Univerzitet u Banjoj Luci

Elektrotehnički fakultet

Osnovi elektrotehnike 1

Teoreme električnih mreža

Predavanje: 11. blok

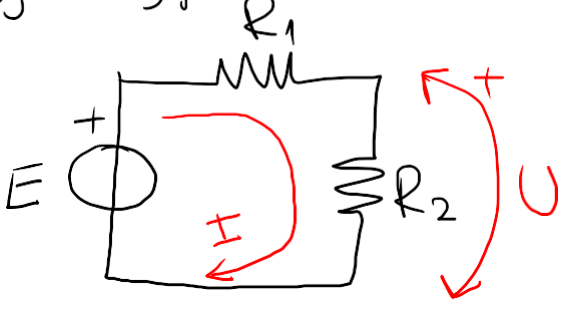
ТЕОРЕМЕ ЛИНЕАРНОСТИ

– теорема пропорционалности само један сигнал у колу (НТ или СТ)

E, I_g побуда — U, I одзив

$U = aE$
$I = bE$

$U = c \cdot I_g$
$I = d \cdot I_g$



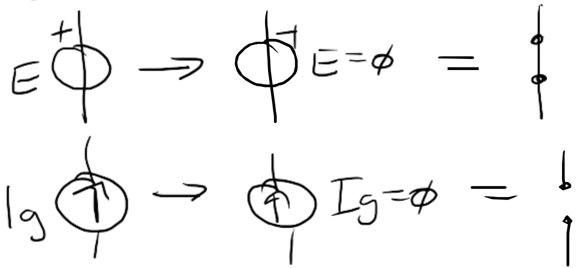
$$I = \frac{E}{R_1+R_2} = \frac{1}{R_1+R_2} E = aE$$

$$U = \frac{R_2}{R_1+R_2} E = bE$$

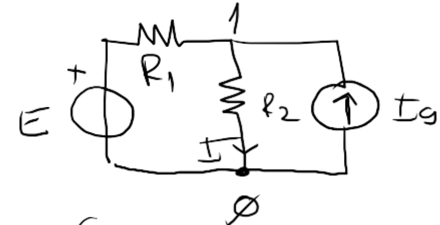
U – одзив, E – побуда

- теорема суперпозиције

$E = 0$, $I_g = 0$ анулирање побуде



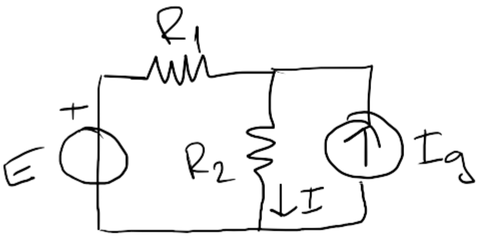
$$I = I(E) + I(I_g)$$



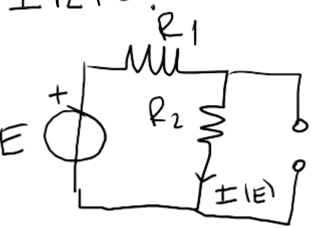
$$\left(\frac{1}{R_1} + \frac{1}{R_2}\right) V_1 = \frac{E_1}{R_1} + I_g$$

$$\frac{R_1 + R_2}{R_1 R_2} V_1 = \frac{E}{R_1} + I_g$$

$$V_1 = \frac{R_2}{R_1 + R_2} E + \frac{R_1 R_2}{R_1 + R_2} I_g$$

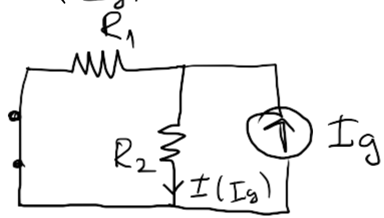


$I(E) = ?$



$$I(E) = \frac{E}{R_1 + R_2}$$

$I(I_g) = ?$



$$I(I_g) = I_g \frac{R_1}{R_1 + R_2}$$

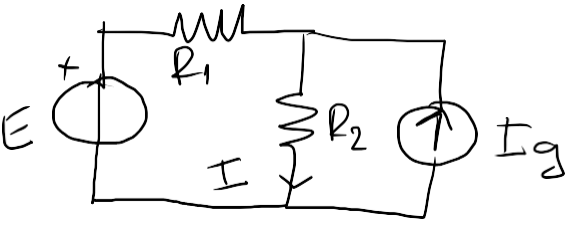
$$\Rightarrow I = \frac{E}{R_1 + R_2} + I_g \frac{R_1}{R_1 + R_2}$$

