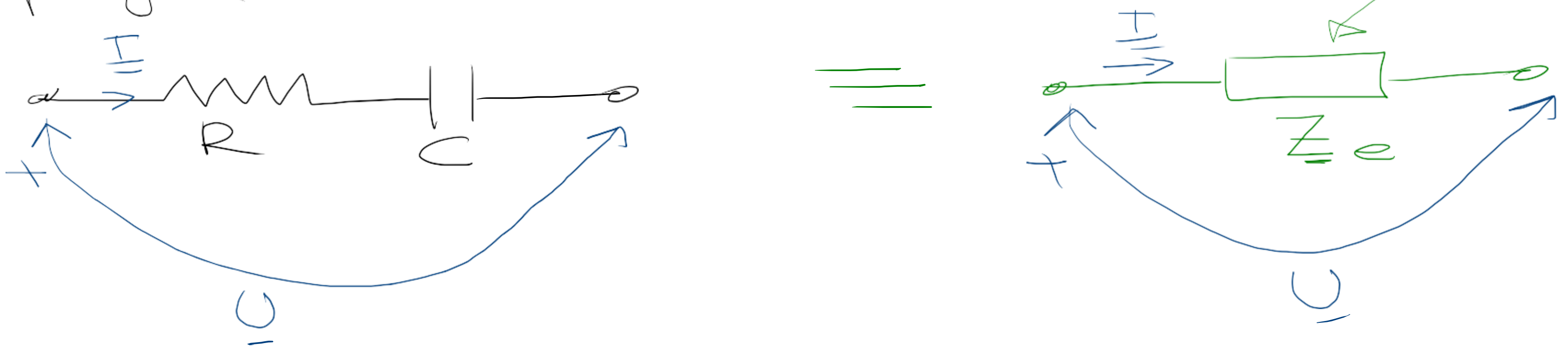
Рјешавање сложених кола у комплексном домену.

Основи електротехнике 2

Предавање: 9. блок

ТРАНСФИГУРАЦИЈЕ МРЕЖА ПАСИВНИХ ЕЛЕМЕНАТА

1. редна веза



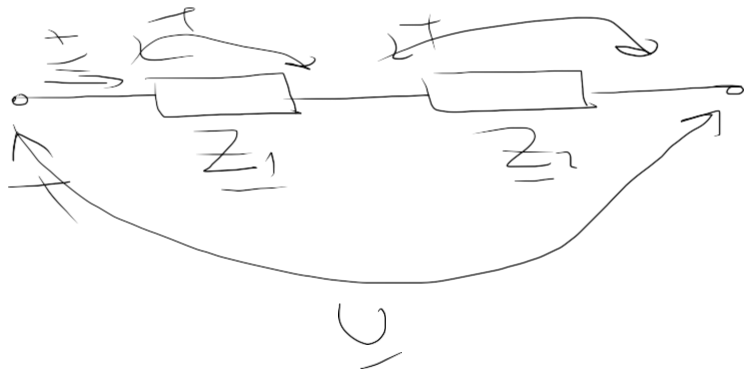
ШЕМАТСКИ СИМБОЛ ЗА ИМПЕДАНСУ

$$\underline{U} = R\underline{I} + \frac{1}{j\omega C}\underline{I} = \underline{Z}_e \cdot \underline{I}$$

$$\underline{Z}_e = R + \frac{1}{j\omega C} = R - j\frac{1}{\omega C} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}\, e^{j\,\mathrm{arctg}\frac{-\frac{1}{\omega C}}{R}}$$

$$= \underbrace{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}_{Z_e}\, e^{\underbrace{-j\,\mathrm{arctg}\frac{1}{\omega RC}}}$$

$$\phi_e = -\mathrm{arctg}\frac{1}{\omega RC}$$



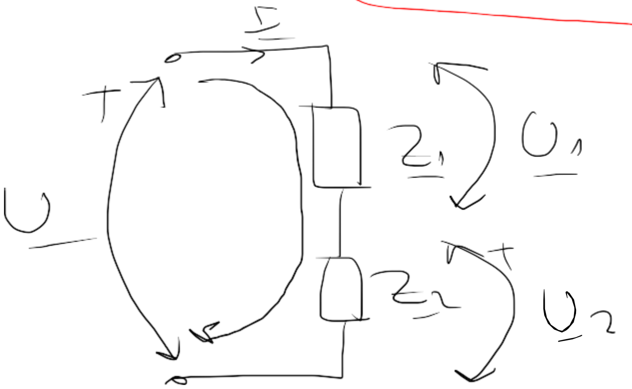
$$=$$

$$\underline{U} = \underline{Z}_1 \cdot \underline{I} + \underline{Z}_2 \cdot \underline{I} = \underline{Z}_e \cdot \underline{I} \Rightarrow \boxed{\underline{Z}_e = \underline{Z}_1 + \underline{Z}_2}$$

$$\boxed{\underline{Z}_1 \oplus \underline{Z}_2 = \underline{Z}_1 + \underline{Z}_2}$$

$$\underline{Z}_e = \sum_{k=1}^{n} \underline{Z}_k$$

за n импеданси



напонски разделник

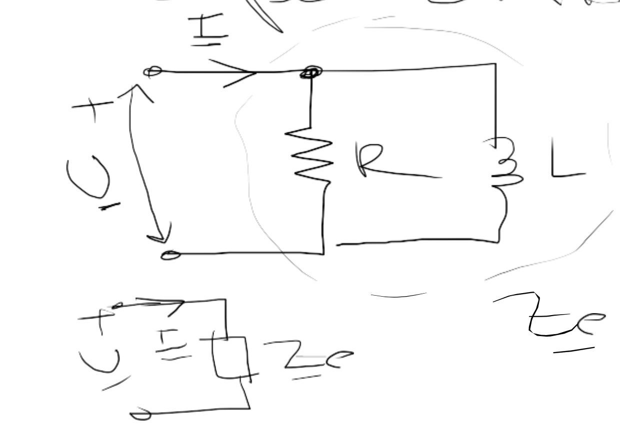
$$\underline{U} = \underline{U}_1 + \underline{U}_2$$

$$\underline{U}_1 = \underline{Z}_1 \cdot \underline{I} \qquad \underline{U}_2 = \underline{Z}_2 \cdot \underline{I}$$

$$\underline{U} = (\underline{Z}_1 + \underline{Z}_2) \cdot \underline{I} \Rightarrow \underline{I} = \frac{\underline{U}}{\underline{Z}_1 + \underline{Z}_2}$$

$$\underline{U}_1 = \frac{\underline{Z}_1}{\underline{Z}_1 + \underline{Z}_2} \underline{U} \qquad \underline{U}_2 = \frac{\underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2} \underline{U}$$

