Supplementary Material for Chapter 4

1. Comparison of tractable SEM with soft SEM

Table 1: Comparison of the methods in dataset Sachs. For each method, the number of instances (N. inst) and percentage of missing values (Missing %), the BIC score in the training dataset (BIC), the log-likelihood in the test dataset (LL), the imputation accuracy (Acc), the learning time in seconds (Time), the treewidth, the number of changes in the structure (N. Changes), the number of computations of the expected sufficient statistics (N. ESS) and the proportion of local changes performed that improved the expected score (Prop) are shown. The best results are denoted in boldface.

Method	N. inst	Missing (%)	BIC	LL	Acc	Time	Treewidth	N. Changes	N. ESS	Prop
TSEM	500	30	-3007.81	-4098.35	**0.72**	**0.05**	1	8	20	0.4
TSEM-poly	500	30	-3007.81	-4098.35	**0.72**	0.05	1	8	20	0.4
soft SEM	500	30	**-3007.55**	**-4096.14**	**0.72**	0.27	1	8	216	1
TSEM	500	50	-2363.76	-4892.04	**0.63**	0.03	0	0	10	-
TSEM-poly	500	50	-2363.76	-4892.04	**0.63**	**0.02**	0	0	10	-
soft SEM	500	50	**-2360.18**	**-4871.09**	**0.63**	0.05	0	0	113	-
TSEM	500	70	-1436.34	-5098.36	**0.62**	0.03	0	0	2	-
TSEM-poly	500	70	-1436.34	-5098.36	**0.62**	**0.02**	0	0	2	-
soft SEM	500	70	**-1404.96**	**-4920.89**	**0.62**	0.05	0	0	114	-
TSEM	2000	30	**-11389.9**	-14911.37	0.71	0.13	2	11	39	0.26
TSEM-poly	2000	30	**-11389.9**	-14911.37	0.71	**0.13**	2	11	35	0.29
soft SEM	2000	30	-11409.57	**-14903.03**	**0.72**	0.44	2	12	277	1
TSEM	2000	50	**-8545.22**	**-15258.25**	**0.69**	0.12	1	9	26	0.31
TSEM-poly	2000	50	**-8545.22**	**-15258.25**	**0.69**	**0.11**	1	9	22	0.36
soft SEM	2000	50	-8552.54	-15294.14	0.69	0.3	1	8	225	1
TSEM	2000	70	-5691.51	-18437.5	**0.63**	0.11	1	1	28	0.04
TSEM-poly	2000	70	-5691.51	-18437.5	**0.63**	0.1	1	1	26	0.04
soft SEM	2000	70	**-5668.78**	**-18345.37**	**0.63**	**0.08**	1	1	126	1
TSEM	5000	30	**-27510.51**	**-36511.37**	**0.74**	0.33	2	15	38	0.34
TSEM-poly	5000	30	**-27510.51**	**-36511.37**	**0.74**	**0.32**	2	15	36	0.36
soft SEM	5000	30	-27971.87	-37463.5	0.73	0.62	2	15	304	1
TSEM	5000	50	**-21018.92**	**-38094.89**	**0.7**	0.35	2	10	49	0.16
TSEM-poly	5000	50	**-21018.92**	**-38094.89**	**0.7**	**0.35**	2	10	45	0.18
soft SEM	5000	50	-21063.53	-38407.09	0.7	0.42	2	9	242	1
TSEM	5000	70	**-13508.83**	**-41054.11**	**0.67**	0.42	1	7	23	0.09
TSEM-poly	5000	70	**-13508.83**	**-41054.11**	**0.67**	0.27	1	7	23	0.09
soft SEM	5000	70	-13916.89	-44615.15	0.64	**0.16**	1	2	142	1

Table 2: Comparison of the methods in dataset Child. For each method, the number of instances (N. inst) and percentage of missing values (Missing %), the BIC score in the training dataset (BIC), the log-likelihood in the test dataset (LL), the imputation accuracy (Acc), the learning time in seconds (Time), the treewidth, the number of changes in the structure (N. Changes), the number of computations of the expected sufficient statistics (N. ESS) and the proportion of local changes performed that improved the expected score (Prop) are shown. The best results are denoted in boldface.

Method	N. inst	Missing (%)	BIC	LL	Acc	Time	Treewidth	N. Changes	N. ESS	Prop
TSEM	500	30	**-5176.96**	**-6608.17**	0.78	0.14	1	19	45	0.36
TSEM-poly	500	30	-5186.33	-6638.15	**0.78**	**0.13**	2	19	44	0.36
soft SEM	500	30	-5318.98	-6974.27	0.76	1.97	1	13	662	1
TSEM	500	50	**-4134.34**	**-7749.98**	**0.7**	0.13	1	11	44	0.11
TSEM-poly	500	50	**-4134.34**	**-7749.98**	**0.7**	**0.12**	1	11	38	0.13
soft SEM	500	50	-4299.5	-8235.01	0.65	0.75	1	4	465	1
TSEM	500	70	-2672.06	**-8544.87**	**0.62**	0.07	1	2	8	0.12
TSEM-poly	500	70	-2672.06	**-8544.87**	**0.62**	**0.06**	1	2	8	0.12
soft SEM	500	70	**-2667.54**	-8559.02	0.62	0.33	1	1	405	1
TSEM	2000	30	**-19728.69**	-25225.49	0.78	0.38	2	24	59	0.37
TSEM-poly	2000	30	-19816.7	**-25220.92**	**0.78**	**0.38**	2	23	53	0.4
soft SEM	2000	30	-19885.32	-25361.82	0.77	3.16	2	21	855	1
TSEM	2000	50	**-14998.59**	**-25956.1**	**0.73**	0.3	1	19	47	0.36
TSEM-poly	2000	50	-15012.73	-25969.07	0.73	**0.28**	2	19	45	0.38
soft SEM	2000	50	-15347.69	-26798.49	0.72	2.05	1	13	666	1
TSEM	2000	70	-10481.08	-33446.57	0.62	0.19	1	3	28	0.07
TSEM-poly	2000	70	-10481.08	-33446.57	0.62	**0.18**	1	3	28	0.07
soft SEM	2000	70	**-10427.59**	**-32919.87**	**0.62**	0.64	1	3	451	1
TSEM	5000	30	**-47316.55**	**-61464.47**	**0.78**	1.05	3	25	64	0.38
TSEM-poly	5000	30	**-47316.55**	**-61464.47**	**0.78**	**0.98**	3	25	64	0.38
soft SEM	5000	30	-47994.97	-62614.5	0.77	4.57	2	26	996	1
TSEM	5000	50	**-36177.66**	**-62447.51**	**0.74**	**1.07**	2	21	66	0.27
TSEM-poly	5000	50	**-36177.66**	**-62447.51**	**0.74**	1.07	2	21	60	0.3
soft SEM	5000	50	-37333.55	-65253.46	0.73	3.23	2	18	783	1
TSEM	5000	70	**-24376.47**	**-71189.98**	**0.67**	1.48	1	16	80	0.05
TSEM-poly	5000	70	-24465.27	-71269.95	0.67	**0.93**	1	14	64	0.06
soft SEM	5000	70	-25563	-80055.53	0.63	1.25	1	6	521	1

