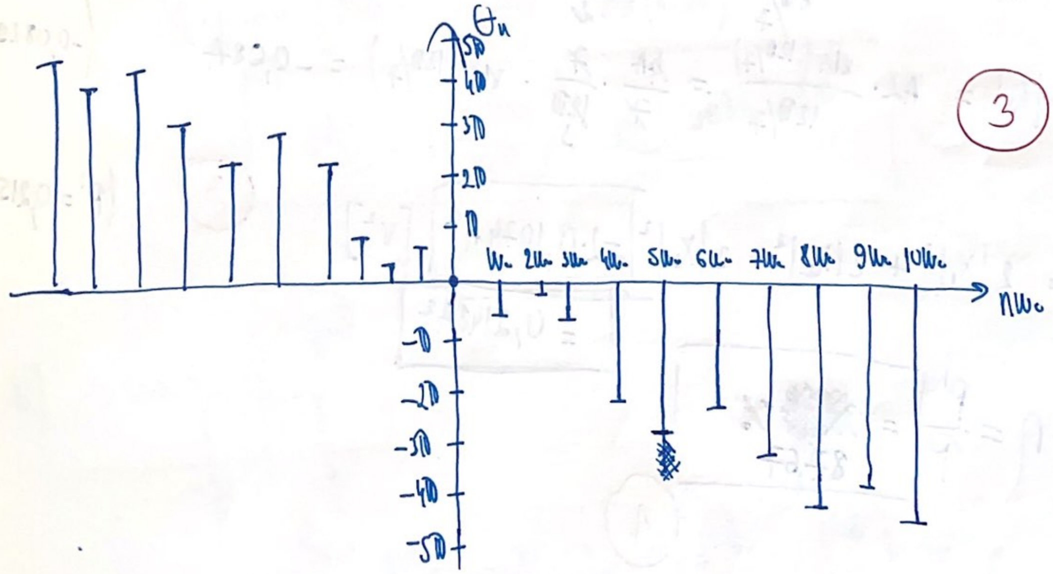
$$\theta_6 = -\frac{24\pi}{7} + \pi = -\frac{17\pi}{7}$$

$$\theta_7 = -\frac{28\pi}{7} + \pi = -3\pi$$

$$\theta_8 = -\frac{32\pi}{7} + 0 = -\frac{32\pi}{7}$$

$$\theta_9 = -\frac{36\pi}{7} + \pi = -\frac{29\pi}{7}$$

$$\theta_{10} = -\frac{40\pi}{7} + \pi = \frac{-33\pi}{7}$$



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$$P = \frac{1}{T}\int_T x^2(t)\,dt = \frac{1}{T}\left[\int_0^{\theta}\frac{9A^2}{49}dt + \int_{\theta}^{T}\frac{16A^2}{49}dt\right] =$$

$$= \frac{1}{T}\left[\frac{9A^2}{49}\theta + \frac{16A^2}{49}T - \frac{16A^2}{49}\theta\right] = \frac{1}{T}\left(-\frac{7A^2}{49}\theta + \frac{16A^2}{49}T\right)$$

$$= A^2\left(-\frac{1}{7}\cdot\frac{\theta}{T} + \frac{16}{49}\right) = A^2\left(-\frac{1}{7}\cdot\frac{4}{7} + \frac{16}{49}\right) = A^2\left(-\frac{4}{49} + \frac{16}{49}\right) =$$

$$\boxed{= A^2\cdot\frac{12}{49}\,[V^2]} = \boxed{0{,}245\,A^2}$$

$(0{,}2448A^2)$

3

$$P' = \sum_{n=-3}^{3}|X_n|^2$$

$X_0 = 0$

$$|X_1| = A\alpha\cdot\frac{\sin(4\pi/7)}{4\pi/7} = \frac{4A}{\cancel{7}}\cdot\frac{\cancel{7}}{4\pi}\cdot\sin(4\pi/7) = 0{,}31A \qquad (0{,}3103A)$$