$$I = \frac{V_1}{R_2}$$

$$= \quad I = \frac{1}{R_1 + R_2} E + \frac{R_1}{R_1 + R_2} I_g$$

– теорема линейности

$U = a_1 E_1 + b_2 I_{g2} + a_3 E_3 + \dots$

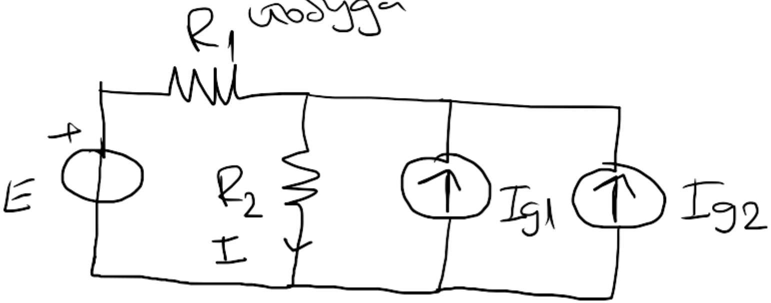


$$I = \underbrace{\frac{1}{R_1+R_2}}_{a} E + \underbrace{\frac{R_1}{R_1+R_2}}_{b} I_g = a E + b I_g$$

- теорема линеарне зависности одзива од побуде

$$U = aE + b$$

одзив (U); посматрана побуда (E); b ← дејство свих осталих побуда



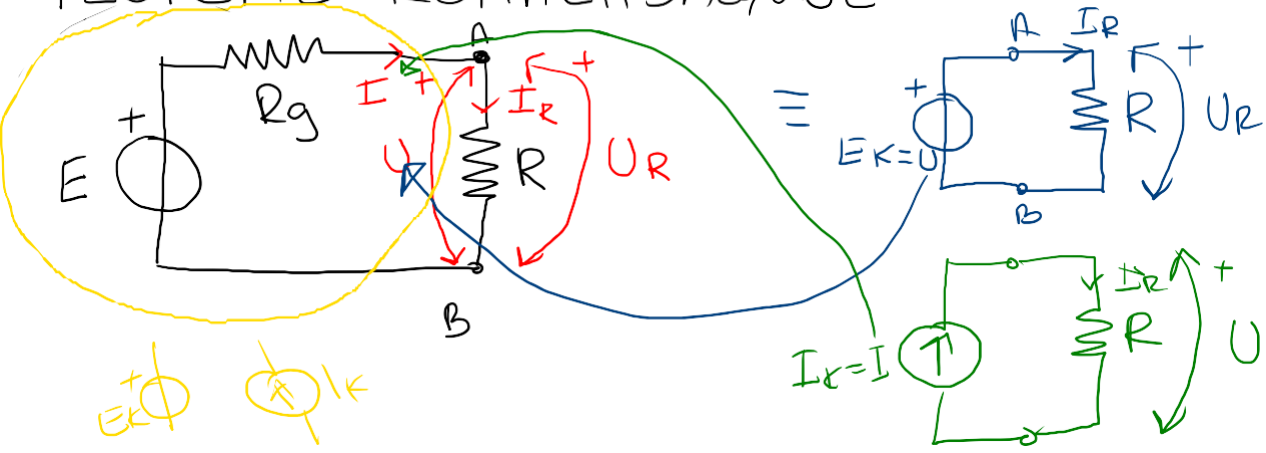
$$I = \frac{E}{R_1+R_2} + I_{g1}\frac{R_1}{R_1+R_2} + I_{g2}\frac{R_1}{R_1+R_2} = \underbrace{\frac{1}{R_1+R_2}}_{a}E + \underbrace{(I_{g1}+I_{g2})\frac{R_1}{R_1+R_2}}_{b}$$