Table 3: Comparison of the methods in dataset Insurance. For each method, the number of instances (N. inst) and percentage of missing values (Missing %), the BIC score in the training dataset (BIC), the log-likelihood in the test dataset (LL), the imputation accuracy (Acc), the learning time in seconds (Time), the treewidth, the number of changes in the structure (N. Changes), the number of computations of the expected sufficient statistics (N. ESS) and the proportion of local changes performed that improved the expected score (Prop) are shown. The best results are denoted in boldface.

Method	N. inst	Missing (%)	BIC	LL	Acc	Time	Treewidth	N. Changes	N. ESS	Prop
TSEM	500	30	-6038.33	**-7767.24**	0.79	0.22	1	23	57	0.37
TSEM-poly	500	30	-6038.33	**-7767.24**	0.79	**0.21**	1	23	55	0.38
soft SEM	500	30	**-6019.74**	-7789.55	**0.79**	7.18	2	22	1372	1
TSEM	500	50	**-4696.51**	**-8250.78**	**0.74**	0.19	1	18	48	0.21
TSEM-poly	500	50	**-4696.51**	**-8250.78**	**0.74**	**0.18**	1	18	46	0.22
soft SEM	500	50	-5150.63	-9708.79	0.68	2.43	1	7	930	1
TSEM	500	70	**-3378.26**	**-10749.63**	**0.65**	0.11	1	1	17	0
TSEM-poly	500	70	**-3378.26**	**-10749.63**	**0.65**	**0.09**	1	1	17	0
soft SEM	500	70	-3380.46	-10846.73	0.65	0.47	0	0	707	-
TSEM	2000	30	**-22716.91**	**-27919.38**	**0.81**	**0.84**	4	32	88	0.31
TSEM-poly	2000	30	**-22716.91**	**-27919.38**	**0.81**	0.84	4	32	88	0.31
soft SEM	2000	30	-23045.4	-28911.77	0.8	9.22	2	27	1506	1
TSEM	2000	50	**-17674.59**	**-28661.37**	**0.78**	0.64	2	27	69	0.3
TSEM-poly	2000	50	-17722.78	-28844.98	0.77	**0.63**	3	26	66	0.32
soft SEM	2000	50	-18124.95	-30456.04	0.76	7.12	2	21	1313	1
TSEM	2000	70	**-12892.57**	**-39886.61**	**0.66**	**0.34**	1	5	71	0.04
TSEM-poly	2000	70	**-12892.57**	**-39886.61**	**0.66**	0.34	1	5	71	0.04
soft SEM	2000	70	-12896.73	-40159.54	0.65	1.43	1	3	796	1
TSEM	5000	30	**-53680.54**	**-66986.16**	**0.82**	4.59	4	40	117	0.32
TSEM-poly	5000	30	-53982.36	-67991.37	0.82	**3.69**	4	37	103	0.34
soft SEM	5000	30	-55230.01	-69839.39	0.81	16.66	4	39	1857	1
TSEM	5000	50	**-42353.8**	**-70780.07**	**0.78**	1.91	3	30	99	0.26
TSEM-poly	5000	50	**-42353.8**	**-70780.07**	**0.78**	**1.9**	3	30	99	0.26
soft SEM	5000	50	-43089.11	-73471.61	0.77	9.87	2	25	1437	1
TSEM	5000	70	**-28633.3**	**-78700.94**	**0.72**	1.35	2	23	81	0.11
TSEM-poly	5000	70	**-28633.3**	**-78700.94**	**0.72**	**1.33**	2	23	78	0.12
soft SEM	5000	70	-30481.63	-88165	0.69	4.62	1	11	1033	1

Table 4: Comparison of the methods in dataset Water. For each method, the number of instances (N. inst) and percentage of missing values (Missing %), the BIC score in the training dataset (BIC), the log-likelihood in the test dataset (LL), the imputation accuracy (Acc), the learning time in seconds (Time), the treewidth, the number of changes in the structure (N. Changes), the number of computations of the expected sufficient statistics (N. ESS) and the proportion of local changes performed that improved the expected score (Prop) are shown. The best results are denoted in boldface.

Method	N. inst	Missing (%)	BIC	LL	Acc	Time	Treewidth	N. Changes	N. ESS	Prop
TSEM	500	30	**-5231.62**	**-6687.93**	**0.82**	0.21	2	20	56	0.34
TSEM-poly	500	30	-5231.88	-6688.18	**0.82**	**0.16**	2	19	45	0.42
soft SEM	500	30	-5252.33	-6734.24	0.82	9.38	2	19	1732	1
TSEM	500	50	-4100.12	-7617.22	**0.78**	0.13	1	8	35	0.2
TSEM-poly	500	50	-4100.12	-7617.22	**0.78**	**0.12**	1	8	33	0.21
soft SEM	500	50	**-4080.23**	**-7544.04**	0.78	5.16	1	10	1334	1
TSEM	500	70	**-2617.51**	**-8298.96**	**0.77**	0.11	1	3	24	0.04
TSEM-poly	500	70	**-2617.51**	**-8298.96**	**0.77**	**0.11**	1	3	24	0.04
soft SEM	500	70	-2632.38	-8484.32	0.76	1.15	1	1	1028	1
TSEM	2000	30	**-19867.08**	**-25854.3**	0.82	0.49	2	26	74	0.32
TSEM-poly	2000	30	-19873.9	-25874.11	0.82	**0.47**	2	25	63	0.37
soft SEM	2000	30	-19983.42	-25994.89	**0.82**	11.02	2	22	1778	1
TSEM	2000	50	**-15037.52**	**-26003.25**	**0.8**	0.49	2	20	65	0.22
TSEM-poly	2000	50	**-15037.52**	**-26003.25**	**0.8**	**0.48**	2	20	57	0.25
soft SEM	2000	50	-15284.91	-27064.8	0.79	10.27	2	20	1724	1
TSEM	2000	70	**-9833.88**	**-30273.81**	**0.77**	0.4	1	9	48	0.08
TSEM-poly	2000	70	-9893.01	-30760.43	0.77	**0.29**	1	7	41	0.1
soft SEM	2000	70	-9849.18	-30413.94	0.76	3.45	1	6	1200	1
TSEM	5000	30	**-48746.62**	**-64706.8**	**0.83**	0.99	2	29	75	0.37
TSEM-poly	5000	30	-48753.24	-64731.47	**0.83**	**0.73**	3	27	66	0.38
soft SEM	5000	30	-48944.94	-64941.27	0.83	15.72	2	27	1935	1
TSEM	5000	50	**-36709.91**	**-65013.85**	**0.81**	1.21	2	25	63	0.37
TSEM-poly	5000	50	**-36709.91**	**-65013.85**	**0.81**	**1.04**	2	25	63	0.37
soft SEM	5000	50	-36942.13	-65344.68	0.8	12.33	2	23	1823	1
TSEM	5000	70	-24148.49	-73568.13	0.77	0.67	2	11	57	0.14
TSEM-poly	5000	70	-24148.49	-73568.13	0.77	**0.67**	2	11	52	0.15
soft SEM	5000	70	**-24055.35**	**-73465.74**	**0.77**	5.73	1	10	1340	1

