1

$$xy' - 4y - x^2\sqrt{y} = 0$$

$$\Leftrightarrow xy' - 4y = x^2\sqrt{y} \quad /: x, \quad x \neq 0 \quad \dots (1)$$

$$y' - \frac{4}{x}y = x y^{\frac{1}{2}} \Rightarrow \text{Bernulijeva d.j.}$$

$$\alpha = \frac{1}{2} \Rightarrow \quad t = y^{1-\alpha} = y^{\frac{1}{2}} \Rightarrow$$

$$t' = \frac{1}{2} \cdot y^{-\frac{1}{2}} y'$$

$$\frac{1}{2} y^{-\frac{1}{2}} y' - \frac{4}{x} \cdot y \cdot \frac{1}{2} \cdot y^{-\frac{1}{2}} = x \cancel{y^{\frac{1}{2}}} \cdot \frac{1}{2} \cancel{y^{-\frac{1}{2}}} \Rightarrow$$

$$t' - \frac{2}{x} t = \frac{x}{2}$$

$$t = e^{\int \frac{2}{x}dx} \cdot \left(C + \int \frac{x}{2} e^{\int -\frac{2}{x}dx} dx \right)$$

$$\Leftrightarrow t = e^{2\ln|x|} \cdot \left(C + \int \frac{x}{2} \cdot e^{-2\ln|x|} dx \right)$$

$$\Leftrightarrow t = x^2 \cdot \left(C + \int \frac{\cancel{x}}{2} \cdot \frac{1}{x^{\cancel{2}}} dx \right)$$

$$\Leftrightarrow t = x^2 \cdot \left(C + \frac{\ln|x|}{2} \right) \Rightarrow$$

$$\boxed{\sqrt{y} = x^2 \cdot \left(C + \frac{\ln|x|}{2} \right)}$$

opste rjesenje početne jednačine na intervalu $(-\infty, 0) \cup (0, +\infty)$

U slučaju (1), za $x = 0$, dobijamo:

$$\cancel{0 \cdot y'} - 4y - \cancel{0^2\sqrt{y}} = 0 \Rightarrow y = 0 \Rightarrow$$

$\boxed{\text{Tačka } (0,0) \text{ je singularno rjesenje.}}$

2

$$3y^2y' = 2xy^3 - 16x$$

smjena: $t = y^3 \Rightarrow t' = 3y^2y' \Rightarrow$

$$t' = 2xt - 16x$$

$$\Leftrightarrow \quad t' - 2xt = -16x$$

$$t = e^{\int 2x\,dx} \cdot \left(C + \int -16x\, e^{\int -2x\,dx}\, dx \right)$$

$$\Leftrightarrow \quad t = e^{x^2} \cdot \left(C - 16 \cdot \int x e^{-x^2}\, dx \right)$$

$$\Leftrightarrow \quad t = e^{x^2} \cdot \left(C + 8 \cdot \int e^{-x^2} \cdot (-2x)\, dx \right) \quad \ldots (1)$$

smjena: $-x^2 = u \Rightarrow -2x\,dx = du$

$$I_1 = \int e^{-x^2} (-2x)\, dx = \int e^u\, du = e^u + C_1 = e^{-x^2} + C_1 \overset{(1)}{\Rightarrow}$$

$$t = e^{x^2} \cdot \left(C + 8e^{-x^2} \right) \Rightarrow$$

$$t = C \cdot e^{x^2} + 8 \Rightarrow$$

$$\boxed{y^3 = C \cdot e^{x^2} + 8}$$

opšte rješenje početne diferencijalne jednačine na intervalu $(-\infty, \infty)$

$$y(0) = 0 \Rightarrow 0^3 = C \cdot e^{0^2} + 8 \Rightarrow C + 8 = 0 \Rightarrow C = -8$$

Partikularno rješenje koje sadrži tačku $(0,0)$:

$$y^3 = 8 - 8e^{x^2} \Rightarrow y = \sqrt[3]{8(1 - e^{x^2})}$$

$$\Rightarrow \boxed{y = 2\sqrt[3]{1 - e^{x^2}}}$$

③ $$(4x+3y^2)\,dx + 2xy\,dy = 0$$

$$\Leftrightarrow \quad 4x+3y^2 + 2xy\cdot y' = 0$$

$$\Leftrightarrow \quad y' = -\frac{4x+3y^2}{2xy} \quad ; \quad x \neq 0 \wedge y \neq 0$$