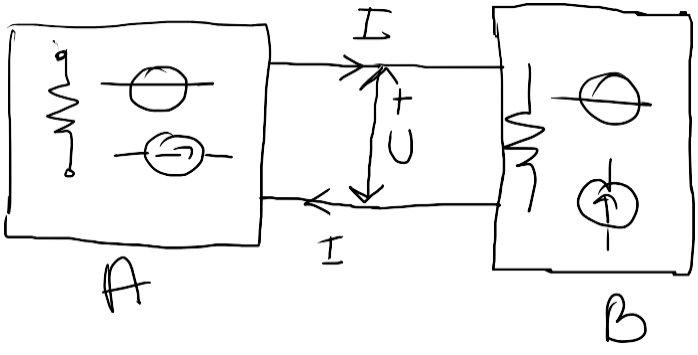
$$I = aE + b \qquad I''\,(E = 30\ V)$$

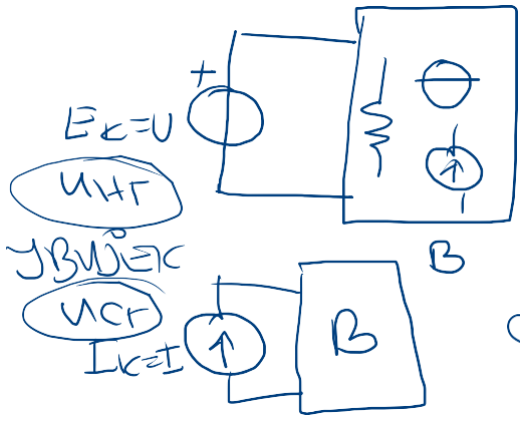
$I'\,(E = 10\ V)$ $\qquad I' = 10\cdot a + b$ $\qquad \Rightarrow a = \frac{I'-I''}{10}$ $\qquad I''' = \frac{I'-I''}{10}\cdot 30 + I''$

$I''\,(E = 0).$ $\qquad I'' = 0\cdot a + b \Rightarrow b = I''$

ТЕОРЕМЕ КОМПЕНЗАЦИЈЕ

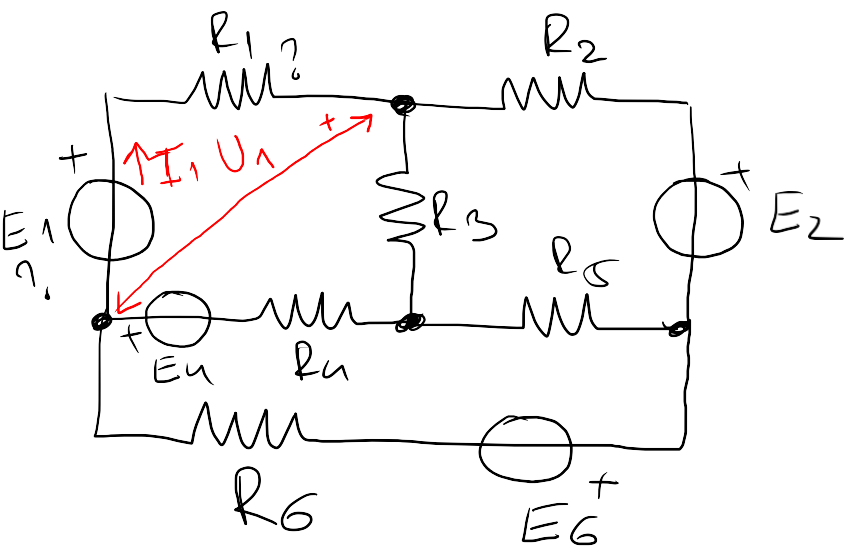






НАПОНСКИ КАЛИТ.

СТРУЈНИ КАЛИТ.

