Inferencia Pregrado (Graduación)

$X \sim N(\mu, \sigma^2)$, $X_1, X_2, \ldots, X_n \overset{iid}{\sim} N(\mu, \sigma^2)$

$\boxed{\Theta = (\mu, \sigma^2)}$ $f_{\mu,\sigma}(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$, $x \in \mathbb{R}$.

$\hat{\Theta} = (\hat{\mu}, \hat{\sigma}^2)$, $\hat{\mu} = \hat{\mu}(X_1, \ldots, X_n)$

$\hat{\Theta} : \Lambda \longrightarrow \Theta$.

$E(\hat{\theta}) = \Theta$ $n \to \infty$

$E(\hat{\Theta}_n) \longrightarrow ¿?$

Teoría Estadística

$\psi_1 : (x, f_{\mu,\sigma}) \longrightarrow P_{\mu_0 \sigma_0^2} \rightarrow$ Paramétrico Clásico

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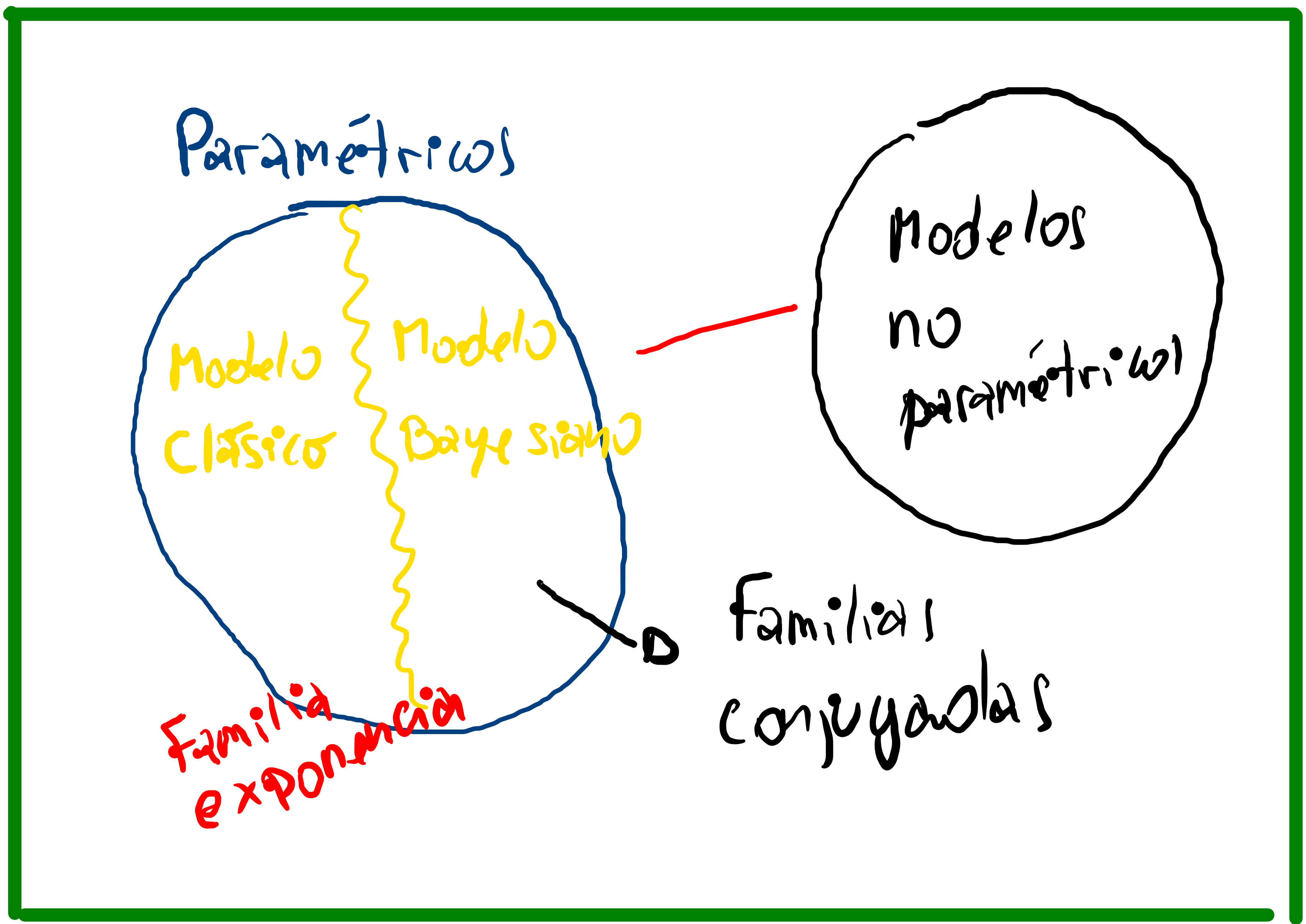
Modelo $\rightarrow$ • Bayesiano

