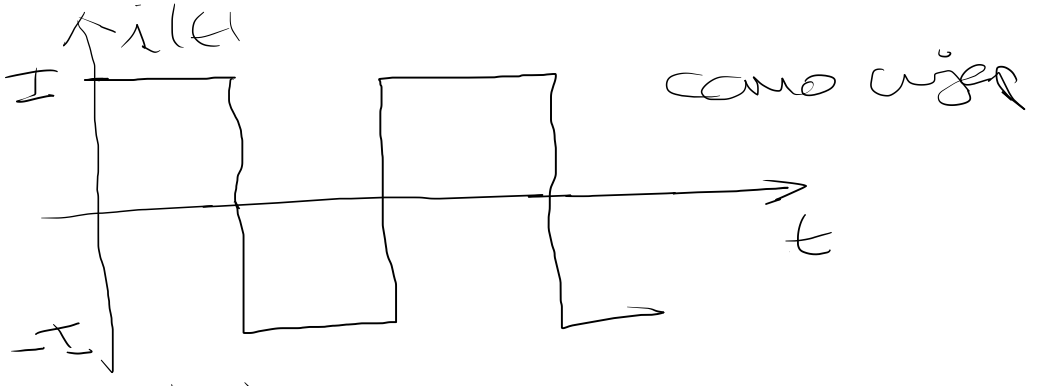
ЕЛЕКТРИЧНЕ МРЕЖЕ СА ВРЕМЕ-ПРОМЕНЉИВИМ СТРУЈАМА

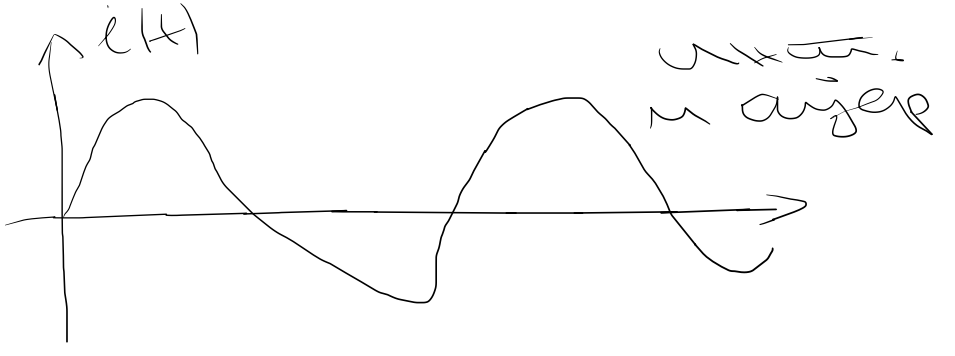
- Опште карактеристике и дефиниције

Струје и напони се мењају у функцији времена

јачина струје се мења

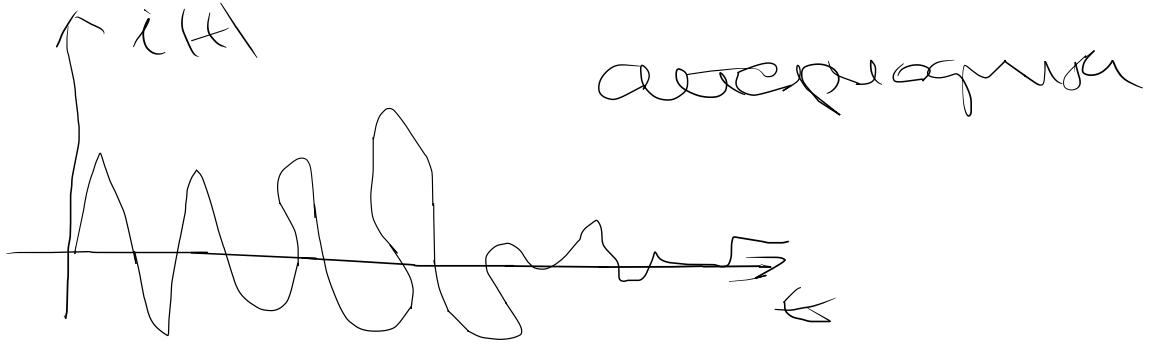


само иде



мења и иде

Променливите големини се делат на
- апериодични
- периодични

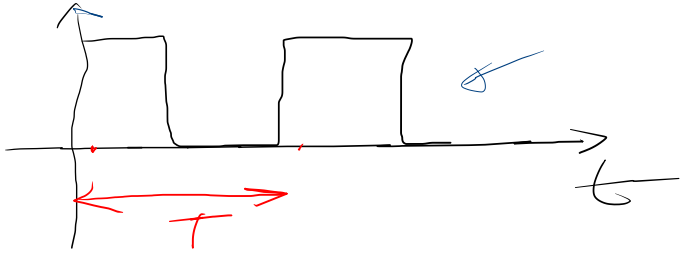


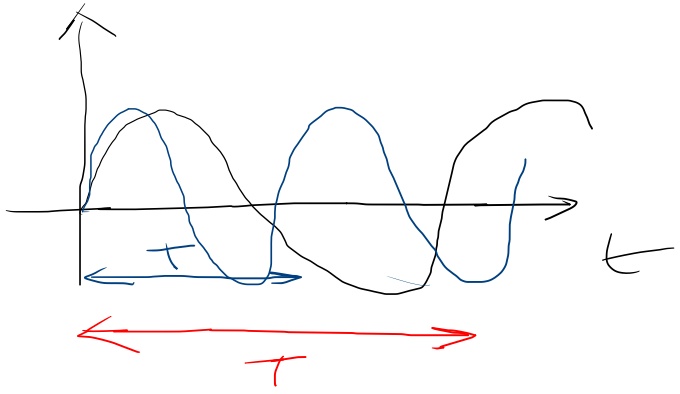
апериодична

$f(t)$

T – период на периодична фја

$f(t+T) = f(t)$





симетрични периодични функции

$$f\left(t+\frac{T}{2}\right)=-f(t)$$

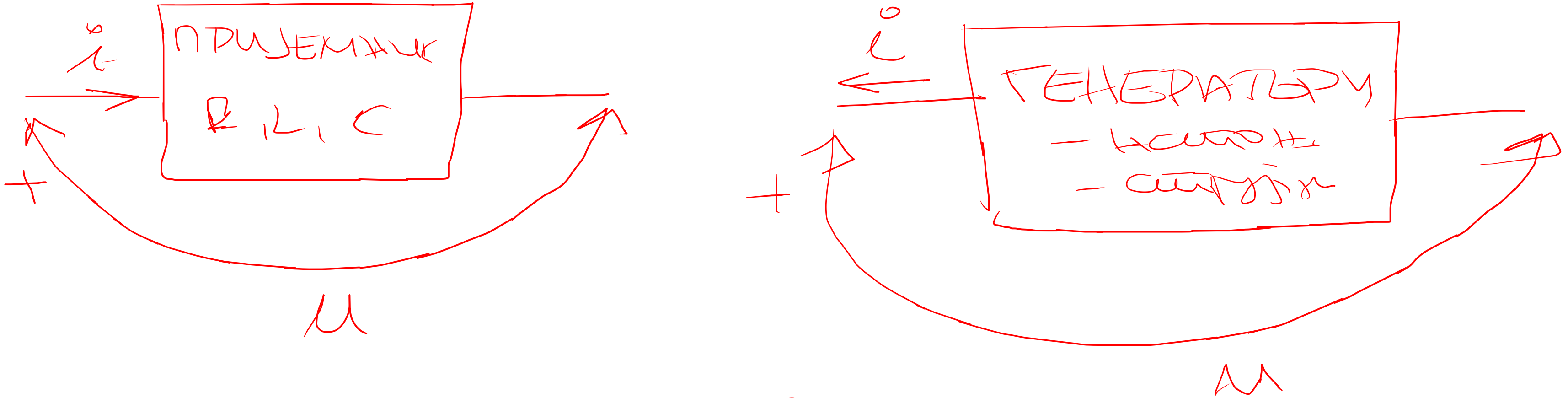
T – период

– полупериодични функции → имат само
синуси или косинуси

Основни елементи у шемама са временски промењљивим струјама

- Генератори (напонски, струјни)
- Пријемници: отпорник (R)
- кондензатори са лин. диел. (C)
- калемови без феромагн. језгра (односно оне са феромагн. језгром у лин. режиму) (L)

