ГОДИНА: 2016.

# ОЕТ 2(МАГНЕТИЗАМ)

## ~РЕШЕНА ЗБИРКА И КОЛОКВИЈУМИ~

АУТОР:
ЗОРАН ЛАЗИЋ

СТУДЕНТСКА ОРГАНИЗАЦИЈА ЕЛЕКТРОН

# Стално магнетско поље у вакууму

① $Q_1, Q_2, |\vec{v}_1|, |\vec{v}_2| << c$

$r$ | $\vec{F}_e, \vec{F}_m = ?$

$$\vec{F}_e = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{r^2} \vec{r}_{012}$$

$$\vec{F}_m = \frac{\mu_0}{4\pi} \frac{Q_2\vec{v}_2 \times (Q_1\vec{v}_1 \times \vec{r}_{012})}{r^2}$$

② $I_1, I_2$

a) $$d\vec{B}_1 = \frac{\mu_0}{4\pi} \frac{dI\, d\vec{l} \times \vec{r}_{012}}{r^2}$$

$I_1 \; d\vec{l}_1$

$$d\vec{F}_m = Q_2\vec{v}_2 \times d\vec{B}_1$$

$$Q_2\vec{v}_2 = \frac{d\vec{l}_2}{dt} = I_2 d\vec{l}_2$$

$$\boxed{d\vec{F}_m = I_2 d\vec{l}_2 \times \frac{\mu_0}{4\pi} \frac{I_1 d\vec{l}_1 \times \vec{r}_{012}}{r_{12}^2}}$$

б) $\vec{F}_m = \oint d\vec{F}_m$

$$\vec{F}_m = \frac{\mu_0}{4\pi} \oint_{C_2} I_2 d\vec{l}_2 \times \oint_{C_1} \frac{I_1 d\vec{l}_1 \times \vec{r}_{012}}{r_{12}^2}$$

$$\vec{F}_m = \frac{\mu_0}{4\pi} \oint_{C_2} \oint_{C_1} \frac{I_2 d\vec{l}_2 \times I_1 d\vec{l}_1 \times \vec{r}_{012}}{r_{12}^2}$$

в) У општем случају не важи

③ $Q = 10\,nC$

$\vec{v} = 10\,\vec{i}_y\; km/s$

$\vec{E} = 10\,\vec{i}_x\; \frac{V}{m}$

$\vec{B} = 2\,\vec{i}_z\; mT$

$\vec{F} = ?$

$$\vec{F} = \vec{F}_e + \vec{F}_m$$

$$\vec{F}_e = Q\cdot\vec{E} = 0{,}1\,\mu N\,\vec{i}_x$$

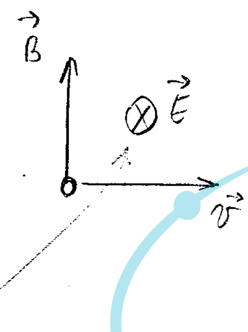
$$\vec{F}_m = Q\vec{v}\times\vec{B} = 10nC\cdot 10\cdot 10^3 \tfrac{m}{s}\,\cancel{\vec{i}_y \times \vec{i}_z}\; 2\cdot 10^{-3}\,T = 0{,}2\,\mu N\,\vec{i}_x$$

$$\boxed{\vec{F} = 0{,}3\,\mu N\,\vec{i}_x}$$

④ $Q, v, \vec{B}\; (\vec{B} \perp \vec{v})$

$\vec{E} = ?$

$\vec{F} = 0$




$$\vec{F}_e + \vec{F}_m = 0$$

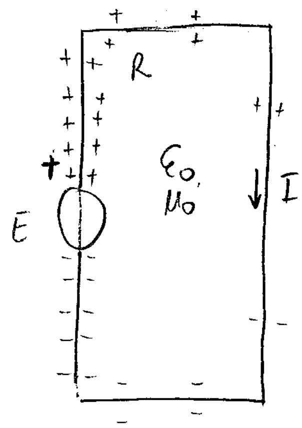
$$Q\cdot\vec{E} + Q\vec{v}\times\vec{B} = 0$$

$$Q E + \cancel{Q} v B = 0 \Rightarrow E = -vB \Rightarrow \vec{E} = -\vec{v}\times\vec{B}$$

⑤ $R, E, Q_p$

a) $v = 0$

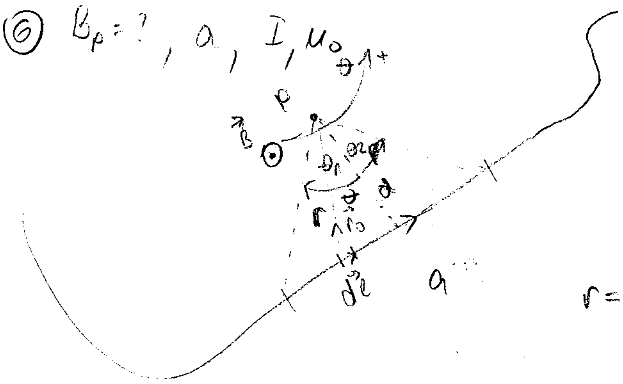
b) $v$




a) $\vec{Fe} = Q \cdot \vec{E}$

b) $\vec{Fe} = Q \cdot \vec{E}$ ; $\vec{F_m} = Q\, \vec{v} \times \vec{B}$

⑥ $B_p = ?$ , $a$, $I$, $\mu_0$




$$B = \frac{\mu_0}{4\pi} \int \frac{I\, d\theta}{r}$$

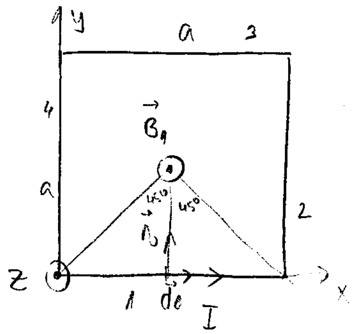
$$B = \frac{\mu_0 I}{4\pi d} \int_{\theta=\theta_1}^{\theta_2} \cos\theta$$

$$r = \frac{d}{\cos\theta}$$

$$\boxed{B = \frac{\mu_0 I}{4 d \pi}\left(\sin\theta_2 - \sin\theta_1\right)}$$

⑦ $a = 0{,}1\,m$

$I = 10\,A$




$B_1 = B_2 = B_3 = B_4$

$\vec{B} = 4 \cdot \vec{B_1}$

$$\vec{B} = 4 \frac{\mu_0 I}{4\pi \frac{a}{2}} \left(\sin 45° - \sin(-45°)\right) \vec{i_z}$$

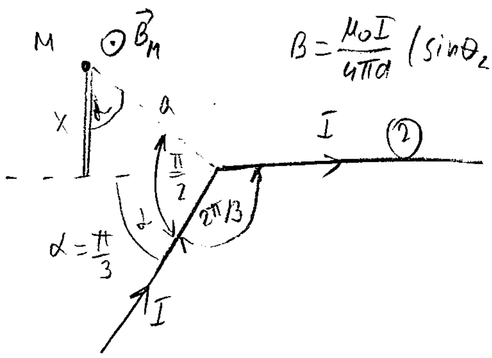
$\sqrt{2}$

$$\boxed{B = \frac{2\mu_0 I \sqrt{2}}{\pi a} \approx 113\,mT}$$

⑧ $I = 50\,A$

$B_M = ?$

$a = 80\,mm$




$$B = \frac{\mu_0 I}{4\pi d}\left(\sin\theta_2 - \sin\theta_1\right)$$

$$B_M = B_1 + B_2$$

$$B_1 = \frac{\mu_0 I}{4\pi a}\left(\sin 0 - \sin\left(-\frac{\pi}{2}\right)\right) = \frac{\mu_0 I}{4\pi a}$$

$$B_2 = \frac{\mu_0 I}{4\pi x}\left(\sin\frac{\pi}{2} - \sin\alpha\right)$$

$$x = a\cos\alpha = \frac{a}{2}$$

$$B_2 = \frac{\mu_0 I}{4\pi \frac{a}{2}}\left(1 - \frac{\sqrt{3}}{2}\right)$$

$$B_2 = \frac{2\mu_0 I}{4\pi a}\left(1 - \frac{\sqrt{3}}{2}\right) = \frac{\mu_0 I}{4\pi a}\left(2 - \sqrt{3}\right)$$

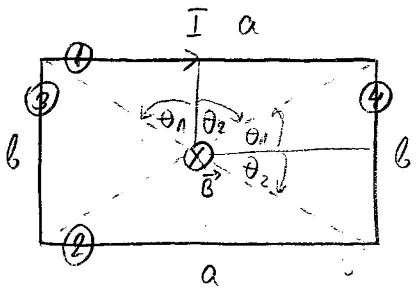
$$B = \frac{\mu_0 I}{4\pi a}\left(2 - \sqrt{3} + 1\right)$$

$$\boxed{B = \frac{\mu_0 I}{4\pi a}\left(3 - \sqrt{3}\right)}$$

⑨ $a = 100\,mm$

$b = 50\,mm$

$I = 20A$




$\mu_0$

$B_1 = B_2$

$B_3 = B_4$

$\vec{B} = 2\vec{B_1} + 2\vec{B_3}$

$$B = \frac{\mu_0 I}{4\pi d}(\sin\theta_2 - \sin\theta_1)$$

($B_1$) $d = \frac{b}{2}$

$$\sin\theta_2 = -\sin\theta_1 = \frac{\frac{a}{2}}{\sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}}$$

$$B_1 = \frac{\mu_0 I}{\cancel{4}\pi \frac{b}{\cancel{2}}} \cancel{2} \frac{\frac{a}{\cancel{2}}}{\frac{\sqrt{a^2+b^2}}{\cancel{2}}} = \frac{\mu_0 I}{\pi b}\frac{a}{\sqrt{a^2+b^2}}$$

($B_2$) $d = \frac{a}{2}$

$$\sin\theta_2 = -\sin\theta_1 = \frac{\frac{b}{2}}{\sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}}$$

$$B_3 = \frac{\mu_0 I}{\cancel{4}\pi \frac{a}{2}} \cancel{2} \frac{\frac{b}{\cancel{2}}}{\frac{\sqrt{a^2+b^2}}{\cancel{2}}} = \frac{\mu_0 I}{a\pi}\frac{b}{\sqrt{a^2+b^2}}$$

$$B = 2B_1 + 2B_3 = 2\frac{\mu_0 I}{\pi}\left(\frac{a}{b\sqrt{a^2+b^2}} + \frac{b}{a\sqrt{a^2+b^2}}\right) = 2\frac{\mu_0 I}{\pi}\frac{\sqrt{a^2+b^2}\,\cancel{(a^2+b^2)}}{ba\,\cancel{(a^2+b^2)}}$$

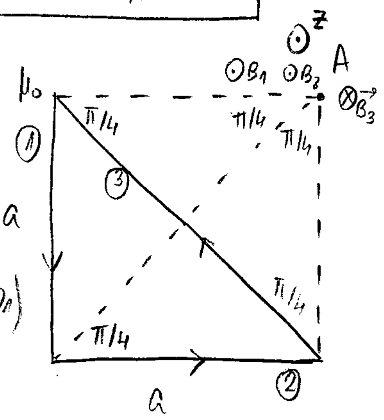
$$\boxed{B = \frac{2\mu_0 I\sqrt{a^2+b^2}}{ab\pi}}$$

⑩ $a = 250\,mm$

$I = 100\,mA$

$B_A = ?$

$$B = \frac{\mu_0 I}{4\pi d}(\sin\theta_2 - \sin\theta_1)$$




1) $B_1 = \frac{\mu_0 I}{4\pi a}\left(\sin \overset{\frac{\sqrt{2}}{2}}{\cancel{45^\circ}} - \sin \overset{0}{\cancel{0}}\right) \Rightarrow$

$$\vec{B_1} = \frac{\mu_0 I\sqrt{2}}{8\pi a}\vec{i_z}$$

2) $B_2 = \frac{\mu_0 I}{4\pi a}\left(\sin\overset{0}{\cancel{0}} - \sin\overset{-\frac{\sqrt{2}}{2}}{\cancel{(-45^\circ)}}\right) \Rightarrow$

$$\vec{B_2} = \frac{\mu_0 I\sqrt{2}}{8\pi a}\vec{i_z}$$

3) $B_3 = \frac{\mu_0 I}{4\pi \frac{a\sqrt{2}}{\cancel{2}}}\left(\sin 45^\circ - \sin(-45^\circ)\right)$

$$B_3 = \frac{\mu_0 I}{2\pi a\cancel{\sqrt{2}}}\cancel{\sqrt{2}} \qquad \boxed{\vec{B_3} = -\frac{\mu_0 I}{2\pi a}\vec{i_z}}$$

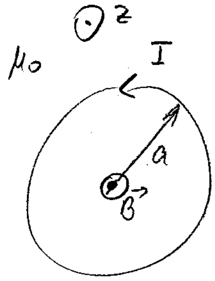
$$\vec{B} = \frac{\mu_0 I}{4\pi a}\sqrt{2}\,\vec{i_z} - \frac{\mu_0 I}{2\pi a}\vec{i_z}$$

$$\boxed{\vec{B} = \frac{\mu_0 I}{4\pi a}\left(\sqrt{2} - 2\right)\vec{i_z}}$$

$$\boxed{B = 23{,}4\,nT}$$

(11) $a = 0{,}1\,m$

$I = 50\,A$




$$B = \frac{\mu_0}{4\pi}\int_0 \frac{I\,d\theta}{r}$$

$$B = \frac{\mu_0 I}{4\pi a}\int_{\theta=0}^{2\pi} d\theta$$

$$\boxed{\vec{B} = \frac{\mu_0 I}{2a}\vec{i}_z}$$

$B \approx 314\,\mu T$

(12) $a, Q, \vec{\omega}$

a) $\vec{E}_0 = ?$

б) $\vec{B}_0 = ?$

a) $\vec{E} = 0$ (због симетрије

б) $\vec{B} = \frac{\mu_0 I}{2a}\vec{i}_z$

$$I = \frac{dQ}{dt} = \frac{Q\cdot\frac{dl}{2a\pi}}{dt} = \frac{Q}{2a\pi}\cdot v = \frac{Q}{2a\pi}\omega a$$

$$I = \frac{Q}{2\pi}\omega$$

$$\boxed{\vec{B} = \frac{\mu_0 Q\vec{\omega}}{4a\pi}}$$

(13) $a = 0{,}1\,m$

$d = 1\cdot 10^{-3}\,m$ $\quad I = const$

$N = 8{,}47\cdot 10^{28}\,m^{-3}$

$|\vec{v}| = 1\cdot 10^{-3}\,\frac{m}{s}$ $\quad e = 1{,}602\cdot 10^{-19}\,C$

$\vec{B} = ?$

$$J = Nqv = Nev$$

$$I = J\cdot S = Nev\cdot\frac{d^2}{4}\pi$$

$$B = \frac{\mu_0 I}{2a} = \frac{\mu_0 N e v d^2\pi}{8a} = 67\,\mu T$$

(14) $N = 10$

$a = 0{,}1\,m$

$I = 25\,A$

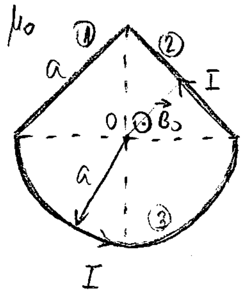
$B = ?$

$$dB = \frac{\mu_0 I}{2a} \qquad B = N\,dB \Rightarrow B = \frac{N\mu_0 I}{2a} \qquad \boxed{B = 1{,}57\,mT}$$

(15) $a, I$

$B_0 = ?$

$B_0 = B_1 + B_2 + B_3$




1) $B_1 = \frac{\mu_0 I}{4\pi\frac{a\sqrt{2}}{2}}\left(\sin 45^\circ - \sin(-45^\circ)\right) = \frac{\mu_0 I}{2a\pi}$

2) $B_2 = B_1 = \frac{\mu_0 I}{2a\pi}$

3) $B_3 = \frac{\mu_0 I}{4\pi a}\int_{\theta=0}^{\pi} d\theta = \frac{\mu_0 I}{4a}$

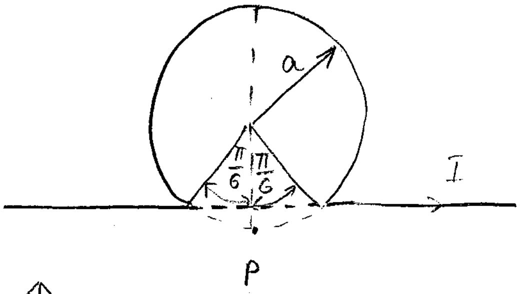
$$B = \frac{\mu_0 I}{a\pi} + \frac{\mu_0 I}{4a} = \frac{\mu_0 I}{a}\left(\frac{1}{\pi} + \frac{1}{4}\right)$$

(16) $a = 0{,}1\,m$

$I = 100\,A$

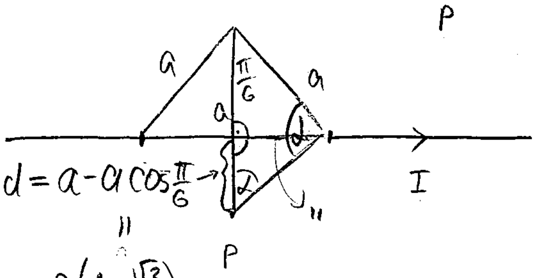
$B = ?$

$$\boxed{B = B_1 + B_2 = 253{,}5\,mT}$$




$B = B_1 + B_2$

(1)




$d = a - a\cos\frac{\pi}{6} = a\left(1 - \frac{\sqrt{3}}{2}\right)$

$$B_1' = \frac{\mu_0 I}{4\pi d}\left(\sin\frac{\pi}{2} - \sin\left(-\frac{\pi}{2}\right)\right)$$

$$B_1' = \frac{\mu_0 I}{2\pi a\left(1 - \cos\frac{\pi}{6}\right)} = \frac{\mu_0 I}{2\pi a\left(1 - \frac{\sqrt{3}}{2}\right)}$$

$B_1 = B_1' - B_1''$

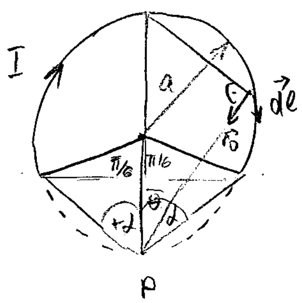
$$B_1'' = \frac{\mu_0 I}{4\pi d}\left(\sin\alpha - \sin(-\alpha)\right) = \frac{\mu_0 I}{4\pi d}2\sin\alpha = \frac{\mu_0 I}{2d\pi}\sin\alpha = \frac{\mu_0 I \sin\frac{5\pi}{12}}{2\pi a\left(1 - \frac{\sqrt{3}}{2}\right)}$$

$2\alpha + \frac{\pi}{6} = \pi$

$2\alpha = \frac{5\pi}{6} \Rightarrow \alpha = \frac{5\pi}{12}$

$$\boxed{B_1 = \frac{\mu_0 I}{2\pi a\left(1 - \frac{\sqrt{3}}{2}\right)}\left(1 - \sin\frac{5\pi}{12}\right)} \qquad B_1 = 50{,}8\,mT$$

(2)




$r = 2a\cos\alpha$

$$B_2 = \frac{\mu_0}{4\pi}\int_{\theta}\frac{I\,d\theta}{r} = \frac{\mu_0 I}{4\pi 2a}\int_{\theta=-\alpha}^{\alpha}\frac{d\theta}{\cos\theta}$$

$$\int\frac{d\theta}{\cos\theta} = \int\frac{\cos\theta\,d\theta}{1 - \sin^2\theta} = \left|\begin{matrix}\sin\theta = t\\ \cos\theta\,d\theta = dt\end{matrix}\right| = \int\frac{dt}{(1-t)(1+t)} =$$

$$\frac{1}{(1-t)(1+t)} = \frac{1}{2}\frac{1}{1-t} + \frac{1}{2}\frac{1}{1+t} \qquad = \frac{1}{2}\left(\int\frac{dt}{1-t} + \int\frac{dt}{1+t}\right) =$$

$t \to 1$ $\qquad t \to -1$

$$= \frac{1}{2}\ln\frac{1+\sin\theta}{1-\sin\theta}$$

$$B_2 = \frac{\mu_0 I}{8a\pi}\cdot\frac{1}{2}\left(\ln\frac{1+\sin\frac{5\pi}{12}}{1-\sin\frac{5\pi}{12}} - \ln\frac{1+\sin\left(-\frac{5\pi}{12}\right)}{1-\sin\left(-\frac{5\pi}{12}\right)}\right) = \frac{\mu_0 I}{16a\pi}\ln\left(\frac{1+\sin\frac{5\pi}{12}}{1-\sin\frac{5\pi}{12}}\cdot\frac{1+\sin\frac{5\pi}{12}}{1-\sin\frac{5\pi}{12}}\right) =$$

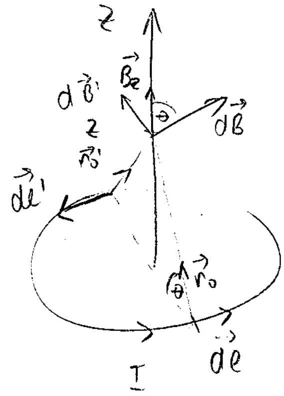
$$= \frac{\mu_0 I}{\cancel{16}_{8} a\pi}\cancel{2}\ln\frac{1+\sin\frac{5\pi}{12}}{1-\sin\frac{5\pi}{12}}$$

$$\boxed{B_2 = \frac{\mu_0 I}{8a\pi}\ln\frac{1+\sin\frac{5\pi}{12}}{1-\sin\frac{5\pi}{12}}} \qquad B_2 = 202{,}7\,mT$$

(17) $I = 1A$

$z = 0{,}1\,m$

$a = ? \Leftrightarrow B = B_{max}$




$$dB = \frac{\mu_0}{4\pi} \frac{I\, d\vec{l} \times \vec{r}_0}{r^2}$$

$$r = \sqrt{a^2+z^2}$$

$$dB = \frac{\mu_0}{4\pi} \frac{I\, dl}{r^2}$$

$$|d\vec{l} \times \vec{r}_0| = dl\, \cancel{r_0} \sin\angle(d\vec{l}, \vec{r}_0) = dl$$

$$B = B_z = \int dB \cos\theta, \quad \cos\theta = \frac{a}{r}$$

$$B = \frac{\mu_0}{4\pi} \int_l \frac{I a\, dl}{r^3} = \frac{\mu_0 I a \cdot 2a\pi}{4\pi (a^2+z^2)^{3/2}}$$

$$\boxed{\vec{B} = \frac{\mu_0 I a^2}{2(a^2+z^2)^{3/2}} \vec{i}_z}$$

$B = B_{max} \Leftrightarrow B' = 0$ $\quad f(a) = \frac{(a^2+z^2)^{3/2}}{a^2}$ → ako ova f-ja ima minimum, B ima maksimum

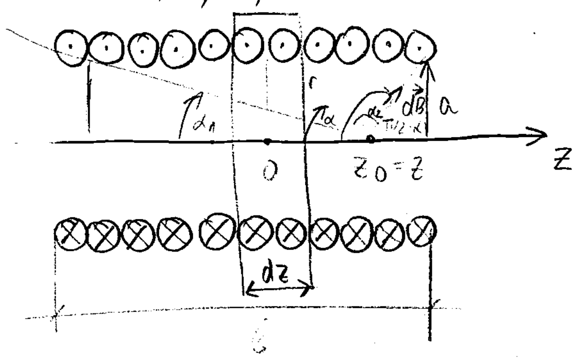
$$f'(a) = \frac{\frac{3}{2}(a^2+z^2)^{1/2}\, 2a\, a^2 - 2a(a^2+z^2)^{3/2}}{a^4}$$

$$3\cancel{(a^2+z^2)^{1/2}} a^2 = 2(a^2+z^2)\cancel{(a^2+z^2)^{1/2}}$$

$$3a^2 = 2a^2 + 2z^2$$

$$\boxed{a = \pm\sqrt{2}\, z}$$

~~(18)~~ (19) $a, b, N, I$




$$\frac{dN}{dz} = \frac{N}{b} \Rightarrow dN = \frac{N}{b} dz$$

$$dB = \frac{\mu_0 I a^2}{2r^3} \quad (1) \qquad \underline{dI = I\, dN}$$

$$dB = \frac{\mu_0 I a^2}{2r^3} \cdot \frac{N}{b} dz = \frac{\mu_0 I \cancel{a^2} N \cancel{a} d\alpha}{2b \frac{\cancel{a^3}}{\sin^3\alpha} \cancel{\sin^2\alpha}}$$

$$ctg\,\alpha = -\frac{z}{a}$$

$$\cancel{-}\frac{d\alpha}{\sin^2\alpha} = \cancel{-}\frac{dz}{a} \Rightarrow dz = \frac{a\, d\alpha}{\sin^2\alpha}$$

$$r = \frac{a}{\sin\alpha}$$

$$B = \int dB = \int_{\alpha_1}^{\alpha_2} \frac{\mu_0 I N}{2b} \sin\alpha\, d\alpha$$

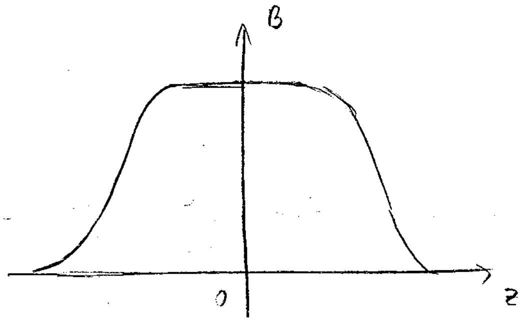
$$\boxed{\vec{B} = \frac{\mu_0 I N}{2b}(\cos\alpha_1 - \cos\alpha_2)\vec{i}_z}$$

⑳ $a = 2\,cm$

$b = 20\,cm$

$N = 200$

$I = 10\,A$




$$\vec{B} = \frac{\mu_0 I N}{2b}\left(\cos\alpha_1 - \cos\alpha_2\right)\vec{i}_z \qquad B = B_{max} \Leftrightarrow B' = 0 \qquad \left(-\sin\alpha_1 + \sin\alpha_2\right) = 0$$

$$\sin\alpha_1 = \sin\alpha_2$$

$$\alpha \in (0, 180°) \Downarrow$$

$$\alpha_1 = \alpha_2\ ;\ \alpha_1 = \pi - \alpha_2$$

㉑ $a, b, N$

$$B_b = \frac{\mu_0 I N}{2b}\left(\cancel{\cos\alpha_1}^{\,1} - \cancel{\cos\alpha_2}^{\,-1}\right) \qquad \boxed{B_b = \frac{\mu_0 I N}{2b}\,2} = \frac{\mu_0 I N}{b}$$

$$\cos\alpha_1 = \frac{\frac{b}{2}}{r}$$

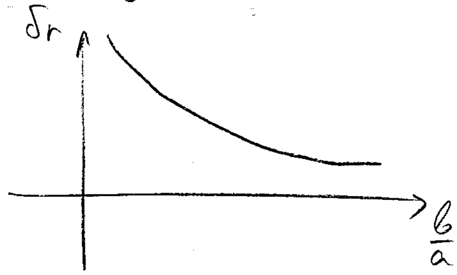
$$\cos\alpha_2 = \frac{-\frac{b}{2}}{r}$$

$$\cos\alpha_1 - \cos\alpha_2 = \frac{2\,\frac{b}{2}}{r} \qquad r = \sqrt{a^2 + \left(\frac{b}{2}\right)^2}$$

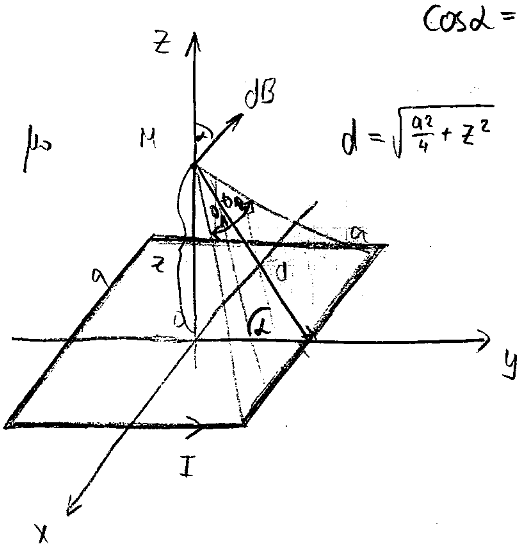
$$\cos\alpha_1 - \cos\alpha_2 = \frac{b}{\sqrt{a^2 + \left(\frac{b}{2}\right)^2}} \qquad B_k = \frac{\mu_0 I N}{2b}\,\frac{b}{\sqrt{a^2 + \left(\frac{b}{2}\right)^2}}$$

$$\delta_r = \frac{B_b - B_k}{B_k} = \frac{2 - \frac{b}{\sqrt{a^2 + \frac{b^2}{4}}}}{\frac{b}{\sqrt{a^2 + \frac{b^2}{4}}}} = \frac{2\sqrt{a^2 + \frac{b^2}{4}} - b}{b} = \frac{2a\sqrt{1 + \frac{b^2}{a^2}\frac{1}{4}} - b}{b} =$$

$$= \frac{2\sqrt{1 + \frac{b^2}{a^2}\frac{1}{4}}}{\frac{b}{a}} - 1 = \frac{\sqrt{4 + \frac{b^2}{a^2}}}{\frac{b}{a}} - 1$$




*㉒ $a, I$




$$d = \sqrt{\frac{a^2}{4} + z^2}$$

$$\cos\alpha = \frac{\frac{a}{2}}{\sqrt{\frac{a^2}{4} + z^2}} = \frac{a}{\sqrt{a^2 + 4z^2}}$$

$$\sin\theta_2 = -\sin\theta_1 = \frac{\frac{a}{2}}{\sqrt{z^2 + \left(\frac{a\sqrt{2}}{2}\right)^2}} = \frac{a}{\sqrt{2a^2 + z^2}}$$

$$B = \frac{\mu_0 I}{4\pi d}\left(\sin\theta_2 - \sin\theta_1\right)$$

$$B_z = B\cos\alpha = \frac{\mu_0 I}{\cancel{2}\,4\pi d}\cdot\cancel{2}\,\frac{a}{\sqrt{2a^2 + 4z^2}}\cdot\frac{a}{\sqrt{a^2 + 4z^2}} =$$

$$= \frac{\mu_0 I a^2}{\pi\sqrt{2a^2 + 4z^2}\left(a^2 + 4z^2\right)}$$

$$\boxed{B = 4B_z = \frac{4\mu_0 I a^2}{\pi\sqrt{2a^2 + 4z^2}\left(a^2 + 4z^2\right)}}$$

*(23) $a, b, I$ $\qquad a^2\pi = b^2$

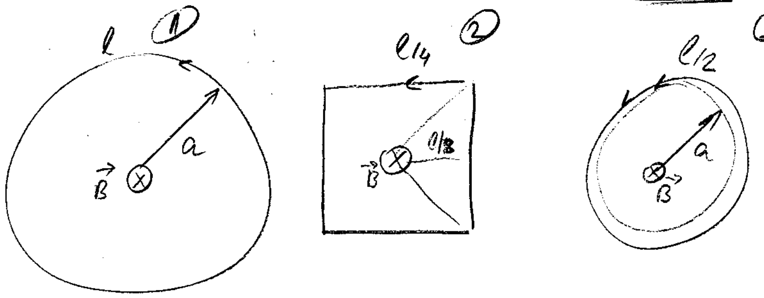
$\rightarrow$ (17) $B_{kr} = \dfrac{\mu_0 I a^2}{2(a^2+z^2)^{3/2}}$ ; $z \to +\infty$ $\qquad B_{kr} = \dfrac{\mu_0 I a^2 \pi}{\pi\, 2|z^3|} = \dfrac{\mu_0 I b^2}{2\pi |z^3|} = \dfrac{\mu_0 I b^2}{2\pi z^2 |z|}$

$\rightarrow$ (22) $B_{kv} = \dfrac{4\mu_0 I b^2}{\pi(4z^2+b^2)\sqrt{4z^2+2b^2}}$ ; $z \to +\infty$ $\qquad B_{kv} = \dfrac{\cancel{4}\mu_0 I b^2}{\pi \cancel{4}z^2 \cdot 2|z|} = \dfrac{\mu_0 I b^2}{2\pi z^2 |z|}$

*(24) $l = 100\,mm$

$I = 10\,A$

$r = 1\,m \quad r \gg a$



$2a\pi = l \Rightarrow$ $\qquad a = \dfrac{l}{4} \qquad a = \dfrac{l}{4\pi}$

$a = \dfrac{l}{2\pi}$

a) $B_1 = \dfrac{\mu_0}{4\pi}\int\limits_{\vartheta} \dfrac{I\,d\vartheta}{r} =$

$= \dfrac{\mu_0 I}{4\pi a}\int\limits_0^{2\pi} d\vartheta = \dfrac{\mu_0 I}{2a} = \dfrac{\mu_0 I \cancel{2}\pi}{\cancel{2}l} = \dfrac{\mu_0 I \pi}{l} \approx 0{,}395\,mT$ $\qquad B_3 = 2\,\dfrac{\mu_0 I}{4\pi \frac{l}{4\pi}}\, 2\pi = \dfrac{4\mu_0 I \pi}{l} = 1{,}58\,mT$

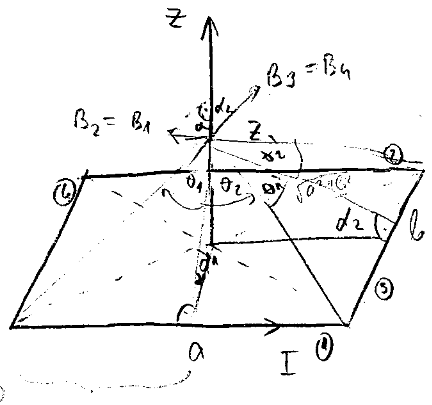
$B_2 = 4\,\dfrac{\mu_0 I}{4\pi \frac{l}{8}}\left(\dfrac{\sqrt{2}}{2} - \left(-\dfrac{\sqrt{2}}{2}\right)\right) = \dfrac{8\mu_0 I \sqrt{2}}{\pi l} \approx 0{,}453\,mT$

б) $B_1 = \dfrac{\mu_0 I a^2}{2(a^2+z^2)^{3/2}} = \dfrac{\mu_0 I a^2}{2|r^3|} = \dfrac{\mu_0 I \frac{l^2}{4\pi^2}}{2|r^3|} = \dfrac{\mu_0 I l^2}{8\pi^2 r^3} = 1{,}59\,nT$

23. $B_2 = \dfrac{\mu_0 I \left(\frac{l}{4}\right)^2}{2\pi r^3} = \dfrac{\mu_0 I l^2}{32\pi r^3} = 1{,}25\,nT$

$B_3 = \cancel{2}\,\dfrac{\mu_0 I \left(\frac{l}{4\pi}\right)^2}{\cancel{2} r^3} = \dfrac{\mu_0 I l^2}{16\pi^2 r^3} = 0{,}80\,nT$

*(25)



$B_1 = B_2 = \dfrac{\mu_0 I}{4\pi d_1}(\sin\theta_2 - \sin\theta_1)$

$d_1 = \sqrt{\left(\frac{b}{2}\right)^2 + z^2}$

$\sin\theta_2 = -\sin\theta_1 = \dfrac{\frac{a}{2}}{\sqrt{\left(\frac{\sqrt{a^2+b^2}}{2}\right)^2 + z^2}} =$

$= \dfrac{a}{\sqrt{4z^2+a^2+b^2}}$

$\cos\alpha_1 = \dfrac{\frac{b}{2}}{d_1}$

$B = 2B_1\cos\alpha_1 + 2B_3\cos\alpha_2$

10 years and counting...

$$B_3 = B_4 = \frac{\mu_0 I}{4\pi d_2}\left(\sin\theta_2' - \sin\theta_1'\right)$$

$$d_2 = \sqrt{\left(\frac{a}{2}\right)^2 + z^2}$$

$$\sin\theta_2 = -\sin\theta_1 = \frac{\frac{b}{2}}{\sqrt{\left(\frac{\sqrt{a^2+b^2}}{2}\right)^2 + z^2}} = \frac{b}{\sqrt{4z^2+a^2+b^2}}$$

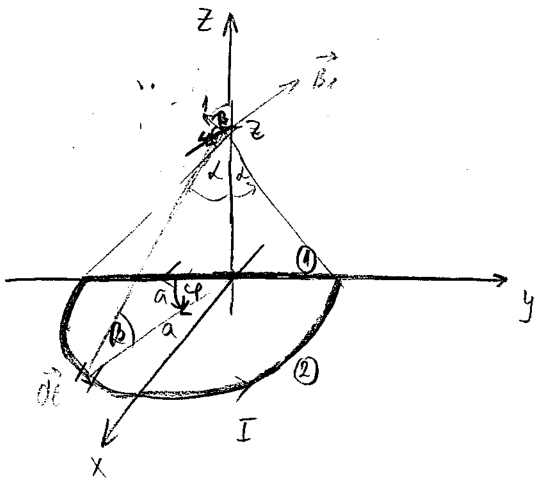
$$\cos\alpha_2 = \frac{\frac{a}{2}}{d_2}$$

$$B = \cancel{2}\,\frac{\mu_0 I}{\cancel{4}\pi d_1}\,\cancel{2}\,\frac{a}{\sqrt{4z^2+a^2+b^2}}\,\frac{\frac{b}{2}}{d_1} + \cancel{2}\,\frac{\mu_0 I}{\cancel{4}\pi d_2}\,\cancel{2}\,\frac{b}{\sqrt{4z^2+a^2+b^2}}\,\frac{\frac{a}{2}}{d_2} =$$

$$= \frac{\mu_0 I a b}{2\pi\sqrt{4z^2+a^2+b^2}}\left(\frac{1}{d_1^2} + \frac{1}{d_2^2}\right) = \frac{\mu_0 I a b}{2\pi\sqrt{4z^2+a^2+b^2}}\left(\overset{\frac{4}{b^2+4z^2}}{\cancel{\frac{1}{\frac{b^2}{4}+z^2}}} + \overset{\frac{4}{a^2+4z^2}}{\cancel{\frac{1}{\frac{a^2}{4}+z^2}}}\right) \Rightarrow$$

$$\boxed{\vec{B} = \frac{2\mu_0 I ab}{\pi\sqrt{4z^2+a^2+b^2}}\left(\frac{1}{b^2+4z^2} + \frac{1}{a^2+4z^2}\right)\vec{i}_z}$$

*26 $I,\ a,\ B = ?$




① $B_1 = \frac{\mu_0 I}{4\pi z}\left(\sin\alpha - \sin(-\alpha)\right) = \frac{\mu_0 I}{\underset{2}{4\pi z}}\,\cancel{2}\sin\alpha$

$$\sin\alpha = \frac{a}{\sqrt{a^2+z^2}}$$

$$\boxed{\vec{B}_{1x} = \frac{-\mu_0 I}{2\pi z}\,\frac{a}{\sqrt{a^2+z^2}}\,\vec{i}_x}$$

② $B_{2y} = 0$

$dB_{2z} = dB_2 \cos\beta$

$dB_2 = \frac{\mu_0}{4\pi}\frac{I|d\vec{l}\times\vec{r}_0|}{r^2}$

$|d\vec{l}\times\vec{r}_0| = |d\vec{l}|\cdot\cancel{|\vec{r}_0|}\cdot\overset{\sin 90^\circ}{\cancel{\sin\angle(\vec{r}_0, d\vec{l})}} = dl$

$r = \text{const}$ $r = \sqrt{a^2+z^2}$

$\cos\beta = \frac{a}{r}$

$$dB_2 = \frac{\mu_0 I}{4\pi(a^2+z^2)}\,dl$$

$$dB_{2z} = \frac{\mu_0 I\,dl}{4\pi(a^2+z^2)}\,\frac{a}{r} \Rightarrow B_{2z} = \frac{\mu_0 I a}{4\pi(a^2+z^2)^{3/2}}\int^{a\pi} \cancel{dl}$$

$$\boxed{\vec{B}_{2z} = \frac{\mu_0 I a^2}{4(a^2+z^2)^{3/2}}\,\vec{i}_z}$$

$$dB_{2H} = dB_2\sin\beta = \frac{\mu_0 I\,dl}{4\pi(a^2+z^2)}\,\frac{z}{r} = \frac{\mu_0 I\,dl\,z}{4\pi(a^2+z^2)^{3/2}}$$

$$\boxed{dl = a\,d\varphi}$$

$$dB_{2x} = dB_{2H}\sin\varphi \Rightarrow B_{2x} = \frac{\mu_0 I z a}{4\pi(a^2+z^2)^{3/2}}\int_0^{\pi}\sin\varphi\,d\varphi =$$

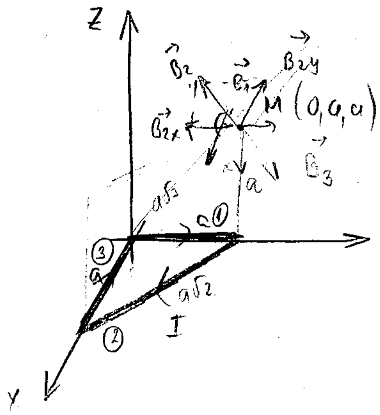
$$= \frac{\mu_0 I z a}{\underset{2}{\cancel{4}}\pi(a^2+z^2)^{3/2}}\cdot\left(\cancel{-\cos\pi + \cos 0}\right)^{2} \Rightarrow \boxed{\vec{B}_{2x} = \frac{\mu_0 I z a}{2\pi(a^2+z^2)^{3/2}}\,\vec{i}_x}$$

$$\vec B = \vec B_{1x} + \vec B_{2x} + \vec B_{2z} = \frac{-\mu_0 I a}{2\pi z\underbrace{\sqrt{a^2+z^2}}_{r}}\vec i_x + \frac{\mu_0 I a^2}{4\underbrace{(a^2+z^2)^{3/2}}_{r^3}}\vec i_z + \frac{\mu_0 I z a}{2\pi\underbrace{(a^2+z^2)^{3/2}}_{r^3}} =$$

$$= \frac{\mu_0 I a^2}{2r^3}\left(\frac{1}{2}\vec i_z + \left(\frac{z}{a\pi} - \frac{r^2}{a\pi z}\right)\vec i_x\right) = \frac{\mu_0 I a^2}{2r^3}\left(\frac{1}{2}\vec i_z + \frac{\cancel{z^2} - a^2 - \cancel{z^2}}{\cancel{a}\pi z}\vec i_x\right) \Rightarrow$$

$$\boxed{\vec B = \frac{\mu_0 I a^2}{2(a^2+z^2)^{3/2}}\left(\frac{1}{2}\vec i_z + \frac{a}{\pi z}\vec i_x\right)}$$

*(27)




$$\vec B = \vec B_1 + \vec B_2 + \vec B_3$$

① $$\vec B_1 = \frac{\mu_0 I}{4\pi a}\left(\sin 0 - \sin\left(-\frac{\pi}{4}\right)\right)\vec i_x = \frac{\mu_0 I}{4\pi a}\left(+\frac{\sqrt2}{2}\right)\vec i_x$$

② $$B_{2x} = B_{2y} = \frac{\sqrt2}{2}B_2 = \frac{\sqrt2}{2}\frac{\mu_0 I}{4\pi a}\left(\frac{a\sqrt2}{a\sqrt3} - \sin 0\right) = \frac{\sqrt2}{2}\frac{\mu_0 I}{4\pi a}\frac{\sqrt2}{\sqrt3}$$

$$\vec B_{2x} = -B_2\vec i_x \qquad \vec B_{2y} = -B_2\vec i_y$$

③ $$B_{3y} = B_{3z} = \frac{\sqrt2}{2}B_3 = \frac{\sqrt2}{2}\frac{\mu_0 I}{4\pi a\sqrt2}\left(\sin 0 + \frac{a}{a\sqrt3}\right) = \frac{\sqrt2}{2}\frac{\mu_0 I}{4\pi a}\frac{1}{\sqrt6}$$

$$\vec B_{3y} = B_{3y}\vec i_y \qquad \vec B_{3z} = -B_{3z}\vec i_z$$

$$\vec B = \frac{\mu_0 I}{4a\pi}\left(\frac{\sqrt2}{2}\vec i_x - \frac{\sqrt2}{2}\frac{\sqrt2}{\sqrt3}\vec i_x - \frac{\sqrt2}{2}\frac{\sqrt2}{\sqrt3}\vec i_y + \frac{\sqrt2}{2}\frac{1}{\sqrt6}\vec i_y - \frac{\sqrt2}{2}\frac{1}{\sqrt6}\vec i_z\right) =$$

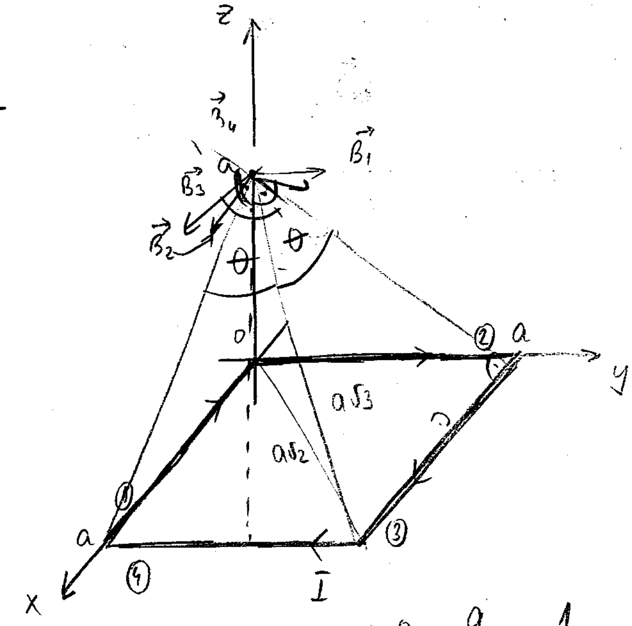
$$= \frac{\mu_0 I}{4a\pi}\left(\frac{\sqrt2}{2}\left(1 - \frac{\sqrt2}{\sqrt3}\right)\vec i_x - \vec i_y\left(\frac{1}{\sqrt3} - \frac{1}{2\sqrt3}\right) - \frac{1}{2\sqrt3}\vec i_z\right) \Rightarrow$$

$$\boxed{\vec B = \frac{\mu_0 I}{4a\pi}\left(\frac{\sqrt2}{2}\left(1 - \frac{\sqrt2}{\sqrt3}\right)\vec i_x - \frac{1}{2\sqrt3}\left(\vec i_y + \vec i_z\right)\right)}$$

*(28) $a = 1\,m$

$I = 10A$

$B = ?$




① $\vec{B}_1 = \frac{\mu_0 I}{4\pi a}\left(\sin 0 - \sin\left(-\frac{\pi}{4}\right)\right)\vec{i}_y = \frac{\mu_0 I}{4\pi a}\frac{\sqrt{2}}{2}\vec{i}_y = \frac{\mu_0 I\sqrt{2}}{8\pi a}\vec{i}_y$

② $\vec{B}_2 = \frac{\mu_0 I}{4\pi a}\left(\sin\frac{\pi}{4} - \sin 0\right)\vec{i}_x = \frac{\mu_0 I}{4\pi a}\frac{\sqrt{2}}{2}\vec{i}_x = \frac{\mu_0 I\sqrt{2}}{8\pi a}\vec{i}_x$

③ $B_{3z} = B_{3y} = \frac{\mu_0 I}{4\pi a\sqrt{2}}\frac{\sqrt{2}}{2}\left(\sin\theta - \cancel{\sin 0}^{\,0}\right) = \frac{\mu_0 I}{4\pi a}\frac{1}{2}\frac{1}{\sqrt{3}} = \frac{\mu_0 I\sqrt{3}}{24\pi a}$

$\sin\theta = \frac{a}{a\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\vec{B}_{3z} = B_{3z}\vec{i}_z(-1) \quad \vec{B}_{3y} = B_{3y}\vec{i}_y(-1)$

④ $B_{4x} = B_{4z} = B_4\frac{\sqrt{2}}{2} = \frac{\mu_0 I}{4\pi a\sqrt{2}}\frac{\sqrt{2}}{2}\left(\sin 0 - \sin(-\theta)\right) =$

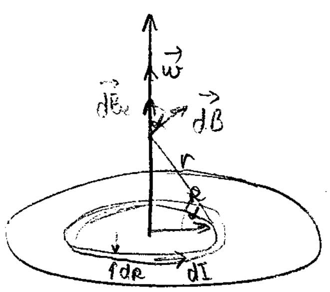
$= \frac{\mu_0 I}{8\pi a}\frac{\sqrt{3}}{3} = \frac{\mu_0 I\sqrt{3}}{24 a\pi}$ $\quad \vec{B}_{4x} = B_{4x}\vec{i}_x(-1) \quad \vec{B}_{4z} = B_{4z}\vec{i}_z(-1)$

$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4 = \frac{\mu_0 I\sqrt{2}}{8\pi a}\vec{i}_y - \frac{\mu_0 I\sqrt{3}}{24 a\pi}\vec{i}_x + \frac{\mu_0 I\sqrt{2}}{8\pi a}\vec{i}_x - \frac{\mu_0 I\sqrt{3}}{24\pi a}\vec{i}_y -$

$- 2\frac{\mu_0 I\sqrt{3}}{24\pi a}\vec{i}_z \Rightarrow$

$$\boxed{\vec{B} = \frac{\mu_0 I}{24 a\pi}\left(3\sqrt{2} - \sqrt{3}\right)\vec{i}_x + \frac{\mu_0 I}{24 a\pi}\left(3\sqrt{2} - \sqrt{3}\right)\vec{i}_y - \frac{\mu_0 I\sqrt{3}}{12\pi a}\vec{i}_z}$$

*(29) $\varepsilon_r \approx 1$, $a$, $\rho_s$, $\omega$ a) $B_z = ?$ b) $B_\infty(\vec{m}) = ?$




$J_s = \rho_s v = \rho_s \omega R$

$dI = J_s \cdot dR = \rho_s \omega R\, dR$

$dB_z = \frac{\mu_0\, dI\, R^2}{2(R^2+z^2)^{3/2}}$

$B_z = \int dB_z = \int_{R=0}^{a} \frac{\mu_0 \rho_s \omega R^3 dR}{2(R^2+z^2)^{3/2}} =$

$= \frac{\mu_0 \rho_s \omega}{2}\int_{R=0}^{a}\frac{R^3 dR}{(R^2+z^2)^{3/2}}$

$\int_0^a \frac{R^3 dR}{\left(\sqrt{R^2+z^2}\right)^3} = \left| \begin{array}{l} R^2+z^2 = t^2 \\ \cancel{2}R\,dR = \cancel{2}t\,dt \\ R\,dR = t\,dt \\ R^2 = t^2 - z^2 \\ R=0 \Rightarrow t = |z| \\ R=a \Rightarrow t = \sqrt{a^2+z^2} \end{array} \right| = \int_{|z|}^{\sqrt{a^2+z^2}} (t^2 - z^2)\, \cancel{t^{-3}}^{\,t^{-2}}\, \cancel{t}\, dt = \int_{|z|}^{\sqrt{a^2+z^2}} dt - \int_{|z|}^{\sqrt{a^2+z^2}} \frac{z^2}{t^2}dt =$

$= \int_0^a R^2 (R^2+z^2)^{-\frac{3}{2}} R\,dR =$

$= \sqrt{a^2+z^2} - |z| - z^2\left(\frac{t^{-1}}{-1}\right)\Bigg|_{|z|}^{\sqrt{a^2+z^2}} =$

$$= \sqrt{a^2+z^2} - |z| + \frac{z^2}{\sqrt{a^2+z^2}} - \frac{\cancel{z^2}^{\,|z|}}{\cancel{|z|}} = \frac{a^2+2z^2}{\sqrt{a^2+z^2}} - 2|z|$$

$$\vec{B}_z = B_z \vec{i}_z \qquad \boxed{\vec{B}_z = \frac{\mu_0 \rho_s \vec{\omega}}{2}\left(\frac{a^2+2z^2}{\sqrt{a^2+z^2}} - 2|z|\right)}$$

б) $z \gg a$ $\quad dm = dI\,ds = \rho_s \omega R\,dR \cdot R^2\pi = \rho_s \omega R^3 \pi\, dR$

$$m = \int_{R=0}^{a} dm = \rho_s \omega \pi \frac{a^4}{4}$$

$$B_z = \frac{\mu_0 \rho_s \omega}{2} \cdot \frac{a^2+2z^2 - 2|z|\sqrt{a^2+z^2}}{\sqrt{a^2+z^2}} \cdot \frac{a+2z^2+2|z|\sqrt{a^2+z^2}}{a+2z^2+2|z|\sqrt{a^2+z^2}} =$$

$$= \frac{\mu_0 \rho_s \omega}{2} \cdot \frac{a^4 + \cancel{4a^2z^2} + \cancel{4z^4} - \cancel{4z^2a^2} - \cancel{4z^4}}{\sqrt{a^2+z^2}\,(a+2z^2+2|z|\sqrt{a^2+z^2})} = \frac{\mu_0 \rho_s \omega a^4 \pi \cdot 2}{2\pi 2 \sqrt{a^2+z^2}\,(a+2z^2+2|z|\sqrt{a^2+z^2})} =$$

$$= \frac{2\mu_0 m}{\pi} \cdot \frac{1}{\sqrt{a^2+z^2}\,(a+2z^2+2|z|\sqrt{a^2+z^2})} \overset{z\gg a}{=} \frac{2\mu_0 m}{\pi} \cdot \frac{1}{|z|\,(2z^2+2|z||z|)} =$$

$$= \frac{2\mu_0 m}{\pi} \cdot \frac{1}{4z^2|z|} \Rightarrow \boxed{\vec{B}_\infty(\vec{m}) = \frac{\mu_0 \vec{m}}{2\pi z^2 |z|}}$$

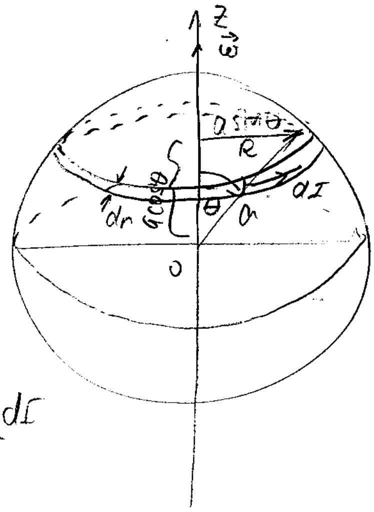
*(30) $\varepsilon_r \approx 1,\ a,\ \rho_s,\ \omega$

a) $\vec{B}_0 = ?$

б) $\vec{B}_\infty(\vec{m}) = ?$

$J_s = \rho_s v = \rho_s \omega R$

$dI = J_s\,dr = \rho_s \omega R\, da$




$R = a\sin\theta$

$dR = a\,d\theta$

a) $$dB = \frac{\mu_0 (a\sin\theta)^2 dI}{2(R^2+a^2-R^2)^{3/2}} = \frac{\mu_0 a^2 \sin^2\theta\, dI}{2|a|^3}$$

$$dB = \frac{\mu_0 \sin^2\theta\, dI}{2a} = \frac{\mu_0 \rho_s \omega a^2 \sin^3\theta\, d\theta}{2a}$$

$$B = \int_{\theta=0}^{\pi} dB = \frac{\mu_0 \rho_s \omega a}{2} \int_{\theta=0}^{\pi} \sin^3\theta\, d\theta = \frac{2\mu_0 \rho_s \omega a}{3}$$

$$\int_0^{\pi} \sin^3\theta\, d\theta = \int_0^{\pi} (1-\cos^2\theta)\sin\theta\, d\theta =$$

$$= \left| \begin{matrix} \cos\theta = t & \theta=0 & t=1 \\ -\sin\theta\, d\theta = dt & \theta=\pi & t=-1 \end{matrix} \right| =$$