– паралелна веза



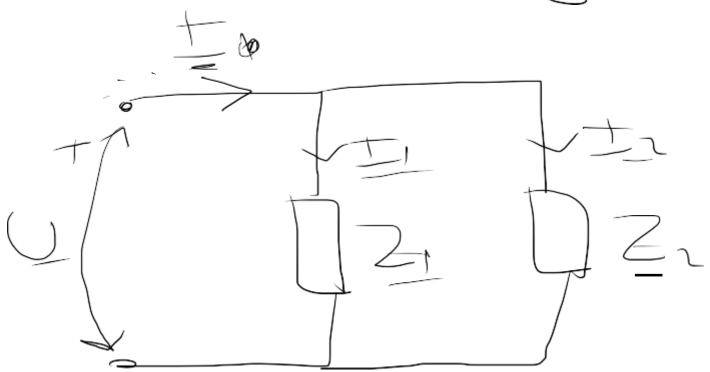
$$\underline{I}_R = \frac{\underline{U}}{R} \qquad \underline{I}_L = \frac{\underline{U}}{j\omega L}$$

$$\underline{I} = \underline{I}_R + \underline{I}_L$$

$$\underline{I} = \frac{\underline{U}}{R} + \frac{\underline{U}}{j\omega L} = \underline{U}\,\frac{R + j\omega L}{j\omega L \cdot R}$$

$$\underline{U} = \frac{R \cdot j\omega L}{R + j\omega L}\,\underline{I} = \underline{Z}_e \cdot \underline{I}$$

$$\underline{Z}_e = \frac{R \cdot j\omega L}{R + j\omega L}$$



$$\underline{I} = \underline{I}_1 + \underline{I}_2 = \frac{\underline{U}}{\underline{Z}_1} + \frac{\underline{U}}{\underline{Z}_2} = \underline{U}\,\frac{\underline{Z}_1 + \underline{Z}_2}{\underline{Z}_1 \cdot \underline{Z}_2}$$

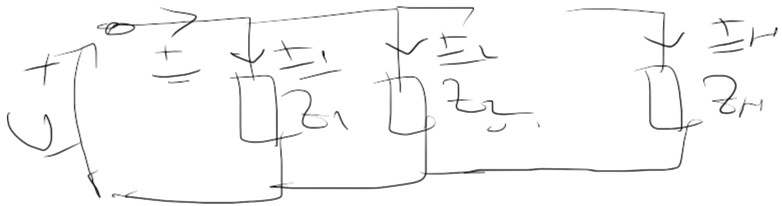
$$\underline{I} = \frac{\underline{U}}{\underline{Z}_e} \Rightarrow \boxed{\underline{Z}_e = \frac{\underline{Z}_1 \cdot \underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2}}$$

$\underline{Z}_1 \| \underline{Z}_2$

Струјни раздјелник

$$\underline{I}_1 = \frac{\underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2}\,\underline{I} \qquad \underline{I}_2 = \frac{\underline{Z}_1}{\underline{Z}_1 + \underline{Z}_2} \cdot \underline{I}$$

За n паралелно врзани:

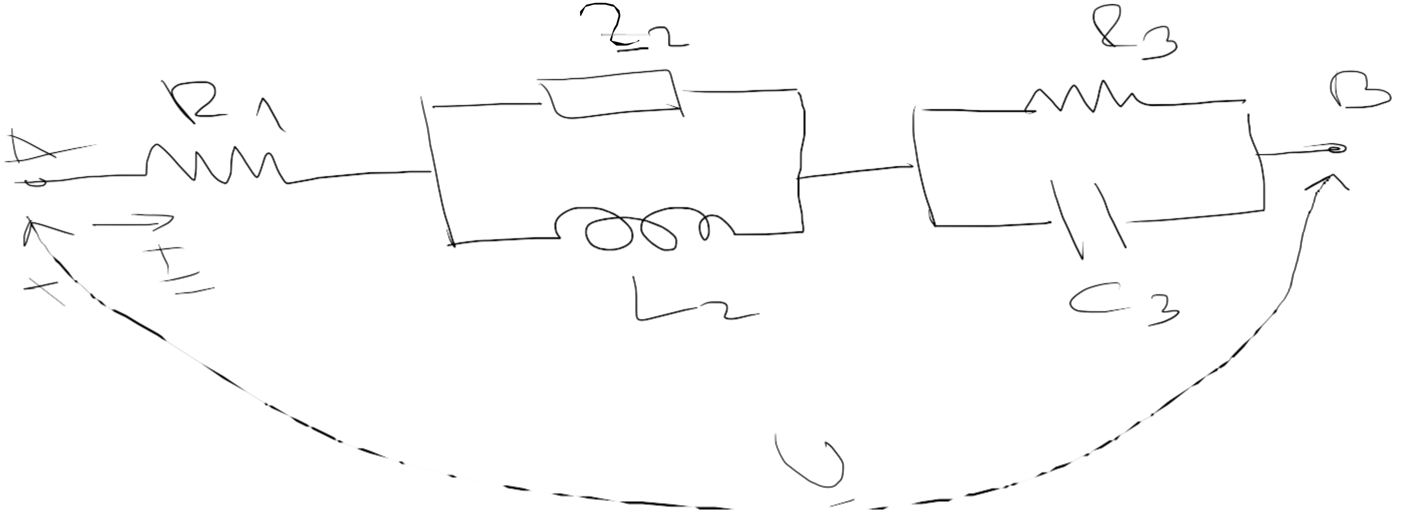


$$\underline{I} = \underline{I}_1 + \underline{I}_2 + \dots + \underline{I}_n = \frac{\underline{U}}{\underline{Z}_1} + \frac{\underline{U}}{\underline{Z}_2} + \dots + \frac{\underline{U}}{\underline{Z}_n}$$