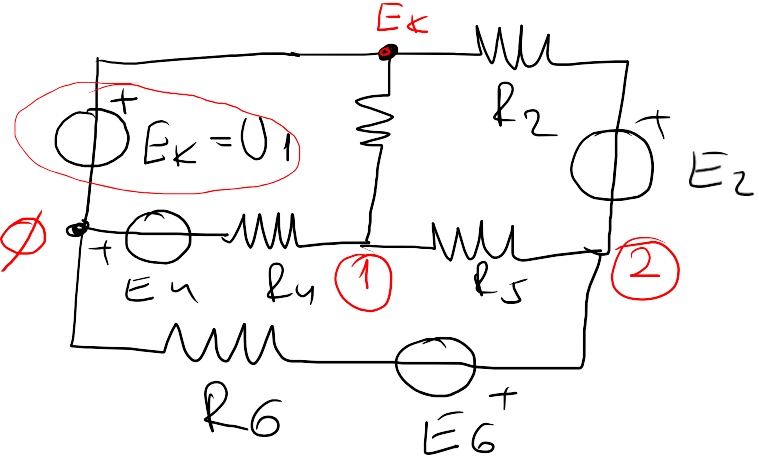


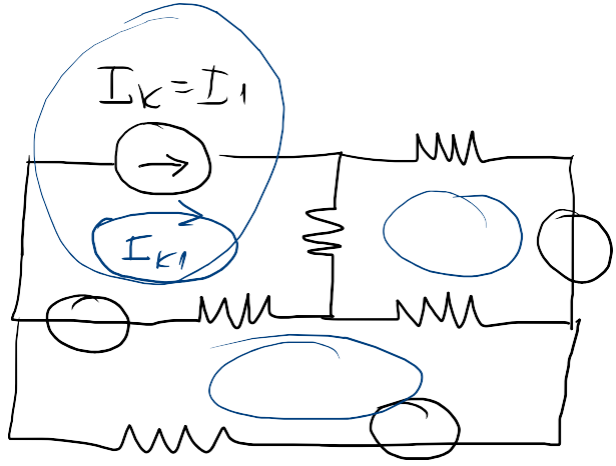
$n_u = 4$

$n_g = 6$

Konturne enačbe: 3 kos

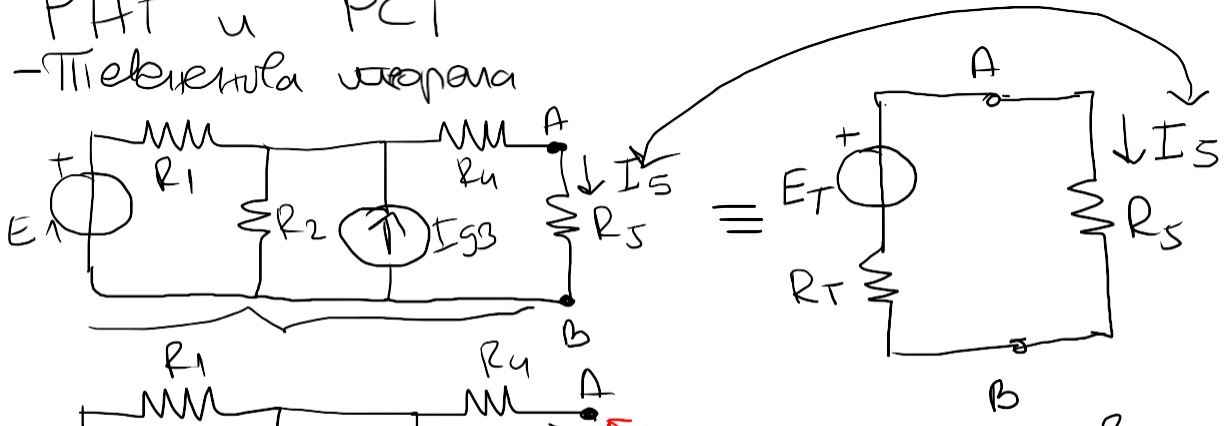
Uzlov enačbe: 3 kos

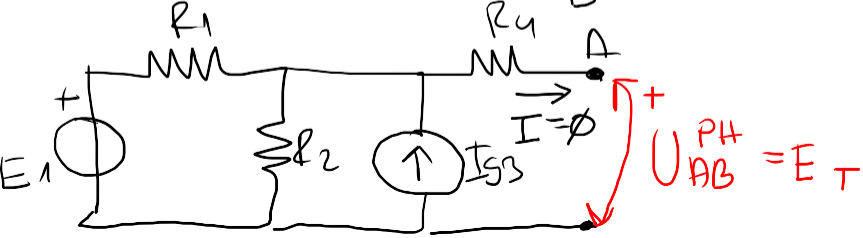




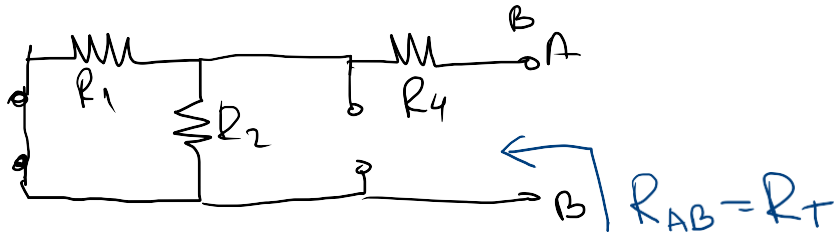
ТЕВЕНЕНОВА И НОРТОНОВА ТЕОРЕМА

РНТ и РСТ
- Тевенена теорема

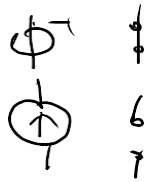




$$E_T = \frac{R_2}{R_1+R_2}E_1 + \frac{R_1R_2}{R_1+R_2}I_{g3}$$

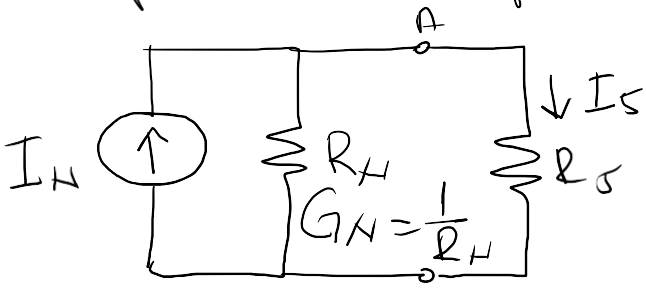


