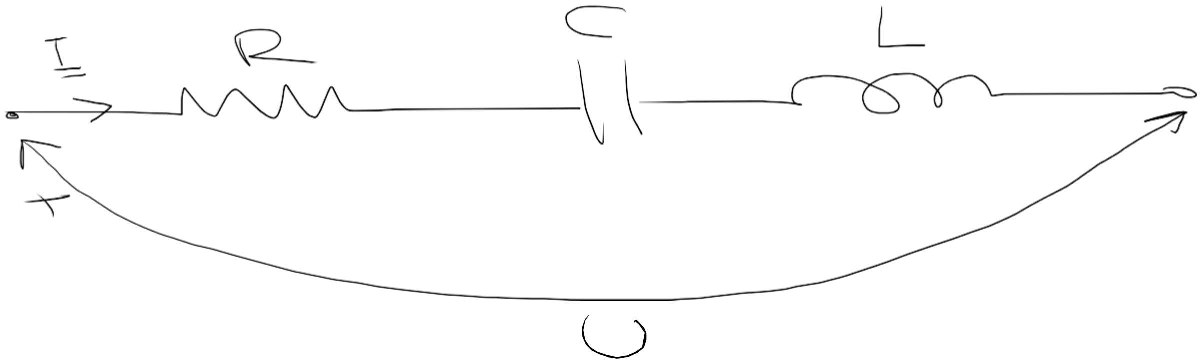
Теореме кола у простопериодичном режиму.

Основи електротехнике 2

Предавање: 10. блок

РЕДНО РЕЗОНАНТНО КОЛО



фазор $\underline{U} = U \angle \phi$ (ω) x

$\underline{U} = U$

$\underline{Z} = Z e^{j\phi}$

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{U e^{j0}}{Z e^{j\phi}} = \frac{U}{Z} e^{-j\phi}$$

$$\underline{Z} = R + j\omega L + \frac{1}{j\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$= \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}\, e^{j\,\mathrm{arctg}\frac{\omega L - \frac{1}{\omega C}}{R}} = Z e^{j\phi}$$

Ефективна вредност струје у колу је

$$I = \frac{U}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \qquad \underline{I} = I e^{j\psi}$$

$$\theta - \psi = \phi$$

ако је $\theta = 0 \Rightarrow \psi = -\phi = -\operatorname{arctg}\dfrac{\omega L - \frac{1}{\omega C}}{R}$

$$I = \frac{U}{\sqrt{R^2 + \underbrace{\left(\omega L - \frac{1}{\omega C}\right)^2}_{0}}} \longrightarrow \max$$

$$\omega L - \frac{1}{\omega C} = 0 \Rightarrow \omega L = \frac{1}{\omega C}$$

1° $\omega = const$, а L и C су изабрани тако да је испуњен услов $\omega L = \frac{1}{\omega C}$

2° мијења кружна учестаност генератора

$$\omega_r = \frac{1}{\sqrt{LC}}$$

$$I(\omega=\omega_r)=\frac{U}{R}$$
$$\phi=-\psi=0$$

ефективна вредност струје достиже макс. и струја је у фази са прикљученим напоном

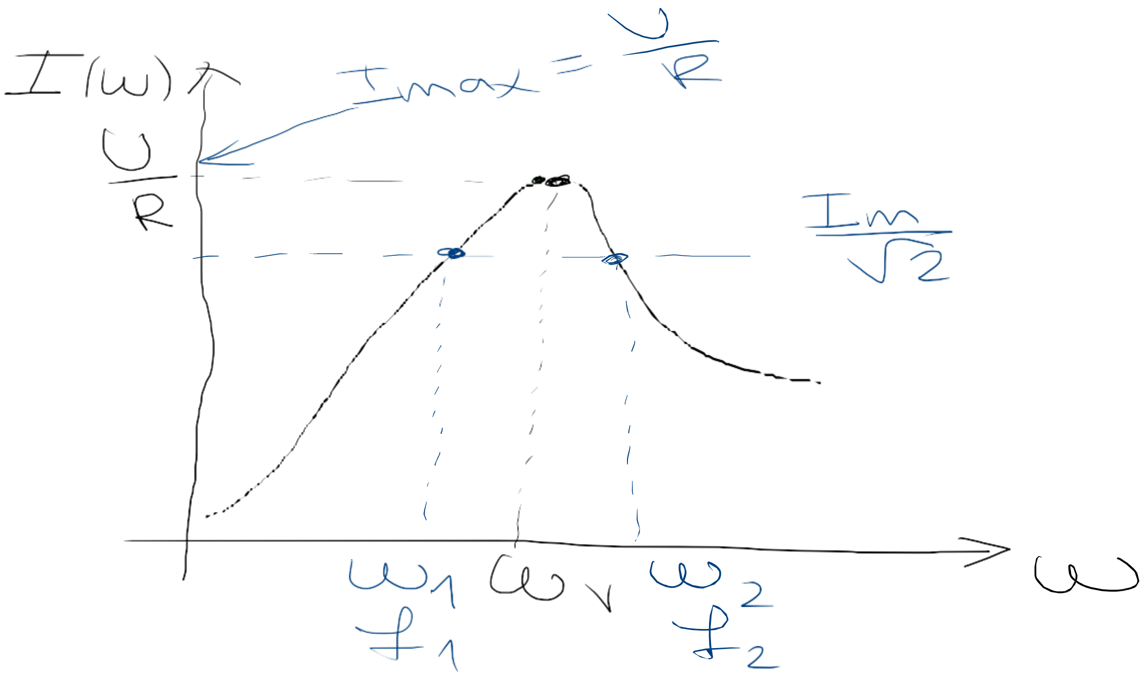
Каже се да је коло у резонанцији.

$$\omega_r=\frac{1}{\sqrt{LC}}$$ ~~кружна~~ резонантна кружна учестаност

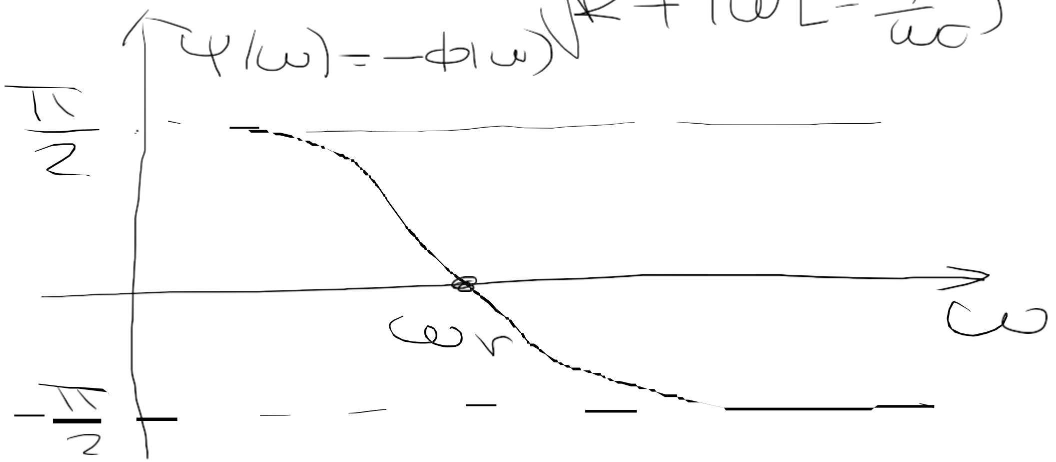
$$f_r=\frac{1}{2\pi\sqrt{LC}}$$

При $\omega=\omega_r$: $\underline{U}_L+\underline{U}_C=j\omega_r L\underline{I}+\frac{1}{j\omega_r C}\underline{I}=j\underbrace{\left(\omega_r L-\frac{1}{\omega_r C}\right)}_{0}=0$

$$\underline{U}_L=j\omega_r L\underline{I}$$
$$\underline{U}_C=\frac{1}{j\omega_r C}\underline{I}$$
$$U_L=\omega_r L I\neq 0$$



$$I(\omega) = \frac{U}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$



f_1 и f_2 се називају граничне фреквенције а њихова разлика $\boxed{\Delta f = f_2 - f_1}$ пропусни опсег.