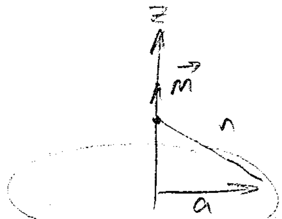
$$= \int_{-1}^{1} (1-t^2)\, dt = 2 - \left(\frac{1}{3}+\frac{1}{3}\right) = \frac{4}{3}$$

$$\boxed{\vec{B} = \frac{2\mu_0 \rho_s \vec{\omega} a}{3}}$$

б) $dm = dI\,S = \underbrace{\rho_s \omega R\, dR}_{dI}\ \underbrace{R^2\pi}_{S} = \rho_s \omega \pi R^3 dR = \rho_s \omega \pi a^4 \sin^3\theta\, d\theta$

$$m = \int_{\theta=0}^{\pi} dm = \int_0^{\pi} \rho_s \omega \pi a^4 \sin^3\theta\, d\theta = \frac{4}{3}\rho_s \omega \pi a^4$$

$$\boxed{\vec{m} = \frac{4}{3}\rho_s \pi a^4 \vec{\omega}}$$




$$dB = \frac{\mu_0\, dI\, a^2}{2 r^3} \qquad r^2 = a^2 + z^2$$

$$dm = dI\, a^2 \pi$$

$$dB = \frac{\mu_0\, dI\, a^2 \pi}{\pi\, 2 |z|^3} \quad dm$$

$$\boxed{dB = \frac{\mu_0\, dm}{2\pi |z|^3}} \qquad \boxed{z - a\cos\theta \approx z}$$

$$B = \int dB = \frac{\mu_0}{2\pi |z - a\cos\theta|^3} \int dm \Rightarrow \boxed{\vec{B} = \frac{\mu_0 \vec{m}}{2\pi |z|^3}}$$

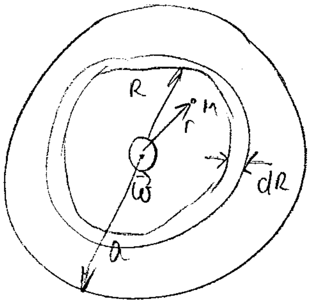
$|z|^3$ , $m$

$$\vec{B} = \frac{\mu_0 \frac{4}{3}\rho_s \pi a^4 \vec{\omega}}{2\pi |z|^3}$$

$$\boxed{\vec{B} = \frac{4\mu_0 \rho_s a^4 \vec{\omega}}{2|z|^3}}$$

*(37) $\varepsilon_r \approx 1,\ a,\ \rho,\ \omega,\ \mu_0$

$dB = dJ_s \mu_0 \rightarrow$ магнетско поље на оси соленоида



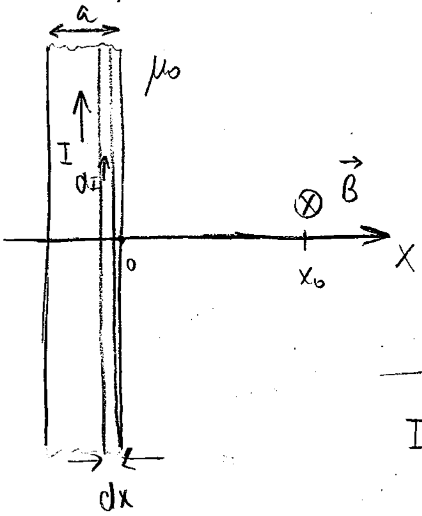

— посматрајмо ваљак као низ шупљих цилиндара
— један такав цилиндар је полупречника R и дебљине dR и поље које он ствара одговара пољу њему еквивалентног соленоида; поље ван соленоида је нула, те је оно нула и ван цилиндра. Такође, како је цилиндар бесконачно дугачак, поље у њему је хомогено. Пољу на растојању r од центра доприносе само цилиндри на растојању од r до a.

$$J = \rho \cdot v = \rho \omega R$$

$$dJ_s = J\,dR = \rho \omega R\, dR$$

$$B = \int_R dB = \int_R dJ_s \mu_0 = \mu_0 \rho \omega \int_{R=r}^{a} R\, dR = \underline{\underline{\mu_0 \rho \omega \frac{a^2 - r^2}{2}}} \qquad \boxed{0 \le r \le a}$$

** ~~32~~ 33 $\mu_0,\ a,\ \delta << a,\ I,\ B(x)=?;\ x = x_0$




$$dB = \frac{\mu_0\, dI}{\cancel{4}\pi (x_0 - x)}\left(\sin\frac{\pi}{2} - \sin\left(-\frac{\pi}{2}\right)\right)$$

$$dB = \frac{\mu_0\, dI}{2\pi (x_0 - x)}$$

$$B = \int_{x=-a}^{0} \frac{\mu_0\, dI}{2\pi (x_0 - x)} = \int_{-a}^{0} \frac{\mu_0 I\, dx}{2\pi a (x_0 - x)} = \frac{\mu_0 I}{2\pi a}\int_{0}^{-a} \frac{d(x_0 - x)}{x_0 - x} =$$

$$I = a\frac{dI}{dx} \Rightarrow dI = \frac{I\, dx}{a}$$

$$J_s = \frac{I}{a};\ dI = J_s\, dx$$

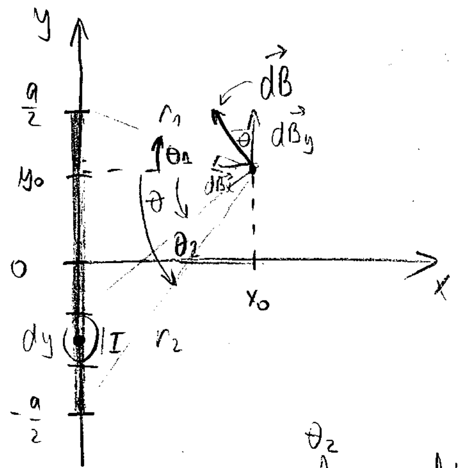
$$= \frac{\mu_0 I}{2\pi a} \ln\frac{x_0 + a}{x_0}$$

* 34 $a,\ I,\ \mu_0,\ B = ?$

$$J_s = \frac{I}{a}$$

$$dI = J_s\, dy$$

$$dI = \frac{I}{a} dy$$




$$dB = \frac{\mu_0\, dI}{\cancel{4}\pi\sqrt{(y_0 - y)^2 + x_0^2}}\left(\sin\frac{\pi}{2} - \sin\left(-\frac{\pi}{2}\right)\right)$$

$$dB = \frac{\mu_0\, dI}{2\pi\sqrt{x_0^2 + (y - y_0)^2}} \qquad r = \sqrt{(y_0 - y)^2 + x_0^2}$$

$$dB_x = dB \sin\theta \qquad ;\ d\vec{B}_x = -dB_x \vec{i}_x$$

$$dB_y = dB \cos\theta \qquad d\vec{B}_y = dB_y \vec{i}_y$$

$$B_x = \int_{\theta=\theta_1}^{\theta_2} dB_x = \int \frac{\mu_0 J_s\, dy}{2\pi r}\sin\theta = \int_{\theta_1} \frac{\mu_0 J_s \frac{\cancel{x_0}\, d\theta}{\cos^{\cancel{2}}\theta}}{2\pi \frac{\cancel{x_0}}{\cancel{\cos\theta}}}\sin\theta$$

$$tg\,\theta = \frac{y_0 - y}{x_0} \qquad \cos\theta = \frac{x_0}{r}$$

$$\cancel{+}\frac{d\theta}{\cos^2\theta} = \frac{\cancel{+}dy}{x_0} \qquad r = \frac{x_0}{\cos\theta}$$

$$dy = \frac{x_0\, d\theta}{\cos^2\theta}$$

$$B_x = \frac{\mu_0 J_s}{2\pi}\int_{\theta_1}^{\theta_2} \frac{\sin\theta}{\cos\theta}\, d\theta = \left|\begin{array}{c} \cos\theta = t \\ -d\theta \sin\theta = dt \end{array}\right| =$$

$$= \frac{\mu_0 J_s}{2\pi}\int_{\theta_2}^{\theta_1} \frac{dt}{t} = \frac{\mu_0 I}{2a\pi} \ln\frac{\cos\theta_1}{\cos\theta_2} =$$

$$B_y = \int dB_y = \int_{\theta_1}^{\theta_2} \frac{\mu_0 J_s \frac{\cancel{x_0}\, d\theta}{\cancel{\cos^2\theta}}}{2\pi \frac{\cancel{x_0}}{\cancel{\cos\theta}}}\cancel{\cos\theta}$$

$$= \frac{\mu_0 I}{2a\pi} \ln\frac{\frac{\cancel{x_0}}{r_1}}{\frac{\cancel{x_0}}{r_2}} = \frac{\mu_0 I}{2a\pi} \ln\frac{r_2}{r_1}$$

$$B_y = \frac{\mu_0 I}{2a\pi}(\theta_2 - \theta_1)$$

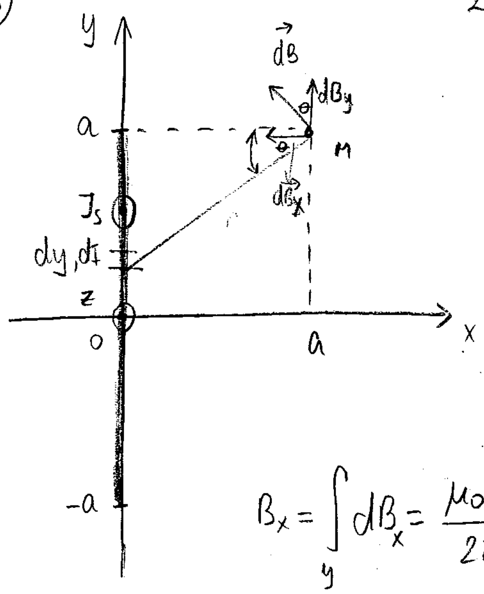
$$r_1 = \sqrt{\left(\frac{a}{2} - y_0\right)^2 + x_0^2}$$

$$\boxed{\vec{B} = \frac{\mu_0 I}{2a\pi} \ln\frac{r_2}{r_1}\vec{i}_x - \frac{\mu_0 I}{2a\pi}(\theta_2 - \theta_1)\vec{i}_y}$$

$$r_2 = \sqrt{\left(\frac{a}{2} + y_0\right)^2 + x_0^2}$$

10 years and counting...

⁎ (35)




$$2a,\quad J_s(y)=J_{s0}\frac{y}{a},\quad J_{s0}=const.,\quad B_M=?$$

$$dB=\frac{\mu_0\,dI}{2\pi r}\qquad dI=J_s\,dy=J_{s0}\frac{y}{a}\,dy$$

$$dB=\frac{\mu_0 J_{s0}\,y\,dy}{2\pi a\sqrt{(a-y)^2+a^2}}$$

$$-dB_x=dB\sin\theta=dB\frac{a-y}{r}=\frac{\mu_0 J_{s0}}{2\pi a}\frac{y(a-y)\,dy}{(a-y)^2+a^2}$$

$$dB_y=dB\cos\theta=dB\frac{a}{r}=\frac{\mu_0 J_{s0}\cancel{a}}{2\pi\cancel{a}}\frac{y\,dy}{(a-y)^2+a^2}$$

$$B_x=\int_y dB_x=\frac{\mu_0 J_{s0}}{2\pi a}\overbrace{\left(\int_{y=-a}^{a}\frac{(ay-y^2)\,dy}{(a-y)^2+a^2}\right)}^{I}\cdot(-1)$$

$$I=\frac{\cancel{a}}{\cancel{a}}\underbrace{\int_{-a}^{a}\frac{y\,dy}{(a-y)^2+a^2}}_{I_B}-\underbrace{\frac{1}{a}\int_{-a}^{a}\frac{y^2\,dy}{(a-y)^2+a^2}}_{I_A}$$

$$I_A=\frac{1}{a}\int_{y=-a}^{a}\frac{y^2\,dy}{(a-y)^2+a^2}=\left|\begin{array}{l}a-y=t\\ dy=-dt\\ y=a-t\\ y=-a\Rightarrow t=2a\\ y=a\Rightarrow t=0\end{array}\right|=\frac{1}{a}\int_{2a}^{0}\frac{(a-t)^2(-dt)}{t^2+a^2}=\frac{1}{a}\int_0^{2a}\frac{a^2-2at+t^2}{t^2+a^2}\,dt=$$

$$=\frac{1}{a}\left(\int_0^{2a}dt-2a\int_0^{2a}\frac{t}{t^2+a^2}\,dt\right)=\frac{1}{\cancel{a}}2\cancel{a}-\frac{1}{a}\int_0^{2a}\frac{\frac{1}{2}\overbrace{d(t^2+a^2)}^{2t\,dt}}{t^2+a^2}=$$

$$=2-\frac{\cancel{2a}}{\cancel{2a}}\ln\frac{5\cancel{a^2}}{\cancel{a^2}}=\underline{\underline{2-\ln 5}}$$

$$I_B=\int_{-a}^{a}\frac{y\,dy}{(a-y)^2+a^2}=\left|\begin{array}{ll}a-y=t & y=-a\Rightarrow t=2a\\ dy=-dt & y=a\Rightarrow t=0\\ y=(a-t) & \end{array}\right|=\int_{2a}^{0}\frac{(a-t)(-dt)}{t^2+a^2}=a\int_0^{2a}\frac{dt}{t^2+a^2}-\int_0^{2a}\frac{t\,dt}{t^2+a^2}=$$

$$=\cancel{a}\frac{1}{\cancel{a}}\operatorname{arctg}\frac{t}{a}\Big|_{t=0}^{2a}-\frac{1}{2}\ln 5=\operatorname{arctg}2-\frac{1}{2}\ln 5$$

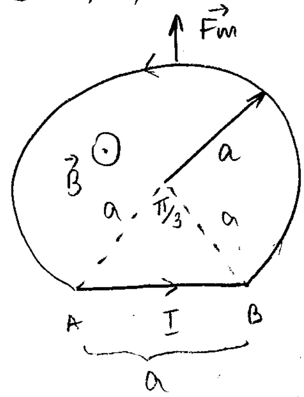
$$B_x=I_A-I_B=2-\ln 5-\operatorname{arctg}2+\frac{1}{2}\ln 5=\underline{\underline{2-\operatorname{arctg}2-\ln\sqrt{5}}}$$

$$B_y=\int_y dB_y=\frac{\mu_0 J_{s0}}{2\pi}\overbrace{\int_{-a}^{a}\frac{y\,dy}{(a-y)^2+a^2}}^{I_B}=\frac{\mu_0 J_{s0}}{2\pi}\left(\operatorname{arctg}2-\ln\sqrt{5}\right)$$

$$\boxed{\vec{B}=B_x\vec{i_x}+B_y\vec{i_y}}$$

** ~~36~~ 37 $a, I, B$ $\quad \vec{F}_m = ?$

$\mu_0$




$$d\vec{F}_m = I\, d\vec{l} \times \vec{B}$$

$$\vec{F}_m = \int_l d\vec{F}_m = I \int_B^A d\vec{l} \times \vec{B} = \underline{I\, \vec{BA} \times \vec{B}}$$

$$\boxed{F_m = I a B}$$

$|AB| = a$

38 $d = 10\,mm$

$D = 100\,mm$

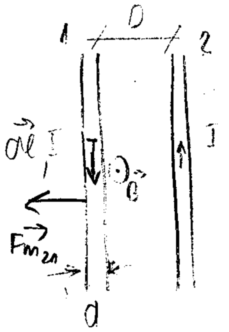
$J = 2{,}5\ A/mm^2$

$F_m' = ?$

$$I = J \cdot S = J \frac{d^2}{4}\pi = 2{,}5 \cdot \frac{100}{4} \cdot 3{,}14 = 2{,}5 \cdot 78{,}5 = \underline{196\,A}$$

- магнетна индукција која учествује у сили је магнетна инд. од другог проводника

$$B = \frac{\mu_0 I}{2\pi D}$$



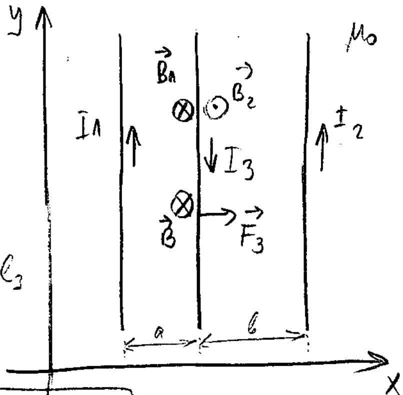

$$|d\vec{F}_m| = I\,|d\vec{l} \times \vec{B}| = I\, dl B$$

$$F_m' = \frac{dF_m}{dl} = I B = \underline{\frac{\mu_0 I^2}{2\pi D} \simeq 0{,}08\ N/m}$$

$xy \neq xy$

39 $I_1, I_2, I_3$ a) $F_3' = ?$

б) $F_3' = 0;\ \frac{I_1}{I_2} = ?$




$$\vec{B} = \vec{B}_1 + \vec{B}_2 \qquad B = B_1 - B_2$$

$$B_1 = \frac{\mu_0 I_1}{2\pi a} \qquad B_2 = \frac{\mu_0 I_2}{2\pi b}$$

$$B = \frac{\mu_0}{2\pi}\left(\frac{I_1}{a} - \frac{I_2}{b}\right)$$

$$\vec{B} = -\frac{\mu_0}{2\pi}\left(\frac{I_1}{a} - \frac{I_2}{b}\right)\vec{i}_z$$

$$(-\vec{i}_y) \times (-\vec{i}_z) = \vec{i}_x$$

a) $d\vec{F}_{m_3} = I_3\, dl_3 (-\vec{i}_y) \times \vec{B} \,/ : dl_3$

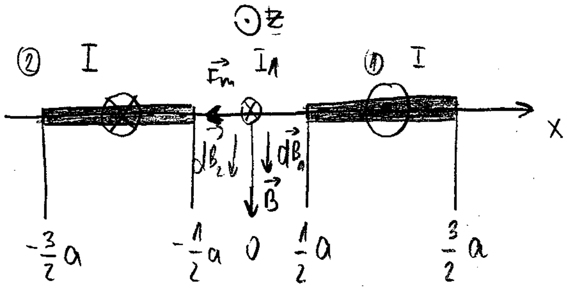
$$\vec{F}_{m_3}' = I_3 (-\vec{i}_y) \times \vec{B}$$

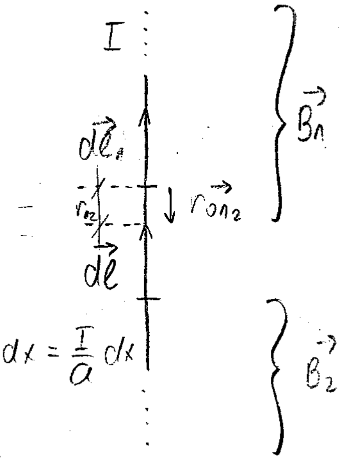
$$\boxed{\vec{F}_{m_3}' = \frac{\mu_0 I_3}{2\pi} \cdot \left(\frac{I_1}{a} - \frac{I_2}{b}\right)\vec{i}_x}$$

б) $\vec{F}_{m_3} = 0 \Rightarrow \frac{I_1}{a} = \frac{I_2}{b}$

$$\boxed{\frac{I_1}{I_2} = \frac{a}{b}}$$

(40)




$$dI = I'dx = \frac{I}{a}dx$$

$$\vec{B}_1 = \frac{\mu_0}{4\pi} I \frac{\cancel{d\vec{l}_1 \times \vec{r}_{012}}}{r_{12}^2} \to 0$$

$$\vec{B}_1 = 0 \Rightarrow \vec{B}_2 = 0$$

– вектор магнетске индукције бесконачног проводника на њему је једнак нули ако нема спољашњег магн. поља.

① $d\vec{B}_1 = \dfrac{\mu_0 I dx}{2\pi x a}(-\vec{i}_y)$

$$B_1 = \frac{\mu_0 I}{2\pi a}\int_{\frac{1}{2}a}^{\frac{3}{2}a}\frac{dx}{x} = \frac{\mu_0 I}{2\pi a}\ln\frac{\frac{3}{2}\cancel{a}}{\frac{1}{2}\cancel{a}}$$

$$\boxed{\vec{B}_1 = \frac{\mu_0 I}{2\pi a}\ln 3(-\vec{i}_y)}$$

$$d\vec{F}_m = I_1 d\vec{l} \times \vec{B} \quad / : dl$$

$$\vec{F}_m' = I_1(-\vec{i}_z) \times \vec{B}$$

② $d\vec{B}_2 = \dfrac{\mu_0 I dx}{2\pi a(-x)}(-\vec{i}_y)$

$$\vec{B}_2 = \int_{-\frac{3}{2}a}^{-1/2\,a}\frac{\mu_0 I\,dx}{a2\pi(-x)}(-\vec{i}_y) = \frac{\mu_0 I}{2\pi a}\ln\frac{+\frac{3}{2}a}{+\frac{1}{2}a}(-\vec{i}_y)$$

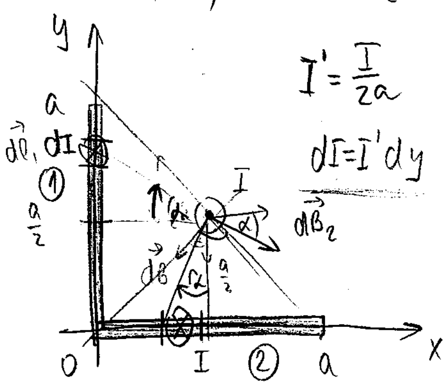
$$\vec{F}_m' = I_1\frac{\mu_0 I}{\pi a}\ln 3\,(-\vec{i}_z)\times(-\vec{i}_y)$$

$$\boxed{\vec{F}_m' = \frac{\mu_0 I I_1}{\pi a}\ln 3(-\vec{i}_x)}$$

$$\boxed{\vec{B}_2 = \frac{\mu_0 I}{2\pi a}\ln 3(-\vec{i}_y)}$$

$$\vec{B} = \vec{B}_1 + \vec{B}_2 \qquad \boxed{\vec{B} = \frac{\mu_0 I}{\pi a}\ln 3\,(-\vec{i}_y)}$$

*(41) a) $2a, I$; $x_0 = y_0 = \frac{a}{2}$




$$I' = \frac{I}{2a}$$

$$dI = I'dy$$

$1°$ $dB_1 = \dfrac{\mu_0 dI}{2\pi r}$

$$-d\vec{B}_{1x} = dB_1 \sin\alpha\, \vec{i}_x$$

$$-d\vec{B}_{1y} = dB_1 \cos\alpha\, \vec{i}_x$$

$$\cos\alpha = \frac{\frac{a}{2}}{r} \Rightarrow r = \frac{a}{2\cos\alpha}$$

$$\mathrm{tg}\,\alpha = \frac{a-y}{\frac{a}{2}} \Big/ d \qquad \cancel{+}\frac{d\alpha}{\cos^2\alpha} = \frac{\cancel{-}dy}{\frac{a}{2}}$$

$$dy = \frac{a\,d\alpha}{2\cos^2\alpha}$$

$$dB_{1x} = dB_1\sin\alpha = \frac{\mu_0 I\,dy}{2a\,2\pi r}\sin\alpha = \frac{\mu_0 I}{4a\pi}\frac{\frac{a\,d\alpha}{2\cos^2\alpha}}{\frac{a}{2\cos\alpha}}\sin\alpha$$

$$B_{1x} = \frac{\mu_0 I}{4a\pi}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{\sin\alpha}{\cos\alpha}d\alpha = \left|\begin{matrix}\cos\alpha = t\\ -\sin\alpha\,d\alpha = dt\\ \alpha = -\frac{\pi}{4} \Rightarrow t = \frac{\sqrt{2}}{2}\\ \alpha = \frac{\pi}{4} \Rightarrow t = \frac{\sqrt{2}}{2}\end{matrix}\right| = \frac{\mu_0 I}{4a\pi}\int_{\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}}\frac{dt}{t} = 0$$

(симетрију)

$$dB_{1y} = dB_1\cos\alpha = \frac{\mu_0 I\,dy}{2a\,2\pi r}\cos\alpha = \frac{\mu_0 I}{4a\pi}\frac{\frac{a\,d\alpha}{2\cos^2\alpha}}{\frac{a}{2\cos\alpha}}\cos\alpha$$

$$B_{1y} = \frac{\mu_0 I}{4a\pi}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}d\alpha = \frac{\mu_0 I}{4a\pi}\frac{\pi}{2} = \frac{\mu_0 I}{8a}$$

$$\boxed{\vec{B}_{1y} = \frac{\mu_0 I}{8a}(-\vec{i}_y)}$$

$$\vec{B} = \vec{B}_1 + \vec{B}_2$$

$$\vec{B}_1 = \vec{B}_{1x} + \vec{B}_{1y}$$

$\underline{2^\circ}$ $\quad dB_2 = \dfrac{\mu_0\, dI}{2\pi r}$

$d\vec{B}_{2x} = dB_{2x}\,\vec{i_x} = dB_2 \cos\alpha\, \vec{i_x}$ $\qquad \vec{B_2} = \vec{B}_{2x} + \vec{B}_{2y}$

$dB_{2y} = 0$ (симетрија)

$$B_{2x} = \int_\alpha \frac{\mu_0 I}{4a\pi} \cdot \frac{\frac{a\,d\alpha}{\cancel{\cos^2\alpha}}}{\frac{\cancel{a}}{\cancel{\cos\alpha}}} \cancel{\cos\alpha} = \frac{\mu_0 I}{4a\pi} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} d\alpha \quad \frac{\pi}{2} \qquad \vec{B}_{2x} = \frac{\mu_0 I}{8a}\vec{i_x}$$

$$\boxed{\vec{B} = \frac{\mu_0 I}{8a}\left(\vec{i_x} - \vec{i_y}\right)}$$

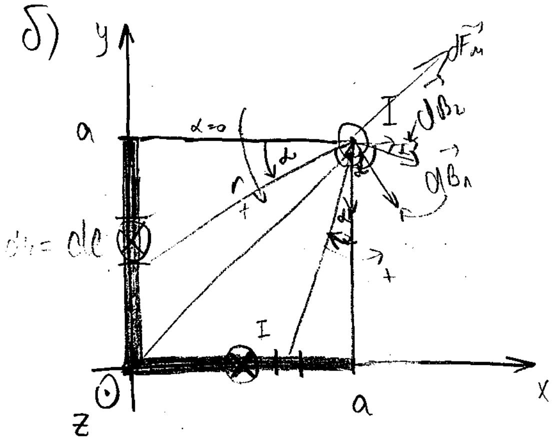
$x\,y\,z\,x\,y$

$d\vec{F} = I\, d\vec{l} \times \vec{B} = I\, dl\, \vec{i_z} \times \vec{B} \quad /:dl$

$$\vec{F'} = I \frac{\mu_0 I}{8a}\, \vec{i_z} \times \left(\vec{i_x} - \vec{i_y}\right) = \frac{\mu_0 I^2}{8a}\left(\vec{i_z}\times\vec{i_x} - \vec{i_z}\times\vec{i_y}\right) =$$

$$= \underline{\frac{\mu_0 I^2}{8a}\left(\vec{i_y} + \vec{i_x}\right)}$$

б)




$dI = \dfrac{I}{2a}\, dl$

$$dB_1 = \frac{\mu_0\, dI}{2\pi r} = \frac{\mu_0}{2\pi}\,\frac{I}{2a}\,\frac{\frac{\cancel{a}\, d\alpha}{\cos^2\alpha}}{\frac{\cancel{a}}{\cancel{\cos\alpha}}} = \underline{\frac{\mu_0 I}{4a\pi}\,\frac{d\alpha}{\cos\alpha}}$$

$r = \dfrac{a}{\cos\alpha}$

$\operatorname{tg}\alpha = \dfrac{a-y}{a}$ $\qquad \boxed{dy = \dfrac{a\,d\alpha}{\cos^2\alpha}}$

$$\vec{B}_{1x} = \int_0^{+\frac{\pi}{4}} dB_1 \sin\alpha\, \vec{i_x} = \frac{\mu_0 I}{4a\pi}\,\vec{i_x} \int_{+\frac{\pi}{4}}^{0} \frac{\sin\alpha}{\cos\alpha}\, d\alpha =$$

$$= \frac{\mu_0 I}{4a\pi}\,\vec{i_x} \int_{\frac{\sqrt{2}}{2}}^{1} \frac{dt}{t} = \frac{\mu_0 I}{4a\pi}\left(\cancel{\ln 1} - \ln\frac{\sqrt{2}}{2}\right)\vec{i_x} =$$

$$= \underline{\frac{\mu_0 I}{4a\pi}\left(-\ln\frac{\sqrt{2}}{2}\right)\vec{i_x} = \frac{\mu_0 I}{4a\pi}\,\frac{1}{2}\ln 2\, \vec{i_x}}$$

$$\vec{B}_{1y} = \int_0^{+\frac{\pi}{4}} dB_1 \cos\alpha \left(-\vec{i_y}\right) = \underline{\frac{\mu_0 I}{4a\pi}\left(+\frac{\pi}{4}\right)\left(-\vec{i_y}\right)}$$

$$\vec{B}_{2x} = \int_{-\frac{\pi}{4}}^{0} dB_2 \cos\alpha\, \vec{i_x} = \underline{\frac{\mu_0 I}{4a\pi}\,\frac{\pi}{4}\,\vec{i_x}}$$

$\ln\dfrac{\sqrt{2}}{2} = \ln 2^{-\frac{1}{2}} = -\dfrac{1}{2}\ln 2$

$$\vec{B}_{2y} = \int_{-\frac{\pi}{4}}^{0} dB_2 \sin\alpha\left(-\vec{i_y}\right) = \underline{\frac{\mu_0 I}{4a\pi}\left(-\ln\frac{\sqrt{2}}{2}\right)\left(-\vec{i_y}\right)} = \frac{\mu_0 I}{4a\pi}\,\frac{1}{2}\ln 2\left(-\vec{i_y}\right)$$

$$\vec{B} = \frac{\mu_0 I}{8a\pi}\ln 2\left(\vec{i_x} - \vec{i_y}\right) + \frac{\mu_0 I}{16a\pi}\,\pi\left(\vec{i_x} - \vec{i_y}\right) = \underline{\frac{\mu_0 I}{16a\pi}\left(2\ln 2 + \pi\right)\left(\vec{i_x} - \vec{i_y}\right)}$$

$x\overleftarrow{y}\underrightarrow{z}xy$

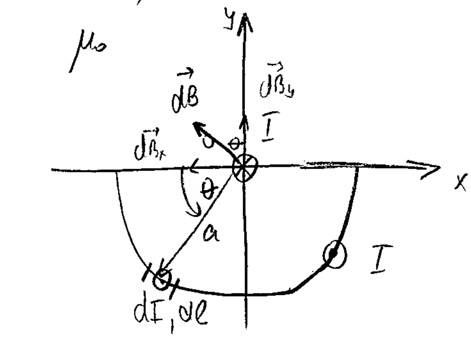
$$d\vec{F} = I\,dl\,\vec{i_z}\times\vec{B}$$

$$\vec{F'} = \frac{d\vec{F}}{dl} = I\,\vec{i_z}\times B = \frac{\mu_0 I^2}{16a\pi}\left(2\ln 2+\pi\right)\vec{i_z}\times\left(\vec{i_x}-\vec{i_y}\right)$$

$$\boxed{\vec{F'} = \frac{\mu_0 I^2}{16a\pi}\left(2\ln 2+\pi\right)\left(\vec{i_x}+\vec{i_y}\right)}$$

(42) $a, I, F' = ?$

$\mu_0$




$$d\vec{F_m} = I\,dl\,(-\vec{i_z})\times\vec{B}$$

$$\vec{F'} = \frac{d\vec{F_m}}{dl} = I\,(-\vec{i_z})\times\vec{B}$$

$$dB = \frac{\mu_0\,dI}{2\pi a} \qquad dI = \frac{I}{\pi a}dl = \frac{I}{\pi \cancel{a}}\cancel{a}\,d\theta = \frac{I}{\pi}d\theta$$

$$dB = \frac{\mu_0 I\,d\theta}{2a\pi^2}$$

$dB_y = 0$ (симетрија) $\quad -d\vec{B_x} = dB\sin\theta\,\vec{i_x}$

$$d\vec{B_x} = \frac{\mu_0 I\,d\theta}{2a\pi^2}\sin\theta(-\vec{i_x}) \;;\; \vec{B} = \int_0 d\vec{B_x}$$

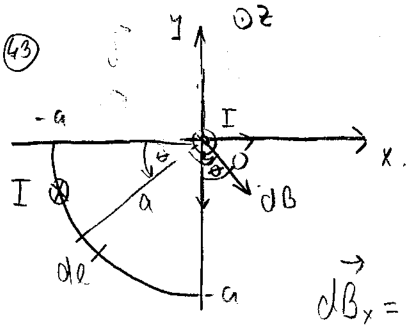
$$\vec{B} = \frac{\mu_0 I}{2a\pi^2}(-\vec{i_x})\int_0^{\pi}\sin\theta\,d\theta = \frac{\mu_0 I}{2a\pi^2}(-\vec{i_x})\left(\underbrace{-\cos\pi+\cos 0}_{2}\right)$$

$$\boxed{\vec{B} = \frac{\mu_0 I}{a\pi^2}(-\vec{i_x})}$$

$xy\underrightarrow{z}xy$

$$\vec{F'} = I\left(-\vec{i_z}\right)\times\left(-\vec{i_x}\right)\frac{\mu_0 I}{a\pi^2} \Rightarrow \boxed{\vec{F'} = \frac{\mu_0 I^2}{a\pi^2}\vec{i_y}}$$

(43)




$a, I, F' = ?$

$$\vec{F'} = I\,\vec{i_z}\times\vec{B_0}$$

$$dB = \frac{\mu_0\,dI}{2\pi a} \;;\; dI = \frac{I}{\frac{a\pi}{2}} = \frac{2I}{a\pi}\cancel{dl}\,a\,d\theta \Rightarrow dB = \frac{\mu_0 I}{a\pi^2}d\theta$$

$$d\vec{B_x} = dB\sin\theta(-\vec{i_x}) = \frac{\mu_0 I}{a\pi^2}\sin\theta\,d\theta\,(+\vec{i_x})$$

$$\vec{B_x} = \int_\theta d\vec{B_x} = \int_0^{\pi/2}\frac{\mu_0 I}{a\pi^2}\sin\theta\,d\theta\,(+\vec{i_x}) = \frac{\mu_0 I}{a\pi^2}\left(-\underbrace{\cos\frac{\pi}{2}}_{0}+\cos 0\right)(+\vec{i_x}) =$$

$$= \frac{\mu_0 I}{a\pi^2}(+\vec{i_x})$$

$$\vec{B_y} = \int_\theta d\vec{B_y} = \int_0^{\pi/2}\frac{\mu_0 I}{a\pi^2}\cos\theta\,d\theta(-\vec{i_y}) = \frac{\mu_0 I}{a\pi^2}\left(\underbrace{\sin\frac{\pi}{2}}_{1}-\underbrace{\sin 0}_{0}\right)(-\vec{i_y}) =$$

$$= \frac{\mu_0 I}{a\pi^2}\quad(-\vec{i_y})$$

$$\boxed{B = \frac{\mu_0 I}{a\pi^2}\left(\vec{i_x}-\vec{i_y}\right)}$$

$xyzxy$

$$\vec{F'} = \frac{\mu_0 I^2}{a\pi^2}\vec{i_z}\times\left(\vec{i_x}-\vec{i_y}\right) =$$

$$= \frac{\mu_0 I^2}{a\pi^2}\left(\vec{i_x}+\vec{i_y}\right)$$

(44) $a = 25\,mm$

$I = 2\,A$

$B = 100\,mT$

$\theta = \frac{\pi}{4}$

$\vec{F_m}$

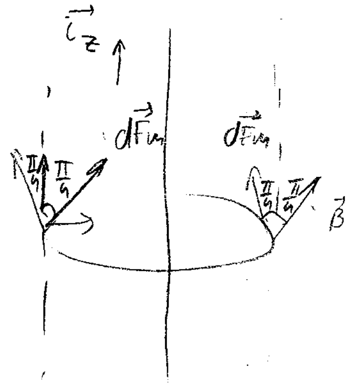
$$dF_m' = I\left|\vec{dl}\times\vec{B}'\right|$$

$$F_m' = \int I\,\vec{dl}\times\vec{B} \quad I\left(\int \vec{dl}\right)\times\vec{B} = I\,2a\pi\,B$$

$$\vec{F_m} = F_m'\sin\theta\,\vec{i_z}$$

$$\boxed{\vec{F_m} = I\,2a\pi\,B\sin\theta\,\vec{i_z}}$$

$$F_m = 0{,}22\,N$$




(45) $B_0 = 0{,}8\,T$

$N = 12;\ d = 10\,mm$

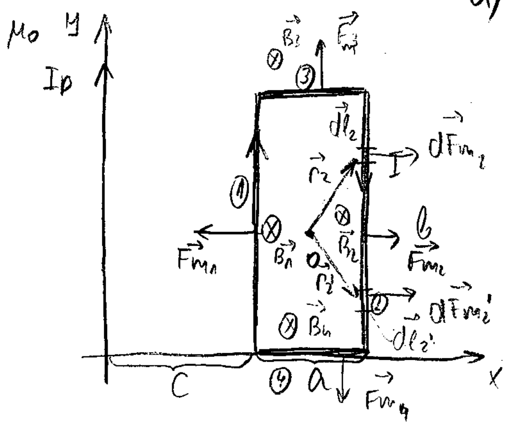
$I = 100\,mA$

$F_m = ?$

$$d\vec{F_m} = I\,\vec{dl}\times\vec{B_0} \Rightarrow \vec{F_m} = I\int \vec{dl}\times\vec{B_0} = I N d\pi B_0\,\vec{i_z}$$

$$|\vec{F_m}| = 30\,mN$$

(46) $I_p, a, b, c, I$




a) $B = \frac{\mu_0 I_p}{2\pi r}$

① $B_1 = \frac{\mu_0 I_p}{2\pi c}$; $\vec{F_{m1}} = I\left(\int_{L_1}\vec{dl}\right)\times\vec{B_1} = \frac{\mu_0 I_p I b}{2\pi c}(-\vec{i_x})$

$$F_{m1} = 32{,}5\,\mu N$$

② $B_2 = \frac{\mu_0 I_p}{2\pi(c+a)} \Rightarrow \vec{F_{m2}} = \frac{\mu_0 I_p I b}{2\pi(c+a)}\,\vec{i_x}$

$$F_{m2} = 20\,\mu N$$

③ $B_3 = \frac{\mu_0 I_p}{2\pi r}$; $dF_{m3} = I\,dl\,B \Rightarrow F_{m3} = \frac{\mu_0 I I_p}{2\pi}\int_c^{a+c}\frac{dl^{\,dr}}{r} = \frac{\mu_0 I I_p}{2\pi}\ln\frac{a+c}{c}$

$\vec{F_{m3}} = F_{m3}\,\vec{i_y}$ $\quad F_{m3} = 19{,}42\,\mu N$

④ $B_4 = \frac{\mu_0 I_p}{2\pi r}$; $F_{m4} = F_{m3} = \frac{\mu_0 I I_p}{2\pi}\ln\frac{a+c}{c} = 19{,}42\,\mu N$

$$\vec{F_{m4}} = F_{m4}(-\vec{i_y})$$

б) $\vec{F_m} = \vec{F_{m1}} + \vec{F_{m2}} + \vec{F_{m3}} + \vec{F_{m4}} = 12{,}5\,\mu N\,(-\vec{i_x})$

$M_m$ у односу на центар:

$$\vec{M_m} = \int_L \vec{r}\times d\vec{F_m}$$

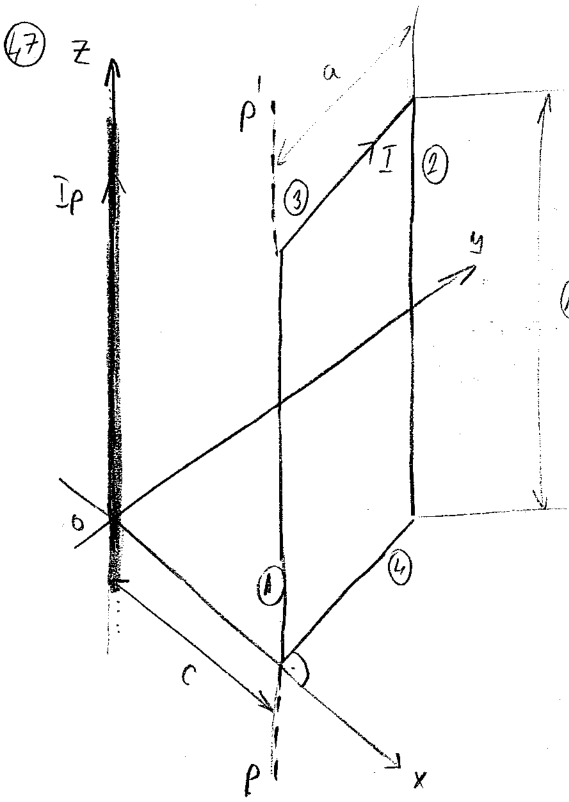
①,② : ако узмемо два дела $dl_2$ и $dl_2'$ видимо да се њихови моменти поништавају па је $M_{m2} = 0 \Rightarrow M_{m1} = 0$

③,④ како су $F_{m3}$ и $F_{m4}$ једнаке, на основу симетрије имамо

$$M_{m3} = M_{m4} = 0$$

$$\boxed{M_m = 0}$$

47




$$B = \frac{\mu_0 I_p}{2\pi r}$$

xyzxy

a) ① $\vec B_1 = \frac{\mu_0 I_p}{2\pi c}\vec i_y$ $\quad d\vec F_{m1} = I\,dl\,\cancel{\vec i_z \times \vec i_y}\,\frac{\mu_0 I_p}{2\pi c}$ $\quad -\vec i_x$

$$\vec F_{m1} = \int_{L_1} d\vec F_{m1} = \frac{\mu_0 I I_p}{2\pi c}\, b\,(-\vec i_x)$$

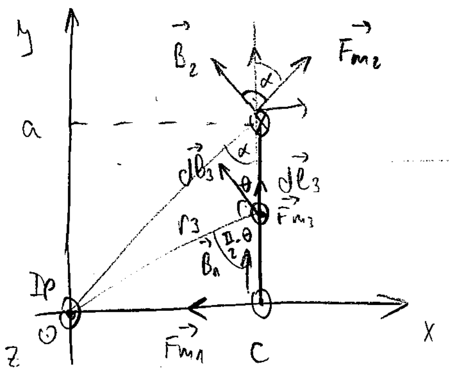
$$\underline{|\vec F_m| = 32{,}5\,\mu N}$$

② $B_2 = \frac{\mu_0 I_p}{2\pi\sqrt{a^2+c^2}}$

$$F_{m2} = \int_{L_2} dF_{m2} = \int_{L_2} I\,dl\,B_2 = \frac{\mu_0 I_p I b}{2\pi\sqrt{a^2+c^2}}$$

$$\vec F_{m2x} = F_{m2}\sin\alpha\,\vec i_x = \frac{\mu_0 I I_p b}{2\pi\sqrt{a^2+c^2}}\,\frac{c\,\vec i_x}{\sqrt{a^2+c^2}} = \underline{23{,}37\,\mu N\,\vec i_x}$$

$$\vec F_{m2y} = F_{m2}\cos\gamma\,\vec i_y = \frac{\mu_0 I I_p b}{2\pi\sqrt{a^2+c^2}}\,\frac{a\,\vec i_y}{\sqrt{a^2+c^2}} = \underline{14{,}61\,\mu N\,\vec i_y}$$




③ $dB_3 = \frac{\mu_0 I_p}{2\pi\sqrt{c^2+y^2}}$ $\quad d\vec F_{m3} = I\,d\vec l\times\vec B_3$ $\quad dl = dy$

$$F_{m3} = \int_{L_3} dF_{m3} = \int_{L_3}\frac{\mu_0 I_p I\,dy}{2\pi\sqrt{y^2+c^2}}\,\sin\angle(d\vec B_3, d\vec l_3)$$

$$\sin\theta = \cos\left(\frac{\pi}{2}-\theta\right) = \frac{y}{\sqrt{y^2+c^2}}$$

$$F_{m3} = \frac{\mu_0 I_p I}{2\pi}\int_0^a \frac{y\,dy}{y^2+c^2} = \frac{\mu_0 I_p I}{2\pi}\int_0^a \frac{\frac{1}{2}d(y^2+c^2)}{y^2+c^2} = \frac{\mu_0 I I_p}{2\pi}\,\frac{1}{2}\ln\frac{a^2+c^2}{c^2} \Rightarrow$$

$$\boxed{\vec F_{m3} = \frac{\mu_0 I I_p}{2\pi}\ln\frac{\sqrt{a^2+c^2}}{c}\,\vec i_z} \qquad F_{m3} = 6{,}595\,\mu N$$

④ - на основу симетрије:

$$\underline{\vec F_{m4} = -\vec F_{m3} = -\frac{\mu_0 I I_p}{2\pi}\ln\frac{\sqrt{a^2+c^2}}{c}\,\vec i_z} \qquad F_{m4} = 6{,}595\,\mu N$$

б) $\vec F_m = ?$ $\quad \vec F_m = \vec F_{m1} + \vec F_{m2} + \vec F_{m3} + \vec F_{m4}$

$$\vec F_m = -32{,}5\,\mu N\,\vec i_x + 23{,}37\,\mu N\,\vec i_x + 14{,}61\,\mu N\,\vec i_y + \cancel{6{,}595\,\mu N\,\vec i_z} - \cancel{6{,}595\,\mu N\,\vec i_z}$$

$$\boxed{\vec F_m = -9{,}13\,\mu N\,\vec i_x + 14{,}61\,\mu N\,\vec i_y}$$

$\vec M_m = \vec M_{m1} + \vec M_{m2} + \vec M_{m3} + \vec M_{m4}$   нормална компонента силе F

$\vec M_m = \int \vec r \times d\vec F_m = r\, F_{mn}\, \vec i_M$

$M_{m1} = 0 \to (r=0)$

$M_{m3} = M_{m4} = 0 \quad (F_{m3n} = F_{m4n} = 0)$

$M_{m2} = a \cdot F_{m2}$

$M_m = aF_{m2}$   $\vec i_m = -\vec i_z$ (смер ротације по правилу десне руке)

$\vec M_m = -aF_{m2}\,\vec i_z$

(48) $a = 2\,cm$

$I = 2A$

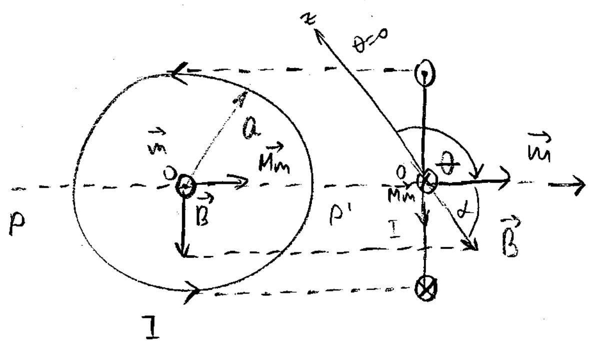
$|\vec B| = 0{,}6T$

$\alpha = \frac{\pi}{6}$

а) $\vec M_m(\alpha) = ?$

б) ПСР = ?

в) $A(\alpha \to ПСР)$




а) $\vec M_m = \vec m \times \vec B$

$|\vec m| = IS = Ia^2\pi$

$\vec M_m = mB\sin\angle(\vec m, \vec B)\,\vec i_{pp'}$

$\vec M_m = Ia^2\pi B \sin\alpha\,\vec i_{pp'}$

$\sin\alpha = \sin(\pi - \alpha) = \sin\theta$

$\boxed{\vec M_m = Ia^2\pi B\sin\theta\,\vec i_{pp'}}$   $M_m = 754\,\mu Nm$

б) ПСР $\Rightarrow M_m = 0 \Leftrightarrow \sin\theta = 0 \Rightarrow \theta = 0$ ПЛР B и m су супротни

$\boxed{\theta = \pi}$ ПСР B и m се поклапају

в) $A_m = \int_{\theta_0}^{ПСР} dA_m = \int_{5\pi/6}^{\pi} M_m\, d\theta = Ia^2\pi B \int_{5\pi/6}^{\pi} \sin\theta$

$A_m = Ia^2\pi B\left(\cos\frac{5\pi}{6} - \cos\pi\right) \Rightarrow \boxed{A_m = Ia^2\pi B\left(1 - \frac{\sqrt3}{2}\right)}$

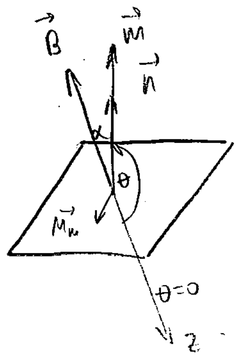
$A_m = 200\,\mu J$

(49) a, N, I, B, α

а) $M_{m\,max} = ?$

б) ПСР = ?

в) ПЛР = ?