Table 5: Comparison of the methods in dataset Mildew. For each method, the number of instances (N. inst) and percentage of missing values (Missing %), the BIC score in the training dataset (BIC), the log-likelihood in the test dataset (LL), the imputation accuracy (Acc), the learning time in seconds (Time), the treewidth, the number of changes in the structure (N. Changes), the number of computations of the expected sufficient statistics (N. ESS) and the proportion of local changes performed that improved the expected score (Prop) are shown. The best results are denoted in boldface.

Method	N. inst	Missing (%)	BIC	LL	Acc	Time	Treewidth	N. Changes	N. ESS	Prop
TSEM	500	30	**-22016.26**	**-29008.44**	**0.4**	0.21	1	11	27	0.19
TSEM-poly	500	30	**-22016.26**	**-29008.44**	**0.4**	**0.19**	1	11	27	0.19
soft SEM	500	30	-22436.82	-30429.3	0.36	5.35	1	5	1371	1
TSEM	500	50	-16740.19	-31789.25	**0.32**	0.29	0	0	6	-
TSEM-poly	500	50	-16740.19	-31789.25	**0.32**	**0.1**	0	0	6	-
soft SEM	500	50	**-16690.62**	**-31618.13**	**0.32**	3.17	0	0	1194	-
TSEM	500	70	-10922.69	-33589.87	**0.33**	0.11	0	0	2	-
TSEM-poly	500	70	-10922.69	-33589.87	**0.33**	**0.1**	0	0	2	-
soft SEM	500	70	**-10589.91**	**-32275.22**	**0.33**	2.22	0	0	1196	-
TSEM	2000	30	**-81794.11**	**-110040.58**	**0.42**	1.08	1	21	56	0.29
TSEM-poly	2000	30	**-81794.11**	**-110040.58**	**0.42**	**1.07**	1	21	52	0.31
soft SEM	2000	30	-82995.53	-114206.21	0.41	15.92	1	14	1745	1
TSEM	2000	50	**-60582.81**	-114093.51	0.38	1.1	1	13	42	0.21
TSEM-poly	2000	50	-60588	**-114082.7**	**0.38**	**1.06**	1	12	37	0.22
soft SEM	2000	50	-62360.05	-121002.11	0.35	8.54	1	5	1377	1
TSEM	2000	70	-40271.47	-130852.49	**0.33**	**0.4**	0	0	14	-
TSEM-poly	2000	70	-40271.47	-130852.49	**0.33**	0.41	0	0	14	-
soft SEM	2000	70	**-38832.72**	**-125354.54**	**0.33**	5.06	0	0	1198	-
TSEM	5000	30	**-193273.36**	**-253930.01**	**0.45**	4.35	1	28	66	0.39
TSEM-poly	5000	30	**-193273.36**	**-253930.01**	**0.45**	**4.15**	1	28	60	0.43
soft SEM	5000	30	-200572.22	-273587.94	0.42	42.24	1	21	1977	1
TSEM	5000	50	**-145069.68**	**-269646.2**	**0.4**	3.89	1	24	66	0.23
TSEM-poly	5000	50	**-145069.68**	**-269646.2**	**0.4**	**3.69**	1	24	58	0.26
soft SEM	5000	50	-149710.07	-283249.68	0.39	26.7	1	12	1645	1
TSEM	5000	70	**-94639.66**	**-305762.2**	**0.34**	**1.83**	1	3	32	0
TSEM-poly	5000	70	**-94639.66**	**-305762.2**	**0.34**	1.84	1	3	30	0
soft SEM	5000	70	-94714.88	-308679.54	0.33	14.75	0	0	1198	-

Table 6: Comparison of the methods in dataset Alarm. For each method, the number of instances (N. inst) and percentage of missing values (Missing %), the BIC score in the training dataset (BIC), the log-likelihood in the test dataset (LL), the imputation accuracy (Acc), the learning time in seconds (Time), the treewidth, the number of changes in the structure (N. Changes), the number of computations of the expected sufficient statistics (N. ESS) and the proportion of local changes performed that improved the expected score (Prop) are shown. The best results are denoted in boldface.