- Za $x=0$ imamo:

$$\cancel{4\cdot 0} + 3y^2 + \cancel{2\cdot 0\cdot y\cdot y'} = 0 \quad \Rightarrow \quad y = 0$$

Dobijamo singularno rješenje - tačku $(x,y)=(0,0)$

- Za $y=0$ imamo:

$$4x + \cancel{3\cdot 0^2} + \cancel{2x\cdot 0\cdot 0'} = 0 \quad \Rightarrow \quad x = 0$$

Dobijamo opet isto singularno rješenje - tačku $(0,0)$.

- Za $x \neq 0$ i $y \neq 0$ imamo:

$$y' = -\frac{2}{y} - \frac{3}{2x}y$$

$$\Leftrightarrow \quad y' + \frac{3}{2x}y = -2y^{-1} \; /2y \Rightarrow \text{ Bernulijeva d.j.}$$

$$2y\cdot y' + \frac{3}{x}y^2 = -4$$

smjena: $t = y^2 \Rightarrow t' = 2y\cdot y' \Rightarrow$

$$t' + \frac{3}{x}t = -4$$

$$t = e^{-\int \frac{3}{x}dx}\cdot\left(C_1 + \int -4\, e^{-\int -\frac{3}{x}dx}\,dx\right)$$

$$t = e^{-3\ln|x|}\cdot\left(C_1 + \int -4\, e^{3\ln|x|}\,dx\right)$$

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$$\Leftrightarrow \quad t = |x|^{-3} \cdot \left(C_1 - 4 \int |x|^3 \, dx \right)$$

$$\Leftrightarrow \quad t = x^{-3} \cdot sgn^{-3}(x) \cdot \left(C_1 - 4 \cdot sgn^3(x) \cdot \int x^3 \, dx \right)$$

$$\Leftrightarrow \quad t = C_1 x^{-3} \cdot sgn^{-3}(x) - \cancel{4} x^{-3} \cdot \frac{x^4}{\cancel{4}}$$

$$\Leftrightarrow \quad t = C_1 \cdot sgn^{-3}(x) \cdot x^{-3} - x \qquad \dots (\star)$$

$$\Leftrightarrow \quad t = C x^{-3} - 2x \quad , \quad C = C_1 \cdot sgn^{-3}(x)$$

$$\Rightarrow \quad \boxed{y^2 = \frac{C}{x^3} - 2x}$$ opšte rješenje na intervalu $(-\infty, 0) \cup (0, +\infty)$

★ DISKUSIJA:

$$\left.\begin{array}{l} sgn^3(x) = (sgn(x))^3 = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases} \\ sgn(x^3) = \begin{cases} 1, & x^3 > 0 \Leftrightarrow x > 0 \\ 0, & x^3 = 0 \Leftrightarrow x = 0 \\ -1, & x^3 < 0 \Leftrightarrow x < 0 \end{cases} \end{array}\right\} \Rightarrow sgn^3(x) = sgn(x^3)$$

Ako je $x > 0$ onda je $(\star)$

$$t = C_1 x^{-3} - x = \frac{C_1}{x^3} - x$$ opšte rješenje na $(0, +\infty)$

Ako je $x < 0$ onda je $(\star)$

$$t = -C_1 x^{-3} - x = \frac{-C_1}{x^3} - x$$ opšte rješenje na $(-\infty, 0)$

4

$$y' + \frac{y}{x+1} + y^2 = \theta \quad , \quad x \neq -1$$

$$\Leftrightarrow \quad y' + \frac{y}{x+1} = -y^2 \quad \Rightarrow \text{ Bernulijeva d.j.}$$

smjena: $t = y^{-1} \Rightarrow t' = -y^{-2} y' \; , \; y \neq \theta$

- Ako je $y = \theta$ imamo

$$\theta' + \frac{\theta}{x+1} + \theta^2 = \theta \quad \Leftrightarrow \quad \theta = \theta \quad T$$

