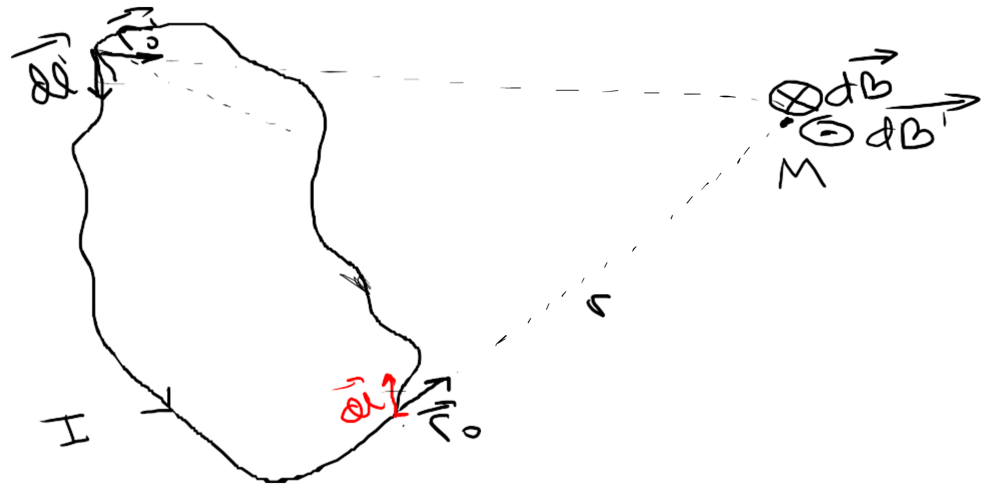
Стално магнетско поље у вакууму (Био-Саваров закон. Флукс)

Основи електротехнике 2

Предавање: 2. блок

МАГНЕТСКА ИНДУКЦИЈА КОПЛАНАРНОГ СИСТЕМА



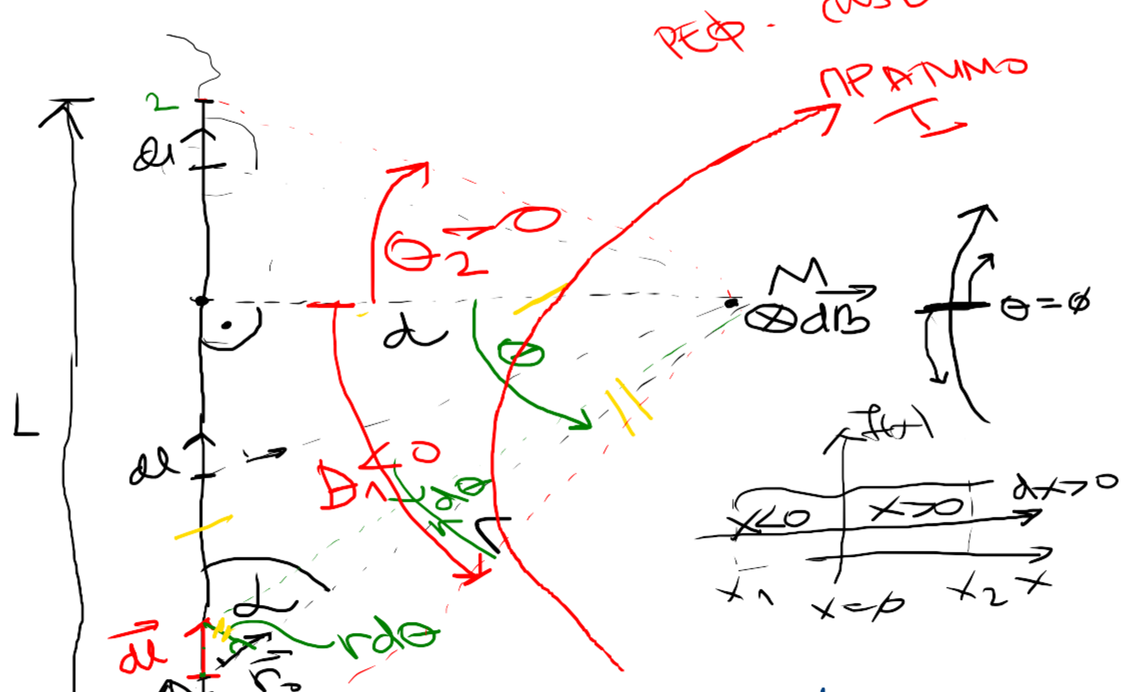
$$d\vec{B} = \frac{\mu_0}{4\pi}\,\frac{I\,d\vec{l}\times\vec{r}_0}{r^2}$$

$$dB = \frac{\mu_0}{4\pi}\,\frac{I\,dl\,\sin\angle(d\vec{l},\vec{r}_0)}{r^2}$$

$$\angle(d\vec{l},\vec{r}_0) = \alpha$$

$$dB = \frac{\mu_0 I\,dl}{4\pi r^2}\sin\alpha$$

ПРИМЈЕР:



$$\vec{dB} = \frac{\mu_0}{4\pi}\frac{I\,\vec{dl}\times\vec{r_0}}{r^2}$$

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl}{r^2}\sin\alpha$$

Сви dB имају исти смјер и леже на истом правцу

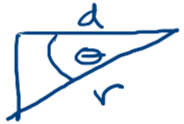
$B = \int dB$ гдје је $\vec{B}$ ⊗

$$B = \int dB = \int \frac{\mu_0}{4\pi}\frac{I\,dl}{r^2}\sin\alpha$$

дуж segmenta

$\Downarrow$

$$\cos\theta = \frac{d}{r} \Rightarrow r = \frac{d}{\cos\theta}$$



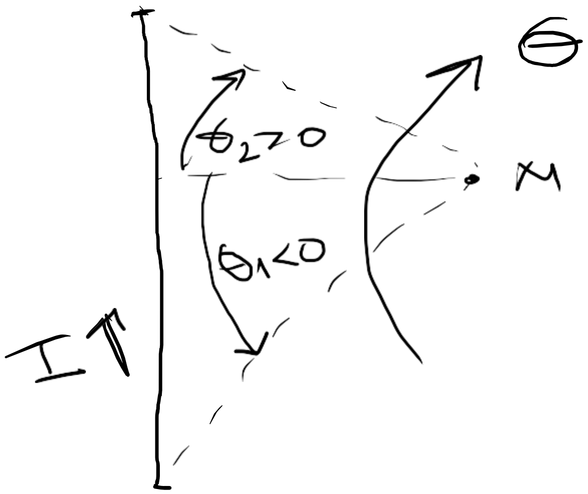
$$B = \frac{\mu_0 I}{4\pi}\int\frac{\cancel{dl}}{\cancel{r^2}}\cdot\frac{\cancel{r}\,d\theta}{\cancel{dl}}$$

$$\sin\alpha = \frac{r\,d\theta}{dl}$$

$$B = \frac{\mu_0 I}{4\pi}\int\frac{d\theta}{\frac{d}{\cos\theta}} = \frac{\mu_0 I}{4\pi d}\int_{\theta_1}^{\theta_2}\cos\theta\,d\theta$$

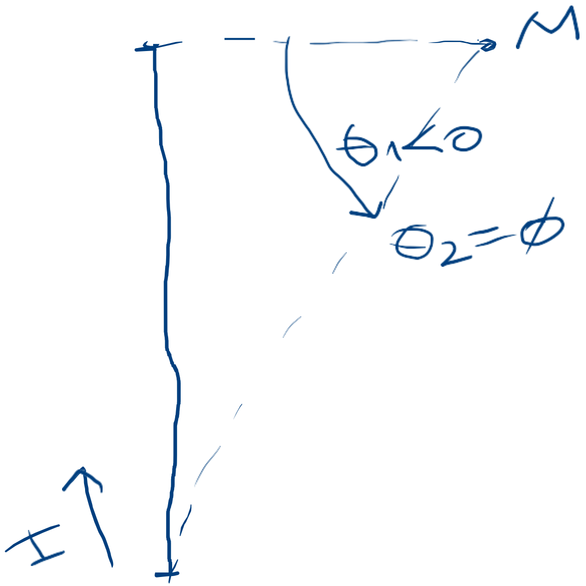
$$\boxed{B = \frac{\mu_0 I}{4\pi d}\left(\sin\theta_2 - \sin\theta_1\right)}$$