− селективност

широк Δf → коло је мање селективно

узак Δf → коло је веће селективности фактор доброте кола (Q-фактор)

$$Q = \frac{f_r}{\Delta f}$$

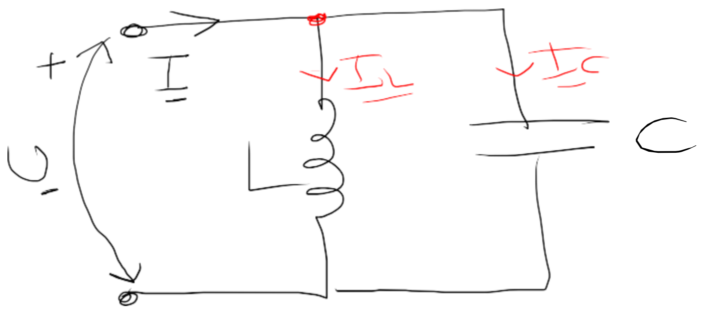
за редно резонантно коло

$$Q = \frac{\omega_r \cdot L}{R} = \frac{1}{\omega_r R C}$$

* општија дефиниција Q-фактора

$$Q = 2\pi \frac{\text{ЕНЕРГИЈА САДРЖАНА У КОЛУ ПРИ РЕЗ. УЧЕСТАНОСТИ}}{\text{ЕНЕРГИЈА ЏУЛОВИХ ГУБИТАКА У ТОКУ ЈЕДНОГ ПЕРИОДА ПРИ РЕЗ. УЧЕСТАНОСТИ}}$$

Паралелно резонантно коло



$\underline{U} = U$ узимамо напон и [illegible]

$$\underline{I} = \underline{I}_L + \underline{I}_C = \frac{\underline{U}}{j\omega L} + \frac{\underline{U}}{\frac{1}{j\omega C}}$$

$$\underline{I} = \frac{\underline{U}}{j\omega L} + j\omega C\underline{U} = j\left(\omega C - \frac{1}{\omega L}\right)\underline{U} = I e^{j\varphi}$$

$$\underline{I} = \left(\omega C - \frac{1}{\omega L}\right)e^{j\frac{\pi}{2}} \cdot U e^{j0} = \underbrace{U\left(\omega C - \frac{1}{\omega L}\right)}_{}e^{j\frac{\pi}{2}} = I e^{j\varphi}$$

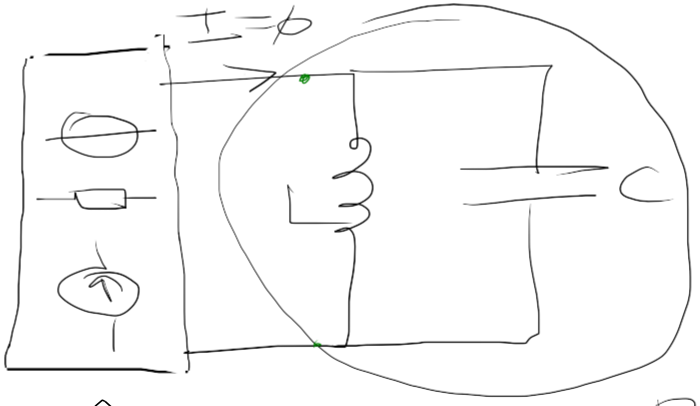
ако је $\omega C - \frac{1}{\omega L} = 0 \Rightarrow I = U\left(\omega C - \frac{1}{\omega L}\right) = 0$

1° L, C бирамо тако да је $\omega C = \frac{1}{\omega L}$

2° $\omega = \omega_0 = \frac{1}{\sqrt{LC}}$ антирезонантна учестаност

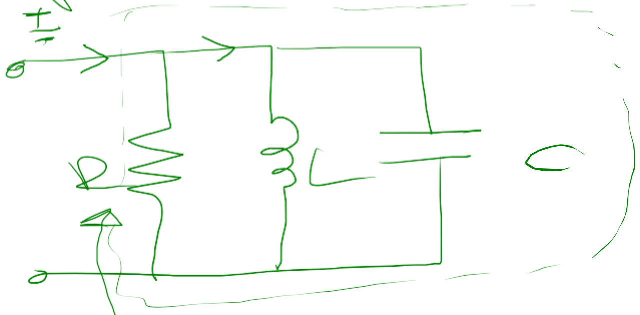
При антирезонанцији, струје калема и кондензатора нису једнаке нули али њихов збир јесте

$$I_C + I_L = 0$$



МАТ $\Leftrightarrow$ ЕЛ.

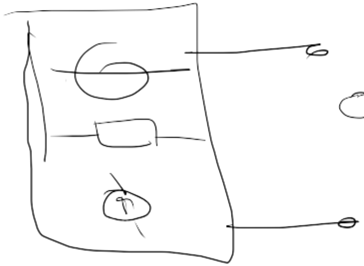
Практично:



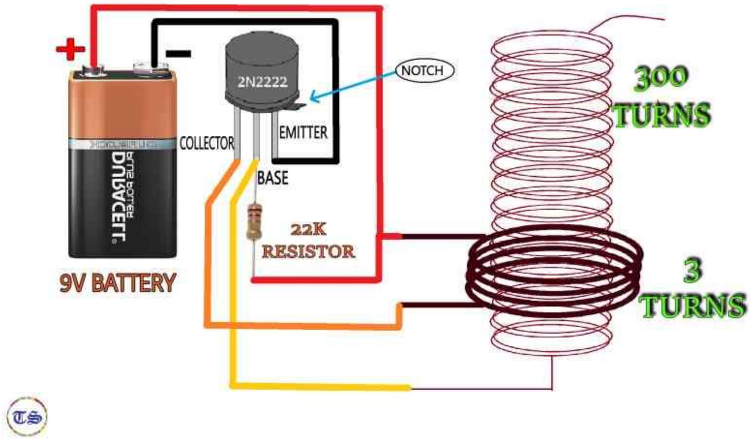
представља губитке

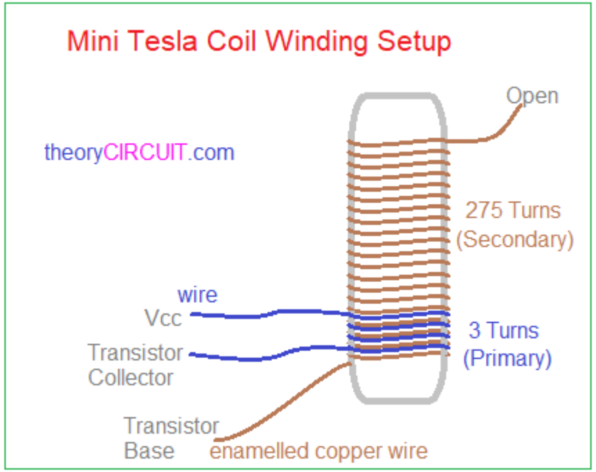
остатак кола

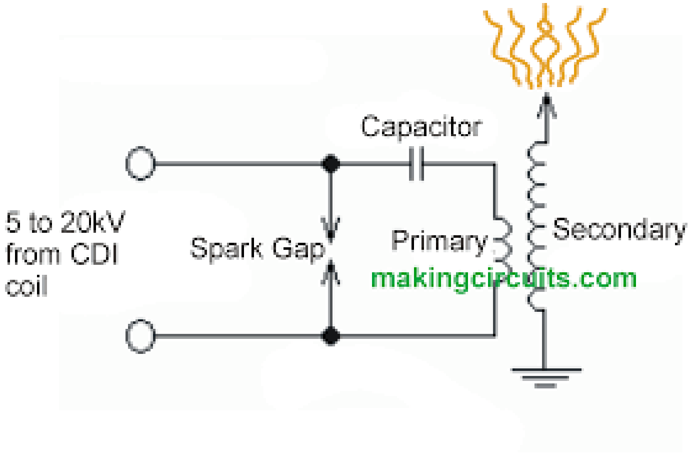
размјена енергије: калем се празни а конд. се пуни и обрнуто