$$\boxed{\frac{1}{\underline{Z}_e} = \frac{1}{\underline{Z}_1} + \frac{1}{\underline{Z}_2} + \frac{1}{\underline{Z}_3} + \dots + \frac{1}{\underline{Z}_n}}$$

$$\boxed{\underline{Y}_e = \underline{Y}_1 + \underline{Y}_2 + \dots + \underline{Y}_n}$$

impedanța loca



$$\underline{Z}_{AB} = \frac{\underline{U}}{\underline{I}}$$

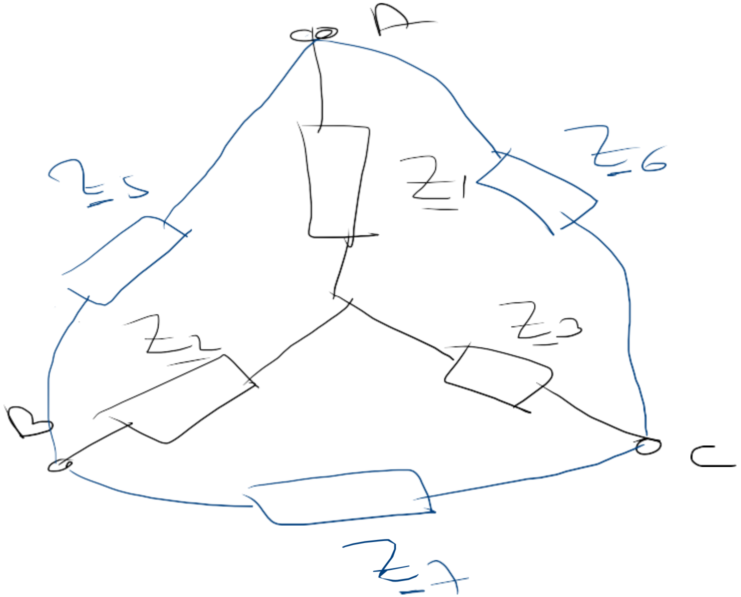
$$\underline{Z}_e = R_1 + \frac{j\omega L_2 \cdot \underline{Z}_2}{j\omega L_2 + \underline{Z}_2} + \frac{R_3 \cdot \frac{1}{j\omega C_3}}{R_3 + \frac{1}{j\omega C_3}}$$

$$\underline{Z}_e = R_1 \oplus (L_2 \parallel \underline{Z}_2) \oplus (R_3 \parallel C_3)$$

$$R_1 \oplus (j\omega L_2 \parallel \underline{Z}_2) \oplus \left(R_3 \parallel \frac{1}{j\omega C_3}\right) \checkmark$$

ТРАНСФИГУРАЦИЈЕ ЗВИЈЕЗДА-ТРОУГАО ПАСИВНИХ ЕЛЕМЕНАТА

1. Познате импедансе звијезде



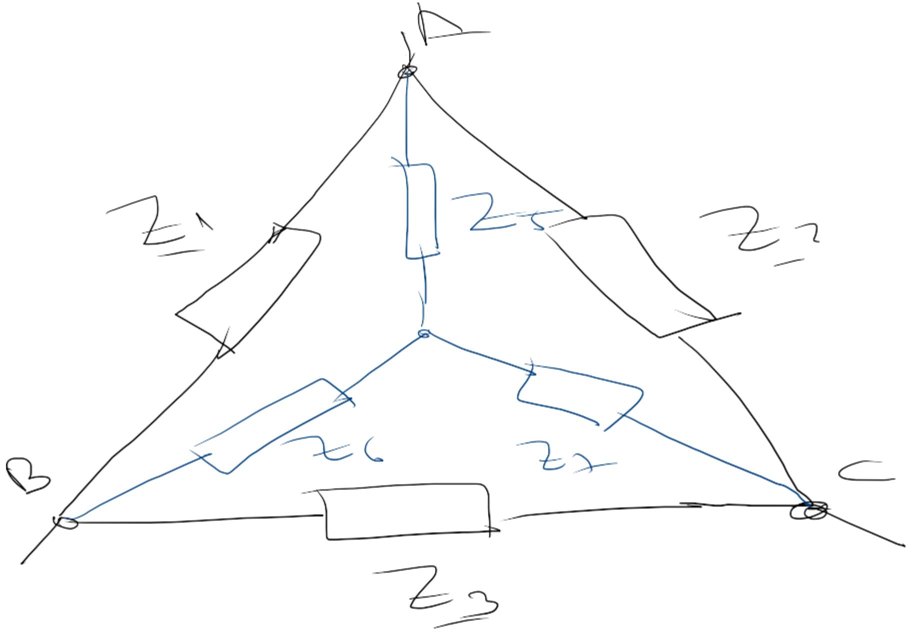
$\underline{Z}_1, \underline{Z}_2, \underline{Z}_3 \rightarrow$ импедансе св. грана

$$\underline{Z}_5 = \underline{Z}_1 + \underline{Z}_2 + \frac{\underline{Z}_1 \cdot \underline{Z}_2}{\underline{Z}_3}$$

$$\underline{Z}_6 = \underline{Z}_1 + \underline{Z}_3 + \frac{\underline{Z}_1 \cdot \underline{Z}_3}{\underline{Z}_2}$$

$$\underline{Z}_7 = \underline{Z}_2 + \underline{Z}_3 + \frac{\underline{Z}_2 \cdot \underline{Z}_3}{\underline{Z}_1}$$

2. Познате импедансе троугла $\underline{Z}_1, \underline{Z}_2, \underline{Z}_3 \to$ Y



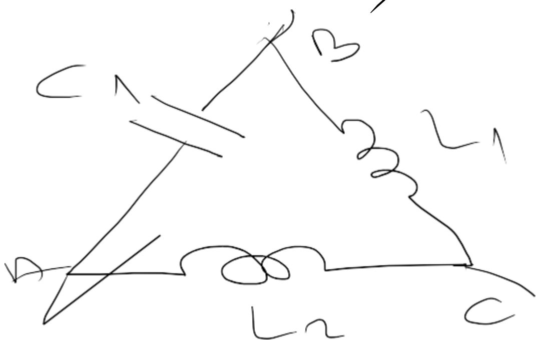
$$\underline{Z}_5 = \frac{\underline{Z}_1 \cdot \underline{Z}_2}{\underbrace{\underline{Z}_1 + \underline{Z}_2 + \underline{Z}_3}_{\underline{Z}_\Delta}}$$

$$\underline{Z}_6 = \frac{\underline{Z}_1 \cdot \underline{Z}_3}{\underline{Z}_\Delta}$$

$$\underline{Z}_7 = \frac{\underline{Z}_2 \cdot \underline{Z}_3}{\underline{Z}_\Delta}$$

