

Formalne metode u softverskom inženjerstvu

07 RE; Gramatike

ETFB 24-25

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$$r = b(ab)^* \mid ab^*a$$

$$\begin{aligned} r &= r_1 \mid r_2 \\ r_1 &= b(ab)^* \\ r_2 &= ab^*a \end{aligned}$$

$$\begin{aligned} r_1 &= r_3 r_4 \\ r_3 &= b \\ r_4 &= (ab)^* \end{aligned}$$

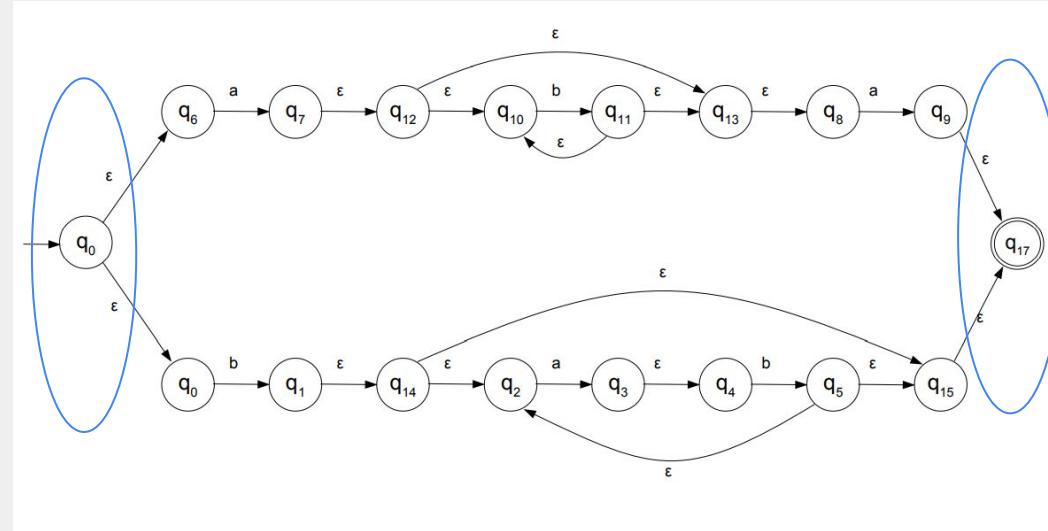
$$\begin{aligned} r_4 &= r_5^* \\ r_5 &= ab \end{aligned}$$

$$\begin{aligned} r_5 &= r_6 r_7 \\ r_6 &= a \\ r_7 &= b \end{aligned}$$

$$\begin{aligned} r_2 &= r_8 r_9 \\ r_8 &= a \\ r_9 &= b^*a \end{aligned}$$

$$\begin{aligned} r_9 &= r_{10} r_{11} \\ r_{10} &= b^* \\ r_{11} &= a \end{aligned}$$

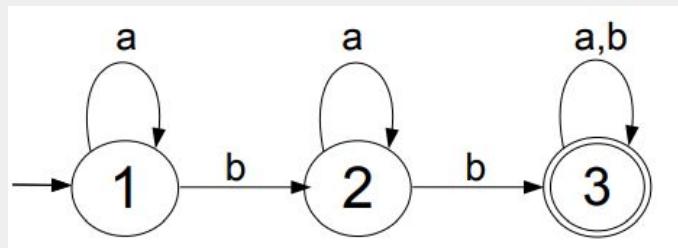
$$\begin{aligned} r_{10} &= r_{12}^* \\ r_{12} &= b \end{aligned}$$



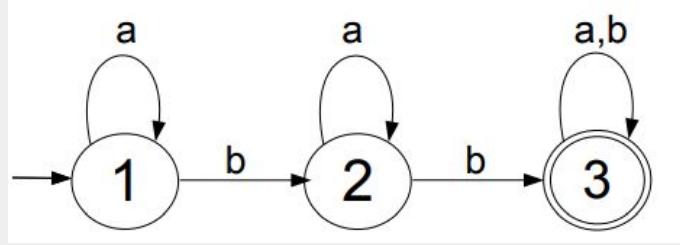
RE → e-NKA → NKA → DKA

RE ← e-NKA ← NKA ← DKA ??

Konstrukcija RE na osnovu automata



Regularni izraz?