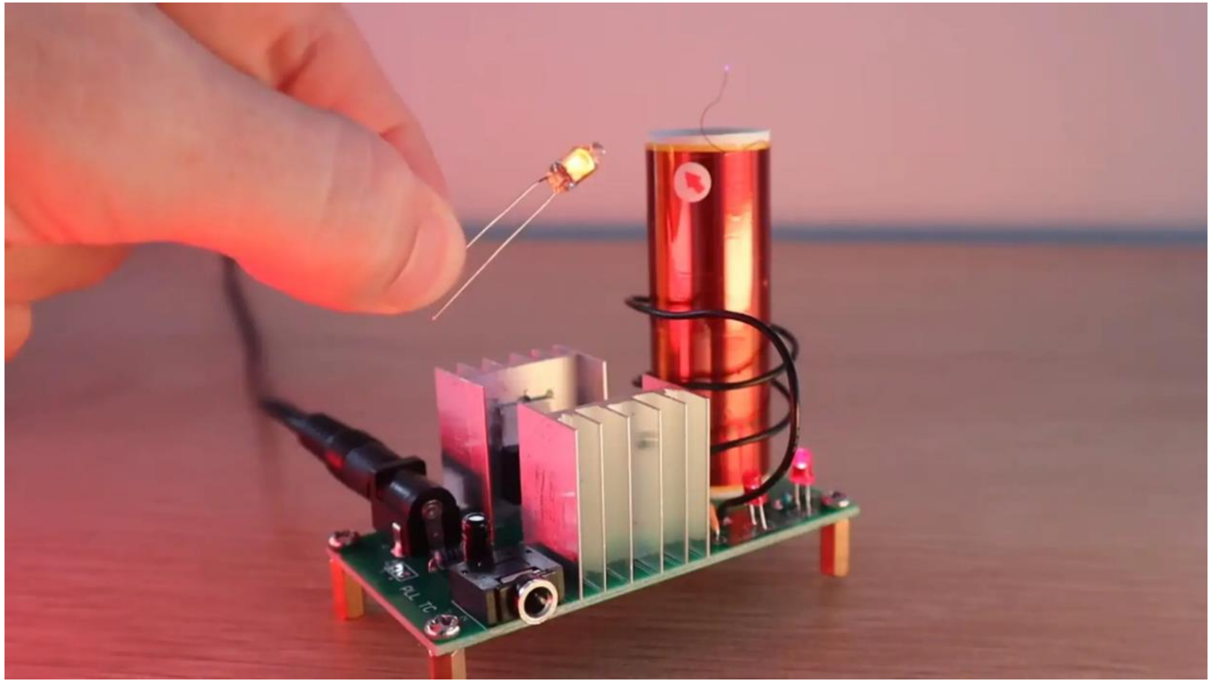


отворена веза


NOTCH
2N2222
COLLECTOR
EMITTER
BASE
22K RESISTOR
300 TURNS
3 TURNS
9V BATTERY


Mini Tesla Coil Winding Setup
theoryCIRCUIT.com
Open
275 Turns (Secondary)
wire
Vcc
Transistor Collector
3 Turns (Primary)
Transistor Base
enamelled copper wire


Capacitor
5 to 20kV from CDI coil
Spark Gap
Primary
Secondary
makingcircuits.com

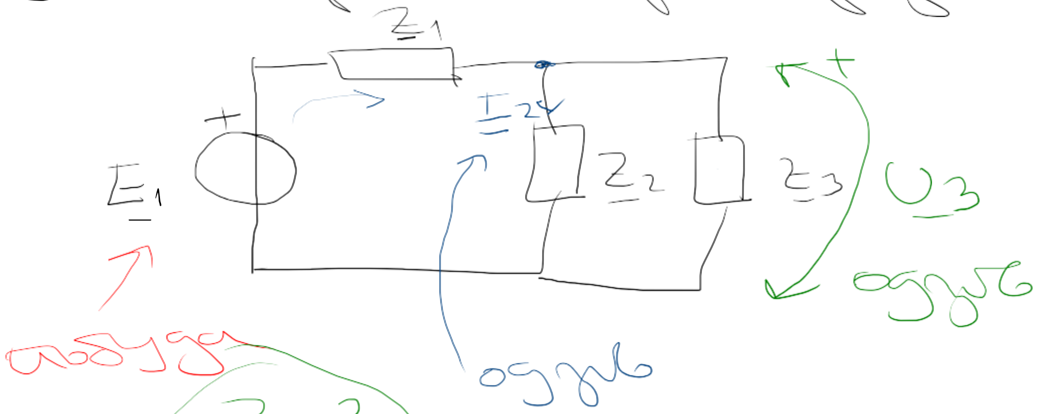


ТЕОРЕМЕ ЕЛ. КОЛА У КОМПЛЕКСНОМ ДОМЕНУ

Кола су у простопериодичном режиму али анализу вршимо у компл. домену.

– ТЕОРЕМА ЛИНЕАРНОСТИ

1. теорема пропорционалности



$$\underline{U}_3 = \frac{\frac{\underline{Z}_2 \cdot \underline{Z}_3}{\underline{Z}_2+\underline{Z}_3}}{\underline{Z}_1 + \frac{\underline{Z}_2\underline{Z}_3}{\underline{Z}_2+\underline{Z}_3}} \cdot \underline{E}_1 = \underline{b}_2 \cdot \underline{E}_1$$

$$\underline{I}_2 = \frac{\underline{E}_1}{\underline{Z}_1 + \frac{\underline{Z}_2 \cdot \underline{Z}_3}{\underline{Z}_2+\underline{Z}_3}} \cdot \frac{\underline{Z}_3}{\underline{Z}_2+\underline{Z}_3}$$

$$\underline{I}_2 = \frac{\underline{Z}_3}{(\underline{Z}_2+\underline{Z}_3)\left(\underline{Z}_1 + \frac{\underline{Z}_2\underline{Z}_3}{\underline{Z}_2+\underline{Z}_3}\right)} \underline{E}_1$$

ако се промени: $\underline{E}_1 \rightarrow 2\underline{E}_1$

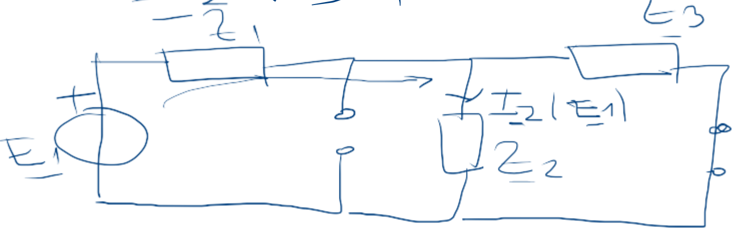
$$\underline{I}_2' = \frac{\underline{Z}_3}{(\underline{Z}_2+\underline{Z}_3)\left(\underline{Z}_1 + \frac{\underline{Z}_2\underline{Z}_3}{\underline{Z}_2+\underline{Z}_3}\right)} 2\underline{E}_1$$

$$\underline{I}_2 = \underline{a}_1 \cdot \underline{E}_1$$

2. теорема суперпозиције

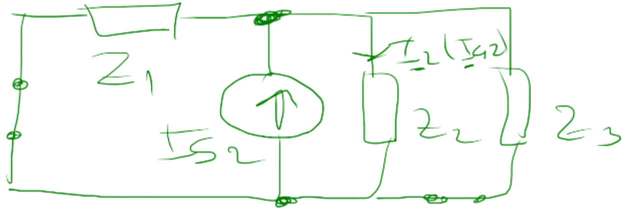
$$\underline{I}_2 = \underline{I}_2(E_1) + \underline{I}_2(I_{g2}) + \underline{I}_2(E_3)$$