$$|X_2| = A\alpha\cdot\frac{\sin(8\pi/7)}{8\pi/7} = \frac{4A}{\cancel{7}}\cdot\frac{\cancel{7}}{8\pi_{2}}\cdot\sin(8\pi/7) = -0{,}07A \qquad (-0{,}069A)$$

$$|X_3| = A\alpha\cdot\frac{\sin(12\pi/7)}{12\pi/7} = \frac{4A}{\cancel{7}}\cdot\frac{\cancel{7}}{12\pi_{3}}\cdot\sin(12\pi/7) = -0{,}08A \qquad (-0{,}0829A)$$

$$P' = 2|X_1|^2 + 2|X_2|^2 + 2|X_3|^2 \boxed{= 2\cdot 0{,}1074A^2}\,[V^2]$$

$$\boxed{= 0{,}2148A^2}$$

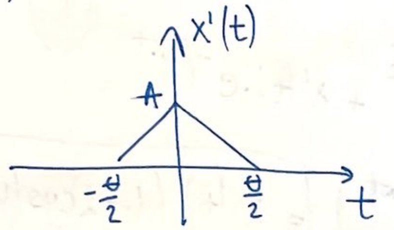
3

$(P' = 0{,}2158A^2)$

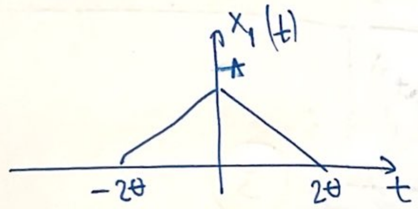
$$\eta = \frac{P'}{P}\,\boxed{= 87{,}67\,\%}$$

1

2. a)

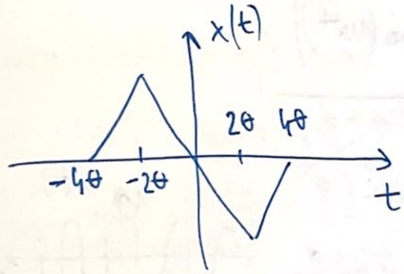


$$\longleftrightarrow \quad X'(j\omega) = \frac{A\theta}{2} \cdot \left(\frac{\sin\left(\omega \frac{\theta}{4}\right)}{\omega \frac{\theta}{4}} \right)^2$$



$$\longleftrightarrow \quad X_1(j\omega) = 2A\theta \cdot \left(\frac{\sin(\omega\theta)}{\omega\theta} \right)^2$$

3



$$x(t) = x_1(t+2\theta) - x_1(t-2\theta)$$

$$X(j\omega) = X_1(j\omega) \cdot e^{j\omega 2\theta} - X_1(j\omega) \cdot e^{-j\omega 2\theta}$$

4

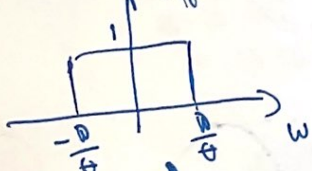
$$X(j\omega) = 2A\theta \cdot \left(\frac{\sin(\omega\theta)}{\omega\theta} \right)^2 \left(e^{j\omega 2\theta} - e^{-j\omega 2\theta} \right) \cdot \frac{2j}{2j}$$

$$X(j\omega) = j4A\theta \cdot \left(\frac{\sin(\omega\theta)}{\omega\theta} \right)^2 \cdot \sin(2\omega\theta)$$

$$\boxed{= 4A\theta \cdot \left(\frac{\sin(\omega\theta)}{\omega\theta} \right)^2 \cdot \sin(2\omega\theta) \cdot e^{j\frac{\pi}{2}}}$$

3

б)



$$x'(t) = \frac{1}{2\pi} \int_{-\frac{\pi}{\theta}}^{\frac{\pi}{\theta}} 1 \cdot e^{j\omega t}\, d\omega = \frac{1}{2\pi} \cdot \frac{1}{jt} \cdot \left(e^{jt\frac{\pi}{\theta}} - e^{-jt\frac{\pi}{\theta}} \right) =$$

$$= \frac{1}{\pi t} \cdot \sin\left(\frac{\pi}{\theta} t\right) \Big| \cdot \frac{\frac{1}{\theta}}{\frac{1}{\theta}}$$

$$\boxed{= \frac{1}{\theta} \cdot \frac{\sin\left(\frac{\pi}{\theta} t\right)}{\frac{\pi}{\theta} t}}$$