$m = NIa^2\pi$

$\vec M_m = \vec m \times \vec B$

$M_m = mB\sin\angle(\vec m, \vec B) = NIa^2\pi B\sin\theta$

а) $M_m = M_{m\,max} \Leftrightarrow M_m' = 0$

$\cos\theta = 0 \Rightarrow \theta = k\frac{\pi}{2} = \frac{\pi}{2}$

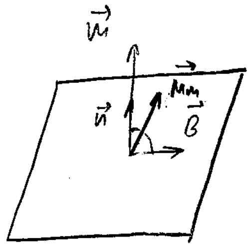
$\boxed{\alpha = \pi/2}$

$M_m = 0 \Leftrightarrow \sin\theta = 0 \Rightarrow \theta = 0\ (\alpha = \pi)$ ПЛР б)

$\theta = \pi\ (\alpha = 0)$ ПСР в)

*50 $a$, $B(t) = B_m \cos\omega t$

$i(t) = I_m \cos\left(\omega t - \frac{\pi}{4}\right)$




$M_{m\,sr} = ?$

$$M_m = mB \sin\measuredangle(\vec{m}, \vec{B})$$

$$M_m(t) = \underbrace{a^2\, i(t)\, B(t)}\, \cancel{\sin}\frac{\pi}{2}^{1}$$

$$M_m(t) = a^2 I_m B_m \cos\omega t \cos\left(\omega t - \frac{\pi}{4}\right)$$

$$M_{sr} = \frac{1}{T}\int_0^T M_m(t)dt = \frac{1}{T}\int_0^T a^2 I_m B_m \cos\omega t \cos\left(\omega t - \frac{\pi}{4}\right) dt$$

$$= \frac{1}{T}a^2 I_m B_m \int_0^T \frac{1}{2}\left(\cos\overbrace{\left(\cancel{\omega t} - \left(\cancel{\omega t} - \frac{\pi}{4}\right)\right)}^{\frac{\sqrt{2}}{2}} + \cos\left(\omega t + \omega t - \frac{\pi}{4}\right)\right) dt =$$

$$= \frac{1}{2T}a^2 I_m B_m \left(\frac{\sqrt{2}}{2}\int_0^T dt + \int_0^T \cos\left(2\omega t - \frac{\pi}{4}\right)\frac{1}{2\omega}\, d\left(2\omega t - \frac{\pi}{4}\right)\right) =$$

$$= \frac{1}{2T}a^2 I_m B_m \left(\frac{\sqrt{2}}{2}T + \frac{1}{2\omega}\sin\left(2\omega t - \frac{\pi}{4}\right)\Big|_0^T\right) = \qquad \omega = \frac{2\pi}{T}$$

$$= \frac{1}{2T}a^2 I_m B_m \left(\frac{\sqrt{2}}{2}T + \frac{T}{4\pi}\left(\sin\left(\frac{4\pi}{\cancel{T}}\cancel{t} - \frac{\pi}{4}\right) - \sin\left(-\frac{\pi}{4}\right)\right)\right) =$$

$$= \frac{1}{2T}a^2 I_m B_m \left(\frac{\sqrt{2}}{2}T + \frac{T}{4\pi}\left(\sin\left(2\pi + \frac{7\pi}{4}\right) + \sin\frac{\pi}{4}\right)\right) =$$

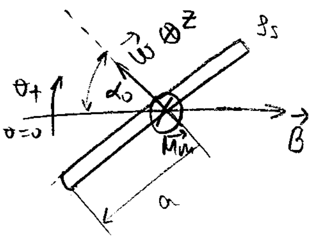
$$= \frac{1}{2T}a^2 I_m B_m \left(\frac{\sqrt{2}}{2}T + \frac{T}{4\pi}\left(\cancel{-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}}^{0}\right)\right) =$$

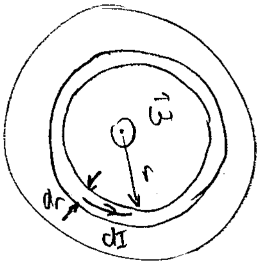
$$= \frac{a^2 I_m B_m \sqrt{2}}{4}$$

*51 $a, \rho_s, \omega, B$

a) $M_m = ?$

d) $A_m = ?$




$$J_s = v\rho_s = \omega r \rho_s$$

$$dI = J_s\, dr$$

$$dI = \omega \rho_s r\, dr$$

$$dm = dI \cdot S = r^2\pi\, \omega \rho_s r\, dr$$

$$d\vec{m} = \vec{\omega}\rho_s \pi r^3 dr$$

$$\vec{m} = \int_0^a d\vec{m} = \vec{\omega}\rho_s\pi\frac{a^4}{4}$$

$$\vec{M} = \vec{m} \times \vec{B} = \rho_s\pi\frac{a^4}{4}\,\vec{\omega} \times \vec{B}$$

$$|\vec{\omega} \times \vec{B}| = \omega B \sin\measuredangle(\vec{\omega}, \vec{B}) = \omega B \sin(\pi - \theta) = \omega B \sin\theta$$

$$\boxed{\vec{M} = \rho_s\pi\frac{a^4}{4}\omega B \sin\theta \cdot \vec{i}_z}$$

a) $\theta = \alpha_0 \Rightarrow \boxed{\vec{M} = \frac{1}{4}\rho_s \pi a^4 \omega B \sin\alpha_0\, \vec{i}_z}$

б) $A_m\Big|_{\alpha_0}^{\text{пср}} = ?$ $\quad \theta = \text{ПСР} \Leftrightarrow M_m = 0 \quad \sin\theta = 0 \Rightarrow \theta = 0$ ПНР

$\boxed{\theta = \pi}$ ПСР

$$A_m = \int_{\theta=\alpha_0}^{\text{пср}} M_m\, d\theta = \frac{1}{4}\rho_s \omega B a^4 \pi \int_{\alpha_0}^{\pi} \sin\theta =$$

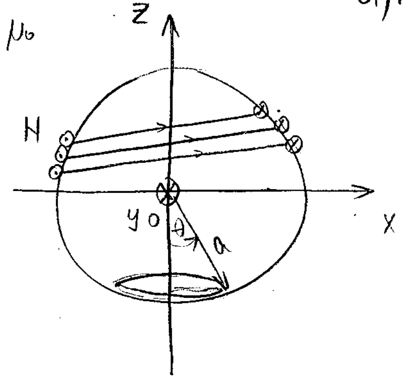
$$= \frac{1}{4}\rho_s \omega B a^4 \pi \left(\cos\alpha_0 - \cos\pi\right)$$

$$\boxed{A_m = \frac{1}{4}\rho_s \omega B a^4 \pi \left(1 + \cos\alpha_0\right)}$$

*52 $a, N, i(t) = I_m \cos\omega t$, $\vec{B}(t) = \frac{B_m}{2}\left(\vec{i}_x + \vec{i}_z\sqrt{3}\right)\sin\omega t$. $\qquad dz = -a\sin\theta\, d\theta$

$M(t) = ?$

$\mu_0$




a) $\vec{M}(t) = \vec{m}(t) \times \vec{B}(t) \qquad \cos\theta = \frac{-z}{a} \qquad -\sin\theta\, d\theta = -\frac{dz}{a}$

$$\vec{m}(t) = \int d\vec{m}(t)$$

$$dN = \frac{N}{\pi} d\theta; \quad d\vec{m} = dI\, S\, \vec{i}_z = \underbrace{i(t)\, dN}_{dI} (a\sin\theta)^2 \pi\, \vec{i}_z$$

$$\vec{m} = \int_\theta d\vec{m} = i(t) \frac{N}{\cancel{\pi}} a^2 \cancel{\pi} \int_0^{\pi} \sin^2\theta\, d\theta\ \vec{i}_z =$$

$$\int_0^{\pi} \sin^2 d\theta = \int_0^{\pi} \frac{1-\cos 2\theta}{2} d\theta = \frac{1}{2}\int_0^{\pi} d\theta - \frac{1}{2}\int_0^{\pi} \cos 2\theta \frac{1}{2} d(2\theta) = \frac{1}{2}\pi - \cancel{\frac{1}{2}\sin 2\theta\Big|_0^{\pi}}^{\,0} = \frac{\pi}{2}$$

$$\boxed{\vec{m} = I_m N a^2 \frac{\pi}{2} \cos\omega t\ \vec{i}_z}$$

xyzxy

$$\vec{M}(t) = \vec{m} \times \vec{B} = I_m N a^2 \frac{\pi}{2} \cos\omega t\ \vec{i}_z \times \left(\vec{i}_x + \vec{i}_z\sqrt{3}\right) \frac{B_m}{2} \sin\omega t =$$

$$= \frac{1}{4} I_m B_m N a^2 \pi \underbrace{\cos\omega t \sin\omega t}_{\frac{1}{2}\sin 2\omega t}\ \vec{i}_y$$

$$\boxed{\vec{M}(t) = \frac{1}{8} I_m B_m N a^2 \pi \sin 2\omega t\ \vec{i}_y}$$

б) $\vec{M}_{sr} = \frac{1}{T}\int_0^T \vec{M}(t)\, dt = \frac{1}{T}\frac{1}{8} I_m B_m N a^2 \pi \int_0^T \sin 2\omega t\, dt\ \vec{i}_y$ $\qquad \omega = \frac{2\pi}{T}$

$$\frac{1}{T}\int_0^T \sin 2\omega t\, dt = \frac{1}{T}\int_0^T \sin 2\omega t \frac{1}{2\omega} d(2\omega t) = \frac{1}{2T\omega}\left(-\cos 2\omega t\right)\Big|_0^T =$$

$$= \frac{1}{4\pi}\left(\cancel{\cos 0}^{\,1} - \cancel{\cos 4\pi}^{\,1}\right) = 0$$

$$\boxed{\vec{M}_{sr} = 0}$$

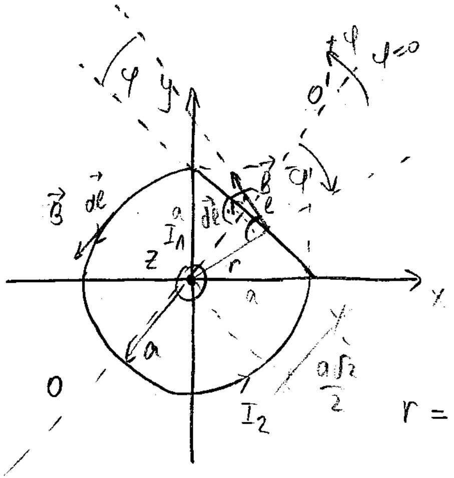
*(53) $I_1 = 1\,kA$

$a = 0{,}1\,m$

$I_2 = 2\,kA$

а) $F_m = ?$

б) $M_{OO'} = ?$



а) $B = \frac{\mu_0 I_1}{2\pi r}$

– Магнетна сила на кружни део контуре је нула јер су вектори $\vec{B}$ и $d\vec{l}$ колинеарни па је

$\sin\angle(\vec{B}, d\vec{l}) = \sin 0 = 0$

$r = \frac{a\sqrt{2}}{2\cos\varphi} \Rightarrow B = \frac{\mu_0 I_1}{2\pi a\sqrt{2}} 2\cos\varphi = \underline{\underline{\frac{\mu_0 I_1 \cos\varphi\sqrt{2}}{2a\pi}}}$

$\operatorname{tg}\varphi = \frac{l}{\frac{a\sqrt{2}}{2}}$

$d\vec{F}_m = I_2\, d\vec{l} \times \vec{B} = I_2\, dl\, B \sin\varphi$

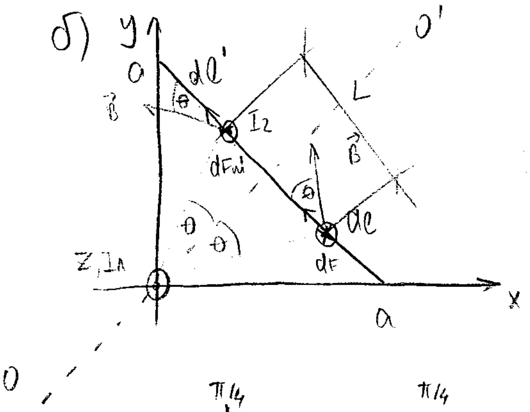
$\frac{d\varphi}{\cos^2\varphi} = \frac{2\,dl}{a\sqrt{2}} = \frac{dl\sqrt{2}}{a} \Rightarrow dl = \frac{a\sqrt{2}\, d\varphi}{2\cos^2\varphi}$

$t = \cos\varphi$

$F_m = \int_\varphi dF_m = \int_{\varphi=-\frac{\pi}{4}}^{\pi/4} I_2 \frac{\cancel{a\sqrt{2}}}{\cancel{2}} \frac{d\varphi}{\cos^2\varphi} \frac{\mu_0 I_1 \cancel{\cos\varphi}\cancel{\sqrt{2}}}{2\cancel{a}\pi} \sin\varphi = \frac{\mu_0 I_1 I_2}{2\pi} \int_{-\pi/4}^{\pi/4} \frac{\sin\varphi}{\cos\varphi} d\varphi =$

$= \frac{\mu_0 I_1 I_2}{2\pi} \int_{+\sqrt{2}/2}^{+\sqrt{2}/2} \frac{dt}{t} = 0$

б)



– ако посматрамо два симетрично постављена струјна елемента $dl$ и $dl''$ имамо да је

$dM_m = dF_z\, L$

$L = \cancel{2} \cdot \frac{a\sqrt{2}}{\cancel{2}} \operatorname{tg}\varphi = a\sqrt{2}\operatorname{tg}\varphi$

$M_m = \int_0^{\pi/4} dM_m$

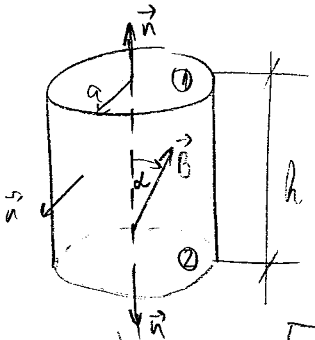
$M_m = \frac{\mu_0 I_1 I_2 a\sqrt{2}}{2\pi} \int_0^{\pi/4} \operatorname{tg}^2\varphi\, d\varphi$

$\int_0^{\pi/4} \operatorname{tg}^2\varphi\, d\varphi = \int_0^{\pi/4} \frac{\sin^2\varphi}{\cos^2\varphi} d\varphi = \int_0^{\pi/4} \left(\frac{1}{\cos^2\varphi} - 1\right) d\varphi = (\operatorname{tg}\varphi - \varphi)\Big|_0^{\pi/4} = \operatorname{tg}\frac{\pi}{4} - \frac{\pi}{4} = 1 - \frac{\pi}{4}$

$$\boxed{\vec{M} = \frac{\mu_0 I_1 I_2 a\sqrt{2}}{2\pi}\left(1 - \frac{\pi}{4}\right)\vec{i}_{OO'}}$$

(54) $a, h, \vec{B}, \alpha$

$\Psi_{OM} = ?$



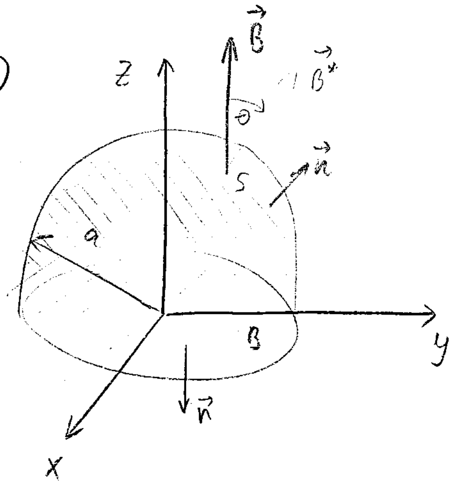
$\Psi_{OM} + \Psi_{B_1} + \Psi_{B_2} = 0$

$\Psi_{B_1} = -\Psi_{B_2}$ (симетрија) $\Rightarrow \boxed{\Psi_{OM} = 0}$

$\Psi_{B_1} = a^2\pi B\cos\alpha$

$\Psi_{B_2} = a^2\pi B\cos(\pi - \alpha) = -a^2\pi B\cos\alpha$

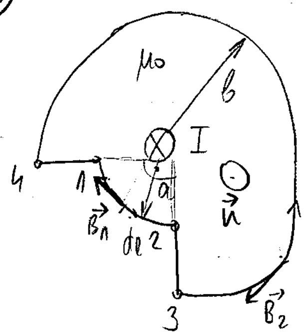
(35)




$$\Psi_S = -\Psi_B = -a^2\pi B \cos\pi = \underline{\underline{a^2\pi B}}$$

* $\underline{\Psi_S = a^2\pi B^* \cos\theta}$

(56)




a) $$\oint_1^1 \vec B d\vec l = \int_1^2 \vec B d\vec l + \int_2^3 \vec B d\vec l + \int_3^4 \vec B \cdot d\vec l + \int_4^1 \vec B d\vec l$$

$$|\vec B_1| = \frac{\mu_0 I}{2\pi a}$$

$$\int_1^2 \vec B_1 d\vec l = -B_1 \int_1^2 dl = -\frac{\mu_0 I}{2\cancel{\pi a}} \frac{\cancel{\pi a}}{2} = \frac{-\mu_0 I}{4}$$

$$\int_2^3 \vec B d\vec l = \int_4^1 \vec B d\vec l = 0 \quad \left(\angle(\vec B, d\vec l) = 90^\circ\right)$$

$$\int_3^4 \vec B d\vec l = -B_2 \int_3^4 dl = \frac{-\mu_0 I}{2\cancel{\pi b}} \frac{3\cancel{\pi b}}{2} = -\frac{3}{4}\mu_0 I$$

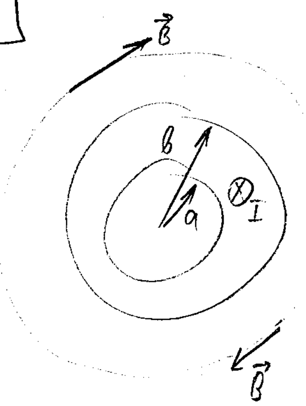
$$\oint_1^1 \vec B d\vec l = \underline{\underline{-\mu_0 I}}$$

б) $\boxed{\oint_C \vec B d\vec l = -\mu_0 I}$

(57) $a = 10mm$

$b = 15mm$

$\underline{I = 500A}$




$$J = \frac{I}{(b^2-a^2)\pi} \qquad \angle(\vec B, d\vec l) = 0^\circ$$

1) $r < a \Rightarrow \boxed{B = 0}$

$\left(\oint_C \vec B d\vec l = \mu_0 \cancelto{0}{\sum I} \Rightarrow B = 0\right)$

2) $r \in [a, b]$ $$\oint_C \vec B d\vec l = \mu_0 \frac{I}{(b^2-a^2)\cancel{\pi}}(r^2-a^2)\cancel{\pi}$$

$$B \cdot 2r\pi = \mu_0 \frac{I(r^2-a^2)}{(b^2-a^2)}$$

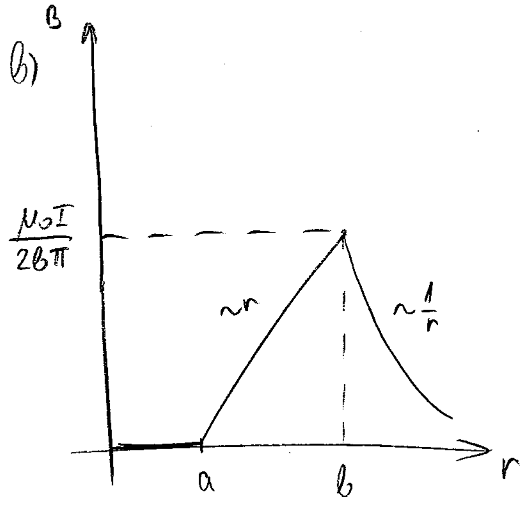
$$\boxed{B = \mu_0 \frac{I(r^2-a^2)}{2r\pi(b^2-a^2)}}$$

3) $r > b$ $$\oint_C \vec B d\vec l = \mu_0 \sum I$$

$$B \cdot 2r\pi = \mu_0 I$$

$$\boxed{B = \frac{\mu_0 I}{2r\pi}}$$

b)



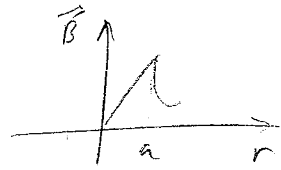

(58) $a = 0{,}5\,m$

$I = 10A$

$B_{max} = ?$

$$B = \frac{\mu_0 I'}{2\pi r} = \frac{\mu_0 \frac{I}{a^2\cancel{\pi}} r^{\cancel{2}}\cancel{\pi}}{2\pi \cancel{r}} = \frac{\mu_0 I r}{2\pi a^2} \quad r<a$$

$$B = \frac{\mu_0 I}{2\pi r}\ ;\ r>a$$

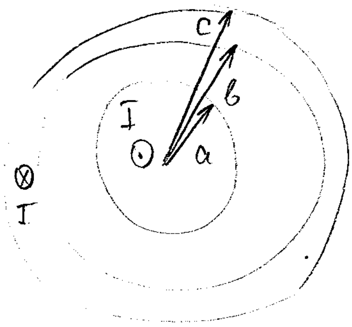



$B = B_{max} \iff r = a$

$$B_{max} = \frac{\mu_0 I}{2\pi a}$$

(59) $a, b, c, I$

$B = ?$




1) $r<a \quad I' = \frac{I}{a^2\cancel{\pi}} r^2\cancel{\pi} = \frac{I r^2}{a^2}$

$$B = \frac{\mu_0 I'}{2r\pi} = \frac{\mu_0 I r}{2\pi a^2} \checkmark$$

2) $r \in [a, b) \quad I' = I$

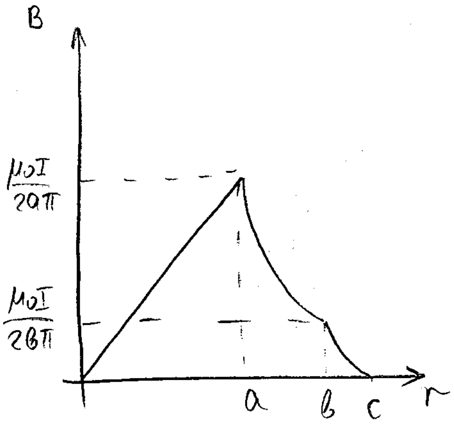
$$\boxed{B = \frac{\mu_0 I}{2r\pi}}$$

3) $r \in [b, c);\ I' = I - \frac{I(r^2-b^2)\pi}{(c^2-b^2)\pi}$

$$B = \frac{\mu_0}{2r\pi} I \frac{c^2 \cancel{-b^2} - r^2 \cancel{+b^2}}{c^2-b^2}$$

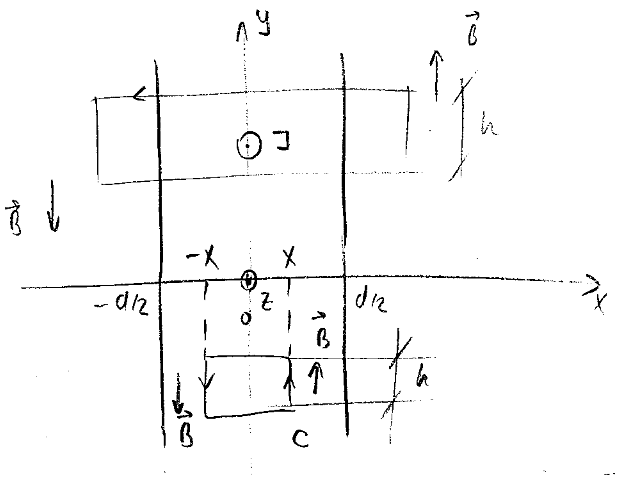
$$\boxed{B = \frac{\mu_0 I}{2r\pi} \frac{c^2-r^2}{c^2-b^2}}$$

4) $r>c \quad \boxed{B=0} \quad (I-I=0)$



$\vec{B}(x) = -\vec{B}(-x)$ $\qquad$ $\vec{B}(x) = B(x)\cdot\vec{i_y}$

(60)




1) $|x| > \frac{d}{2}$ $\quad 2B\cancel{h} = \mu_0 J d \cancel{h}$

$$\boxed{B = \frac{\mu_0 J d}{2}}$$

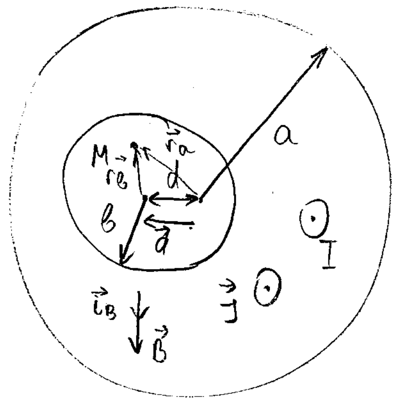
2) $|x| < \frac{d}{2}$ $\quad 2B\cancel{h} = \mu_0 J 2x \cancel{h}$

$$\oint_C \vec{B}\cdot d\vec{l} = \mu_0 \Sigma I$$

$$\boxed{B = \mu_0 J x}$$

(61) $a, b, d, I$

$$J = \frac{I}{\pi(a^2-b^2)}$$




$J_1 = J$ $\qquad$ $J_2 = -J$

$B_1 \cdot 2 r_a \cancel{\pi} = J_1 r_a^2 \cancel{\pi} \mu_0$ $\qquad$ $B_2 \cdot 2 \cancel{r_b} \cancel{\pi} = J_2 r_b^{\cancel{2}} \cancel{\pi} \mu_0$

$B_1 = \frac{\mu_0 J_1 r_a}{2}$ $\qquad$ $B_2 = \frac{\mu_0}{2}(-J)\, r_b$

$$B_1 = \frac{\mu_0}{2} J r_a$$

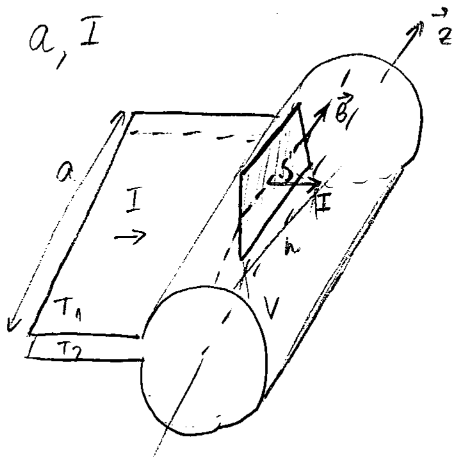
$$B = \frac{\mu_0 J}{2}(r_a - r_B)$$

$$\vec{B} = \frac{\mu_0}{2}\vec{J}\times\underbrace{(\vec{r_a} - \vec{r_b})}_{\vec{d}} = \frac{\mu_0}{2}\vec{J}\times\vec{d}$$

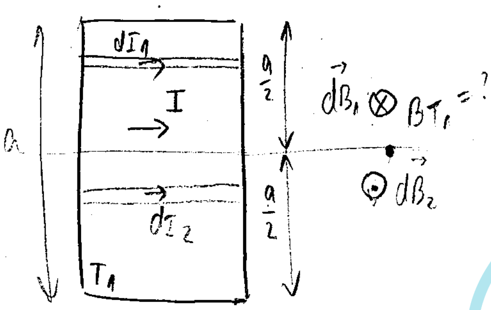
$$\sphericalangle(\vec{J},\vec{d}) = 90^\circ$$

$$\boxed{B = \frac{\mu_0}{2}\frac{I d}{\pi(a^2-b^2)}} \qquad \vec{B} = B\,\vec{i_B}$$

(62) $a, I$




$$\vec{B} = \vec{B}_{T_1} + \vec{B}_{T_2} + \vec{B}_V$$




$\Rightarrow \boxed{B_{T_1} = 0}$

$\Downarrow$

$\boxed{B_{T_2} = 0}$

(V) $\vec{B} = J_s \mu_0 \vec{i_z}$

$J_s = \frac{I}{a} \Rightarrow \boxed{\vec{B} = \frac{I}{a}\mu_0 \vec{i_z}}$

$$\oint_C \vec{B}\,d\vec{l} = \mu_0 \sum I$$

$$B\cdot \cancel{h} = \mu_0 J_s \cdot \cancel{h}$$

~~63~~ 64

1) $\oint_C \vec{B}\,d\vec{l} = \mu_0 \int_S \vec{J}\,d\vec{s}$

2) $\oint_S \vec{B}\,d\vec{s} = 0$

~~65~~ ~~66~~ ~~67~~

## 2. Стално магнетско поље у присуству материјала

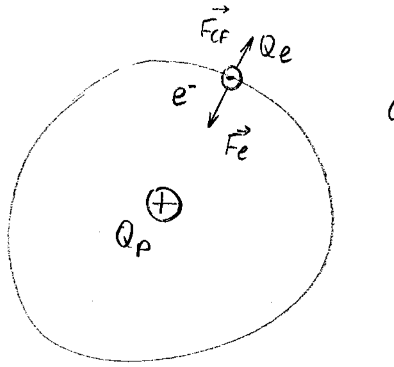
68 $\underline{r = 5{,}28 \cdot 10^{-11}\,m}$

a) $v = ?$

б) $T = ?$

в) $m = ?$

г) $\frac{m}{L} = ?$




$e = 1{,}6 \cdot 10^{-19}\,C$
$m = 9{,}1 \cdot 10^{-31}\,kg$

a) $F_e = \frac{Q_p Q_e}{4\pi\varepsilon_0 r^2} = \frac{e^2}{4\pi\varepsilon_0 r^2}$

$F_{CF} = m a_n = m\frac{v^2}{r} \Rightarrow v = \sqrt{\frac{F_{CF}\, r}{m}}$

$F_e = F_{CF} \Rightarrow v = \sqrt{\frac{e^2}{4\pi\varepsilon_0 m r}} \approx \underline{\underline{2{,}2 \cdot 10^6\,\frac{m}{s}}}$

б) $T = \frac{2\pi}{\omega} = \frac{2\pi r}{v}$ $\boxed{T \approx 1{,}51 \cdot 10^{-16}\,s}$

в) $m = I \cdot S$ $\quad I = \frac{q}{t} = \frac{e}{T}$ $\quad S = r^2\pi$

$m = \frac{e}{T} r^2\pi$ $\boxed{m \approx 9{,}3 \cdot 10^{-24}\,Am}$

г) $L = m v r \approx \underline{\underline{1{,}06 \cdot 10^{-34}\,\frac{kg\,m^2}{s}}}$ $\boxed{\frac{m}{L} = 8{,}8 \cdot 10^{10}\,\frac{C}{kg}}$

69 $\underline{r = 5{,}28 \cdot 10^{-11}\,m}$

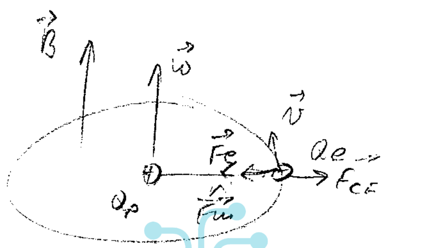
a) $\Delta\omega = ?$

б) $\Delta m = ?$

$(B=0)$ $F_e = F_{CF}$

$\frac{e^2}{4\pi\varepsilon_0 r^2} = m\frac{v^2}{r} = m\omega^2 r$

$\boxed{\omega = \sqrt{\frac{e^2}{4\pi\varepsilon_0 m r^3}}}$




$\vec{F}_m = q\vec{v} \times \vec{B} =$
$= -e\,\vec{v} \times \vec{B} =$
$= \underline{e\vec{B} \times \vec{v}}$

(в) $F_e + F_m = F_{CF}$

$\frac{e^2}{4\pi\varepsilon_0 r^2} + e\overset{\omega_1 r}{\cancel{v_1}}B = m\omega_1^2 r$

$\omega_1^2 m r - \omega_1 e B r = \frac{e^2}{4\pi\varepsilon_0 r^2} \quad /: mr$

$\omega_1^2 - \omega_1\frac{eB}{m} - \underbrace{\frac{e^2}{4\pi\varepsilon_0 m r^3}}_{\omega^2} = 0$

$\omega_1 > 0 \Rightarrow$

$\Rightarrow \omega_1 = \frac{\frac{eB}{m} + \sqrt{\frac{e^2B^2}{m^2} + 4\omega^2}}{2}$

$\boxed{\omega_1 = \frac{eB}{2m} + \sqrt{\left(\frac{eB}{2m}\right)^2 + \omega^2}}$

$\omega \gg \frac{eB}{2m}$

$\boxed{\omega_1 \approx \frac{eB}{2m} + \omega}$

$\boxed{\Delta\omega = \frac{eB}{2m}}$

б) $T=\frac{2\pi}{\omega}$ $\quad T_1=\frac{2\pi}{\omega_1}$

$m = Is = \frac{e}{T}s$ $\quad m_1 = \frac{e}{T_1}s$

$$\Delta m = es\left(\frac{1}{T_1}-\frac{1}{T}\right) = \frac{\frac{2\pi(\omega_1-\omega)}{\omega_1\omega_2}}{\frac{2\pi}{\omega}\frac{2\pi}{\omega_1}}\, er^2\pi = \frac{\omega_1-\omega}{2}er^2$$

70) $m = 9{,}3\cdot 10^{-24}\ Am^2$

$\vec{m} = const.$

$\rho_{Fe} = 7{,}8\ kg/dm^3 = 7800\ kg/m^3$

$A_{Fe} = 56\ u$ ; $u = 1{,}66\cdot 10^{-27}\ kg$

$M = ?$ $\quad \vec{M} = \frac{\sum_{dv}\vec{m}}{dv}$

$\vec{m} = const \Rightarrow \sum_{dv}\vec{m} = N\cdot\vec{m}$

$\rho_{Fe}dv \rightarrow$ маса $dv$

$A_{Fe} \rightarrow$ маса атома

$N = \frac{\rho_{Fe}dv}{A_{Fe}} \rightarrow$ бр. атома

$M = \frac{N m}{dv} = \frac{\rho_{Fe}}{A_{Fe}} m$ $\quad \boxed{M = 7{,}8\cdot 10^5\ A/m}$

71) прочитати

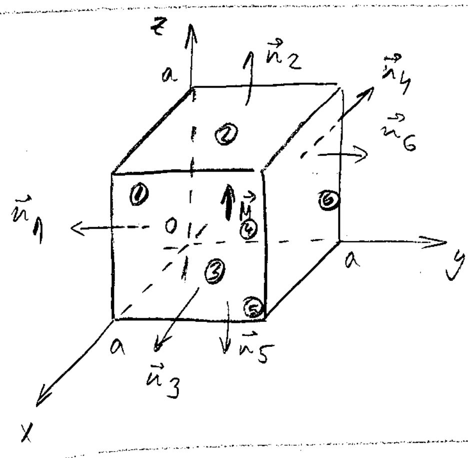
72) прочитати

73) прочитати

74) а, $\vec{M}(x) = M_0\frac{x}{a}\vec{i}_z$ ;

а) $\vec{J}_{As} = ?$ ~~б)~~

$\vec{J}_{As} = \vec{M}\times\vec{n}$



$J_{As} = M\sin\measuredangle(\vec{M},\vec{n})$

$J_{As_2} = J_{As_3} = J_{As_4} = J_{As_5} = 0$

јер је $\measuredangle(\vec{M},\vec{n}) \in \{0°, 180°\}$

$\vec{J}_{As_1} = M_0\frac{x}{a}\vec{i}_x$

$\vec{J}_{As_6} = -M_0\frac{x}{a}\vec{i}_x$

75) нелинеарна

1) $\oint_C \vec{H}\cdot d\vec{l} = \int_S \vec{J}d\vec{S}$

2) $\oint_S \vec{B}d\vec{S} = 0$

3) $\vec{B} = \vec{B}(\vec{H})$

линеарна нехомогена

2) -||-

3) $\vec{B} = \mu(r)\vec{H}$

2) $\oint_C \frac{\vec{B}}{\mu(r)}d\vec{l} = \int_S \vec{J}d\vec{S}$

лин. хомогена

2) -||- $\quad$ 3) $\vec{B} = \mu_0\mu_r\vec{H}$

3) $\oint_C \frac{\vec{B}}{\mu_0\mu_r}d\vec{l} = \int_S \vec{J}d\vec{S}$

$\mu$

вакуум

$\mu_r = 1$

2) -||-

3) $B = \mu_0 H$

1) $\oint_C \vec{B}d\vec{l} = \mu_0\int_S \vec{J}d\vec{S}$

$\oint \vec{B}d\vec{l} = \mu\int_S \vec{J}d\vec{S}$

(76) $\mu_r = 10$

$H = 1000\ A/m$

$B, \vec{M} = ?$

$$B = \mu_0 \mu_r H = 4\pi \cdot 10^{-7} \cdot 10 \cdot 1000 = \underline{\underline{4\pi\ mT}}$$

$$M = (\mu_r - 1) H = \underline{\underline{9000\ A/m}}$$

(77) $\vec{M}, \vec{J}$

$$\oint_C \vec{B}\, d\vec{l} = ? \qquad \oint_C \vec{H}\, d\vec{l} = \int_S \vec{J}\, d\vec{s} \qquad \vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

$$\oint_C \frac{\vec{B}}{\mu_0} d\vec{l} - \oint_C \vec{M}\, d\vec{l} = \int_S \vec{J}\, d\vec{s}$$

$$\boxed{\oint_C \vec{B}\, d\vec{l} = \mu_0 \left[ \int_S \vec{J}\, d\vec{s} + \oint_C \vec{M}\, d\vec{l} \right]}$$

(78) $I_A = \oint_C \vec{M}\, d\vec{l} \qquad \vec{M} = \text{const} \qquad I_A = M \cancel{\oint_C d\vec{l}} \Rightarrow \boxed{I_A = 0}$

$$I_A = \int_S \vec{J}_A\, d\vec{s} \simeq \vec{J}_A \cdot \vec{S} = J_{A_n} S \Longrightarrow \boxed{J_{A_n} = 0}$$

(79) $\mu_r, J = 0$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M} \qquad \vec{H}\mu_r - \vec{H} = \vec{M} \Rightarrow \underline{\underline{\vec{M} = \vec{H}(\mu_r - 1)}}$$

$$\vec{B} = \mu_0 \mu_r \vec{H} \qquad \oint_C \vec{M}\, d\vec{l} = (\mu_r - 1) \oint_C \vec{H}\, d\vec{l} = 0$$

$$\oint_C \vec{H}\, d\vec{l} = \int_S \underbrace{\vec{J}}_{0}\, d\vec{s} = 0$$

$$\boxed{I_A = \oint_C \vec{M}\, d\vec{l} = 0}$$

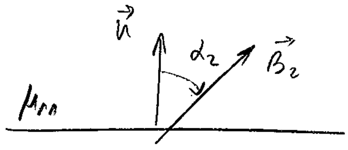
(80) $\mu_{r1} = 100$

$\mu_{r2} = 500$

$B_2 = \sqrt{2}\,T$

$\alpha_2 = 45^\circ$

$\vec{B}_1 = ?$




Гр. услови: $B_{1n} = B_{2n}$

$H_{1t} = H_{2t}$

$$B_{2t} = B_{2n} = B_2 \cos\alpha_2$$

$$B_{2n} = \sqrt{2}\,\frac{\sqrt{2}}{2} = 1\,T$$

$$\boxed{B_{1n} = 1\,T}$$

$H_{1t} = H_{2t}$ $\qquad B = \mu_0 \mu_r H$

$$\frac{B_{1t}}{\cancel{\mu_0}\mu_{r1}} = \frac{B_{2t}}{\cancel{\mu_0}\mu_{r2}}$$

$$B_{1t} = \frac{\mu_{r1}}{\mu_{r2}} B_{2t} = \frac{1}{5}\,T$$

$$B_1 = \sqrt{B_{1n}^2 + B_{1t}^2} = \sqrt{1 + \frac{1}{25}}\;T = \frac{\sqrt{26}}{5}\,T$$

$$\operatorname{tg}\alpha_1 = \frac{B_{1t}}{B_{1n}} = \frac{1}{5} \Rightarrow \alpha_1 = \operatorname{arctg}\frac{1}{5} = 11^\circ 18' \qquad \alpha_1 = \sphericalangle(\vec{n}, \vec{B}_1)$$

(81) $\mu_r = 1000$

$\mu_0$ і $\mu_{r_v} \approx 1$

$\alpha_2 = ?$ $\quad \alpha_1 \leq 1^\circ$

$$\frac{\operatorname{tg}\alpha_2}{\operatorname{tg}\alpha_1} = \frac{\mu_{r2}}{\mu_{r1}} = \mu_{r2}$$

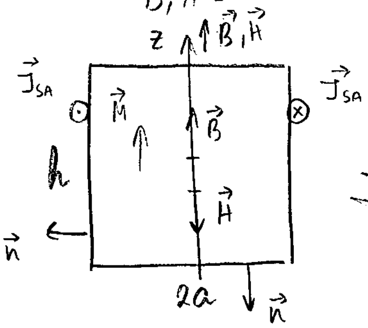
$\mu_{r1} = 1$

$$\operatorname{tg}\alpha_2 = 1000\,\operatorname{tg}\alpha_1$$

$$\alpha_2 \leq \operatorname{arctg}(1000\,\operatorname{tg}1^\circ) = 86{,}72^\circ$$

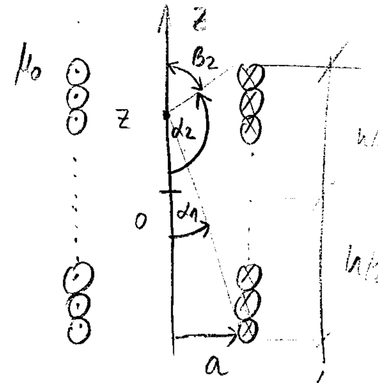
** ~~(82)~~ *(83) $\vec{M}$, $a$, $h$

$\vec{B}, \vec{H} = ?$




$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

$$\vec{J}_{SA} = \vec{M} \times \vec{n} \Rightarrow |\vec{J}_{SA}| = M$$




$$\vec{B} = \frac{\mu_0 J_{AS}}{2}(\cos\alpha_1 - \cos\alpha_2)\vec{i}_z$$

$$\cos\alpha_1 = \frac{\frac{h}{2} + z}{\sqrt{\left(\frac{h}{2}+z\right)^2 + a^2}}$$

$$-\cos\alpha_2 = +\cos\beta_2 = \frac{\frac{h}{2} - z}{\sqrt{\left(\frac{h}{2}-z\right)^2 + a^2}}$$

$$\vec{B} = \frac{\mu_0 \vec{M}}{2}\left(\frac{\frac{h}{2}+z}{\sqrt{\left(\frac{h}{2}+z\right)^2 + a^2}} + \frac{\frac{h}{2}-z}{\sqrt{\left(\frac{h}{2}-z\right)^2 + a^2}}\right)\vec{i}_z$$

1) $|z| < \frac{h}{2}$ $\quad \vec{H}$ и $\vec{B}$ имају супротне смерове

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

$$\vec{H} = \frac{\vec{M}}{2}\underbrace{\left(\overbrace{\frac{\frac{h}{2}+z}{\sqrt{\left(\frac{h}{2}+z\right)^2 + a^2}}}^{<1} + \overbrace{\frac{\frac{h}{2}-z}{\sqrt{\left(\frac{h}{2}-z\right)^2 + a^2}}}^{<1} - 2\right)}_{<0}\vec{i}_z$$

2) $|z| > \frac{h}{2}$ $\quad \vec{H}$ и $\vec{B}$ имају ист. смер

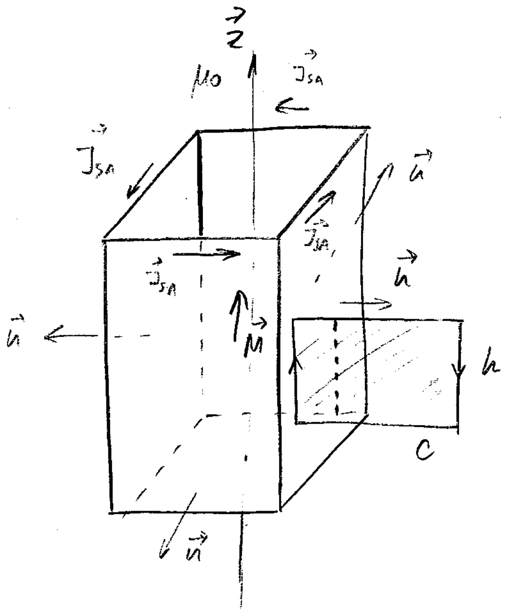
$$\vec{H} = \frac{\vec{B}}{\mu_0}$$

$$\vec{H} = \frac{\vec{M}}{2}\underbrace{\left(\frac{\frac{h}{2}+z}{\sqrt{\left(\frac{h}{2}+z\right)^2 + a^2}} + \frac{\frac{h}{2}-z}{\sqrt{\left(\frac{h}{2}-z\right)^2 + a^2}}\right)}_{>0}\vec{i}_z$$

(84) $a, \vec{M}, \mu_0$

a) $\vec{J}_{SA} = ?$

б) $\vec{H}_0, \vec{B}_0 = ?$




a) $\vec{J}_{SA} = \vec{M} \times \vec{n}$

$|\vec{J}_{SA}| = M$

б) $\oint_c \vec{B} d\vec{l} = \mu_0 \int_l \vec{J}_s d\vec{l}$

$\|$

$B \cdot \cancel{h} = \mu_0 J_s \cancel{h}$

$$\boxed{\vec{B}_0 = \mu_0 J_s \vec{i}_z}$$

$$\boxed{\vec{B}_0 = \mu_0 \vec{M}}$$

$$\vec{H}_0 = \frac{\vec{B}_0}{\mu_0} - \vec{M} \Rightarrow \boxed{\vec{H}_0 = 0}$$

(85) $a = 100\ mm$

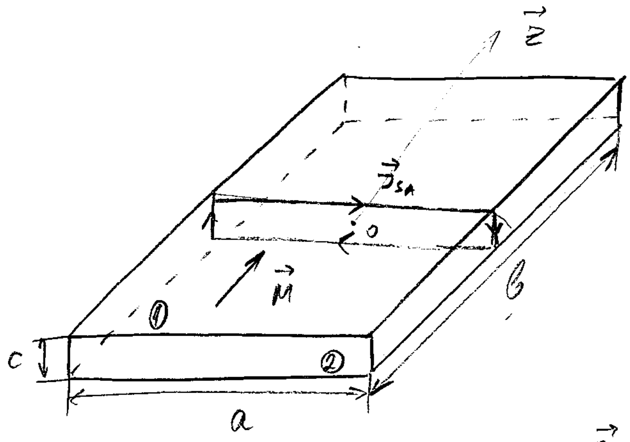
$b = 50\ mm$

$c = 2\ mm$

$M = 10\ \frac{kA}{m}$

a) $J_s = ?$

б) $\vec{B}_0 = ?$




$|\vec{J}_{SA}| = M$

$a, b >> c$

$\vec{B} = \vec{B}_1 + \vec{B}_2$

струјни плашт

$$\vec{B}_1 = \frac{\mu_0 J_s}{2} \vec{i}_z$$

$$\vec{B}_2 = \frac{\mu_0 J_s}{2} \vec{i}_z$$

$$\boxed{\vec{B} = \mu_0 J_s \vec{i}_z}$$

$$\boxed{\vec{B} = \mu_0 \vec{M}}$$

(86) $\mu = 1\ \frac{mH}{m}$

$N' = 1000\ m^{-1}$

$I = 10 A$

$J_{AS} = ?$

$$\oint \vec{H} d\vec{l} = NI$$

$$H \cdot l = NI \Rightarrow \boxed{H = N'I}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M} \qquad \vec{B} = \mu \vec{H}$$

$$N'I = \frac{\mu N' I}{\mu_0} - M \Rightarrow M = N'I \frac{\mu - \mu_0}{\mu_0}$$

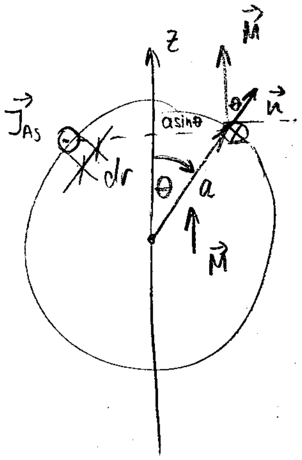
$$J_{AS} = M = N'I \frac{\mu - \mu_0}{\mu_0}$$

$$\vec{J}_{AS} = \vec{M} \times \vec{n}$$

(87) $a, \vec{M} = const$

a) $J_{AS} = ?$ $J_A = ?$

б) $H_0, B_0 = ?$




$\underline{J_{AS} = M \sin\theta}$ $J_A = 0$ ($M = const$)

$$dB = \frac{\mu_0 a'^2}{2r^3} dI$$

$a' = a\sin\theta$

$r = a$

$dI = J_{AS} dr = \underbrace{M \sin\theta}_{J_{AS}} \underbrace{a\,d\theta}_{dr}$

$$B_0 = \int_\theta dB = \int_{\theta=0}^{\pi} \frac{\mu_0 \cancel{a^2} \sin^2\theta \, M \sin\theta \, \cancel{a} \, d\theta}{2 \cancel{a^3}}$$

$$B_0 = \frac{\mu_0 M}{2} \cdot \int_0^{\pi} \sin^3\theta \, d\theta = \frac{2}{3}\mu_0 M \qquad \boxed{\vec{B}_0 = \frac{2}{3}\mu_0 \vec{M}}$$

$$\vec{H}_0 = \frac{\vec{B}_0}{\mu_0} - \vec{M} \qquad \boxed{\vec{H}_0 = -\frac{1}{3}\vec{M}}$$

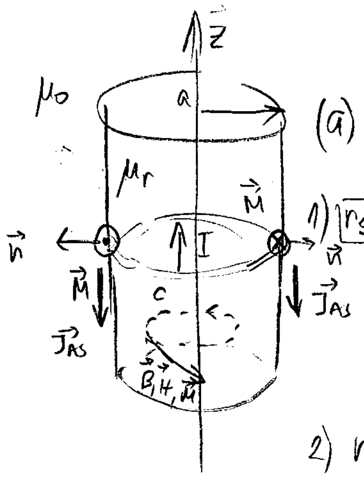
(88) $\mu_r, a, I, \mu_0$

(a) $\vec{B}(r) = ?$

$\vec{H}(r) = ?$

(б) $M = ?$

(в) $J_{AS} = ?$




(a) $\oint_C \vec{H} d\vec{l} = \sum I$

1) $\boxed{r \leq a}$ $H \cdot 2r\cancel{\pi} = \frac{I}{a^2\pi} r^2 \cancel{\pi}$

$$\boxed{H = \frac{Ir}{2a^2\pi}} \qquad \underline{\vec{H} = H \vec{i}_\varphi}$$

2) $r > a$ $H \cdot 2r\pi = I$ $\Rightarrow$ $\boxed{H = \frac{I}{2r\pi}}$

1) $r \leq a$ $B = \mu H = \mu_0 \mu_r H$ $\boxed{B = \frac{\mu_0 \mu_r I r}{2a^2\pi}}$

2) $r > a$ $B = \mu_0 H$ $\boxed{B = \frac{\mu_0 I}{2\pi r}}$ $\underline{\vec{B} = B \vec{i}_\varphi}$

(б) $\vec{H} = \frac{\vec{B}}{\mu_0} - M \Rightarrow \vec{M} = \frac{\vec{B}}{\mu_0} - \vec{H}$

$\boxed{r \leq a}$

$$\vec{M} = \left(\frac{\mu_r I r}{2a^2\pi} - \frac{Ir}{2a^2\pi}\right)\vec{i}_\varphi = \underline{\frac{Ir}{2a^2\pi}(\mu_r - 1)\vec{i}_\varphi}$$

$\boxed{r > a}$: $\vec{M} = 0$

в) $\vec{J}_{AS} = \vec{M} \times \vec{n} \Rightarrow |\vec{J}_{AS}| = M$

$\vec{J}_{AS} = |\vec{J}_{AS}| (-\vec{i}_z)$

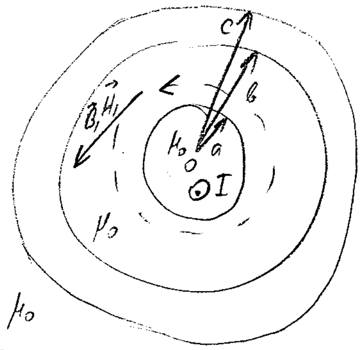
$\boxed{r = a}$

$$\boxed{\vec{J}_{AS} = -\frac{I}{2a\pi}(\mu_r - 1)\vec{i}_z}$$

(89) Cu $\Rightarrow \mu_r \approx 1$, $a$, $b$, $c$, $\mu_0$, $I$

$$B = \frac{k_1 H}{k_2 + H}, \quad k_1, k_2 > 0$$

$\vec{B}, \vec{H} = ?$ $\quad \vec{M} = ?$



$\boxed{\vec{H}}$

1) $r < a$ $\quad \oint \vec{H} d\vec{l} = \sum I$

$$H \cdot 2r\pi = \frac{I}{a^2\pi} r^2\pi$$

$$\boxed{\vec{H} = \frac{Ir}{2a^2\pi} \vec{i}_\varphi}$$

2) $r \geq a$ $\quad \oint \vec{H} d\vec{l} = \sum I$

$$H \cdot 2r\pi = I$$

$$\boxed{\vec{H} = \frac{I}{2r\pi} \vec{i}_\varphi}$$

$\boxed{\vec{B}}$

1) $r < a$ $\quad \vec{B} = \mu_0 \vec{H}$

$$\boxed{\vec{B} = \frac{\mu_0 I r}{2a^2\pi} \vec{i}_\varphi}$$

2) $a \leq r < b$ $\quad \vec{B} = \mu_0 \vec{H}$

$$\boxed{\vec{B} = \frac{\mu_0 I}{2r\pi} \vec{i}_\varphi}$$

3) $b \leq r < c$ $\quad \vec{B} = \frac{k_1 H}{k_2 + H} \vec{i}_\varphi$

$$\boxed{\vec{B} = \frac{k_1 I}{2r\pi k_2 + I} \vec{i}_\varphi}$$

4) $r \geq c$ $\quad \vec{B} = \mu_0 \vec{H}$

$$\boxed{\vec{B} = \frac{\mu_0 I}{2r\pi} \vec{i}_\varphi}$$

$\boxed{\vec{M}}$

1) $r < b$ $\quad \boxed{\vec{M} = 0}$

2) $b \leq r < c$ $\quad \vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$

$$\vec{M} = \frac{\vec{B}}{\mu_0} - \vec{H} = \left( \frac{k_1 I}{\mu_0 (2r\pi k_2 + I)} - \frac{I}{2r\pi} \right) \vec{i}_\varphi$$

3) $r \geq c$ $\quad \boxed{\vec{M} = 0}$

(90) $l = 300\,mm$

$S = 2\,cm^2$

$\mu_r = 500$

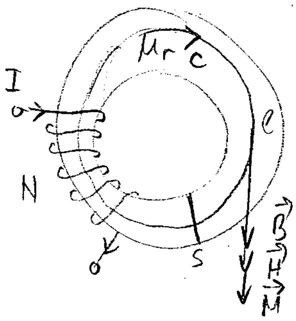
$N = 400$

$I = 250\,mA$

(a) $\vec{B}, \vec{H}, \vec{M} = ?$

(б) $\Phi = ?$

(в) $J_{As} = ?$



(a) $\oint \vec{H} d\vec{l} = \sum I$

$Hl = NI \Rightarrow$ $\boxed{\vec{H} = \frac{NI}{l} \vec{i}_\varphi}$ $\quad \underline{H = 333 \frac{A}{m}}$

$\vec{B} = \mu_0 \mu_r \vec{H}$ $\quad \boxed{\vec{B} = \frac{\mu_0 \mu_r N I}{l} \vec{i}_\varphi} \approx \underline{209\,mT}$

$\vec{M} = \frac{\vec{B}}{\mu_0} - \vec{H}$

$\boxed{\vec{M} = \frac{NI}{l}(\mu_r - 1) \vec{i}_\varphi}$ $\quad \underline{M = 166{,}3 \frac{kA}{m}}$

б) $\Phi_J = BS = \frac{\mu_0 \mu_r N I S}{l} = \underline{41{,}8\,\mu Wb}$

$$\Phi = \Phi_J \cdot N = \frac{\mu_0 \mu_r N^2 I S}{l} \approx \underline{16{,}7\,mWb}$$

$$J_{SA} = M = 166{,}3\ \frac{kA}{m} \qquad J_{SA} = \frac{NI}{l}(\mu_r - 1)$$

$$J_{SA} = N\frac{I_e}{l} \Rightarrow I_e = \frac{J_{SA}\, l}{N} = \frac{\cancel{N} I \cancel{l}}{\cancel{l}\cancel{N}}(\mu_r - 1) = I(\mu_r - 1)$$

$$\underline{\underline{I_e = 124{,}75\,A}}$$

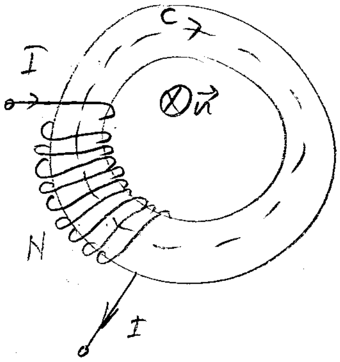
(91) $N = 400$

$l = 200\ mm$

$I = 0{,}5\,A$

$B = 0{,}9\,T$

$\vec{H}, \vec{M}, \vec{J}_{AS} = ?$




$$\oint_c \vec{H}\, d\vec{l} = \Sigma I$$

$$Hl = NI$$

$$\boxed{H = \frac{NI}{l}} \qquad \underline{\underline{\vec{H} = 1000\ \frac{A}{m}\ \vec{i}_4}}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - M$$

$$\vec{M} = \vec{H} - \frac{\vec{B}}{\mu_0} \qquad \boxed{\vec{M} = 715\ \frac{kA}{m}\ \vec{i}_4}$$

$$\boxed{J_{SA} = |\vec{M}| = 715\ \frac{kA}{m}}$$

(92) $l = 0{,}2\,m$

$S = 10\,cm^2$

$N = 1000$

$I = 2{,}5\,A$

$\phi = ?$

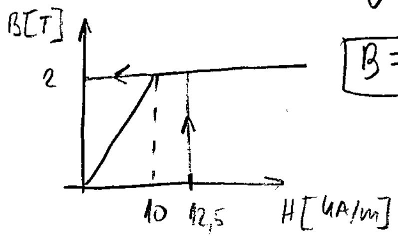
$$Hl = NI$$

$$H = \frac{NI}{l}$$

$$\underline{\underline{H = 12{,}5\,kA/m}}$$

$$\Downarrow$$

$$\boxed{B = 2\,T}$$

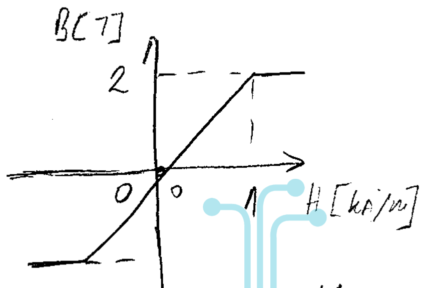



$$\phi_1 = BS$$

$$\phi = N\,\phi_1$$

$$\phi = NBS = 2 \cdot \cancel{10^3} \cdot \cancel{10} \cdot \cancel{10^{-4}}$$

$$\boxed{\phi = 2\,Wb}$$



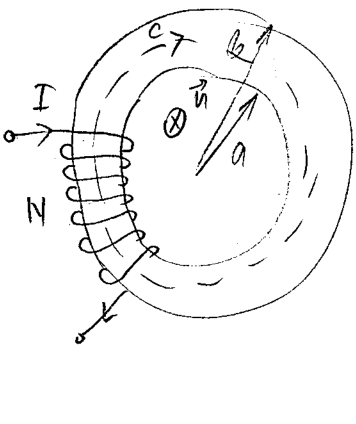

(93) $a = 150\ mm$

$b = 200\ mm$

$h = 100\ mm$

$N = 1000$

$I_{max} = ?$




$$\oint_c \vec{H}\, d\vec{l} = \Sigma I$$

$$H \cdot 2r\pi = NI$$

$$H = \frac{NI}{2r\pi}$$

$$H_{max} = 1\ \frac{kA}{m}$$

$$H = H_{max} \Leftrightarrow r = a$$

$$H_{max} = \frac{N I_{max}}{2a\pi} \Rightarrow \boxed{I_{max} = \frac{H_{max}\, 2a\pi}{N}}$$

$$\underline{\underline{I_{max} = 0{,}94\,A}}$$

10 years and counting...

(94)* (а) струја $I$ $\rightarrow$ флукс $(\phi)$ $\quad \left(J = \frac{I}{S} \rightarrow B = \frac{\phi}{S}\right)$

(б) напон $U$ $\rightarrow$ $\phi \cdot R_m = B\cancel{S} \cdot \frac{l}{\mu \cancel{S}} = \frac{Bl}{\mu} = H\,l = \oint_C \vec{H}\,d\vec{l}$

(в) ЕМС: $E$ $\rightarrow$ $N \cdot I$ – магнетомоторна сила

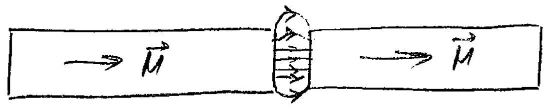
(г) I КЗ : $\sum I = 0 \rightarrow \oint_S \vec{B}\,d\vec{s} = 0 \quad (\sum \phi = 0)$

(д) II КЗ : $\sum U = 0 \rightarrow \oint_C \vec{H}\,d\vec{l} = \int_S \vec{J}\,d\vec{s}$

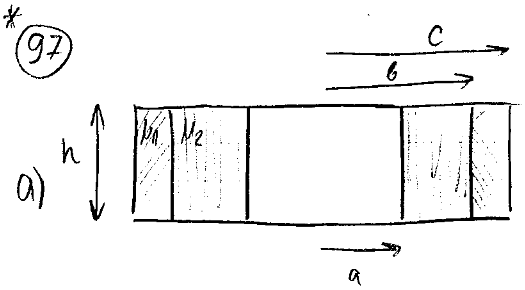
(95)* Ако је материјал линеаран онда је и $R_m = \frac{l}{\mu S}$ линеарно ел. у пољу па важе све теореме.

Ако је материјал нелинеаран не важе: линеарности, суперпозиција, Тевененова и Нортонова теорема.