– Екв. импеданса има нег. реални део
– не смета за прорачун и можемо
даље функционисати али ипак ово
значи да се тражена импеданса не
може реализовати помоћу R, L и C
елемената

– $\underline{Z}_\Delta = 0$



$$\underline{Z}_\Delta = j\omega L_1 + j\omega L_2 + \frac{1}{j\omega C_2}$$

$$= j\left(\omega(L_1 + L_2) - \frac{1}{\omega C_2}\right)$$

$$\omega(L_1 + L_2) = \frac{1}{\omega C_2}$$

КОМПЛЕКСНА ИМПЕДАНСА (РЕЗИСТАНСА, РЕАКТАНСА) И КОМП. АДМИТАНСА (КОНДУКТАНСА, СУСЦЕПТАНСА)

За сериско RL коло:

$$\underline{Z}_e = R + j\omega L$$

За сериско RC коло

$$\underline{Z}_e = R + \frac{1}{j\omega C} = R - j\frac{1}{\omega C} = \underbrace{R} + j\underbrace{\left(-\frac{1}{\omega C}\right)}$$

За паралелно RC коло

$$\underline{Z}_e = \frac{R\cdot\frac{1}{j\omega C}}{R+\frac{1}{j\omega C}} = \frac{R}{1+j\omega RC}\cdot\frac{1-j\omega RC}{1-j\omega RC} = \underbrace{\frac{R}{1+(\omega RC)^2}} + j\,\underbrace{\frac{-\omega R^2 C}{1+(\omega RC)^2}}$$

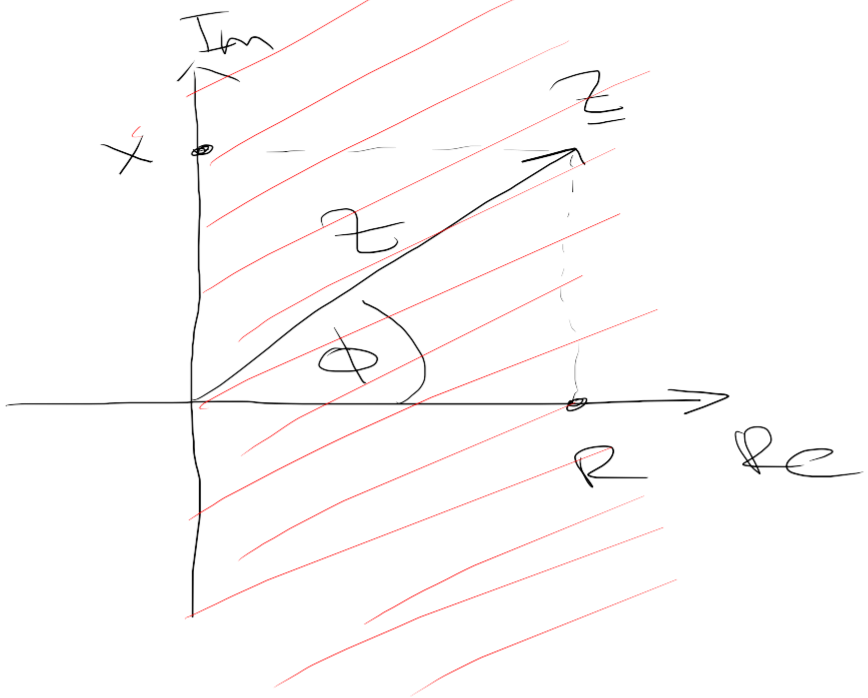
$$Z = R + jX$$

РЕЗИСТАНЦА

РЕАКТАНЦА

$$R = \mathrm{Re}\{Z\} \quad [\Omega]$$

$$X = \mathrm{Im}\{Z\} \quad [\Omega]$$



$$R > 0$$

$$X < 0, \; X > 0, \; X = 0$$

ВАЖНО!

$$\underline{Z} = R + jX = \sqrt{R^2 + X^2}\, e^{j\,\mathrm{arctg}\frac{X}{R}} = Z e^{j\phi} = Z\cos\phi + jZ\sin\phi$$

$$R = Z \cdot \cos\phi$$

$$X = Z \cdot \sin\phi$$

$$-\frac{\pi}{2} \le \phi \le \frac{\pi}{2} \quad \Rightarrow \quad \phi = \begin{cases} \mathrm{arctg}\frac{X}{R}, & R > 0 \\ \frac{\pi}{2}, & R = 0 \text{ и } X > 0 \\ -\frac{\pi}{2}, & R = 0 \text{ и } X < 0 \\ 0, & R = 0 \text{ и } X = 0 \end{cases}$$

$$\cos\phi \ge 0$$

$$\underline{Y} = \mathrm{Re}\{\underline{Y}\} + j\,\mathrm{Im}\{\underline{Y}\} = \underbrace{G}_{[S]} + j\underbrace{B}_{[S]}$$

Кондуктанса (G) Сусцептанса (B)

$$\underline{Y} = Y e^{j\theta} = \frac{1}{Z} e^{-j\phi} = Y\cos\phi - jY\sin\phi$$