1° $\underline{I}_2(E_1) = ?$



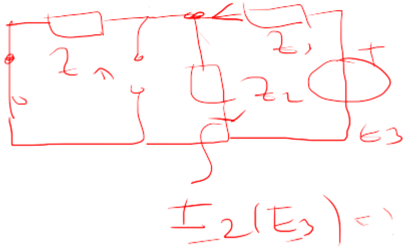
$$\underline{I}_2(E_1) = \frac{E_1}{\underline{Z}_1 + \frac{\underline{Z}_2 \cdot \underline{Z}_3}{\underline{Z}_2 + \underline{Z}_3}} \cdot \frac{\underline{Z}_3}{\underline{Z}_2 + \underline{Z}_3}$$

2° $\underline{I}_2(I_{g2}) = ?$



$$\underline{I}_2(I_{g2}) = \frac{I_{g1} \cdot (\underline{Z}_1 \| \underline{Z}_2 \| \underline{Z}_3)}{\underline{Z}_2}$$

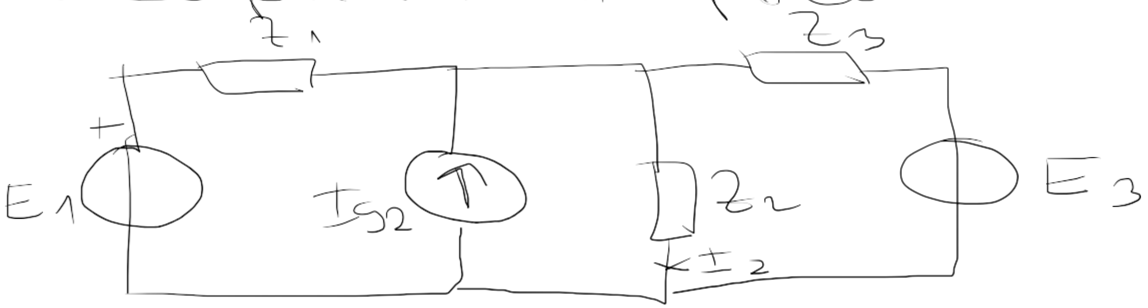
3° $\underline{I}_2(E_3) = ?$



$$\underline{I}_2(E_3) = \frac{\underline{Z}_1}{\underline{Z}_1 + \underline{Z}_2} \cdot \frac{E_3}{\underline{Z}_3 + \frac{\underline{Z}_1 \cdot \underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2}}$$

$$\underline{I} = \underline{I}_2(E_1) + \underline{I}_2(I_{g2}) + \underline{I}_2(E_3) = \underline{\text{Након од методу}}$$

3. Теорема линеарности

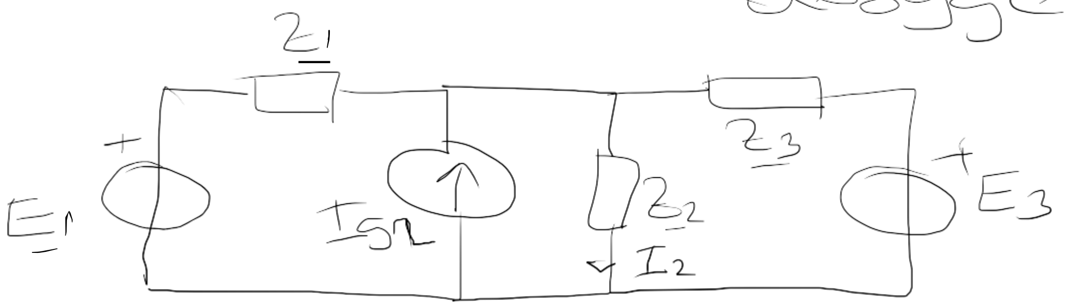


$\underline{I}_2 = ?$ посматрати одзив

$$\underline{I}_2 = \underbrace{\frac{\underline{Z}_3}{\underline{Z}_2+\underline{Z}_3}\cdot\frac{1}{\underline{Z}_1+\frac{\underline{Z}_2\cdot\underline{Z}_3}{\underline{Z}_2+\underline{Z}_3}}}_{\underline{a}_1}\underline{E}_1 + \underline{I}_{g2}\underbrace{\frac{(\underline{Z}_1\|\underline{Z}_2\|\underline{Z}_3)}{\underline{Z}_2}}_{\underline{a}_2} + \underbrace{\frac{\underline{Z}_1}{\underline{Z}_1+\underline{Z}_2}\cdot\frac{1}{\underline{Z}_3+\frac{\underline{Z}_1\underline{Z}_2}{\underline{Z}_1+\underline{Z}_2}}}_{\underline{a}_3}\underline{E}_3$$

$$\boxed{\underline{I}_2 = \underline{a}_1\cdot\underline{E}_1 + \underline{a}_2\,\underline{I}_{g2} + \underline{a}_3\,\underline{E}_3}$$

4. Теорема линеарне зависности одзива од побуде



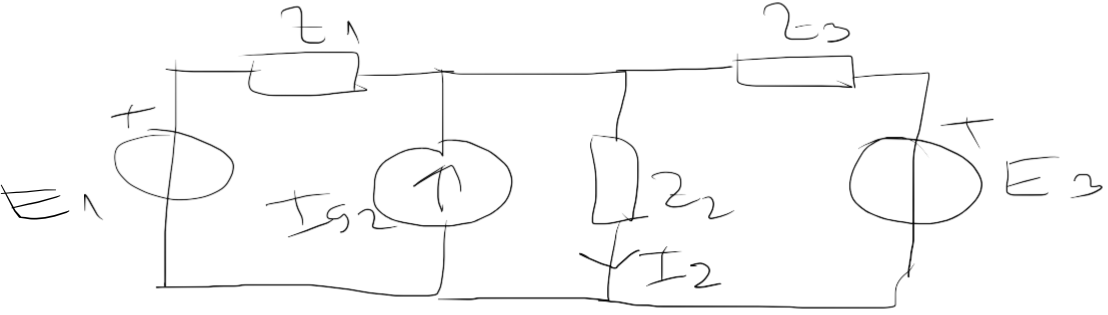
$$\underline{I}_2 = \underline{a}_1\cdot\underline{E}_1 + \underline{a}_2\cdot\underline{I}_{g2} + \underline{a}_3\underline{E}_3$$

ако се у колу мијења нпр. само $\underline{I}_{g2}$

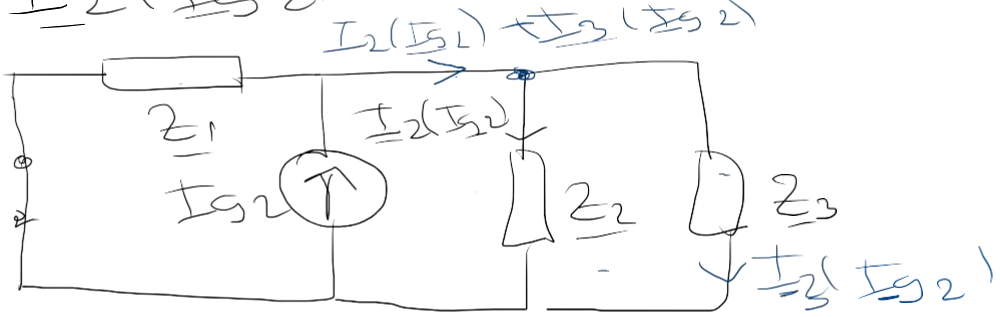
$$\underline{I}_2 = \underline{a}_2\cdot\underline{I}_{g2} + \underbrace{(\underline{a}_1\underline{E}_1 + \underline{a}_2\underline{E}_3)}_{\text{не мијењају}} = \underline{a}\cdot\underline{I}_{g2} + \underline{b}$$