(96)



*(97)



$\mu_{r1}, \mu_{r2}$

$\vec{B} = B\vec{i}_\varphi$ ; $\vec{H} = \vec{H}\vec{i}_\varphi$ ; $\vec{M} = M\vec{i}_\varphi$

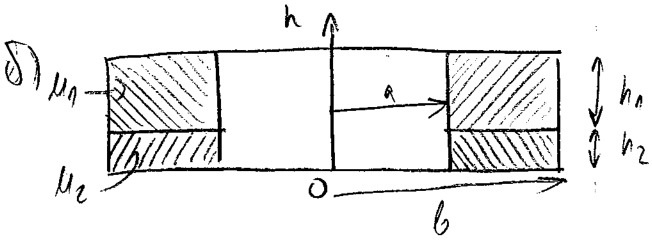
$r \in [a,c] \Rightarrow H = \frac{NI}{2\pi r} \qquad r \notin [a,c] \Rightarrow H = 0$

$$B = \mu H = \begin{cases} \frac{\mu_0 \mu_{r2} NI}{2r\pi}, & r \in (a,b) \\ \frac{\mu_0 \mu_{r1} NI}{2r\pi}, & r \in (b,c) \end{cases}$$

$r \notin [a,c] \Rightarrow B = 0$

$$M = \frac{B}{\mu_0} - H = \begin{cases} \frac{NI}{2r\pi}(\mu_{r2} - 1), & r \in (a,b) \\ \frac{NI}{2r\pi}(\mu_{r1} - 1), & r \in (b,c) \end{cases}$$

$M = 0 \quad r \notin [a,c]$



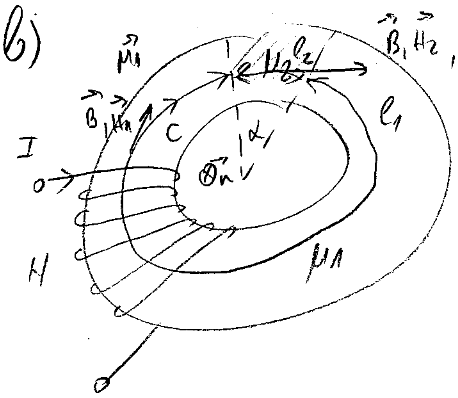
$$H = \frac{NI}{2r\pi}$$

$$B = \begin{cases} \frac{\mu_1 NI}{2r\pi}, & h \in (h_2, h_1 + h_2) \\ \frac{\mu_2 NI}{2r\pi}, & h \in (0, h_2) \end{cases}$$

$$M = \frac{B}{\mu_0} - H = \left(\frac{\mu}{\mu_0} - 1\right)\frac{NI}{2r\pi}$$

$$M = \begin{cases} \frac{\mu_1 - \mu_0}{\mu_0} & h \in (h_2, h_2 + h_1) \\ \frac{\mu_2 - \mu_0}{\mu_0}, & h \in (0, h_2) \end{cases}$$

b)




$\vec{B}_1, \vec{H}_2, \vec{M}_2$ $\quad \oint \vec{H} d\vec{l} = \Sigma I$

$(1) \quad H_1 l_1 + H_2 l_2 = NI$

$S_1 = S_2 \Rightarrow B_1 = B_2 = B \qquad B = \mu_1 H_1 = \mu_2 H_2$

$$(**) \quad l_1 = \cancel{2} r\pi \frac{2\pi - \alpha}{\cancel{2}\pi} = r(2\pi - \alpha) \qquad H_2 = \frac{\mu_1}{\mu_2} H_1 \quad (*)$$

$$l_2 = 2r\pi \frac{\alpha}{2\pi} = r\alpha$$

$(**), (*) \to (1) \qquad H_1 r(2\pi - \alpha) + \frac{\mu_1}{\mu_2} H_1 r\alpha = NI$

$$\boxed{H_1 = \frac{NI}{r(2\pi - \alpha) + \frac{\mu_1}{\mu_2} r\alpha}} \qquad \boxed{H_2 = \frac{NI}{\frac{\mu_2}{\mu_1} r(2\pi - \alpha) + r\alpha}}$$

$$\boxed{B = \mu_1 H_1 = \frac{NI}{\frac{r(2\pi - \alpha)}{\mu_1} + \frac{r\alpha}{\mu_2}}} \qquad r \in [a, b]$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M} \qquad \vec{M}_1 = \frac{\vec{B}}{\mu_0} - H_1 = \frac{\mu_1 \vec{H}_1}{\mu_0} - \vec{H}_1 = \vec{H}_1 \frac{\mu_1 - \mu_0}{\mu_0}$$

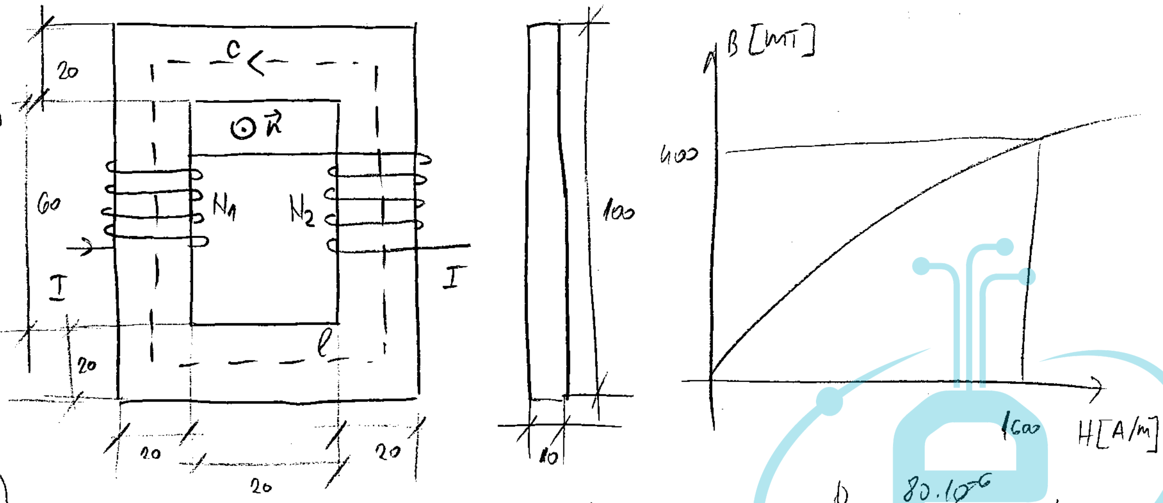
$$\boxed{|\vec{M}_1| = \frac{NI(\mu_1 - \mu_0)}{\mu_0 r\left((2\pi - \alpha) + \frac{\mu_1}{\mu_2}\alpha\right)}}$$

$$\boxed{M_2 = \frac{NI(\mu_2 - \mu_0)}{\mu_0 r\left(\frac{\mu_2}{\mu_1}(2\pi - \alpha) + \alpha\right)}}$$

(98) $I = 1{,}2\,A$

$\Phi = 80\,\mu Wb$

$N = ?$




$\oint \vec{H} d\vec{l} = \Sigma I$

$Hl = (N_1 + N_2) I$

$$\boxed{N_1 + N_2 = \frac{Hl}{I}}$$

$l = 40 + 40 + 80 + 80$

$\boxed{l = 240\,mm}$

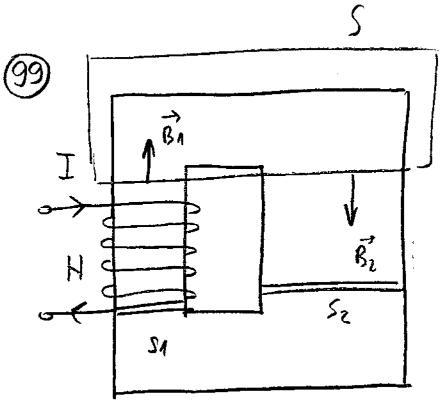
$\boxed{S = 200\,mm^2}$

$\Phi = BS \Rightarrow B = \frac{\Phi}{S} = \frac{80 \cdot 10^{-6}}{200 \cdot 10^{-6}} = 400\,mT$

$\boxed{H = 1600\,A/m}$

$$N_1 + N_2 = \frac{1600 \cdot 240 \cdot 10^{-3}}{1{,}2} = 320$$

(99)



а) $I \nearrow$ $\oint_S \vec{B}\,d\vec{s} = 0$

$-B_1 S_1 + B_2 S_2 = 0$

$B_1 S_1 = B_2 S_2$

$B_1 = \frac{S_2}{S_1} B_2$ $\Rightarrow (S_2 > S_1)$ $\boxed{B_1 > B_2}$

$B_2 = \frac{S_1}{S_2} B_1$

– таьи дго држне упази у зоситеве

б) да :)

(100) $H = 1000$

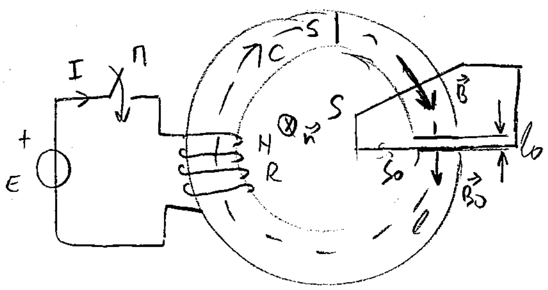
$R = 50\,\Omega$ ; $E = 150\,V$

$l = 1\,m$ ; $l_0 = \pi\,mm$

$S = S_0 = 1\,cm^2$

---

$H, H_0 = ?$



$I = \frac{E}{R} = 3A$

$\oint \vec{H}\,d\vec{l} = \Sigma I$

$Hl + H_0 l_0 = NI$

$\oint_S \vec{B}\,d\vec{s} = 0$

$-BS + B_0 S_0 = 0$

$BS = B_0 S_0 \Rightarrow (S = S_0)$ $\boxed{B = B_0}$

$H_0 = \frac{B_0}{\mu_0}$

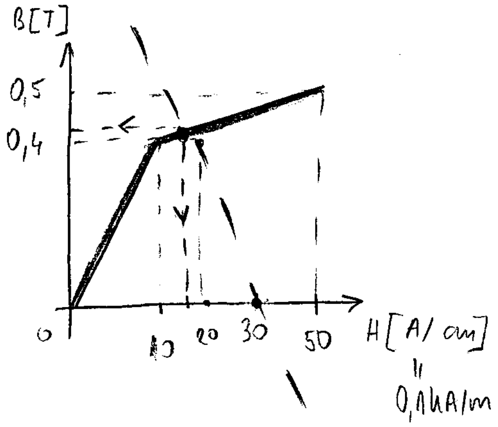
$Hl + \frac{B}{\mu_0} l_0 = NI$

$H \cdot 1 + \frac{B}{4\pi 10^{-7}} \pi \cdot 10^{-3} = 3000$

$H + 2500 B = 3000$

$B = 0 \Rightarrow H = 3000\,kA/m$

$H = 2000 \Rightarrow B = \frac{1000}{2500} = 0{,}4$



$\boxed{H \approx 19\,kA}$

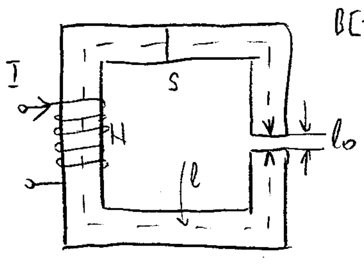
$\boxed{B \simeq 0{,}42\,T}$

$\boxed{H_0 = \frac{B_0}{\mu_0} = 337\,kA/m}$

(101) $S = 5\,cm^2$ ; $l = 250\,mm$

$l_0 = 0{,}1\,mm$ ; $N = 500$

$I = 5A$ ; $H, H_0 = ?$



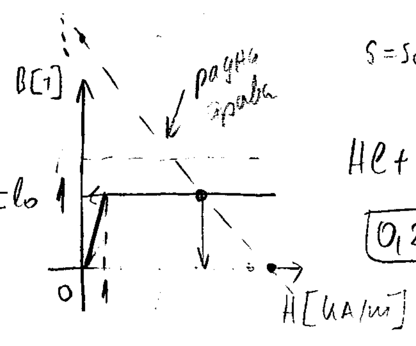
$\oint \vec{H}\,d\vec{l} = \Sigma I \Rightarrow Hl + H_0 l_0 = NI$

$-BS + B_0 S_0 = 0$

$S = S_0 \Rightarrow \boxed{B = B_0}$

$Hl + \frac{B}{\mu_0} l_0 = NI$

$\boxed{0{,}25 H + 79{,}577 B = 2500}$



$\boxed{B = 1T}$ $H = \frac{10000}{79{,}57} \simeq 9{,}68\,\frac{kA}{m}$

$\boxed{H_0 = \frac{B}{\mu_0} = 796\,kA/m}$

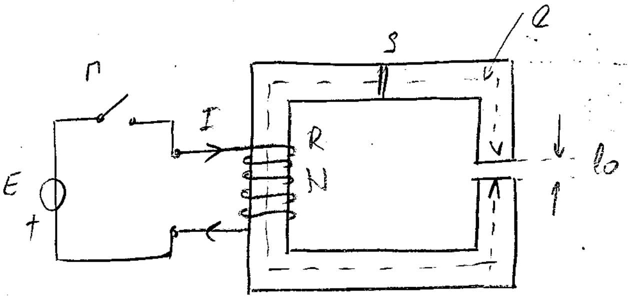
$B = 0 \Rightarrow H = 10000$

$H = 0 \Rightarrow B = \frac{2500}{80} = 31{,}25\,T$

*(102) $S = 1 cm^2$ $\quad N = 1000$

$l = 0,1 m$ $\quad R = 100\Omega$

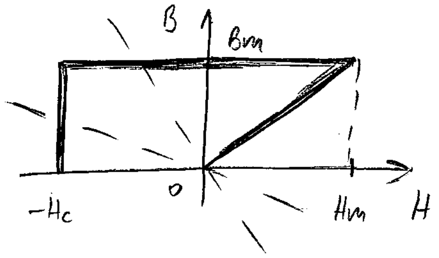
$l_0 = 1 mm$ $\quad E = 50V$



$1^\circ\ \vec{B} = \vec{H} = \vec{M} = 0$

$2^\circ\ I = \frac{E}{R} = 0,5A$

$$\left.\begin{array}{l}\frac{B_m}{H_m} = 0,001\frac{H}{m} \\ H_c = \frac{B_m}{0,001\frac{H}{m}}\end{array}\right\} \Rightarrow \boxed{H_c = H_m}$$



$$\Pi \overset{1^\circ}{(0)} \to \Pi \overset{2^\circ}{(3)} \to \Pi \overset{3^\circ}{(0)}$$

$B_0(3^\circ) = ?$ $\quad 2^\circ\ B = B_0\ (S = S_0)$

$$\oint_c \vec{H}d\vec{l} = \sum I$$

$Hl + H_0 l_0 = NI \quad (2^\circ)$

$Hl + \frac{B}{\mu_0} l_0 = NI \quad / : l$

$$H + \frac{B \cdot 10^{-3}}{0,1 \cdot 10^{-7} \cdot 4\pi} = \frac{1000 \cdot 0,5}{0,1}$$

$\boxed{H + 7957,74\, B = 5000}\ / : H$

ако је $H = H_m \Rightarrow B = B_m$

$$1 + 7957,74 \underbrace{\frac{B_m}{H_m}}_{0,001\frac{H}{m}} = \frac{5000}{H_m} \Rightarrow \boxed{H_m = 555\frac{A}{m}}$$

$\boxed{B_m = 0,555 T}$

$3^\circ\ I = 0$

$H \cdot l + H_0 l_0 = 0$

$\boxed{Hl + \frac{B_0}{\mu_0} l_0 = 0}$

једначина праве

за $B > 0$, $H < 0$; тада права мора сећи или праву $B = B_m$ или $H = -H_c$

- претпоставимо да сече $H = -H_c$.

- тада је $H = -H_c = -H_m$

$$B_0 = \frac{-(-H_c) l \mu_0}{l_0} = \frac{H_m l \mu_0}{l_0} \approx 69,74\ mT$$

- претпоставимо да сече $B = B_m$

- тада је $B = B_m$:

$$H = -\frac{B_m l_0}{\mu_0 l} = -4416,54\frac{A}{m} < -H_c,$$ што је немогуће

Дакле $B_0$ је $\underline{69,74\ mT}$.

(103) $l = 20\,cm$

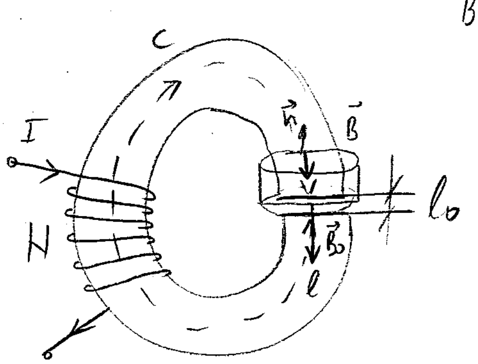
$l_0 = 2\,mm$

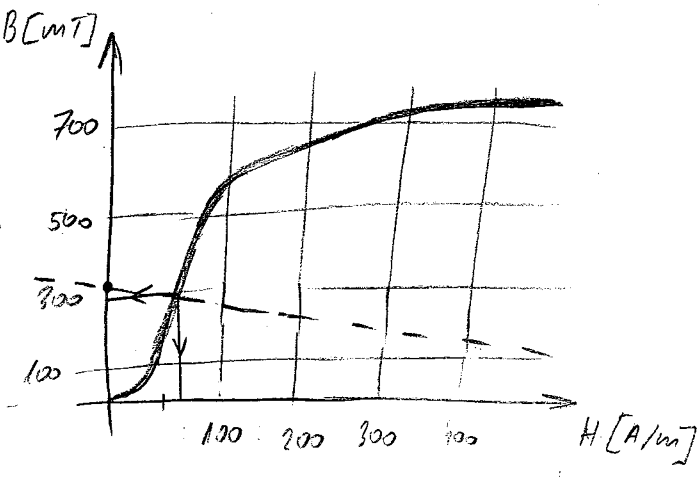
$l_0 << l$

$N = 250$

$I = 2A$

$B, B_0, H, H_0 = ?$




$$\oint_C \vec{H}\,d\vec{l} = \sum I$$

$$Hl + H_0 l_0 = NI$$

$$\oint_S \vec{B}\,d\vec{s} = 0$$

$$-BS + B_0 S_0 = 0$$

$$S = S_0 \Rightarrow \boxed{B = B_0}$$

$$H \cdot l + \frac{B}{\mu_0} l_0 = NI \quad / : l$$

$$H + \frac{B\, l_0}{\mu_0 l} = \frac{NI}{l}$$

$$H + B\frac{2 \cdot 10^{-3}}{4\pi \cdot 10^{-7}\, 0{,}2} = \frac{250 \cdot 2}{0{,}2}$$

$$H + 8000\,B = 2500$$

$$H = 0 \Rightarrow B = \frac{2500}{8000} = 0{,}3125$$

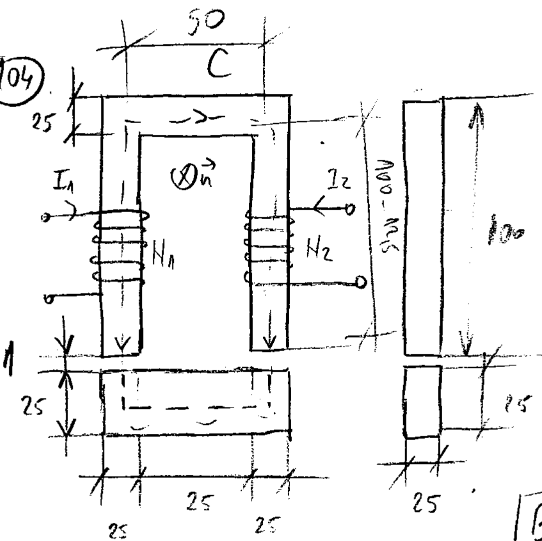
$$B = 0 \Rightarrow H = 2500$$

$$\boxed{B \approx 0{,}3T} \qquad \boxed{H \approx 60\,A/m}$$

$$B_0 = B \qquad H_0 = \frac{B_0}{\mu_0} = \frac{0{,}3}{4\pi} \cdot 10^7 = \underline{\underline{238\,kA/m}}$$

$$\boxed{B_0 = 0{,}3T}$$

(104)




$N_1 = 40$ $\quad N_2 = 100$

$I_1 = 8A$ $\quad I_2 = 0$

$$\oint \vec{H}\,d\vec{l} = \sum I$$

$$H\left[(100 - 12{,}5) \cdot 2 + 50 + 2 \cdot 12{,}5 + 50\right] +$$

$$+ H_0\left[2 \cdot 1\right] = N_1 I_1 - N_2 I_2$$

$$0{,}3\,H + 0{,}002\,H_0 = 320A - 100 I_2$$

$$\boxed{B_0 = 1{,}2T}$$

$$H_0 = \frac{B_0}{\mu_0} = 955\,kA$$

H je sa grafika:

$(B = B_0 = 1{,}2T)$

$H = 800\ A/m$

$$0{,}3 \cdot 800 + 0{,}002 \cdot 955000 = 320 - 100 I_2$$

$$100 I_2 = 320 - 2150$$

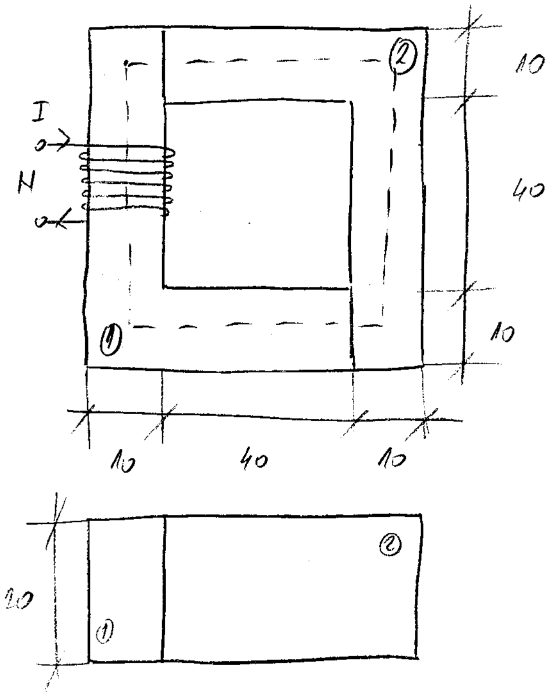
$$\boxed{I_2 = -18{,}3\,A}$$

(105) $N = 200$

$I = 0{,}15A$

---

$\phi = ?$




| $\phi$ [μWb] | B [mT] | $H_1$ [A/m] | $H_2$ [A/m] | NI [A] |
| --- | --- | --- | --- | --- |
| 0 | 0 | 0 | 0 | 0 |
| 10 | 50 | 35 | 75 | 11 |
| 20 | 100 | 50 | 95 | 14,5 |
| 30 | 150 | 62 | 108 | 17 |
| 40 | 200 | 72 | 120 | 19,2 |
| 50 | 250 | 80 | 132 | 21,2 |
| 60 | 300 | 95 | 144 | 23,9 |
| 70 | 350 | 130 | 157 | 28,7 |
| 80 | 400 | 202 | 177 | [illegible] |
| 90 | 450 | 320 | 200 | [illegible] |
| 100 | 500 | 483 | 228 | 71,1 |

$$\oint_C \vec{H}\, d\vec{l} = \sum I$$

$$H_1 l_1 + H_2 l_2 = NI$$

$$l_1 = l_2 = 100\ mm$$

$$S = 200\ mm^2 = 200 \cdot 10^{-6}\ m^2$$

$$BS = \phi$$

$$B = \frac{\phi}{S}$$

$$\phi = 72\ \mu Wb$$

(106) $\mu_{r1} = 1200$

$\mu_{r2} = 800$

$N_1 = 300$

$N_2 = 400$

$I_1 = 0{,}3A$

$I_2 = 0{,}5A$

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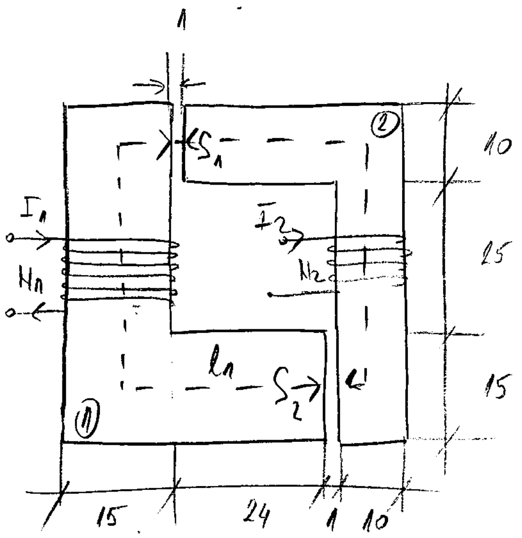


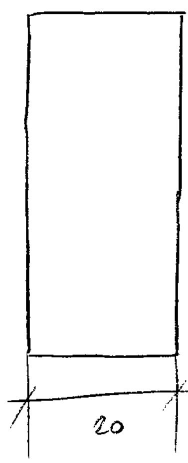


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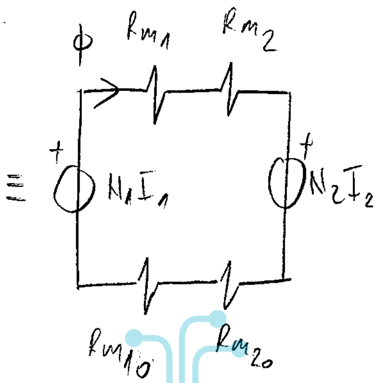



$$R_m = \frac{l}{\mu S}$$

$S_1 = 200\,mm^2$, $S_2 = 300\,mm^2$; $l_{01} = l_{02} = 1mm$

$$R_{m10} = \frac{l_{01}}{\mu_0 S_1} \qquad R_{m1} = \frac{l_1}{\mu_{r1}\mu_0 S_2}$$

$$R_{m20} = \frac{l_{02}}{\mu_0 S_2} \qquad R_{m2} = \frac{l_2}{\mu_{r2}\mu_0 S_1}$$

$$l_1 = 15 + 24 + 30 + 7{,}5 = 76{,}5\,mm$$

$$l_2 = 5 + 37{,}5 + 5 + 24 = 71{,}5\,mm$$

$$\phi = \frac{N_1 I_1 - N_2 I_2}{R_{m1} + R_{m2} + R_{m10} + R_{m20}} \qquad \underline{\phi \approx -15{,}37\,\mu Wb}$$

** ~~107~~ ** ~~108~~ ** ~~109~~ ** ~~110~~ ** ~~111~~ ** ~~112~~

# 3. Променљиво електромагнетно поље

(113) $\vec{E}_{ind} = \vec{v} \times \vec{B}$

$$e_{ind\,dyn} = \oint_C \vec{E}_{ind}\, d\vec{l} = \underline{\oint_C (\vec{v} \times \vec{B})\, d\vec{l}}$$

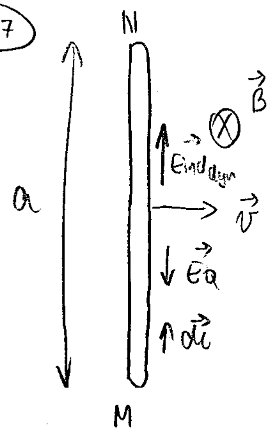
$$e_{ind} = -\frac{d\Phi}{dt} = -\frac{d\left(\int_S \vec{B}\, d\vec{s}\right)}{dt} \qquad \text{стати} \rightarrow s = const\,,\ v = 0$$

$$e_{ind} = \oint_C \vec{E}\, d\vec{l} \qquad \boxed{e_{ind} = -\int_S \frac{d\vec{B}}{dt}\, d\vec{s}}$$

$$\boxed{e_{ind} = -\int_S \frac{d\vec{B}}{dt}\, d\vec{l} + \oint_C (\vec{v} \times \vec{B})\, d\vec{l}}$$

(114), (115), *(116) → прочитати

(117)



$\vec{E}_{ind\,dyn} = \vec{v} \times \vec{B}$ ; $E_{ind\,dyn} = vB$

$\vec{E}_Q = -\vec{E}_{ind\,dyn}$ ; $E_Q = vB$

$$V_M - V_N = \int_M^N \vec{E}_Q\, d\vec{l} = -E_Q \int_M^N dl = \underline{\underline{-vBa}}$$

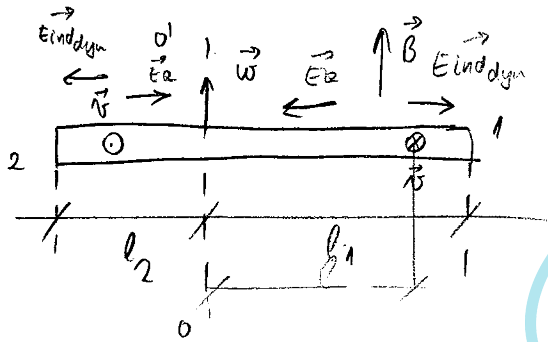
(118) $\omega = 1000\ rad/s$

$\vec{B} = \vec{const}$ $\quad B = 0{,}1T$

$l_1 = 0{,}05m\,,\ l_2 = 0{,}02m$

a) $e_{ind} = ?$

б) $V_1 - V_2 = ?$



$\vec{E}_{ind\,dyn} = \vec{v} \times \vec{B}$

$E_{ind\,dyn} = vB = \omega l B$

$E_Q = \omega l B$

$$e_{ind} = \oint_C \vec{E}\, d\vec{l} = -\int_{-l_2}^{0} E_Q(-dl) + \left[-\int_0^{l_1} (-E_Q)\, dl\right]$$

$$\boxed{e_{ind\,dyn} = \frac{1}{2}\omega B\left(l_1^2 - l_2^2\right)}$$

$$V_1-V_2=\oint_C E_Q\,dl = \int_{l_2}^{0}\vec{E_Q}\,d\vec{l} + \int_{0}^{l_1}\vec{E_Q}\,d\vec{l} = \frac{1}{2}\omega(-l_2^2)B + \frac{1}{2}\omega B(l_1^2) =$$

$$= \frac{1}{2}\omega B(l_1^2-l_2^2)$$

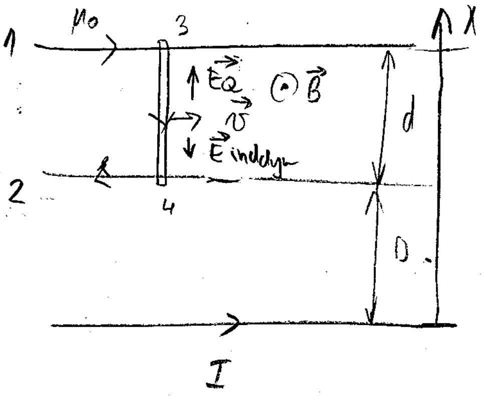
**119)** $I = 200\,A$

$D = 50\,mm$

$d = 20\,mm$

$v = 36\,\frac{km}{h} = 3{,}6\,m/s$

$V_1 - V_2 = ?$




$\vec E_{inddyn} = \vec v \times \vec B \; (-vB)$

$E_Q = vB$

$B = \frac{\mu_0 I}{2\pi x}$

$$V_1 - V_2 = \int_1^2 \vec{E_Q}\,d\vec l$$

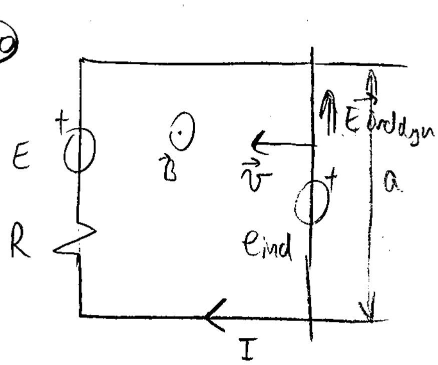
$\vec{E_Q}$ и $d\vec l$ су супротног смера

$dl$ је супротно од $dx$

$$V_1 - V_2 = \cancel{\int_1^3 \vec{E_Q}\,d\vec l} + \int_3^4 \vec{E_Q}\,d\vec l + \cancel{\int_4^2 \vec{E_Q}\,d\vec l} = \int_3^4 -vB\cdot dl = -v\int_{D+d}^{D}\frac{\mu_0 I}{2\pi}\frac{(-dx)}{x}$$

$$V_1 - V_2 = -\frac{\mu_0 I}{2\pi}v\ln\frac{D+d}{D} = \frac{\mu_0 I}{2\pi}v\ln\frac{D}{D+d}$$

***120)**




а) $P_m = F_m v = \frac{E-vBa}{R}vBa$

$\vec F_m = \int_l I\,d\vec l\times\vec B = IaB = \frac{E-vBa}{R}Ba$

$I = \frac{E-e}{R} = \frac{E-vBa}{R}$

$\vec E_{inddyn} = \vec v\times\vec B$

$e_{ind} = \int_l \vec E_{inddyn}\times d\vec l = vBa$

$F_{meh} = 0 \Rightarrow \boxed{I=0}$

$E = e_{ind} = v_0 Ba$

б) $F_{meh}$ само савлађује $F_m$

$$P_m = \frac{1}{R}(EvBa - v^2B^2a^2)$$

$$P_m = P_{m\,max} \Leftrightarrow P_m' = 0$$

$$P_m' = \frac{1}{R}(EBa - 2B^2a^2v) = 0 \Rightarrow$$

$$2B^{\cancel{2}}a^{\cancel{2}}v = E\cancel{Ba}$$

$$\boxed{v = \frac{E}{2Ba}} = \frac{v_0\cancel{Ba}}{2\cancel{Ba}} = \frac{v_0}{2}$$

$F_{meh} = \frac{E-vBa}{R}Ba$

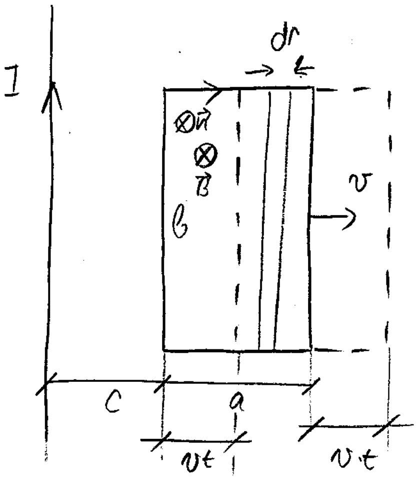
$\boxed{F_{meh} = 0{,}5\,N}$

$\boxed{P_m = 2{,}5\,W}$

(121) $I, a, b, v, c$

$e_{ind} = ?$

$e_{ind} = -\frac{d\phi}{dt}$




$B = \frac{\mu_0 I}{2\pi r}$

$\phi = \int \vec{B}\,d\vec{s} = \int\limits_{c+vt}^{c+a+vt} \frac{\mu_0 I}{2\pi r}\, b\,dr$

$\phi = \frac{\mu_0 I b}{2\pi} \ln \frac{c+a+vt}{c+vt}$

$e_{ind} = -\frac{d}{dt}\left(\frac{\mu_0 I b}{2\pi}\left(\ln(c+a+vt) - \ln(c+vt)\right)\right) =$

$= -\frac{\mu_0 I b}{2\pi}\left(\frac{v}{c+a+vt} - \frac{v}{c+vt}\right) = -\frac{\mu_0 I b}{2\pi} v \frac{-a}{(c+a+vt)(c+vt)}$

$$\boxed{e_{ind} = \frac{\mu_0 I b a v}{2\pi} \frac{1}{(c+a+vt)(c+vt)}}$$

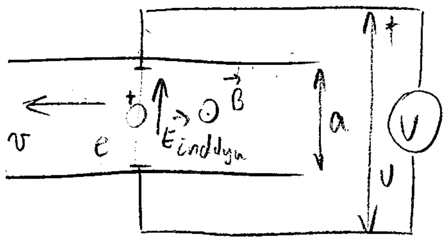
(122) $\sigma = 1\ mS/m$

$B = 10\ mT$

$a = 10\ mm$

$U = 2\ mV$

$v = ?$



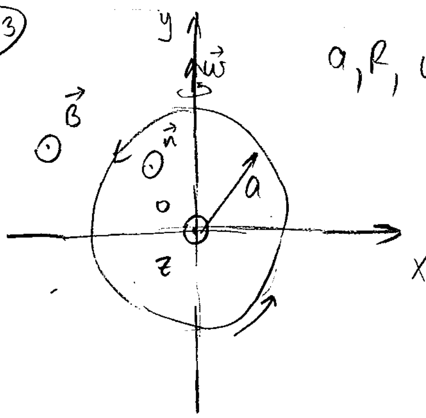

$U = e = \int\limits_{l} \vec{E}_{inddyn}\, d\vec{l} =$

$= vBa$

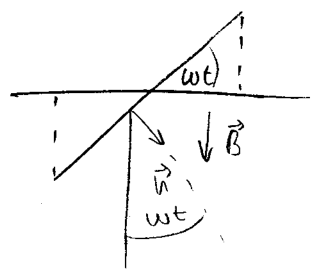
$U = vBa \Rightarrow v = \frac{U}{Ba}$

$v = \frac{2\cdot 10^{-3}}{10\cdot 10^{-3}\cdot 10\cdot 10^{-3}} = 20\ m/s$

(123)




$a, R, \omega, B;\ P_J = ?$




$\phi(t) = BS\cos\omega t =$

$= Ba^2\pi \cos\omega t$

$e_{ind} = -\frac{d\phi}{dt} = Ba^2\pi\omega \sin\omega t$

$i(t) = \frac{e_{ind}(t)}{R}$

$P_J = i^2(t) R = \frac{1}{R}\left(Ba^2\pi\omega \sin\omega t\right)^2 = \frac{1}{R} B^2 a^4 \pi^2 \omega^2 \sin^2\omega t$

период је $\frac{\pi}{\omega}$

$P_J = \frac{1}{T}\int\limits_0^T p_J(t)\,dt = \frac{B^2 a^4 \pi^2 \omega^2}{R} \frac{1}{\frac{\pi}{\omega}} \int\limits_0^{\frac{\pi}{\omega}} \sin^2\omega t$

$$\int_0^{\pi/\omega} \sin^2 \omega t \, dt = \frac{1}{\omega}\int_0^{\pi/\omega} \frac{1-\cos 2\omega t}{2} \, d(\omega t) = \frac{1}{2\omega}\left[ \omega t \Big|_0^{\frac{\pi}{\omega}} - \frac{1}{2}\sin 2\omega t \Big|_0^{\frac{\pi}{\omega}} \right] =$$

$$= \frac{1}{2\omega}\left[\pi - \frac{1}{2}\cancel{\sin 2\pi}\right] = \frac{\pi}{2\omega}$$

$$P_J = \frac{B^2 a^4 \pi^2 \omega^2}{R} \, \frac{\cancel{\omega}}{\cancel{\pi}} \, \frac{\cancel{\pi}}{2\cancel{\omega}}$$

$$\boxed{P_J = \frac{B^2 a^4 \pi^2 \omega^2}{2R}}$$

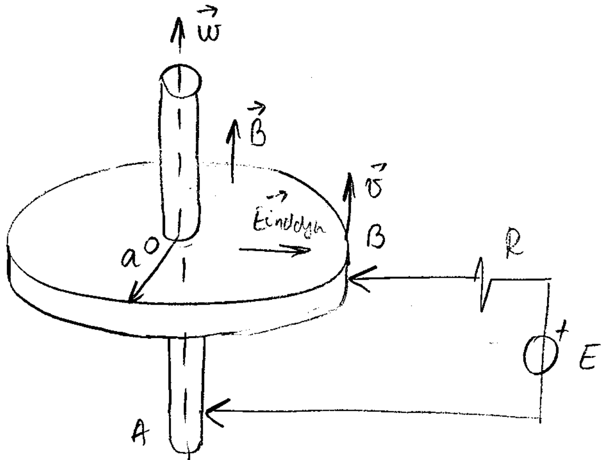
(124) $a = 20\,mm$

$E = 500\,mV$

$R = 1\,\Omega$

$B = 1\,T$

$\omega = ?$



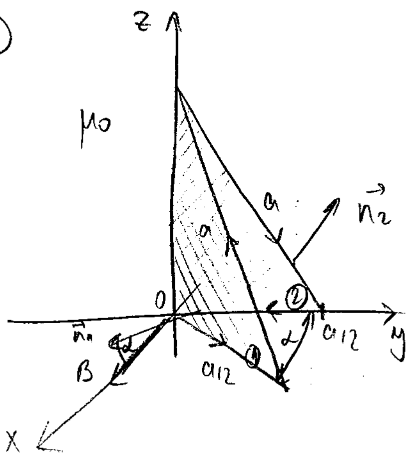

$$e_{ind} = \int_A^B \vec{E}_{inddyn} \cdot \vec{dl} = \cancel{\int_A^0 \vec{E}_{inddyn} \cdot \vec{dl}} + \int_0^B \vec{E}_{inddyn} \cdot \vec{dl}$$

$$e_{ind} = \int_0^B \vec{E}_{inddyn} \, \vec{dl} = B\int_0^a \omega r \, dr = \underline{\underline{B\omega \frac{a^2}{2}}}$$

$$i = \frac{E - e_{ind}}{R} = 0 \text{ (у празном ходу)} \qquad E = e_{ind} = B\omega\frac{a^2}{2}$$

$$\omega = \frac{2E}{Ba^2} = \underline{\underline{2500\,rad/s}}$$

(125)




$$a, \alpha, \vec{B}(t) = B_m \cos \omega t \, \vec{i}_x$$

$$\phi = \int_S \vec{B} \cdot \vec{S}$$

$$\phi_2 = \frac{a^2\sqrt{3}}{4}\,\frac{1}{2}\,B(t)\,(-1)$$

$$\phi_1 = \frac{a^2\sqrt{3}}{4}\cdot\frac{1}{2}\,B(t)\cos\alpha$$

$$\phi = \phi_1 + \phi_2 = B(t)\,\frac{a^2\sqrt{3}}{8}\,(\cos\alpha - 1)$$

$$e_{ind} = -\frac{d\phi}{dt} = \frac{a^2\sqrt{3}}{8}(\cos\alpha - 1)\left[-\frac{d}{dt}\left(B_m \cos \omega t\right)\right]$$

$$\boxed{e_{ind} = B_m \omega \frac{a^2\sqrt{3}}{8} \sin\omega t \,(\cos\alpha - 1)}$$

(126) $b = 40cm$

$a = 2\,cm$

$N = 400$

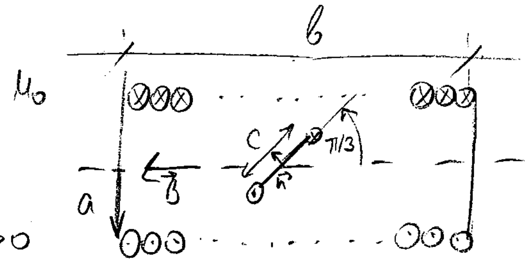
$i = I_0 e^{-\frac{t}{\tau}}, \; t>0$

$I_0 = 4A$

$\tau = 150\mu s$

$c = 1cm$

$e_{ind} = ?$



$c << b \Rightarrow B = \mu_0 i N'$

$$B = \frac{\mu_0 N}{b} I_0 e^{-\frac{t}{\tau}}$$

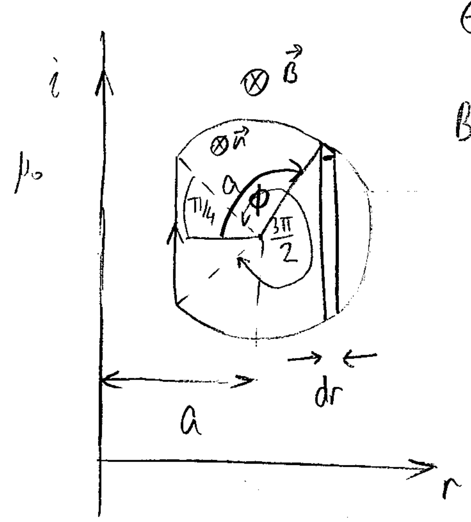
$$e = -\frac{d\phi}{dt} \; ; \; \phi(t) = \int_S \vec{B}\, d\vec{s}$$

$$\phi = \int_S \vec{B}\cdot d\vec{s} = \int_S B ds \cos(180^\circ - 90^\circ - 60^\circ) = \frac{\sqrt{3}}{2} B c^2$$

$$e_{ind} = -\frac{\mu_0 N I_0}{b} c^2 \frac{\sqrt{3}}{2}\left(-\frac{1}{\tau}\right) e^{-\frac{t}{\tau}}$$

$$\boxed{e_{ind} = \frac{\mu_0 N I_0 c^2 \sqrt{3}}{2 b \tau} e^{-\frac{t}{\tau}}} \qquad \underline{\underline{e_{ind} = 2{,}902 \cdot 10^{-3} e^{-\frac{t}{\tau}} \, V}}$$

(127) $i(t) = I_m \sin\omega t$



$$e_{ind} = -\frac{d\phi}{dt}$$

$$B = \frac{\mu_0 i}{2\pi r} \qquad \phi = \int_S \vec{B}\, d\vec{s} = \int \frac{\mu_0 i}{2\pi r}$$

$$ds = 2a\sin\varphi \, \cancel{dr}^{\,a\sin\varphi\, d\varphi} = 2a^2 \sin^2\varphi \, d\varphi$$

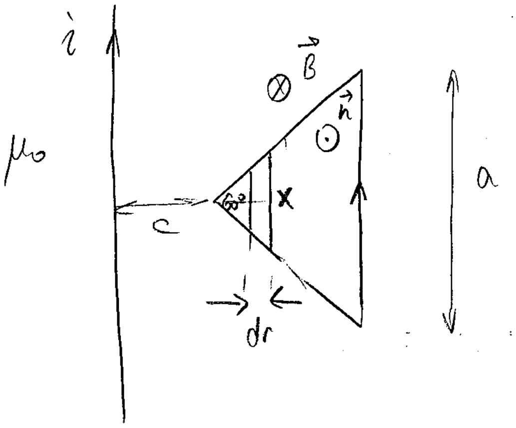
$$r = a - a\cos\varphi$$

$$\phi = \frac{\mu_0 i}{\cancel{2}\pi} \int_{\pi/4}^{\pi} \frac{\cancel{2}a^{\cancel{2}} \sin^2\varphi\, d\varphi}{\cancel{a}(1-\cos\varphi)} = \frac{\mu_0 i a}{\pi} \int_{\pi/4}^{\pi} \frac{\sin^2\varphi\, d\varphi}{1-\cos\varphi}$$

$$\phi = \frac{\mu_0 i a}{\pi} \int_{\pi/4}^{\pi} \frac{\cancel{(1-\cos\varphi)}(1+\cos\varphi)}{\cancel{1-\cos\varphi}} d\varphi = \frac{\mu_0 i a}{\pi}\left[\pi - \frac{\pi}{4} + \sin\pi - \sin\frac{\pi}{4}\right] =$$

$$= \frac{\mu_0 i a}{\pi}\left(\frac{3\pi}{4} - \frac{\sqrt{2}}{2}\right) \qquad \boxed{e_{ind} = -\frac{\mu_0 a}{\pi}\left(\frac{3\pi}{4} - \frac{\sqrt{2}}{2}\right) I_m \omega \cos\omega t}$$

(128) $a, c$ $\quad e_{ind} = ? = -\frac{d\Phi}{dt}$



$$(r-c) = x\frac{\sqrt{3}}{2} \Rightarrow x = \frac{2(r-c)}{\sqrt{3}}$$

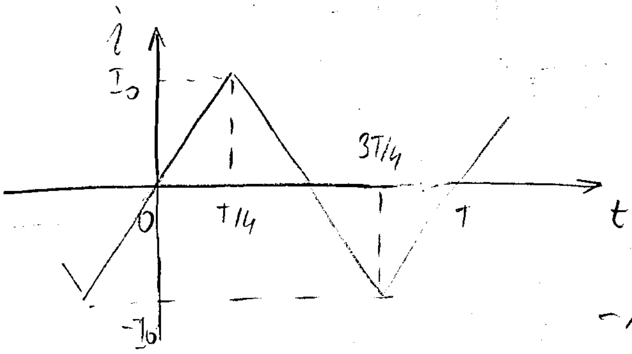
$$\Phi = \int_S \vec{B}\,d\vec{s} = -\int_c^{c+\frac{a\sqrt{3}}{2}} \frac{\mu_0 i}{2\pi r}\underbrace{x\,dr}_{ds}$$

$$\Phi = -\int_{r=c}^{c+\frac{a\sqrt{3}}{2}} \frac{\mu_0 i}{2\pi}\frac{2}{\sqrt{3}}\frac{r-c}{r}\,dr$$

$$\Phi = -\frac{2\mu_0 i}{2\pi\sqrt{3}}\left(\frac{a\sqrt{3}}{2} - c\ln\frac{c+\frac{a\sqrt{3}}{2}}{c}\right)$$

$$\Phi = -\frac{\sqrt{3}\mu_0 i}{3\pi}\left(\frac{a\sqrt{3}}{2} - c\ln\frac{2c+a\sqrt{3}}{2c}\right)$$

$$e_{ind} = -\frac{d\Phi}{dt} = +\frac{\sqrt{3}}{3}\frac{\mu_0}{\pi}\left(\frac{a\sqrt{3}}{2} - c\ln\frac{2c+a\sqrt{3}}{2c}\right)\frac{di}{dt}$$



$$\frac{di}{dt} = \frac{I_0}{T/4} = \frac{4I_0}{T}\ ; \quad t \in \left(0, \frac{T}{4}\right) \cup \left(\frac{3T}{4}, T\right)$$

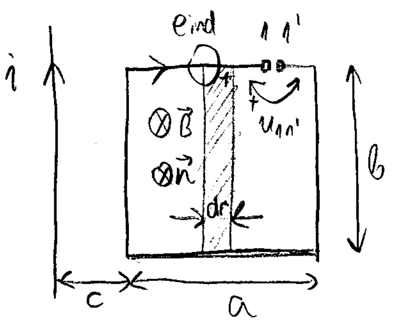
$$\frac{di}{dt} = -\frac{4I_0}{T}\ ; \quad t \in \left(\frac{T}{4}, \frac{3T}{4}\right)$$

– максимална $e_{ind}$ се добија за $\frac{di}{dt} = \frac{4I_0}{T}$

$$\boxed{e_{ind\,max} = \frac{4I_0\mu_0\sqrt{3}}{3\pi T}\left(\frac{a\sqrt{3}}{2} - c\ln\frac{2c+a\sqrt{3}}{2c}\right)}$$

(129) $i(t) = I_m \sin\omega t$

$U_{11'} = ?$



$$e_{ind} = -\frac{d\Phi}{dt} \qquad B = \frac{\mu_0 i}{2r\pi}$$

$$\Phi(t) = \int_S B\,ds = \int_{r=c}^{c+a}\frac{\mu_0 i}{2\pi r}b\,dr = \frac{\mu_0 i b}{2\pi}\ln\frac{c+a}{c}$$

$$e_{ind} = -\frac{\mu_0 b}{2\pi}\ln\frac{c+a}{c}\frac{di}{dt} = -\frac{\mu_0 b}{2\pi}\omega\ln\frac{c+a}{c}I_m\cos\omega t$$

$$\boxed{U_{11'} = e_{ind}}$$

(130) $a = 30\,mm$

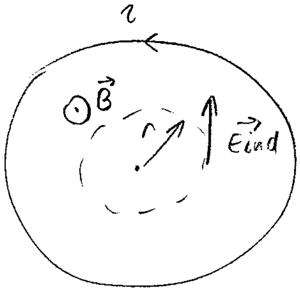
$N' = 1000\ 1/m$

$i(t) = I_m \cos\omega t$

$I_m = 5A\ ;\ \omega = 1000\,s^{-1}$

$E_{ind} = ?$

$$B = \mu_0 N' i = \underbrace{\mu_0 N' I_m}_{B_m}\cos\omega t = B_m \cos\omega t$$




$$\oint_C \vec{E}_{ind}\, d\vec{l} = -\frac{d\Phi}{dt}$$

1) $E_{ind}\cdot \cancel{2}r\cancel{\pi} = -\frac{d}{dt}\left(B r^{\cancel{2}} \cancel{\pi}\right)$

$$E_{ind} = \cancel{-}\frac{1}{2}\, r B_m \omega\,(\cancel{-}\sin\omega t) \Rightarrow$$

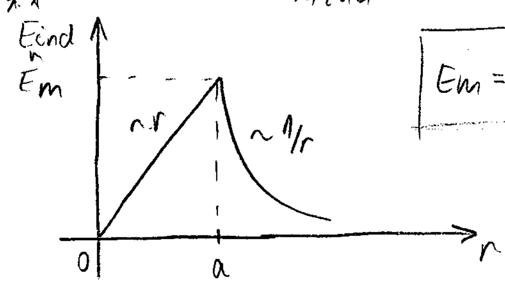
$$E_{ind} = \underbrace{\frac{1}{2}\mu_0 N' I_m r \omega}_{E_{m\,ind} \to \text{амплитуда}} \sin\omega t\ ,\ r<a$$

2) $r>a$

$$E_{ind}\cdot 2r\cancel{\pi} = -\frac{d}{dt}\left(B a^2 \cancel{\pi}\right)$$

$$E_{ind} = \underbrace{\frac{1}{2r}\, a^2 \mu_0 N' I_m \omega}_{E_{m\,ind}} \sin\omega t\ ,\ r>a$$

**

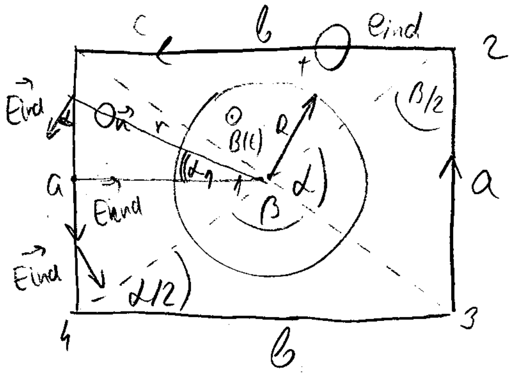



$$\boxed{E_m = \frac{1}{2}\mu_0 N' I_m a \omega \sin\omega t}$$

(131) $R, a, b\ ,\ B(t) = B_m \sin\omega t$

a) $e_{ind} = ?$

*б) $e_{ind}$ у страницама = ?




$$tg\frac{\alpha}{2} = \frac{a}{b}$$

$$\alpha = 2\,arctg\frac{a}{b}$$

$$tg\frac{\beta}{2} = \frac{b}{a}$$

$$\beta = 2\,arctg\frac{b}{a}$$

а) $e_{ind} = -\frac{d\Phi}{dt}$

$$\Phi = \int_S \vec{B}\, d\vec{s} = B\cdot R^2\pi = B_m R^2 \pi \sin\omega t$$

$$\frac{a}{b} = tg\frac{\alpha}{2}$$

$$\boxed{e_{ind} = -B_m R^2 \pi\,\omega \cos\omega t}$$

б) $e_{ind_a} = -\frac{d\Phi_{\Delta 123}}{dt}\qquad \Phi_{\Delta 123} = \frac{\alpha}{2\pi} B R^2 \pi$

$$e_{ind_a} = -\frac{\alpha}{2\pi} B_m R^2 \pi\,\omega \cos\omega t = \frac{\alpha}{2\pi} e_{ind}\ ;\ \alpha = 2\,arctg\frac{a}{b}$$

$e_{ind_b} = -\frac{d\Phi_{\Delta 134}}{dt}\qquad \Phi_{\Delta 134} = \frac{\beta}{2\pi} B R^2 \pi$

$$e_{ind_b} = -\frac{\beta}{2\pi} B_m R^2 \pi\,\omega \cos\omega t = \frac{\beta}{2\pi} e_{ind}\ ;\ \beta = 2\,arctg\frac{a}{b}$$

II начин: $e_{ind_a} = \oint_C \vec{E}_{ind} d\vec{l} = \int_1^3 \vec{E}_{ind}\, d\vec{l} + \int_3^2 \vec{E}_{ind}\, d\vec{l} + \int_2^1 \vec{E}_{ind}\, d\vec{l}$

$\sphericalangle(\vec{E}_{ind}, d\vec{l}) = 90^\circ$

$E_{ind} \cdot 2r\pi = -\frac{d}{dt}\left(B \cdot R^2\pi\right)$

$E_{ind} = \frac{-1}{2r} \cdot R^2 B_m \omega \cos\omega t$

$e_{ind_a} = \int -\frac{1}{2r} \cdot R^2 B_m \omega \cos\omega t \cdot dl \cdot \cos\alpha_1$, $\quad dl = r\cos\alpha_1 d\alpha_1$

$e_{ind_a} = -R^2 B_m \omega \cos\omega t \int_{-\alpha}^{\alpha} \frac{1}{2r} r \cos^2\alpha_1 d\alpha_1$

$$\int_{-\alpha}^{\alpha} \frac{1}{2}\cos^2\alpha_1 d\alpha_1 = \frac{1}{2}\int_{-\alpha}^{\alpha} \frac{1+\cos 2\alpha_1}{1} d\alpha_1 = \frac{1}{4}\left[2\alpha + \frac{1}{2}\sin 2\alpha_1\Big|_{-\alpha}^{+\alpha}\right] =$$

$$= \frac{\alpha}{2} + \frac{1}{8}\cancel{\left(\sin 2\alpha_1 - \sin(-2\alpha_1)\right)} = \frac{\alpha}{2}$$

$$e_{ind_a} = -R^2 B_m \omega\cos\omega t \cdot \frac{\alpha}{2}\cdot\frac{\pi}{\pi} = \underbrace{-R^2 B_m \pi\,\omega\cos\omega t}_{e_{ind}} \cdot \frac{\alpha}{2\pi}$$

$\boxed{e_{ind_a} = e_{ind}\frac{\alpha}{2\pi}}$ ; $\alpha = 2\operatorname{arctg} a/B$

*(132) $a = 5$ cm

$\mu_r = 1000$

$N' = 1000\,m^{-1}$

$i(t) = I\sqrt{2}\cos\omega t$

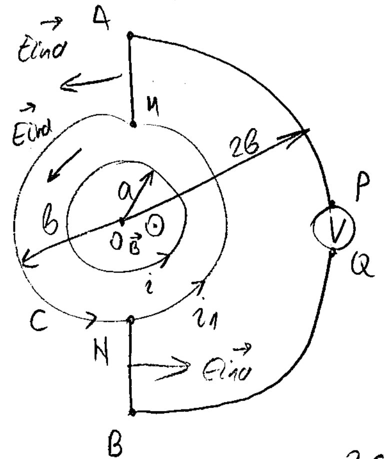
$I = 0{,}2$ A

$\omega = 100\,s^{-1}$

$b = 10$ cm

$s = 1\,mm^2$

$\sigma = 58$ MS/m




a) $B = \mu N' i = \mu_0\mu_r N' I\sqrt{2}\cos\omega t$

$r > a$; $E_{ind}\cdot 2r\pi = -\frac{d}{dt}(Ba^2\pi)$

$E_{ind} = \frac{1}{2r}\mu_0\mu_r N' I\sqrt{2}\omega a^2 \sin\omega t$

$J = \sigma E = \sigma(E_{ind} + E_q)$

- за контуру C $E_q = 0$ → нема нагомилавања наелектрисања

$J = \sigma E_{ind}(b)$

$i_1(t) = Js = \frac{\mu_0\mu_r N' I\omega a^2 s}{2b}\sqrt{2}\sin\omega t$

$i_1(t) = I_1\sqrt{2}\sin\omega t$ $\quad\boxed{I_1 \approx 18{,}??\,A}$

10 years and counting...