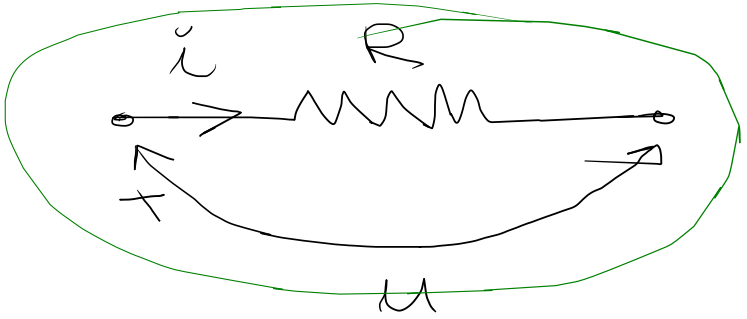
ПРИЈЕМНИЦИ

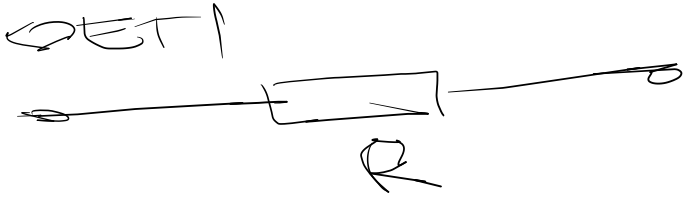


Говоримо о тренутној вредности у посматраном тренутку времена =>

- Омметрия

$$u(t) = R\,i(t) \quad \Rightarrow \quad i(t) = \frac{u(t)}{R}$$



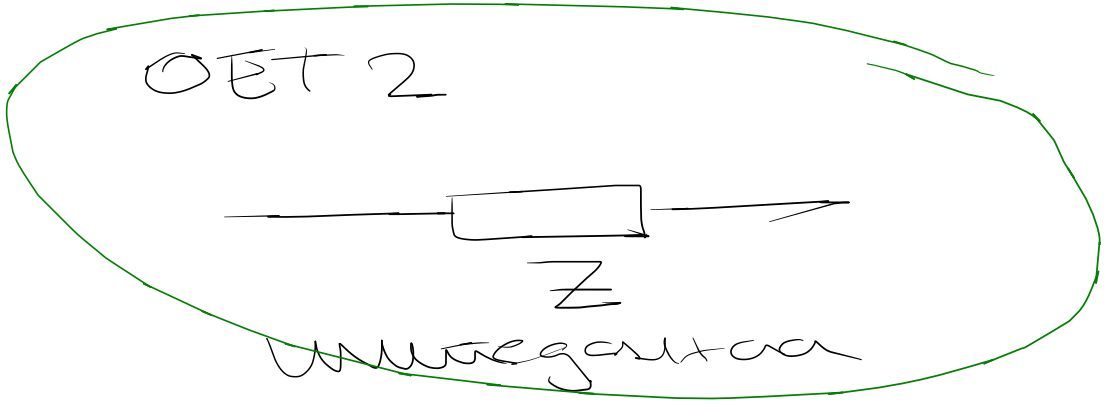


$$i(t) = G\,u(t)$$

$$RG = 1$$

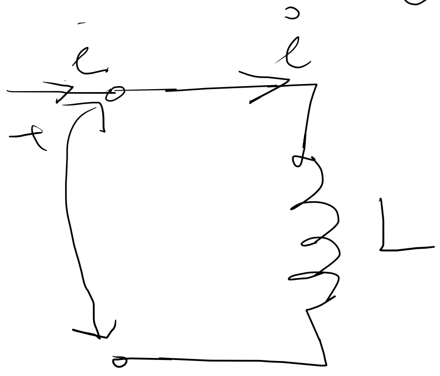
$$R \neq f(t)$$

$$R\,[\Omega]$$



– індуктивні кола (індуктивностей)
– котушки або з'єднання

$$u(t) = L\frac{di(t)}{dt} \Rightarrow i(t) = \frac{1}{L}\int u(t)dt + I_0$$



$L(t)$

$L \neq f(t)$

— kondenzator

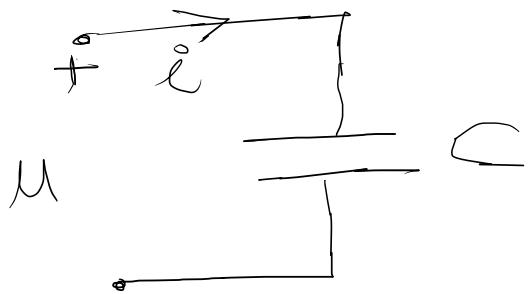
$$Q = C \cdot U \qquad q = \int i\,dt$$

$$Q(t) = C \cdot u(t)$$

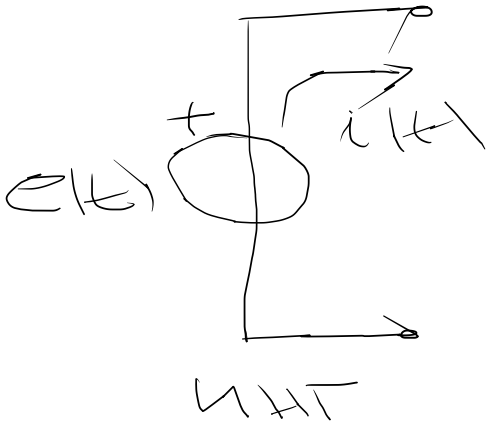
$$i(t) = \frac{dQ(t)}{dt}$$

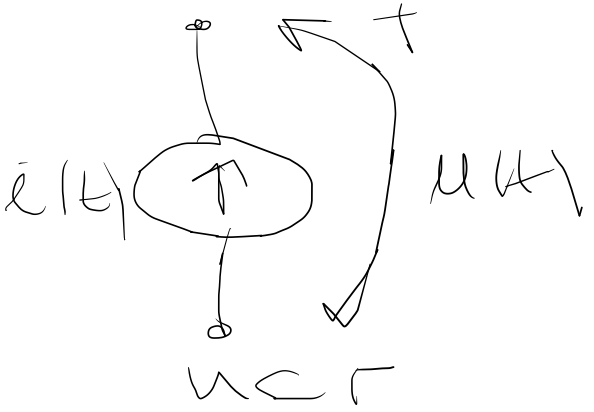
$$i(t) = \frac{d}{dt}(C\,u(t)) = C\,\frac{du}{dt}$$

TEK



$$\Rightarrow u(t) = \frac{1}{C}\int i(t)\,dt + U_0$$


+
e(t)
i(t)


+
i(t)
u(t)

КИРХОФОВИ ЗАКОНИ ЗА КОЛА СА ВРЕМ. ПРОМЈЕНЉИВИМ СТРУЈАМА

$$\sum_{k=1}^{n} i_k(t) = 0$$

ПРВИ КИРХОФОВ ЗАКОН
(у сваком тренутку)

- „+" ка чвору
- „−" од чвора

а може и обрнуто

$$u_{AB}(t) = \sum_{\text{од } A \text{ до } B} u(t)$$

$$\sum_{k=1}^{n} u_k(t) = 0$$

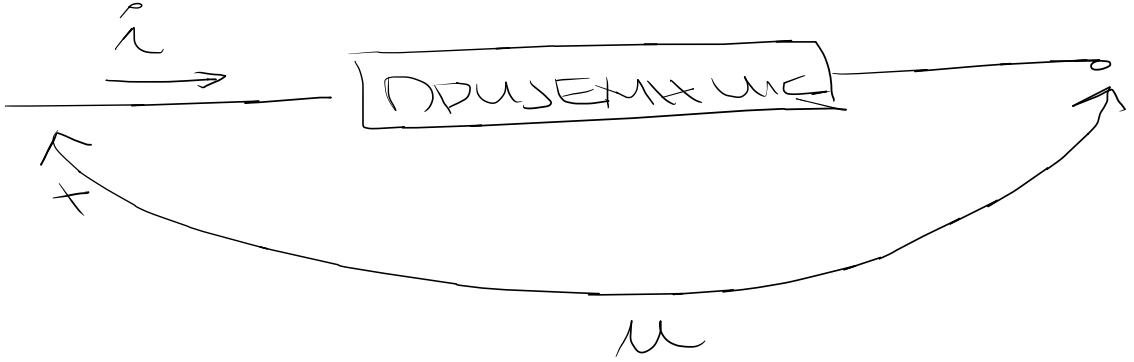
ДРУГИ КИРХОФОВ ЗАКОН
(у сваком тренутку времена)