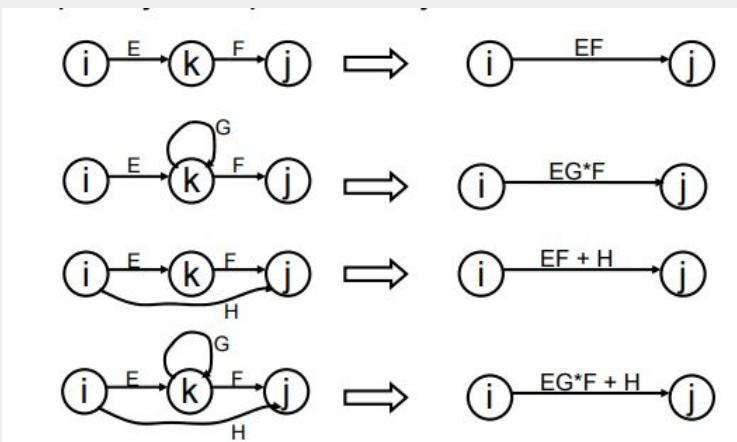


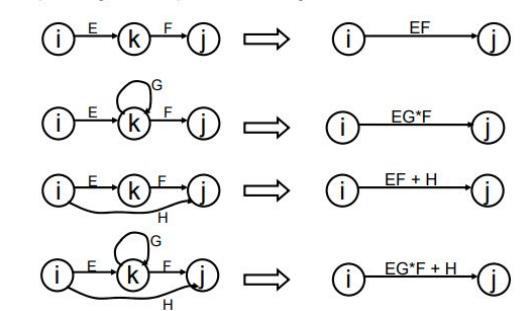
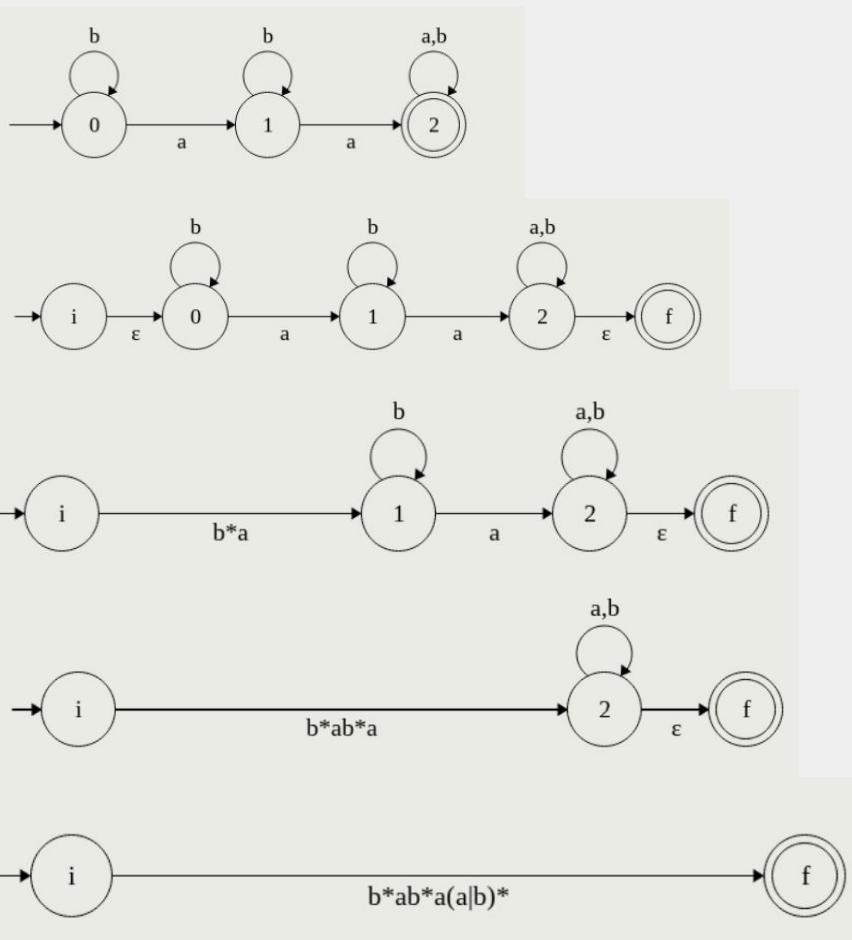
$a^*ba^*b(a \mid b)^*$

Složeniji automat?