пасивизација кола: $E=0$, $I_g=0$



$$R_T = R_4 + \frac{R_1R_2}{R_1+R_2}$$

– Нортонова теорема



$R_N = R_T$

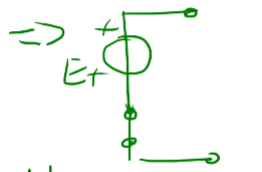
$I_N = \frac{E_T}{R_T}$

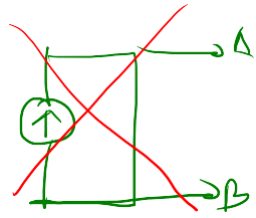
$R_T = R_N$

$E_T = I_N \cdot R_N$

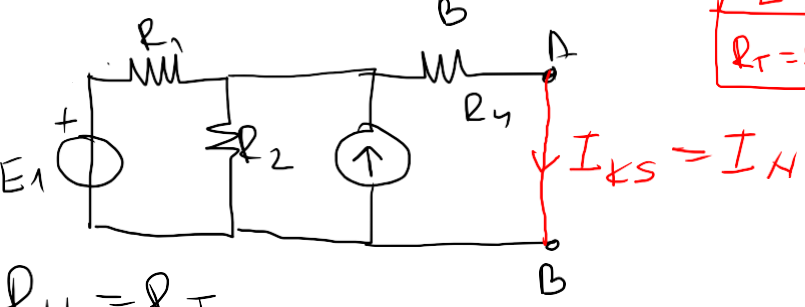
$R_T = R_N = E_T / I_N$

* $R_{AB} = \infty$



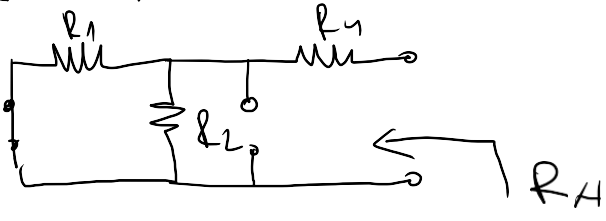


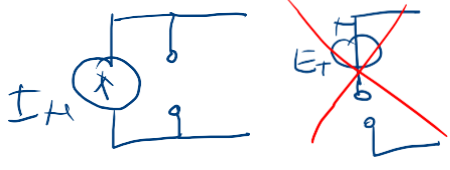
Може се определи само тевенов ЕСН.