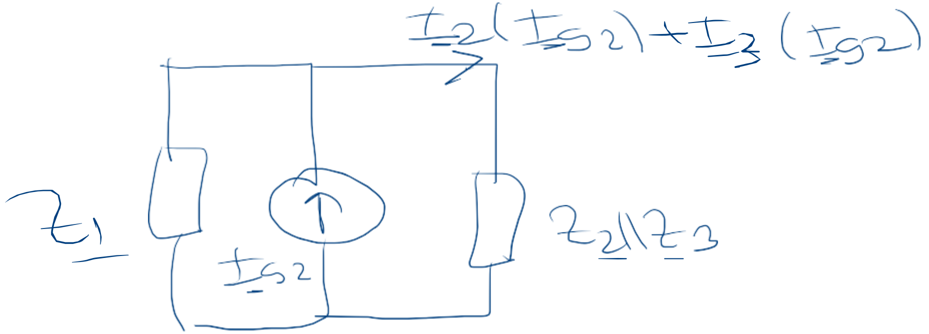
побуда која се мијења

збирно дејство осталих побуда које се не мијењају



$\underline{I}_2(\underline{I}_{g2}) = ?$





$$\underline{I}_2(\underline{I}_{g2}) = \underline{I}_{g2}\,\frac{\underline{Z}_1}{\underline{Z}_1+\frac{\underline{Z}_2\underline{Z}_3}{\underline{Z}_2+\underline{Z}_3}} \cdot \frac{\underline{Z}_3}{\underline{Z}_2+\underline{Z}_3}$$

$$= \underline{I}_{g2}\,\frac{\underline{Z}_1\cancel{(\underline{Z}_2+\underline{Z}_3)}}{\underline{Z}_1\underline{Z}_2+\underline{Z}_1\underline{Z}_3+\underline{Z}_2\underline{Z}_3} \cdot \frac{\underline{Z}_3}{\cancel{\underline{Z}_2+\underline{Z}_3}} = \boxed{\frac{\underline{Z}_1\cdot\underline{Z}_3}{\underline{Z}_1\underline{Z}_2+\underline{Z}_1\underline{Z}_3+\underline{Z}_2\underline{Z}_3}\,\underline{I}_{g2}}$$

*

$$I_2(I_{S2}) = \frac{I_{S2} \cdot Z_1 \| Z_2 \| Z_3}{Z_2}$$

$(Z_1 \| Z_2) \| Z_3$

$$\frac{1}{Z_e} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}$$

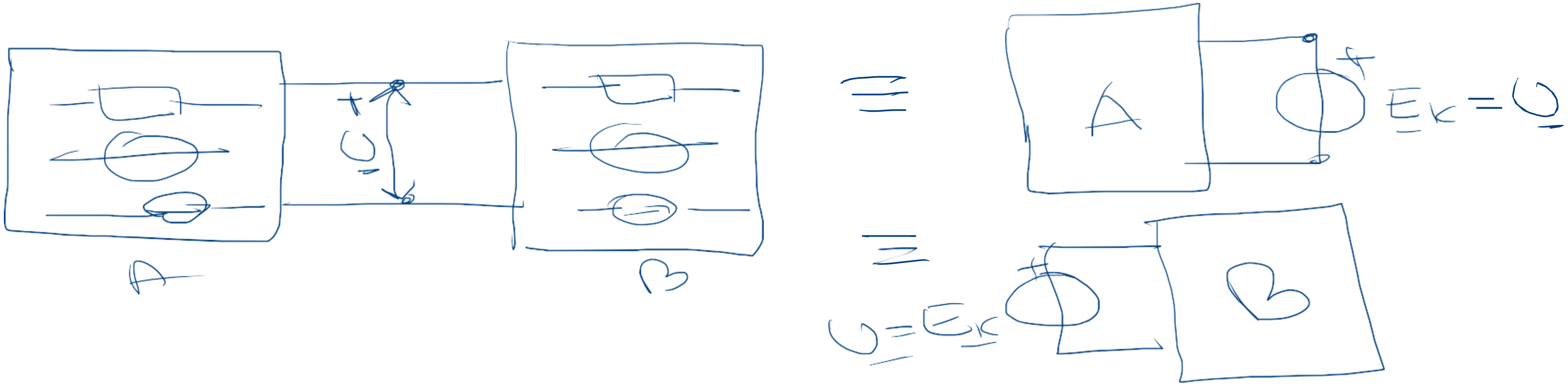
$$Z_e = \frac{\frac{Z_1 \cdot Z_2}{Z_1+Z_2} \cdot Z_3}{\frac{Z_1 \cdot Z_2}{Z_1+Z_2} + Z_3} = \frac{Z_1 \cdot Z_2 \cdot Z_3}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}$$

$$I_2(I_{S2}) = I_{S2} \frac{1}{\cancel{Z_2}} \frac{Z_1 \cancel{Z_2} Z_3}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}$$

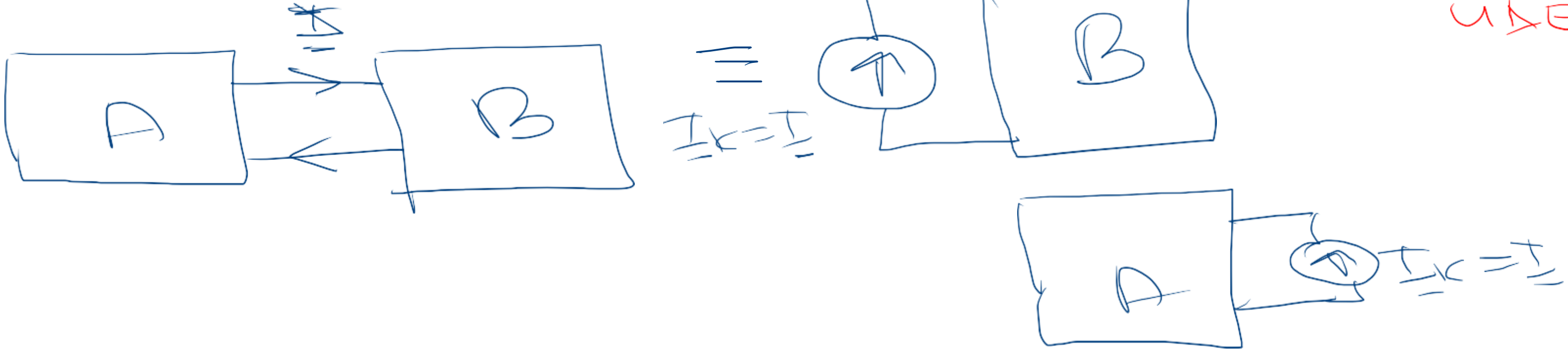
$$I_2(I_{S2}) = \frac{Z_1 Z_3}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3} \cdot I_{S2}$$