- „+" ако се стрелица нађе на „+" напона
- „−"

Снага у мрежама са хармонијски променљивим сигналима

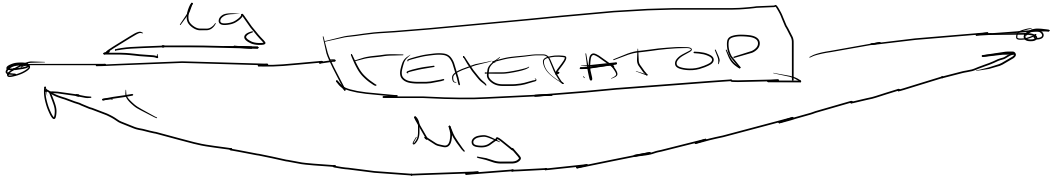
Тренутна вредност снаге пријемника је

$$p(t) = u(t) \cdot i(t)$$



Тренутна вредност снаге генератора је

$$p_g(t) = u_g(t) \cdot i_g(t)$$



ПРОСТОПЕРИОДИЧНИ ВЕЛИЧИНИ

$y = \sin t$

$y = \cos t$

$\sin x = \cos\left(x - \frac{\pi}{2}\right) = \cos\left(\frac{\pi}{2} - x\right)$

$-\infty < t < \infty$

$i(t) = I_m \cos(\omega t + \psi)$

$i(t)$ — моментна стойност

I_m — амплитуда (неотрицателна const.)