Dakle, $\boxed{y = \theta \text{ je singularno rješenje.}}$

- Ako je $y \neq \theta$ imamo

$$-y^{-2} \cdot y' + \frac{y}{x+1} \cdot (-y^{-2}) = -y^2 \cdot (-y^{-2})$$

$$\overset{t=y^{-1}}{\Longrightarrow} \quad t' - \frac{1}{x+1} t = 1$$

$$\Leftrightarrow \quad t = e^{-\int -\frac{1}{x+1} dx} \cdot \left(C_1 + \int 1 \cdot e^{\int -\frac{1}{x+1} dx} dx \right)$$

$$\Leftrightarrow \quad t = e^{\ln|x+1|} \cdot \left(C_1 + \int e^{-\ln|x+1|} dx \right)$$

$$\Leftrightarrow \quad t = |x+1| \cdot \left(C_1 + \int \frac{dx}{|x+1|} \right)$$

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$$\Leftrightarrow \quad t = (x+1)\cdot \operatorname{sgn}(x+1)\cdot\left(C_1 + \frac{1}{\operatorname{sgn}(x+1)}\int\frac{dx}{x+1}\right)$$

$$\Leftrightarrow \quad t = \underbrace{C_1 \operatorname{sgn}(x+1)}_{C}\cdot (x+1) + (x+1)\ln|x+1|$$

$$\Rightarrow \quad y^{-1} = C(x+1) + (x+1)\ln|x+1|$$

$$\Rightarrow \quad \boxed{\frac{1}{y} = (x+1)\cdot\left(C + \ln|x+1|\right)}$$

opste rješenje na intervalu $(-\infty, -1)\cup(-1, +\infty)$

(5)

$$(e^y - y')\cdot x = 2$$

$$\Rightarrow \quad e^y - y' = \frac{2}{x}, \quad x \neq 0$$

- Ako je $x = 0$, početna diferencijalna jednačina nema rješenje jer je $(e^y - y')\,0 = 0 \neq 2$, $(\forall y \in \mathbb{R})$
- Sada kada je $x \neq 0$ imamo

$$y' - e^y = -\frac{2}{x} \quad \ldots(1)$$

Posljednja jednačina nije linearna i linearnost kvari e^y.

Kako je $(e^y)' = e^y \cdot y'$, jednačinu (1) ćemo pomnožiti sa e^y; $e^y > 0$ $(\forall y \in \mathbb{R})$ pa dobijamo

$$e^y y' - (e^y)^2 = -\frac{2}{x} e^y$$

Uvođenjem smjene $e^y = t$ dobijamo:

$$t' - t^2 = -\frac{2}{x} t$$

$$\Leftrightarrow \quad t' + \frac{2}{x} t = t^2 \quad \Rightarrow \text{ Bernulijeva d.j.}$$

smjena: $u = t^{-1} \Rightarrow u' = -t^{-2} \cdot t' \Rightarrow$

$$-t^{-2} t' - \frac{2}{x} \cdot t \, t^{-2} = t^2 \cdot (-t^{-2})$$

$$\Rightarrow \quad u' - \frac{2}{x} u = -1$$

$$u = e^{-\int -\frac{2}{x} dx} \cdot \left(C + \int -1 \cdot e^{\int -\frac{2}{x} dx} dx \right)$$

$$\Leftrightarrow \quad u = e^{2\ln|x|} \cdot \left(C - \int e^{-2\ln|x|} dx \right)$$

$$\Leftrightarrow \quad u = |x|^2 \cdot \left(C - \int \frac{dx}{|x|^2} \right)$$

$$\Leftrightarrow \quad u = x^2 \cdot \left(C + \int \frac{-1}{x^2} dx \right)$$

$$\Leftrightarrow \quad u = x^2 \cdot \left(C + \frac{1}{x} \right)$$

$$\Leftrightarrow \quad u = Cx^2 + x \quad \Rightarrow$$

$$t^{-1} = Cx^2 + x$$

$$\Leftrightarrow \quad \frac{1}{t} = Cx^2 + x$$

$$\Leftrightarrow \quad \frac{1}{e^y} = Cx^2 + x$$

$$\Leftrightarrow \quad e^y = \frac{1}{Cx^2 + x}$$

$$\Leftrightarrow \quad \boxed{y = \ln\left(\frac{1}{Cx^2 + x}\right)}$$

opšte rješenje na intervalu $(-\infty, 0) \cup (0, +\infty)$

6.