Теорема компензације

1. Напонска компензација



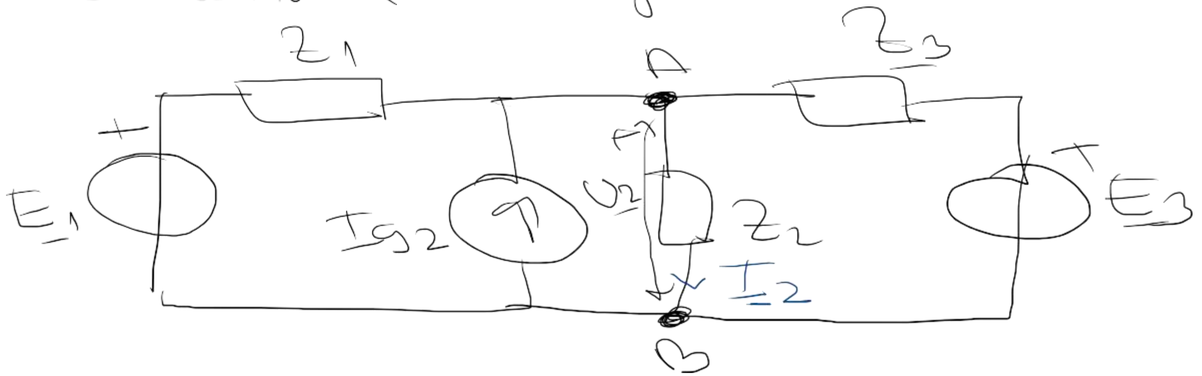
2. Струјна компензација

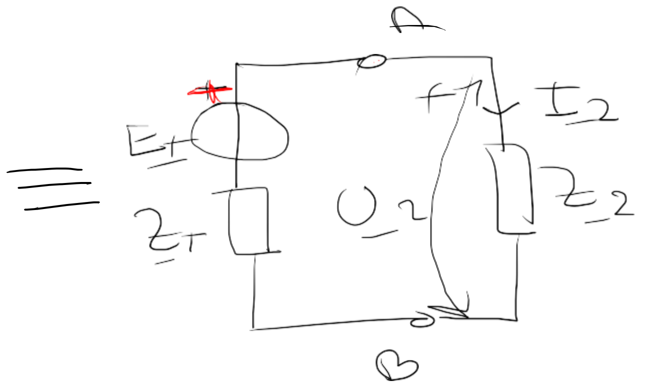


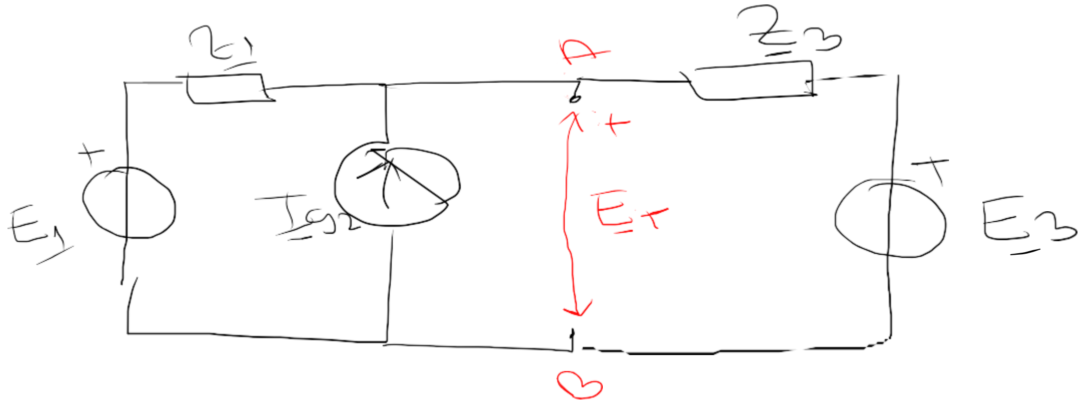
КОМПЕНЗАЦИОНИ ГЕНЕРАТОРИ СУ ИДЕАЛНИ!