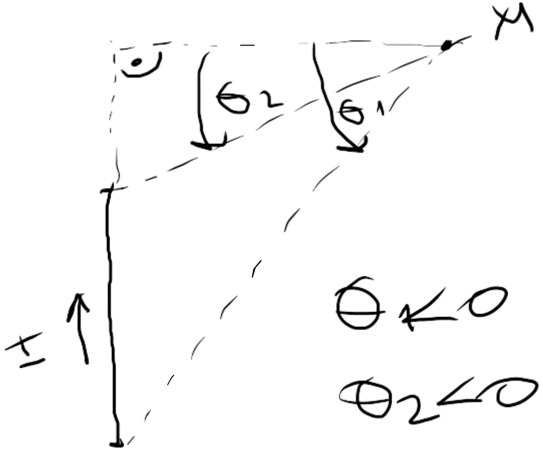
$\vec{B}$ ⊗



$\Theta < 0$ и $\Theta_2 > 0$

[illegible]





$\Theta < 0$

$\Theta_2 < 0$

* $L \to \infty$

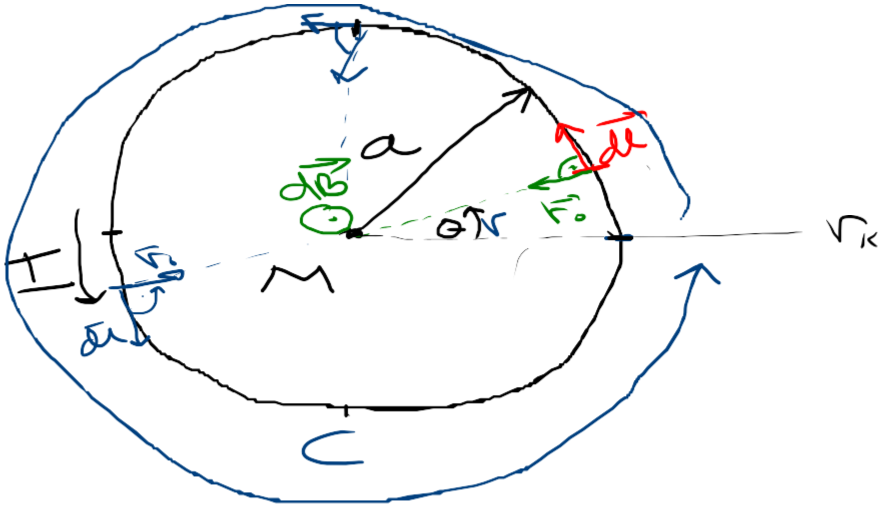
$\theta \simeq \frac{\pi}{2}$

$L \to \infty$

$$\left.\begin{aligned} \theta_1 &\simeq -\frac{\pi}{2} \\ \theta_2 &\simeq \frac{\pi}{2} \end{aligned}\right\}$$

$$B = \frac{\mu_0 I}{2\pi d}$$

ПРИМЈЕР: Магнетска индукција у центру кружне контуре



dl иде дуж $2\pi a$

$$d\vec{B} = \frac{\mu_0 I \, d\vec{l} \times \vec{r}_0}{4\pi r^2}$$

$d\vec{l}$ тангенцијално на кружницу

$$dB = \frac{\mu_0 I \, dl}{4\pi a^2} \sin(\underbrace{\cancel{\measuredangle d\vec{l}, \vec{r}_0}}_{\frac{\pi}{2}}) \rightarrow 1$$

$$dB = \frac{\mu_0 I \, dl}{4\pi a^2}$$

$$B = \int dB \qquad \vec{B} \odot$$

$$B = \oint_C \frac{\mu_0 I \, dl}{4\pi a^2} = \frac{\mu_0 I}{4\pi a^2} 2\pi a$$

$$\boxed{B = \frac{\mu_0 I}{2a}}$$

$$B = \frac{\mu_0 I}{4\pi a^2} \oint_C dl$$

$$dl = a \cdot d\theta$$

$$B = \frac{\mu_0 I}{4\pi a^2} \int_0^{2\pi} a \, d\theta$$

$$B = \frac{\mu_0 I}{4\pi a} 2\pi = \frac{\mu_0 I}{2a}$$

Сила и моменат на струјну контуру у маг. пољу

$$d\vec{F}_{12} = \frac{\mu_0}{4\pi}\,\frac{I_2\, d\vec{l}_2 \times (I_1\, d\vec{l}_1 \times \vec{r}_{012})}{r^2}$$

$$d\vec{F}_{12} = I_2\, d\vec{l}_2 \times \left(\frac{\mu_0}{4\pi}\,\frac{I_1\, d\vec{l}_1 \times \vec{r}_{012}}{r^2}\right) = I_2\, d\vec{l}_2 \times \vec{B}_1$$

$$\boxed{d\vec{F}_m = I\, d\vec{l} \times \vec{B}}$$

$$\vec{F}_m = \int d\vec{F}_m = \int I\, d\vec{l} \times \vec{B}$$

дуж контуре

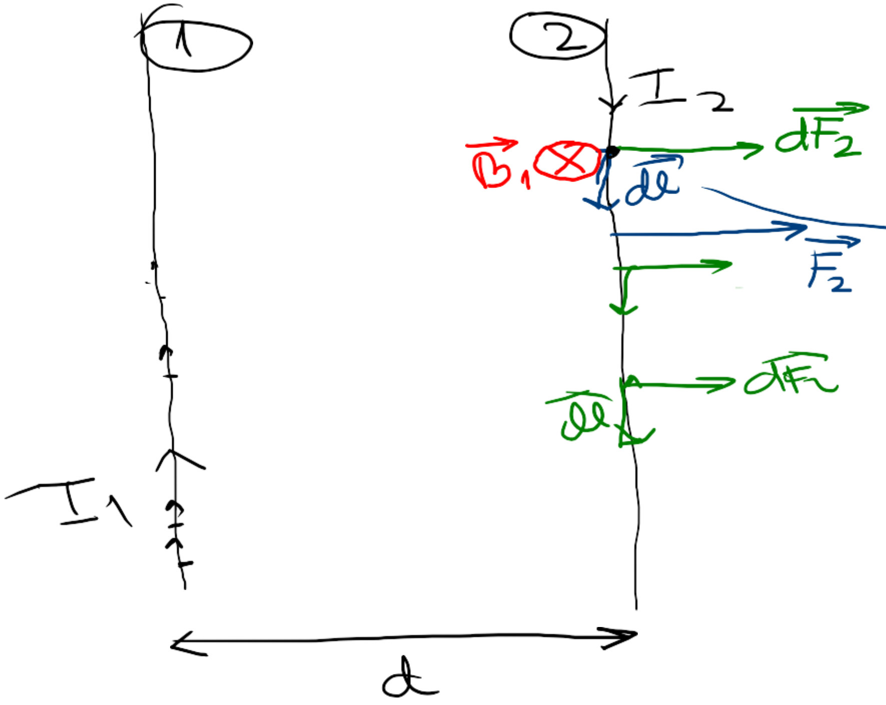
спољашње маг. поље

$$\vec{F}_m = I \int d\vec{l} \times \vec{B}$$

ако се I не мења

$$F_m = \int I\, dl\, B \sin\angle(d\vec{l}, \vec{B})$$

у општем случају не може



$$d\vec{F}_2 = I_2 d\vec{l} \times \vec{B}_1$$

B_1 је од проводника са I_1 у тачки овде ће деловати $I_2 d\vec{l}$.