Method	N. inst	Missing (%)	BIC	LL	Acc	Time	Treewidth	N. Changes	N. ESS	Prop
TSEM	500	30	-5276.81	-6491.66	0.9	0.31	1	30	75	0.4
TSEM-poly	500	30	-5276.81	-6491.66	0.9	**0.29**	1	30	73	0.41
soft SEM	500	30	**-5213.23**	**-6425.98**	**0.9**	22.68	2	31	2602	1
TSEM	500	50	**-4288.6**	**-6821.84**	**0.88**	**0.31**	2	29	69	0.28
TSEM-poly	500	50	**-4288.6**	**-6821.84**	**0.88**	0.31	2	29	69	0.28
soft SEM	500	50	-4607.07	-7490.56	0.87	13.87	1	18	2047	1
TSEM	500	70	**-3286.79**	**-10292.78**	**0.8**	0.15	1	2	38	0
TSEM-poly	500	70	**-3286.79**	**-10292.78**	**0.8**	**0.14**	1	2	34	0
soft SEM	500	70	-3312.43	-10660.62	0.79	1.08	0	0	1337	-
TSEM	2000	30	**-18687.05**	**-22921.24**	**0.92**	1.29	3	48	143	0.3
TSEM-poly	2000	30	-18724.15	-22986.74	0.92	**1.21**	3	46	132	0.33
soft SEM	2000	30	-19018.1	-23555.97	0.92	34.13	2	43	3101	1
TSEM	2000	50	**-15075.2**	**-24082.48**	0.9	0.91	2	34	103	0.3
TSEM-poly	2000	50	-15080.31	-24087.14	**0.9**	**0.88**	2	34	100	0.3
soft SEM	2000	50	-15414.12	-25210.23	0.9	21.27	1	28	2440	1
TSEM	2000	70	**-10766.92**	-28707.21	**0.86**	0.75	1	23	81	0.15
TSEM-poly	2000	70	**-10766.92**	**-28707.21**	**0.86**	**0.73**	1	23	81	0.15
soft SEM	2000	70	-11897.25	-34981.18	0.83	8.64	1	11	1760	1
TSEM	5000	30	**-44921.58**	**-54464.68**	**0.93**	3.77	3	51	168	0.29
TSEM-poly	5000	30	-45091.5	-54626.1	0.93	**3.61**	3	50	153	0.31
soft SEM	5000	30	-46002.27	-55802.63	0.92	47.49	3	54	3543	1
TSEM	5000	50	**-36083.19**	**-56039.34**	**0.91**	3.61	3	42	148	0.24
TSEM-poly	5000	50	**-36083.19**	**-56039.34**	**0.91**	**3.52**	3	42	144	0.24
soft SEM	5000	50	-37180.39	-59626.44	0.9	34.79	3	42	3042	1
TSEM	5000	70	**-25509.29**	**-64106.14**	0.87	1.86	1	30	97	0.22
TSEM-poly	5000	70	-25540.24	-64142.53	**0.87**	**1.85**	2	29	92	0.22
soft SEM	5000	70	-26502.27	-66838.23	0.86	18.71	1	23	2286	1

Table 7: Comparison of the methods in dataset Barley. For each method, the number of instances (N. inst) and percentage of missing values (Missing %), the BIC score in the training dataset (BIC), the log-likelihood in the test dataset (LL), the imputation accuracy (Acc), the learning time in seconds (Time), the treewidth, the number of changes in the structure (N. Changes), the number of computations of the expected sufficient statistics (N. ESS) and the proportion of local changes performed that improved the expected score (Prop) are shown. The best results are denoted in boldface.

Method	N. inst	Missing (%)	BIC	LL	Acc	Time	Treewidth	N. Changes	N. ESS	Prop
TSEM	500	30	**-27597.68**	**-35830.73**	**0.43**	0.36	1	15	48	0.17
TSEM-poly	500	30	**-27597.68**	**-35830.73**	**0.43**	**0.34**	1	15	44	0.18
soft SEM	500	30	-28263.68	-38334.96	0.38	12.33	1	6	2558	1
TSEM	500	50	**-20868.44**	**-39853.52**	**0.33**	0.25	1	3	18	0
TSEM-poly	500	50	**-20868.44**	**-39853.52**	**0.33**	**0.24**	1	3	18	0
soft SEM	500	50	-21044.81	-40683.73	0.32	3.02	0	0	2260	-
TSEM	500	70	-13428.41	-42859.44	**0.32**	0.12	0	0	2	-
TSEM-poly	500	70	-13428.41	-42859.44	**0.32**	**0.11**	0	0	2	-
soft SEM	500	70	**-12982.4**	**-41037.49**	**0.32**	3.16	0	0	2263	-
TSEM	2000	30	**-99496.71**	**-128841.89**	**0.49**	**1.53**	1	32	75	0.41
TSEM-poly	2000	30	**-99496.71**	**-128841.89**	**0.49**	1.54	1	32	71	0.44
soft SEM	2000	30	-100336.88	-131241.43	0.48	51.34	1	26	3652	1
TSEM	2000	50	**-76908.89**	-140364.97	**0.4**	1.78	1	19	72	0.1
TSEM-poly	2000	50	-76919.12	**-140342.58**	0.4	**1.41**	1	17	61	0.11
soft SEM	2000	50	-79430.49	-150145.24	0.36	15.69	1	6	2558	1
TSEM	2000	70	-51488.5	-168928.02	**0.33**	0.34	0	0	8	-
TSEM-poly	2000	70	-51488.5	-168928.02	**0.33**	**0.33**	0	0	8	-
soft SEM	2000	70	**-49643.1**	**-162128.93**	**0.33**	7.16	1	1	2311	1
TSEM	5000	30	**-240196.14**	**-316400.47**	**0.5**	**3.77**	1	37	82	0.4
TSEM-poly	5000	30	**-240196.14**	**-316400.47**	**0.5**	3.8	1	37	82	0.4
soft SEM	5000	30	-241806.92	-321050.37	0.49	84.43	1	29	3757	1
TSEM	5000	50	**-180813.06**	**-321135.47**	**0.46**	4.91	1	31	78	0.33
TSEM-poly	5000	50	**-180813.06**	**-321135.47**	**0.46**	**4.9**	1	31	74	0.35
soft SEM	5000	50	-190203.25	-349153.78	0.41	51.29	1	14	2971	1
TSEM	5000	70	-122108.26	**-402895.87**	**0.33**	1.67	1	2	41	0
TSEM-poly	5000	70	-122108.26	**-402895.87**	**0.33**	**1.67**	1	2	41	0
soft SEM	5000	70	**-122001.31**	-404526.49	0.33	12.26	0	0	2264	-

Table 8: Comparison of the methods in dataset Hailfinder. For each method, the number of instances (N. inst) and percentage of missing values (Missing %), the BIC score in the training dataset (BIC), the log-likelihood in the test dataset (LL), the imputation accuracy (Acc), the learning time in seconds (Time), the treewidth, the number of changes in the structure (N. Changes), the number of computations of the expected sufficient statistics (N. ESS) and the proportion of local changes performed that improved the expected score (Prop) are shown. The best results are denoted in boldface.