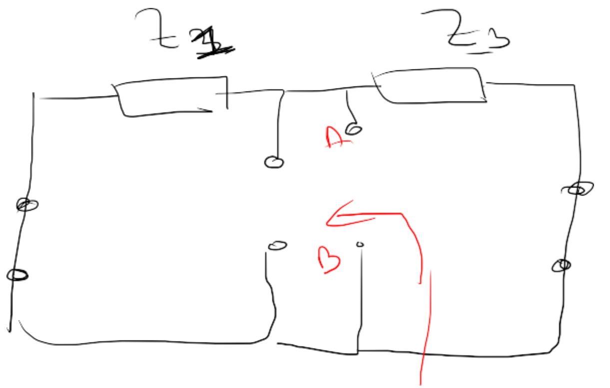
ТЕВЕНЕНОВА И НОРТОНОВА ТЕОРЕМА

Тевененова теорема



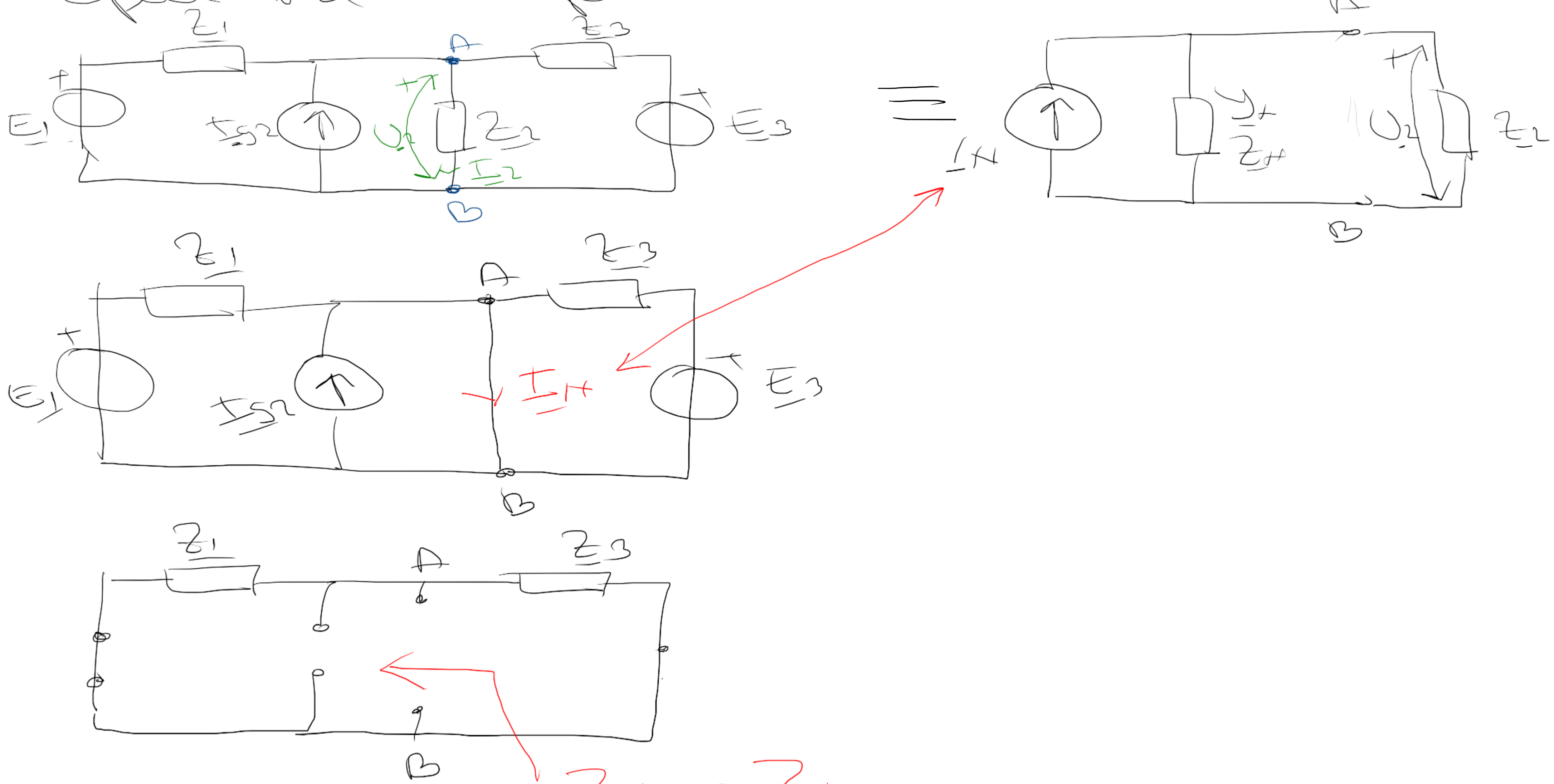






$Z_T = Z_{AB}$

Нортонова теорема



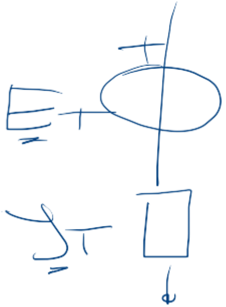
$Z_H = Z_{AB}$

Ваше релације

$$\underline{I}_H = \frac{\underline{E}_T}{\underline{Z}_T}$$

$$\underline{Z}_H = \underline{Z}_T$$

$$\underline{Y}_H = \frac{1}{\underline{Z}_T}$$



и

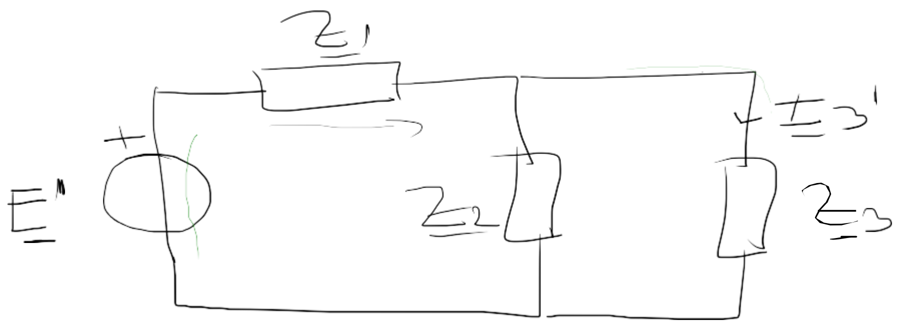
$$\underline{E}_T = \underline{I}_H \cdot \underline{Z}_H = \underline{I}_H / \underline{Y}_H$$

$$\underline{Z}_T = \underline{Z}_H$$

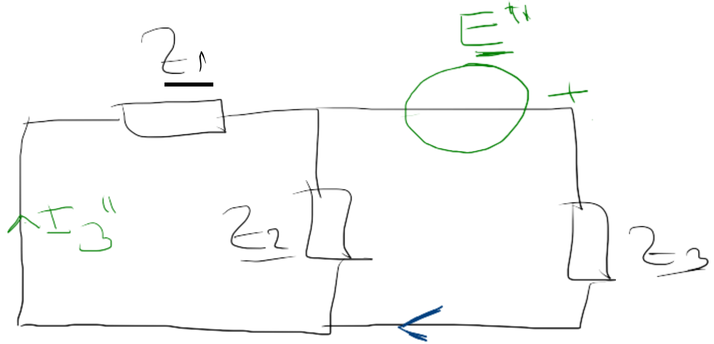
$$\underline{Z}_T = \frac{1}{\underline{Y}_H}$$

$$\underline{Y}_T = \frac{1}{\underline{Z}_T} = \underline{Y}_H$$

ТЕОРЕМА РЕЦИПРОЦИТЕТА



$\longleftrightarrow$



ако је $\underline{E}' = \underline{E}''$ онда је $\underline{I}_3'' = \underline{I}_3'$

$$\underline{I}_3' = \frac{\underline{Z}_2}{\underline{Z}_2+\underline{Z}_3} \cdot \frac{\underline{E}'}{\underline{Z}_1 + \frac{\underline{Z}_2\underline{Z}_3}{\underline{Z}_2+\underline{Z}_3}}$$

$$= \frac{\underline{Z}_2}{\cancel{\underline{Z}_2+\underline{Z}_3}} \cdot \frac{\cancel{(\underline{Z}_2+\underline{Z}_3)}\,\underline{E}'}{\underline{Z}_1\underline{Z}_2+\underline{Z}_1\underline{Z}_3+\underline{Z}_2\underline{Z}_3}$$

$$\underline{I}_3' = \frac{\underline{Z}_2 \cdot \underline{E}'}{\underline{Z}_1\underline{Z}_2+\underline{Z}_1\underline{Z}_3+\underline{Z}_2\underline{Z}_3}$$

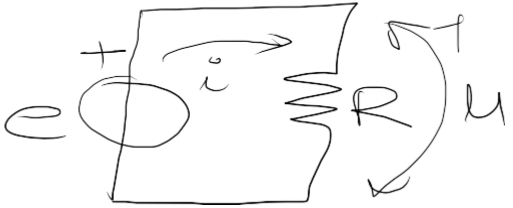
$$\underline{I}_3'' = \frac{\underline{Z}_2}{\underline{Z}_1+\underline{Z}_2} \cdot \frac{\underline{E}''}{\underline{Z}_3 + \frac{\underline{Z}_1 \cdot \underline{Z}_2}{\underline{Z}_1+\underline{Z}_2}}$$

$$= \frac{\underline{Z}_2}{\cancel{\underline{Z}_1+\underline{Z}_2}} \cdot \frac{\cancel{(\underline{Z}_1+\underline{Z}_2)}\,\underline{E}''}{\underline{Z}_1\underline{Z}_3+\underline{Z}_2\underline{Z}_3+\underline{Z}_1\underline{Z}_2}$$

$$\underline{I}_3'' = \frac{\underline{Z}_2 \cdot \underline{E}''}{\underline{Z}_1\underline{Z}_3+\underline{Z}_2\underline{Z}_3+\underline{Z}_1\underline{Z}_2}$$

ТЕОРЕМА ОДРЖАЊА СНАГЕ

$$\boxed{\sum p_g(t) = \sum p_p(t)}$$ при усаглашеним реф. смеровима



$$e(t) = \sqrt{2} E \cos \omega t$$

$$i(t) = \frac{e(t)}{R} = \sqrt{2}\frac{E}{R}\cos \omega t$$

$$p_g(t) = e(t) \cdot i(t) = \sqrt{2} E \cos \omega t \, \sqrt{2}\frac{E}{R} \cos \omega t$$

$$p_p(t) = u(t) \cdot i(t) = R\, i(t)\, i(t) = R\, i^2(t) = \cancel{R} \cdot 2\frac{E^2}{R^{\cancel{2}}} \cos^2 \omega t$$

$$p_g(t) = 2\frac{E^2}{R} \cos^2 \omega t = p_p(t) = 2\frac{E^2}{R} \cos^2 \omega t$$

Теорема одржања комплексне снаге

$$\boxed{\begin{aligned} \sum \underline{S}_g &= \sum \underline{S}_p \\ \sum P_g &= \sum P_p \\ \sum Q_g &= \sum Q_p \end{aligned}}$$

За уподобљене ред. одређене

$$\sum \underline{S}_g \neq \sum \underline{S}_p$$

$\underline{E}$ $\quad \underline{I}$ $\quad \underline{Z} = R + jX$ $\quad \underline{U}$

$$\underline{E} = E$$

$$\underline{I} = \frac{\underline{E}}{R+jX} = \frac{E(R-jX)}{R^2+X^2}$$

$$\underline{S}_g = \underline{E} \cdot \underline{I}^* = \frac{E \cdot E(R+jX)}{R^2+X^2} \Rightarrow P_g + jQ_g = \frac{RE^2}{R^2+X^2} + j\frac{XE^2}{R^2+X^2}$$