$$B_1 = \frac{\mu_0 I_1}{2\pi d}$$

$$\vec{F}_2 = \int d\vec{F}_2 \qquad F_2 = \int dF_2$$

$\vec{F}_2$ као да сабиру

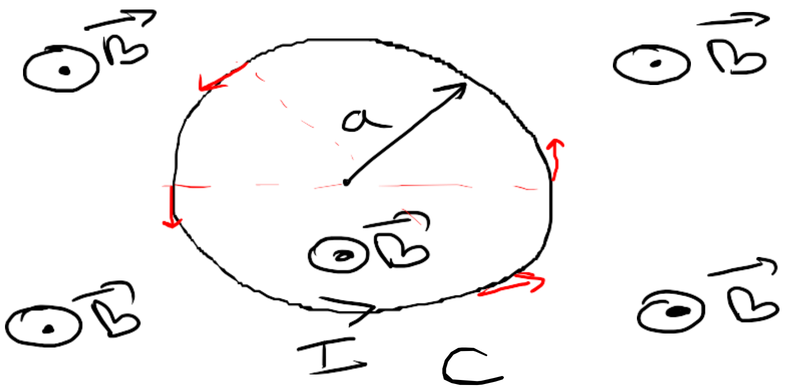
$$F_2 = I_2 \cdot l_2 \cdot B_1$$

дужина проводника ②

$$F_2' = \frac{F_2}{l_2} = \frac{\mu_0 I_1 I_2}{2\pi d} \left[\frac{N}{m}\right]$$

$$d\vec{F}_m = I\,d\vec{l} \times \vec{B}$$

хомогено маг. поле инд. $\vec{B}$

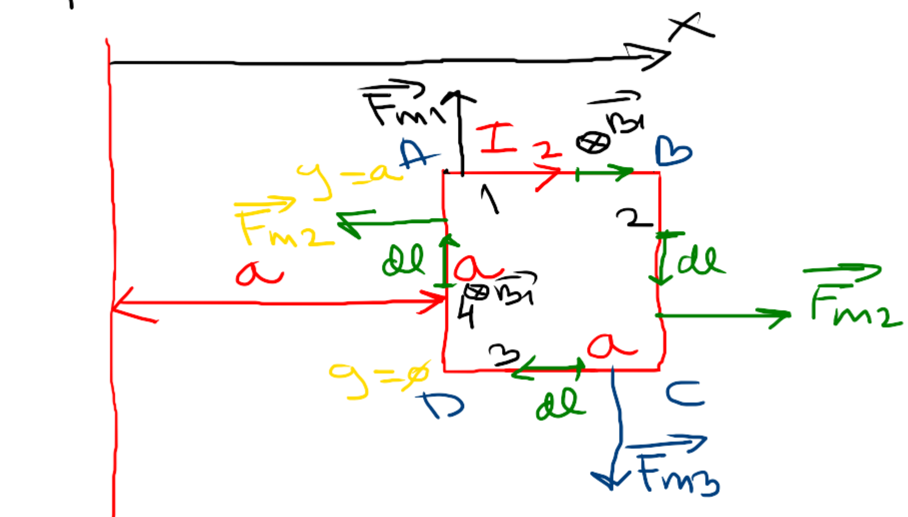


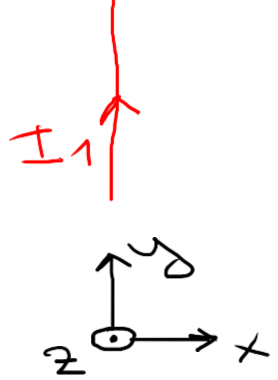
$$\vec{F}_m = \oint_C I\,d\vec{l} \times \vec{B}$$

$$\vec{F}_m = I\left(\oint_C d\vec{l}\right) \times \vec{B} = 0$$

$$\boxed{\vec{F}_m = 0 \text{ у хомогено маг. поле}}$$

Пример:





$$\vec F_{m2} = \oint I_2\, d\vec l \times \vec B_1 = ?$$

1° $\vec F_{m1} = \int_A^B I_2\, d\vec l \times \vec B_1$

$$\vec B_1 = \frac{\mu_0 I_1}{2\pi x}(-\vec i_z)$$

$$d\vec l = dx \cdot \vec i_x$$

$$\vec F_{m1} = \int_A^B I_2 \cdot dx \cdot B_1 \, (\vec i_y)$$

$$F_{m1} = \int_a^{2a} I_2\, dx \frac{\mu_0 I_1}{2\pi x} = \frac{\mu_0 I_1 I_2}{2\pi} \ln 2$$

$$\vec F_{m1} = \frac{\mu_0 I_1 I_2}{2\pi} \ln 2 \, (\vec i_y)$$

2° $\vec F_{m2} = \int_B^C I_2\, d\vec l \times \vec B_1$

3° $\vec F_{m3} = \int_C^D I_2\, d\vec l \times \vec B_1$

$$\vec F_{m3} = -\frac{\mu_0 I_1 I_2}{2\pi} \ln 2 \, (\vec i_y)$$

4° $\vec F_{m4} = \int_D^A I_2\, d\vec l \times \vec B_1$

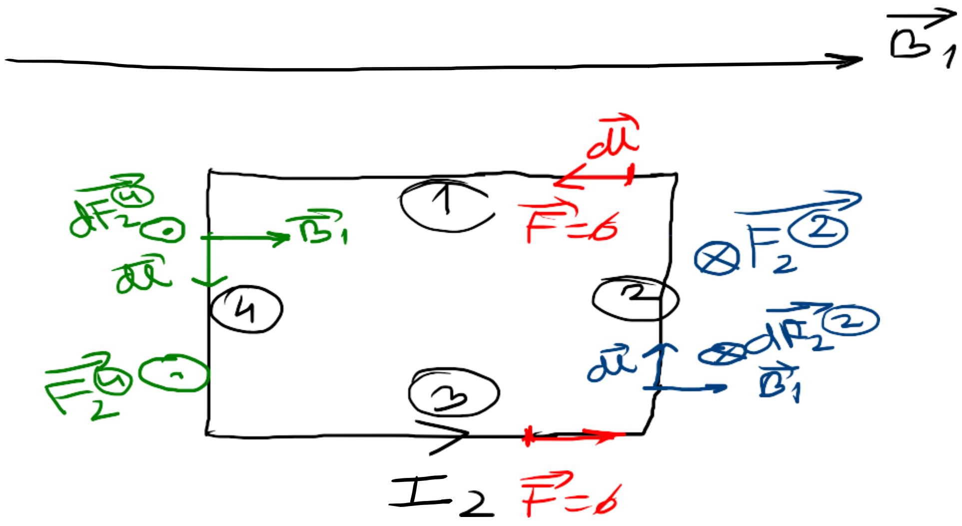
$$B_1 = \frac{\mu_0 I_1}{2\pi a}$$

$$\vec F_{m4} = \int_0^a I_2\, dy (\vec i_y) \times \frac{\mu_0 I_1}{2\pi a}(-\vec i_z)$$

$$\vec F_{m4} = \frac{\mu_0 I_1 I_2}{2\pi \cancel{a}}(-\vec i_x) \int_0^a dy \quad \cancel{a}$$