$\omega t + \psi$ — моментна фаза

$\omega > 0$ [rad/s] — кръгова честота

ψ за $t = 0$ — начална фаза ($-\pi, \pi$], ψ до 2π

$u(t) = U_m \cos(\omega t + \theta)$

θ — начална фаза

фаза за $t = 0$

КАНОНИЧЕН ОБЕКТ

$$i(t) = I_m \cos\left(\omega t + \psi - \frac{\pi}{2}\right)$$

$$I_m \sin(\omega t + \psi)$$

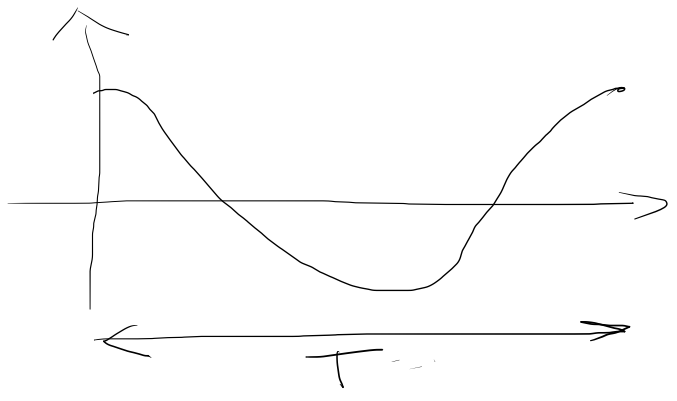
Исто је

$f\,[Hz]$

као

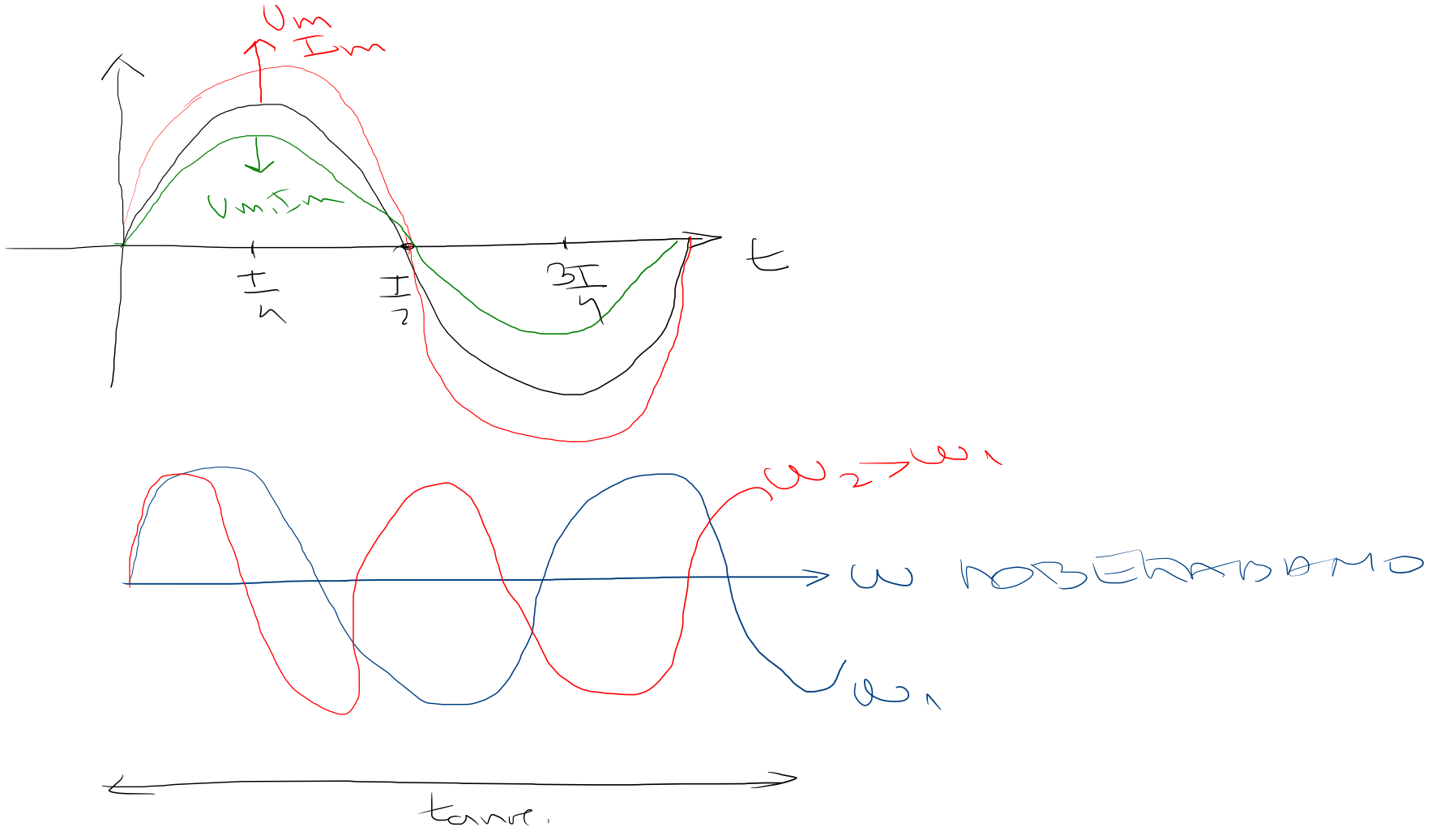
$$\omega = 2\pi f$$

Период



$$T = \frac{2\pi}{\omega}$$

$$f = \frac{1}{T}$$ фреквенца


V_m
I_m
$V_m \cdot I_m$
$\frac{T}{4}$
$\frac{T}{2}$
$\frac{3T}{4}$
t
$\omega_2 \to \omega_1$
ω ROBERTSON
ω_1
tempo

$i(t) = I_m \cos(\omega t + \psi)$

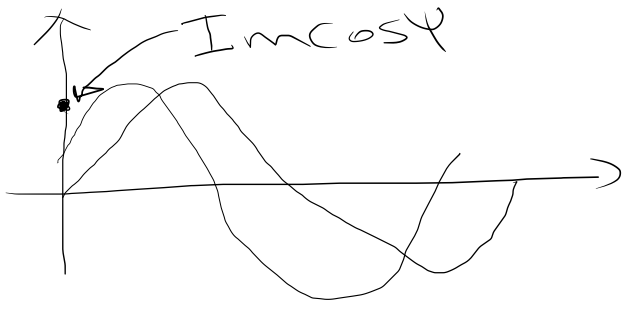
$i(t) \to max$ ako je $\cos(\omega t + \psi) = 1$

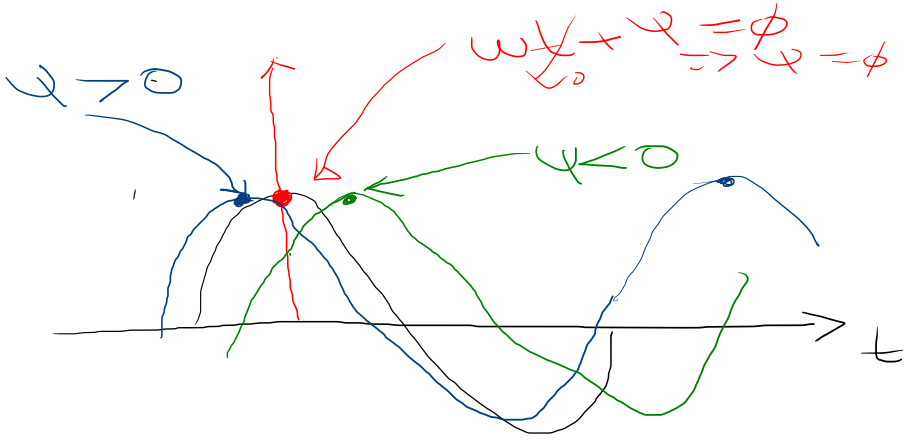
I_m

$\omega t + \psi = \phi$