— $\rightarrow$ Modelo No paramétrico

Modelos



Paramétricos
Modelo Clásico
Modelo Bayesiano
Familia exponencial
Familias conjugadas
Modelos no paramétricos

Modelo de Poisson: "$X \sim \text{Poisson}(\theta)$"

$$f_\theta(x) = \frac{e^{-\theta}\theta^{x}}{x!}, \quad x \in \mathbb{N}, \ \theta > 0.$$

$$P_\theta : 2^{\mathbb{N}} \to [0,1]$$

$$P_\theta(A) = \sum_{k \in A} f_\theta(k).$$

• Modelo Normal

$$f_\theta(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad x \in \mathbb{R}, \ \mu \in \mathbb{R}, \ \sigma^2 > 0$$

$$P_\theta : \mathcal{B} \to [0,1]$$

$$P_\theta(A) = \int_A f_\theta(x)\, dx .$$

Modelo de Probabilidad Normal

$$\left(\mathbb{R}, \mathcal{B}, P^{N}_{\theta_0}\right), \quad \begin{matrix} \mu_0 = 10 \\ \sigma_0 = 1 \end{matrix}, \quad \theta_0 = (\mu_0, \sigma_0)$$

→ Incertidumbre: $X = x$

Modelo de Prob. Poisson

$$\left(\mathbb{N}, 2^{\mathbb{N}}, P^{P}_{\theta_0}\right), \quad \theta_0 = 2{,}5.$$

$X = x$!!

Modelo ESTADISTICO Normal

$$\left(\mathbb{R}, \mathcal{B}, \{P_\theta^N : \theta=(\mu,\sigma^2)\in \mathbb{R}\times\mathbb{R}_+\}\right)$$

Incertidumbre: $P_\theta^N = ¿?$

Modelo estadístico de Poisson

$$\left(\mathbb{N}, 2^{\mathbb{N}}, \{P_\theta^P : \theta\in \mathbb{R}_+\}\right)$$

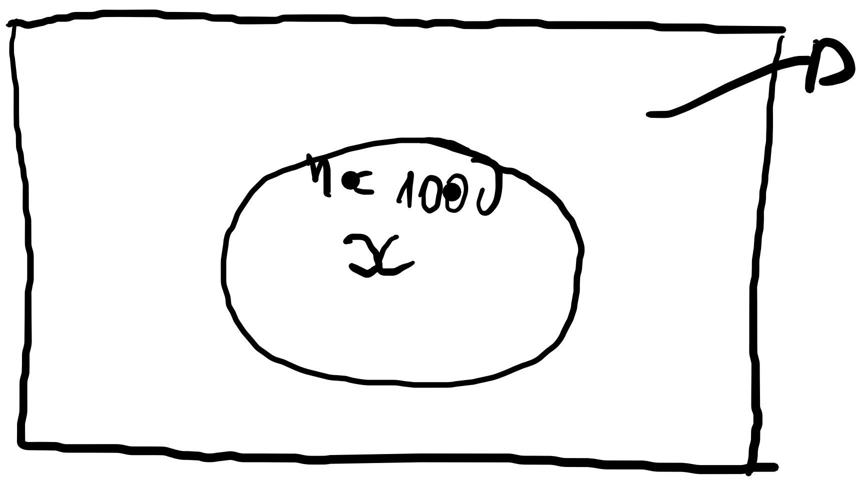
$n = 1000$

A: candidato

B: ______

$X_i = \begin{cases} 1, & A, \\ 0, & B. \end{cases}$

$x = (x_1, x_2, \dots, x_{1000})$



Población

$N = 50.000$

$P^*_\theta \in \mathcal{P} = \{P_\theta : - \}$

X : var

Ω : espacio muestral

$\mathcal{G} = \{A : A \subset \Omega + \text{Prop}\}$

V

Medida de Lebesgue

Medida de Conteo.