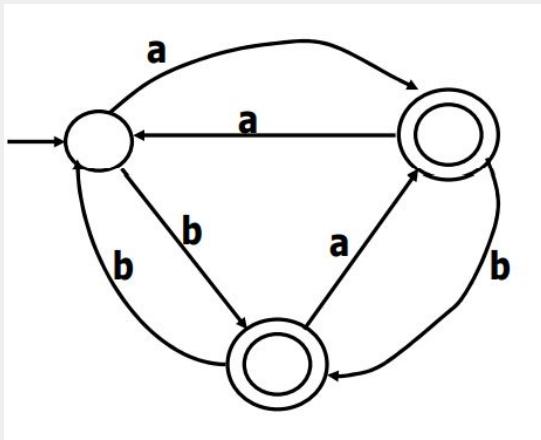
1. Dodajemo jedno novo početno stanje sa ϵ -prelazom do starog početnog
2. Dodajemo jedno novo završno stanje sa ϵ -prelazima od svih završnih
3. odaberemo stanje za eliminaciju (k)

Zamenjujemo na sledeći način:

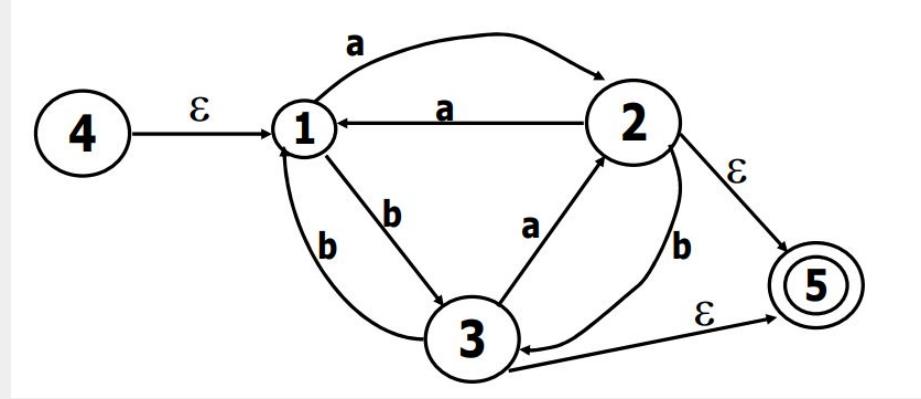
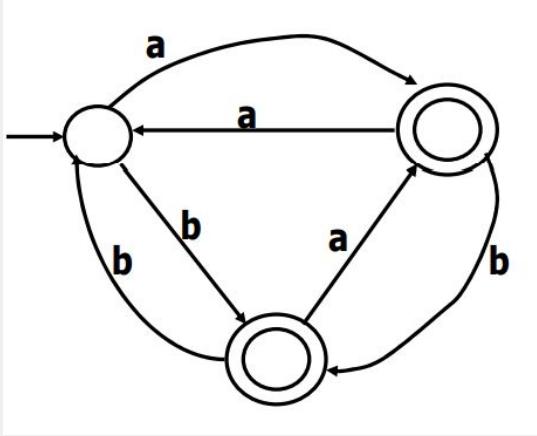




Konačan regularan izraz zavisi od izbora redosleda stanja koji se eliminišu

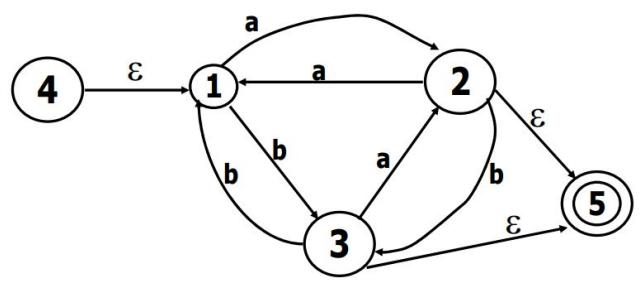


Složeniji automat?
Idea u nastavku.