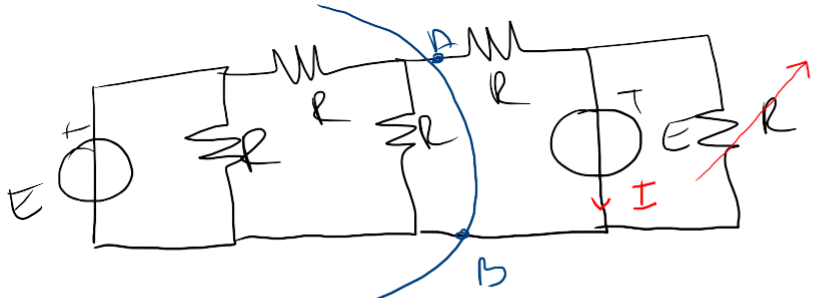
** $R_{AB} \to \infty$

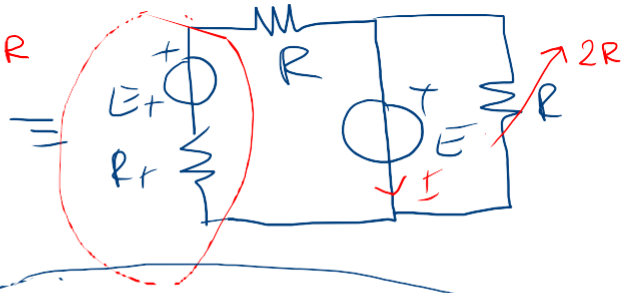
$R_N = R_T$

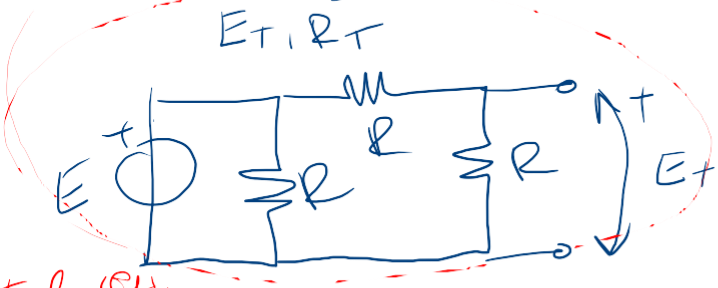


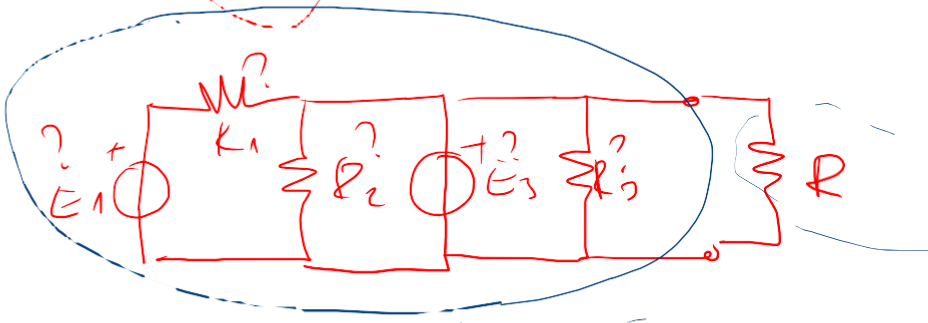


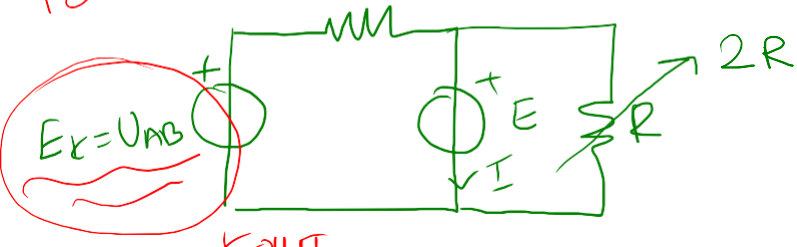
Само Нортонов ЕСН. се може определи

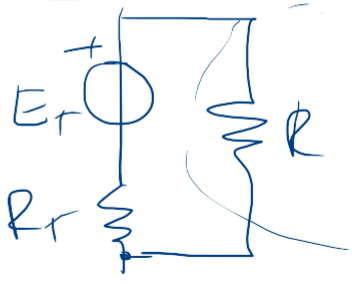

E
R
R
R
I
E
R
2R
a
b


E
R
R
I
E
R
2R


E
R
R
R
E


R
R
R
R

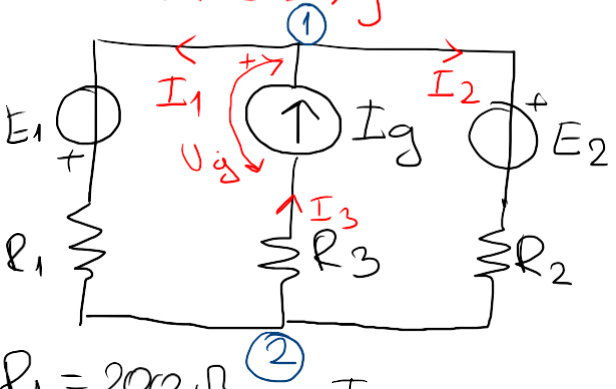

E
R
R
R
I
E
R
2R


R
R

ТЕОРЕМА ОДРЖАЊА СНАГЕ

$$\sum P_g = \sum P_p$$

$$\sum (UI)_g = \sum (UI)_p$$



$R_1 = 200\ \Omega$

$R_2 = 1000\ \Omega$

$R_3 = 400\ \Omega$