$\omega t = -\psi$

Ako je $\psi > 0$ $\Rightarrow$





$\psi < 0$ $\omega t = -\psi$

$\cos(\omega t + \psi) = 1$

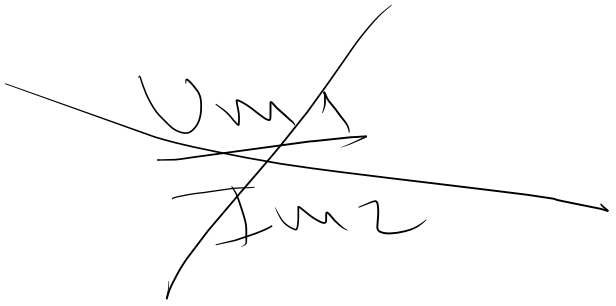
$u_1(t) = U_{m1}\cos(\omega t + \theta_1)$

$u_2(t) = U_{m2}\cos(\omega t + \theta_2)$

$i_1(t) = I_{m1}\cos(\omega t + \psi_1)$

$i_2(t) = I_{m2}\cos(\omega t + \psi_2)$

$\frac{\underline{U_{m1}}}{U_{m2}}$ и $\frac{\underline{I_{m1}}}{I_{m2}}$



$(\omega t + \theta_1) - (\omega t + \theta_2) = \theta_1 - \theta_2$

$\theta_1 = \theta_2$ — у фазу

$\theta_1 > \theta_2$ — u_1 фазою попереду u_2

$\theta_1 < \theta_2$ — u_1 фазою відстає за u_2

$\theta_1 - \theta_2 = \pi$ — у протифазі

$\theta_1 - \theta_2 = \pm\frac{\pi}{2}$ — у квадратурі

$(\omega t + \theta_1) - (\omega t + \psi_2) = \theta_1 - \psi_2$

Можемо имати фазно померање кола у односу на струју.