Numerišu se stanja

Dodaju se novo početno i završno i numerišu se sa n-1 i n



Eliminacija stanja 1

2-1-2 2-(aa)-2

2-1-3 2-(b+ab)-3

2-1-4 \emptyset

2-1-5 \emptyset

3-1-2 3-(ba+a)-2

3-1-3 3-(bb)-3

3-1-4 \emptyset

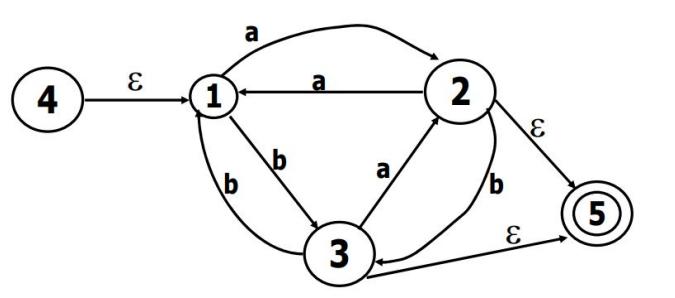
3-1-5 \emptyset

4-1-2 4-(a)-2

4-1-3 4-(b)-3

4-1-4 \emptyset

4-1-5 \emptyset



2-1-2 2-(aa)-2

2-1-3 2-(b+ab)-3

2-1-4 \emptyset

2-1-5 \emptyset

3-1-2 3-(ba+a)-2

3-1-3 3-(bb)-3

3-1-4 \emptyset

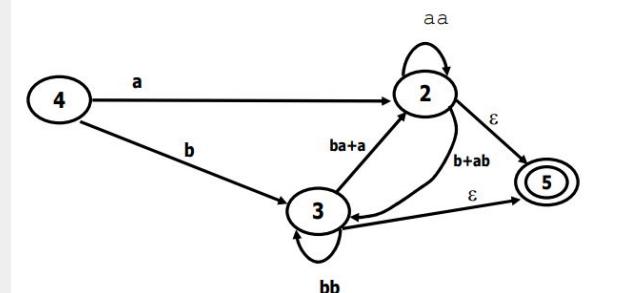
3-1-5 \emptyset

4-1-2 4-(a)-2

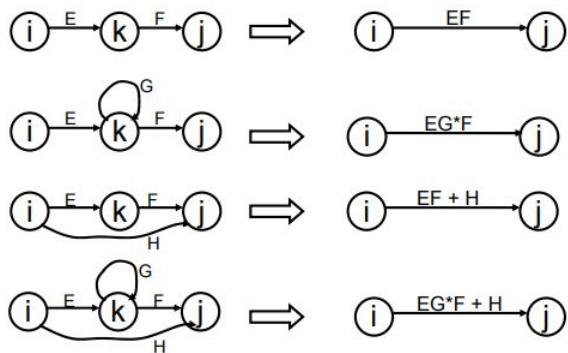
4-1-3 4-(b)-3

4-1-4 \emptyset

4-1-5 \emptyset



Može se nastaviti proces za novodobijeni automat.
Eliminiše se naredno stanje (2).



Od i do j uvek imamo:

Ili se direktno prelazi iz i u j bez prolaska kroz stanje k .

Ili se prolazi kroz k

- prvo ide prelaz do k
pa eventualno petlja u k
pa nastavak do j .

Uopštavanje:

Zamislimo da ima više prelaza od i do k , k do k , i od k do j , da nisu u pitanju nužno direktni prelazi.

Ili se prelazi iz i u j bez prolaska kroz stanje k .

Ili se prolazi kroz k

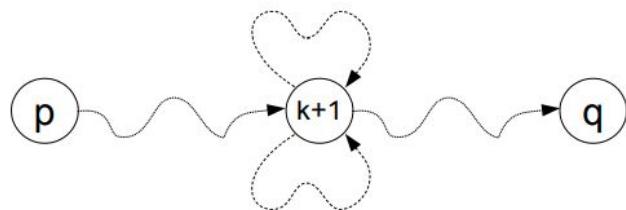
- prvo idu prelazi do k
pa eventualno "petlja/ciklus" u k
pa prelazi do j .