$$y' \operatorname{tg} y + 4x^3 \cos y = 2x$$

$$\Leftrightarrow \quad \frac{\sin y}{\cos y} \cdot y' + 4x^3 \cos y = 2x \quad ; \quad \cos y \neq 0$$

$$\Leftrightarrow \quad \sin y \cdot y' + 4x^3 \cos^2 y = 2x \cos y$$

smjena: $\cos y = t \Rightarrow$

$-\sin y \cdot y' = t' \Rightarrow$

$$-t' + 4x^3 t^2 = 2xt$$

$$\Leftrightarrow \quad t' + 2xt = 4x^3 t^2 \quad \Rightarrow \text{ Bernulijeva d.j.} \quad / \cdot (-t^{-2})$$

smjena: $u = t^{-1} \Rightarrow u' = -t^{-2} t'$

$$-t^{-2}\cdot t' + 2xt\cdot(-t^{-2}) = 4x^3t^2\,(-t^{-2})$$

$$\Rightarrow \quad u' - 2xu = -4x^3$$

$$u = e^{-\int -2x\,dx}\cdot\left(C + \int -4x^3\cdot e^{\int -2x\,dx}\,dx\right)$$

$$\Leftrightarrow \quad u = e^{x^2}\cdot\left(C + \int -4x^3 e^{-x^2}\,dx\right) \quad \dots (1)$$

$$I_1 = \int -4x^3 e^{-x^2}\,dx = \begin{cases} -x^2 = w \\ -2x\,dx = dw \end{cases}$$

$$= \int 2x^2\cdot e^{-x^2}\,(-2x\,dx)$$

$$= \int -2we^{w}\,dw = \begin{cases} u_1 = -2w & v_1 = e^{w} \\ du_1 = -2dw & dv_1 = e^{w}dw \end{cases}$$

$$= -2we^{w} + \int 2e^{w}dw$$

$$= -2we^{w} + 2e^{w} + C_1$$

$$= 2e^{w}(1-w) + C_1$$

$$= 2e^{-x^2}(1+x^2) + C_1 \quad \overset{(1)}{\Longrightarrow}$$

$$u = e^{x^2}\cdot\left(C + 2e^{-x^2}(1+x^2)\right) \quad \Rightarrow$$

$$t^{-1} = Ce^{x^2} + 2(1+x^2)$$

$$\Leftrightarrow \boxed{\frac{1}{\cos y} = Ce^{x^2} + 2(1+x^2)}$$ opste rjesenje diferencijalne jednacine $\cos y \neq 0$

(7) $$(\sin^2 y + x\,\mathrm{ctg}\, y)\cdot y' = 1$$

$$\Leftrightarrow \quad y' = \frac{1}{\sin^2 y + x\,\mathrm{ctg}\, y}$$

$$\Leftrightarrow \quad x' = \sin^2 y + \mathrm{ctg}\, y \cdot x$$

$$\Leftrightarrow \quad x' - \mathrm{ctg}\, y \cdot x = \sin^2 y \quad ; \quad \sin y \neq 0$$

$$x = e^{-\int -\mathrm{ctg}\, y\, dy} \cdot \left(C_1 + \int \sin^2 y \cdot e^{\int -\mathrm{ctg}\, y\, dy}\, dy \right)$$

$$\Leftrightarrow \quad x = e^{\int \frac{\cos y\, dy}{\sin y}} \cdot \left(C_1 + \int \sin^2 y \cdot e^{-\int \frac{\cos y\, dy}{\sin y}}\, dy \right)$$

$$\Leftrightarrow \quad x = e^{\ln|\sin y|} \cdot \left(C_1 + \int \sin^2 y \cdot e^{-\ln|\sin y|}\, dy \right)$$

$$\Leftrightarrow \quad x = |\sin y| \cdot \left(C_1 + \int \sin^2 y \cdot \frac{1}{|\sin y|}\, dy \right)$$

$$\Leftrightarrow \quad x = \sin y \cdot \mathrm{sgn}(\sin y) \cdot \left(C_1 + \frac{1}{\mathrm{sgn}(\sin y)} \cdot \int \sin y\, dy \right)$$

$$\Leftrightarrow \quad x = \underbrace{C_1 \operatorname{sgn}(\sin y)}_{C} \cdot \sin y + \sin y \cdot \int \sin y \, dy$$

$$\Leftrightarrow$$

$$x = C \sin y - \sin y \cos y$$

$$\Leftrightarrow \quad \boxed{x = \sin y \cdot (C - \cos y)}$$ opšte rješenje, $\sin y \neq 0$