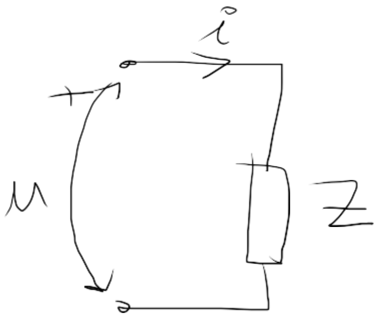
$$G = Y \cdot \cos\phi \qquad Y = \sqrt{G^2 + B^2}$$

$$B = -Y \cdot \sin\phi$$

$$\phi = \begin{cases} -\mathrm{arctg}\dfrac{B}{G}, & G > 0 \\ -\dfrac{\pi}{2}, & G = 0 \text{ и } B > 0 \\ \dfrac{\pi}{2}, & G = 0 \text{ и } B < 0 \\ 0, & G = 0 \text{ и } B = 0 \end{cases}$$

$$\boxed{\begin{array}{c} G \geq 0 \\ B > 0 \\ B < 0 \\ B = 0 \end{array}}$$

СНАГЕ У ПРОСТОПЕРИОДИЧНОМ РЕЖИМУ



$$p(t) = u(t)\cdot i(t)$$

$$u(t) = \sqrt{2}\,U\cos(\omega t + \theta)$$

$$i(t) = \sqrt{2}\,I\cos(\omega t + \psi)$$

$$\Rightarrow p(t) = 2UI\cos(\omega t + \theta)\cdot\cos(\omega t + \psi)$$

~~уводимо~~ де $\phi = \theta - \psi \Rightarrow \psi = \theta - \phi \quad \cos\phi \geq 0$

$$p(t) = UI\cos\phi + UI\cos\phi\cos(2\omega t + 2\theta) + UI\sin\phi\cdot\sin(2\omega t + 2\theta)$$

$P = UI\cos\phi$ [W] ≥ 0 активна (средња) снага

$Q = UI\sin\phi$ [var] РЕАКТИВНА СНАГА

волтамперреактивни

$S = UI$ [VA]

Фактор снаге пријемника

$$K = \cos\phi = \frac{P}{S}, \quad 0 \le K \le 1$$

Фактор реактивности

$$K_r = \sin\phi = \frac{Q}{S}, \quad -1 \le K_r \le 1$$

$K_r < 0$ претежно капацитивни пријемник

$K_r > 0$ претежно индуктивни пријемник

$$K^2 + K_r^2 = 1 = \cos^2\phi + \sin^2\phi \Rightarrow K^2 = 1 - K_r^2$$

$$K = \pm\sqrt{1-K_r^2}$$

увијек

$$K_r = \pm\sqrt{1-K^2}$$

Комплексна снага пријемника

$$\underline{S} = P + jQ = UI\cos\phi + jUI\sin\phi \quad [VA]$$

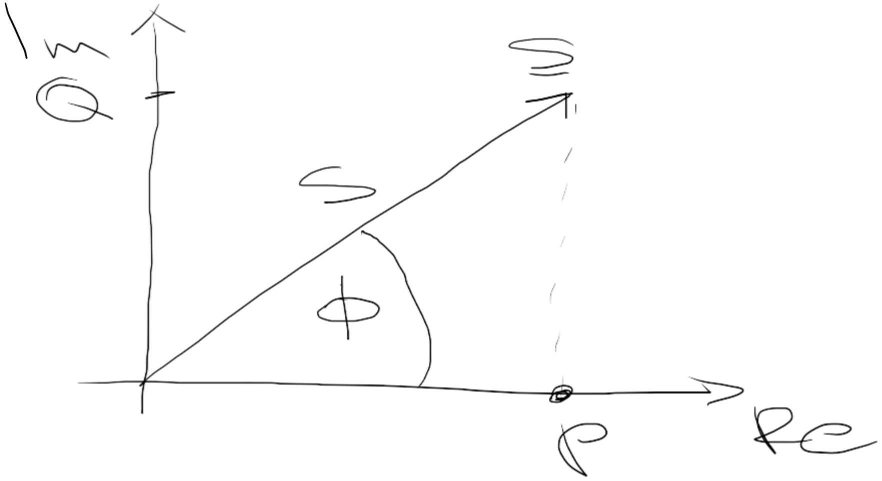
$$\underline{S} = UI(\cos\phi + j\sin\phi) = UIe^{j\phi} = Se^{j\phi}$$

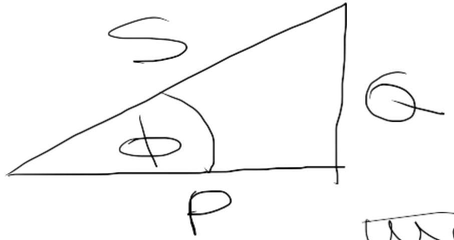
$$\underline{U} \cdot \underline{I} = Ue^{j\theta} Ie^{j\psi} = UIe^{j(\theta+\psi)} \neq UIe^{j\phi}$$

$$\phi = \theta - \psi$$

$$\underline{U} \cdot \underline{I}^{*} = Ue^{j\theta} Ie^{-j\psi} = UIe^{j(\theta-\psi)}$$

$$\boxed{\underline{S} = \underline{U} \cdot \underline{I}^{*}}$$





треуголь силе

$$S = \sqrt{P^2 + Q^2}$$

– Сила генератора

Снага генератора

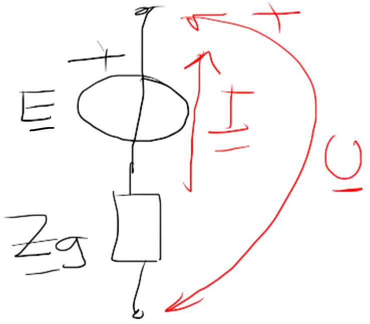
$$-\pi < \phi < \pi$$

$$P > 0$$

$$P < 0$$

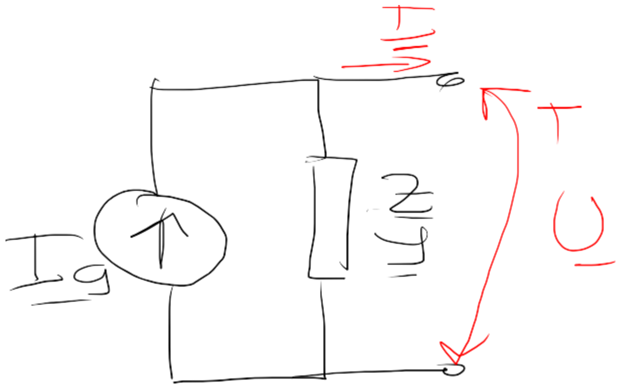
$$\underline{S} = \underline{E} \cdot \underline{I}^* = \underline{U} \cdot \underline{I}^*$$