Method	N. inst	Missing (%)	BIC	LL	Acc	Time	Treewidth	N. Changes	N. ESS	Prop
TSEM	500	30	**-21438.93**	**-26933.86**	**0.6**	0.75	2	45	116	0.29
TSEM-poly	500	30	-21450.43	-26937.85	**0.6**	**0.68**	2	43	98	0.34
soft SEM	500	30	-21984.52	-28523.57	0.55	65.33	1	25	4537	1
TSEM	500	50	**-16808.66**	**-30946.04**	**0.48**	0.44	1	22	73	0.18
TSEM-poly	500	50	**-16808.66**	**-30946.04**	**0.48**	**0.42**	1	22	69	0.19
soft SEM	500	50	-17324.79	-32731.32	0.44	18.51	1	7	3494	1
TSEM	500	70	-10952.1	**-35042.55**	**0.39**	**0.18**	1	1	14	0
TSEM-poly	500	70	-10952.1	**-35042.55**	**0.39**	0.2	1	1	14	0
soft SEM	500	70	**-10930.98**	-35139.2	0.39	4.22	0	0	3086	-
TSEM	2000	30	**-78896.48**	**-101414.9**	**0.64**	2.15	2	57	134	0.4
TSEM-poly	2000	30	-78938	-101475.65	0.64	**1.95**	2	56	123	0.42
soft SEM	2000	30	-79613.98	-103525.6	0.63	143.61	2	50	6142	1
TSEM	2000	50	**-60867.41**	**-104960.56**	**0.58**	1.87	2	45	112	0.35
TSEM-poly	2000	50	**-60867.41**	**-104960.56**	**0.58**	**1.78**	2	45	104	0.38
soft SEM	2000	50	-63435.23	-113974.36	0.53	62.34	1	25	4605	1
TSEM	2000	70	-42016.66	-135048.1	**0.42**	0.68	1	4	64	0.03
TSEM-poly	2000	70	-42016.66	-135048.1	**0.42**	**0.65**	1	4	64	0.03
soft SEM	2000	70	**-41870.59**	**-133653.07**	0.41	11.72	1	3	3264	1
TSEM	5000	30	**-191323.93**	-249688.83	0.64	6.81	2	66	174	0.36
TSEM-poly	5000	30	-191329.74	**-249688.24**	**0.64**	**6.47**	2	62	144	0.42
soft SEM	5000	30	-192553.72	-252546.92	0.64	161.1	2	57	6512	1
TSEM	5000	50	**-146352.79**	**-252816.54**	**0.6**	6.25	2	57	156	0.33
TSEM-poly	5000	50	-146484.5	-253466.19	0.6	**5.86**	2	56	140	0.37
soft SEM	5000	50	-151228.39	-268204.99	0.57	122.65	2	44	5697	1
TSEM	5000	70	**-97467.99**	**-279774.68**	**0.49**	9.06	1	30	132	0.1
TSEM-poly	5000	70	**-97467.99**	**-279774.68**	**0.49**	**5.03**	1	30	126	0.1
soft SEM	5000	70	-102727.67	-320843.58	0.43	34.5	1	11	3794	1

Table 9: Comparison of the methods in dataset Hepar II. For each method, the number of instances (N. inst) and percentage of missing values (Missing %), the BIC score in the training dataset (BIC), the log-likelihood in the test dataset (LL), the imputation accuracy (Acc), the learning time in seconds (Time), the treewidth, the number of changes in the structure (N. Changes), the number of computations of the expected sufficient statistics (N. ESS) and the proportion of local changes performed that improved the expected score (Prop) are shown. The best results are denoted in boldface.

Method	N. inst	Missing (%)	BIC	LL	Acc	Time	Treewidth	N. Changes	N. ESS	Prop
TSEM	500	30	**-12205.9**	**-16920.73**	0.79	0.58	1	32	105	0.23
TSEM-poly	500	30	**-12205.9**	**-16920.73**	0.79	**0.53**	1	32	93	0.26
soft SEM	500	30	-12249.48	-16971.51	**0.8**	83.19	1	19	6199	1
TSEM	500	50	**-8874.23**	**-17135.52**	**0.79**	0.41	1	14	82	0.07
TSEM-poly	500	50	**-8874.23**	**-17135.52**	**0.79**	**0.4**	1	14	74	0.08
soft SEM	500	50	-8945.86	-17280.34	0.79	36.82	1	6	5258	1
TSEM	500	70	-5502.17	**-17846.78**	**0.78**	0.26	1	1	39	0
TSEM-poly	500	70	-5502.17	**-17846.78**	**0.78**	**0.26**	1	1	39	0
soft SEM	500	70	**-5492.57**	-17879.45	0.78	6.65	0	0	4835	-
TSEM	2000	30	**-47315.45**	**-65701.7**	**0.8**	1.25	2	47	136	0.3
TSEM-poly	2000	30	-47352.95	-65831.91	0.8	**1.15**	2	46	119	0.35
soft SEM	2000	30	-47462.46	-66110.58	0.8	163.17	2	36	7522	1
TSEM	2000	50	**-34478.53**	**-66364.83**	**0.79**	1.27	2	31	107	0.18
TSEM-poly	2000	50	-34495.3	-66425.48	0.79	**1**	1	28	89	0.21
soft SEM	2000	50	-34658.52	-67136.75	0.79	77.8	1	17	6125	1
TSEM	2000	70	**-21186.03**	**-69190.37**	**0.78**	0.63	1	4	58	0.03
TSEM-poly	2000	70	**-21186.03**	**-69190.37**	**0.78**	**0.62**	1	4	56	0.04
soft SEM	2000	70	-21191.21	-69341.99	0.78	15.46	1	2	4977	1
TSEM	5000	30	**-117433.91**	-164160.63	**0.8**	2.74	3	65	164	0.33
TSEM-poly	5000	30	-117455.04	**-164149.87**	0.8	**2.58**	3	61	150	0.34
soft SEM	5000	30	-117665.41	-164717.28	0.8	252.65	2	55	8966	1
TSEM	5000	50	**-85215.75**	**-165031.85**	**0.79**	2.92	2	42	137	0.23
TSEM-poly	5000	50	-85228.66	-165137.46	0.79	**2.31**	2	40	120	0.26
soft SEM	5000	50	-85394.71	-166016.23	0.79	145.79	2	32	7182	1
TSEM	5000	70	**-52528.12**	**-171007.88**	**0.78**	1.94	1	10	97	0.04
TSEM-poly	5000	70	**-52528.12**	**-171007.88**	**0.78**	**1.92**	1	10	95	0.04
soft SEM	5000	70	-52677.89	-171991.74	0.78	34.37	1	6	5267	1

Table 10: Comparison of the methods in dataset Win95pts. For each method, the number of instances (N. inst) and percentage of missing values (Missing %), the BIC score in the training dataset (BIC), the log-likelihood in the test dataset (LL), the imputation accuracy (Acc), the learning time in seconds (Time), the treewidth, the number of changes in the structure (N. Changes), the number of computations of the expected sufficient statistics (N. ESS) and the proportion of local changes performed that improved the expected score (Prop) are shown. The best results are denoted in boldface.