⑧ $$(2x^2 y \ln y - x)\, y' = y \; ; \qquad y > 0 \text{ (zbog } \ln y)$$

$$\Leftrightarrow \quad y' = \frac{y}{2x^2 y \ln y - x}$$

$$\Leftrightarrow \quad x' = \frac{2x^2 y \ln y - x}{y}$$

$$\Leftrightarrow \quad x' = 2 \ln y \, x^2 - \frac{1}{y} \cdot x$$

$$\Leftrightarrow \quad x' + \frac{1}{y} x = 2 \ln y \; x^2 \Rightarrow$$ Bernulijeva d.j.

smjena $t = x^{-1} \Rightarrow t' = -x^{-2} x'$ $\qquad /(-x^{-2})$

$$-x^{-2} x' + \frac{1}{y} \cdot x \cdot (-x^{-2}) = 2 \ln y \cdot x^2 (-x^{-2})$$

$$\Leftrightarrow \quad t' - \frac{1}{y} t = -2 \ln y$$

$$t = e^{-\int -\frac{1}{y}dy} \cdot \left(C + \int -2\ln y \; e^{\int -\frac{1}{y}dy} dy \right)$$

$$\Leftrightarrow \quad t = e^{\ln|y|} \cdot \left(C - 2\int \ln y \cdot e^{-\ln|y|} dy \right)$$

Kako je $y>0$ vrijedi $|y| = y$ pa je:

$$t = e^{\ln(y)} \cdot \left(C - 2\int \ln y \cdot e^{\ln(y^{-1})} dy \right)$$

$$\Leftrightarrow \quad t = y \cdot \left(C - 2\int \ln y \cdot \frac{1}{y} dy \right) \quad \ldots (1)$$

$$I_1 = \int \ln y \cdot \frac{1}{y} dy = \left\{ \begin{array}{l} \ln y = w \\ \frac{1}{y}dy = dw \end{array} \right.$$

$$= \int w\,dw$$

$$= \frac{1}{2} w^2$$

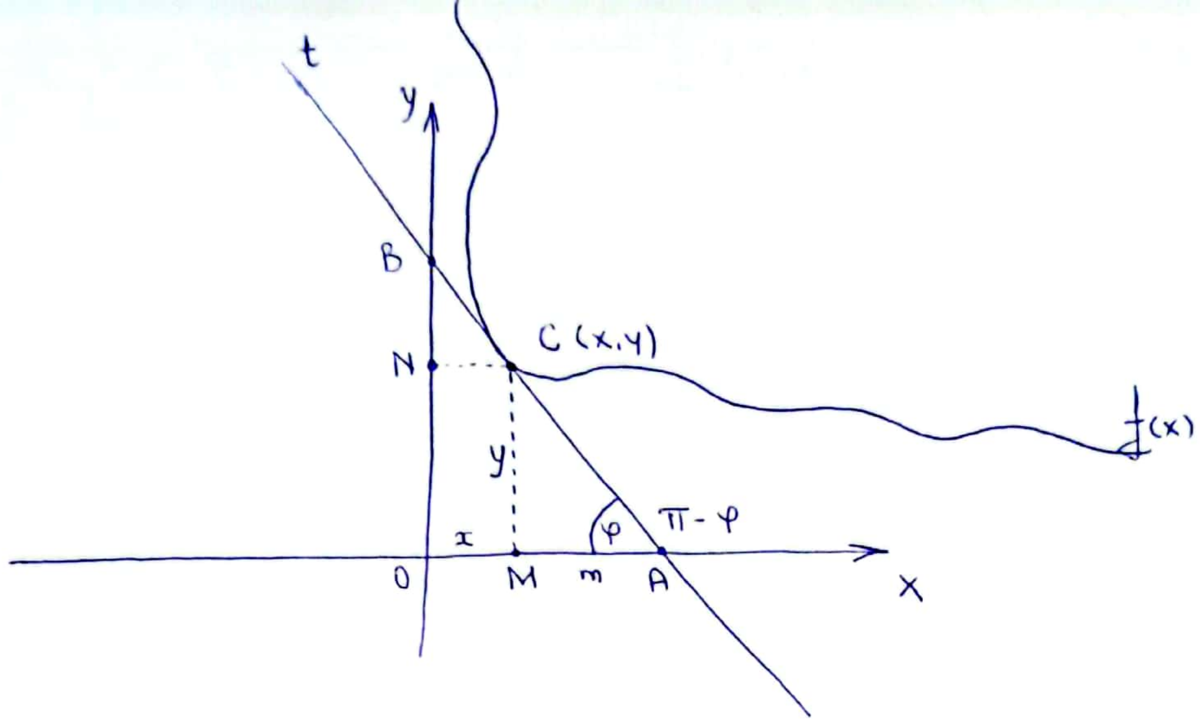
$$= \frac{1}{2}\ln^2 y \quad \overset{(1)}{\Longrightarrow}$$