3

$$X(j\omega) = X'(j\omega) + X'(j(\omega-\omega_0)) + X'(j(\omega+\omega_0))$$

4

$$x(t) = x'(t) + x'(t)\cdot e^{j\omega_0 t} + x'(t)\cdot e^{-j\omega_0 t}$$

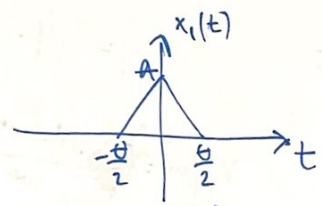
$$= x'(t)\left(1 + e^{j\omega_0 t} + e^{-j\omega_0 t}\right) \boxed{= x'(t)\left(1+2\cos(\omega_0 t)\right)}$$

$$\boxed{x(t) = \frac{1}{\theta}\cdot\frac{\sin\left(\frac{\pi}{\theta}t\right)}{\frac{\pi}{\theta}t}\left(1+2\cos(\omega_0 t)\right)}$$

3

$$\boxed{= \frac{1}{\theta}\cdot \operatorname{sinc}\left(\frac{t}{\theta}\right)\left(1+2\cos(\omega_0 t)\right)}$$

b) $x(t) =$



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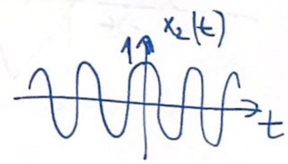
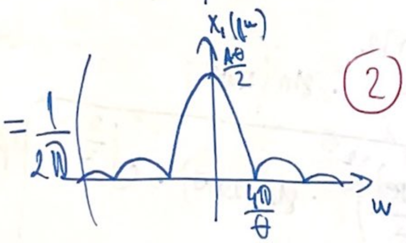


$X(j\omega) = \frac{1}{2\pi}\Bigg($



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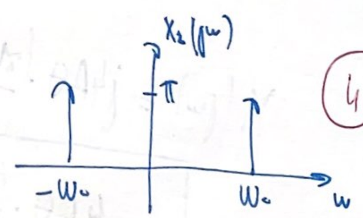

$\Bigg)$

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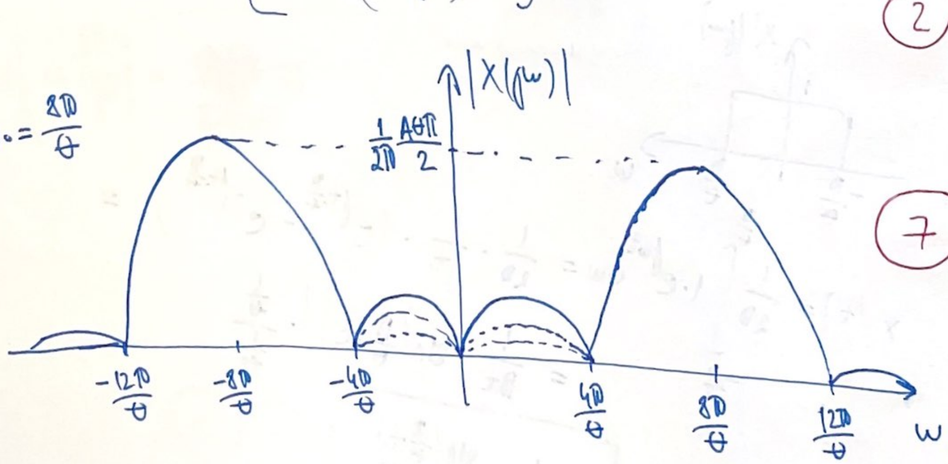
$$X(j\omega) = \frac{1}{2\pi}\left[\frac{A\theta}{2}\cdot\left(\frac{\sin\left(\omega\frac{\theta}{4}\right)}{\frac{\omega\theta}{4}}\right)^2\right] * \left[\pi\,\delta(\omega-\omega_0) + \pi\,\delta(\omega+\omega_0)\right]$$