$$\vec F_{m4} = -\frac{\mu_0 I_1 I_2}{2\pi} \vec i_x$$

$$\vec F_{m2} = \frac{\mu_0 I_1 I_2}{2\pi} \vec i_x$$



$$\vec{F}_2 = \oint_{C_2} I_2 \, d\vec{l} \times \vec{B}_1$$

1

$$d\vec{F}_2^{(1)} = I_2 \, d\vec{l} \times \vec{B}_1$$

$$\angle(\vec{dl}, \vec{B}_1) = \emptyset$$

$$\vec{F}_2^{(1)} = \emptyset$$

3

$$\angle(\vec{dl}, \vec{B}_1) = \emptyset \qquad \vec{F}_2^{(3)} = \emptyset$$

2

$$d\vec{F}_2^{(2)} = I_2 \, d\vec{l} \times \vec{B}_1$$

4

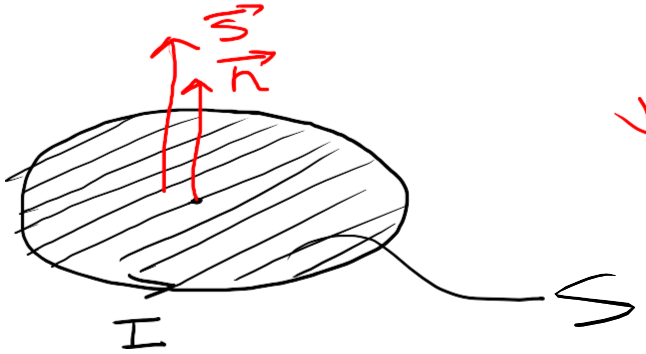
$$d\vec{F}_2^{(4)} = I_2 \, d\vec{l} \times \vec{B}_1$$

Удобно сдвоят сила: $\vec{F}_2^{(2)}$ и $\vec{F}_2^{(4)}$ стремят да завъртят контура

$$\vec{M} = \vec{m} \times \vec{B}$$

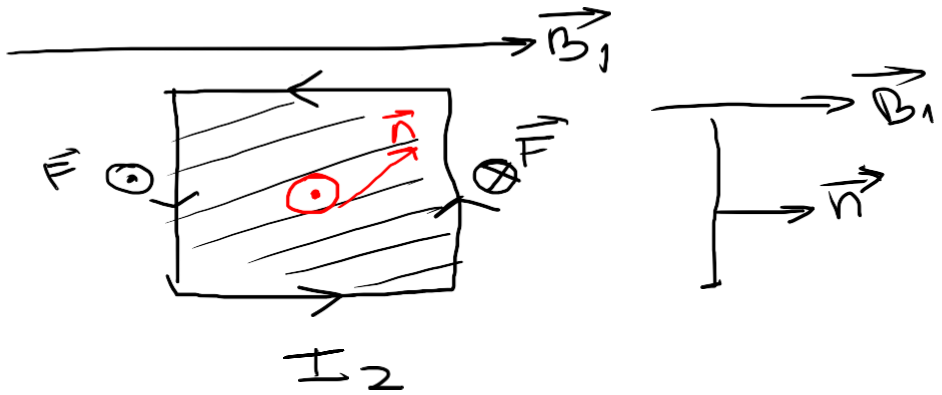
маг. момент контура $\vec{m} = I \cdot \vec{S}$

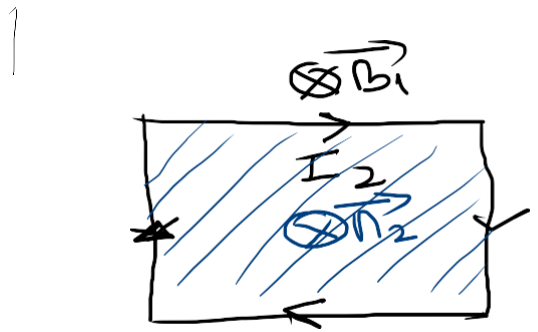
$$\vec{m} = I\vec{S}$$



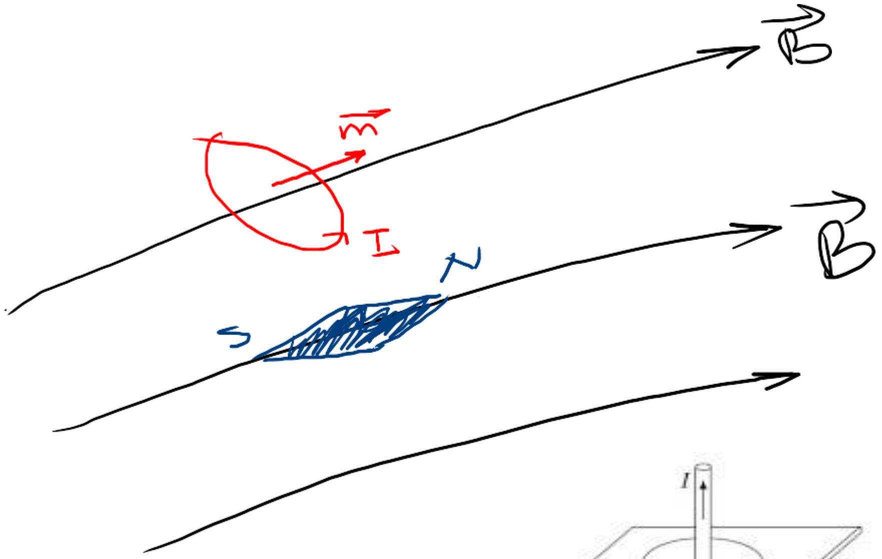
Маг. момент контуре

$\vec{n}$ – нормала на површину основицу на контуру. Одређено је правилом десне руке: прстима обухватимо струју I а палац показује $\vec{n}$ тј. $\vec{S}$.