СРЕДЊА ВРЕДНОСТ $f(t)$ на делу интервала (a,b):

$$f_{sr} = \frac{1}{b-a} \int_a^b f(t)\,dt$$

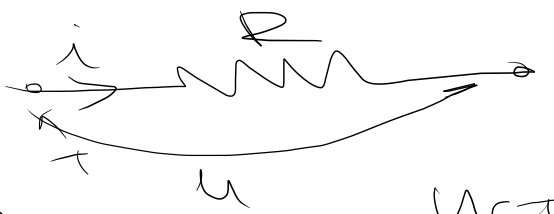
$i(t) = I_m \cos \omega t$ на једном периоду T је

$$I_{sr} = \frac{1}{T} \int_0^T i(t)\,dt = \frac{1}{T} \int_0^T I_m \cos \omega t\,dt = \frac{I_m}{T} \cdot \frac{1}{\omega} (\sin \omega t) \Big|_0^T$$

$$= \frac{I_m}{\omega T} \left(\sin \frac{2\pi}{T} \cdot T - 0 \right) = 0$$

ЕФЕКТИВНА ВРЕДНОСТ

I i(t) R



Исто ли е?

$$p(t) = u(t) \cdot i(t) = R \cdot i^2(t)$$

$$P_{SR} = \frac{1}{T}\int_0^T p(t)\,dt = \frac{1}{T}\int_0^T R i^2(t)\,dt = R I^2$$

$$I^2 = \frac{1}{T}\int_0^T i^2(t)\,dt \Rightarrow I = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt}$$

$$i(t) = I_m \cos \omega t \Rightarrow I = \sqrt{\frac{1}{T}\int_0^T I_m^2 \cos^2 \omega t\,dt}$$

$$I^2 = \frac{1}{T}\int_0^T I_m^2 \frac{1+\cos 2\omega t}{2}\,dt = \frac{1}{T}\frac{I_m^2}{2}T \Rightarrow$$