$$t = y \cdot \left(C - \cancel{2} \cdot \frac{1}{\cancel{2}} \ln^2 y \right)$$

$$\Rightarrow \quad x^{-1} = y \cdot \left(C - \ln^2 y \right)$$

$$\Rightarrow \quad \boxed{\frac{1}{x} = y \cdot \left(C - \ln^2 y \right)} \qquad \text{opšte rješenje za } y>0$$

⑨



Tačka presjeka tangente i apscise je $\{A\}$.

Iz uslova zadatka vrijedi:

$$\overline{AO} = \overline{AC} \quad \dots (1)$$

Uz oznake kao na slici vrijedi

$$y' = tg(\pi - \varphi) = -tg\,\varphi = -\frac{\overline{BO}}{\overline{AO}} \quad \dots (2)$$

Neka je $AM = m$. Vrijedi $\overline{OM} = x$ i $\overline{ON} = \overline{MC} = y$

$\Delta ACM \sim \Delta ABO \Rightarrow$

$$\frac{\overline{AM}}{\overline{AO}} = \frac{\overline{MC}}{\overline{OB}} \Rightarrow \frac{m}{\overline{AO}} = \frac{y}{\overline{OB}} \quad \dots (3)$$

Primjenom Pitagorine teoreme u pravouglom trouglu ACM imamo:

$$\overline{AC}^2 = \overline{AM}^2 + \overline{MC}^2$$

odnosno: $\overline{AC} = \sqrt{m^2+y^2} \xRightarrow{(1)}$

$$m+x = \sqrt{m^2+y^2} \quad /^2$$

$$\Leftrightarrow \cancel{m^2} + 2mx + x^2 = \cancel{m^2} + y^2$$

$$\Leftrightarrow 2mx = y^2 - x^2 \Rightarrow$$

$$m = \frac{y^2-x^2}{2x} \quad \ldots \quad (4)$$

Iz izraza (3) imamo:

$$\frac{\overline{OB}}{\overline{AO}} = \frac{y}{m} = \frac{y}{\frac{y^2-x^2}{2x}} = \frac{2xy}{y^2-x^2} \ldots (5)$$

pa uvrštavanjem izraza (5) u izraz (2) dobijamo:

$$y' = -\frac{2xy}{y^2-x^2}$$

$$\Leftrightarrow y' = \frac{\cancel{x^2}\cdot\left(2\frac{y}{x}\right)}{\cancel{x^2}\left(1-\left(\frac{y}{x}\right)^2\right)} \Rightarrow \text{homogena d.j.}$$

$$y' = \frac{2\frac{y}{x}}{1-\left(\frac{y}{x}\right)^2}$$

smjena: $\frac{y}{x} = t \Rightarrow y = tx \Rightarrow y' = t'x + t$

$$t'x + t = \frac{2t}{1-t^2}$$

$$\Leftrightarrow \quad t'x = \frac{2t - t(1-t^2)}{1-t^2}$$

$$\Leftrightarrow \quad \frac{dt}{dx} x = \frac{2t - t + t^3}{1-t^2}$$

$$\Leftrightarrow \quad \frac{dt}{\frac{t+t^3}{1-t^2}} = \frac{dx}{x}$$

$$\Leftrightarrow \quad \int \frac{1-t^2}{t(1+t^2)}\,dt = \int \frac{dx}{x} \qquad \ldots (6)$$