б) $V_P - V_Q = \int_P^Q \vec{E}_Q\, d\vec{l} = \int_P^M \vec{E}_Q\, d\vec{l} + \int_M^N \vec{E}_Q \cdot d\vec{l} + \int_N^Q \vec{E}_Q\, d\vec{l} = -\int_P^M \vec{E}_{ind}\, d\vec{l} - \int_N^Q \vec{E}_{ind}\, d\vec{l}$

отпорност волтметра је велика па је $i \approx 0$ $\Rightarrow \boxed{E_Q = -E_{ind}}$

$E_Q = 0$
(затворена контура кроз коју тече струја па нема нагомилавања наелектрисања)

– од A до M и од N до A је $\measuredangle(\vec{E}_{ind}, d\vec{l}) = 90°$ па је $\int \vec{E}_{ind}\, d\vec{l} = 0$

$$V_P - V_Q = \int_A^B \vec{E}_{ind} \cdot d\vec{l} = E_{ind}(2b) \cdot \pi \cdot 2b = \cancel{2\pi b} \cdot \frac{1}{\cancel{4b}_{2}} \mu_0 \mu_r N' I \omega a^2 \sqrt{2} \sin\omega t$$

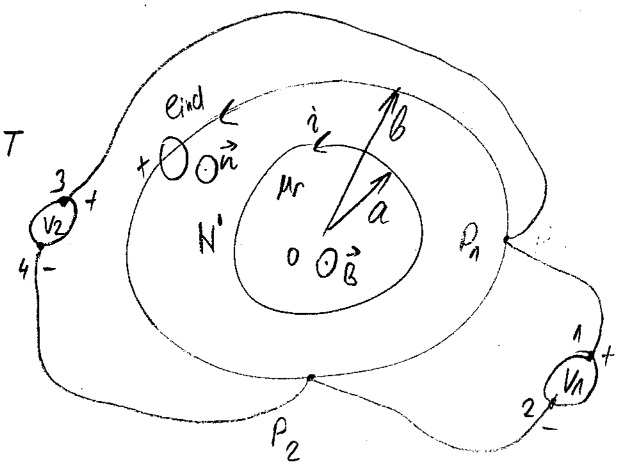
$$V_P - V_Q = \frac{1}{2} \mu_0 \mu_r \pi N' I \omega a^2 \sqrt{2} \sin\omega t = \underbrace{0{,}098}_{\text{ефективна вредност}} \sqrt{2} \sin\omega t \ V$$

$$\boxed{V = 0{,}1\,V}$$

*(133) $\mu_r$, $N'$

$$i(t) = \begin{cases} 0, & t < 0 \\ I_0 \frac{t}{T}, & 0 \le t \le T \\ I_0, & t > T \end{cases}$$

$b$, $R'$, $\measuredangle P_1 O P_2 = \pi/2$




$B = \mu_0 \mu_r N' i(t)$

$B(t) = \mu_0 \mu_r N' I_0 \frac{t}{T}$

$$e_{ind} = -\frac{d\Phi}{dt} = -\frac{d}{dt}(B a^2 \pi) = -\mu_0 \mu_r N' I_0 \frac{a^2 \pi}{T}$$

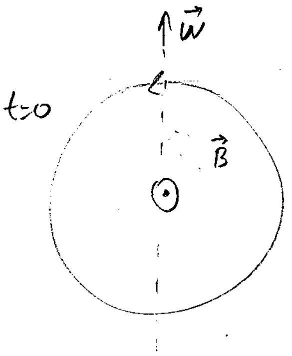
$$V_3 - V_4 = -\frac{3}{4} e_{ind} \qquad V_1 - V_2 = \frac{1}{4} e_{ind}$$

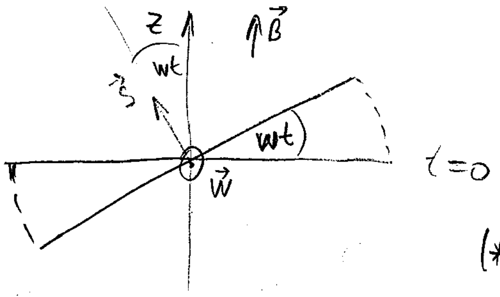
$$(V_3 - V_4) + (V_1 - V_2) = \frac{1}{2} e_{ind} = -\frac{1}{2} \mu_0 \mu_r N' I_0 \frac{a^2 \pi}{T}$$

$$(V_3 - V_4) - (V_1 - V_2) = -e_{ind} = \mu_0 \mu_r N' I_0 \frac{a^2 \pi}{T}$$

** ~~134~~ 135 $a, w, \vec{B}(t) = B_m \cos \omega t \, \vec{i_z}, \quad w = \omega$

$e_{ind}, e_{ind_{st}}, e_{ind_{dyn}} = ?$




$$e_{ind} = -\frac{d\phi}{dt} = -\frac{d}{dt}\int_S \vec{B}\,d\vec{s} = -\frac{d}{dt}\underbrace{(BS\cos Wt)}_{\phi(t)}$$

$$(*)\ \phi(t) = B_m a^2\pi \cos \omega t \cos Wt$$

$$\omega = W$$

$$\phi(t) = B_m a^2 \pi \cos^2 \omega t$$

$$e_{ind} = B_m a^2 \pi \, 2 \cos \omega t \sin \omega t \cdot \omega$$

$$\boxed{e_{ind} = B_m a^2 \pi \omega \sin 2\omega t}$$

– за $e_{ind_{st}}$ контура мирује па се за фактор $\cos Wt$ из (*) може узети да је константан приликом диференцирања

$$e_{ind_{st}} = -\frac{d\phi_{st}}{dt} = B_m a^2\pi \omega \sin \omega t \cos Wt = $$

$$= \frac{1}{2} B_m a^2 \pi \omega \sin 2\omega t$$

– на исти начин:

$$e_{ind_{dyn}} = -\frac{d\phi_{dyn}}{dt} = B_m a^2 \pi W \sin Wt \cos \omega t = \frac{1}{2} B_m a^2 \pi \omega \sin 2\omega t$$

136 $a = 100\ mm$

$R' = 10\ \Omega/m$

$\omega = 100\pi\ rad/s$

$B(t) = B_m \cos 2\pi f t$

$B_m = 0{,}1\,T;\ f = 50\,Hz$

$P_J = ?$

$\boxed{\omega = 2\pi f}$

$$R = R' \cdot 2a\pi = 6{,}28\,\Omega$$

$$i = \frac{e}{R} \qquad p = i^2 R = \frac{e^2}{R}$$

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}\int_S \vec{B}\,d\vec{s} = -\frac{d}{dt}(B a^2\pi \sin \omega t)$$

$$e = -\Big(B_m \cos(2\pi f t) a^2 \pi \cos(\omega t) \cdot \omega + B_m a^2 \pi \sin \omega t \,(-\sin 2\pi f t)\, 2\pi f\Big) =$$

$$= B_m a^2 \pi \omega \big(\underbrace{\cos^2 \omega t - \sin^2 \omega t}_{-\cos 2\omega t}\big)$$

$$\boxed{e = -B_m a^2 \pi \omega \cos 2\omega t}$$

$$p = \frac{1}{R} B_m^2 a^4 \pi^2 \omega^2 \cos^2 2\omega t$$

$$P_J = \frac{1}{T}\int_0^T p(t)\,dt = \frac{1}{R} B_m^2 a^4 \pi^2 \omega^2 \frac{1}{T}\int_0^T \cos^2 2\omega t\, dt$$

$$\frac{1}{T}\int_0^T \cos^2 2\omega t\,dt = \frac{1}{T}\int_0^T \frac{1+\cos 4\omega t}{2}\,dt = \frac{1}{T}\cdot\frac{1}{2}\left[\int_0^T dt + \frac{1}{4\omega}\int_0^T \cos 4\omega t\, d(4\omega t)\right]$$

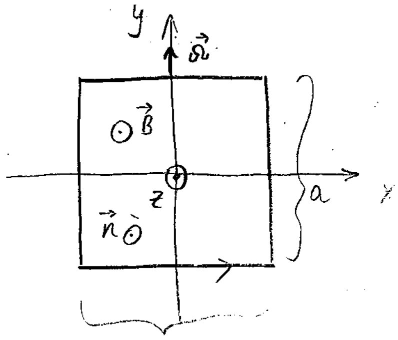
$$= \frac{1}{2T}\cdot\left[T + \frac{1}{4\frac{2\pi}{T}}\left(\cancel{\sin 4\frac{2\pi}{T}\cdot T}^{\,0} - \cancel{\sin 0}_{\,0}\right)\right] = \frac{1}{2T}\left[T + \frac{T}{8\pi}\cdot 0\right] = \frac{1}{2}$$

$$\boxed{P_J = \frac{B_m^2 a^4 \pi^2 \omega^2}{2R}} \qquad \boxed{P_J \approx 78\,mW}$$

(137) $a, \Omega, \vec{B}(t) = B_m \cos \omega t\, \vec{i}_z, R$

a) $e_{st} = ?$ $e_{dyn} = ?$

б) $\omega = \Omega$, $P_J = ?$




$$\phi = \int_S \vec{B}\, d\vec{S} = B a^2 \cos\Omega t = B_m a^2 \cos\omega t \cos\Omega t \quad a$$

$$e_{st} = -\frac{d\phi_{st}}{dt} = -B_m a^2 \cos(\Omega t)\,\omega\,(-\sin\omega t) = \underline{B_m a^2 \omega \cos\Omega t \sin\omega t}$$

$$e_{dyn} = -\frac{d\phi_{dyn}}{dt} = -B_m a^2 \sin(\omega t)\,\Omega\,(-\sin\Omega t) = \underline{B_m a^2 \Omega \cos\omega t \sin\Omega t}$$

б) $\omega = \Omega \Rightarrow e_{st} = e_{dyn}$

$$e = e_{st} + e_{dyn} = 2B_m a^2 \omega \sin(\omega t)\cos(\omega t) = B_m a^2 \omega \sin(2\omega t)$$

$$i = \frac{e}{R} \qquad p(t) = i^2(t) R = \frac{e^2(t)}{R} \qquad p(t) = \frac{B_m^2 a^4 \omega^2}{R}\sin^2(2\omega t)$$

$$P_J = \frac{1}{T}\int_0^T p(t)\,dt = \frac{B_m^2 a^4 \omega^2}{R}\underbrace{\frac{1}{T}\int_0^T \sin^2(2\omega t)\,dt}$$

$$\frac{1}{T}\int_0^T \sin^2(2\omega t)\,dt = \frac{1}{T}\int_0^T \frac{1-\cos(2\cdot 2\omega t)}{2}\,dt = \frac{1}{2T}\left[\int_0^T dt - \int_0^T \cos 4\omega t\, dt\right] =$$

$$= \frac{1}{2T}\left[T - \cancel{\frac{1}{4\omega}\sin 4\omega t\Big|_0^T}^{\,0}\right] = \underline{\frac{1}{2}}$$

$$\boxed{P_J = \frac{B_m^2 a^4 \omega^2}{2R}}$$

(138) $b = 1m$

$a = 5cm$

$N = 800$

$i(t) = I_m \cos \omega t$

$I_m = 1A$

$\omega = 2\pi f = 100\pi s^{-1}$

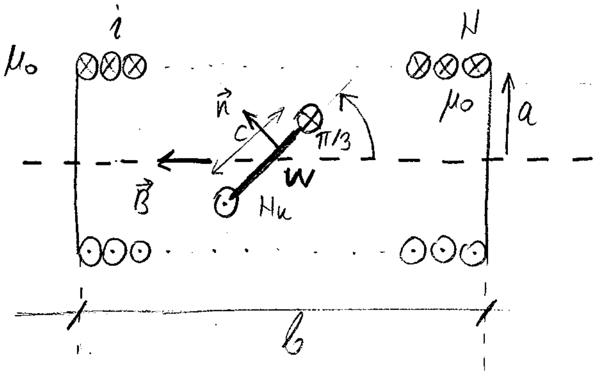
$f = 50 Hz$

$C = 1cm$

$N_u = 200$

$W = \omega$

$e_{ind} = ?$




$B = \mu_0 N' i = \mu_0 N' I_m \cos \omega t \quad ; N' = \frac{N}{b}$

$\Phi_j(t) = \int_S \vec{B} d\vec{s} = Bs \cos\left(\pi - \frac{\pi}{2} - \frac{\pi}{3} - \omega t\right) =$

$= \mu_0 N' I_m c^2 \cos \omega t \cos\left(\pi/6 - \omega t\right)$

$\phi(t) = N_u \phi_j(t)$

$e_{ind} = -\frac{d\phi}{dt} = \mu_0 N' I_m c^2 N_u$

$\cos\alpha \cdot \cos\beta = \frac{1}{2}\left[\cos(\alpha-\beta) + \cos(\alpha+\beta)\right]$

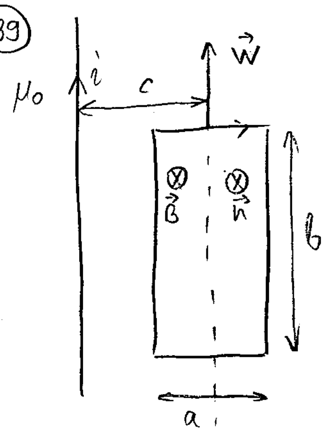
$\cos \omega t \cdot \cos\left(\pi/6 - \omega t\right) = \frac{1}{2}\left[\cos\frac{\pi}{6} + \cos\left(2\omega t - \frac{\pi}{6}\right)\right]$

$e_{ind} = \mu_0 N' I_m c^2 N_u \cdot \frac{1}{\cancel{2}} \sin\left(2\omega t - \frac{\pi}{6}\right) \cdot \cancel{2}\omega$

$$\boxed{e_{ind} = \underbrace{\frac{N}{b} \mu_0 N_u I_m c^2 \omega}_{E_m} \sin\left(2\omega t - \frac{\pi}{6}\right)}$$

$\boxed{E_m = 6,32 mV}$

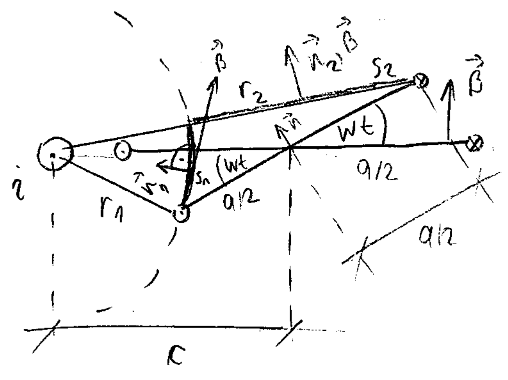
(139)




$i(t) = I_m \cos \omega t$

$a, b, W = \omega$

$e_{ind} = ?$




$r_2^2 = c^2 + \frac{a^2}{4} - 2\frac{a}{2}c\cos(\pi - \omega t)$

$r_1^2 = c^2 + \frac{a^2}{4} - 2\frac{a}{2}c\cos \omega t$

$r_2 = \sqrt{c^2 + \frac{a^2}{4} + ac\cos\omega t}$

$r_1 = \sqrt{c^2 + \frac{a^2}{4} - ac\cos\omega t}$

$\phi = \int_S \vec{B} d\vec{s} = \int_{S_1} \vec{B} d\vec{s} + \int_{S_2} \vec{B} d\vec{s}$

$\overset{0}{\measuredangle}(\vec{B}, d\vec{s}) = 90°$

$$\phi = \int_{r=r_1}^{r_2} \underbrace{\frac{\mu_0 i}{2r\pi}}_{B} \underbrace{b\,dr}_{ds} = \frac{\mu_0 i b}{2\pi} \ln\frac{r_2}{r_1} = \frac{\mu_0 b}{2\pi} I_m \cos(\omega t) \ln\frac{r_2(t)}{r_1(t)}$$

$$e_{ind} = -\frac{d\phi}{dt} \overset{!!}{=} (-)\frac{\mu_0 b}{2\pi} I_m \left[ \ln\frac{r_2}{r_1} \underbrace{\frac{d}{dt}(\cos\omega t)}_{A} + \cos(\omega t) \underbrace{\frac{d}{dt}\left(\ln\frac{r_2}{r_1}\right)}_{B} \right]$$

$$A = \frac{d}{dt}\cos\omega t \overset{!!}{=} (-)\omega \sin\omega t$$

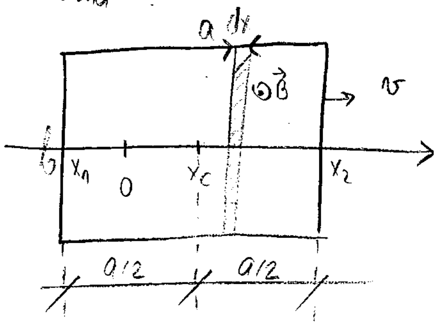
$$B = \frac{d}{dt}(\ln r_2 - \ln r_1) = \frac{-ac\omega \sin\omega t}{2\left(\sqrt{c^2+\frac{a^2}{4}+ac\cos\omega t}\right)^2} - \frac{+ac\omega\sin\omega t}{2\left(\sqrt{c^2+\frac{a^2}{4}-ac\cos\omega t}\right)^2} \overset{!!}{=} \frac{(-)ac\omega\sin\omega t}{2}\left(\frac{1}{r_2^2}+\frac{1}{r_1^2}\right)$$

$$\boxed{e_{ind} = \frac{\mu_0 b}{2\pi} I_m \left[ \ln\frac{r_2}{r_1}\,\omega \sin\omega t + \cos(\omega t)\frac{ac\omega \sin\omega t}{2}\left(\frac{1}{r_2^2}+\frac{1}{r_1^2}\right)\right]}$$

** ↓ $e_{ind_{st}}$ ** ↓ $e_{ind_{dyn}}$

(140) $a, b, v, B(x,t) = B_0 \cos\frac{\pi x}{a} \cos\frac{\pi v t}{a}$

$e_{ind} = ?$




$$\phi = \int_S \vec{B}\,d\vec{s} \qquad \phi(t) = \int_{x_1}^{x_2} B_0 \cos\frac{\pi x}{a} \cos\frac{\pi v t}{a} \underbrace{b\,dx}_{ds}$$

$$\phi(t) = B_0 \cos\left(\frac{\pi v t}{a}\right) b \frac{a}{\pi}\left(\sin\frac{\pi x_2}{a} - \sin\frac{\pi x_1}{a}\right)$$

$$x_2 = vt + \frac{a}{2} \qquad x_c = vt$$

$$x_1 = vt - \frac{a}{2}$$

$$\phi(t) = B_0 b \frac{a}{\pi} \cos\left(\frac{\pi v t}{a}\right)\left(\sin\left(\frac{\pi}{a}vt + \frac{\pi}{2}\right) - \sin\left(\frac{\pi}{a}vt - \frac{\pi}{2}\right)\right) =$$

$$= B_0 b \frac{a}{\pi} \cos\left(\frac{\pi v t}{a}\right)\left(\cos\frac{\pi v t}{a} - \left(-\cos\frac{\pi v t}{a}\right)\right)$$

$$\boxed{\phi(t) = 2 B_0 b \frac{a}{\pi} \cos^2\left(\frac{\pi v t}{a}\right)}$$

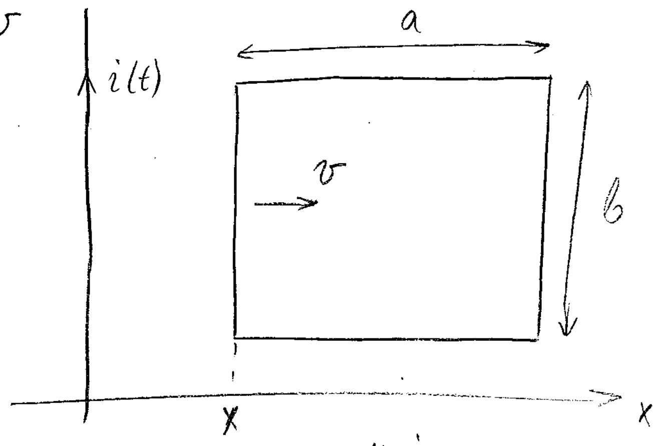
$$e_{ind} = -\frac{d\phi}{dt} = -2B_0 b \cancel{\frac{a}{\pi}}\, 2\cos\frac{\pi v t}{a}\left(-\sin\frac{\pi v t}{a}\right)\cdot \cancel{\frac{\pi v}{a}}$$

$$\boxed{e_{ind} = 2B_0 b v \sin\left(2\frac{\pi v t}{a}\right)}$$

(141) $i(t) = I_m \sin\omega t, \; a, b, v$

$x = x_0 = 5a$

$e_{ind_{st}}, \; e_{ind_{dyn}}, \; e_{ind} = ?$




$$e_{ind} = -\frac{d\phi}{dt}$$

$$\phi(t) = \int_S \vec{B}\, d\vec{s}$$

$$B = \frac{\mu_0 i}{2\pi x}$$

$$\phi(t) = \int_{x+vt}^{x+a+vt} \frac{\mu_0 i}{2\pi x}\, b\, dx$$

$$\phi(t) = \frac{\mu_0 I_m b}{2\pi} \sin\omega t \ln\frac{x+a+vt}{x+vt}$$

$$e_{ind_{st}} = \frac{\mu_0 I_m b}{2\pi} \ln\frac{x+a+vt}{x+vt}\left(-\frac{d}{dt}\sin(\omega t)\right) = \underline{\underline{\frac{-\mu_0 I_m b}{2\pi}\, \omega \cos\omega t \ln\frac{6a+vt}{5a+vt}}}$$

$$e_{ind_{dyn}} = \frac{\mu_0 I_m b}{2\pi} \sin\omega t \left[-\frac{d}{dt}\left(\ln(6a+vt) - \ln(5a+vt)\right)\right]$$

$$e_{ind_{dyn}} = \frac{\mu_0 I_m b}{2\pi} \sin\omega t \left[-\frac{v}{6a+vt} + \frac{v}{5a+vt}\right] = \underline{\underline{\frac{\mu_0 I_m b}{2\pi} \sin\omega t \, \frac{va}{(6a+vt)(5a+vt)}}}$$

$$\boxed{e_{ind} = e_{ind_{st}} + e_{ind_{dyn}}}$$

*(142) $a = 10\,mm$

$h = 1\,mm$

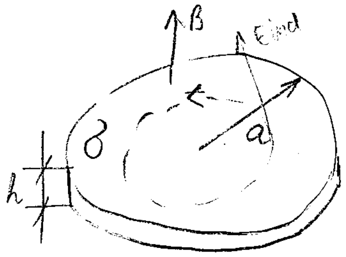
$B = B_m \cos\omega t$

$B_m = 2T$

$\omega = 2\pi f; \; f = 50\,Hz$

$\sigma = 10\,MS/m$

$P_J = ?$




$$\oint_C \vec{E}_{ind}\, d\vec{l} = -\frac{d\phi}{dt}$$

$$E_{ind} \cdot 2\cancel{r}\cancel{\pi} = -\frac{d}{dt}\left(B \cdot r^{\cancel{2}}\cancel{\pi}\right)$$

$$E_{ind} = \frac{1}{2} B_m \omega r \sin\omega t$$

$$\vec{J} = \vec{E}_{ind} \cdot \sigma$$

$$p_J = \vec{J} \cdot \vec{E} = \sigma E_{ind}^2 = \sigma \frac{1}{4} B_m^2 \omega^2 r^2 \sin^2\omega t$$

$$P_J = \int_V p_J\, dv = \underbrace{\int_{r=0}^{a} \frac{1}{4} \sigma B_m^2 \omega^2 r^2 \sin^2\omega t \, \underbrace{2\pi r h dr}_{dv}}_{P_J}$$

$$P_J = \frac{1}{4}\sigma B_m^2 \omega^2 \sin^2(\omega t)\, h \frac{a^4}{\cancel{4}_2}\cancel{\pi}, \quad P_J(t) = \frac{1}{8}\sigma B_m^2 \omega^2 h a^4 \pi \sin^2\omega t$$

$$P_{J_{sr}} = \frac{1}{T}\int_0^T P_J(t)\,dt = \frac{1}{8}\sigma B_m^2 \omega^2 h \pi a^4 \frac{1}{T}\int_0^T \sin^2\omega t\, dt$$

$$\frac{1}{T}\int_0^T \sin^2(\omega t)\,dt = \frac{1}{T}\int_0^T \frac{1-\cos 2\omega t}{2}\,dt = \frac{1}{2T}\left[T - \frac{1}{2\omega}\sin 2\omega t\Big|_0^T\right] = \frac{1}{2}$$

$$\boxed{P_{J_{sr}} = \frac{1}{16}\gamma B_m^2\omega^2 h\pi a^4}$$

*(143)

$a = 20\,mm$

$b = 1\,m$

$N = 1000$

$a_1 = 9\,mm$

$a_2 = 10\,mm$

$\gamma = 58\,MS/m$

$I = 10\,A$

$f = 50\,Hz$

$P_J = ?$

$$i = I\sqrt{2}\cos 2\pi f t$$

$$B \approx \mu_0 N' i = \mu_0 \frac{N}{b} I\sqrt{2}\cos 2\pi f t$$

$$\oint_C \vec{E}_{ind}\, d\vec{l} = -\frac{d\Phi}{dt}$$

$$\Phi = B S = B r^2\pi$$

$$E_{ind}\cancel{2r\pi} = \frac{\mu_0 I N\sqrt{2}}{b}\, r\cancel{\pi}\,\cancel{2\pi} f \sin(2\pi f t)$$

$$p_J = \vec{J}\,\vec{E} = \underbrace{\gamma E}_{J}\cdot E = \gamma E_{ind}^2$$

$$p_J = \gamma\frac{\mu_0^2 I^2 N^2 2}{b^2} r^2\pi^2 f^2\sin^2(2\pi f t)$$

$$P_J = \int_v p_J\,dv = \int_{a_1}^{a_2} p_J\cdot 2r\pi b\,dr$$

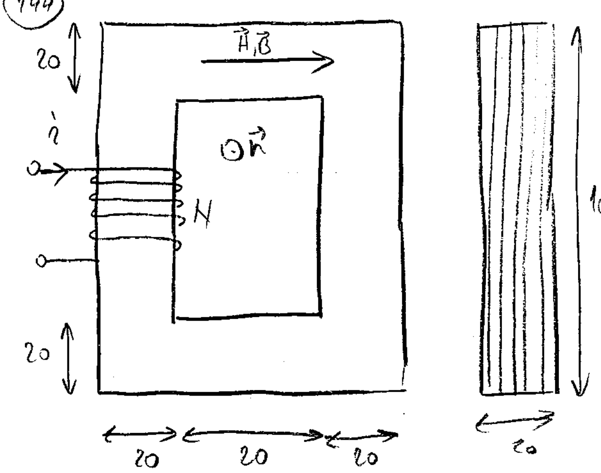
$$P_J = \gamma\frac{\mu_0^2 I^2 N^2 \cancel{4}\pi^3 f^2}{b}\sin^2(2\pi f t)\,\frac{a_2^4 - a_1^4}{\cancel{4}}$$

$$P_{J_{sr}} = \frac{1}{T}\int_0^T P_J(t)\,dt \qquad P_{J_{sr}} = \frac{1}{2}\frac{P_J(t)}{\sin^2(2\pi f t)}$$

$$\boxed{P_{J_{sr}} = \frac{\gamma\mu_0^2 I^2 N^2\pi^3 f^2}{2b}\left(a_2^4 - a_1^4\right)}$$

$$\underline{\underline{P_{J_{sr}} = 1{,}22\,W}}$$

*(144)




$\delta = 0{,}5\,mm$

$\mu_r = 400$

$\rho = 0{,}6\,\mu\Omega m$

$N = 600$

$I = 1\,A$

$f = 50\,Hz$

$P_J = ?$

$$H\cdot l = NI \Rightarrow H = \frac{NI}{l}$$

$$B = \mu_0\mu_r H = \mu_0\mu_r\frac{NI}{l}$$

$$B_m = B\sqrt{2} \simeq 1{,}76\,T$$

$$E_{ind} \cdot \cancel{2d} = -\frac{d}{dt}\left(B_m \cos 2\pi f t \cdot \cancel{2d} x\right)$$

$$E_{ind} = B_m\, 2\pi f \sin(2\pi f t)\, x$$

$$p_J = \delta E_{ind}^2 = \frac{1}{\rho} E_{ind}^2 = \frac{1}{\rho} 4\pi^2 f^2 \sin^2(2\pi f t)\, x^2 B_m^2$$

$$P_J = \int_V p_J\, dV = \frac{1}{\rho} 4\pi^2 f^2 \sin^2(2\pi f t) B_m^2 \int_{\delta} x^2 \cdot \underbrace{db\,dx}_{?}$$

$$P_J = \frac{1}{\rho} \cancel{4}\pi^2 f^2 \sin^2(2\pi f t)\, B_m^2\, db\, 2 \cdot \frac{\delta^3}{24_3}$$

$$P_J = \frac{1}{\rho}\pi^2 f^2 B_m^2 \frac{db\delta^3}{3} \sin^2(2\pi f t)$$

$$\boxed{P_{Jsr} = \frac{B_m^2 \pi^2 f^2 \delta^3 d b}{6\rho}}$$

$d = 20 \cdot 20 = 400\ mm^2 = 4\ cm^2$

(145) $N = 500$

$a = 30\ mm$

$b = 500\ mm$

$c = 20\ mm$

a) $L_{12} = ?$

б) $i(t) = I_m \cos \omega t$

$I_m = 10A$

$\omega = 2\pi f$ ; $f = 50 Hz$

$E = ?$

a) $B_1 = \mu_0 N' I_1$

$\Phi_2 = B_1 c^2 \pi$

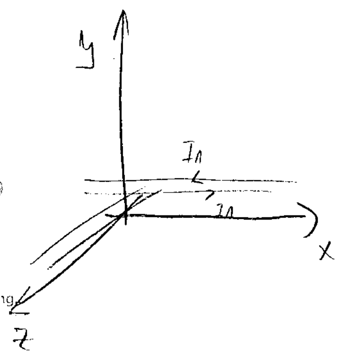
$L_{21} = \frac{\Phi_2}{I_1} = \mu_0 N' c^2 \pi$

$\boxed{L_{21} = \frac{1}{b} \mu_0 N c^2 \pi} \approx 1{,}58\ \mu H$

б) $e_{ind} = -L_{21}\frac{di}{dt} = L_{21} I_m \omega \sin \omega t =$

$= \frac{\mu_0 N c^2 \pi}{b} I_m \omega \sin \omega t$ $\Rightarrow \boxed{E = \frac{\mu_0 N c^2 \pi}{b} I_m \omega} \approx 5\ mV$

(146)




$\measuredangle B_1 ds = 90°$

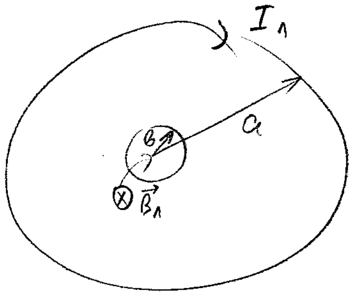
$\Downarrow$

$\Phi_2 = 0 \Rightarrow L_{21} = 0$

*(147) $a = 200m$

$b = 10 mm$

$L_{12} = ?$




$$B_1 = \frac{\mu_0 I_1 a^2}{2r^3} = \frac{\mu_0 I_1}{2a}$$

$b << a \Rightarrow B = const$ на целој [illegible] области полупречника $b$

$$\Phi_2 = B_1 b^2 \pi = \frac{\mu_0 I_1}{2a} b^2 \pi$$

$$L_{21} = \frac{\Phi_2}{I_1} = \frac{\mu_0 b^2 \pi}{2a}$$

$$\boxed{L_{21} = 987 pH}$$

*(148) $a = 20mm$

$d = 100mm$

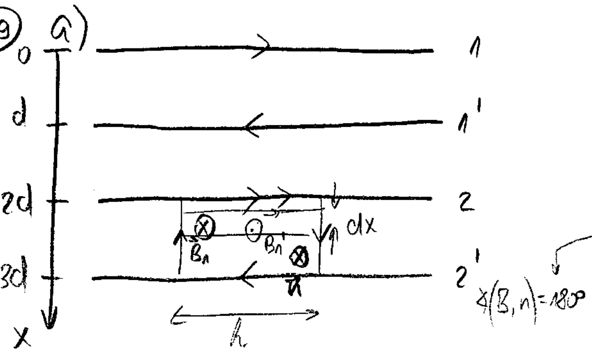
$L_{12} = ?$

$$B_1 = \frac{\mu_0 I_1 a^2}{2 r^3} \qquad r = \sqrt{a^2 + d^2}$$

$$B_1 \approx const \qquad \Phi_2 = B_1 a^2 \pi \qquad L_{21} = \frac{\Phi_2}{I_1}$$

$$\boxed{L_{21} \approx \frac{\mu_0 a^4 \pi}{2(a^2+d^2)^{3/2}}}$$

(149) a)



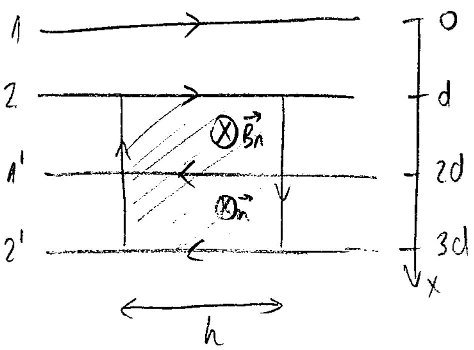

$$B_1 = B_1' - B_1 = \frac{\mu_0 I}{2\pi(x-d)} - \frac{\mu_0 I}{2\pi x}$$

$$\Phi_2 = -\int_{2d}^{3d} B_1 \, dx \cdot h = -\frac{\mu_0 I}{2\pi} h \left[ \ln \frac{3d-d}{2d-d} - \ln \frac{3d}{2d} \right]$$

$$\Phi_2' = \frac{\Phi_2}{h} = -\frac{\mu_0 I}{2\pi} \ln \frac{2}{\frac{3}{2}} = -\frac{\mu_0 I}{2\pi} \ln \frac{4}{3}$$

$$L_{12}' = \frac{\Phi_2'}{I} \qquad \boxed{L_{12}' = -\frac{\mu_0 I}{2\pi} \ln \frac{4}{3}}$$

б)



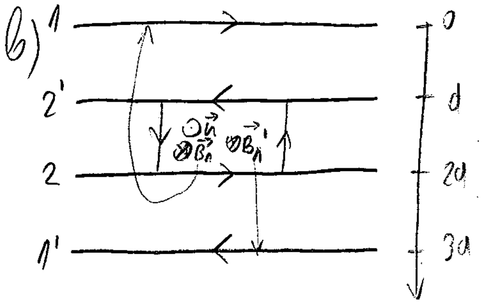

$$\Phi_2(B_1') = 0 \qquad \Phi_2 = \Phi_2(B_1) \qquad B_1 = \frac{\mu_0 I}{2\pi x}$$

$$\Phi_2 = \int_S \vec{B_1} d\vec{s} = + \int_{x=d}^{3d} \frac{\mu_0 I}{2\pi x} dx \cdot h$$

$$\Phi_2' = + \frac{\mu_0 I}{2\pi} \ln 3$$

$$\boxed{L_{21}' = \frac{\mu_0 \ln 3}{2\pi}}$$

b)




$$B = B_1 + B_1' = \frac{\mu_0 I}{2\pi x} + \frac{\mu_0 I}{2\pi(3d-x)}$$

$$\Phi_2 = -h\frac{\mu_0 I}{2\pi}\left(\ln x - \ln(3d-x)\right)\Big|_d^{2d}$$

$$\Phi_2 = -\frac{\mu_0 I}{2\pi} h\left[\ln 2 + \ln 2\right]$$

$$\Phi_2' = -\frac{\mu_0 I}{\pi}\ln 2$$

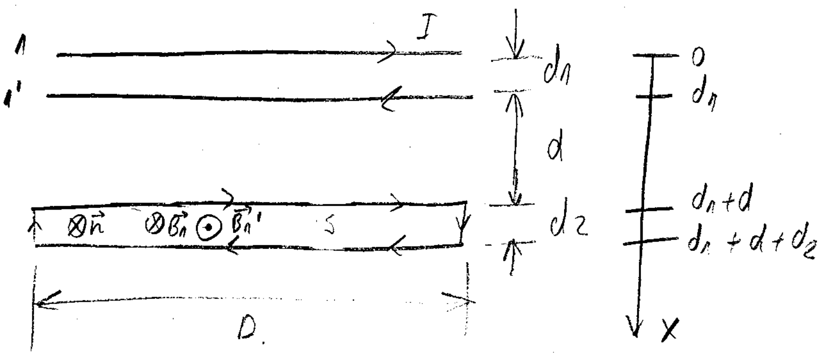
$$\boxed{L_{12}' = -\frac{\mu_0}{\pi}\ln 2}$$

159 $D = 20\,m$

$d_1 = 10\,mm$
$d_2 = 3\,mm$
$d = 50\,mm$

$I = 20\,A$, $f = 50\,Hz$

$E = ?$




$$e_{ind} = -L_{12}\frac{di}{dt}$$

$$i = I\sqrt{2}\cos 2\pi f t$$

$$B_1 = B_1 - B_1' = \frac{\mu_0 I}{2\pi x} - \frac{\mu_0 I}{2\pi(x-d_1)}$$

$$\Phi_2 = \int_S \vec{B}_1 d\vec{s} = \int_{x=d_1+d}^{d_1+d+d_2} B_1 D\,dx$$

$$\Phi_2 = D\frac{\mu_0 I}{2\pi}\left(\ln x - \ln(x-d_1)\right)\Big|_{d_1+d}^{d_1+d+d_2}$$

$$\Phi_2 = \frac{\mu_0 I D}{2\pi}\left[\ln\frac{d_1+d_2+d}{d_1+d} - \ln\frac{d+d_2}{d}\right] = \frac{\mu_0 I D}{2\pi}\ln\frac{d(d_1+d_2+d)}{(d_1+d)(d+d_2)}$$

$$L_{21} = L_{12} = \frac{\Phi_2}{I} = \frac{\mu_0 D}{2\pi}\ln\frac{d(d_1+d+d_2)}{(d_1+d)(d_2+d)}$$

$$e_{ind} = +\frac{\mu_0 D}{\cancel{2}\pi}\ln\frac{d(d_1+d_2+d)}{(d_1+d)(d_2+d)}\,I\sqrt{2}\cdot\cancel{2}\pi f\sin 2\pi f t$$

$$\boxed{E = \left|\mu_0 D I f\ln\frac{d(d_1+d_2+d)}{(d_1+d)(d_2+d)}\right|}$$

151) $\mu_r = 500$

$S = 2\,cm^2$

$l = 15\,cm$

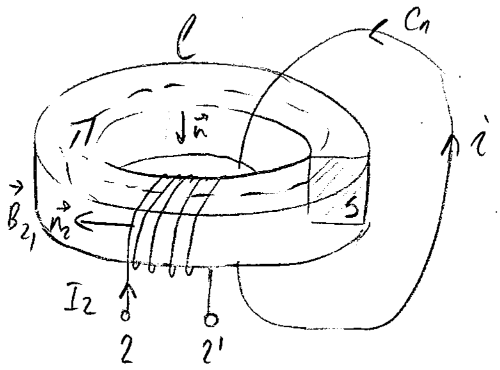
$N = 300$

$I = 100\,A$

$f = 50\,Hz$

$e_{ind} = ?$ ($E_{ind}$)

$i = I\sqrt{2}\cos(2\pi f t)$



$Hl = NI_2$

$B_2 = \mu H = \mu_0 \frac{NI_2}{l}\mu_r$

$\Phi_1 = \oint_{c_1} \vec{B_2}\,\vec{ds} = B_2 S$

$\Phi_1 = \mu_0 \frac{NI_2}{l} S \mu_r$

$L_{12} = \frac{\Phi_1}{I_2} = \frac{\mu_0\mu_r N S}{l}$

$$e_{ind} = -L_{12}\frac{di}{dt} = L_{12} I\sqrt{2}\,2\pi f \sin 2\pi f t$$

$$E_{ind} = L_{12} I \cdot 2\pi f = \frac{\mu_0\mu_r N S}{l} I\,2\pi f \qquad \boxed{E_{ind} = 7{,}9\,V}$$

* $E_{ind}' = N_n E_{ind}$ $(\Phi_1' = N_n \Phi_1 \ldots)$

152) $l = 400\,mm$

$S = 10\,cm^2$

$N = 1000$

$I = 500\,mA$

$\mu_r = 1000$

$L = ?$

$L = \frac{\Phi}{I}$

$\oint_c \vec{H}\vec{dl} = \sum I$

$Hl = NI \Rightarrow H = \frac{NI}{l}$

$B = \mu H = \mu_0\mu_r \frac{NI}{l}$

$\Phi = NBS = \mu_0\mu_r \frac{N^2 I S}{l}$

$L = \frac{\Phi}{I} \qquad \boxed{L = \mu_0\mu_r \frac{N^2 S}{l}}$

153) $N = 300$

$a = 20\,mm$

$b = 30\,mm$

$h = 25\,mm$

$L = ?$

$H \cdot 2r\pi = NI$

$H = \frac{NI}{2r\pi}$

$B = \mu_0\mu_r H = \mu_0\mu_r \frac{NI}{2r\pi}$

$$\Phi = N\int_S \vec{B}\vec{ds} = N\int_a^b \mu_0\mu_r \frac{NI}{2r\pi} \underbrace{h\,dr}_{ds} = \mu_0\mu_r \frac{N^2 I}{2\pi} h \ln\frac{b}{a}$$

$L = \frac{\Phi}{I} \qquad \boxed{L = \mu_0\mu_r h \frac{N^2 \ln\frac{b}{a}}{2\pi}}$

$\mu_r \approx 1$

$\boxed{L = \mu_0 \frac{N^2 h}{2\pi} \ln\frac{b}{a}}$

** ~~154~~ 155 $\mu, l_0, l, s, N$

$L = ?$

$$\oint_C \vec{H}\,d\vec{l} = \Sigma I$$

$$Hl + H_0 l_0 = NI$$

$$S = S_0 \Rightarrow B = B_0$$

$$B = \mu_0 \mu_r H$$

$$\frac{B_0}{\mu_0\mu_r} l + \frac{B_0}{\mu_0} l_0 = NI$$

$$B\left[\frac{l}{\mu_0\mu_r} + \frac{l_0}{\mu_0}\right] = NI$$

$$\boxed{B = \frac{NI}{\left[\frac{l}{\mu_0\mu_r} + \frac{l_0}{\mu_0}\right]}}$$

$$\phi = NBS = \frac{N^2 I S}{\frac{l}{\mu} + \frac{l_0}{\mu_0}} \qquad L = \frac{\phi}{I} \Rightarrow \boxed{L = \frac{N^2 S}{\frac{l}{\mu} + \frac{l_0}{\mu_0}}}$$

156 $b = 0{,}1\,m$

$S = 1\,cm^2$

$N = 200$

$I = 0{,}1\,A$

$\mu_r = 1000$

$B = ?$ $H = ?$

$M = ?$ $\phi_j = ?$

$L = ?$

$$B = \mu N' I$$

$$B = \mu_0 \mu_r \frac{N}{b} I$$

$$H = \frac{B}{\mu} \; ; \; H = \frac{N}{b} I$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

$$M = \frac{B}{\mu_0} - H = \frac{N}{b} I (\mu_r - 1)$$

$$\phi_j = BS = \mu_0 \mu_r \frac{NIS}{b}$$

$$\boxed{\phi = N\phi_j = \mu_0 \mu_r \frac{N^2 I S}{b}}$$

$$\boxed{L = \frac{\phi}{I} = \mu_0 \mu_r \frac{N^2 S}{b}}$$

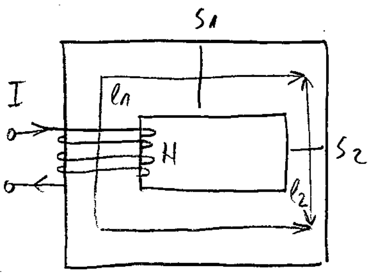
157 $S_1 = 4\,cm^2$ ; $l_1 = 20\,cm$

$S_2 = 2\,cm^2$ ; $l_2 = 5\,cm$

$\mu_r = 1000$ ; $N = 200$

$I = 1\,A$

$B_2 = ?$ $L = ?$




$$\oint_C \vec{H}\,d\vec{l} = NI$$

$$H_1 l_1 + H_2 l_2 = NI$$

$$\frac{B_1}{\mu} l_1 + \frac{B_2}{\mu} l_2 = NI$$

$$B_1 S_1 = B_2 S_2$$

$$B_1 = \frac{B_2 S_2}{S_1}$$

$$\frac{B_2 S_2 l_1}{\mu S_1} + \frac{B_2 l_2}{\mu} = NI \;/\cdot \mu S_1$$

$$B_2 (S_2 l_1 + l_2 S_1) = NI\mu S_1$$

$$\boxed{B_2 = \frac{NI\mu S_1}{S_2 l_1 + S_1 l_2}}$$

$$\boxed{B_2 = 1{,}676\,T}$$

$$L = \frac{\phi}{I} = \frac{NB_2 S_2}{I} = \frac{N^2 \mu S_1 S_2}{l_1 S_2 + l_2 S_1}$$

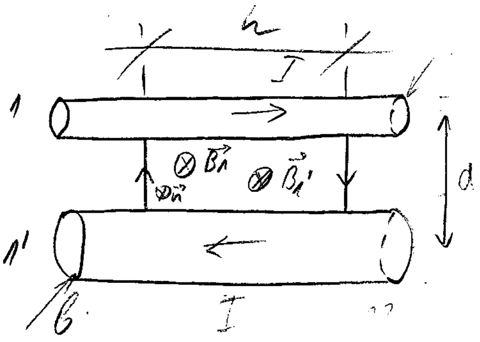
$$\boxed{L \approx 67{,}2\,mH}$$

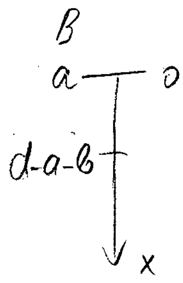
(158) $a = 1mm$

$b = 2mm$

$d = 20mm$

$L_e' = ?$




$B = \frac{\mu_0 I}{2\pi r}$

$$B = B_1 + B_1' = \frac{\mu_0 I}{2\pi x} + \frac{\mu_0 I}{2\pi (d-x-a)} = \frac{\mu_0 I}{2\pi}\left(\frac{1}{x} + \frac{1}{d-x-a}\right)$$

$$\phi = \int_S \vec{B}\,d\vec{s} = \int_a^{d-b-a} \underbrace{\frac{\mu_0 I}{2\pi}\left(\frac{1}{x} + \frac{1}{d-a-x}\right)}_{B} dx \cdot h = \frac{\mu_0 I h}{2\pi}\left[\ln\frac{d-b-a}{a} - \ln\frac{\cancel{d}-a-\cancel{d}+b+a}{d-a-a}\right]$$

$d >> a, b$

$d-2a \simeq d$

$d-b-a \simeq d$

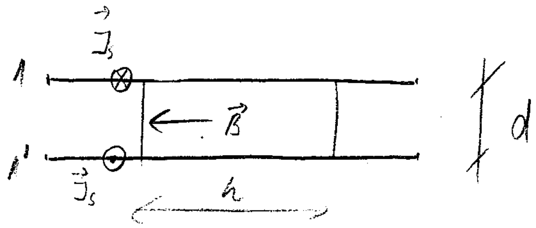
$$\phi' = \frac{\phi}{h} = \frac{\mu_0 I}{2\pi}\left[\ln\frac{d}{a} + \ln\frac{d}{b}\right] = \frac{\mu_0 I}{2\pi}\ln\frac{d^2}{ab} = \frac{\mu_0 I}{2\pi} 2\ln\frac{d}{\sqrt{ab}}$$

$$L_e' = \frac{\phi'}{I} \qquad \boxed{L_e' = \frac{\mu_0}{2\pi}\ln\frac{d}{\sqrt{ab}}}$$

(159) $a = 10mm$

$d = 0,1 mm$

$L' = ?$




$B_1 = \frac{\mu_0 J_s}{2}$

$J_s = \frac{I}{a}$

$$B = 2B_1 = \mu_0 J_s = \mu_0 \frac{I}{a}$$

$$\phi = \int_S \vec{B}\,d\vec{s} = Bhd$$

$$\phi' = \frac{\phi}{h} = Bd = \frac{\mu_0 I d}{a} \qquad L_e' = \frac{\phi'}{I} \qquad \boxed{L_e' = \frac{\mu_0 d}{a}} \qquad L_e' \approx 12,6 \frac{nH}{m}$$

(160) $a = 2mm$

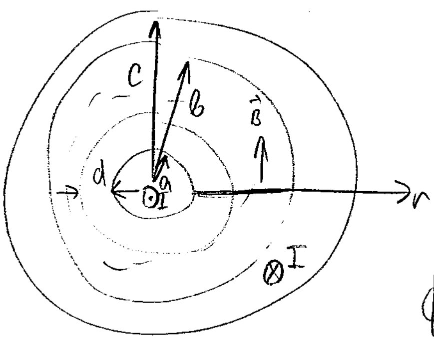
$b = 6mm$

$c = 6,5mm$

$\mu_r = 100$

$d = 2mm$

$\Delta L_e' = ?$




1) nema ferita

$H \cdot 2r\pi = I$

$H = \frac{I}{2r\pi}$

$B = \mu_0 H = \frac{\mu_0 I}{2r\pi}$

$$\phi = \int_S \vec{B}\,d\vec{s} = \int_a^b B\,h\,dr$$

$$\phi' = \frac{\phi}{h} = \frac{\mu_0 I}{2\pi}\ln\frac{b}{a}$$

$$\boxed{L_{e1}' = \frac{\mu_0}{2\pi}\ln\frac{b}{a}}$$

2) постави се ферити

$$B=\frac{\mu_0 I}{2\pi r}, \quad a+d<r<b$$

$$B=\frac{\mu_r\mu_0 I}{2\pi r}, \quad a<r\le a+d$$

$$\phi=\int_S \vec{B}\,d\vec{s}=\int_{a+d}^{b}\frac{\mu_0 I}{2\pi r}\,dr\,h+\int_{a}^{a+d}\frac{\mu_r\mu_0 I}{2\pi r}\,dr\,h$$

$$\phi'=\frac{\phi}{h}=\frac{\mu_0 I}{2\pi}\ln\frac{b}{a+d}+\frac{\mu_0\mu_r I}{2\pi}\ln\frac{a+d}{a}$$

$$\boxed{L_{e_2}'=\frac{\phi'}{I}=\frac{\mu_0}{2\pi}\left[\ln\frac{b}{a+d}+\mu_r\ln\frac{a+d}{a}\right]}$$

$$\Delta L_e'=L_{e_2}'-L_{e_1}'$$

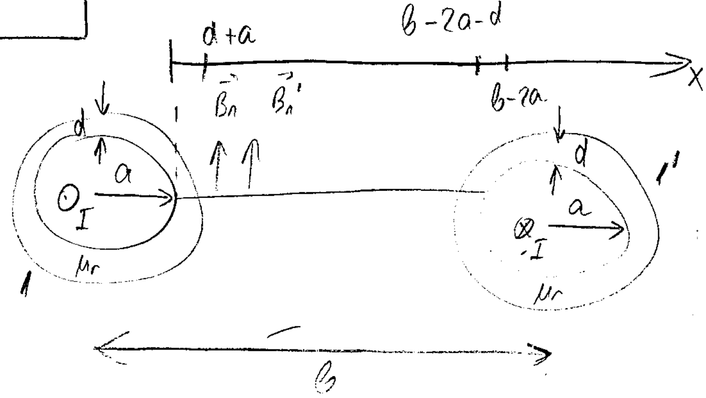
$$\Delta L_e'=\frac{\mu_0}{2\pi}\left[\ln\frac{b}{a+d}+\mu_r\ln\frac{a+d}{a}-\ln\frac{b}{a}\right]$$

$$\ln\frac{b}{a}=\ln\frac{b(a+d)}{a(a+d)}=\ln\frac{b}{a+d}+\ln\frac{a+d}{a}$$

$$\boxed{\Delta L_e'=\frac{\mu_0}{2\pi}\ln\frac{a+d}{a}(\mu_r-1)}$$

*(161) $a, b, \mu_r, d$

$L_e'=?$




$$B=\frac{\mu_0 I}{2r\pi}$$

$$B_1=\frac{\mu_0\mu_r I}{2\pi x},\ x\in(a,d+a)\cup(b-2a-d,b-2a) \qquad B_1=\frac{\mu_0 I}{2\pi x}\ r\in(d,b-2a-d)$$

$$B_2=\frac{\mu_0\mu_r I}{2\pi(b-a-x)},\ \text{—}\,\|\,\text{—} \qquad B_2=\frac{\mu_0 I}{2\pi(b-a-x)},\ \text{—}\,\|\,\text{—}$$

$$\phi'=\int_0^d(B_1+B_2)\,dx+\int_{d+a}^{b-2a-d}(B_1+B_2)\,dx+\int_{b-2a-d}^{b-2a}(B_1+B_2)\,dx$$