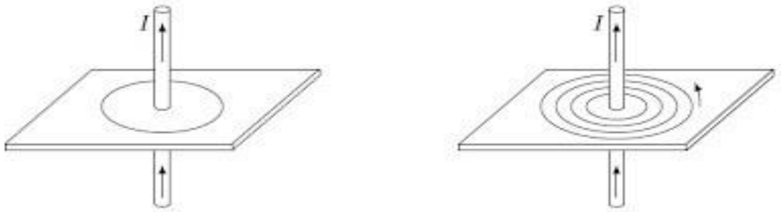

$\vec{B}_1$
$\vec{F}$
$\vec{n}$
$\vec{F}$
I_2
$\vec{B}_1$
$\vec{n}$

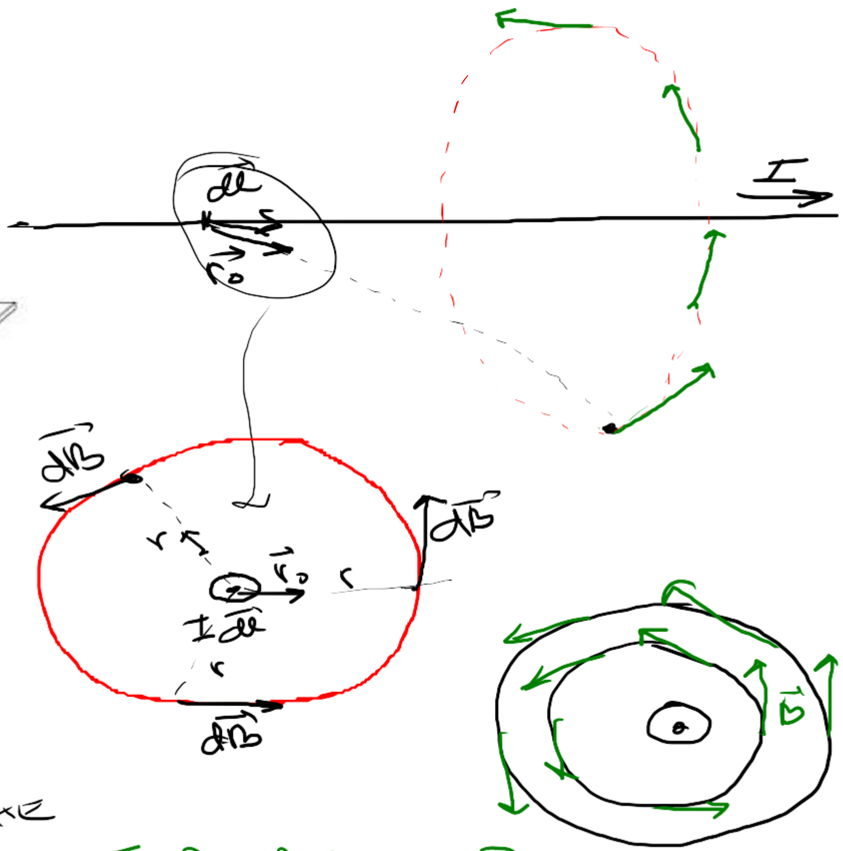

$\vec{B}_1$
I_2
$\vec{n}_2$

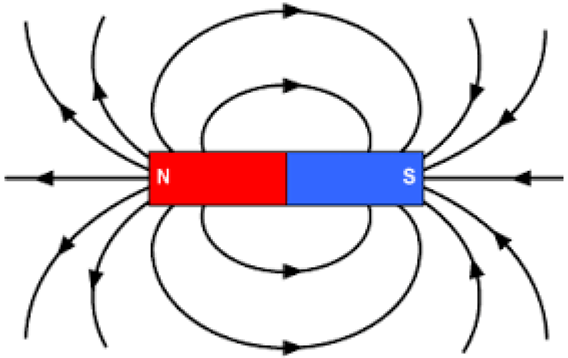
Линије вектора маг. индукције

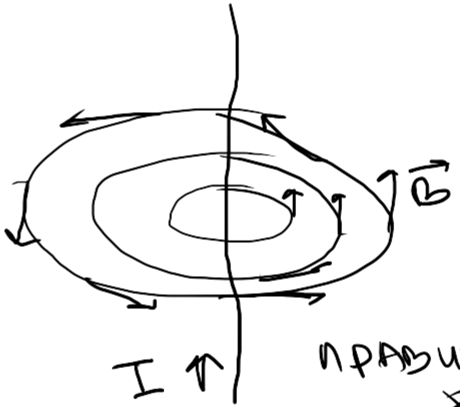


$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{l} \times \vec{r_0}}{r^2}$$