$$\underline{S} = \underline{U} \cdot \underline{I}_g^*$$



$$\underline{S} = \underline{U} \cdot \underline{I}^*$$

$$\underline{U} = \underline{E} - \underline{Z}_g \cdot \underline{I}$$



$$\underline{S} = \underline{U} \cdot \underline{I}^*$$

$$\underline{I} = \underline{I}_g - \frac{\underline{U}}{\underline{Z}}$$

РЈЕШАВАЊЕ СЛОЖЕНИХ КОЛА У КОМПЛЕКСНОМ ДОМЕНУ

$$\boxed{\sum \underline{I} = 0 \quad \text{за} \quad n_č - 1}$$

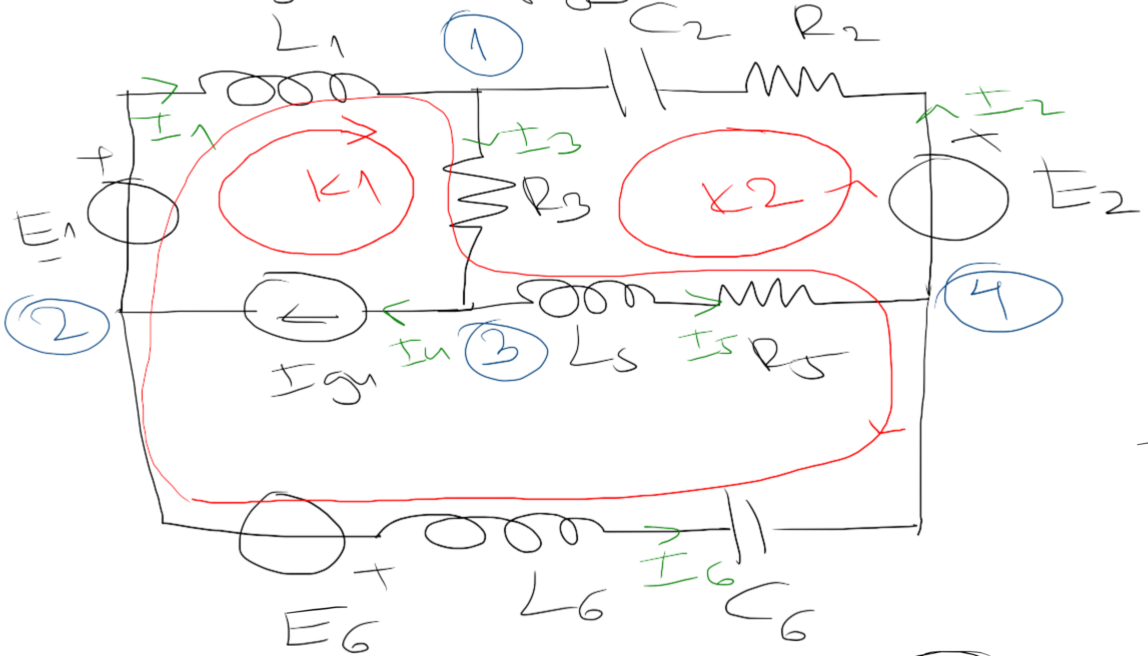
$n_č$ – број чворова

$$\sum \underline{U} = 0 \quad \text{за} \quad n_k = n_g - (n_č - 1)$$

број грана

број независних контура

Написати систем једначина: непосредно је на Кирхофа и на контура



$n_č - 1 = 3$

$$\text{(1)}\quad -\underline{I}_1 - \underline{I}_2 + \underline{I}_3 = 0$$

$$\text{(2)}\quad \underline{I}_1 - \underline{I}_4 + \underline{I}_6 = 0$$

$$\text{(3)}\quad -\underline{I}_3 + \underline{I}_4 + \underline{I}_5 = 0$$

$n_g - (n_č - 1) = 6 - 3 = 3$ независне контуре

$$K1:\ \underline{U}_{21} + \underline{U}_{13} + \underline{U}_{32} = 0$$

$$K2:\ \underline{U}_{41} + \underline{U}_{13} + \underline{U}_{34} = 0$$

$$K3:\ \underline{U}_{13} + \underline{U}_{34} + \underline{U}_{42} + \underline{U}_{21} = 0$$

$$\underline{U}_{21} = -\underline{U}_{12} = -\underline{E}_1 + j\omega L_1 \underline{I}_1$$

$$\underline{U}_{13} = R_3 \underline{I}_3 = -\underline{U}_{31}$$

$$\underline{U}_{14} = -\underline{U}_{41} = \underline{E}_2 - \underline{I}_2\left(R_2 + \frac{1}{j\omega C_2}\right)$$

$$\underline{U}_{34} = -\underline{U}_{43} = (j\omega L_5 + R_5)\,\underline{I}_5$$

$\underline{U}_{23} \leftarrow$ ист.! $\Rightarrow \underline{I}_4 = \underline{I}_{g4}$

$$\underline{U}_{42} = \underline{E}_6 - j\omega L_6 \underline{I}_6 - \frac{1}{j\omega C_6}\underline{I}_6$$

Редуковани систем

→ n_d једначина у којима од непознатих струје грана

$n_{\check{c}} - 1 = 3$

1. $\underline{I}_1 + \underline{I}_2 = \underline{I}_3$
2. $\underline{I}_1 + \underline{I}_6 = \underline{I}_4$
3. $\underline{I}_4 + \underline{I}_5 = \underline{I}_3$

$n_g - (n_{\check{c}} - 1) = 3$

ИСТ

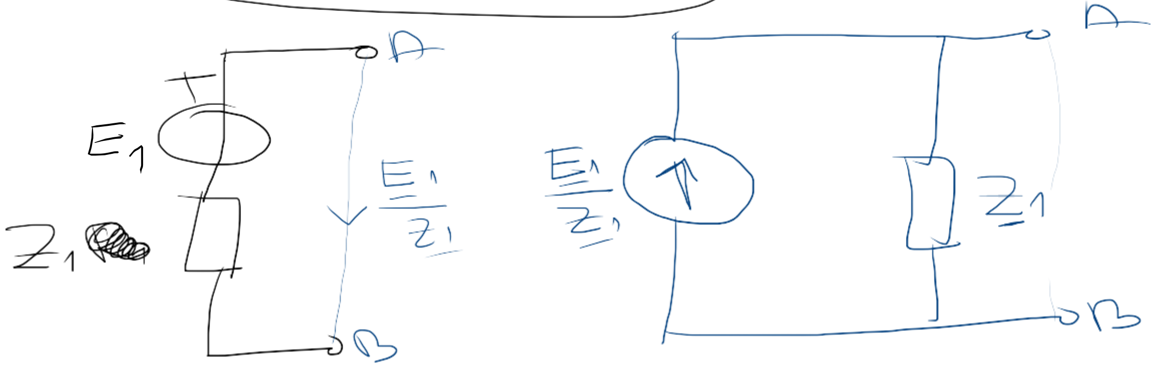
K1 $\quad -\underline{E}_1 + j\omega L_1 \underline{I}_1 + R_3 \underline{I}_3 + \boxed{\overset{?}{\underline{U}_{32}}} = 0$ ✕ $\quad \underline{I}_4 = \underline{I}_{g4}$