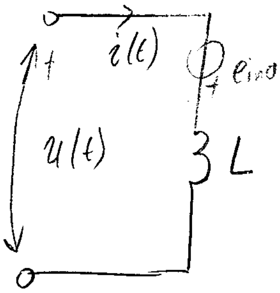
$$\phi'=\frac{\mu_0 I}{2\pi}\left[\mu_r\left(\ln\frac{d+a}{a}+\underbrace{\ln\frac{b-\cancel{d}-a}{b-a-d+a}}_{0}\right)+\ln\frac{b-2a-d}{d+a}+\ln\frac{b-a-d-a}{\cancel{b}-a-\cancel{b}+2a+d}+\right.$$

$$\left.+\mu_r\left(\underbrace{\ln\frac{b-2a}{b-2a-d}}_{0}+\ln\frac{\cancel{b}-a-\cancel{b}+2a+d}{\cancel{b}-a-\cancel{b}+2a}\right)\right]$$

$$\phi' = \frac{\mu_0 I}{2\pi}\left[2\mu_r \ln\frac{d+a}{a} + \ln\frac{b}{d+a} + \ln\frac{b}{d+a}\right] = \frac{\mu_0 I}{\pi}\left[\mu_r \ln\frac{d+a}{a} + \ln\frac{b}{d+a}\right]$$

$$L_e' = \frac{\phi'}{I} \qquad \boxed{L_e' = \frac{\mu_0}{\pi}\left[\mu_r \ln\frac{d+a}{a} + \ln\frac{b}{d+a}\right]}$$

~~** 162~~ ~~** 163~~ 164 $i(t) = I_0\left(1 - e^{-t/\tau}\right), L, \tau, I_0$




a) $e_{ind} = ?$

b) $u = ?$

$$e_{ind} = -L\frac{di}{dt} = -L\frac{d}{dt}\left(I_0 - I_0 e^{-\frac{t}{\tau}}\right) =$$

$$= -L \cdot (-I_0) \cdot \left(-\frac{1}{\tau} e^{-\frac{t}{\tau}}\right)$$

$$\boxed{e_{ind} = -\frac{L I_0}{\tau} e^{-\frac{t}{\tau}}}$$

b) $u = -e_{ind}$ $\qquad \boxed{u(t) = \frac{L I_0}{\tau} e^{-\frac{t}{\tau}}}$

~~** 165~~ 166 $l = 200\,mm$

$S = 2\,cm^2$; $I_m = 0{,}1\,A$

$N = 300$; $\omega = 1000\,s^{-1}$

$i_g(t) = I_m \cos\omega t$

$u = ?$

$$H \cdot l = NI$$

$$B = \mu_0 H = \mu_0 \frac{NI}{l}$$

$$\phi = NBS = \mu_0 \frac{N^2 I S}{l}$$

$$L = \frac{\phi}{I} = \frac{\mu_0 N^2 S}{l}$$

$$u = L\frac{di}{dt}$$

$$\boxed{u(t) = -\frac{\mu_0 N^2 S}{l} I_m \omega \sin\omega t}$$

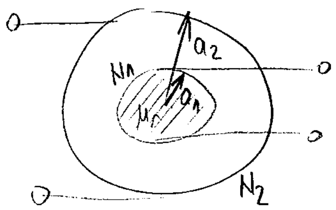
$$u(t) = U_m \cos\left(\omega t + \frac{\pi}{2}\right)$$

$$U_m = \frac{\mu_0 N^2 S}{l} I_m \omega$$

167 $a_1, a_2, l, N_1, N_2, \mu_r$

$k = ?$

$$k = \frac{|L_{12}|}{\sqrt{L_1 L_2}}$$




$$L_1 = \frac{\phi_1}{I_1}; \quad B_1 = \mu_r \mu_0 H_1' I_1 = \frac{\mu_r \mu_0}{l} N_1 I_1$$

$$\phi_1 = N_1 B_1 S_1 = \frac{\mu_r \mu_0 N_1^2 I_1 S_1}{l} = \frac{\mu_r \mu_0 N_1^2 I_1 a_1^2 \pi}{l}$$

$$\boxed{L_1 = \frac{\mu_r \mu_0 N_1^2 a_1^2 \pi}{l}}$$

$$\phi_{21} = N_2 B_1 S_1 = \frac{\mu_0 \mu_r N_1 N_2 a_1^2 \pi I_1}{l}$$

$$L_{21} = L_{12} = \frac{\phi_{21}}{I_1}$$

$$\boxed{L_{21} = \frac{\mu_0 \mu_r N_1 N_2 a_1^2 \pi}{l}}$$

$$B_2 = \mu_0\mu_r N_2' I_2 = \mu_0\mu_r \frac{N_2}{l} I_2, \quad r\in(0,a_1)$$

$$B_2 = \mu_0 N_2' I_2 = \mu_0 \frac{N_2}{l} I_2, \quad r\in(a_1,a_2)$$

$$\Phi_2 = N_2\int_S \vec B\, d\vec s = \mu_0\mu_r \frac{N_2^2}{l} I_2\, a_1^2\pi + \mu_0\frac{N_2^2}{l} I_2 (a_2^2-a_1^2)\pi$$

$$\Phi_2 = \mu_0 \frac{N_2^2}{l} I_2 \pi(\mu_r a_1^2 + a_2^2 - a_1^2) =$$

$$= \mu_0 \frac{N_2^2}{l} I_2 \pi\left(a_1^2(\mu_r-1) + a_2^2\right)$$

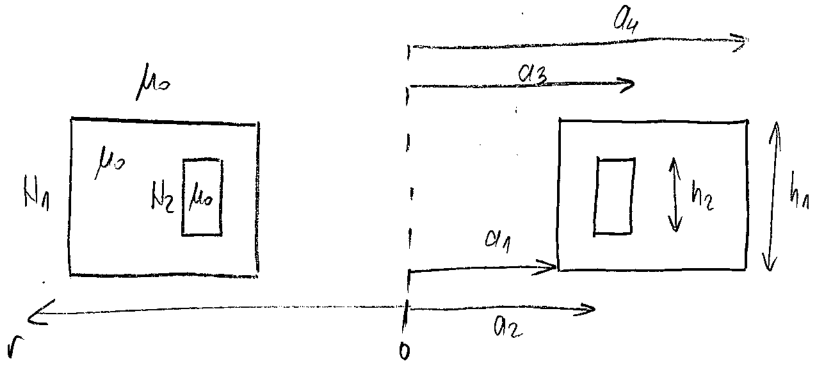
$$L_2 = \frac{\Phi_2}{I_2} = \mu_0 \frac{N_2^2}{l}\pi\left(a_1^2(\mu_r-1)+a_2^2\right)$$

$$k = \frac{\frac{\mu_0\mu_r N_1 N_2 a_1^2\pi}{l}}{\sqrt{\frac{\mu_0\mu_r N_1^2 a_1^2\pi}{l}\,\frac{\mu_0 N_2^2\pi}{l}\left(a_1^2(\mu_r-1)+a_2^2\right)}} = \frac{\cancel{\frac{\mu_0\mu_r N_1 N_2 a_1^2\pi}{l}}}{\cancel{\frac{\mu_0 N_1 N_2 \pi a_1}{l}}\sqrt{\mu_r\left(a_1^2(\mu_r-1)+a_2^2\right)}}$$

$$\boxed{k = \sqrt{\frac{\mu_r}{\mu_r - 1 + \left(\frac{a_2}{a_1}\right)^2}}}$$

(163) $a_1, a_2, a_3, a_4, h_1, h_2, N_1, N_2$

$k = ?$




(2) $\oint_C \vec H\, d\vec l = \Sigma I$

$H_2 \cdot 2r\pi = N_2 I_2$

$$B_2 = H_2\mu_0 = \mu_0 \frac{N_2 I_2}{2r\pi}$$

$$\Phi_2 = N_2\int_{S_2} \vec B_2\, d\vec s_2 = \frac{\mu_0 N_2^2 I_2}{2\pi} h_2 \int_{a_2}^{a_3}\frac{dr}{r}$$

$ds_2 = h_2\, dr$

$$\Phi_2 = \frac{\mu_0 N_2^2 I_2 h_2}{2\pi}\ln\frac{a_3}{a_2}$$

$$L_2 = \frac{\Phi_2}{I_2} \Rightarrow \boxed{L_2 = \frac{\mu_0 N_2^2 h_2}{2\pi}\ln\frac{a_3}{a_2}}$$

(1) $\oint_C \vec H\, d\vec l = \Sigma I$

$H_1\, 2r\pi = N_1 I_1; \quad B_1 = \mu_0 H_1$

$$\Phi_1 = N_1\int_{S_1}\vec B_1\, d\vec s_1 = \frac{\mu_0 N_1^2 I_1}{2\pi} h_1 \int_{a_1}^{a_4}\frac{dr}{r}$$

$ds_1 = h_1\, dr$

$$\Phi_1 = \frac{\mu_0 N_1^2}{2\pi} h_1 \ln\frac{a_4}{a_1}$$

$$L_1 = \frac{\Phi_1}{I_1} \Rightarrow \boxed{L_1 = \frac{\mu_0 N_1^2 h_1}{2\pi}\ln\frac{a_4}{a_1}}$$

$$L_{12} = \frac{\Phi_1(I_2)}{I_2} = \frac{N_1 \int_{S_2} B_2 \, ds_2}{I_2} = \frac{\mu_0 N_1 N_2 h_2}{2\pi} \ln \frac{a_3}{a_2}$$

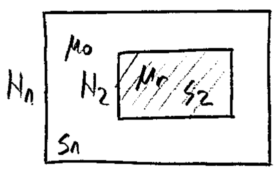
$$k = \frac{|L_{12}|}{\sqrt{L_1 L_2}} = \frac{\cancel{\frac{\mu_0 N_1 N_2}{2\pi}} h_2 \ln \frac{a_3}{a_2}}{\sqrt{\cancel{\frac{\mu_0 N_1^2}{2\pi}} h_1 \ln \frac{a_4}{a_1} \; \cancel{\frac{\mu_0 N_2^2}{2\pi}} h_2 \ln \frac{a_3}{a_2}}}$$

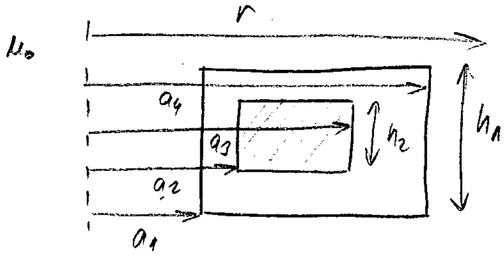
$$k = \sqrt{\frac{h_2^{\cancel{2}} \ln^{\cancel{2}} \frac{a_3}{a_2}}{h_1 \ln \frac{a_1}{a_1} \cancel{h_2 \ln \frac{a_3}{a_2}}}} \qquad \boxed{k = \sqrt{\frac{h_2 \ln \frac{a_3}{a_2}}{h_1 \ln \frac{a_4}{a_1}}}}$$

(169) $a_1, a_2, a_3, a_4, h_1, h_2, N_1, N_2, \mu_r$

$L_1, L_2, L_{12}, k, U_{m1} = ?$

$E_m = 10V, \; f = 1kHz$




($L_2$) $\oint_C \vec{H} d\vec{l} = \sum I$

$H_2 2r\pi = N_2 I_2$

$B_2 = \mu_0 \mu_r H_2 = \mu_0 \mu_r \frac{N_2 I_2}{2r\pi}$

$\Phi_2 = N_2 \int_{S_2} \vec{B}_2 d\vec{s}_2 = \mu_0 \mu_r \frac{N_2^2 I_2}{2\pi} h_2 \ln \frac{a_3}{a_2}$

$L_2 = \frac{\Phi_2}{I_2}$ $\quad \boxed{L_2 = \mu_0 \mu_r \frac{N_2^2 h_2}{2\pi} \ln \frac{a_3}{a_2}}$

($L_1$) $\oint_C \vec{H} d\vec{l} = \sum I$

$H_1 2r\pi = N_1 I_1$

$B_1' = \mu_0 \mu_r \frac{N_1 I_1}{2r\pi}, \quad a_2 < r < a_3$

$B_1 = \mu_0 \frac{N_1 I_1}{2r\pi}$

$\Phi_1 = N_1 \left[ \int_{S_1} \vec{B}_1 d\vec{s}_1 - \int_{S_2} \vec{B}_1 d\vec{s}_2 + \int_{S_2} \vec{B}_1' d\vec{s}_2 \right]$

$\Phi_1 = \frac{\mu_0 N_1^2 I_1}{2\pi} \left[ h_1 \ln \frac{a_4}{a_1} - h_2 \ln \frac{a_3}{a_2} + \mu_r h_2 \ln \frac{a_3}{a_2} \right]$

$$\boxed{\Phi_1 = \frac{\mu_0 N_1^2 I_1}{2\pi} \left[ h_1 \ln \frac{a_4}{a_1} + h_2 \ln \frac{a_3}{a_2} (\mu_r - 1) \right]}$$

$L_1 = \frac{\Phi_1}{I_1} \Rightarrow \boxed{L_1 = \frac{\mu_0 N_1^2}{2\pi} \left[ h_1 \ln \frac{a_4}{a_1} + h_2 (\mu_r - 1) \ln \frac{a_3}{a_2} \right]}$

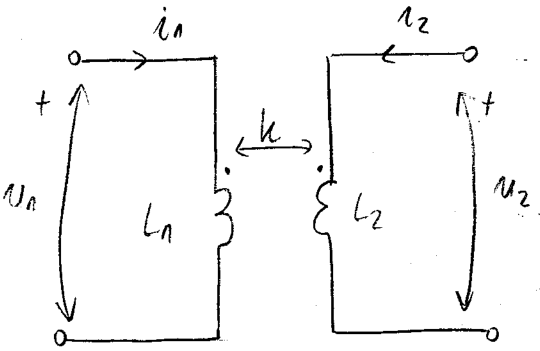
($L_{12}$) $L_{12} = \frac{\Phi_{12}}{I_2} = \frac{N_1 \int_{S_2} \vec{B}_2 d\vec{s}_2}{I_2}$

$L_{12} = \frac{N_1 \mu_0 \mu_r \frac{N_2 I_2}{2\pi} h_2 \ln \frac{a_3}{a_2}}{I_2}$

$$\boxed{L_{12} = \mu_0 \mu_r \frac{N_1 N_2 h_2}{2\pi} \ln \frac{a_3}{a_2}}$$

($k$) $k = \frac{|L_{12}|}{\sqrt{L_1 L_2}}$

$$k = \frac{\cancel{\frac{\mu_0 \mu_r N_1 N_2}{2\pi}} h_2 \ln \frac{a_3}{a_2}}{\cancel{\frac{\mu_0 N_1 N_2}{2\pi}} \sqrt{\mu_r h_2 \ln \frac{a_3}{a_2} \left[ h_1 \ln \frac{a_4}{a_1} + h_2 (\mu_r - 1) \ln \frac{a_3}{a_2} \right]}} = \frac{\mu_r h_2 \ln \frac{a_3}{a_2}}{\sqrt{\mu_r h_2 \ln \frac{a_3}{a_2} \left[ h_1 \ln \frac{a_4}{a_1} + h_2 (\mu_r - 1) \ln \frac{a_3}{a_2} \right]}}$$




$$u_2 = E_m \cos(2\pi f t)$$

$$i_1 = 0$$

$$u_1 = L_1 \cancel{\frac{di_1}{dt}}^{\,0} + L_{12}\frac{di_2}{dt}$$

$$u_2 = L_{12}\cancel{\frac{di_1}{dt}}_{\,0} + L_2\frac{di_2}{dt} = E_m\cos(2\pi f t)$$

$$\frac{di_2}{dt} = \frac{E_m}{L_2}\cos(2\pi f t)$$

$$\boxed{u_1 = L_{12}\frac{E_m}{L_2}\cos(2\pi f t)}$$

$$U_{1m} = \frac{L_{12}E_m}{L_2} = \frac{\cancel{\frac{\mu_0\mu_r N_1 N_2 b_2}{2\pi}\ln\frac{a_3}{a_2}}\, E_m}{\cancel{\frac{\mu_0\mu_r N_2^2 b_2}{2\pi}\ln\frac{a_3}{a_2}}} = \underline{\underline{\frac{N_1}{N_2}E_m}}$$

(170) $L_1 = 10\,mH$

$L_2 = 40\,mH$

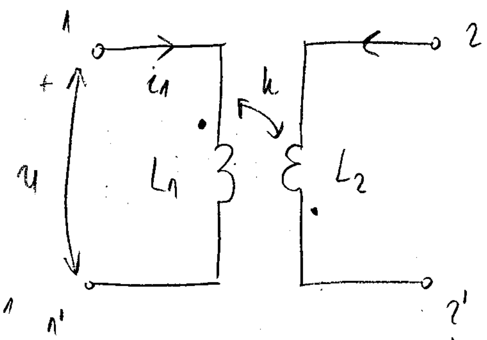
$k = 0{,}5$

$i(t) = I_m \sin\omega t$

$I_m = 2A$ ; $\omega = 10^4 s^{-1}$

$u_{22'}(t) = ?$

$i_2(t) = 0 \rightarrow \frac{di_2}{dt} = 0$




$$k = -\frac{L_{12}}{\sqrt{L_1L_2}}$$

$$L_{12} = -k\sqrt{L_1L_2} = \underline{\underline{-10\,mH}}$$

$$u_1(t) = L_1\frac{di_1}{dt} + L_{12}\cancel{\frac{di_2}{dt}}^{\,0}$$

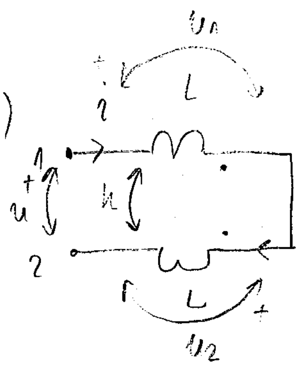
$$u_2(t) = L_{12}\frac{di_1}{dt} + L_2\cancel{\frac{di_2}{dt}}^{\,0}$$

$$u_2(t) = L_{12}\frac{di_1}{dt} = L_{12} I_m\,\omega\cos\omega t$$

$$u_2(t) = -200\cos\omega t \ \ V$$

$$\boxed{U_{2m} = 200\,V}$$

(171) a)




$i_1 = i_2 = i$

$u = u_1 + u_2$

$$u_1 = L\frac{di}{dt} + L_{12}\frac{di}{dt}$$

$$u_2 = L_{12}\frac{di}{dt} + L\frac{di}{dt}$$

$$u = L\frac{di}{dt} - kL\frac{di}{dt} - kL\frac{di}{dt} + L\frac{di}{dt}$$

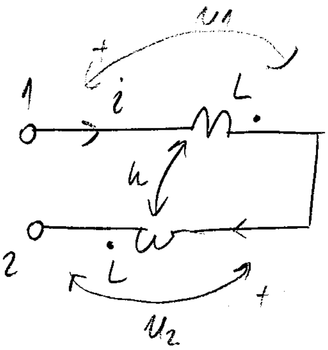
$$k = -\frac{L_{12}}{\sqrt{LL}}$$

$$L_{12} = -kL$$

$$u = \underbrace{2L(1-k)}_{L_e}\frac{di}{dt}$$

$$\boxed{L_e = 2L(1-k)}$$

δ)



$i_1=i_2=i$

$u=u_2+u_1$

$\underline{L_{12}=kL}$

$u_1=L\frac{di}{dt}+L_{12}\frac{di}{dt}$

$u_2=L_{12}\frac{di}{dt}+L\frac{di}{dt}$

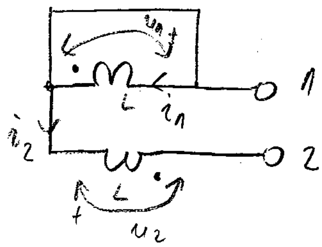
$u=\underbrace{2L(1+k)}_{Le}\frac{di}{dt}$

$\boxed{Le=2L(1+k)}$

(172) $L=4mH$

$k=0{,}5$

$Le=?$



$L_{12}=kL$

$u_1=0$

$u=u_2$

$u_1=L\frac{di_1}{dt}+L_{12}\frac{di_2}{dt}=0$

$\Downarrow$

$\frac{di_1}{dt}=-\frac{L_{12}}{L}\frac{di_2}{dt}$

$u=u_2=L_{12}\frac{di_1}{dt}+L\frac{di_2}{dt}=\underbrace{\left(-\frac{L_{12}^2}{L}+L\right)}_{Le}\frac{di_2}{dt}$

$\boxed{Le=L(1-k^2)=3mH}$

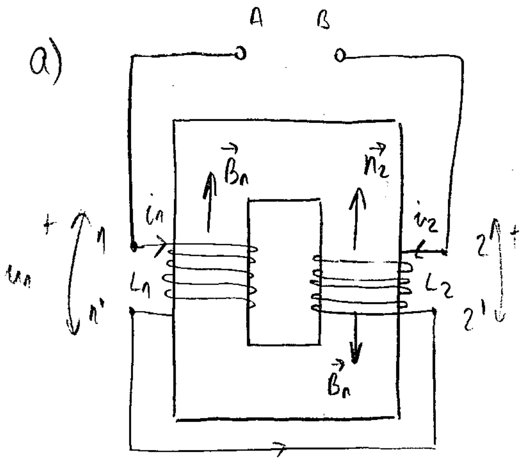
(173) $L_1=1H$

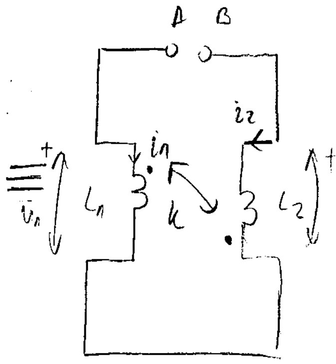
$L_2=2H$

$k=0{,}95$

$Le=?$

a)





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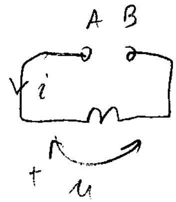


$k=\frac{|L_{12}|}{\sqrt{L_1L_2}}$ $\boxed{L_{12}=-k\sqrt{L_1L_2}}$

$i_2=-i_1=-i$

$u_1=\frac{di_1}{dt}L_1+\frac{di_2}{dt}L_{12}$

$u_2=\frac{di_2}{dt}L_2+\frac{di_1}{dt}L_{12}$

$u=u_1-u_2$

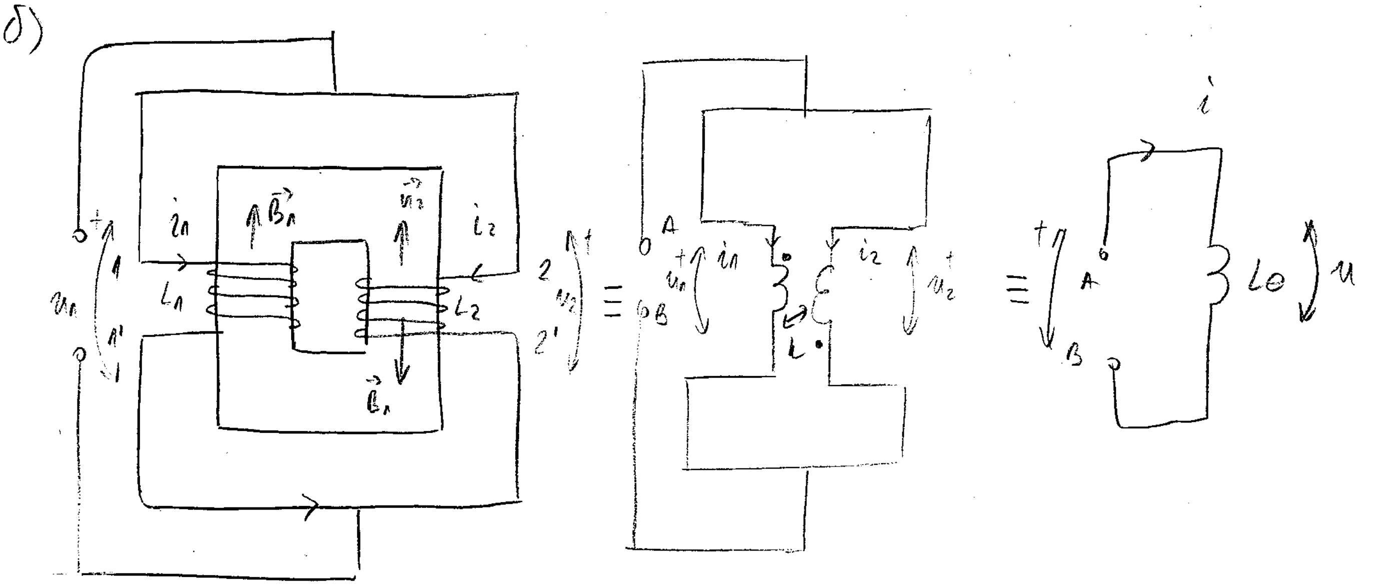
$u=\frac{di}{dt}L_1-\frac{di}{dt}L_{12}+\frac{di}{dt}L_2-\frac{di}{dt}L_{12}$

$u=\frac{di}{dt}\underbrace{(L_1+L_2-2L_{12})}_{Le}$

$\boxed{Le=L_1+L_2+k\sqrt{L_1L_2}}$

$\boxed{Le=5{,}687H}$

δ)




$$k = \frac{-L_{12}}{\sqrt{L_1 L_2}} \Rightarrow \boxed{L_{12} = -k\sqrt{L_1 L_2}}$$

$$u_1 = u_2 = u$$

$$i = i_1 + i_2 \qquad \frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$$

$$\Downarrow$$

$$\frac{di_2}{dt} = \frac{di}{dt} - \frac{di_1}{dt}$$

$$u_1 = L_1 \frac{di_1}{dt} + L_{12} \frac{di_2}{dt}$$

$$u_2 = L_2 \frac{di_2}{dt} + L_{12} \frac{di_1}{dt}$$

$$u_1 = u_2 \Rightarrow L_1 \frac{di_1}{dt} + L_{12}\frac{di_2}{dt} = L_{12}\frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

$$\frac{di_2}{dt}(L_2 - L_{12}) = \frac{di_1}{dt}(L_1 - L_{12})$$

$$\frac{di_2}{dt} = \frac{di_1}{dt}\,\frac{L_1 - L_{12}}{L_2 - L_{12}} \qquad \frac{di}{dt} = \frac{di_1}{dt}\,\frac{L_2 - L_{12} + L_1 - L_{12}}{L_2 - L_{12}}$$

$$\Downarrow$$

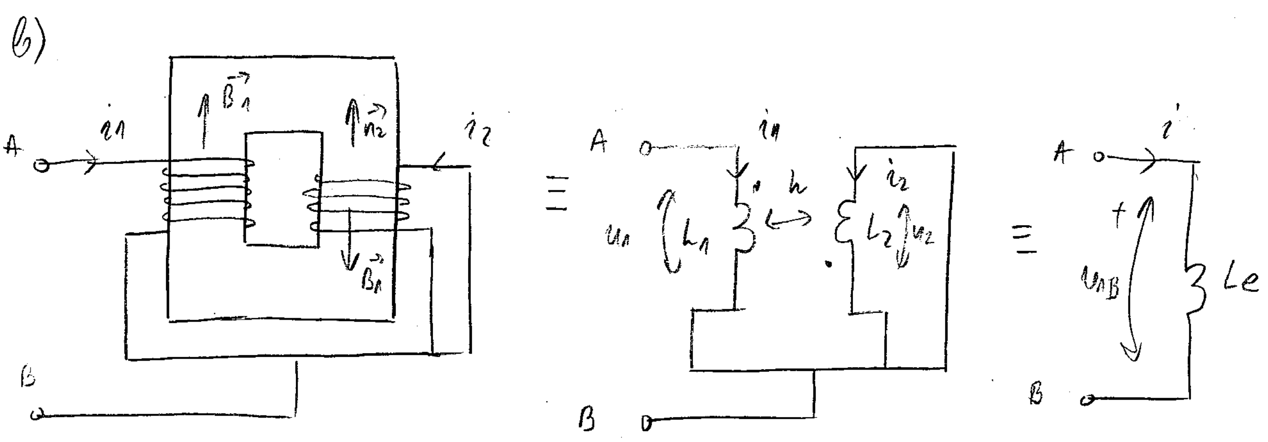
$$\frac{di_1}{dt} = \frac{di}{dt}\,\frac{L_2 - L_{12}}{L_1 + L_2 - 2L_{12}}$$

$$u_{AB} = u_1 = L_1 \frac{di_1}{dt} + L_{12}\,\frac{L_1 - L_{12}}{L_2 - L_{12}}\,\frac{di_1}{dt}$$

$$u_{AB} = \frac{di}{dt}\underbrace{\frac{\cancel{L_2 - L_{12}}}{L_1 + L_2 - 2L_{12}}\;\frac{L_1 L_2 - \cancel{L_{12}L_1} + \cancel{L_{12}L_1} - L_{12}^2}{\cancel{L_2 - L_{12}}}}_{L_e}$$

$$\boxed{L_e = \frac{L_1 L_2 - L_{12}^2}{L_1 + L_2 - 2L_{12}}}$$

b)




$u_2 = 0 \qquad i = i_1 \qquad u = u_1 \qquad\qquad L_{12} = -k\sqrt{L_1 L_2}$

$$0 = u_2 = L_2 \frac{di_2}{dt} + L_{12}\frac{di_1}{dt} \Rightarrow \frac{di_2}{dt} = -\frac{L_{12}}{L_2}\frac{di_1}{dt}$$

$$u = u_1 = \frac{di_1}{dt}L_1 + \frac{di_2}{dt}L_{12} = \frac{di_1}{dt}\left(L_1 - \frac{L_{12}^2}{L_2}\right) = \frac{di}{dt}\underbrace{\left(L_1 - \frac{L_{12}^2}{L_2}\right)}_{L_e}$$

$$\boxed{L_e = L_1 - \frac{L_{12}^2}{L_2}}$$

$$L_e = L_1 - \frac{k^2 L_1 \cancel{L_2}}{\cancel{L_2}} \qquad \boxed{L_e = L_1(1-k^2)}$$

(174) $l = 0{,}2\,m$

$S = 4\,cm^2$

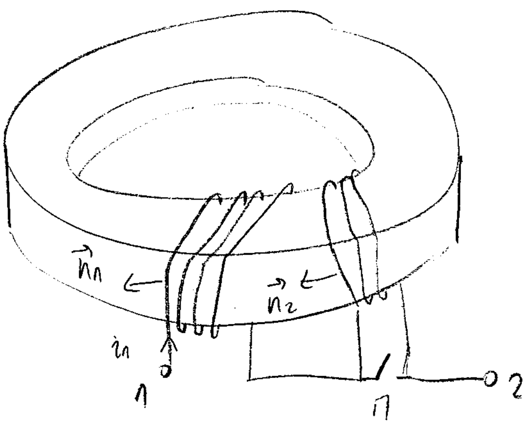
$\mu_r = 1000$

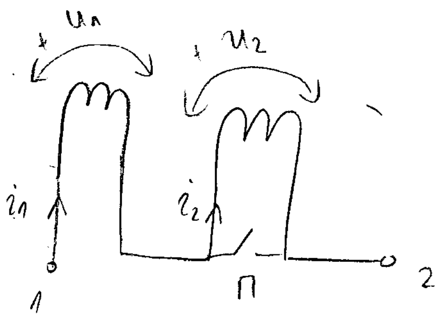
$N_1 = N_2 = 500$

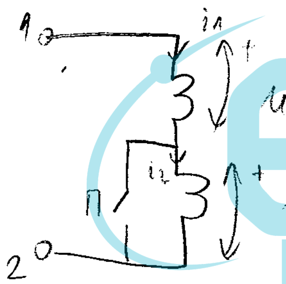
a) $\Pi_{отв}$

б) $\Pi_{зат}$

$L_e = ?$




a) $i_1 = i_2 = i$

$$u = u_1 + u_2 = L_1\frac{di_1}{dt} + L_{12}\frac{di_2}{dt} + L_2\frac{di_2}{dt} + L_{12}\frac{di_1}{dt} =$$

$$= \frac{di}{dt}\underbrace{\left(L_1 + L_2 + 2L_{12}\right)}_{L_e} \qquad \boxed{L_e = L_1 + L_2 + 2L_{12}}$$

$$L_{12} = \frac{\Phi_{12}}{I_2} = \frac{N_1 B_2 S}{I_2} = \frac{N_1 \mu \frac{N_2 I_2}{l} S}{I_2} \qquad \underline{\underline{L_{12} = \frac{N_1 N_2 \mu S}{l}}}$$

$$H_2 l = N_2 I_2$$

$$H_2 = \frac{N_2 I_2}{l}$$

$$B_2 = \mu H_2$$

$$L_1 = \frac{\Phi_1}{I_1} = \frac{N_1 B_1 S}{I_1} = \underline{\underline{\frac{N_1^2 \mu S}{l}}} \qquad \underline{\underline{L_2 = \frac{N_2^2 \mu S}{l}}}$$

$$L_e = \frac{\mu S}{l}\left(N_1^2 + N_2^2 + 2N_1N_2\right) \qquad \boxed{L_e = \frac{\mu S}{l}(N_1+N_2)^2}$$

б) $u_2 = 0 \quad u = u_1 \;;\; i = i_1$

$$L_2 \frac{di_2}{dt} + L_{12}\frac{di_1}{dt} = 0$$

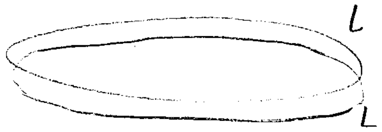
$$u = u_1 = L_1 \frac{di_1}{dt} + L_{12}\frac{di_2}{dt} = \underbrace{\left(L_1 - \frac{L_{12}^2}{L_2}\right)}_{L_e} \frac{di}{dt}$$

$$\frac{di_2}{dt} = -\frac{L_{12}}{L_2}\frac{di_1}{dt}$$

$$\boxed{L_e = L_1 - \frac{L_{12}^2}{L_2}}$$

$$L_e = \frac{L_1L_2 - L_{12}^2}{L_2} = \frac{\frac{N_1^2N_2^2\mu^2S^2}{l^2} - \frac{N_1^2N_2^2\mu^2S^2}{l}}{\frac{N_2^2\mu S}{l}} = 0 \qquad \boxed{L_e = 0}$$

(175) $L_1, L_{12} = ?$



$$L_{12} = \frac{\Phi_{12}}{I_2} = \frac{B_2 S}{I_2} = \frac{\overbrace{\frac{\mu_0 I_2}{2a}}^{B_2} a^2\pi}{I_2} = \frac{\mu_0 I a \pi}{2}$$

$$L_1 = \frac{\Phi_1}{I_1} = \frac{B_1 S}{I_1} = \frac{\frac{\mu_0 I_1}{2a} a^2\pi}{I_1} = \frac{\mu_0 I a \pi}{2}$$

$$L_{12} = L_1 \qquad \boxed{|L_{12}| \approx L}$$

(176) $L, L_e = ?$

$L_e = 0$ (нема промене флукса у унутрашњој контури)

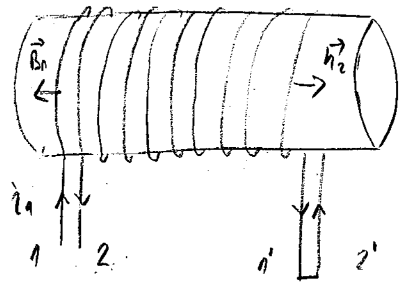
(177) $L = 100 \mu H$

$k = 0,9$

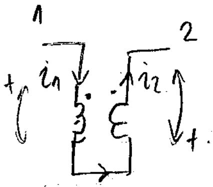
а) $L_{12}$ $1'2'$ спојени

б) $L_{1'2}$ $12'$ спојени

в) $L_{11'}$ $12$ и $1'2'$ спојени



а) $u = \underbrace{L_1 \frac{di_1}{dt} + L_{12}\frac{di_2}{dt}}_{u_1} + \underbrace{L_2 \frac{di_2}{dt} + L_{12}\frac{di_1}{dt}}_{u_2}$



$L_{12} = -k\sqrt{L_1 L_2} = -kL$

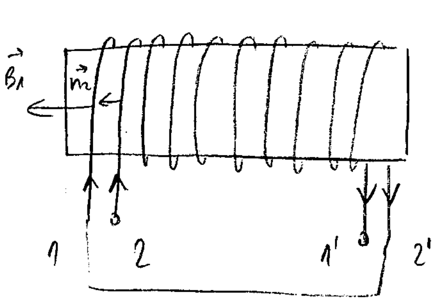
$u_1 = u_1 + u_2$ $\qquad i_2 = i_1 = i$

$u = L_1 \frac{di}{dt} + L_{12}\frac{di}{dt} + L_2 \frac{di}{dt} + L_{12}\frac{di}{dt}$

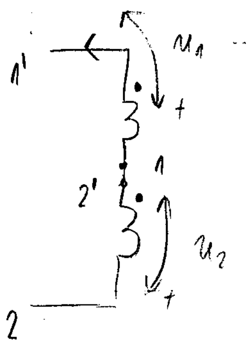
$u = \underbrace{(L_1 + L_2 + 2L_{12})}_{Le} \frac{di}{dt}$

$Le = L_1 + L_2 + 2L_{12}$ $\qquad Le = 2L - 2kL = 2L(1-k)$

б)



$L_{12} = k\sqrt{L_1 L_2} = kL$

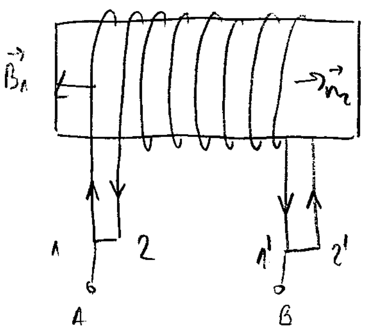


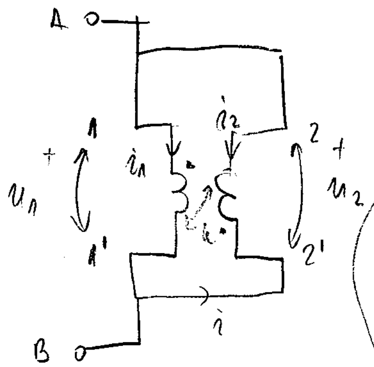
$u = u_1 + u_2$ $\qquad i = i_1 = i_2$

$u = \underbrace{(L_1 + L_2 + 2L_{12})}_{Le} \frac{di}{dt}$

$Le = L_1 + L_2 + 2L_{12} = 2(1+k)L$

в)





$i = i_1 + i_2 \Rightarrow i_1 = \frac{1}{2} i$

$u_1 = u_2 = u$ $\qquad \frac{di_1}{dt} = \frac{1}{2}\frac{di}{dt}$

$L_1 \frac{di_1}{dt} + L_{12}\frac{di_2}{dt} = L_{12}\frac{di_1}{dt} + L_2 \frac{di_2}{dt}$

$\cancel{(L+kL)}\frac{di_1}{dt} = \frac{di_2}{dt}\cancel{(L+kL)}$

$\boxed{i_1 = i_2}$

$L_{12} = -kL$

$u = u_1 = \frac{di_1}{dt}(L + kL) = \frac{1}{2}\frac{di}{dt}(L + kL)$

$\boxed{Le = \frac{1}{2} L(1+k)}$

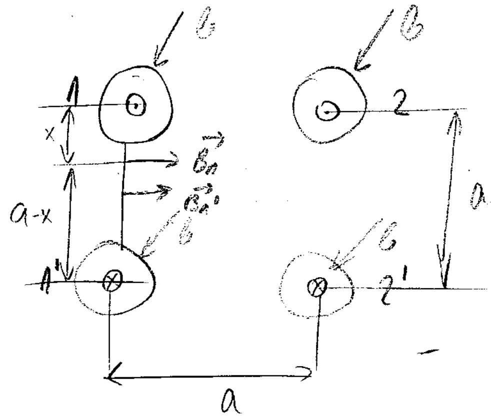
(178) $a = 200\,mm$

$b = 1\,mm$

$D = 1\,km$

$i_1(t) = 2\cos\omega t \; mA$

$\omega = 2\cdot 10^4 s^{-1}$




a) $L_1', L_2', L_{12}' = ?$

б) $i_2 = ?$

$$L_1' = \frac{\Phi_1'}{I_1} \qquad \Phi_1 = \int_{S_1} \vec{B}_{1_U} \vec{ds_1}$$

$$B_{1_U} = B_1 + B_{1'} = \frac{\mu_0 I_1}{2\pi x} + \frac{\mu_0 I_1}{2\pi(a-x)}$$

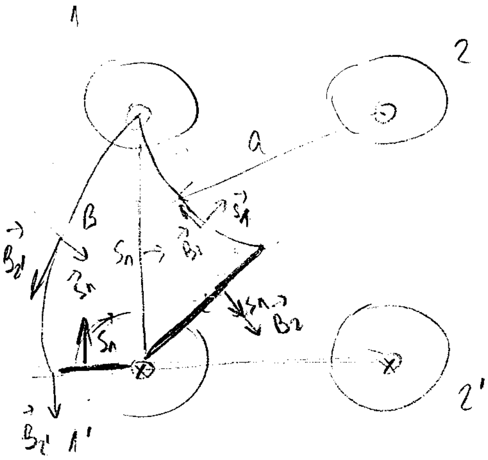
$$ds_1 = D\cdot dx \qquad\qquad a - b \approx a$$

$$\Phi_1 = \int_b^{a-b} \left(\frac{\mu_0 I_1}{2\pi x} + \frac{\mu_0 I_1}{2\pi(a-x)}\right) D\,dx \qquad \Phi_1' = \frac{\Phi_1}{D} = \frac{\mu_0 I_1}{2\pi}\left(\ln\frac{a-b}{b} + \ln\frac{a-b}{a-a+b}\right) =$$

$$= \frac{\mu_0 I_1}{\cancel{2}\pi}\cancel{2}\ln\frac{a}{b} = \frac{\mu_0 I_1}{\pi}\ln\frac{a}{b}$$

$$L_1' = \frac{\Phi_1'}{I_1} \qquad \boxed{L_1' = \frac{\mu_0}{\pi}\ln\frac{a}{b}}$$

$$\boxed{L_2' = \frac{\mu_0}{\pi}\ln\frac{a}{b}}$$




$$\Phi_{12} = \int_{S_1} \vec{B}_{2_U}\vec{ds} = \int_a^{a\sqrt{2}} B_2 D\,dr - \int_{a\sqrt{2}}^{a} B_2' D\,dr =$$

$$= D\left[\int_a^{a\sqrt{2}} \frac{\mu_0 I}{2\pi r}dr - \int_{a\sqrt{2}}^{a}\frac{\mu_0 I}{2\pi r}dr\right] =$$

$$= D\left[\frac{\mu_0 I}{2\pi}\ln\frac{a\sqrt{2}}{\cancel{a}} + \frac{\mu_0 I}{2\pi}\ln\frac{a\sqrt{2}}{\cancel{a}}\right]$$

$$\Phi_{12}' = \frac{\Phi_{12}}{D} = \frac{\mu_0 I}{2\pi}\ln 2$$

$$L_{12}' = \frac{\Phi_{12}'}{I} = \frac{\mu_0}{2\pi}\ln 2$$

б) $i_1 = 2\cos\omega t \ \mu A$

$i_2 = ?$

$u_2 = 0 \rightarrow$ кратак спој

$$u_2 = L_2 \frac{di_2}{dt} + L_{12}\frac{di_1}{dt} = 0$$

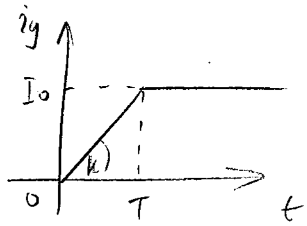
$$\frac{di_2}{\cancel{dt}} = -\frac{di_1}{\cancel{dt}}\frac{L_{12}}{L_2}$$

$$i_2 = -i_1 \frac{L_{12}}{L_2} \approx -131\cos\omega t \ \mu A$$

(179) $L_1, L_2, k, \ i_1 = i_g(t), \ u_2 = 0$

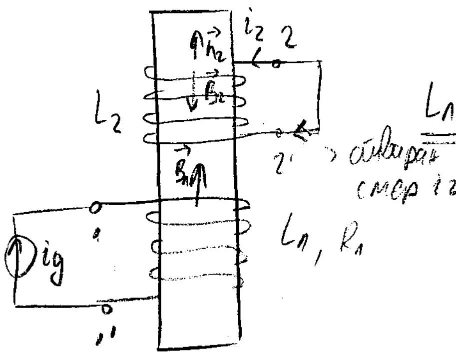
а) $u_1$ ?

б) $i_2$ ?




$k = \frac{I_0}{T}$

$$i_1(t) = i_g(t) = \begin{cases} \frac{I_0}{T}t, & 0 < t < T \\ I_0, & t \geq T \end{cases}$$




$L_{12} = k\sqrt{L_1 L_2}$

$$u_2 = L_2\frac{di_2}{dt} + L_{12}\frac{di_1}{dt}$$

$$u_1 = L_1\frac{di_1}{dt} + L_{12}\frac{di_2}{dt}$$

$$u_2 = 0 \Rightarrow \frac{di_2}{dt} = -\frac{L_{12}}{L_2}\frac{di_1}{dt}$$

$t \in (0,T) \quad \frac{di_1}{dt} > 0; \ i_1 > 0$

$$u_1 = \frac{di_1}{dt}\left(L_1 - \frac{L_{12}^2}{L_2}\right) = \frac{di_1}{dt}\left(L_1 - \frac{k^2 L_1 \cancel{L_2}}{\cancel{L_2}}\right)$$

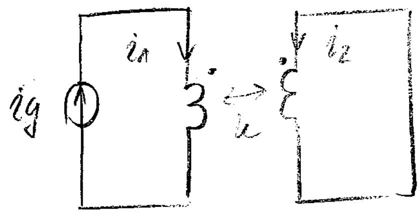
$$u_1 = \frac{di_1}{dt}L_1(\underbrace{1-k^2}_{>0})$$

$u_1(t) > 0 \quad t \in (0,T)$

б) $\frac{di_2(t)}{dt} = -\frac{L_{12}}{L_2}\frac{di_1}{dt} < 0 \quad i_2(t) < 0$

од 2' ка 2

*180




$$u_1 = \frac{di_1}{dt} L_1(1-k^2)$$

$$\boxed{\frac{di_1}{dt} = \frac{I_0}{T}}$$

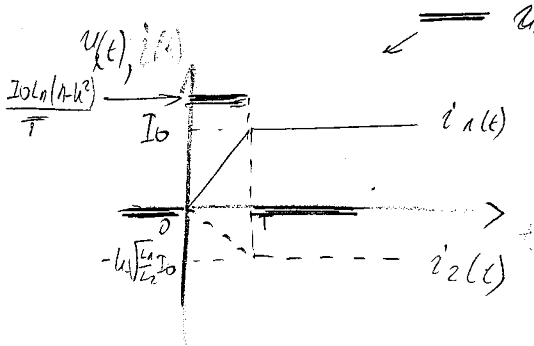
$$u_1 = \frac{I_0}{T} L_1(1-k^2)$$

$$u_1(t) = \begin{cases} \frac{I_0}{T} L_1(1-k^2), & t \in (0,T) \\ 0, & t<0 \\ 0, & t>T \end{cases}$$

$$\frac{di_2}{dt} = -\frac{di_1}{dt}\frac{L_{12}}{L_2} \quad \Big| \int_0^T dt$$

$$i_2(t)\cancel{T} = -\frac{L_{12}}{L_2} i_1(t)\cancel{T}$$

$$\boxed{i_2(t) = -\frac{L_{12}}{L_2} i_1(t)}$$




$$i_2(t) = -\underbrace{\frac{L_{12}}{L_2}}_{<1}\frac{I_0}{T}t = -\frac{k\sqrt{L_1L_2}}{L_2} I_0 \frac{t}{T} = -k\sqrt{\frac{L_1}{L_2}} I_0 \frac{t}{T}$$

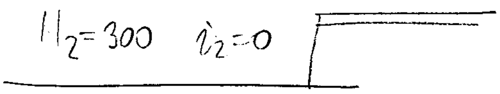
**~~181~~ ~~182~~ ~~183~~ 184***

$l = 0{,}3\,m$

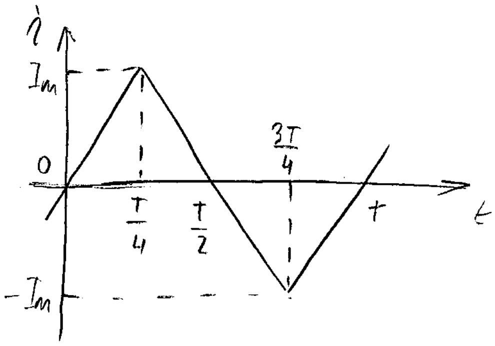
$S = 2\,cm^2$

$N_1 = 600 \quad I_m = 0{,}1\,A \quad T = 4\,ms$

$N_2 = 300 \quad i_2 = 0$



$u_2(t)$

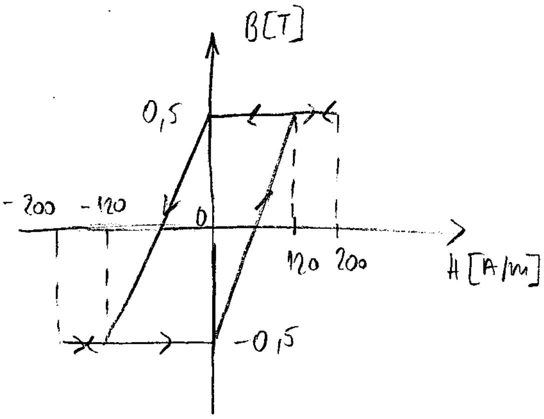



$$\oint_C \vec{H}\, d\vec{l} = \Sigma_H$$

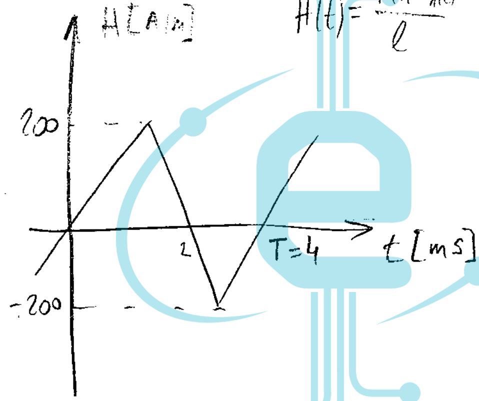
$$H_{1m} l = N_1 I_m$$

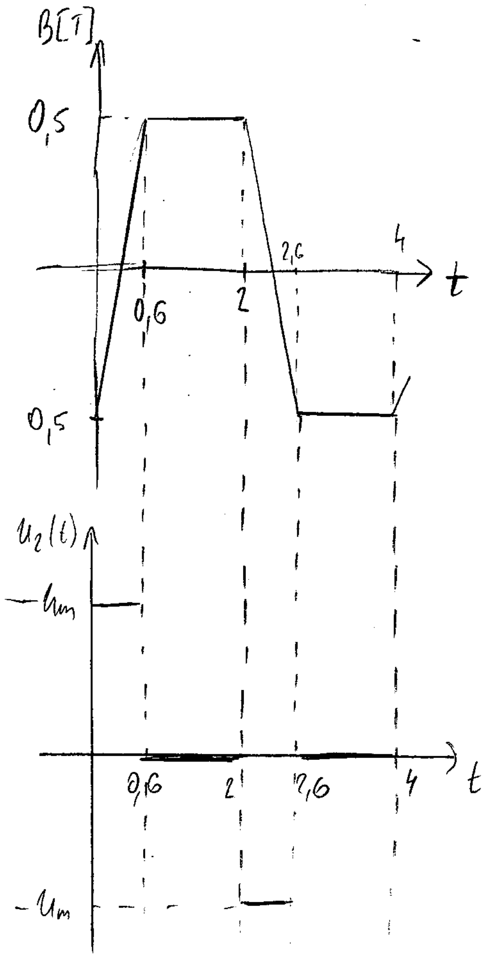
$$H_{1m} = \frac{N_1 I_m}{l}$$

$$\boxed{H_{1m} = 200\,\frac{A}{m}}$$




$$H(t) = \frac{N_1 i_1(t)}{l}$$






$$H_1 = 120\ A/m \Rightarrow \quad i_1(t_0) = \frac{H_1(t_0)\ell}{N_1} = \frac{120\cdot 0{,}3}{600} = \underline{\underline{0{,}06\ A}}$$

$$i_1(t_0) = \frac{4I_m}{T} t_0$$

$$t = \frac{T}{4I_m} \quad i_1(t_0) = 0{,}6s$$

$$H \in (120 \to 0) \quad B = B_m$$
2ms

$$e_{ind} = -\frac{d\phi}{dt} = -\frac{d}{dt}(Bs)$$

$$u_2 = -e_{ind} = \frac{d}{dt}(Bs) = N_2 S \frac{d}{dt} B(t)$$

$$\frac{d}{dt}(B(t)) = \begin{cases} 0, & t \in (0{,}6+n\cdot 2;\ 2+n\cdot 2) \\ \frac{1}{0{,}6} 10^3, & t \in (0+n\cdot 4;\ 0{,}6+n\cdot 4) \\ -\frac{1}{0{,}6} 10^3 & t \in (2+n\cdot 4;\ 2{,}6+n\cdot 4) \end{cases}$$

$$U_m = 2\cdot 10^{-4} \cdot \frac{1}{0{,}6} \cdot 10^3 \cdot 300 = \frac{60}{0{,}6} = \underline{\underline{100V}}$$

~~(185)~~ ~~(186)~~ ~~(187)~~ ~~(188)~~

(189) $a = 30$ nm

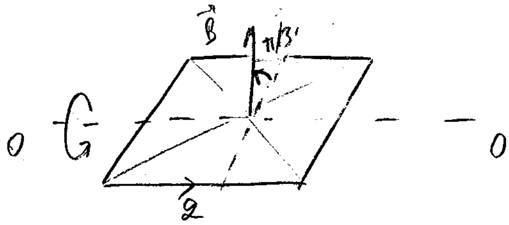
$R = 1{,}5\,\Omega$

$B = 1T$

$\alpha = \frac{\pi}{2}$

---

$q = ?$




$$q = -\frac{\Delta\phi}{R}$$

$$\Delta\phi = \phi^{(2)} - \phi^{(1)}$$

$$\phi^{(1)} = Bs\cos\left(\frac{\pi}{2} - \frac{\pi}{3}\right) = Bs\frac{\sqrt{3}}{2}$$

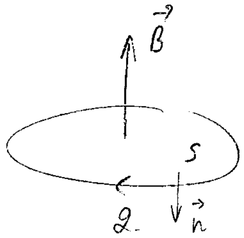
$$\phi^{(2)} = Bs\cos\left(\frac{\pi}{2} + \frac{\pi}{2} - \frac{\pi}{3}\right) = Bs\cos\frac{2\pi}{3} = -\frac{1}{2}Bs$$

$$\Delta\phi = -\frac{1}{2}Bs - \frac{1}{2}Bs\sqrt{3} = -\frac{1}{2}Ba^2\pi(1+\sqrt{3})$$

$$\boxed{q = \frac{1}{2R}Ba^2\pi(1+\sqrt{3})}$$

$$\boxed{q \approx 819\,\mu C}$$

(190) $R, L, B, s$




$$q = -\frac{\Delta\phi}{R}$$

$$\Delta\phi = \phi^{(2)} - \phi^{(1)} = +Bs$$

$$\phi^{(2)} = 0$$

$$\phi^{(1)} = -Bs$$

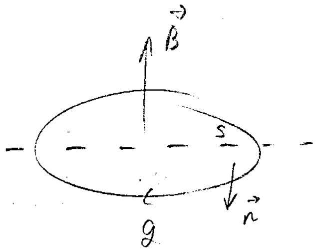
$$\boxed{q = \frac{-Bs}{R}}$$

(191) $R, L, B, s, \varphi = 90°$

$q = ?$

$$q = -\frac{\Delta\phi}{R} = -\frac{0-(-Bs)}{R}$$

$$\boxed{q = -\frac{Bs}{R}}$$

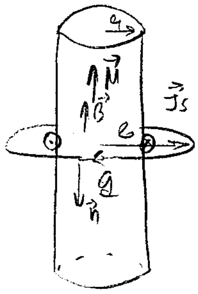



(192) $a, M, b>a, R, q = ?$

$$\oint_C \vec{H}\,d\vec{l} = \Sigma I$$

$$H \cdot 2r\pi = 0$$

$$\boxed{H = 0}$$




$|\vec{J}_{SA}| = |\vec{M}|$

$\vec{J}_A = 0 \quad (\vec{M} = const)$

$$B = \frac{\mu_0 I}{2a\pi}$$

$$B = \mu_0 J_s = \mu_0 M$$

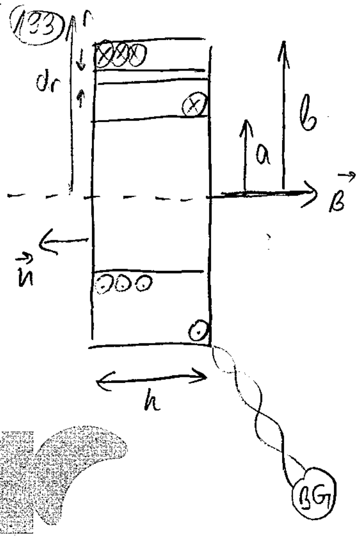
$$\phi^{(1)} = -Bs = -\mu_0 M a^2\pi$$

$$q = -\frac{\Delta\phi}{R}$$

$$\Delta\phi = \cancel{\phi^{(2)}}^{0} - \phi^{(1)} = -\phi^{(1)} = \mu_0 M a^2\pi$$

$$\boxed{q = -\frac{\mu_0 M a^2\pi}{R}}$$

* (193)