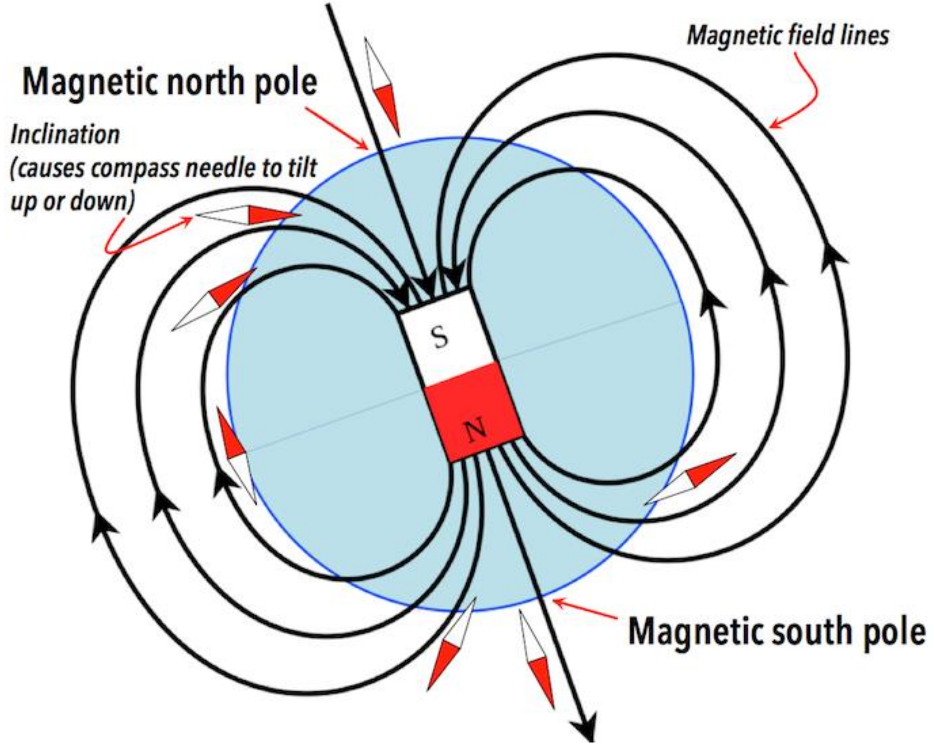


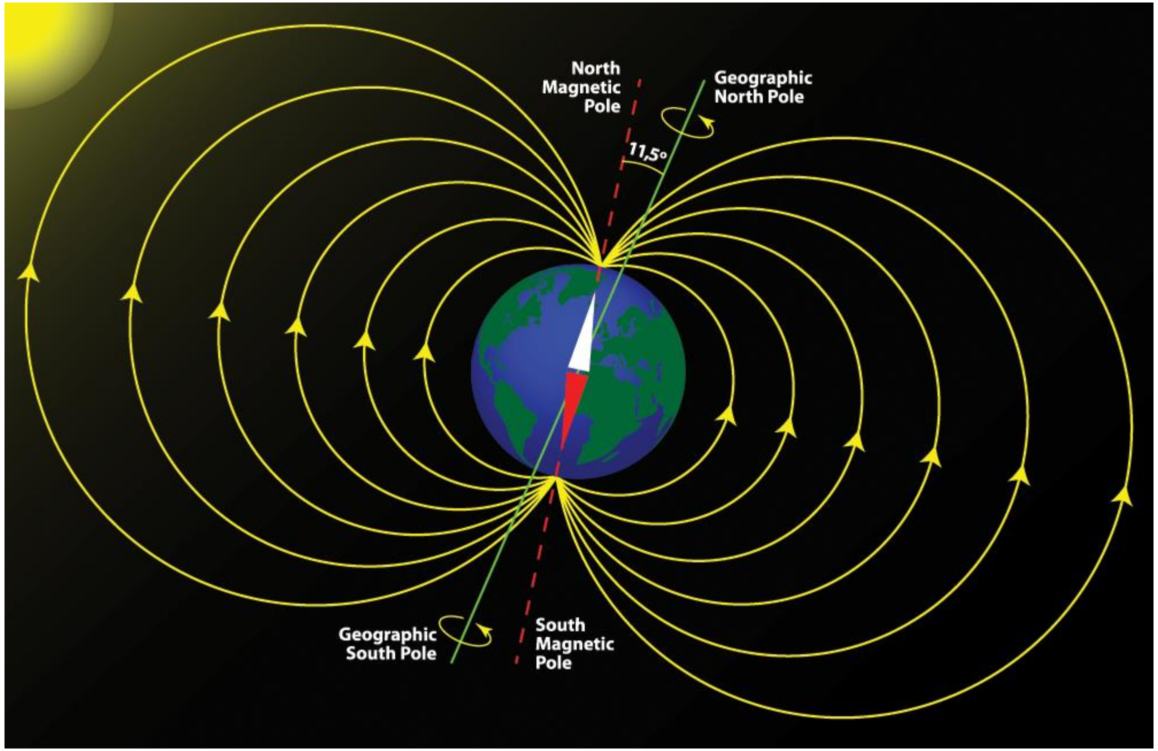


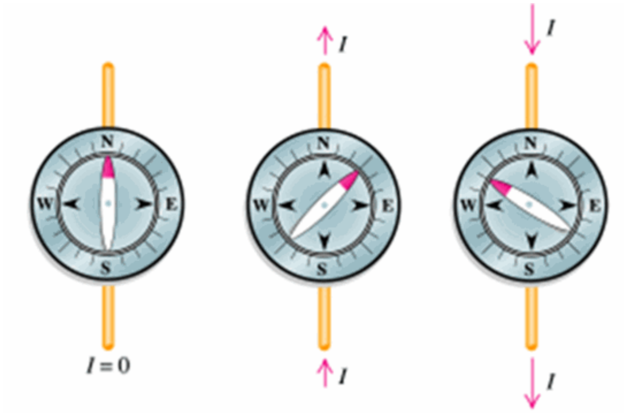


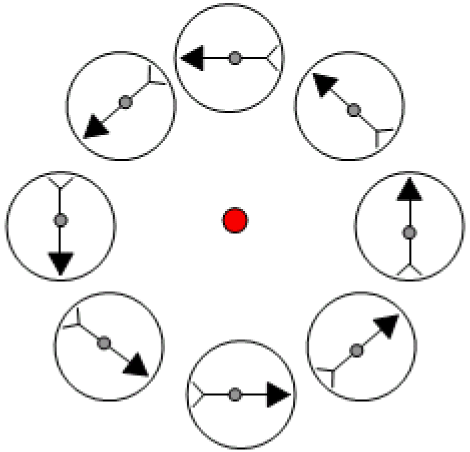
Правило десне руке

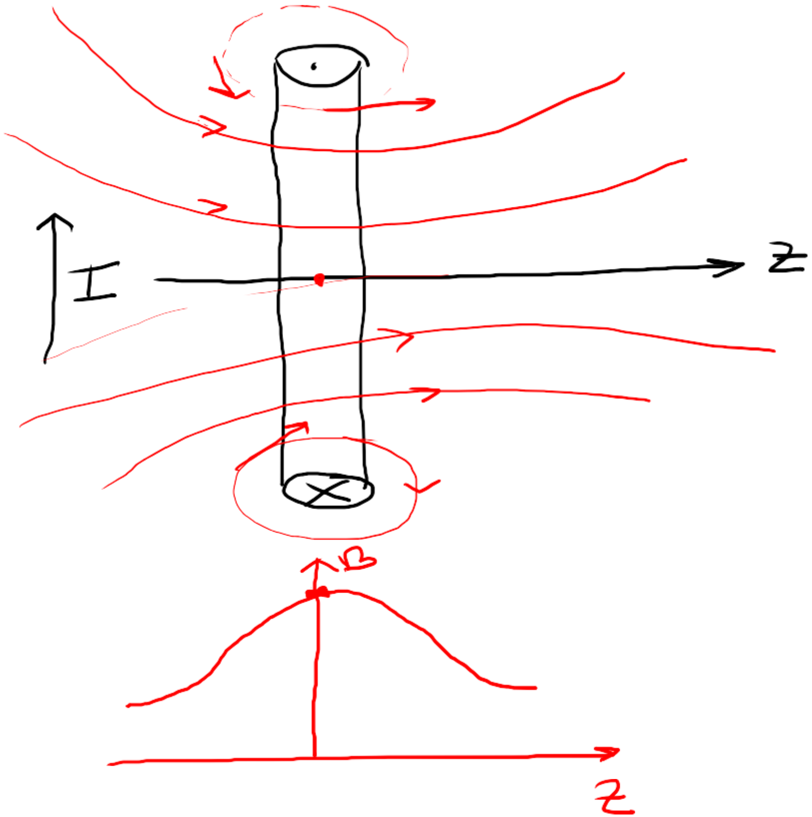
Безизворно

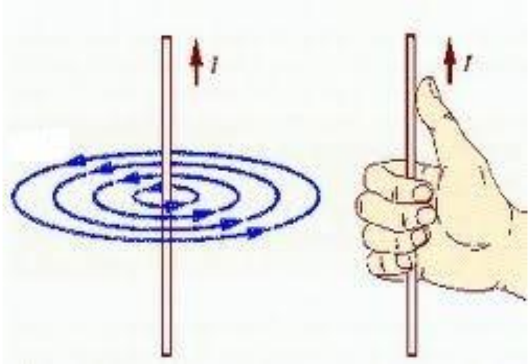

Magnetic field lines
Magnetic north pole
Inclination
(causes compass needle to tilt
up or down)
S
N
Magnetic south pole

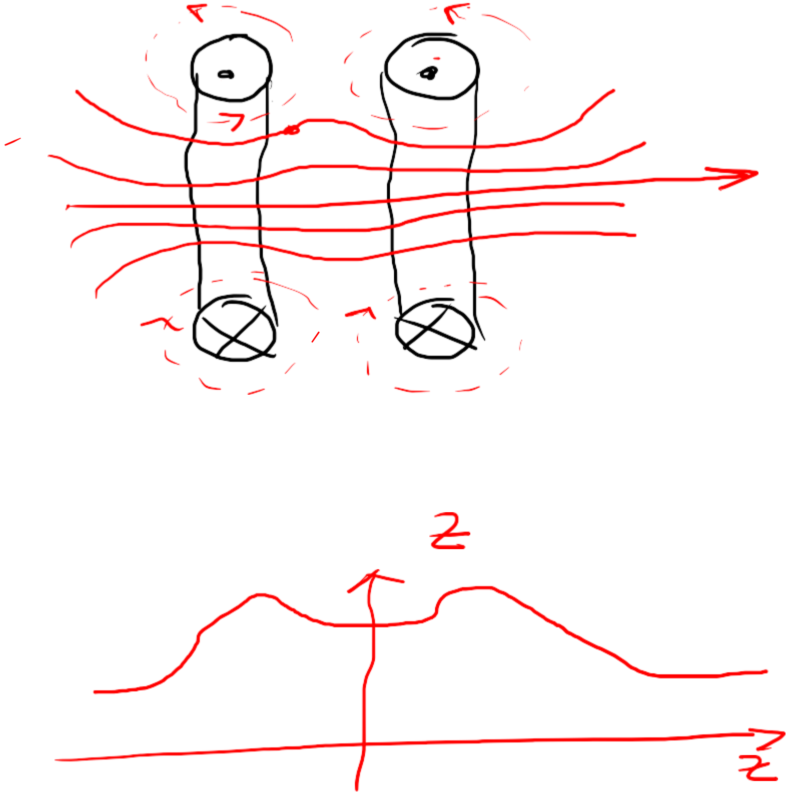

North
Magnetic
Pole
Geographic
North Pole
11,5°
Geographic
South Pole
South
Magnetic
Pole