DKA

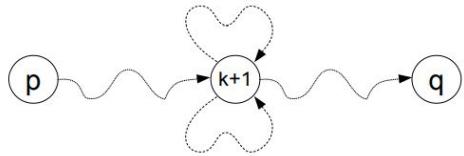
Numerišemo stanja

$L(p, q, k)$ - svi prelazi od p do q ali samo prolaskom kroz stanja $\leq k$

$$L = L(p, k+1, k)L(k+1, k+1, k)^*L(k+1, q, k)$$



$$L = L(p, k+1, k)L(k+1, k+1, k)^*L(k+1, q, k)$$

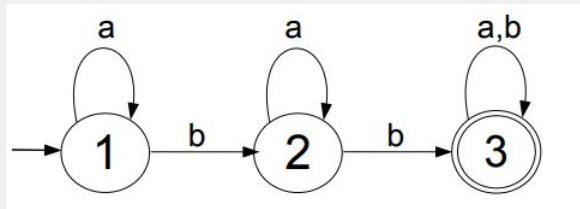


Rekurzivna formula

$$r(p, q, k+1) = r(p, q, k) \mid r(p, k+1, k)(r(k+1, k+1, k))^*r(k+1, q, k)$$

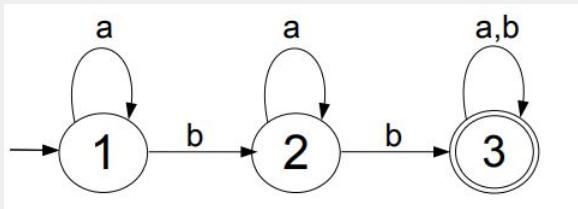
$r(p, q, 0)$ - direktni prelazi

$r(s_0, f, n)$ - konačni rezultat (f - finalno stanje, n - broj stanja)



1 - početno
3 - završno

$$r(1, 3, 3) = ?$$



$$r(p, q, k+1) = r(p, q, k) \mid r(p, k+1, k)r(k+1, k+1, k)*r(k+1, q, k)$$

$$r(1, 3, 3) = r(1, 3, 2) \mid r(1, 3, 2)r(3, 3, 2)*r(3, 3, 2)$$

$$r(1, 3, 2) = r(1, 3, 1) \mid r(1, 2, 1)r(2, 2, 1)*r(2, 3, 1)$$

$$r(1, 3, 1) = r(1, 3, 0) \mid r(1, 1, 0)r(1, 1, 0)*r(1, 3, 0)$$

$$r(1, 2, 1) = r(1, 2, 0) \mid r(1, 1, 0)r(1, 1, 0)*r(1, 2, 0)$$

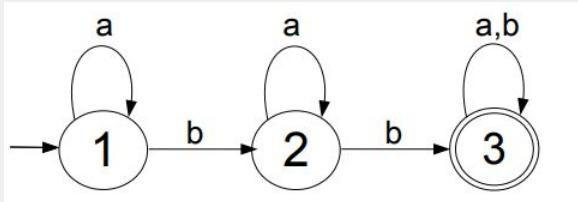
$$r(2, 2, 1) = r(2, 2, 0) \mid r(2, 1, 0)r(1, 1, 0)*r(1, 2, 0)$$

$$r(2, 3, 1) = r(2, 3, 0) \mid r(2, 1, 0)r(1, 1, 0)*r(1, 3, 0)$$

$$r(3, 3, 2) = r(3, 3, 1) \mid r(3, 2, 0)r(2, 2, 1)*r(2, 3, 1)$$

$$r(3, 3, 1) = r(3, 3, 0) \mid r(3, 1, 0)r(1, 1, 0)*r(1, 3, 0)$$

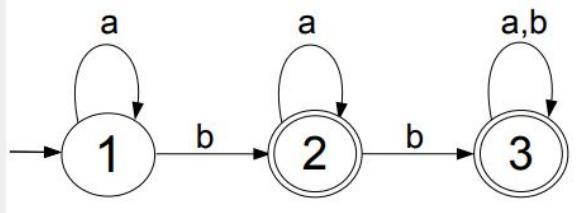
p	r(p,1,0)	r(p,2,0)	r(p,3,0)
1	$\epsilon \mid a$	b	\emptyset
2	\emptyset	$\epsilon \mid a$	b
3	\emptyset	\emptyset	$\epsilon \mid a \mid b$