$N, R, B, q = ?$

$\phi \to B$

$$q = -\frac{\Delta\phi}{R}$$

$$\phi^{(1)} = 0$$

$$dN = \frac{N}{a-b}\,dr$$

$$\measuredangle(\vec{B}, \vec{s}) = 180°$$

$$d\phi^{(2)} = -dN\, B\, r^2\pi$$

$$\phi^{(2)} = \frac{-N}{b-a}\int_a^b B r^2 \pi\, dr$$

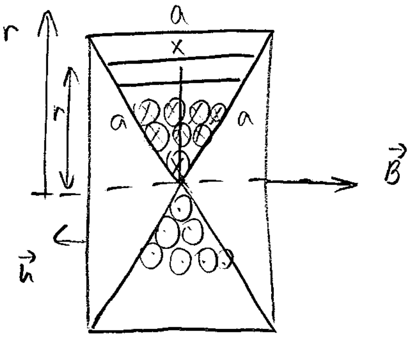
$$\phi^{(2)} = \frac{-NB\pi}{b-a}\,\frac{b^3-a^3}{3} = \frac{1}{3} NB\pi \frac{(b-a)(b^2+ab+a^2)}{b-a}$$

$$\boxed{\phi^{(2)} = -\frac{1}{3} NB\pi\,(b^2+ab+a^2)}$$

$$g = -\frac{\Delta\phi}{R} \qquad \boxed{g = \frac{NB\pi}{3R}\left(a^2+ab+b^2\right)}$$

*(194) $a, N, R, B, g = ?$

$B \to 0$




$$dN = \frac{N}{\frac{a^2\sqrt{3}}{4}} \, x \, dr = \frac{4N}{a^2\sqrt{3}} \cdot \frac{2r}{\sqrt{3}} dr = \frac{8N}{3a^2} r\, dr$$

$$r = x\frac{\sqrt{3}}{2} \Rightarrow x = \frac{2r}{\sqrt{3}}$$

$$\phi^{(1)} = -\int_{r=0}^{\frac{a\sqrt{3}}{2}} dN\, B \underbrace{r^2\pi}_{ds} = -\int_{r=0}^{a\sqrt{3}/2} \frac{8N\pi}{3a^2} B r^3 dr = -\frac{\cancel{8}^{2} N\pi B}{\cancel{3a^2}} \frac{\cancel{9}^{3} a^{\cancel{4}\,2}}{\cancel{4\cdot 16}_{2}}$$

$$\phi^{(1)} = -\frac{3}{8} N\pi B a^2 \qquad \phi^{(2)} = 0$$

$$g = -\frac{\Delta\phi}{R} = \frac{-3}{8R} N\pi B a^2 \qquad \boxed{\Delta\phi = \frac{3}{8} N\pi B a^2}$$

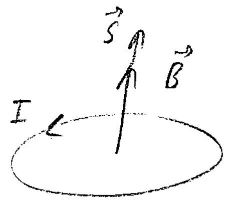
(195) $a, L, B, I = ?$

$$\phi^{(1)} = 0 \qquad \Delta\phi = 0$$

$$\phi^{(2)} = 0$$

$$\phi^{(2)} = LI + Ba^2\pi = 0$$

$$\boxed{I = -\frac{Ba^2\pi}{L}}$$

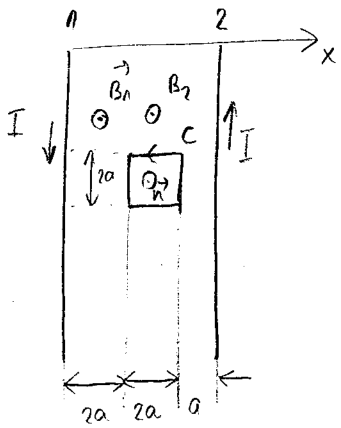



(196) $a = 0{,}1\,m$

$R = 2\Omega$

a) $L_{12} = ?$

б) $g = ?$




a) $B_1 = \frac{\mu_0 I}{2\pi x} \qquad B_2 = \frac{\mu_0 I}{2\pi(5a-x)}$

$$B = B_1 + B_2$$

$$\phi_c = \int_S B\, ds = \int_{2a}^{4a} \left(\frac{\mu_0 I}{2\pi x} + \frac{\mu_0 I}{2\pi(5a-x)}\right) 2a\, dx$$

$$\phi_c = \frac{\mu_0 I}{\pi} a \left[\ln 2 + \ln\frac{5a-2a}{5a-4a}\right]$$

$$\phi_c = \frac{\mu_0 I a}{\pi}\left[\ln 2 + \ln 3\right]$$

$$L_{12} = \frac{\phi_c}{I} \qquad \boxed{L_{12} = \frac{\mu_0 a}{\pi}\left[\ln 2 + \ln 3\right]} \qquad L_{12} = \frac{\mu_0 a}{\pi}\ln 6 = 71{,}7\,nH$$

$$q = -\frac{\Delta\phi}{R} \qquad \phi^{(1)} = 0 \qquad \phi^{(2)} = \phi_c$$

$$\Delta\phi = \phi_c = L_{12} I$$

$$\boxed{q = -\frac{L_{12} I}{R}} \qquad \underline{\underline{q \approx -358\,nC}}$$

(197) $E = 10V$

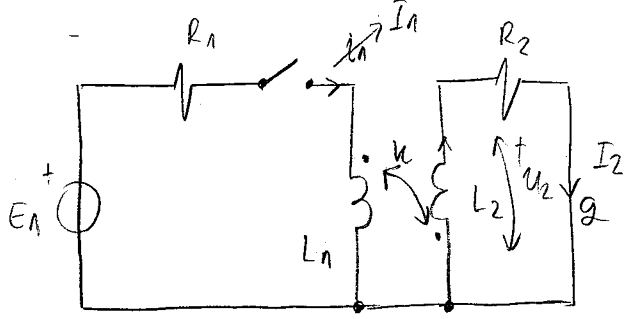
$R = R_1 = R_2 = 5\Omega$

$L = L_1 = L_2 = 0{,}1H$

$k = 0{,}5$

$q = ?$

П отв → зaтв




$L_{12} = +k\sqrt{L_1 L_2} = +0{,}05H$

$$I_1^{(2)} = \frac{E_1}{R_1} = \frac{E}{R}$$

$$u_1 = L_1 \frac{di_1}{dt} + L_{12} \frac{di_2}{dt}$$

$$u_2 = -\left(L_{12} \frac{di_1}{dt} + L_2 \frac{di_2}{dt}\right)$$

$$e_{ind} = -\frac{d\phi}{dt} \qquad u = \frac{d\phi}{dt} \Rightarrow \phi = u\,dt$$

(1) П отв : $I_1^{(1)} = I_2^{(1)} = 0$

$$\phi_2^{(1)} = L_{21} I_1^{(1)} + L_2 I_2^{(1)} = 0$$

(2) П зaтв: $I_1^{(2)} = \frac{E}{R}$ ; $I_2^{(2)} = 0$

$$\Phi_2^{(2)} = L_{12} I_1^{(2)} + \cancel{L_2 I_2^{(2)}}^{=0} = L_{12} I_1^{(2)}$$

$$\Delta\phi_2 = \phi^{(2)} - \phi^{(1)} = -L_{12}\frac{E}{R}$$

$$q = -\frac{\Delta\phi_2}{R}$$

$$\boxed{q = -\frac{kLE}{R^2}}$$

$q = -20mC$

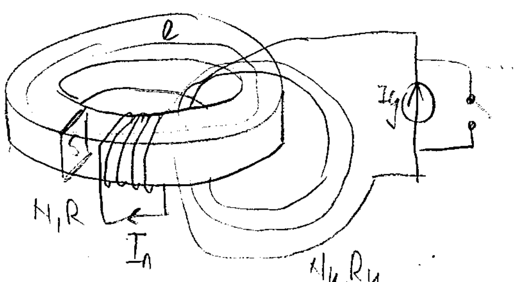
(198) $\mu_r = 500$ $\quad N_k = 3$

$S = 2cm^2$ $\quad R_k = 1\Omega$

$l = 0{,}15m$ $\quad I_g = 2A$

$N = 300$ $\quad$ П $0 \to 3$

$R = 10\Omega$ $\quad q = ?$




$$\oint \vec{H}d\vec{l} = \sum I$$

$$H_1 l = N I_1$$

$$H_1 = \frac{NI_1}{l}$$

$$B_1 = \mu_0 \mu_r H_1 = \mu_0 \mu_r \frac{NI_1}{l}$$

$$L_{12} = \frac{\Phi_{21}}{I_1} = \frac{N_k B_1 S}{I_1} = \underline{\underline{\frac{\mu_0 \mu_r N N_k S}{l}}}$$

(1) П отв: $I_2^{(1)} = I_g$

$\Phi_1^{(1)} = L_{12} I_2^{(1)}$

(2) П затв: $I_2^{(2)} = 0$

$\Phi_1^{(2)} = 0$

$\Delta\Phi = -L_{12} I_g$

$q = \dfrac{L_{12} I_g}{R}$

$$\boxed{q = \frac{\mu_0 \mu_r h_1 N_1 N_2 S}{l R}}$$

(199) $N_1 = 100$ $S = 2cm^2$

$N_2 = 400$ $l = 20cm$

$R_1 = 12\Omega$

$R_2 = 4\Omega$

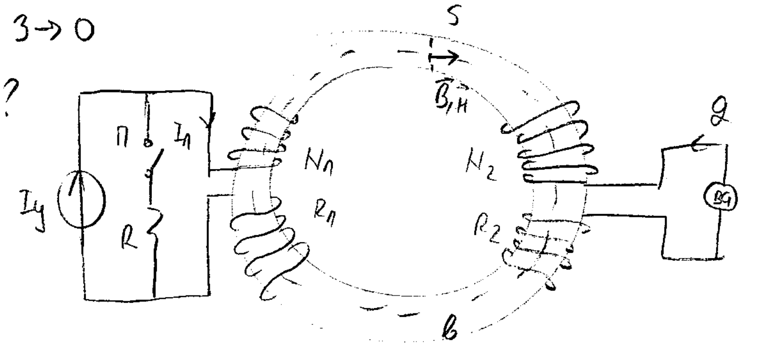
$I_g = 1{,}8A$

$R = 6\Omega$

П: 3 → 0

$q = ?$

| B[T] | 0,6 | 0,75 | 0,86 | 1,03 | 1,16 |
|---|---|---|---|---|---|
| H[A/m] | 200 | 300 | 400 | 600 | 900 |




$\oint_{C_1} \vec{H} d\vec{l} = \Sigma I$

$H_1 l = N_1 I_1^{(1)}$

$H_1 = \dfrac{N_1 I_1}{l}$

(1) П. затв

$I_1^{(1)} = I_g \dfrac{R}{R+R_1} = 1{,}8 \dfrac{6}{18} = 0{,}6A$

$I_2^{(1)} = 0$

$\Phi_2^{(1)} = N_2 B_1^{(1)} S$

(2) П. отв.

$I_1^{(2)} = I_g$

$I_2^{(2)} = 0$

$\Phi_2^{(2)} = N_2 B_1^{(2)} S$

$H_1^{(1)} = \dfrac{N_1 I_1^{(1)}}{l}$

$H_1^{(1)} = 300\,A/m$

$\boxed{B_1^{(1)} = 0{,}75T}$

$H_1^{(2)} = \dfrac{N_1 I_1^{(2)}}{l}$

$H_1^{(2)} = 900\,A/m$

$\boxed{B_1^{(2)} = 1{,}16T}$

$$\Delta\Phi = \Phi_2^{(2)} - \Phi_2^{(1)} = N_2 S (B_1^{(2)} - B_1^{(1)})$$

$$q = -\frac{\Delta\Phi}{R_2} = \frac{N_2 S}{R_2}(B_1^{(1)} - B_1^{(2)})$$

$$q = \frac{400 \cdot 2 \cdot 10^{-4}}{4}(-0{,}41)\,C$$

$$\boxed{q = 8{,}2 \cdot 10^{-3}\,C}$$

(200) $l = 0,2\,m$

$S = 4 \cdot 10^{-4}\,m^2$

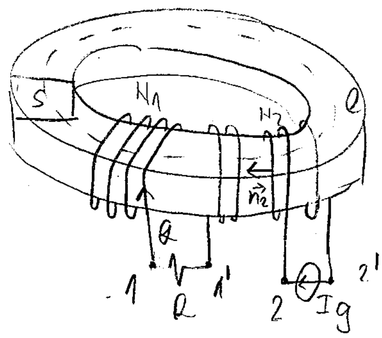
$\mu_r = 1000$

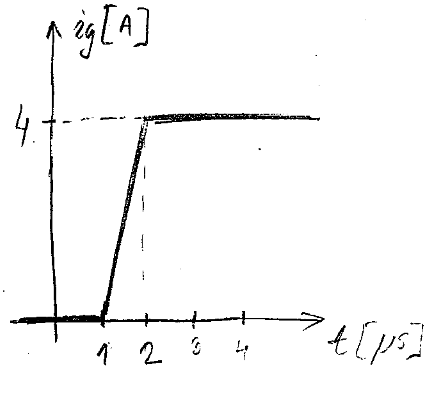
$N_1 = 1000$

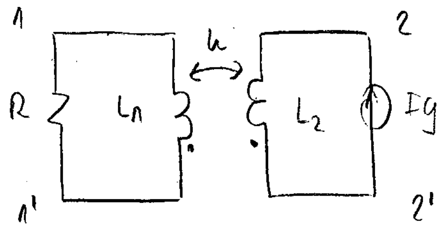
$N_2 = 200$

$R = 10\,\Omega$

$q = ?$




$L_{12} = k\sqrt{L_1 L_2}$

$(k=1)$

$\Uparrow$

$$L_1 = \frac{\mu_0 \mu_r N_1^2 S}{l} \qquad L_2 = \frac{\mu_0 \mu_r N_2^2 S}{l} \qquad L_{12} = \frac{\mu_0 \mu_r N_1 N_2 S}{l} = 0,5H$$

$$q = -\frac{\Delta\phi}{R} \qquad \Delta\phi = \phi_2 - \phi_1 = L_{12} I_2 - L_{12} I_1 = L_{12} I_2$$

$$I_2 = 4A \qquad I_1 = 0$$

$$q = -\frac{L_{12} I_2}{R} = -\frac{0,5 \cdot 4}{10} = \underline{\underline{-0,2C}}$$

(201) $l = 0,5\,m$ $\quad N_1 = 1000$

$l_0 = 2 \cdot 10^{-4}\,m$ $\quad R_1 = 10\,\Omega$

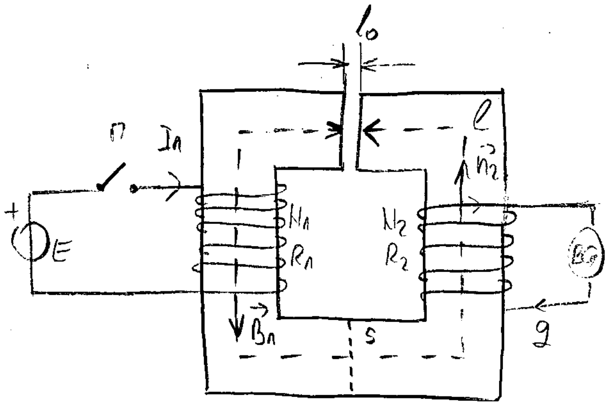
$S = 5 \cdot 10^{-4}\,m^2$ $\quad N_2 = 500$

$\frac{B}{B_m} = arctg \frac{H}{H_0}$ $\quad R_2 = 100\,\Omega$

$B_m = 1,5T$ $\quad П: 0 \to 3$

$H_0 = 1000\,A/m$ $\quad q = 4mC$

$E = ?$




$$I_1^{(1)} = 0 \qquad I_1^{(2)} = \frac{E}{R_1}$$

$H_0$ и $H_0^{(2)}$ није исто, $H_0$ је const!

$$\oint_C \vec{H} d\vec{l} = \sum I$$

$$H_1^{(1)} l + H_0^{(1)} l_0 = 0 \qquad H_1^{(2)} l + H_0^{(2)} l_0 = N_1 I_1$$

$$H_1^{(1)} = 0 \qquad B_0 = \mu_0 H_0 \Rightarrow \boxed{H_0^{(2)} = \frac{B_0}{\mu_0} = \frac{B_1^{(2)}}{\mu_0}}$$

$$B_1^{(1)} = 0$$

$$\frac{B_1^{(2)}}{B_m} = arctg \frac{H_1^{(2)}}{H_0} \Rightarrow \boxed{H_1^{(2)} = H_0\, tg \frac{B_1^{(2)}}{B_m}}$$

$$\phi_2^{(2)} = N_2 B_1^{(2)} S$$

$$q = -\frac{\Delta\phi}{R_2} = \frac{-N_2 B_1^{(2)} S}{R_2}$$

$$B_1^{(2)} = -\frac{q R_2}{N_2 S} = -1{,}6T$$

$$H_0 \, tg \frac{B_1^{(2)}}{B_m} l + \frac{B_1^{(2)}}{\mu_0} l_0 = N_1 \frac{E}{R_1}$$

$$E = \frac{R_1}{N_1}\left[H_0 \, tg \frac{B_1^{(2)}}{B_m} l + \frac{B_1^{(2)}}{\mu_0} l_0\right]$$

$$E = \frac{10}{1000}\left[1000 \cdot 0{,}5 \, tg\left(-\frac{1{,}6}{1{,}5}\right) - \frac{1{,}6}{4\pi \cdot 10^{-7}} \cdot 2 \cdot 10^{-4}\right] V$$

$$E = 0{,}01\left[-1161{,}105\right] V$$

радијани!!

$$\boxed{E \approx -11{,}6V}$$

(202) $L_1 = 1mH$

$L_2 = 4mH$

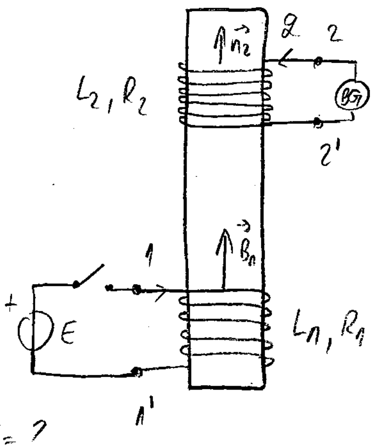
$R_1 = 1\Omega$

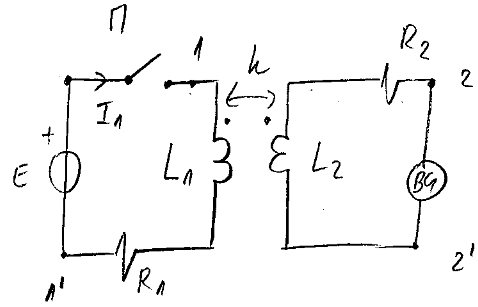
$R_2 = 2\Omega$

$k = 0{,}2$

$E = 10V$

$\Pi: 0 \to 3 \quad q = ?$




$$L_{12} = k\sqrt{L_1 L_2}$$

$$I_1^{(0)} = 0 \qquad \phi_2^{(0)} = \cancel{L_2 I_2^{(0)}} + \cancel{L_{12} I_1^{(0)}} = 0$$

$$I_1^{(3)} = \frac{E}{R_1} \qquad \phi_2^{(3)} = \cancel{L_2 I_2^{(3)}} + L_{12} I_1^{(3)} = L_{12}\frac{E}{R_1}$$

$$\Delta\phi = \phi_2^{(3)} - \phi_2^{(0)} = L_{12}\frac{E}{R_1} \qquad q = -\frac{\Delta\phi}{R_2}$$

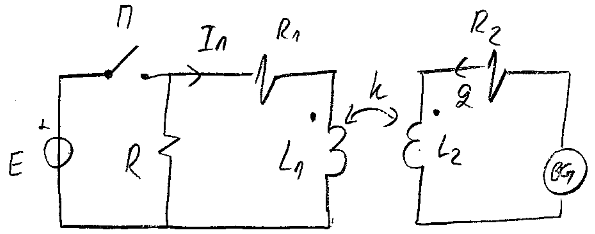
$$\boxed{q = \frac{-k\sqrt{L_1 L_2}\,E}{R_1 R_2}} \qquad q = -2mC$$

(203) $L_1 = 1mH$  $R_1 = 1\Omega$

$L_2 = 4mH$  $R_2 = 2\Omega$

$E = 10V$  $R = 8\Omega$

$q = 1{,}8mC$




$L_{12} = k\sqrt{L_1 L_2}$

б) $k = ?$  $П: 3 \to 0$

$$I_1 = \begin{cases} \frac{E}{R_1}, & П \text{ ЗАТВ.} \\ 0, & П \text{ ОТВ} \end{cases}$$

$$\phi_2^{(3)} = \cancel{I_2^{(3)}}^{\,0} L_2 + I_1^{(3)} L_{12} = I_1^{(3)} L_{12}$$

$$\phi_2^{(0)} = \cancel{I_2^{(0)}}\,\cancel{L_2}^{\,0} + \cancel{I_1^{(0)} L_{12}}^{\,0} = 0$$

$$\Delta\phi = \phi_2^{(0)} - \phi_2^{(3)} = -$$

$$q = -\frac{\Delta\phi}{R_2} = E\frac{1}{R_1}\frac{L_{12}}{R_2} \Rightarrow L_{12} = \frac{qR_1R_2}{E}$$

$$k = \frac{L_{12}}{\sqrt{L_1L_2}} \qquad \boxed{k = \frac{qR_1R_2}{E\sqrt{L_1L_2}}} \qquad \boxed{k \approx 0{,}18}$$

*(204) $a = 2cm$

$b = 4cm$

$h = 1cm$

$N_1 = 157 \simeq 50\pi$

$N_2 = 12$

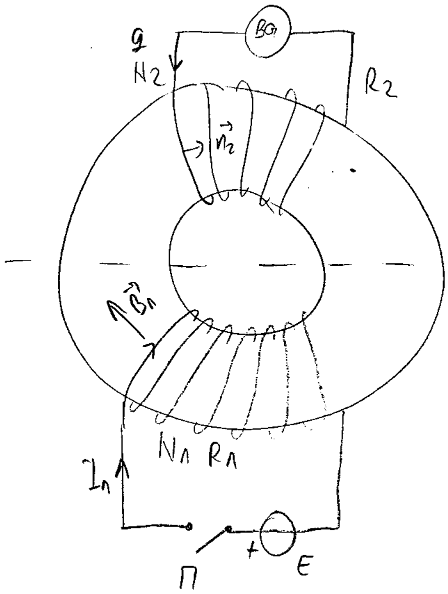
$R_2 = 2\Omega$

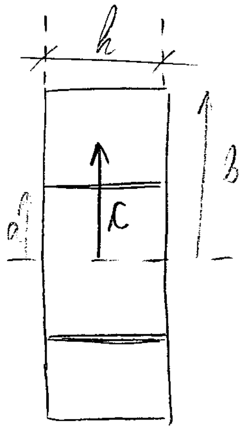
$R_1 = 10\Omega$

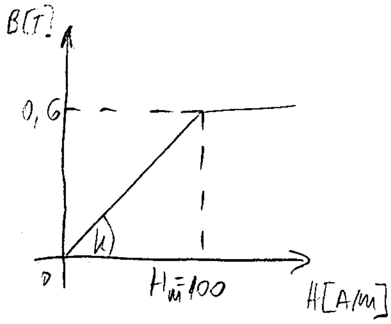
$E = 1V$

$q = ?$

$П: 0 \to 3$




$$q = -\frac{\Delta\phi}{R_2}$$

(1) $П: 0$  $\oint_C \vec{H}\,d\vec{l} = \Sigma I$

$H = 0 \Rightarrow B = 0$

$\phi = 0$

(2) $\oint_C \vec{H}\,d\vec{l} = \Sigma I$

$$H\,2r\pi = N_1 I_1^{(2)} = N_1\frac{E}{R_1}$$

$$H_m \cdot 2c\pi = N_1\frac{E}{R_1}$$

$$c = \frac{N_1 E}{2\pi H_m R_1} = \frac{50\pi \cdot 1}{2\pi \cdot 100 \cdot 10} = 2{,}5cm$$

$$B^{(2)} = \begin{cases} 0, & r<a \\ H\dfrac{B_m}{H_m} = \dfrac{N_1 E}{2r\pi R_1}\dfrac{B_m}{H_m}, & c<r<b \\ B_m = 0{,}6T, & a<r<c \\ 0, & r>b \end{cases}$$

$$\phi^{(2)} = N_2 \int_S \vec{B}^{(2)} d\vec{s} = N_2\left[\int_a^c B_m\, h\cdot dr + \int_c^b \frac{N_1 E B_m}{2\pi R_1 r H_m} h\,dr\right]$$

$$\phi^{(2)} = N_2 B_m h(c-a) + N_2 h\frac{N_1 E B_m}{2\pi R_1 H_m}\ln\frac{b}{c}$$

$$q = -\frac{\phi^{(2)} - \cancel{\phi^{(1)}}^{0}}{R_2} = -\frac{N_2 h\left[B_m(c-a) + \dfrac{N_1 E B_m}{2\pi R_1 H_m}\ln\dfrac{b}{c}\right]}{R_2}$$

$$\boxed{q = -602{,}789\,\mu C}$$

# OET Рокови (колоквијуми)

**2015**

П ① $C_1, C_2, I_1, I_2$

а) $d\vec{F}_{m_2} = ?$

б) $\vec{F}_{m_2} = ?$

а) $d\vec{F}_{m_2} = Q_2\vec{v}_2 \times d\vec{B}_1$

$$d\vec{B}_1 = \frac{\mu_0}{4\pi}\frac{dI\, d\vec{l}_1 \times \vec{r}_{012}}{r_{12}^2}$$

$$Q_2\vec{v}_2 = I_2 \cancel{dt}\frac{d\vec{l}_2}{dt} = I_2 d\vec{l}_2$$

$$d\vec{B}_1 = \frac{\mu_0}{4\pi}\frac{I_1 d\vec{l}_1 \times \vec{r}_{012}}{r_{12}^2}$$

$$d\vec{F}_{m_2} = \frac{\mu_0}{4\pi}\frac{I_2 d\vec{l}_2 \times (I_1 d\vec{l}_1 \times \vec{r}_{012})}{r_{12}^2}$$

б) $$\vec{F}_{m_2} = \frac{\mu_0}{4\pi}\oint_{C_2}\left(I_2 d\vec{l}_2 \times \oint_{C_1}\frac{(I_1 d\vec{l}_1 \times \vec{r}_{012})}{r_{12}^2}\right)$$

② $\mu_r = 100$ $\quad N = 10^{20}\cdot 10^{+6} = 10^{26}\,m^{-3}$

$\vec{H} = 100\,\vec{i}_z$ A/m

а) $\vec{B} = ?$ $\quad \vec{B} = \mu\vec{H} = \mu_0\mu_r\vec{H} = 100\cdot 100\cdot\frac{4\pi}{10^7}\vec{i}_z = \frac{\pi}{250}\vec{i}_z\,T$

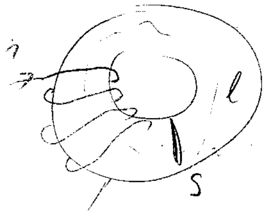
б) $\vec{M} = ?$ $\quad \vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M} \Rightarrow \vec{M} = \frac{\vec{B}}{\mu_0} - \vec{H} = \vec{H}(\mu_r - 1) = 9900\,A/m\,\vec{i}_z$

в) $\vec{J}_A = ?$ $\quad \vec{J}_A = 0 \quad (\vec{M} = const)$

г) $\vec{m} = ?$ $\quad \vec{M} = \sum_i \vec{m} = \vec{m}\cdot N \Rightarrow \vec{m} = \frac{\vec{M}}{N} = \frac{9900\,A/m\,\vec{i}_z}{10^{26}\,m^{-3}} = 9{,}9\cdot 10^{-23}\,Am^2\,\vec{i}_z$

д) $w_m = ?$ $\quad w_m = \frac{1}{2}\vec{B}\cdot\vec{H} = \frac{1}{2}\cdot\frac{\pi}{250}\cdot 100 = \frac{\pi}{5}\,J/m^3$

③ $l, s, N, I_m, f, \frac{B_m}{H_m} = \mu_n$




$P = ?$ $\qquad i(t) = I_m \cos(2\pi f t)$

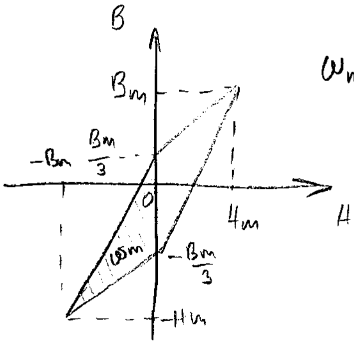
$$P = \frac{1}{T}\int_V w_m\, dv = f\int_V w_m dv = \frac{2}{3} f B_m^2/\mu_n\, s\, l$$

$$\oint_C \vec{H}d\vec{l} = \sum_i I$$

$$H \cdot l = NI$$

$$H = \frac{NI}{l}$$

$$B^2 = \mu_n^2 H^2 = \mu_n^2 \frac{N^2 I_m^2}{l^2}$$




$$w_m = 2\left(\frac{1}{2}\frac{4B_m}{3}H_m - \frac{1}{2}\frac{2B_m}{3}H_m\right) =$$

$$= \frac{2}{3}B_m \frac{B_m}{\mu_n} = \frac{2}{3}\frac{B_m^2}{\mu_n}$$

$$P = \frac{2}{3}f\frac{\mu_n N^2 I_m^2}{l^{\cancel{2}}}s\cancel{l} = \frac{2f\mu_n N^2 I_m^2 s}{3l}$$

④

$$\oint_C \vec{E}d\vec{l} = -\frac{d\Phi}{dt} = -\int_S \frac{d\vec{B}}{dt}d\vec{s} \qquad I$$

$$\oint_S \vec{D}d\vec{s} = Q_{un} = \int_V \rho\, dv \qquad III$$

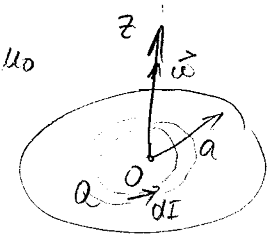
$$\oint_C \vec{H}d\vec{l} = \sum I = \int_S \left(\vec{J} + \frac{\partial\vec{D}}{\partial t}\right)d\vec{s} \qquad II$$

$$\oint_S \vec{B}d\vec{s} = 0 \qquad IV$$

3 ⑴ $Q, a, \omega$ $\qquad \varepsilon_0, \mu_0$

a) $\vec{E} = ?$

б) $\vec{B}_0 = ?$




$$\rho_s = \frac{Q}{a^2\pi}$$

$$J_s = \rho_s \cdot v = \rho_s \omega r$$

$$dI = J_s\, dr = \frac{Q}{a^2\pi}\omega r\, dr$$

a) $$E = \frac{\rho_s}{2\varepsilon_0} = \frac{Q}{2\varepsilon_0 a^2\pi}$$

$$\boxed{\vec{E} = E\vec{i}_z}$$

б) $$dB = \frac{\mu_0}{4\pi}\int_0^{2\pi}\frac{dI}{r}d\theta = \frac{\mu_0 dI}{\cancel{4}_2\pi r}\cancel{2\pi}$$

$$dB = \frac{\mu_0 Q\omega \cancel{r}}{2\cancel{r}a^2\pi}dr$$

$$B = \int_0^a dB = \frac{\mu_0 Q\omega}{2a^2\pi}\int_0^a dr = \frac{\mu_0 Q\omega}{2a\pi}$$

$$\vec{B} = B\vec{i}_z \qquad \boxed{\vec{B} = \frac{\mu_0 Q\vec{\omega}}{2a\pi}}$$

② $a_1 = 20\,mm$

$a_2 = 20\sqrt{2}\,mm$

$r = 40\,mm$

$N_1 = 500$

$N_2 = 1000$

$\mu_r = 48$

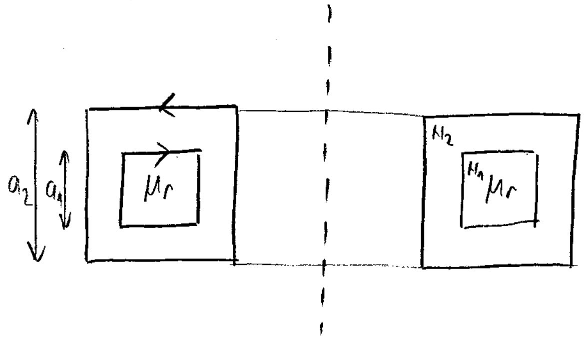
a) $L_1, L_2, L_{12} = ?$

б) $k = ?$

в) $E_m = 49V$

$f = 100\,kHz$

$U_{1m} = ?$




$L_1$: $L_1 = \frac{\Phi_1}{I_1}$ ; $\Phi_1 = N_1 \int_S \vec{B}_1 d\vec{s} = \frac{\mu_0 \mu_r N_1^2 I_1}{2\pi r} a_1^2 = \frac{\mu_0 \mu_r N_1^2 a_1^2 I_1}{2\pi r}$

$\oint_C \vec{H} d\vec{l} = \sum I$

$H_1 2r\pi = N_1 I_1$

$B_1 = \mu_0 \mu_r H_1 = \frac{\mu_0 \mu_r N_1 I_1}{2\pi r}$

$$L_1 = \frac{\Phi_1}{I_1} = \frac{\mu_0 \mu_r N_1^2 a_1^2}{2\pi r}$$

$L_2$: $L_2 = \frac{\Phi_2}{I_2}$ ; $\Phi_2 = N_2 \int \vec{B}_2 d\vec{s}$

$B_2' = \frac{\mu_0 \mu_r N_2 I_2}{2r\pi}$

$B_2 = \frac{\mu_0 N_2 I_2}{2r\pi}$

$\Phi_2 = N_2 \left[ \int_{S_1} B_2 ds_1 - \int_{S_2} B_2 ds_2 + \int_{S_2} B_2' ds_2 \right] =$

$= N_2 \left[ \frac{\mu_0 N_2 I_2}{2r\pi} (a_2^2 - a_1^2) + \frac{\mu_0 \mu_r N_2 I_2}{2r\pi} a_1^2 \right]$

$= \frac{\mu_0 N_2^2 I_2}{2r\pi} \left[ a_2^2 + a_1^2(\mu_r - 1) \right]$

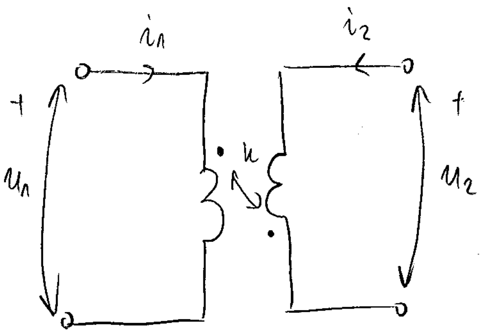
$$\boxed{L_2 = \frac{\Phi_2}{I_2} = \frac{\mu_0 N_2^2}{2\pi r} \left[ a_2^2 + a_1^2(\mu_r - 1) \right]}$$

$L_{12}$: $L_{12} = \frac{\Phi_{12}}{I_2}$

$\Phi_{12} = -N_1 \int_{S_1} \vec{B}_2' d\vec{s}_1 = -\frac{\mu_0 \mu_r N_2 N_1 I_2}{2r\pi} a_1^2$

$$\boxed{L_{12} = L_{21} = \frac{-\mu_0 \mu_r N_1 N_2}{2r\pi} a_1^2}$$

$k$: $\boxed{k = \frac{|L_{12}|}{\sqrt{L_1 L_2}}}$




$$u_1 = L_1 \cancel{\frac{di_1}{dt}}^{\,0} + L_{12}\frac{di_2}{dt}$$

$$u_2 = \cancel{L_{12}\frac{di_1}{dt}}^{\,0} + L_2\frac{di_2}{dt}$$

$$i_1 = 0$$

$$u_2 = L_2\frac{di_2}{dt} = E_m\cos(2\pi f t)$$

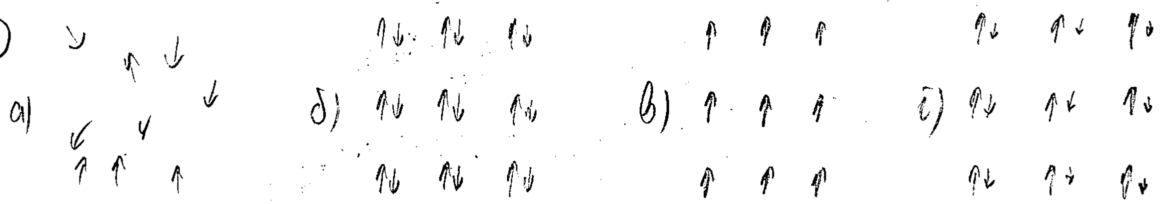
$$\frac{di_2}{dt} = \frac{E_m}{L_2}\cos(2\pi f t)$$

$$u_1 = \underbrace{\frac{L_{12}}{L_2}E_m}_{U_{2m}}\cos(2\pi f t)$$

$$\boxed{U_{2m} = \frac{|L_{12}|}{L_2}E_m}$$

2014

П 1




2 $\frac{l}{l_0} = 100$

$S = 3\,cm^2$

$B_r = 200\mu_0 H_c = 2mT$

$R = 10$

$q = ?$

$$H_c = \frac{2mT}{200\mu_0}$$

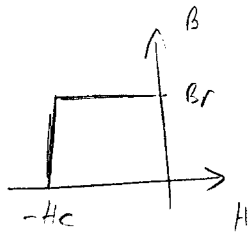
$$\oint_C \vec{H}d\vec{l} = \sum I = 0$$

$$Hl + H_0 l_0 = 0$$

$$Hl + \frac{B}{\mu_0}l_0 = 0 \;/ : l_0$$

$$H\frac{l}{l_0} + \frac{B}{\mu_0} = 0$$

$$100H + \frac{B}{\mu_0} = 0$$




$$q = -\frac{\Delta\Phi}{R} = -\frac{0-\Phi}{R} = -\frac{B_s}{R}$$

$$q = -\frac{1\cdot 3\cdot 10^{-4}}{10}$$

$$\boxed{q = -3\cdot 10^{-8}\,C}$$

$$\boxed{q = -30\,nC}$$

1° $B = B_r \quad -H_c < H < 0$

$$H = -\frac{B_r}{\mu_0}\cdot\frac{1}{100} = -\frac{\overset{2}{200}\mu_0 H_c}{\cancel{\mu_0}\,\cancel{100}} = -2H_c \;\perp$$

2° $H = -H_c \quad 0 < B < B_r$

$$B = -100\mu_0(-H_c) = 100\cancel{\mu_0}\frac{2mT}{\underset{2}{\cancel{200}\cancel{\mu_0}}} = \underline{\underline{1mT}}$$

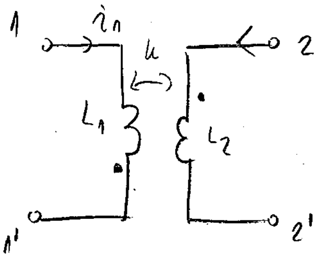
③ $L_1 = 10\,mH$

$L_2 = 90\,mH$

$k = 0{,}5$

$i_1(t) = I_m \sin \omega t$

$I_m = 3A \quad \omega = 10^3 s^{-1}$




a) $L_{12} = ?$

б) $u_{22'} = ?$

в) $U_{22m} = ?$

a) $L_{12} = -k\sqrt{L_1 L_2} = -15\,mH$

б) $u_{11'} = \frac{di_1}{dt} L_1 + \cancel{\frac{di_2}{dt}}^{0} L_2$

$u_{22'} = L_{12}\frac{di_1}{dt} + L_2 \cancel{\frac{di_2}{dt}}^{0}$

б) $u_{22'} = L_{12} I_m \cos\omega t \;\; \omega$

$u_{22'} = -15 \cdot \cancel{10^{-3}} \cdot 3 \cdot \cancel{10^3} = -45\cos\omega t \; V$

в) $U_m = 45V$

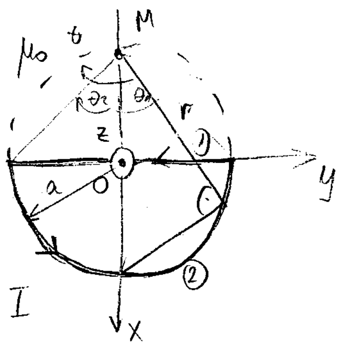
④ I $\oint_C \vec{E}\,d\vec{l} = -\frac{d\phi}{dt} = -\int_S \frac{d\vec{B}}{dt}\,d\vec{s}$

III $\oint_C \vec{H}\,d\vec{l} = \sum I = \int_S \left(\vec{J} + \frac{d\vec{D}}{dt}\right) d\vec{s}$

II $\int_S \vec{D}\,d\vec{s} = Q_{un} = \int_V \rho\, dv$

IV $\oint_S \vec{B}\,d\vec{s} = 0$

3①




① $B_1 = \frac{\mu_0 I}{4\pi d}(\sin\theta_2 - \sin\theta_1) = \frac{\mu_0 I}{4\pi a}\sqrt{2} = \frac{\mu_0 I\sqrt{2}}{4\pi a}$

$d = a$

$\vec{B}_1 = -B_1 \vec{i}_z$

② $\boxed{r = 2a\cos\theta}$

$B_2 = \frac{\mu_0}{4\pi}\int \frac{I\,d\theta}{r} = \frac{\mu_0 I}{4\pi\, 2a}\int_{-\pi/4}^{\pi/4} \frac{d\theta}{\cos\theta} \qquad \vec{B}_2 = B_2 \vec{i}_z$

$\int \frac{d\theta}{\cos\theta}\cdot\frac{\cos\theta}{\cos\theta} = \int \frac{\cos\theta\,d\theta}{1-\sin^2\theta} = \left| \begin{matrix} \sin\theta = t \\ \cos\theta\,d\theta = dt \end{matrix} \right| = \int \frac{dt}{1-t^2} = \int \frac{dt}{(1-t)(1+t)}$

$\frac{1}{1-t^2} = \frac{A}{1-t} + \frac{B}{1+t} = \frac{A + At + B - Bt}{1-t^2}$

$A + B = 1$

$A - B = 0 \Rightarrow A = B = \frac{1}{2}$

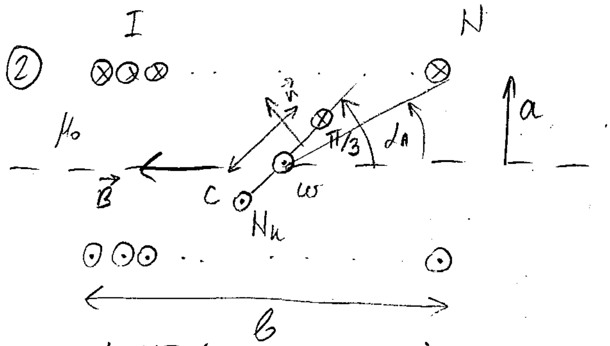
$\vec{B} = \vec{B}_1 + \vec{B}_2$

$\boxed{B = \frac{\mu_0 I}{8a\pi}\left[-2\sqrt{2} + \ln\frac{2+\sqrt{2}}{2-\sqrt{2}}\right]\vec{i}_z}$

$\int \frac{dt}{1-t^2} = \frac{1}{2}\int \frac{1\,dt}{1-t} + \frac{1}{2}\int\frac{1\,dt}{1+t} = \frac{1}{2}\ln\frac{1+t}{1-t} = \frac{1}{2}\ln\frac{1+\sin\theta}{1-\sin\theta}$

$B_2 = \frac{\mu_0 I}{8\pi a}\left[\frac{1}{2}\ln\frac{1+\frac{\sqrt{2}}{2}}{1-\frac{\sqrt{2}}{2}} - \frac{1}{2}\ln\frac{1-\frac{\sqrt{2}}{2}}{1+\frac{\sqrt{2}}{2}}\right] = \frac{\mu_0 I}{8a\pi}\frac{1}{2}\cancel{2}\ln\frac{2+\sqrt{2}}{2-\sqrt{2}} = \frac{\mu_0 I}{8a\pi}\ln\frac{2+\sqrt{2}}{2-\sqrt{2}}$

②




$b = 10\pi\sqrt{3}$ cm

$a = 5\pi$ cm

$N = 800$

$I = 1A$

$C = 1cm$

$N_u = 100$

a) $\phi(t) = ?$

b) $e_{max} = konst$, $\omega = ?$

$$B = \frac{\mu_0 N I}{2b}(\cos\alpha_1 - \cos\alpha_2)$$

$$B = \frac{\mu_0 N I}{2b}\left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right) = \frac{\mu_0 N I\sqrt{3}}{2b}$$

$$\cos\alpha_1 = -\cos\alpha_2 = \frac{b/2}{\sqrt{a^2 + \frac{b^2}{4}}} =$$

$$= \frac{5\pi\sqrt{3}}{\sqrt{25\pi^2 + \frac{300\pi^2}{4}}} = \frac{5\pi\sqrt{3}}{10\pi} = \frac{\sqrt{3}}{2}$$

$$e_{ind} = -\frac{d\phi}{dt}$$

$$\phi = N_u B \cdot S \qquad \underline{S = c^2}$$

$$\phi(t) = N_u B c^2 \cdot \cos\left(\pi - \omega t - \frac{\pi}{2} - \frac{\pi}{3}\right) = \frac{\mu_0 N I N_u c^2 \sqrt{3}}{2b}\cos\left(\omega t - \frac{\pi}{6}\right)$$

$$e = -\frac{d\phi}{dt} = \underbrace{\frac{\mu_0 N I N_u c^2\sqrt{3}}{2b}}_{e_{ind\,max}}\,\omega \sin\left(\omega t - \frac{\pi}{6}\right)$$

$$\omega = \frac{e_{ind\,max}\, 2b}{\mu_0 N I N_u c^2\sqrt{3}} = \frac{40\cdot\cancel{10^{-3}}\cdot 2\cdot 10\cancel{\pi}\cdot\cancel{\sqrt{3}}\cdot\cancel{10^{-2}}}{4\cancel{\pi}\cdot\cancel{10^{-7}}_{10^{-2}}\cdot 800\cdot 1\cdot 100\cdot 1\cdot 10^{-4}\cancel{\sqrt{3}}} = \frac{\cancel{800}}{4\cdot\cancel{800}\cdot 100\cdot 10^{-6}}$$

$$\omega = \frac{10^6}{400} = \frac{10000\cancel{00}}{4\cancel{00}} = \underline{\underline{2500\ rad/s}}$$

2009

1) a) $B = \frac{\mu_0}{4\pi}\int_{\theta} \frac{I\,d\theta}{r}$

б) $r = a$ $\quad B = \frac{\mu_0}{4\pi}\int_0^{2\pi}\frac{I\,d\theta}{a} = \frac{\mu_0 I}{4a\pi}\,2\pi = \underline{\underline{\frac{\mu_0 I}{2a}}}$

в) $B = \frac{4\pi\cdot10^{-7}\cdot 200\cdot 10^{-3}}{2\cdot 3\cdot 10^{-2}} = \frac{400\pi}{3}\,10^{-8}\,T \approx \underline{\underline{4\mu T}}$

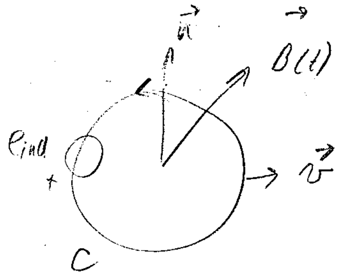
2) $\mu_r = 10$
$H = 1000\,\frac{A}{m}$
$B, M = ?$

$B = \mu_0\mu_r H = \frac{4\pi}{10^7}\cdot 10\cdot 1000 = \frac{4\pi}{10^3}\,T = \underline{\underline{4\pi\,mT}}$

$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M} \Rightarrow M = H\mu_r - H = H(\mu_r - 1) = 9H$ $\quad \boxed{M = 9kA/m}$

$\vec{M}\ \uparrow\ \uparrow\vec{H}\ \uparrow\vec{B}$

3) $B(t)$ a) $e_{ind_{st}} = ?$
б) $e_{ind_{dyn}} = ?$
в) $e_{ind} = ?$




a) $e_{ind_{st}} = -\int_S \frac{d\vec{B}}{dt}\cdot d\vec{s}$

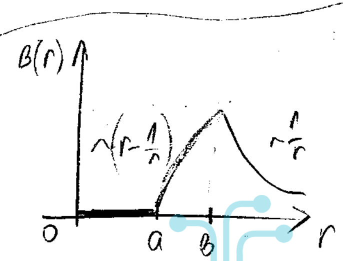
б) $e_{ind_{dyn}} = \oint_C (\vec{v}\times\vec{B})\,d\vec{l}$

в) $e_{ind} = -\frac{d\Phi}{dt} = e_{ind_{st}} + e_{ind_{dyn}} = -\int_S \frac{d\vec{B}}{dt}\,d\vec{s} + \oint_C (\vec{v}\times\vec{B})\,d\vec{l}$

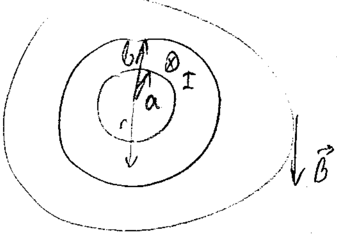
4) I $\oint_C \vec{E}\,d\vec{l} = -\int_S \frac{d\vec{B}}{dt}\,ds$ $\qquad$ II $\oint_C \vec{H}\,d\vec{l} = \sum I = \int_S \left(\vec{J} + \frac{d\vec{D}}{dt}\right) d\vec{s}$

III $\oint_S \vec{D}\,d\vec{s} = Q_{un} = \int_V \rho\,dv$ $\qquad$ IV $\oint_S \vec{B}\,d\vec{s} = 0$

$\vec{D} = \vec{D}(\vec{E})$ $\quad \vec{J} = \vec{J}(\vec{E})$ $\quad \vec{B} = \vec{B}(\vec{H})$




1) $a = 10mm$
$b = 15mm$
$I = 500A$




$\oint_C \vec{B}\,d\vec{l} = \mu_0 I_{ob}$

a)
б) $\vec{B}(r) = ?$

$J_S = \frac{I}{(b^2 - a^2)\pi}$

1) $r < a$
$\boxed{\vec{B} = 0}$ $\quad (I_{ob} = 0)$

2) $a \le r < b$

$B\cdot 2r\pi = \mu_0 \frac{I}{(b^2 - a^2)\cancel{\pi}}(r^2 - a^2)\cancel{\pi}$

$\boxed{B = \frac{\mu_0 I}{2r\pi}\,\frac{r^2 - a^2}{b^2 - a^2}}$

3) $r > b$

$B\cdot 2r\pi = \mu_0 I \Rightarrow \boxed{B = \frac{\mu_0 I}{2r\pi}}$

в) $B(a) = 0$

$B(b) = \frac{\mu_0 I}{2\pi b} = \frac{4\pi\cdot 10^{-7}\cdot 500}{2\pi\cdot 15\cdot 10^{-3}} = \underline{\underline{\frac{20}{3}\,mT}}$

② $l = 0{,}2\,m$

$S = 4\,cm^2$

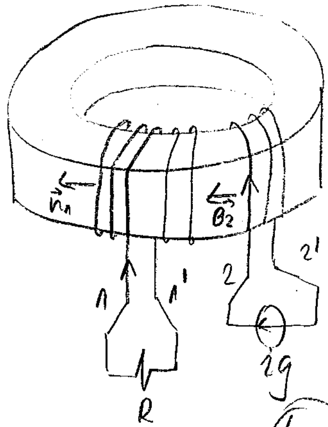
$\mu_r = 1000$

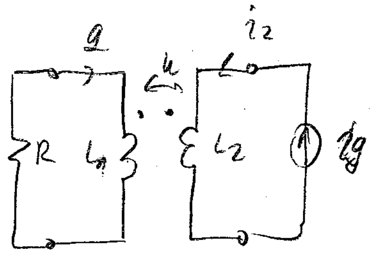
$N_1 = 1000$

$N_2 = 200$

a) $L_1, L_2, L_{12} = ?$

b) $i_2$, $R = 10\,\Omega$ ; $Q = ?$





$L_2$: $$L_2 = \frac{\mu_0 \mu_r N_2^2 S}{l}$$

$L_{12}$: $$L_{12} = \frac{N_1}{I_2} B_2 S = \frac{\mu_0 \mu_r N_1 N_2 S}{l}$$

$L_1$: $$L_1 = \frac{\Phi_1}{I_1} = \frac{\mu_0 \mu_r N_1^2 S}{l}$$

$$\Phi_1 = N_1 \int_{S_1} B_1 \, dS_1 = N_1 B_1 S = \frac{\mu_0 \mu_r N_1^2 I_1 S}{l}$$

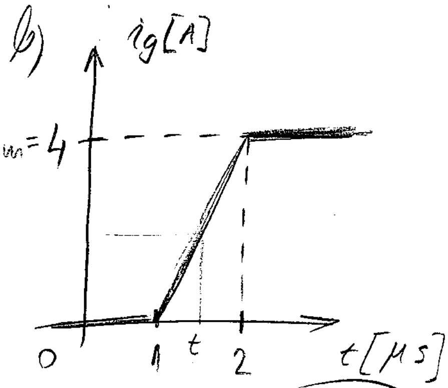
$$\oint_C \vec{H} \, d\vec{l} = \Sigma I$$

$$H_1 l = N_1 I_1$$

$$H_1 = \frac{N_1 I_1}{l}$$

$$B_1 = \mu_0 \mu_r H_1 = \frac{\mu_0 \mu_r N_1 I_1}{l}$$

b)