$$I_1 = \int \frac{1-t^2}{t(1+t^2)}\,dt \qquad \text{integral racionalne funkcije}$$

$$\frac{1-t^2}{t(1+t^2)} = \frac{A}{t} + \frac{Bt+C}{1+t^2} \quad \Big/ \; t(1+t^2)$$

$$\begin{aligned} 1 - t^2 &= A(1+t^2) + (Bt+C)t \\ &= A + At^2 + Bt^2 + Ct \\ &= (A+B)t^2 + Ct + A \end{aligned}$$

$$\begin{aligned} A + B \quad &= -1 \\ C &= 0 \\ A \quad &= 1 \end{aligned}$$

$A = 1,\ B = -2,\ C = 0 \quad \Rightarrow$

$$I_1 = \int \frac{1}{t}\,dt + \int \frac{-2t}{1+t^2}\,dt$$

$$= \ln|t| - \int \frac{2t\,dt}{1+t^2} = \begin{cases} u = 1+t^2 \\ du = 2t\,dt \end{cases}$$

$$= \ln|t| - \ln(1+t^2) + C_1$$

$$= \ln\left|\frac{t}{1+t^2}\right| + C_1 \quad \overset{(6)}{\Longrightarrow}$$

$$\Rightarrow \ln\left|\frac{t}{1+t^2}\right| = \ln|x| + C_2 \Rightarrow$$

$$\Rightarrow \ln\left|\frac{t}{1+t^2}\right| = \ln C_3|x| \quad ; \quad C_2 = \ln C_3$$

$$\Rightarrow \frac{t}{1+t^2} = \pm C_3 x \quad ; C_4 = \pm C_3,\ t = \frac{y}{x} \Rightarrow$$

$$\frac{\frac{y}{x}}{1+\left(\frac{y}{x}\right)^2} = C_4 x \Rightarrow \frac{\frac{y}{\cancel{x^2}}}{\frac{x^2+y^2}{\cancel{x^2}}} = C_4 \Rightarrow$$

$$y = C_4(x^2+y^2) \Rightarrow$$

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$$x^2 + y^2 = \frac{1}{C_4} y \qquad ; \quad \frac{1}{C_4} = C_5 ; \quad C_4 \neq 0$$

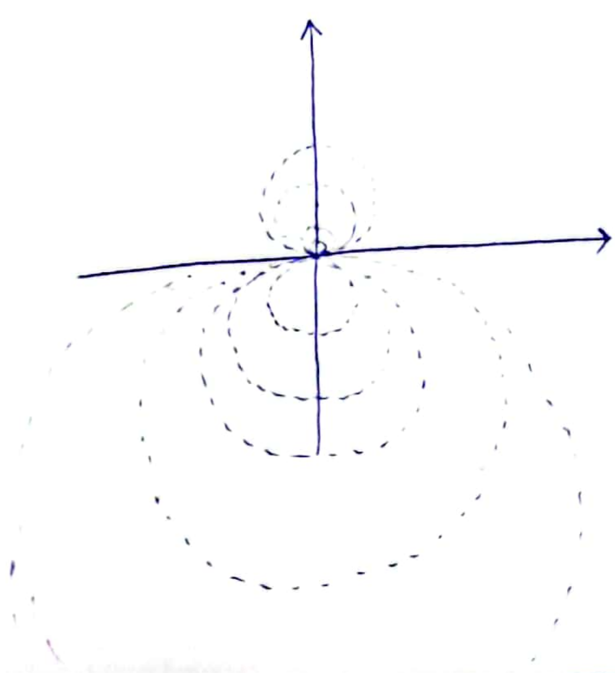
$$\Leftrightarrow \quad x^2 + y^2 - C_5 y = 0$$

$$\Leftrightarrow \quad x^2 + \left(y^2 - 2 \cdot y \cdot \frac{C_5}{2} + \left(\frac{C_5}{2}\right)^2 \right) - \left(\frac{C_5}{2}\right)^2 = 0$$