$E_1 = 7\ V$

$E_2 = 18\ V$

$I_{g3} = -55\ mA$

$I_1 = -0{,}025\ A$

$I_2 = -0{,}03\ A$

$I_3 = -0{,}055\ A$

$U_{12} = -12\ V$

$$\sum P_g = \sum P_p$$

$$E_2(-I_2) + E_1 \cdot I_1 + U_g \cdot I_g = R_1 I_1^2 + R_2 I_2^2 + R_3 I_3^2$$

$$U_{12} = U_g - R_3 I_3 \Rightarrow U_g = -34\ V$$

$P_{E1} = E_1 I_1 = -0{,}175\ W$

$P_{E2} = E_2(-I_2) = 0{,}54\ W$

$P_{Ig} = U_g I_g = 1{,}87\ W$

} $P_g = 2{,}235\ W$

$P_{R1} = 0{,}125\ W$

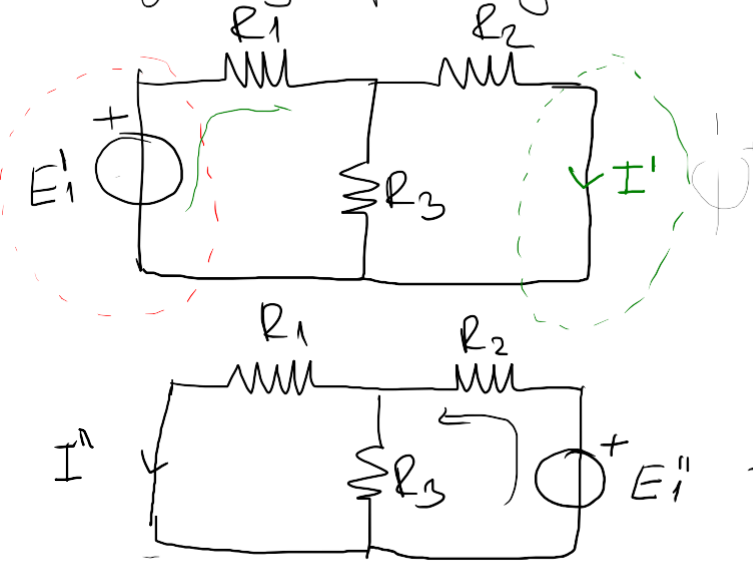
$P_{R2} = 0{,}90\ W$

$P_{R3} = 1{,}21\ W$

} $P_p = 2{,}235\ W$

ТЕОРЕМЕ РЕЦИПРОЦИТЕТА (УЗАЈМНОСТИ)