Method	N. inst	Missing (%)	BIC	LL	Acc	Time	Treewidth	N. Changes	N. ESS	Prop
TSEM	500	30	**-4768.35**	**-5671.2**	**0.96**	1.57	4	90	294	0.27
TSEM-poly	500	30	-4803.48	-5690.79	0.96	**1.26**	4	84	246	0.3
soft SEM	500	30	-4905.66	-5754.25	0.96	424.43	3	72	11530	1
TSEM	500	50	**-3870.55**	**-6160.42**	0.94	1.02	2	63	204	0.24
TSEM-poly	500	50	-3906.45	-6287.56	0.94	**1**	2	59	199	0.23
soft SEM	500	50	-3928.99	-6240.43	**0.94**	243.57	1	44	9253	1
TSEM	500	70	**-2740.88**	**-7324.18**	**0.93**	**0.91**	1	26	193	0.04
TSEM-poly	500	70	**-2740.88**	**-7324.18**	**0.93**	0.91	1	26	185	0.04
soft SEM	500	70	-2838.35	-7651.64	0.92	103.28	1	18	7100	1
TSEM	2000	30	**-16584.25**	**-19579.33**	0.96	3.31	5	105	340	0.29
TSEM-poly	2000	30	-16606.45	-19586.09	**0.97**	**3.15**	5	104	318	0.31
soft SEM	2000	30	-17318.98	-20601.76	0.96	700.32	6	110	14455	1
TSEM	2000	50	**-13746.97**	**-20816.7**	**0.95**	2.82	5	95	324	0.24
TSEM-poly	2000	50	-13780.06	-20948.91	0.95	**2.75**	5	93	319	0.25
soft SEM	2000	50	-14429.89	-22837.64	0.95	456.94	3	77	11812	1
TSEM	2000	70	**-9662.96**	**-23951.56**	**0.94**	2.06	3	55	281	0.15
TSEM-poly	2000	70	-9674.74	-23998.74	0.94	**1.99**	3	55	267	0.15
soft SEM	2000	70	-9992.04	-25057.68	0.93	209.87	2	38	8812	1
TSEM	5000	30	**-40335.92**	**-47695.49**	**0.97**	8.11	5	127	381	0.31
TSEM-poly	5000	30	-40454.04	-47708.85	0.97	**7.67**	5	120	363	0.31
soft SEM	5000	30	-41958.43	-49951.22	0.96	1065.07	7	148	17721	1
TSEM	5000	50	**-33344.81**	**-49641.42**	**0.96**	8.53	5	110	423	0.23
TSEM-poly	5000	50	-33353.61	-49843.98	0.96	**8.01**	5	105	387	0.25
soft SEM	5000	50	-34877.49	-53795.28	0.95	708.68	4	111	14644	1
TSEM	5000	70	**-23564.12**	**-57193.76**	**0.94**	5.52	3	69	386	0.15
TSEM-poly	5000	70	-23564.17	-57214.93	**0.94**	**5.51**	3	69	376	0.15
soft SEM	5000	70	-23883.86	-57987.76	0.94	376.28	3	65	10819	1

Table 11: Comparison of the methods in dataset Pathfinder. For each method, the number of instances (N. inst) and percentage of missing values (Missing %), the BIC score in the training dataset (BIC), the log-likelihood in the test dataset (LL), the imputation accuracy (Acc), the learning time in seconds (Time), the treewidth, the number of changes in the structure (N. Changes), the number of computations of the expected sufficient statistics (N. ESS) and the proportion of local changes performed that improved the expected score (Prop) are shown. The best results are denoted in boldface.

Method	N. inst	Missing (%)	BIC	LL	Acc	Time	Treewidth	N. Changes	N. ESS	Prop
TSEM	500	30	-13933.46	-18037.48	**0.35**	**1.74**	3	71	236	0.19
TSEM-poly	500	30	**-13923.09**	**-18001.91**	0.35	1.74	4	72	238	0.19
soft SEM	500	30	-14382.78	-18826.23	0.35	687.57	2	44	16797	1
TSEM	500	50	**-10858.92**	**-19808.48**	**0.54**	1.83	2	37	260	0.05
TSEM-poly	500	50	**-10858.92**	**-19808.48**	**0.54**	**1.8**	2	37	254	0.05
soft SEM	500	50	-11246.43	-20848.89	0.53	198.28	1	12	13204	1
TSEM	500	70	-7079.07	-22012.55	**0.61**	**0.45**	1	1	41	0.02
TSEM-poly	500	70	-7079.07	-22012.55	**0.61**	0.45	1	1	41	0.02
soft SEM	500	70	**-7004.51**	**-21718.98**	**0.61**	55.79	1	2	11998	1
TSEM	2000	30	**-49193.27**	-64649.06	**0.35**	10.68	5	120	375	0.26
TSEM-poly	2000	30	-49199.58	**-64646.25**	0.35	**10.12**	5	117	354	0.27
soft SEM	2000	30	-50525.59	-67916.06	0.34	1490	4	89	21745	1
TSEM	2000	50	**-37591.7**	**-67967.25**	**0.55**	5.56	5	96	371	0.16
TSEM-poly	2000	50	**-37591.7**	**-67967.25**	**0.55**	**5.55**	5	96	367	0.16
soft SEM	2000	50	-39509.7	-73370.9	0.54	813.35	1	51	17767	1
TSEM	2000	70	**-26211.31**	**-83686.4**	**0.6**	2.04	1	8	206	0.01
TSEM-poly	2000	70	**-26211.31**	**-83686.4**	**0.6**	**2.02**	1	8	206	0.01
soft SEM	2000	70	-26259.44	-84529.06	0.6	137.18	1	7	12551	1
TSEM	5000	30	**-113714.98**	-143338.99	0.37	440.53	5	143	398	0.3
TSEM-poly	5000	30	-113752.76	**-143250.1**	**0.37**	**410.11**	5	138	381	0.3
soft SEM	5000	30	-116866.01	-155062.59	0.36	2728.93	5	142	27897	1
TSEM	5000	50	**-89017.64**	**-158618.49**	**0.56**	76.42	5	128	461	0.19
TSEM-poly	5000	50	-89175.23	-158724.36	0.56	**70.74**	5	124	439	0.2
soft SEM	5000	50	-94252.01	-173341.43	0.55	1285.04	2	76	20391	1
TSEM	5000	70	**-60928.53**	**-187168.66**	**0.62**	12.89	3	53	553	0.03
TSEM-poly	5000	70	**-60928.53**	**-187168.66**	**0.62**	**12.85**	3	53	551	0.03
soft SEM	5000	70	-63407.94	-199987.26	0.61	315.15	1	17	13696	1

2. Comparison of tractable SEM with SEM-kMAX

Table 12: Comparison of the methods in dataset NLTCS. For each method, treewidth bound (Bound) and percentage of missing values (Missing %), the BIC score in the training dataset (BIC), the log-likelihood in the test dataset (LL), the imputation accuracy (Acc), the learning time (Time) and the treewidth (Treewidth) are shown. The best results are denoted in boldface.