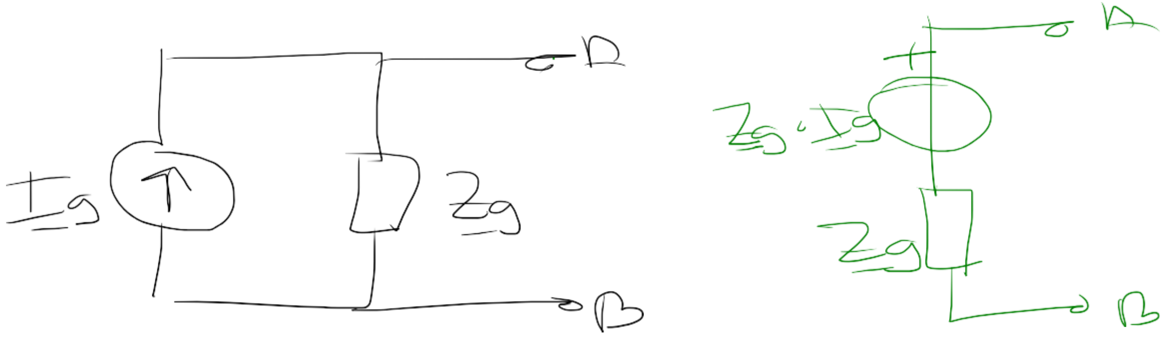
K2 $\quad -\underline{E}_2 + R_2 \underline{I}_2 + \frac{1}{j\omega C_2}\underline{I}_2 + R_3 \underline{I}_3 + j\omega L_5 \underline{I}_5 + R_5 \underline{I}_5 = 0$

K3 $\quad -\underline{E}_1 + j\omega L_1 \underline{I}_1 + R_3 \underline{I}_3 + j\omega L_5 \underline{I}_5 + R_5 \underline{I}_5 - \frac{1}{j\omega C_6}\underline{I}_6 - j\omega L_6 \underline{I}_6 - R_6 \underline{I}_6 = 0$

ЕКВИВАЛЕНЦИЈА НАПОНСКОГ И СТРУЈНОГ ГЕНЕРАТОРА

РНГ и РСГ





МЕТОДА КОНТУРНИХ СТРУЈА У КОМПЛЕКСНОМ ОБЛИКУ

$$n_k = n_g - (n_č - 1)$$

$$\underline{\underline{Z}}_{11} \cdot \underline{I}_{k1} + \underline{\underline{Z}}_{12} \underline{I}_{k2} + \dots + \underline{\underline{Z}}_{1n_k} \cdot \underline{I}_{kn_k} = \underline{E}_{k1}$$

$$\underline{\underline{Z}}_{21} \underline{I}_{k1} + \underline{\underline{Z}}_{22} \underline{I}_{k2} + \dots + \underline{\underline{Z}}_{2n_k} \underline{I}_{kn_k} = \underline{E}_{k2}$$

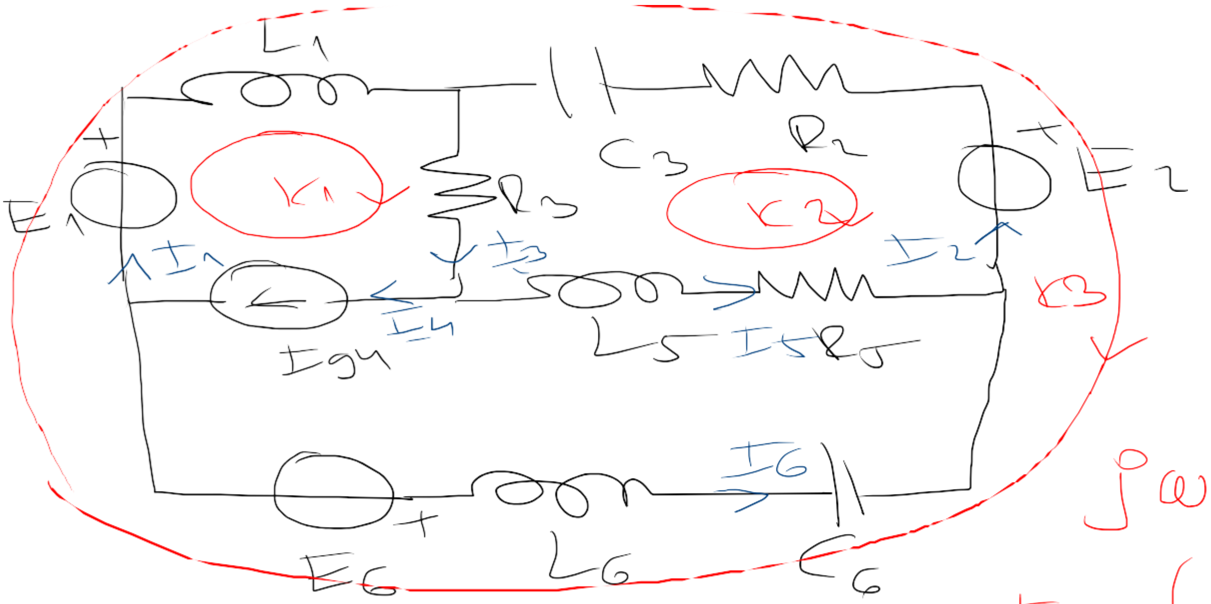
$$\vdots$$

$$\underline{\underline{Z}}_{n_k 1} \underline{I}_{k1} + \underline{\underline{Z}}_{n_k 2} \underline{I}_{k2} + \dots + \underline{\underline{Z}}_{n_k n_k} \underline{I}_{k n_k} = \underline{E}_{k n_k}$$