$\boxed{\Delta I_g = 4A}$

$$Q_1 = -\frac{\Delta \Phi_1}{R}$$

$$u_1 = L_1 \cancel{\frac{di_1}{dt}}^{\,0} + L_{12} \frac{di_2}{dt}$$

$$\Phi_1^{(2)} = L_{12} I_2^{(2)} \;\; (4A) \qquad \Phi_1^{(1)} = L_{12} I_2^{(1)} = 0$$

$$\Delta \Phi_1 = \Phi_1^{(2)} - \Phi_1^{(1)} = L_{12} I_2^{(2)} = L_{12} \cdot \Delta I_g$$

$$\boxed{Q = -\frac{L_{12}}{R} \Delta I_g}$$

2010 ① $\vec{v} = 100\,km/s\ \vec{i}_x$

$\vec{B} = 10\,mT\ \vec{i}_y$

$\vec{E} = ?$ $\vec{a} = 0$

$$\vec{F} = \vec{F}_e + \vec{F}_m = Q\vec{E} + Q\,\vec{v} \times \vec{B}$$

$$\vec{F}_e = -\vec{F}_m$$

$$\cancel{Q}\vec{E} = -\cancel{Q}\,\vec{v} \times \vec{B}$$

$$\vec{E} = -\vec{v} \times \vec{B}$$

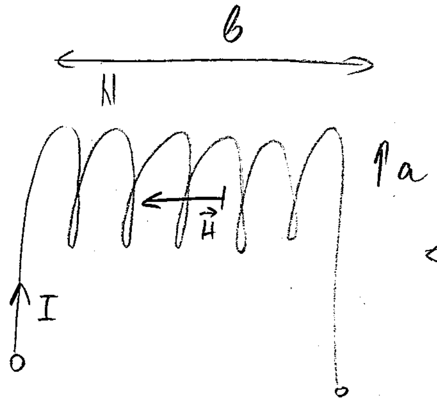
$$\boxed{\vec{E} = -1000 \frac{V}{m}\ \vec{i}_z}$$

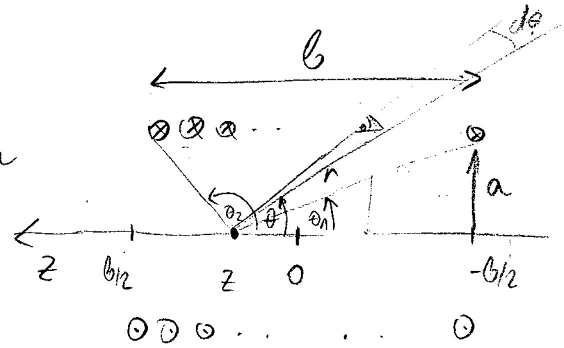
② $N=1000$

$b=0{,}5m$

$a=3\cdot10^{-2}m$

$I=2A$




$$dB=\frac{\mu_0 dN I a^2}{2r^3}$$

$$dN=\frac{N}{b}dz$$

$$ctg\,\theta=-\frac{z}{a}$$

$$\cancel{-}\frac{d\theta}{\sin^2\theta}=\cancel{-}\frac{dz}{a}$$

$$\sin\theta=\frac{a}{r}\Rightarrow r=\frac{a}{\sin\theta}$$

$$dz=\frac{a\,d\theta}{\sin^2\theta}$$

$$B=\int_{\theta_1}^{\theta_2}\frac{\mu_0 I \cancel{a^2}}{2\frac{\cancel{a^3}}{\sin^3\theta}}\frac{N}{b}\frac{\cancel{a}\,d\theta}{\cancel{\sin^2\theta}}=\frac{\mu_0 I N}{2b}(\cos\theta_1-\cos\theta_2)$$

$$\vec{B}=B\vec{i_z}$$

– средина $\theta_1=0$ $\theta_2=\pi$

$$\boxed{B=\frac{\mu_0 I N}{b}}$$

$$H=\frac{B}{\mu_0}=\frac{IN}{b}=4k\frac{A}{m}$$

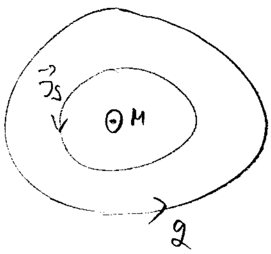
– крај $\theta_1=0$ $\theta_2=\frac{\pi}{2}$

$$\boxed{B=\frac{\mu_0 I N}{2b}}$$

$$H=\frac{IN}{2b}=2k\frac{A}{m}$$

③ $a, M, b, R$

$q=?$




$J_s=M$

$$B=\mu_0 J_s$$

$$\Phi=B\cdot a^2\pi=\frac{\mu_0 J_s}{1}a^2\pi$$

$$\Delta\Phi=-\frac{\mu_0 J_s}{1}a^2\pi$$

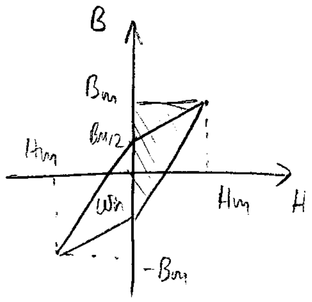
$$q=\frac{-\Delta\Phi}{R}$$

$$q=\frac{\mu_0 J_s a^2\pi}{R}=\frac{\mu_0 M a^2\pi}{R}$$

④ $l, s, N$

$i=I_m\cos(2\pi f t)$

$$\frac{B_m}{H_m}=\mu_n$$




$$w_m=2\left[\frac{\frac{3}{2}B_m H_m}{2}-\frac{\frac{1}{2}B_m H_m}{2}\right]=2\frac{1}{2}B_m H_m=\frac{B_m^2}{\mu_n}$$

$$P_h=\frac{1}{T}\int_v w_m dv=f\,w_m s l$$

$$H_m l=N I_m$$

$$H_m=\frac{N I_m}{l}$$

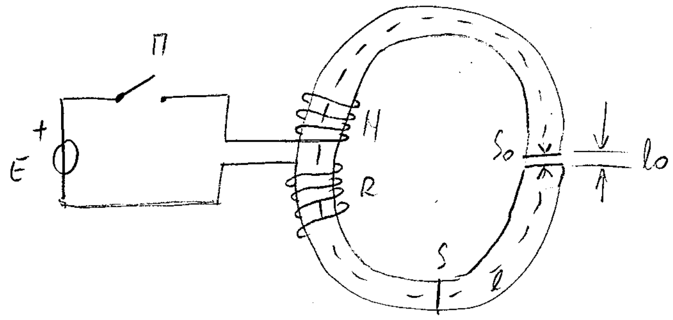
$$P_h=f s l\frac{B_m^2}{\mu_n}$$

$$B_m=\mu_n H_m=\mu_n\frac{N I_m}{l}$$

$$P_h=f s \cancel{l}\frac{\mu_n^{\cancel{2}} N^2 I_m^2}{\cancel{\mu_n} l^{\cancel{2}}}$$

$$\boxed{P_h=\frac{1}{l}f s \mu_n N^2 I_m^2}$$

③ ①




$N = 1000$

$R = 10\,\Omega$

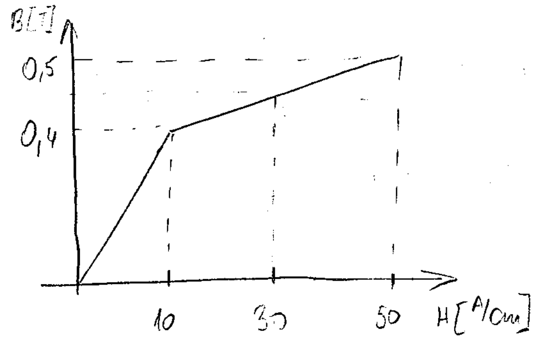
$l = 1\,m$

$l_0 = 3{,}14\ mm$

$S = S_0$

$B_0 = 0{,}45\ T$

$E = ?$




$$\oint_C \vec{H}\,d\vec{l} = \sum I$$

$$Hl + H_0 l_0 = NI$$

$$Hl + \frac{B_0}{\mu_0} l_0 = NI = N\frac{E}{R}$$

$B = B_0 = 0{,}45\,T \Rightarrow H = 30\,\frac{A}{cm}$

$H = 3\,kA/m$

$$\boxed{E = \frac{R}{N}\left(Hl + \frac{B_0}{\mu_0} l_0\right)}$$

$$E = \frac{10}{1000}\left(3000\cdot 1 + \frac{0{,}45}{4\pi\cdot 10^{-7}}\pi\cdot 10^{-3}\right) = \frac{1}{100}\left(3000 + 1125\right) = 41{,}125\,V$$

② $i(t) = I_m \sin\omega t$, $a$

$e_{ind} = ?$

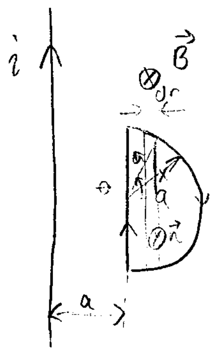
$$B = \frac{\mu_0 i}{2\pi r}$$

$$e_{ind_{st}} = -\frac{d\phi}{dt}$$

$dt$

$$\phi = \int_S \vec{B}\,d\vec{s} = \int_{\theta=-\frac{\pi}{2}}^{0} \frac{\mu_0 i}{2\pi \cancel{a}(1+\cos\theta)}\, 2\cancel{a}^2 \sin^2\theta\, d\theta$$

$$\phi = \frac{\mu_0 i a}{\pi}\int_{-\pi/2}^{0} \frac{(1-\cos\theta)\cancel{(1+\cos\theta)}}{\cancel{1+\cos\theta}}\,d\theta$$




$x = a\cos\theta$

$$ds = 2x\,dr = 2a\sin\theta\,dr = 2a\sin^2\theta\,d\theta$$

$$\phi = \frac{a\mu_0 i}{\pi}\left[\frac{\pi}{2} - \sin 0 + \sin\left(-\frac{\pi}{2}\right)\right]$$

$$\boxed{\phi = \frac{a\mu_0 i}{\pi}\left[\frac{\pi}{2} - 1\right]}$$

$a\,d\theta$, $dr$, θ $\Rightarrow dr = a\sin\theta\,d\theta$

$$r = a + a\cos\theta = a(1+\cos\theta)$$

$$\boxed{e_{ind} = -\frac{d\phi}{dt} = -\frac{\mu_0}{\pi}a\left[\frac{\pi}{2} - 1\right] I_m\omega\cos\omega t}$$

2013 (11)

1) $Q = 10\,\mu C$

$\vec{v} = 10\,\vec{i}_x\ m/s$

$\vec{E} = 10\,\vec{i}_x \cdot 10^{-3}\ \frac{V}{m}$

$\vec{B} = 2\,\vec{i}_z \cdot 10^{-3}\ T$

$\vec{F} = ?$

$\vec{F} = \vec{F}_m + \vec{F}_e$

$\vec{F} = Q\vec{v}\times\vec{B} + Q\vec{E} = 10\cdot 10^{-6}\cdot\left(-20\cdot 10^{-3}\,\vec{i}_y + 10\cdot 10^{-3}\,\vec{i}_x\right) N$

xyzxy

$$\boxed{\vec{F} = \left(-200\,\vec{i}_y + 100\,\vec{i}_x\right) nN}$$

(11) 2) $\mu_r = 100$

$N = 10^{+26}\ m^{-3}$

$\vec{H} = 100\,\vec{i}_z\ A/m$

a) $\vec{B} = \mu_0\mu_r\vec{H} = 4\pi\, 10^{-7}\cdot 100\cdot 100\,\vec{i}_z\ T = \underline{4\pi\ mT\ \vec{i}_z}$

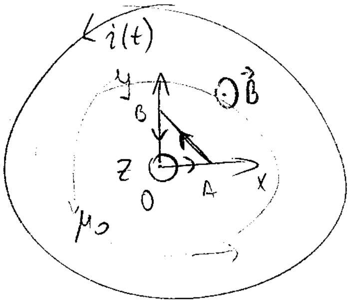
б) $\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M} \Rightarrow \vec{M} = \vec{H}\mu_r - \vec{H} = \vec{H}(\mu_r - 1) = \underline{9{,}9\ kA/m\ \vec{i}_z}$

в) $\underline{\vec{J}_A = 0}$ $\left(\vec{M} = const\right)$

г) $\vec{M} = \frac{\sum_{\Delta v}\vec{m}}{\Delta v} = N\vec{m} \Rightarrow \vec{m} = \frac{\vec{M}}{N} = \underline{9{,}9\cdot 10^{-23}\ A/m\ \vec{i}_z}$

д) $w_m = \frac{1}{2}\vec{B}\vec{H} = \frac{1}{2}\,4\pi\cdot 10^{-3}\cdot 100 = \underline{\frac{\pi}{5}\ J/m^3}$

3) $a,\ N',\ i(t) = I_m\cos\omega t$




$B = \frac{\mu_0 NI}{b} = \mu_0 N' I$

$B(t) = \mu_0 N' i(t)$

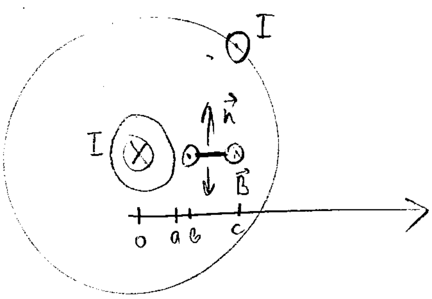
$\phi = B(t)\cdot\frac{b^2}{2}$

a) $\oint_C \vec{E}\,d\vec{l} = -\frac{d\phi}{dt} = \underline{\frac{1}{2} I_m\,\omega\, b^2 \sin\omega t}$

б) $\int_0^A \vec{E}_{ind}\,d\vec{l} = \int_B^0 \vec{E}_{ind}\,d\vec{l} = 0$

$\int_A^B \vec{E}_{ind}\,d\vec{l} = \frac{1}{2} I_m\,\omega\, b^2 \sin\omega t$

4)




$B = \frac{\mu_0 I}{2\pi r}$

$\phi = +\int_S \vec{B}\,d\vec{s} = -\int_b^c \frac{\mu_0 I}{2\pi r}(c-b)\,dr =$

$= \frac{-\mu_0 I}{2\pi}(c-b)\ln\frac{c}{b}$

$L = \frac{\phi}{I}$

$$\boxed{L = \frac{-\mu_0 (c-b)}{2\pi}\ln\frac{c}{b}}$$

(1) $S = 1\,cm^2$

$l = 1\,m$

$N = 1000$

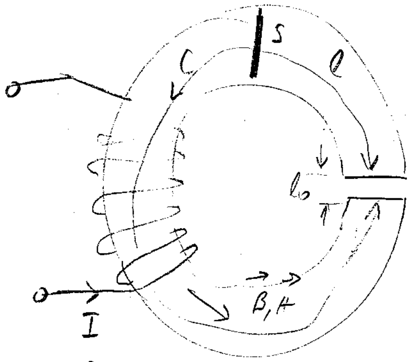
$l_0 = \pi\,mm$

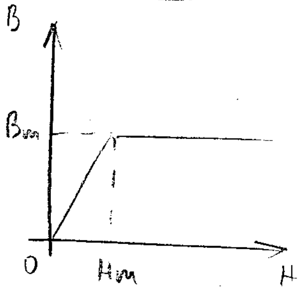
$B_m = 1\,T$

$H_m = 1000\,A/m$

a) $I_{max} = ?$

б) $W_{m_0} = ?$




$$\oint \vec{H} d\vec{l} = \Sigma I$$

$$Hl + H_0 l_0 = NI$$

$$B = B_m \qquad H = H_m \qquad H_0 = \frac{B}{\mu_0}$$

$$I_{max} = \frac{1}{N}\left[H_m l + \frac{B_m}{\mu_0} l_0\right]$$

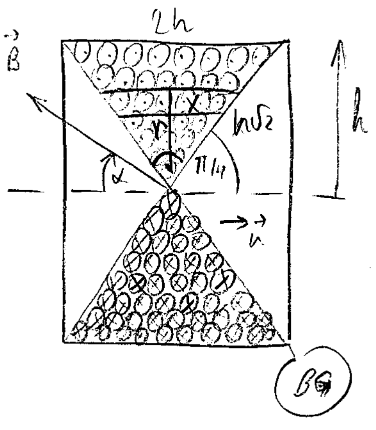
$$I_{max} = 10^{-3}\left[1000 + \frac{1}{4\pi \cdot 10^{-7}}\,\pi \cdot 10^{-3}\right]$$

$$\boxed{I_{max} = 3{,}5\,A}$$

б) $$W_{m_0} = w_{m_0} \underbrace{S\, l_0}_{V} = \frac{1}{2}\vec{B}_0 \cdot \vec{H}_0\, S l_0 = \frac{B_m^2}{2\mu_0} S l_0 = \frac{1}{2 \cdot 4\pi \cdot 10^{-7}}\, 10^{-4} \cdot \pi\, 10^{-3}$$

$$\boxed{W_{m_0} = 0{,}125\,J}$$

(1) $h, N, R, B, \alpha^{(1)} = \pi/6,\ \alpha^{(2)} = \pi/3$




$$N' = \frac{N}{h^2} \qquad dN = N' dS = N' 2x\,dr = \underline{\underline{2r\,dr\,N'}}$$

$$\frac{r}{x} = 1 \Rightarrow r = x$$

$$d\phi = dN\, B\, r^2\pi \cos(\pi - \alpha)$$

$$\phi = \int dN\, B\, r^2 \pi \cos(\pi - \alpha)$$

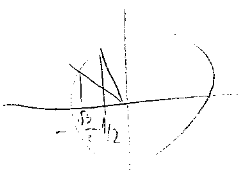
$$\phi = 2N' B \pi \cos(\pi - \alpha) \int_0^h r^3 dr$$

$$\phi = 2\frac{N}{\cancel{h^2}} B\pi \cos(\pi-\alpha)\frac{h^{\cancel{4}}h^2}{4} \qquad \boxed{\phi = \frac{1}{2} N B \pi h^2 \cos(\pi - \alpha)}$$

$$q = -\frac{\Delta\phi}{R}$$

$$q = -\frac{1}{2R} N B \pi h^2 \left[\cos(\underbrace{\pi - \alpha^{(2)}}_{\frac{2\pi}{3}}) - \cos(\underbrace{\pi - \alpha^{(1)}}_{\frac{5\pi}{6}})\right] = -\frac{NB\pi h^2}{2R}\left[-\frac{1}{2} + \frac{\sqrt{3}}{2}\right]$$

$$\boxed{q = -\frac{NB\pi h^2}{4R}(\sqrt{3} - 1)}$$



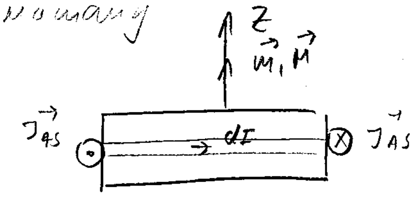
2012 ①① $a, \delta, \delta/a = \frac{1}{100\pi}, N = 10^{22}\,cm^{-3} = 10^{28}\,m^{-3}, \vec{m} = 10^{-23}\vec{i_z}\ Am^2$

а) $\vec{M} = ?$ $\vec{M} = \frac{\sum_v \vec{m}}{dv} = N\vec{m} = 10^5\,A/m\,\vec{i_z}$

б) $\vec{J_A} = 0$ $(\vec{M} = const)$

в) $\vec{J_{As}} = 0$ на базисима $\quad J_{As} = M$ на омотачу

$\vec{J_{As}} = \vec{M} \times \vec{n}$ $\quad \vec{J_{As}} = M\,\vec{i_\varphi}$



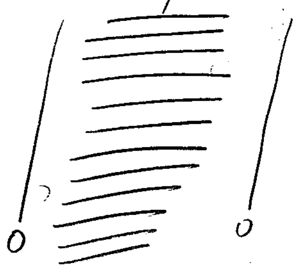

$I = J_{As}\delta$

г) $B_0 = ?$ $\quad a \gg \delta$

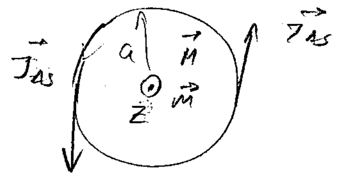
$$B_0 = \int_0^\delta \frac{\mu_0 J_{As} dx}{2a} = \frac{\mu_0 \delta M}{2a} = \frac{4\pi \cdot 10^{-7} \cdot 10^5}{\frac{200\pi}{100}} = 200\,\mu T$$

$dI$

$$dB = \frac{\mu_0 J_{As}\,dx}{2a}$$



д) $\vec{J_{As}}$




② $l = 0{,}2\,m$

$S = 10\,cm^2$

$N = 1000$

$I = 0{,}5A$

а) $\oint_C \vec{H}\,d\vec{l} = \sum I$

$Hl = NI$

$H = \frac{NI}{l} = \frac{500}{0{,}2} = 2500\,A/m$

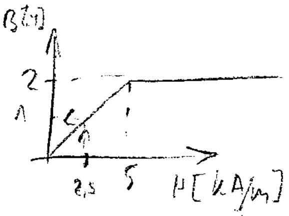
б) $\boxed{B = 1T}$

в) $\phi_j = Bs = 10^{-3}\,Tm^2 = 1\,mWb$

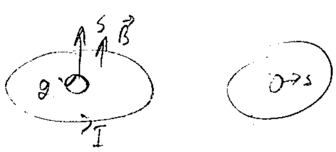
г) $\phi = N\phi_j = 1\,Wb$

д) $W_m = \frac{1}{2}BHSl = 1250 \cdot 10^{-3} \cdot 0{,}2$

$\boxed{W_m = 0{,}25\,J}$



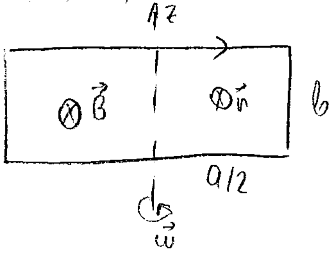

③ $a, s, R, I, q = ?$



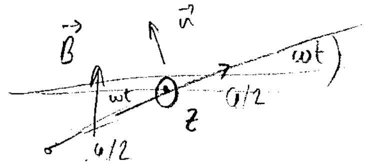

$B = \frac{\mu_0 I}{2a}$ $\quad \phi^{(1)} = Bs$ $\quad \phi^{(2)} = 0$

$q = -\frac{\Delta\phi}{R}$ $\quad q = \frac{\mu_0 I s}{2aR}$

④ $a, b, R, \omega, i(t) = ?$




$e_{ind} = -\frac{d\phi}{dt}$

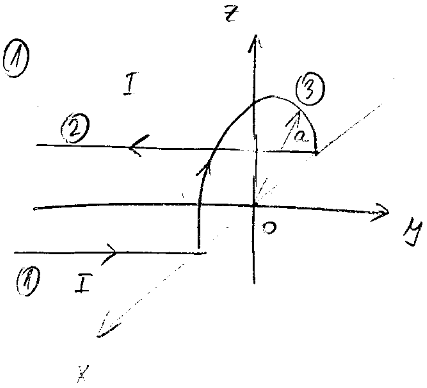



$\phi(t) = Bab\cos\omega t$ ; $e_{ind} = -\frac{d\phi}{dt} = +Bab\omega\sin\omega t$

$i = \frac{e_{ind}}{R}$

$$\boxed{i(t) = \frac{Bab\omega}{R}\sin\omega t}$$

3) ①

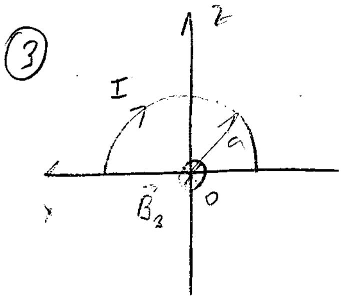



$a, I, \quad B_0 = ?$

① $\vec{B}_1 = \frac{\mu_0 I \vec{i}_z}{4\pi a}\left(\sin\frac{\pi}{2} - \cancel{\sin 0}\right) = \frac{\mu_0 I}{4\pi a}\vec{i}_z$

② $\vec{B}_2 = \frac{\mu_0 I \vec{i}_z}{4\pi a}\left(\sin 0 - \sin\left(-\frac{\pi}{2}\right)\right) = \frac{\mu_0 I}{4\pi a}\vec{i}_z$

③




$B = \frac{\mu_0}{4\pi}\int_0^{\pi}\frac{I\,d\theta}{r} = \frac{\mu_0 I}{4\pi a}\pi$

$\vec{B}_3 = B_3(-\vec{i}_y)$

$\vec{B} = \frac{\mu_0 I}{2\pi a}\vec{i}_z - \frac{\mu_0 I}{4\pi a}\pi\,\vec{i}_y$

$$\boxed{\vec{B} = \frac{\mu_0 I}{4\pi a}\left(2\vec{i}_z - \pi\vec{i}_y\right)}$$

② $N_1 = 200$

$N_2 = 100$

$R_1 = 20\Omega$

$R_2 = 10\Omega$

$E = 3V$

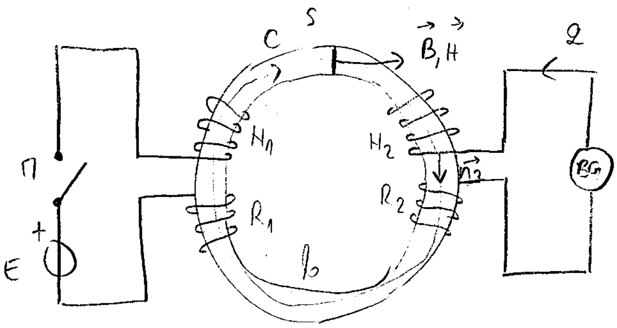
$S = 1\,cm^2$

$b = 0{,}1\,m$

$\mu = 10^{-3}\,H/m$

a) $q = ?$ $\Pi: 0 \to 3$

б) $W_m = ?$




a) $I_1^{(1)} = 0 \qquad I_1^{(2)} = \frac{E}{R_1}$

$\oint_C \vec{H}\,d\vec{l} = \sum I$

$H \cdot b = N_1 I_1$

$B_1 = \mu\frac{N_1 I_1}{b}$

$\Phi_2 = N_2 B_1 S = \mu\frac{N_1 N_2 I_1}{b}S$

$q = -\frac{\Delta\Phi_2}{R_2} = -\frac{\Phi_2^{(2)} - \Phi_2^{(1)}}{R_2}$

$\Phi_2^{(1)} = 0$

$\Phi_2^{(2)} = \frac{\mu N_1 N_2 E S}{b R_1}$

$$\boxed{q = -\frac{\mu N_1 N_2 E S}{b R_1 R_2}}$$

б) $W_m = \frac{1}{2}BHSl = \frac{1}{2}\mu H^2 S b$

$W_m = \frac{1}{2}\mu\frac{N_1^2 E^2 S \cancel{b}}{R_1^2 b^{\cancel{2}}}$

$$\boxed{W_m = \frac{\mu N_1^2 E^2 S}{2R_1^2 b}}$$

(2011) ① $\mu_r = 1000$

$N = 1000$

$R = 4\Omega$

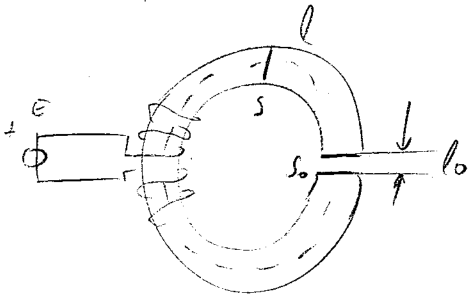
$l_0 = \pi\,mm$

$\frac{l}{l_0} = 2000$

$S = S_0$

$E = 3V$

$B_0 = ?$




$$\oint_C \vec{H}\,d\vec{l} = \sum I$$

$$Hl + H_0 l_0 = NI = N\frac{E}{R}$$

$$\frac{B_0}{\mu_r \mu_0}\, l + \frac{B_0}{\mu_0}\, l_0 = N\frac{E}{R}$$

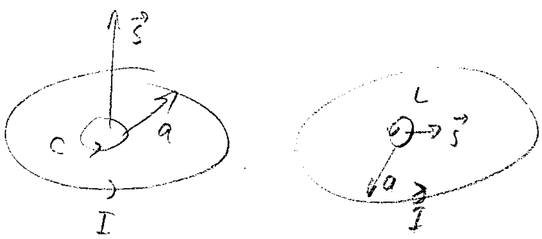
$$B_0\,\frac{l + \mu_r l_0}{\mu_0 \mu_r} = N\frac{E}{R}$$

$$B_0 = \frac{NE\mu_0\mu_r}{R(l + \mu_r l_0)} = \frac{1000 \cdot \cancel{3} \cdot 4\cancel{\pi} \cdot 10^{-7} \cdot \cancel{1000}}{4 \cdot \cancel{3000} \cdot \cancel{\pi} \cdot 10^{-3}} = \underline{\underline{0{,}1\,T}}$$

② a) $\oint_C \vec{E}\,d\vec{l} = -\frac{d\phi}{dt}$ б) $\oint \vec{H}\,d\vec{l} = \sum I = \int_S \vec{J}\,d\vec{s}$ g) $\oint_S \vec{J}\,d\vec{s} = -\frac{d}{dt}\int_V \rho\,dv$

δ) $\oint \vec{D}\,d\vec{s} = Q_0$, $\vec{D} = \vec{D}(E)$ i) $\oint_S \vec{B}\,d\vec{s} = 0$

③



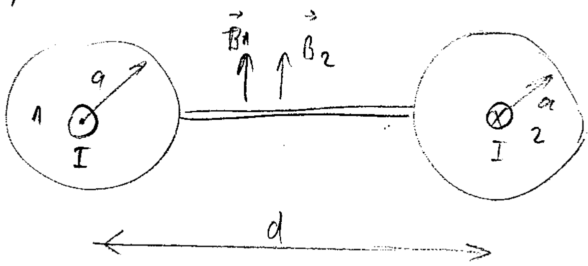

$$\phi_1 = \frac{\mu_0 I}{2a} S$$

$$\Delta\phi = 0 \Rightarrow \phi_2 = \phi_1$$

$$I_1 = \frac{\phi_1}{L} = \underline{\underline{\frac{\mu_0 I S}{2aL}}}$$

④ a, d, I

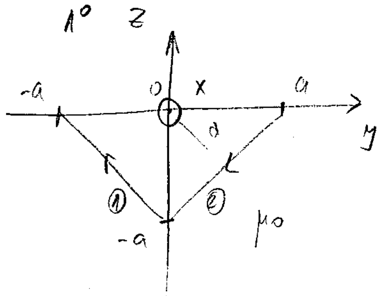
!




$B_1 = \mu_0 I$

3 ①

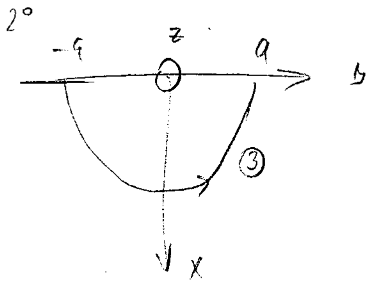
1°




$$B_1 = B_2 = \frac{\mu_0 I}{4\pi a\frac{\sqrt{2}}{2}}\left(\sin 45^\circ - \sin(-45^\circ)\right)$$

$$\vec{B}_1 = \vec{B}_2 = \frac{\mu_0 I}{2\pi a\sqrt{2}}\sqrt{2}\left(-\vec{i}_x\right) = -\frac{\mu_0 I}{2\pi a}\vec{i}_x$$

2°

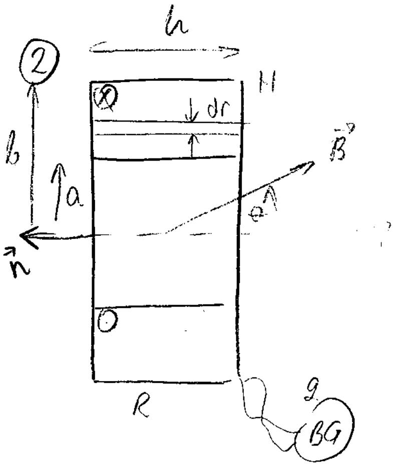



$$B_3 = \frac{\mu_0}{4\pi}\int_0^{\pi}\frac{I d\theta}{r} = \frac{\mu_0 I}{4\pi a}\pi$$

$$\vec{B}_3 = B_3\vec{i}_z$$

$$\vec{B} = \frac{\mu_0 I}{4\pi a}\left(-4\vec{i}_x + \pi\vec{i}_z\right)$$

②




$N, a, b, h, R, \vec{B}$ $\quad \alpha = \theta \to \alpha = 0$

$$dN = \frac{N}{(b-a)h} h\,dr$$

$$dN = \frac{N}{b-a} dr$$

$$d\phi = dN\, B r^2\pi\cos\alpha$$

$$\phi = \int_{r=a}^{b} dN B r^2\pi$$

$$\phi = \frac{N}{b-a} B\pi\int_a^b r^2 dr\cdot\cos\alpha = \frac{NB\pi\cos\alpha}{b-a}\,\frac{b^3 - a^3}{3} \to (b-a)(b^2+ab+a^2)$$

$$\boxed{\phi = \frac{NB\pi}{3}(b^2+ab+a^2)\cos\alpha}$$

$$q = -\frac{\Delta\phi}{R} = +\frac{NB\pi(b^2+ab+a^2)}{3R}\left[1-\cos\alpha\right]$$

# 4. Енергија магнетског поља

**207** $l = 20cm = 0{,}2m$

$a = 1cm \quad S = a^2 = 1\cdot 10^{-4} m^2$

$L = 1mH = 10^{-3} H$

$R = 5\Omega \quad E = 1V$

$W_m = ?$

$w_m = ?$

$$W_m = \frac{1}{2}LI^2 = \frac{1}{2}L\left(\frac{E}{R}\right)^2 = 10^{-3}\cdot\frac{1}{50} = 20\mu J$$

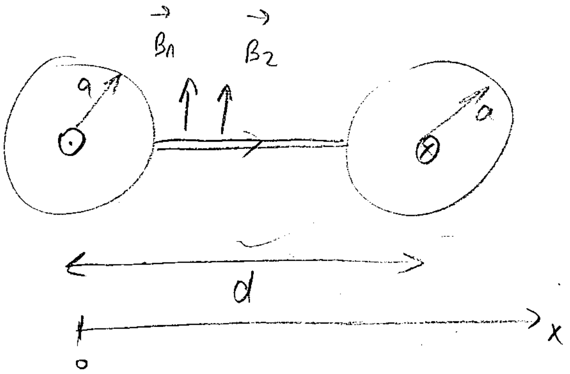
$$w_m = \frac{W_m}{v} = \frac{W_m}{a^2 l} = \frac{20\cdot 10^{-6}}{10^{-4}\cdot 0{,}2} = 1\frac{J}{m^3}$$

**208** a) $L = \frac{\Phi}{I}$ → спољашња

б) $W_m = \frac{1}{2}LI^2 \Rightarrow L = \frac{2W_m}{I^2}$ → спољашња + унутрашња

**209** $\sigma, a, d \gg a, \quad C' = ? \quad Le' = ?$

$R' = ? \quad L_i' = ?$




$$B_1 = \frac{\mu_0 I}{2\pi x} \qquad B_2 = \frac{\mu_0 I}{2\pi(d-x)}$$

$$B = B_1 + B_2$$

$$\phi_e = \int_S \vec{B}d\vec{s} = \int_a^{d-a}\left[\frac{\mu_0 I}{2\pi x} + \frac{\mu_0 I}{2\pi(d-x)}\right]dx\, h$$

$$\phi_e' = \frac{\phi_e}{h} = \frac{\mu_0 I}{2\pi}\ln\frac{d-a}{a} - \frac{\mu_0 I}{2\pi}\ln\frac{d-d+a}{d-a}$$

$$\phi_e' = \frac{\mu_0 I}{\pi}\ln\frac{d}{a} \qquad \boxed{L_e' = \frac{\mu_0}{\pi}\ln\frac{d}{a}}$$

$$Le'\cdot C' = \varepsilon_0\mu_0 = \frac{1}{c_0^2}$$

$$\boxed{C' = \frac{\varepsilon_0\mu_0}{Le'} = \frac{\pi\varepsilon_0}{\ln\frac{d}{a}}}$$

$$B\cdot 2r\pi = \mu_0\frac{I}{a^2\pi}r^2\pi$$

$$\boxed{B = \frac{\mu_0 I}{2a^2\pi}r}$$

$$w_m = \frac{1}{2}\vec{B}\vec{H} = \frac{1}{2}\frac{B^2}{\mu_0} = \frac{1}{2}\frac{\mu_0 I^2 r^2}{4\pi^2 a^4}$$

$$W_m = \int_v w_m\, dv = \int_{r=0}^{a} w_m\, 2r\pi h\, dr$$

$$W_m = \frac{1}{\cancel{8}_4}\frac{\mu_0 \cancel{2} h \cancel{\pi} I^2}{\pi^{\cancel{2}} a^4}\int r^3 dr$$

$$W_m = \frac{1}{4}\cdot\frac{\cancel{a^4}}{4}\frac{\mu_0 h I^2}{\pi \cancel{a^4}} \Rightarrow w_m' = \frac{W_m}{h} = \frac{1}{2}\cdot\frac{\mu_0 I^2}{8\pi}$$

$$W_m' = \frac{1}{2}L_{i1}' I^2$$

$$\Rightarrow L_{i1}' = \frac{\mu_0}{8\pi}$$

$$\boxed{L_i' = 2L_{i1}' = \frac{\mu_0}{4\pi}}$$

* (210) $\sigma' = 58\ \frac{MS}{m}$

$a = 3mm$

$b = 8mm$

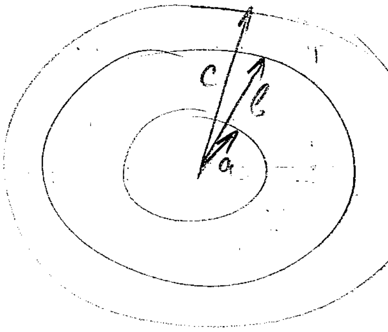
$c = 9mm$

$I = 50A$

а) $P_J' = ?$

б) $W_m' = ?$

в) $L' = ?$




а) $P_J' = R' I$

$R' = R_1' + R_2'$

$R_1 = \rho \frac{h}{a^2\pi} = \frac{1}{\sigma}\frac{h}{a^2\pi}$

$R_1' = \frac{R_1}{h} = \frac{1}{\sigma a^2 \pi}$

$R_2' = \frac{1}{\sigma(c^2-b^2)\pi}$

$$\boxed{P_J' = \frac{I}{\pi\sigma}\left[\frac{1}{a^2} + \frac{1}{(c^2-b^2)}\right]}$$

б) $w_m = \frac{1}{2}\vec{B}\vec{H} = \frac{1}{2}\frac{B^2}{\mu_0}$

$$B = \begin{cases} \frac{\mu_0 I}{2\pi a^2} r, & r<a \\ \frac{\mu_0 I}{2\pi r}, & a<r<b \\ \frac{\mu_0 I}{2\pi r}\frac{c^2-r^2}{c^2-b^2}, & b<r<c \\ 0, & r>c \end{cases}$$

$W_m = \int_V w_m dV$

$dV = 2r\pi\, dr\, h$

$W_m' = \frac{W_m}{h} = \int_0^c \frac{1}{\cancel{2}}\frac{B^2}{\mu_0}\cancel{2}r\pi\, dr$

$$W_m' = \frac{\pi}{\mu_0}\left[\int_0^a \frac{\mu_0^2 I^2}{4\pi^2 a^4} r^2 r\, dr + \int_a^b \frac{\mu_0^2 I^2}{4\pi^2 \cancel{r^2}} \cancel{r}\, dr + \int_b^c \frac{\mu_0^2 I^2}{4\pi^2 r^2}\frac{c^4 - 2c^2r^2 + r^4}{(c^2-b^2)^2} r\, dr\right]$$

$$W_m' = \frac{\cancel{\pi}}{\cancel{\mu_0}} \cdot \frac{\mu_0^{\cancel{2}} I^2}{4\pi^{\cancel{2}}}\left[\frac{1}{\cancel{a^4}}\frac{\cancel{a^4}}{4} + \ln\frac{b}{a} + \frac{\ln\frac{c}{b} - \cancel{2}c^2\frac{c^2-b^2}{\cancel{2}} + \frac{c^4-b^4}{4}}{(c^2-b^2)^2}\right]$$

$$\boxed{W_m' = \frac{\mu_0 I^2}{4\pi}\left[\frac{1}{4} + \ln\frac{b}{a} + \frac{\ln\frac{c}{b} - c^2(c^2-b^2) + \frac{1}{4}(c^4-b^4)}{(c^2-b^2)^2}\right]}$$

в) $W_m' = \frac{1}{2}L' I^2 \Rightarrow \boxed{L' = \frac{2W_m'}{I^2}}$

* (211) $\sigma = 56\ MS/m$ $\quad b = 4mm$

$\varepsilon_r = 2{,}1,\ \mu_r = 1$ $\quad c = 4{,}2mm$

$a = 1mm$ $\quad C', R', L' = ?$

(R') $R' = R_1' + R_2'$ $\quad R_2' = \frac{1}{\sigma}\frac{1}{(c^2-b^2)\pi}$

$R_1' = \frac{1}{\sigma}\frac{1}{a^2\pi}$

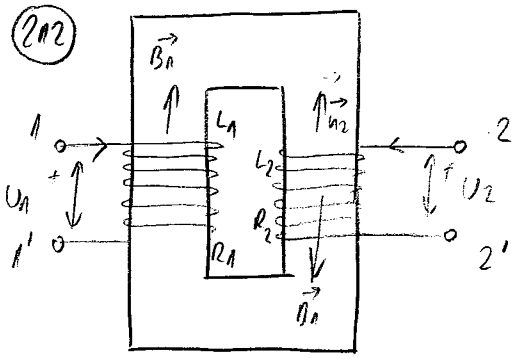
$R = \frac{1}{\sigma\pi}\left[\frac{1}{a^2} + \frac{1}{c^2-b^2}\right]$

$\boxed{L' = L_e' + L_i'}$

$L_e' C' = \varepsilon_0\varepsilon_r\mu_0\mu_r \Rightarrow \boxed{C' = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln\frac{b}{a}}}$

$L_e' = \frac{\mu_0}{2\pi}\ln\frac{b}{a}$

(212)




$L_1 = 4mH \quad R_1 = 3\Omega \quad U_1 = 1,2V$

$L_2 = 9mH \quad R_2 = 4\Omega \quad U_2 = 2,4V$

$k = 0,25$

$W_m = ?$

$$I_1 = \frac{U_1}{R_1} = 0,4A$$

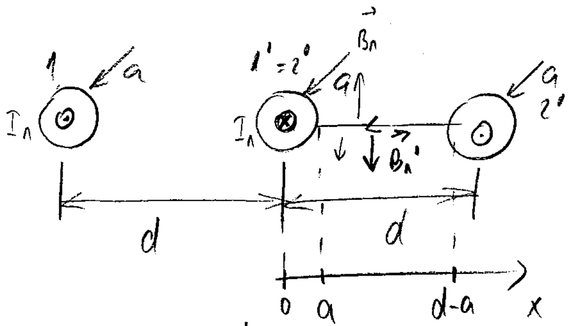
$$I_2 = \frac{U_2}{R_2} = 0,6A$$

$$L_{12} = -k\sqrt{L_1 L_2}$$

$$L_{12} = -1,5mH$$

$$\boxed{W_m = \frac{1}{2}L_1 I_1^2 + L_{12} I_1 I_2 + \frac{1}{2}L_2 I_2^2} = 1,58mJ$$

(213)




$L_{12}' = ?$

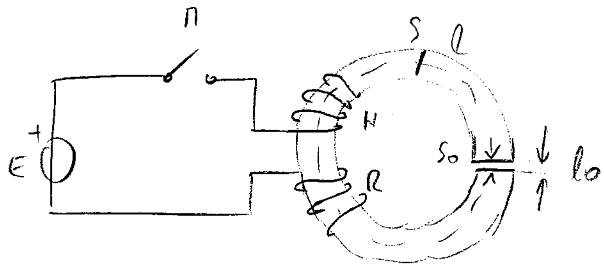
$$L_{12}' = L_{21}' = \frac{\Phi_{21}'}{I_1}$$

$$B_1 = \frac{\mu_0 I_1}{2\pi(d+x)} \qquad B_1' = \frac{\mu_0 I_1}{2\pi x}$$

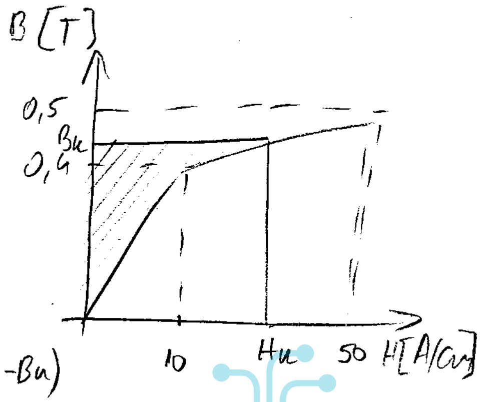
$$\Phi_{21}' = \frac{\Phi}{h} = \int_{x=a}^{d-a} \frac{\mu_0 I_1}{2\pi}\left(\frac{1}{d+x} + \frac{1}{x}\right)dx = \frac{\mu_0 I_1}{2\pi}\left[\ln\frac{d-a}{a} - \ln\frac{d+d-a}{d+a}\right] =$$

$$= \frac{\mu_0 I_1}{2\pi}\ln\frac{d}{2a} \quad \Rightarrow \quad \boxed{L_{12} = \frac{\mu_0}{2\pi}\ln\frac{d}{2a}}$$

(214)




$W_m = ?$




$N = 1000$

$R = 50\Omega$

$l = 1m$

$l_0 = \pi \, mm$

$S = S_0 = 1 \cdot 10^{-4} m^2$

$E = 150V$

$$w_m = \int_0^B H\,dB = \frac{1}{2}B_u H_u + \frac{H + H_u}{2}(B - B_u)$$

$$\oint_C \vec{H}\,d\vec{l} = \Sigma I$$

$$Hl + H_0 l_0 = NI$$

$$Hl + \frac{B}{\mu_0} l_0 = NI$$

$$\boxed{H + 2500B = 3000}$$

$$\boxed{B = 0,42T} \qquad \boxed{H = 1941 \, A/m}$$

$$\boxed{w_m = 234,6 \, J/m^3}$$

$$w_{m0} = \frac{1}{2}B_0 H_0$$

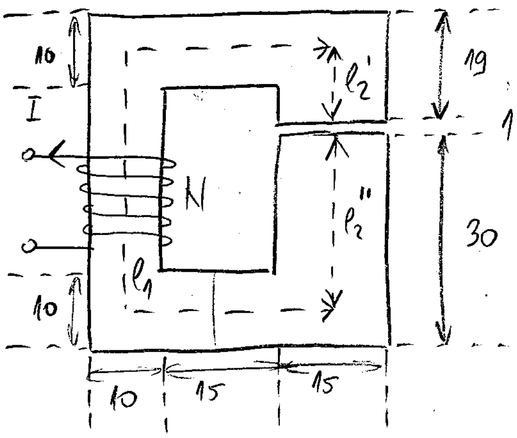
$$\boxed{W_m = w_m S l + w_{m0} S_0 l_0}$$

(215) $N=300$

$I=2{,}5A$

а) $\phi_0=?$

б) $W_m=?$




$$\oint_C \vec{H}\,d\vec{l} = \Sigma I$$

$$H_1 l_1 + H_2 l_2 + H_0 l_0 = NI$$

$$H_1 l_1 + H_2 l_2 + \frac{B_0}{\mu_0} l_0 = NI$$

$l_1=95\ mm$

$l_2'=14mm$

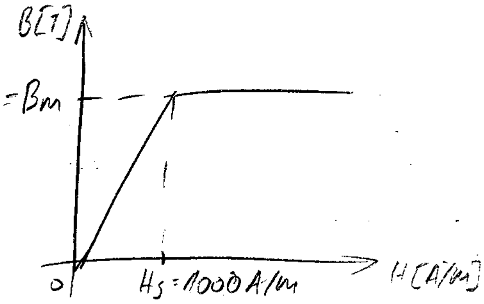
$l_2''=25mm$ $\Big\}(+)=l_2$

$l_0=1mm$

$B_1 S_1 = B_2 S_2 = B_0 S_0$ $\quad 1T = B_m$

$S_1 = 100mm^2$

$S_2 = S_0 = 150mm^2$




а) ПРЕТПОСТАВИМО ДА У УЖЕМ ДЕЛУ НИЈЕ ДОШЛО ДО ЗАСИЋЕЊА

$$B_1 < B_m = 1T$$

$$B_2 = B_0 = \frac{B_1 S_1}{S_2} < \frac{S_1}{S_2} B_m = \frac{2}{3} T$$

$$H_1 < H_s = 1000\ A/m \qquad H_2 = \frac{H_s}{B_m} B_2 < \frac{H_1}{B_1} B_2 = \frac{S_1}{S_2} H_1$$

$$H_2 < \frac{2}{3} H_1$$

$$H_0 = \frac{B_0}{\mu_0} < \frac{2\,T}{3\mu_0} = \frac{2\ 10^7}{3\cdot 4\pi} = \frac{20}{12\pi}\cdot 10^6 = \frac{5}{3\pi}\ 10^6$$

$$H_1 l_1 + H_2 l_2 + H_0 l_0 < 95 + 36 + \frac{5}{3\pi}10^3 = \underline{651\ A/m < NI}$$

- Зове $\boxed{B_1 = B_m}$

$\boxed{B_2 = \frac{2}{3} B_m = B_0}$

$$\phi = B_2 S_2 = B_0 S_0 = \frac{2}{\cancel{3}} \cancel{150}^{50} \cdot 10^{-6}\ Wb$$

$$\boxed{\phi = 100\ \mu Wb}$$

(217) $l=0{,}2m$

$S=4cm^2$

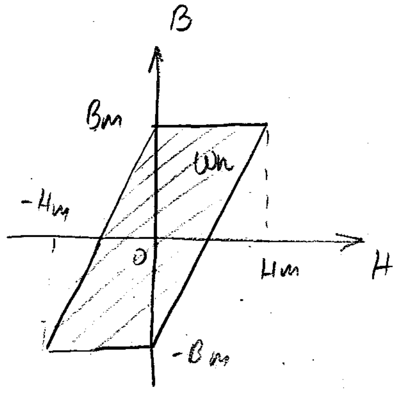
$N=1000$

$I_m=200mA$

$f=50Hz$

$B_m=\mu_n H_m$

$\mu_n=10^{-3}H/m$




$$\oint_C \vec{H}d\vec{l}=\Sigma I$$

$$Hl=NI$$

$$\boxed{H_m=\frac{NI_m}{l}}$$

$$\boxed{B_m=\mu_n H_m}=\mu_n\frac{NI_m}{l}$$

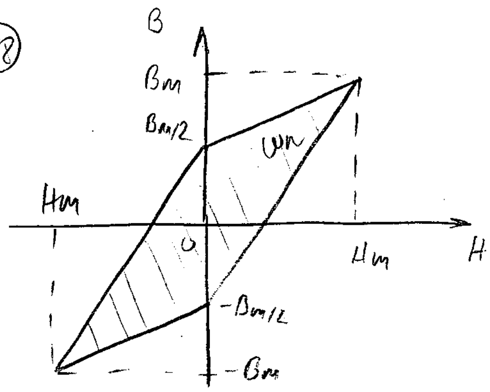
$$P_h=\frac{1}{T}\int_v w_h\,dv=f w_h S l$$

$$w_h=2\frac{1}{2}2B_mH_m=\underline{\underline{2B_mH_m}}=2\mu_n\frac{N^2I_m^2}{l^2}$$

$$P_h=2f\mu_n\frac{N^2I_m^2}{l^{\cancel{2}}}S\cancel{l}\qquad\boxed{P_h=\frac{2f\mu_nN^2I_m^2S}{l}}$$

$$P_h=\frac{2\cdot 50\cdot 10^{-3}\cdot 10^6\;0{,}04\cdot 4\cdot 10^{-4}}{0{,}2}=80\cdot 10^{-1}W\quad\boxed{P_h=8W}$$

(218)




$$H_m=\frac{NI_m}{l}$$

$$B=\mu_n H_m$$

$$P_h=f w_h S l$$

$$w_h=2\frac{1}{2}\frac{3}{2}B_mH_m-2\frac{1}{2}\frac{B_m}{2}H_m$$

$$\boxed{w_h=B_mH_m}$$

$$w_h=\mu_n\frac{N^2I_m^2}{l^2}$$

$$P_h=fS\cancel{l}\mu_n\frac{N^2I_m^2}{l^{\cancel{2}}}$$

$$\boxed{P_h=\frac{\mu S f N^2 I_m^2}{l}}$$

(219) $i(t)=I_m\cos 2\pi f t$

$I_m=2A$

$f_1=25Hz\rightarrow P_1=4W$

$f_2=50Hz\rightarrow P_2=10W$

$P_{h1},P_{J1},P_{h2},P_{J2}=?$

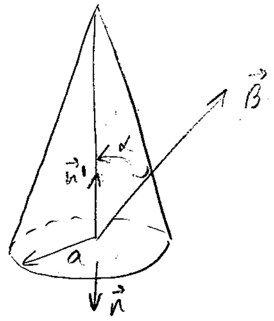
$P_h\sim f\qquad P_h=k_1f$

$P_J\sim f^2\qquad P_J=k_2f^2$

$$k_1f_1+k_2f_1^2=4W$$

пробни колоквијум

П 1




$$\Phi_M = -\Phi_B$$

$$\Phi_B = Ba^2\pi \cancel{\cos}(\pi-\alpha) \rightarrow -\cos\alpha$$

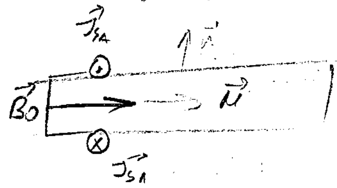
$$\boxed{\Phi_M = Ba^2\pi\cos\alpha}$$

2 $M,\ a,\ L \gg a$

$$B_0 = \frac{\mu_0 J_s}{2}\left(\cos 0 - \cos\frac{\pi}{2}\right)$$

$$B_0 = \frac{\mu_0 J_s}{2} = \frac{\mu_0 M}{2}$$

$$J_s = J_{SA} = M$$

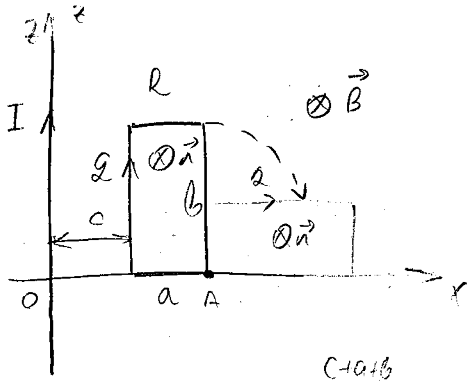



$$\vec{B}_0 = B_0\,\vec{i}_x$$

$$\boxed{\vec{B_0} = \frac{\mu_0 J_s}{2}\,\vec{i}_x}$$

3 $a,\ b,\ R,\ I,\ c$

$q = ?$




$$B = \frac{\mu_0 I}{2\pi r}$$

$$q = -\frac{\Delta\Phi}{R}$$

$$\Phi^{(2)} = \int_S \vec{B}\,d\vec{s} = \int_{c+a}^{c+a+b} \frac{\mu_0 I}{2\pi r}\,a\,dr = \frac{\mu_0 I a}{2\pi}\ln\frac{c+a+b}{c+a}$$

$$\Phi^{(1)} = \int_S \vec{B}\,d\vec{s} = \int_c^{a+c} \frac{\mu_0 I}{2\pi r}\,b\,dr = \frac{\mu_0 I b}{2\pi}\ln\frac{c+a}{c}$$

$$q = -\frac{1}{R}\frac{\mu_0 I}{2\pi}\left[a\ln\frac{c+a+b}{c+a} - b\ln\frac{c+a}{c}\right]$$

4 $l = 0{,}2\,m$

$S = 10\cdot10^{-4}\,m^2 = 10^{-3}\,m^2$

$N = 1000$

$I = ?$ а) $\Phi = 2\cdot10^{-4}\,Wb$

б) $\Phi = 4{,}5\cdot10^{-4}\,Wb$

$$\oint_C \vec{H}\,d\vec{l} = \sum I$$

$$Hl = NI \Rightarrow I = \frac{Hl}{N}$$

а) $\Phi = BS \Rightarrow B = \frac{\Phi}{S} = \frac{2\cdot10^{-4}}{10^{-3}} = 0{,}2\,T \Rightarrow H = 500\,\frac{A}{m}$

$$I = \frac{500\cdot0{,}2}{10^{-3}} = 0{,}1\,A$$

б) $\Phi = BS \Rightarrow B = \frac{\Phi}{S} = 0{,}45\,T \Rightarrow H = 3000\,A/m$

$$\boxed{I = 0{,}6\,A}$$