Method	Bound	Missing (%)	BIC	LL	Acc	Time	Treewidth
TSEM	2	30	**-76159.58**	**-20744.31**	**0.84**	0.95	2
TSEM-poly	2	30	**-76159.58**	**-20744.31**	**0.84**	**0.94**	2
SEM-kMAX	2	30	-82287.85	-22858.25	0.81	114.08	1
TSEM	2	50	**-57422.11**	-20835.91	0.82	1.62	2
TSEM-poly	2	50	-57433.66	**-20829.05**	**0.82**	**1.11**	2
SEM-kMAX	2	50	-64599.72	-25253.83	0.8	172.22	1
TSEM	2	70	**-37356.76**	**-21374.91**	**0.79**	1.35	2
TSEM-poly	2	70	**-37356.76**	**-21374.91**	**0.79**	**1.31**	2
SEM-kMAX	2	70	-50723.09	-32420.91	0.72	191.65	1
TSEM	3	30	**-75118.4**	-20316.49	0.84	1.98	3
TSEM-poly	3	30	-75118.89	**-20316.08**	**0.84**	**1.45**	3
SEM-kMAX	3	30	-77287.48	-21326.04	0.84	133.9	3
TSEM	3	50	**-56802.34**	**-20362.54**	**0.82**	1.63	3
TSEM-poly	3	50	**-56802.34**	**-20362.54**	**0.82**	**1.56**	3
SEM-kMAX	3	50	-62933.37	-23692.38	0.8	134.35	2
TSEM	3	70	**-37139.29**	**-21480.39**	**0.8**	1.81	3
TSEM-poly	3	70	**-37139.29**	**-21480.39**	**0.8**	**1.77**	3
SEM-kMAX	3	70	-48041.56	-34510.3	0.73	410.55	3
TSEM	4	30	**-74471.36**	**-20120.33**	0.84	2.16	4
TSEM-poly	4	30	**-74471.36**	**-20120.33**	0.84	**2.05**	4
SEM-kMAX	4	30	-76073.4	-20733.48	**0.84**	94.81	4
TSEM	4	50	**-56390.91**	**-20224.99**	**0.83**	**2.44**	4
TSEM-poly	4	50	**-56390.91**	**-20224.99**	**0.83**	2.51	4
SEM-kMAX	4	50	-63226.8	-24433.7	0.78	290.85	4
TSEM	4	70	**-36734.7**	**-21089.64**	**0.8**	3.65	4
TSEM-poly	4	70	-36798.58	-21092.7	0.8	**2.62**	4
SEM-kMAX	4	70	-49218.74	-33389.6	0.72	472.74	4
TSEM	5	30	**-73980.01**	**-19826.9**	**0.85**	3.72	5
TSEM-poly	5	30	**-73980.01**	**-19826.9**	**0.85**	**3.15**	5
SEM-kMAX	5	30	-75459.47	-20398.93	0.84	133.34	5
TSEM	5	50	**-56127.82**	-20149.81	0.83	5.61	5
TSEM-poly	5	50	-56162.43	**-20114.17**	**0.83**	**3.96**	5
SEM-kMAX	5	50	-63874.61	-25345.87	0.78	194.13	5
TSEM	5	70	**-36734.7**	**-21089.64**	**0.8**	5.01	4
TSEM-poly	5	70	-36798.58	-21092.7	0.8	**3.38**	4
SEM-kMAX	5	70	-46260.74	-31760.84	0.75	397.01	5

Table 13: Comparison of the methods in dataset MSNBC1. For each method, treewidth bound (Bound) and percentage of missing values (Missing %), the BIC score in the training dataset (BIC), the log-likelihood in the test dataset (LL), the imputation accuracy (Acc), the learning time (Time) and the treewidth (Treewidth) are shown. The best results are denoted in boldface.

Method	Bound	Missing (%)	BIC	LL	Acc	Time	Treewidth
TSEM	2	30	**-1340642.09**	-378664.88	**0.85**	22.63	2
TSEM-poly	2	30	-1340642.22	**-378664.5**	**0.85**	**16.48**	2
SEM-kMAX	2	30	-1381028.21	-390838.49	0.84	206.46	1
TSEM	2	50	**-964681.96**	**-378396.86**	**0.84**	28.63	2
TSEM-poly	2	50	-964946.21	-378520	0.84	**19.19**	2
SEM-kMAX	2	50	-1054080.51	-414493	0.84	212.3	1
TSEM	2	70	**-584712.25**	**-379941.73**	0.84	30.76	2
TSEM-poly	2	70	-584891.38	-380328.74	**0.84**	**20.75**	2
SEM-kMAX	2	70	-725310.16	-474944.09	0.84	170.18	1
TSEM	3	30	**-1336943.65**	**-376657.67**	**0.85**	26.61	3
TSEM-poly	3	30	-1337008.57	-376685.72	0.85	**25.64**	3
SEM-kMAX	3	30	-1371269.35	-386851.69	0.84	440.88	3
TSEM	3	50	**-962106.22**	**-375675.41**	**0.84**	29.81	3
TSEM-poly	3	50	**-962106.22**	**-375675.41**	**0.84**	**28.75**	3
SEM-kMAX	3	50	-1056156.11	-414862.03	0.84	320.34	3
TSEM	3	70	**-583837.14**	**-377893.41**	**0.84**	28.07	3
TSEM-poly	3	70	-583920.19	-378085.4	**0.84**	**26.93**	3
SEM-kMAX	3	70	-725064.55	-473937.57	0.84	131.03	3
TSEM	4	30	**-1329667.62**	**-373235.62**	**0.85**	66.6	4
TSEM-poly	4	30	-1331733.33	-373951.41	0.85	**42.02**	4
SEM-kMAX	4	30	-1364836.46	-384927.38	0.85	356.13	4
TSEM	4	50	**-959850.95**	-373368.17	0.84	55.39	4
TSEM-poly	4	50	-959868.34	**-373365.3**	**0.84**	**50.87**	4
SEM-kMAX	4	50	-1046077.5	-411816.27	0.84	1094.51	4
TSEM	4	70	**-583644.63**	**-377306.49**	0.84	48.1	4
TSEM-poly	4	70	-583729.1	-377513.26	**0.84**	**44.79**	4
SEM-kMAX	4	70	-725666.99	-474574.79	0.84	314.2	4
TSEM	5	30	**-1325847.76**	**-371107.94**	**0.85**	110.83	5
TSEM-poly	5	30	-1328460.71	-372348.21	0.85	**75.95**	5
SEM-kMAX	5	30	-1362877.41	-384842.97	0.85	741	4
TSEM	5	50	**-958042.49**	-371693.35	0.84	118.07	5
TSEM-poly	5	50	-958112.87	**-371692.2**	**0.84**	**92.95**	5
SEM-kMAX	5	50	-1047005.18	-411483.5	0.84	441.25	4
TSEM	5	70	**-583478.29**	**-376858.78**	0.84	70.16	5
TSEM-poly	5	70	-583623.96	-377214.31	**0.84**	**49.27**	5
SEM-kMAX	5	70	-725371.91	-474150.62	0.84	329.48	5