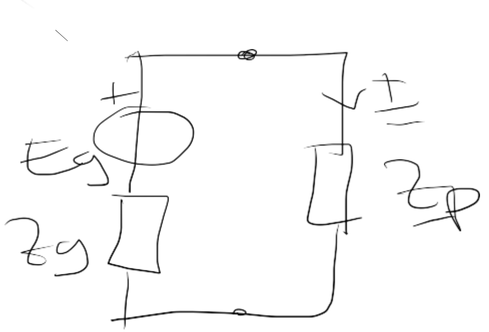
$$\underline{S}_p = \underline{U} \cdot \underline{I}^* = \underline{Z} \cdot \underline{I} \cdot \underline{I}^* = \underline{Z} \cdot I^2$$

$$\underline{I} = \frac{E(R-jX)}{R^2+X^2} \Rightarrow I = \frac{E\sqrt{R^2+X^2}}{R^2+X^2} = \frac{E}{\sqrt{R^2+X^2}}$$

$$\underline{S}_p = \underline{Z} \cdot I^2 = (R+jX)\frac{E^2}{R^2+X^2} = \frac{RE^2}{R^2+X^2} + j\frac{XE^2}{R^2+X^2}$$

$$\Rightarrow \boxed{\begin{aligned} \underline{S}_g &= \underline{S}_p \\ P_g &= P_p \\ Q_g &= Q_p \end{aligned}}$$

ПРИЛАГОЂЕЊЕ ПО СНАЗИ



$$\underline{Z}_p = R_p + jX_p$$
$$\underline{Z}_g = R_g + jX_g$$

$\underline{Z}_p = ?$

$P_p \to max$

$$\underline{S}_p = \underline{Z}_p \cdot I^2 = \overset{R_p + jX_p}{\underline{Z}_p} \frac{E_g^2}{(R_p+R_g)^2 + (X_p+X_g)^2} = \boxed{P_p} + jQ_p$$

$$P_p = \frac{R_p \cdot E_g^2}{(R_p+R_g)^2 + (X_p+X_g)^2} \to max \quad R_p = ? \quad X_p = ? \quad \Rightarrow \boxed{X_p = -X_g}$$

$$P_p = \frac{R_p E_g^2}{(R_p+R_g)^2} \Rightarrow \frac{dP_p}{dR_p} = \frac{1 \cdot E_g^2 (R_p+R_g)^2 - R_p E_g^2 \cdot (R_p^2 + 2R_pR_g + R_g^2)'}{(R_p+R_g)^4}$$

$$= \frac{E_g^2}{(R_p+R_g)^4}\left(R_p^2 + 2R_pR_g + R_g^2 - R_p(2R_p + 2R_g)\right) = 0$$

$$= \frac{E_g^2}{(R_p+R_g)^2}\left(R_p^2 + \cancel{2R_pR_g} + R_g^2 - 2R_p^2 - \cancel{2R_pR_g}\right) = 0$$

$$R_g^2 - R_p^2 = 0$$

$$R_p = \pm R_g$$

$$\boxed{R_p = R_g}$$

Према томе, услов преноса мах. снаге је

$$\boxed{\begin{aligned} R_p &= R_g \\ X_p &= -X_g \\ \text{тј. } \underline{Z}_p &= \underline{Z}_g^* \end{aligned}}$$

$$* \ \underline{Z}_p = R_p + j\cancel{X_p}^{0}$$

$$\underline{Z}_g = R_g + jX_g$$

$$R_p = |\underline{Z}_g|$$

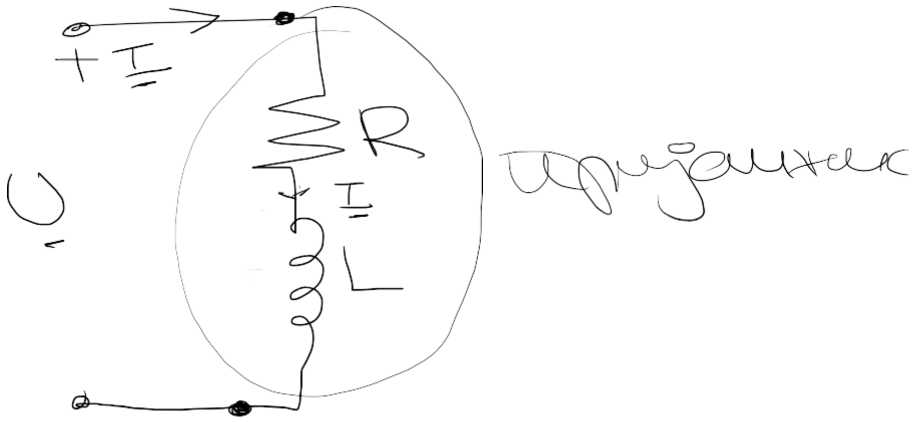
$$R_p = \sqrt{R_g^2 + X_g^2}$$

ПОПРАВКА ФАКТОРА СНАГЕ

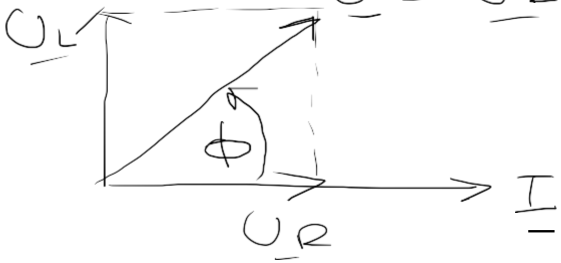
$\cos\phi$

$$\cos\phi = \frac{P}{S}$$

$$\underline{Z}_P = R + j\omega L$$



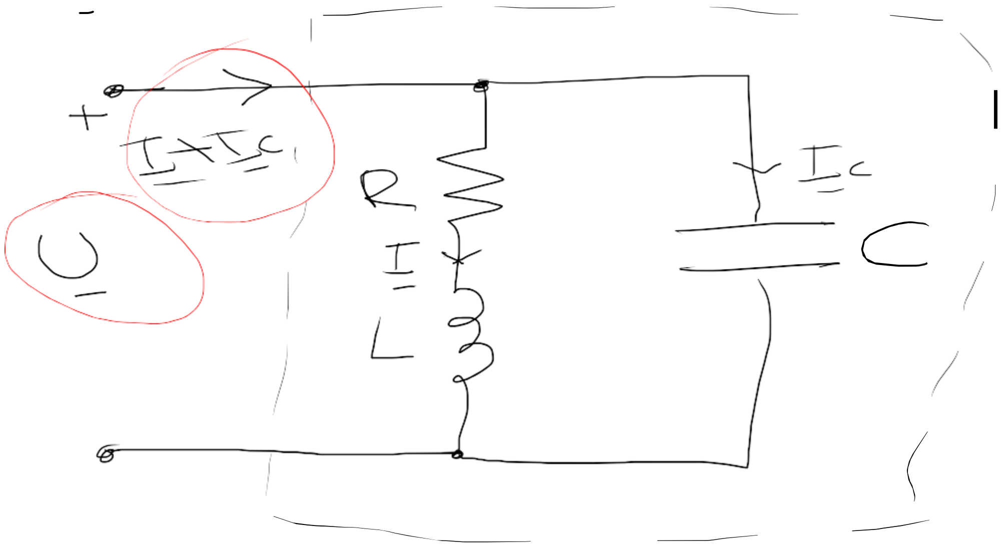
$$\underline{U} = \underline{U}_R + \underline{U}_L$$



$$\cos\phi = \frac{P}{S} \Rightarrow P = S\cos\phi = UI\cos\phi$$

$$I = \frac{P}{U \cdot \cos\phi}$$

$$\phi = \operatorname{arctg}\frac{\omega L}{R}$$



$$I_C = I\sin\phi$$

cos φ გაიზარდა

$$\omega C U = \frac{U}{\sqrt{R^2+(\omega L)^2}} \; \frac{\omega L}{\sqrt{R^2+(\omega L)^2}}$$

$$\underline{I} = \frac{\underline{U}}{R+j\omega L}$$

$$\omega C U = \frac{\omega L U}{R^2+(\omega L)^2}$$

$$\underline{I}_C = \frac{\underline{U}}{\frac{1}{j\omega C}} = j\omega C \underline{U}$$

$$\boxed{C = \frac{L}{R^2+(\omega L)^2}}$$

???