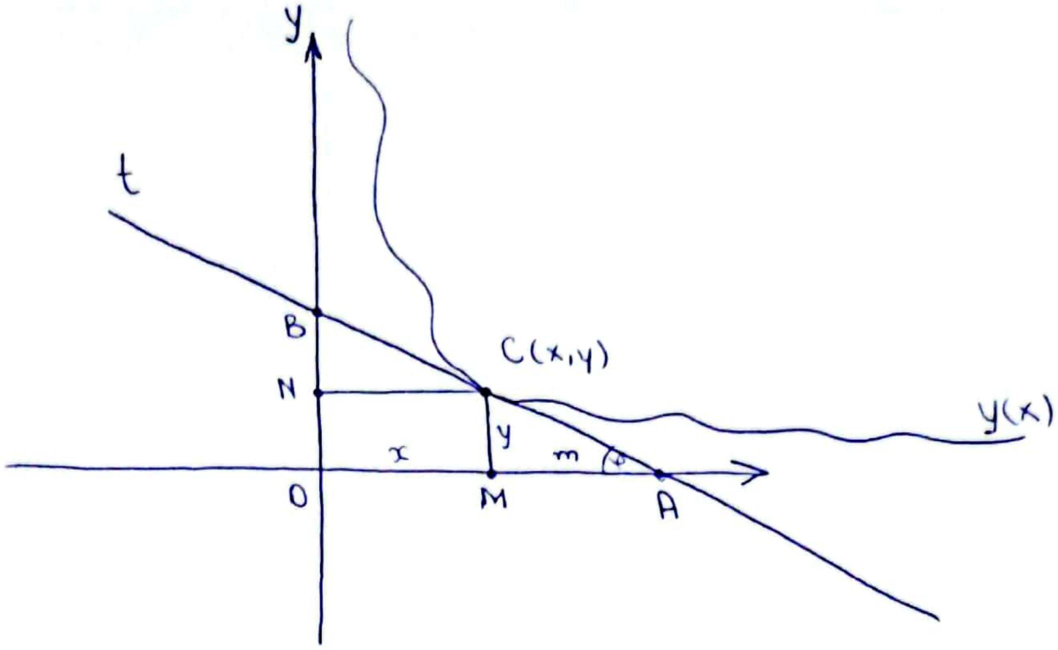
$$\Leftrightarrow \quad x^2 + \left(y - \frac{C_5}{2} \right)^2 = \left(\frac{C_5}{2}\right)^2 \qquad ; \quad \frac{C_5}{2} = C$$

$$\boxed{x^2 + (y - C)^2 = C^2}$$

Geometrijski gledano, skup rješenja diferencijalne jednačine odnosno skup rješenja (krivih u ravni) koje zadovoljavaju početni uslov, jeste skup kružnica sa koordinatama centra $(0, C)$ i poluprečnikom C. Za $C = \infty$ dobijamo „singularno" rješenje $y = 0$.



(10)



Iz uslova zadatka imamo da je površina trougla ACM konstantna i vrijedi

$$P_{\triangle ACM} = P = \frac{\overline{AM} \cdot \overline{CM}}{2} \quad \ldots (1)$$

Neka je $\overline{AM} = m$. Vrijedi $\overline{OM} = x$ i $\overline{CM} = y$.

$$P = \frac{m \cdot y}{2} \Rightarrow m = \frac{2P}{y} \quad \ldots (2)$$

Uz oznake kao na slici vrijedi:

$$y' = tg(\pi - \varphi) = -tg\varphi = -\frac{y}{m} \overset{(2)}{\Longrightarrow}$$

$$y' = -\frac{y}{\frac{2P}{y}} \Rightarrow \frac{dy}{dx} = -\frac{y^2}{2P} \Rightarrow$$

$$-\frac{dy}{y^2} = \frac{dx}{2P} \Rightarrow \int -\frac{1}{y^2}dy = \int \frac{dx}{2P} \Rightarrow$$

$$\frac{1}{y} = \frac{x}{2P} + C_1 \Rightarrow \boxed{y = \frac{1}{\frac{x}{2P} + C_1}}$$