Ако електромоторна сила E_1 у грани ј производи у грани к струју јачине I_1, онда би она производила исту јачину струје у грани ј ако би исто била у грани к.



Важи за кола са само једном електромоторном силом

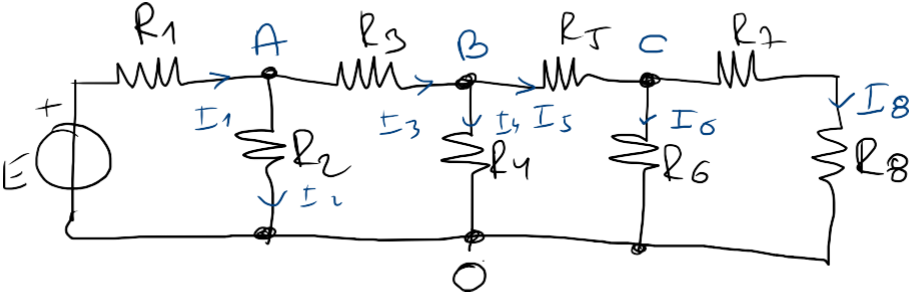
$$I' = \frac{E_1'}{R_1 + \frac{R_2 R_3}{R_2 + R_3}} \cdot \frac{R_3}{R_2 + R_3} = \frac{E_1' (R_2 + R_3) \cdot R_3}{(R_1 R_2 + R_1 R_3 + R_2 R_3)(R_2 + R_3)}$$

ако је $E_1'' = E_1' \Rightarrow I'' = I'$

$$I'' = \frac{E_1''}{R_2 + \frac{R_1 R_3}{R_1 + R_3}} \cdot \frac{R_3}{R_1 + R_3} = \frac{E_1'' (R_1 + R_3) \cdot R_3}{(R_1 + R_3)(R_1 R_2 + R_2 R_3 + R_1 R_3)}$$

$\Rightarrow I'$ и $I'' \Rightarrow$ **$I' = I''$ ако је $E_1' = E_1''$**

МЕТОДА ПРОПОРЦИОНАЛНИХ ВЕЛИЧИНА



Одредимо струје у гранама кола за неку претпостављену вредност емс (E'). Добијене вредности за E' треба помножити са E/E' да би се добиле вредности за E.

$$I = I' \frac{E}{E'}$$

Замислимо да кроз R_7 и R_8 протиче струја јачине $I = 1\,A$. Онда одредимо

$$U_{C0} = (R_7 + R_8)\cdot I_8 \Rightarrow I_6 = \frac{U_{C0}}{R_6}$$

$$I_5 = I_6 + I_6$$

$$U_{BC} = R_5 I_5$$

$$U_{B0} = R_5 \cdot I_5 + R_6 I_6$$

$$I_4 = \frac{U_{B0}}{R_4}$$

$$I_3 = I_4 + I_5$$

$$U_{A0} = R_3 I_3 + R_4 I_4$$

$$I_2 = \frac{U_{A0}}{R_2}$$

$$I_1 = I_2 + I_3$$

$$U_{A0} = E' - R_1 I_1 \Rightarrow E' = U_{A0} + R_1 I_1 = 18\,V$$

ако је $E' = 18\,V$ онда $I_1, \ldots, I_8$

ако је $E = 10\,V$

$\frac{10}{18} I_1, \frac{10}{18} I_2, \ldots, \frac{10}{18} I_8$

КОРИШЋЕЊЕ СИМЕТРИЈЕ СИСТЕМА