$$\boxed{I = \frac{I_m}{\sqrt{2}}}$$

$$i(t) = I_m \cos(\omega t + \psi)$$

$$= \sqrt{2} I \cos(\omega t + \psi)$$

$\sqrt{2} I = I_m$

efektivna vrijednost

faza

početna faza

ugao faze

$$u(t) = \sqrt{2} U \cos(\omega t + \theta)$$

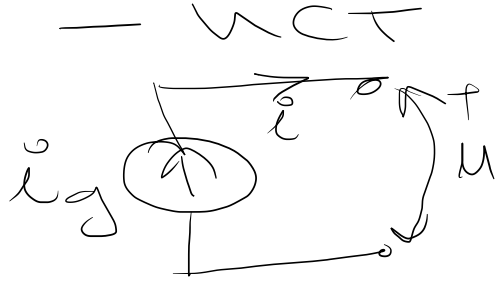
$$\sqrt{2} U = U_m$$

ЕЛЕМЕНТИ КОЛА У ПРОСТОПЕРИОДИЧНОМ РЕЖИМУ

- НАПОН

$$u(t) = e(t) \mp f(i(t))$$

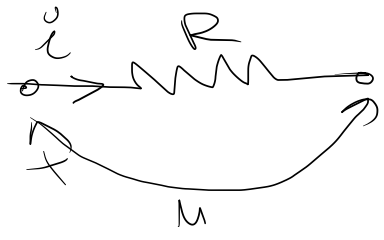
- СТРУЈА



$i(t)$

$$i = i_g \mp f(u)$$

- ОТПОРНИК



$$u = R \cdot i$$

$$i = G u$$

$$p(t) = u(t) \cdot i(t) = R i^2(t) = \frac{u^2(t)}{R}$$

фазори напона и струје

$$i(t) = I_m \cos(\omega t + \psi) = \sqrt{2} I \cos(\omega t + \psi)$$

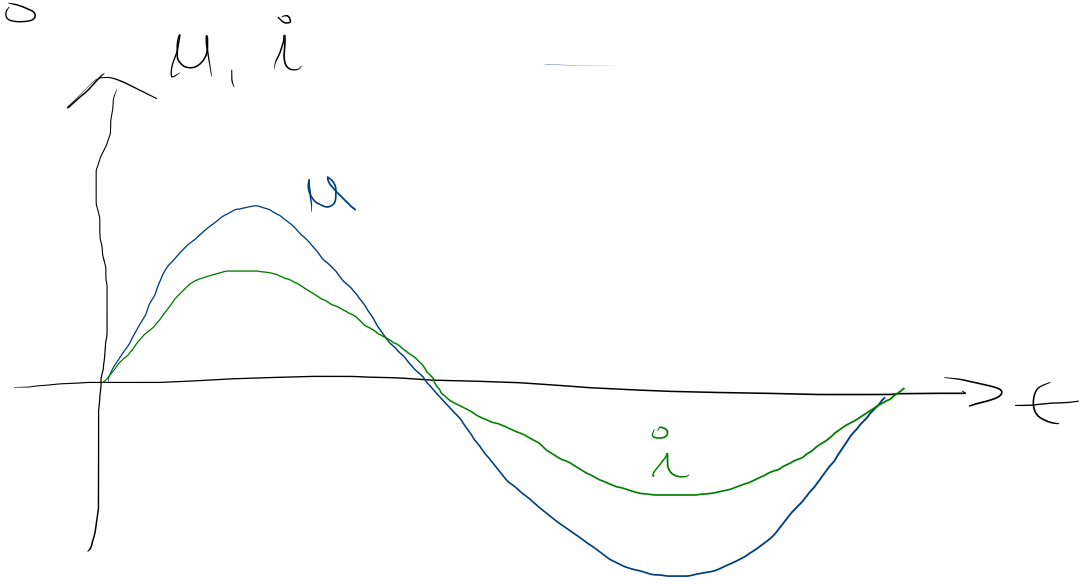
$$u(t) = R i(t) = \sqrt{2} R I \cos(\omega t + \psi) = \sqrt{2} U \cos(\omega t + \theta)$$

$$\Rightarrow U = RI \qquad \theta = \psi \qquad \Rightarrow \varphi = \theta - \psi = 0$$

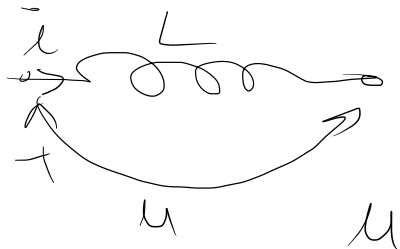
$$p(t) = u(t) \cdot i(t) = 2RI^2 \cos^2(\omega t + \psi)$$

$$= RI^2 (1 + \cos(2\omega t + 2\psi))$$

$$P = \frac{1}{T} \int_0^T p(t)\, dt = \ldots = RI^2$$



- kanar



$$u = L\frac{di}{dt}$$

$$i(t) = \sqrt{2}I\cos(\omega t + \psi)$$

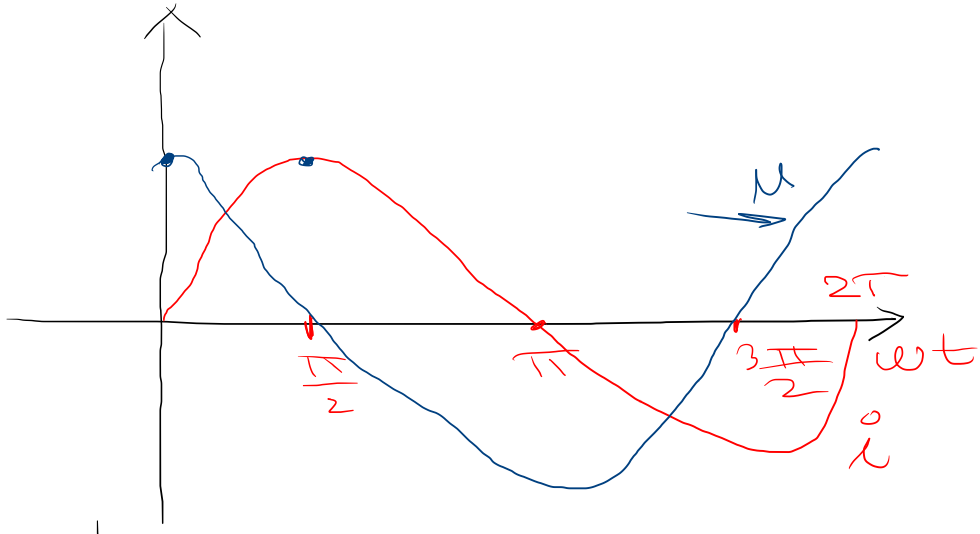
$$u(t) = L\frac{di}{dt} = -\omega L\sqrt{2}I\sin(\omega t + \psi)$$

$$= \sqrt{2}\omega LI\cos\left(\omega t + \psi + \frac{\pi}{2}\right) = \sqrt{2}U\cos(\omega t + \theta)$$

$$\Rightarrow \quad U = \omega LI$$

$$\theta = \psi + \frac{\pi}{2}$$

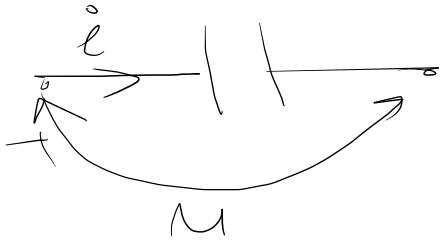
$$\phi = \theta - \psi = \frac{\pi}{2}$$