2

$\omega_0 = \frac{8\pi}{\theta}$



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3. a) $x(t) = A\cos\left(\omega_0 t - \frac{\pi}{4}\right) + 2$

$A = 4V$ $\quad \frac{T}{T_s} = \frac{4}{0{,}5} = 8$ одмерака на периоду

$T = 4s$

$T_s = 0{,}5s$

$q = 5 \Rightarrow \boxed{n=3}$ $\quad \Delta = \frac{6-(-2)}{5} = \frac{8}{5} = \boxed{1{,}6V}$ 1

$x(0) = A\cos\left(-\frac{\pi}{4}\right) + 2 = 4\cdot\cos\left(\frac{\pi}{4}\right) + 2 = 4{,}83V$

$x(T_s) = A\cos\left(2\pi\frac{T_s}{T_0} - \frac{\pi}{4}\right) + 2 = 4\cos\left(\frac{\pi}{4} - \frac{\pi}{4}\right) + 2 = 6V$

$x(2T_s) = 4\cos\left(2\pi\frac{2T_s}{T_0} - \frac{\pi}{4}\right) + 2 = 4\cos\left(\frac{\pi}{2} - \frac{\pi}{4}\right) + 2 = 4{,}83V$

$x(3T_s) = 4\cos\left(2\pi\frac{3T_s}{T_0} - \frac{\pi}{4}\right) + 2 = 4\cos\left(\frac{3\pi}{4} - \frac{\pi}{4}\right) + 2 = 2V$

$x(4T_s) = 4\cos\left(2\pi\frac{4T_s}{T_0} - \frac{\pi}{4}\right) + 2 = 4\cos\left(\pi - \frac{\pi}{4}\right) + 2 = -0{,}83V$

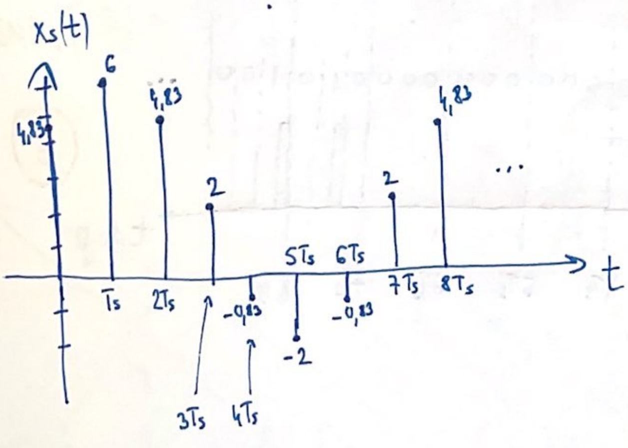
$x(5T_s) = 4\cos\left(2\pi\frac{5T_s}{T_0} - \frac{\pi}{4}\right) + 2 = 4\cos\left(\frac{5\pi}{4} - \frac{\pi}{4}\right) + 2 = -2V$

$x(6T_s) = 4\cos\left(2\pi\frac{6T_s}{T_0} - \frac{\pi}{4}\right) + 2 = 4\cos\left(\frac{6\pi}{4} - \frac{\pi}{4}\right) + 2 = -0{,}83$

$x(7T_s) = 4\cos\left(2\pi\frac{7T_s}{T_0} - \frac{\pi}{4}\right) + 2 = 4\cos\left(\frac{7\pi}{4} - \frac{\pi}{4}\right) + 2 = 2V$