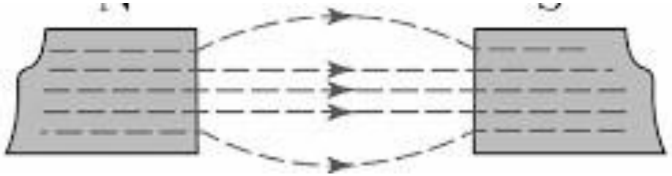

I = 0
I
I
I
I
N
S
W
E

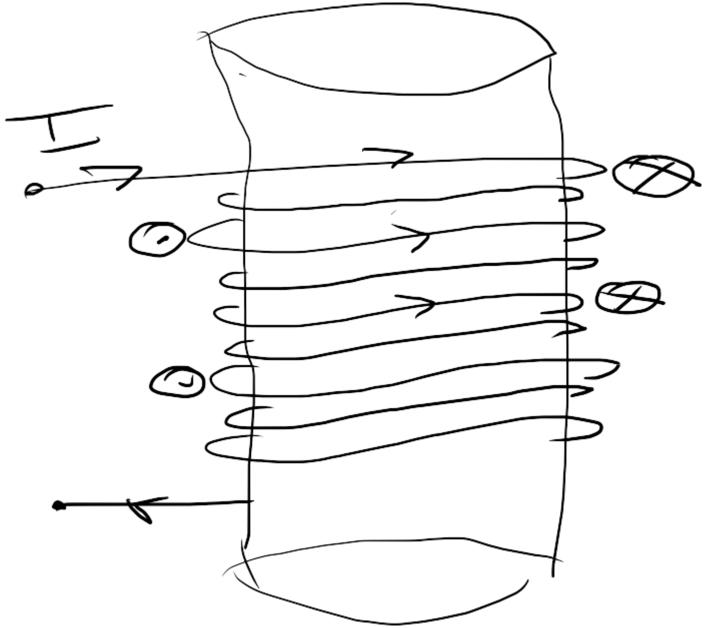


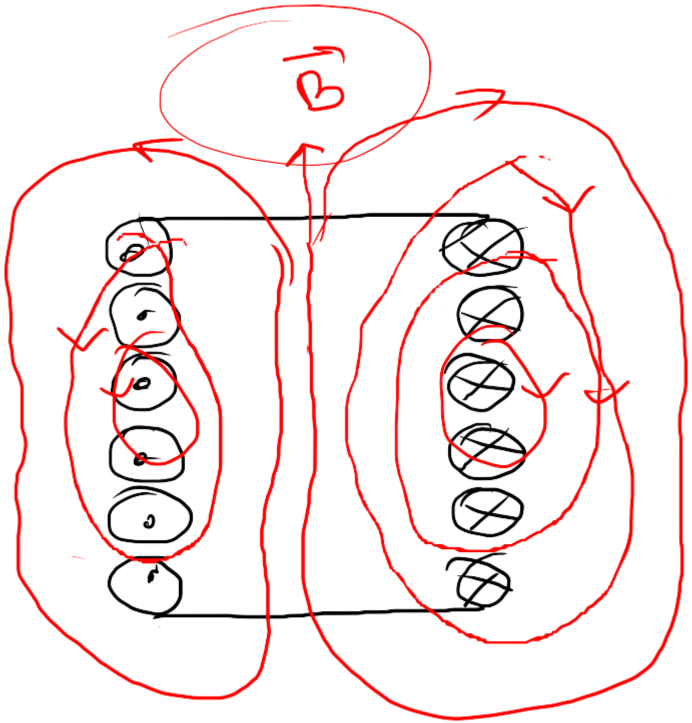

I
z
B
z



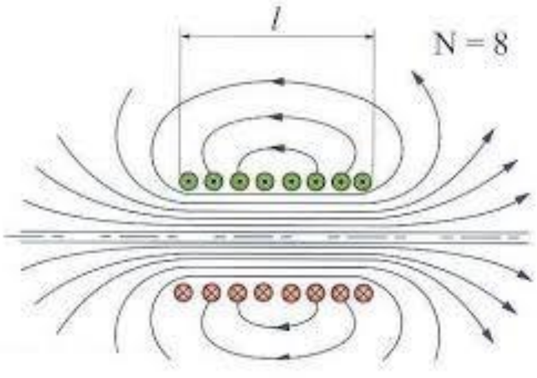

z
z







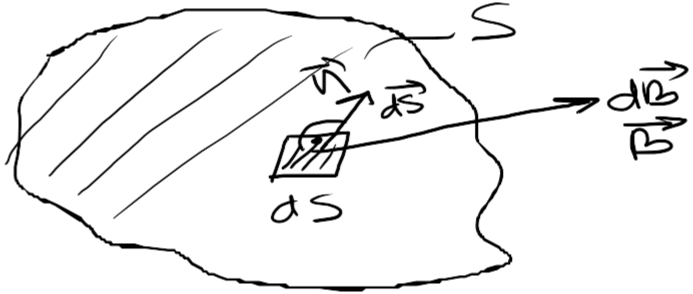
СОЛЕНОИД

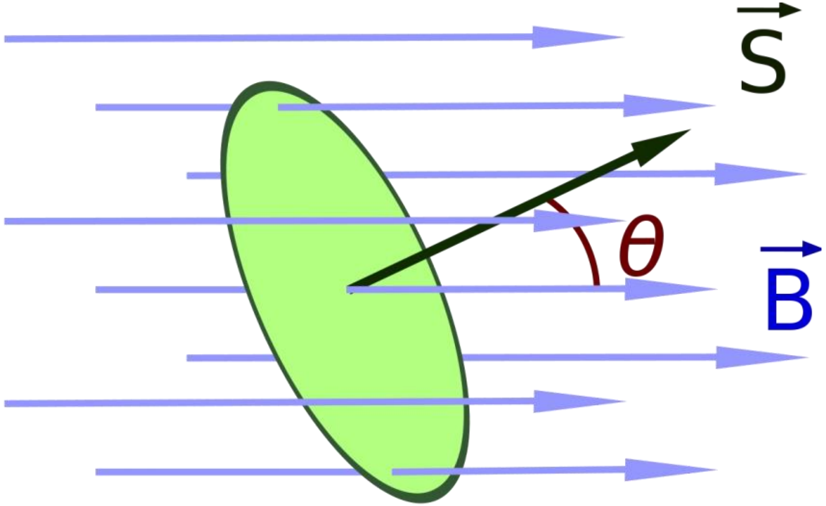


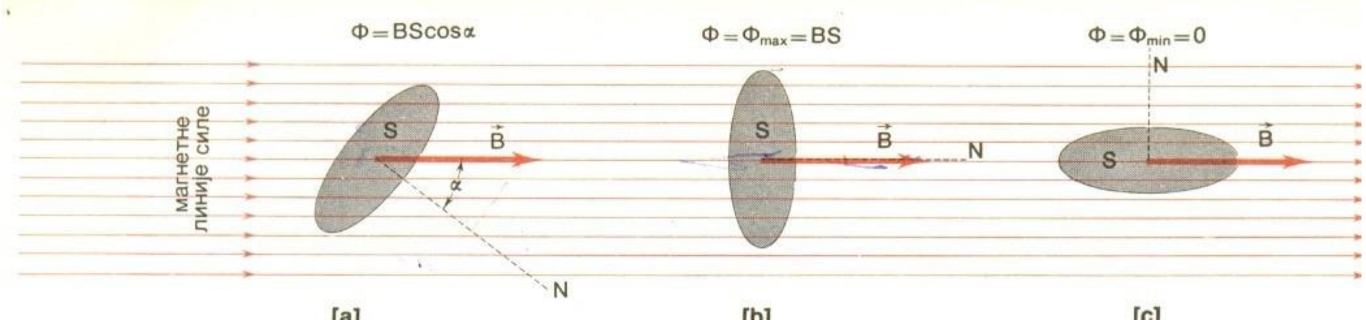
Magnetska indukcija zavojnice (u presjeku)