$P_{sr} = \phi$

$$P(t) = u(t)\cdot i(t) = \ldots = \omega LI^2\sin(2\omega t + 2\theta)$$

Конденсатор



$$i = C \frac{du}{dt}$$

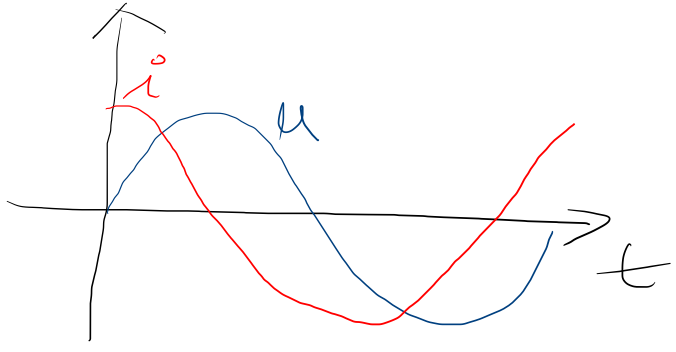
$$u(t) = \sqrt{2}\, U \cos(\omega t + \theta)$$

$$i(t) = \sqrt{2}\,\omega C U \cos\left(\omega t + \theta + \frac{\pi}{2}\right)$$

$$= \sqrt{2}\, I \cos(\omega t + \psi)$$

$$\Rightarrow I = \omega C U \qquad \psi = \theta + \frac{\pi}{2}$$

$$\varphi = \theta - \psi = -\frac{\pi}{2}$$



$$p(t) = u(t) \cdot i(t)$$

$$= \ldots = -\omega C U^2 \sin(2\omega t + 2\theta)$$

$$P_{sr} = \varphi$$

$U = RI$ $\quad U = \omega L I$ $\quad U = \frac{I}{\omega C}$

общий случай $\quad U = Z \cdot I$

↳ обобщение предыдущих

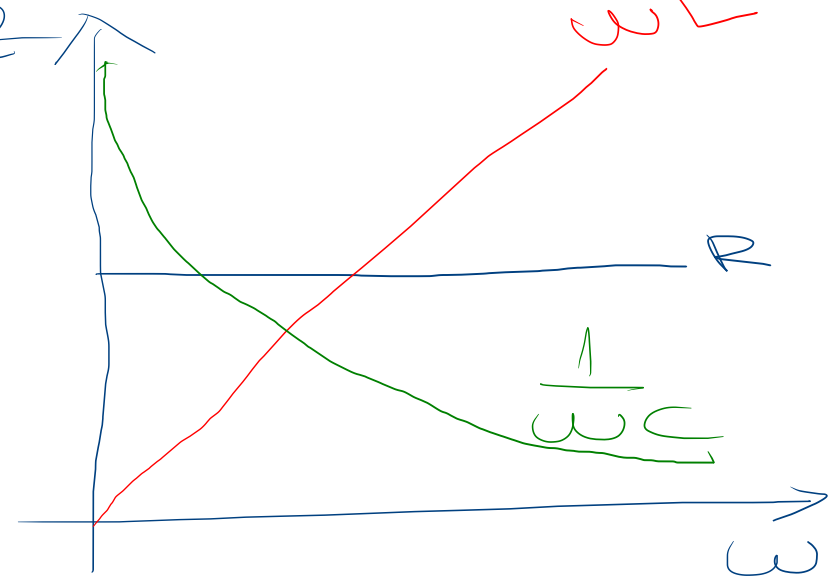
Z [Ом] импеданса

$Z > 0$

$y = \frac{1}{Z}$ адмиттанса

y [См] ← симменс

$y > 0$



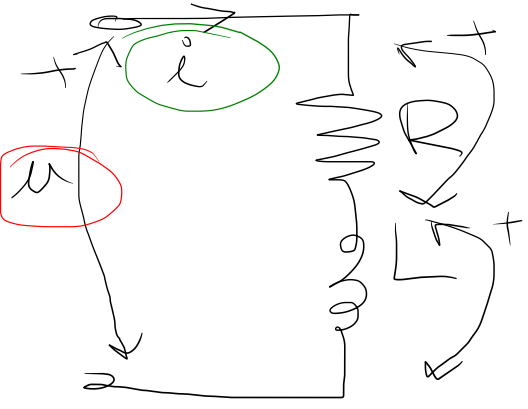
Анализа кола у простопериодичном режиму

$$\sum_{c} i(t) = 0 \qquad \sum_{c} u(t) = 0$$

+ закони везе (i-u) елемената

$$u = Ri \qquad u = L\frac{di}{dt} \qquad i = C\frac{du}{dt}$$

Пример:



$i(t) = \sqrt{2} I \cos\omega t$ познато

$$\left.\begin{aligned} u &= u_R + u_L \\ u_R &= Ri \\ u_L &= L\frac{di}{dt} \end{aligned}\right\} \quad u(t) = \sqrt{2}RI\cos\omega t + L\frac{di}{dt}$$

$$= \sqrt{2}RI\cos\omega t - \omega LI\sqrt{2}\sin\omega t$$

$$= \sqrt{2}\,U\cos(\omega t + \theta)$$

? ?

$$u(t) = \sqrt{2}U\cos(\omega t + \theta)$$

$$= \sqrt{2}U(\cos\omega t \cdot \cos\theta - \sin\omega t \cdot \sin\theta)$$

$$= \sqrt{2}RI\cos\omega t - \sqrt{2}\omega LI\sin\omega t$$

$$\left.\begin{aligned} U\cdot\cos\theta &= RI \\ U\cdot\sin\theta &= \omega LI \end{aligned}\right\} \Rightarrow \begin{aligned} \theta &= ? \\ U &= ? \end{aligned}$$

$$tg\theta = \frac{\omega L}{R} \Rightarrow \theta = arctg\frac{\omega L}{R}$$

$$U^2(\cos^2\theta + \sin^2\theta) = I^2(R^2 + (\omega L)^2)$$

$$U = I\sqrt{R^2 + (\omega L)^2}$$

$$u(t) = \sqrt{2}I\sqrt{R^2 + (\omega L)^2}\cos\left(\omega t + arctg\frac{\omega L}{R}\right)$$