$r(p, q, k+1) = r(p, q, k) \mid r(p, k+1, k)r(k+1, k+1, k)*r(k+1, q, k)$
 $r(1, 3, 3) = r(1, 3, 2) \mid r(1, 3, 2)r(3, 3, 2)*r(3, 3, 2)$
 $r(1, 3, 2) = r(1, 3, 1) \mid r(1, 2, 1)r(2, 2, 1)*r(2, 3, 1)$
 $r(1, 3, 1) = r(1, 3, 0) \mid r(1, 1, 0)r(1, 1, 0)*r(1, 3, 0)$
 $r(1, 2, 1) = r(1, 2, 0) \mid r(1, 1, 0)r(1, 1, 0)*r(1, 2, 0)$
 $r(2, 2, 1) = r(2, 2, 0) \mid r(2, 1, 0)r(1, 1, 0)*r(1, 2, 0)$
 $r(2, 3, 1) = r(2, 3, 0) \mid r(2, 1, 0)r(1, 1, 0)*r(1, 3, 0)$
 $r(3, 3, 2) = r(3, 3, 1) \mid r(3, 2, 0)r(2, 2, 1)*r(2, 3, 1)$
 $r(3, 3, 1) = r(3, 3, 0) \mid r(3, 1, 0)r(1, 1, 0)*r(1, 3, 0)$

p	r(p,1,0)	r(p,2,0)	r(p,3,0)
1	$\epsilon \mid a$	b	\emptyset
2	\emptyset	$\epsilon \mid a$	b
3	\emptyset	\emptyset	$\epsilon \mid a \mid b$

$r(3, 3, 1) = (\epsilon \mid a \mid b) \mid \emptyset (\epsilon \mid a)^* \emptyset = \epsilon \mid a \mid b$
 $r(2, 2, 1) = \epsilon \mid a \mid \emptyset (\epsilon \mid a)^* b = \epsilon \mid a$
 $r(2, 3, 1) = b \mid \emptyset (\epsilon \mid a)^* \emptyset = b$
 $r(1, 2, 1) = b \mid (\epsilon \mid a)(\epsilon \mid a)^* b = b \mid a^* b = (\epsilon \mid a^*)b = a^* b$
 $r(3, 3, 2) = \epsilon \mid a \mid b \mid \emptyset a^* b = \epsilon \mid a \mid b$
 $r(1, 3, 1) = \emptyset \mid (\epsilon \mid a)(\epsilon \mid a)^* \emptyset = \emptyset$
 $r(1, 3, 2) = \emptyset \mid a^* b (\epsilon \mid a)^* b = a^* b (\epsilon \mid a)^* b = a^* b a^* b$
 $r(1, 3, 3) = a^* b a^* b \mid a^* b a^* b (\epsilon \mid a \mid b)^* (\epsilon \mid a \mid b)$
 $= a^* b a^* b \mid (a \mid b)^*$



Sada je i stanje 2 konačno

$$r = r(1,2,3) \mid r(1,3,3)$$

$$r = r(1,2,3) \mid (a^*ba^*b \mid (a \mid b)^*)$$

$$r(p, q, k+1) = r(p, q, k) \mid r(p, k+1, k)r(k+1, k+1, k)^*r(k+1, q, k)$$

$$r(1, 2, 3) = r(1, 2, 2) \mid r(1, 3, 2)r(3, 3, 2)^*r(3, 2, 2)$$

$$r(1, 2, 2) = r(1, 2, 1) \mid r(1, 2, 1)r(2, 2, 1)^*r(2, 2, 1)$$

$$r(3, 2, 2) = r(3, 2, 1) \mid r(3, 2, 1)r(2, 2, 1)^*r(2, 2, 1)$$

$$r(1, 2, 1) = a^*b, \quad r(2, 2, 1) = \varepsilon \mid a$$

$$r(1, 2, 2) = a^*b \mid (a^*b)(\varepsilon \mid a)^*(\varepsilon \mid a) = a^*b \mid a^*b(\varepsilon \mid a)^* = a^*b \mid a^*ba^*$$

$$r(3, 2, 2) = \emptyset \mid \emptyset (\varepsilon \mid a)^*(\varepsilon \mid a) = \emptyset$$

$$r(1, 2, 3) = a^*b \mid a^*ba^* \mid a^*ba^*b(\varepsilon \mid a \mid b)^*\emptyset = a^*b \mid a^*ba^*$$

$$r = a^*b \mid a^*ba^* \mid a^*ba^*b \mid a^*ba^*b(\varepsilon \mid a \mid b)^*(\varepsilon \mid a \mid b)$$