Флукс вектора маг. индукције

$$\phi = \int_S \vec{B} \cdot d\vec{S} \quad [Tm^2] \quad [Wb]$$







ЗАКОН ОДРЖАЊА МАГ. ФЛУКСА

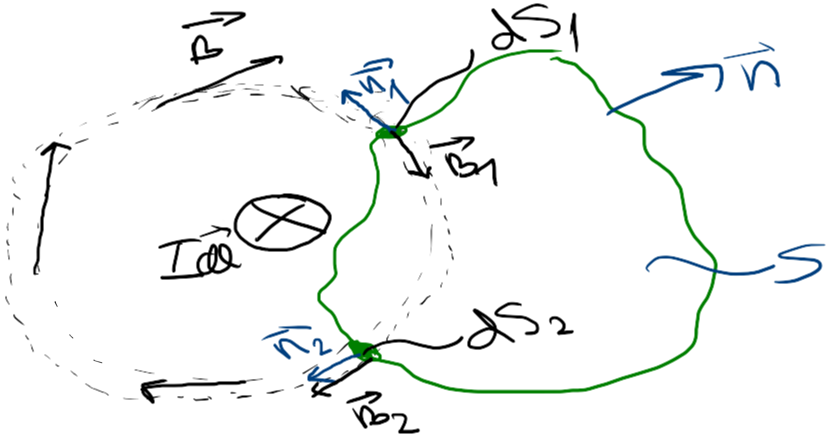
$$\oint_S \vec{B}\,d\vec{S} = 0$$

Закон одржања (конзервације) маг. флукса

$$\vec{B} = \int d\vec{B}$$

од појединих струјних елемената

$$\Phi = \int d\Phi$$

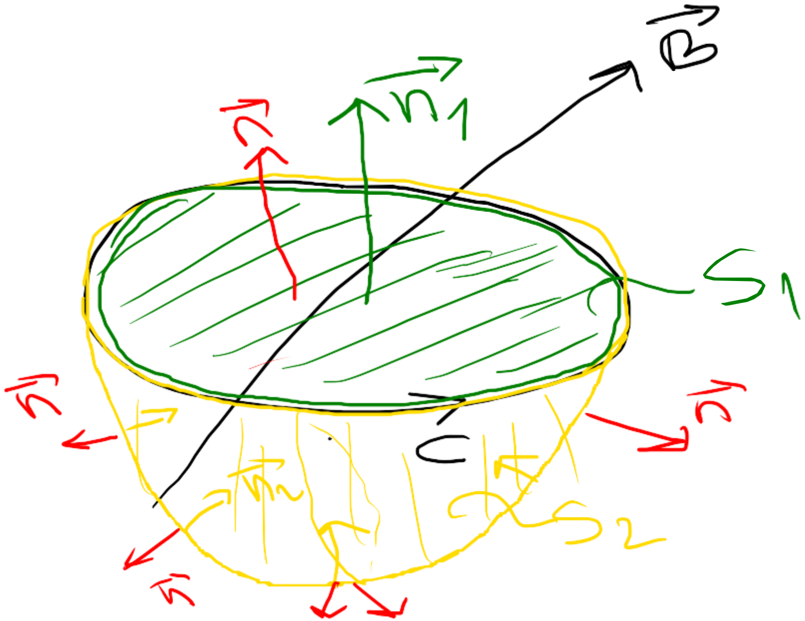


$$|\vec{B}_1| = |\vec{B}_2|$$

$$\vec{B}_2 d\vec{S}_2 = -\vec{B}_1 \cdot d\vec{S}_1$$

$$\sum \Phi = 0$$

Флукс кроз контуру – флукс кроз површ ослоњену на контуру



$$\Phi = \int_{S \text{ на } C} \vec{B}\,d\vec{S}$$

$$\Phi_1 = \int_{S_1 \text{ на } C} \vec{B}\,d\vec{S} \quad \vec{n}_1$$

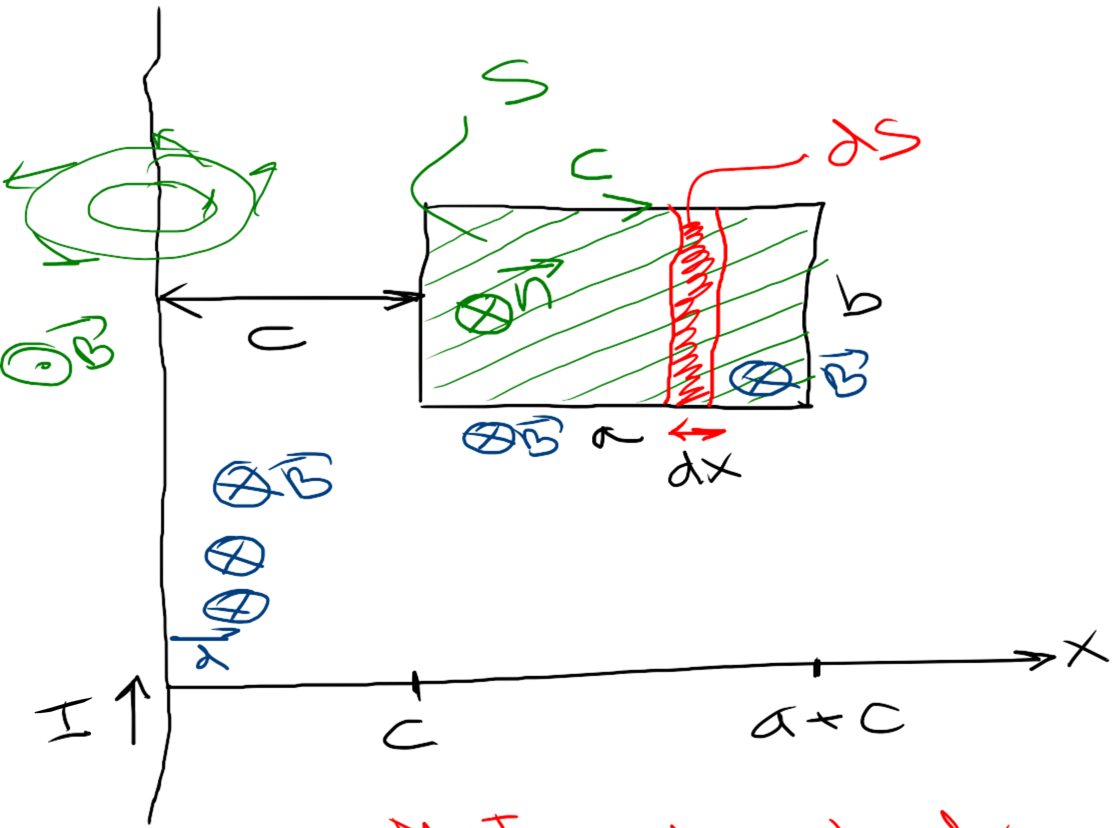
$$\Phi_2 = \int_{S_2 \text{ на } C} \vec{B}\,d\vec{S} \quad \vec{n}_2$$

$\oint_S \vec{B}\,d\vec{S} = 0$ нормале у поље ако је затворена површ

$S_1 + S_2 =$ затворена површ  $= S$

$$\oint_S \vec{B}\,d\vec{S} = \int_{S_1} \vec{B}\,d\vec{S}\,\Big|_{\vec{n} = \vec{n}_1} + \int \vec{B}\,d\vec{S}\,\Big|_{\vec{n} = -\vec{n}_2} = \underbrace{\int \vec{B}\,d\vec{S}}_{\Phi_1} + \underbrace{\int \vec{B}\,d\vec{S}}_{-\Phi_2} = 0 \Rightarrow \boxed{\Phi_1 = \Phi_2}$$

Пример:



$$\Phi = \int_{S_{наC}} \vec{B}\,d\vec{S} = \int_{S_{наC}} B\,dS\cos\angle(\vec{B}, d\vec{S})$$

B морамо одредити само од струје у проводнику

dS бирамо произвољно али да би имао усмереност тачно

$$d\vec{S} = dS\cdot\vec{n}$$

$\vec{n}$ правило десне руке у односу на оријентацију контуре C

$$B = \frac{\mu_0 I}{2\pi d}$$

d = const ⇒ има исти интензитет.

$$B = B(x) = \frac{\mu_0 I}{2\pi x} \qquad dS = b\cdot dx$$

$$\Phi = \int B\cdot b\cdot dx\cdot \cancelto{1}{\cos 0^\circ} = \int_c^{a+c} \frac{\mu_0 I}{2\pi x}\, b\,dx = \frac{\mu_0 I b}{2\pi}\ln\frac{a+c}{c}\left[\frac{Wb}{Tm^2}\right]$$

Промјеном оријентације с контуре C, мијења се и знак флукса.