$x(8T_s) = x(0) = 4{,}83V$

$\vdots$



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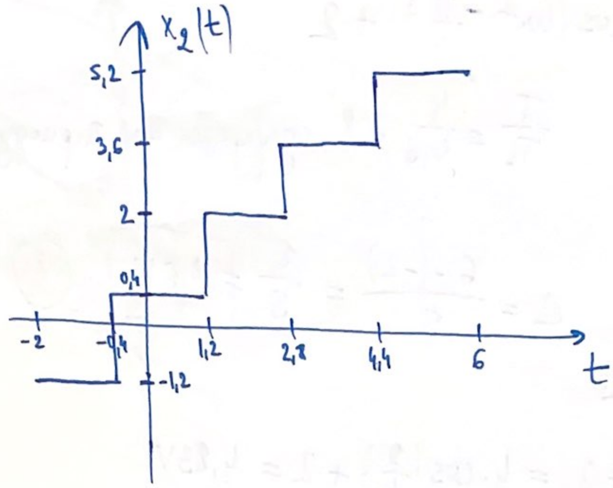

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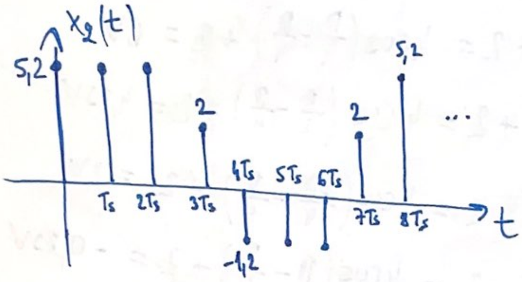

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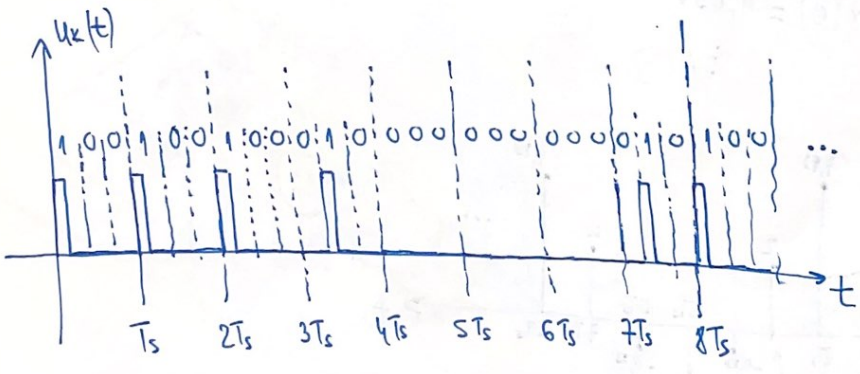
ниво	0	1	2	3	4
ампл.	-1,2	0,4	2	3,6	5,2
кодни зборови	000	001	010	011	100



6

δ) $f_g = 4{,}6\,kHz$

$f_s = 2f_g \boxed{= 9{,}2\,kHz}$ 2

$q = 8$

$$p_x(x) = \begin{cases} \frac{1}{16}, & |x(t)| \leq 8V \\ 0, & \text{иначе} \end{cases}$$

$$\Delta = \frac{8-(-8)}{8} = \frac{16}{8} = 2V$$

$$P_q = \frac{\Delta^2}{12} = \frac{4}{12} \boxed{= \frac{1}{3}\,[V^2]}$$ 2

$$P_s = \int_{-8}^{8} x^2 p(x)\,dx = 2 \cdot \int_0^8 x^2 \cdot \frac{1}{16}\,dx = \frac{1}{8} \cdot \frac{x^3}{3}\Big|_0^8 = \frac{\cancel{512}}{\cancel{24}} = \frac{64}{3}\,[V^2]$$

$$SNR_q = \frac{P_s}{P_q} = \frac{\frac{64}{3}}{\frac{1}{3}} \boxed{= 64}$$ 2 $= 8^2 = q^2 = 2^{2n}$ $(n=3)$

$$SNR_q\,[dB] = 10\log 64 = 10\log 2^{2n} = 6n \boxed{= 18\,dB}$$ 2

$n = 6 \Rightarrow SNR_q = 6n \boxed{= 36\,dB}$ 2