Cristalizar

+

Oportudidades

Familia exponencial

$$\frac{dP_\theta}{d\nu} = f_\theta(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}, =: f(x|\theta)$$

$$= \exp \ln f_\theta(x) \qquad \theta = (\mu, \sigma^2)$$

$$\vdots$$

$$= \frac{1}{\sqrt{2\pi}} \exp\left\{ \frac{\mu x}{\sigma^2} - \frac{1}{2\sigma^2} x^2 - \frac{\mu^2}{2\sigma^2} - \ln\sigma \right\}$$

$$= \exp\left\{ \overbrace{\left(\frac{\mu}{\sigma^2}, \frac{1}{2\sigma^2}\right)}^{\eta(\theta)}{}^{T} \overbrace{(x, -x^2)}^{T(x)} - \overbrace{\left[\frac{\mu^2}{2\sigma^2} + \ln\sigma\right]}^{\xi(\theta)} \right\} \overbrace{\frac{1}{\sqrt{2\pi}}}^{e^{h(x)}}$$

$$\frac{dP_\theta}{d\nu} = f_\theta(x) = \frac{e^{-\theta} \cdot \theta^{x}}{x!} \cdot I_N(x)$$

$$= \exp \ln f_\theta(x)$$

$$= \exp\{\underbrace{\ln(\theta)}_{\eta(\theta)}\,\underbrace{x}_{T(x)} - \underbrace{\theta}_{\xi(\theta)}\}\,\overbrace{\frac{1}{x!}\,I_N(x)}^{h(x)}.$$

$$= \exp\{\eta(\theta)T(x) - \xi(\theta)\}\,h(x).$$

Reparametrización

$$\eta(\theta) = \left(\frac{\mu}{\sigma^2}, \frac{1}{2\sigma^2}\right) =$$

$$\underbrace{\frac{\mu}{\sigma^2}}_{\eta_1} \qquad \underbrace{\frac{1}{2\sigma^2}}_{\eta_2}$$

$$\eta = (\eta_1, \eta_2).$$

$N(\mu, \mu)$

$\downarrow$

No es de rango completo

Modelo Bayesiano Bernoulli

Considerar un vector aleatorio (X, Θ), $X \in \Delta$ y $\Theta \in (0,1)$.

Va a existir una función de densidad conjunta $f(x,\theta)$.

$f_\Theta(x) \neq f(x|\theta)$

$$P(A \cap B) = P(A|B)P(B)$$

$$f(x,\theta) = \underbrace{f(x|\Theta=\theta_0)}_{\text{f.d. datos}} \underbrace{\pi(\theta)}_{\text{priori}} \sim Beta(\underbrace{a_0, b_0}_{\text{hiper parametros.}})$$

Paramétrico

$X \sim \text{Gama}(a, b)$

$X|\theta \sim \text{Modelo}(\theta)$

$\theta \sim \pi(\theta)$

$\mu(F)$: $\rightarrow \int_{\Omega} x \, dF(x) < \infty.$

$\longrightarrow \int_{\Omega} x^2 \, dF(x) < \infty$

$\left.\begin{matrix} \\ \\ \end{matrix}\right\}$ Condición sobre $F(x)$

$S(x)$: Estadístico

Modelo no paramétrico

$$\left(\mathcal{X}, \mathcal{G}, \{ P_s : s \in L \} \right)$$

$\downarrow$

θ

$Y(t)$ es una función aleatoria

$$Y(t) = \int_{-\infty}^{t} f(s)\,ds + \sigma W(t).$$

Modelo Gaussiano continuo de ruido blanco.

$$f(s) \in L^2(\mathbb{R}).$$

$$f_\theta(x) = g(x) \cdot \underbrace{h_\theta(x)}$$

depende de θ

¿? como indexador

Modelo proporcional semi de Cox.