$\underline{\underline{Z}}_{ii}, \; i = 1, \dots, n_k$

$\underline{\underline{Z}}_{ij}, \; i, j = 1, \dots, n_k, \; i \neq j$

$\underline{E}_{ki}, \; i = 1, \dots, n_k$



$$\underline{I}_{k1} = \underline{I}_{g4}$$

$$-R_3 \underline{I}_{k1} + \left(R_2 + R_3 + R_5 + j\omega L_5 + \frac{1}{j\omega C_3}\right)\underline{I}_{k2} + \left(R_2 + \frac{1}{j\omega C_2}\right)\underline{I}_{k3} = -\underline{E}_2$$

$$j\omega L_1 \underline{I}_{k1} + \left(R_2 + \frac{1}{j\omega C_2}\right)\underline{I}_{k2} + \underline{I}_{k3}\left(R_2 + j\omega L_1 + j\omega L_6 + \frac{1}{j\omega C_6}\right) = \underline{E}_1 - \underline{E}_2 - \underline{E}_6$$

$$\Rightarrow \underline{I}_1 = \underline{I}_{k1} + \underline{I}_{k3}$$

$$\underline{I}_2 = -\underline{I}_{k2} - \underline{I}_{k3}$$

$$\underline{I}_3 = \underline{I}_{k1} - \underline{I}_{k2}$$

$$\underline{I}_4 = \underline{I}_{k1} - \underline{I}_{g4}$$

$$\underline{I}_5 = -\underline{I}_{k2}$$

$$\underline{I}_6 = -\underline{I}_{k3}$$

Метода потенцијала чворова у комплексном облику

$n_č - 1$

$$\underline{Y}_{11}\underline{V}_1 + \underline{Y}_{12}\underline{V}_2 + \dots + \underline{Y}_{1(n_č-1)} \cdot \underline{V}_{(n_č-1)} = \underline{I}_{č1}$$

$$\underline{Y}_{21}\underline{V}_1 + \underline{Y}_{22}\underline{V}_2 + \dots + \underline{Y}_{2(n_č-1)} \cdot \underline{V}_{(n_č-1)} = \underline{I}_{č2}$$

$$\vdots$$

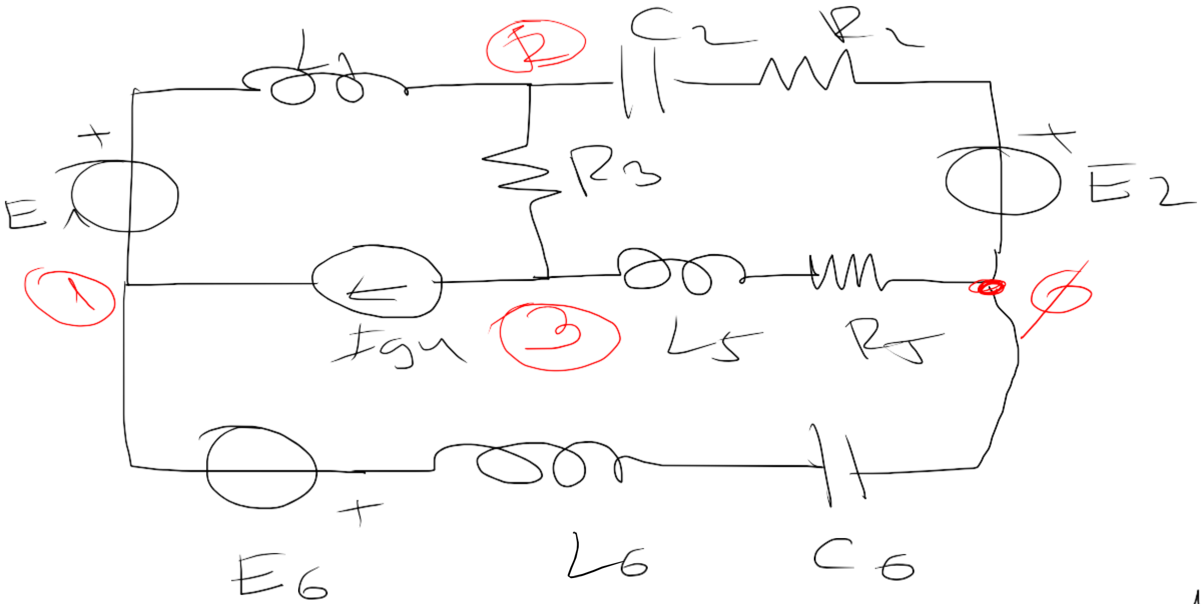
$$\underline{Y}_{(n_č-1)1} \cdot \underline{V}_1 + \underline{Y}_{(n_č-1)2}\underline{V}_2 + \dots + \underline{Y}_{(n_č-1)(n_č-1)}\underline{V}_{(n_č-1)} = \underline{I}_{č(n_č-1)}$$

$\underline{Y}_{ii}, \quad i = 1, \dots, n_č - 1$

$\underline{Y}_{ij}, \quad i, j = 1, \dots, n_č - 1, \quad i \neq j$

$\underline{I}_{či}, \quad i = 1, \dots, n_č - 1$

$\underline{E}|\underline{Z}, \quad \underline{Z}$



$$\underline{V}_1\left(\frac{1}{j\omega L_1}+\frac{1}{j\omega L_6+\frac{1}{j\omega C_6}}\right)-\frac{1}{j\omega L_1}\underline{V}_2=-\frac{E_1}{j\omega L_1}+\underline{I}_{g4}-\frac{E_6}{j\omega L_6+\frac{1}{j\omega C_6}}$$

$$\underline{V}_2\left(\frac{1}{j\omega L_1}+\frac{1}{R_3}+\frac{1}{R_2+\frac{1}{j\omega C_2}}\right)-\frac{1}{j\omega L_1}\underline{V}_1-\frac{1}{R_3}\underline{V}_3=\frac{E_1}{j\omega L_1}+\frac{E_2}{R_2+\frac{1}{j\omega C_2}}$$

$$-\frac{1}{R_3}\underline{V}_2+\underline{V}_3\left(\frac{1}{R_3}+\frac{1}{R_5+j\omega L_5}\right)=\cancel{E_6}-\underline{I}_{g4}$$