Table 14: Comparison of the methods in dataset KDDCup. For each method, treewidth bound (Bound) and percentage of missing values (Missing %), the BIC score in the training dataset (BIC), the log-likelihood in the test dataset (LL), the imputation accuracy (Acc), the learning time (Time) and the treewidth (Treewidth) are shown. The best results are denoted in boldface.

Method	Bound	Missing (%)	BIC	LL	Acc	Time	Treewidth
TSEM	2	30	**-314827.64**	**-78328.56**	**0.99**	74.36	2
TSEM-poly	2	30	-314827.65	-78328.62	**0.99**	**70.77**	2
SEM-kMAX	2	30	-325516.76	-80387.22	0.99	546.74	1
TSEM	2	50	**-229131.26**	-78777.92	0.99	102.95	2
TSEM-poly	2	50	-229132.4	**-78777.64**	**0.99**	**77.84**	2
SEM-kMAX	2	50	-240429.91	-81649.09	0.99	368.55	1
TSEM	2	70	**-141520.39**	**-81451.99**	**0.99**	104.68	2
TSEM-poly	2	70	**-141520.39**	**-81451.99**	**0.99**	**103.06**	2
SEM-kMAX	2	70	-153446.96	-85008.99	0.99	993.17	1
TSEM	3	30	**-312049.76**	-77375.24	**0.99**	177.83	3
TSEM-poly	3	30	-312087.75	**-77374.85**	**0.99**	**149.04**	3
SEM-kMAX	3	30	-317487.48	-78759.88	0.99	856.82	3
TSEM	3	50	**-227429.57**	-77919.43	**0.99**	190.3	3
TSEM-poly	3	50	-227474.57	**-77918.99**	0.99	**182.39**	3
SEM-kMAX	3	50	-235447.84	-80230.11	0.99	486.1	2
TSEM	3	70	**-141088.01**	**-81106.67**	**0.99**	114.57	3
TSEM-poly	3	70	**-141088.01**	**-81106.67**	**0.99**	**113.63**	3
SEM-kMAX	3	70	-151875.52	-83896.09	0.99	508.22	3
TSEM	4	30	**-311099.12**	-76947.91	0.99	294.3	4
TSEM-poly	4	30	-311137.48	**-76944.94**	**0.99**	**270.35**	4
SEM-kMAX	4	30	-317536.81	-78627.78	0.99	659.59	4
TSEM	4	50	**-227130.05**	**-77654.4**	0.99	271.02	4
TSEM-poly	4	50	-227150.21	-77666.04	**0.99**	**227**	4
SEM-kMAX	4	50	-235378.23	-80423.2	0.99	589.69	2
TSEM	4	70	**-140581.08**	**-80957.74**	**0.99**	203.72	4
TSEM-poly	4	70	-140588.3	-80967.79	0.99	**198.96**	4
SEM-kMAX	4	70	-151855.81	-84540.91	0.99	726.15	3
TSEM	5	30	**-310611.96**	-76854.66	0.99	411.29	5
TSEM-poly	5	30	-310681.6	**-76842.88**	**0.99**	**376.57**	5
SEM-kMAX	5	30	-317410.97	-78745.38	0.99	759.15	4
TSEM	5	50	**-227009.77**	**-77759.45**	0.99	422.57	5
TSEM-poly	5	50	-227025.43	-77775.63	**0.99**	**405.31**	5
SEM-kMAX	5	50	-235298.73	-80253.63	0.99	696.03	3
TSEM	5	70	**-140523.87**	**-80779.49**	0.99	324.46	5
TSEM-poly	5	70	-140534.28	-80783.55	**0.99**	**312.95**	5
SEM-kMAX	5	70	-152177.47	-84786.48	0.99	732.7	4

Table 15: Comparison of the methods in dataset Plants. For each method, treewidth bound (Bound) and percentage of missing values (Missing %), the BIC score in the training dataset (BIC), the log-likelihood in the test dataset (LL), the imputation accuracy (Acc), the learning time (Time) and the treewidth (Treewidth) are shown. The best results are denoted in boldface.

Method	Bound	Missing (%)	BIC	LL	Acc	Time	Treewidth
TSEM	2	30	-198656.33	-53076.3	0.93	11.06	2
TSEM-poly	2	30	**-198537.63**	**-53039.02**	**0.93**	**10.32**	2
SEM-kMAX	2	30	-217243.18	-59280.26	0.92	405.48	1
TSEM	2	50	**-154398.83**	**-54130.34**	**0.92**	13.36	2
TSEM-poly	2	50	-154629.48	-54338.19	0.92	**12.83**	2
SEM-kMAX	2	50	-172768.74	-63187.33	0.9	652.21	1
TSEM	2	70	**-105060.64**	**-57068.57**	**0.91**	11.49	2
TSEM-poly	2	70	-105134.75	-57143	0.91	**11.39**	2
SEM-kMAX	2	70	-126623.63	-76829.39	0.88	656.74	1
TSEM	3	30	**-191908.33**	**-50771.14**	0.93	25.85	3
TSEM-poly	3	30	-191958.89	-50776.37	**0.93**	**23.88**	3
SEM-kMAX	3	30	-199625.18	-53342.82	0.93	488.96	3
TSEM	3	50	-149790.49	-51849.69	0.92	26.68	3
TSEM-poly	3	50	**-149478.7**	**-51727.56**	**0.92**	**26.11**	3
SEM-kMAX	3	50	-161555.46	-58557.66	0.91	575.05	3
TSEM	3	70	-103237.67	-55237.89	0.91	**31.71**	3
TSEM-poly	3	70	**-102659.63**	**-55016.27**	**0.91**	34.22	3
SEM-kMAX	3	70	-133745.3	-87585.45	0.85	497.63	3
TSEM	4	30	**-187941.26**	-49452.91	**0.93**	48.53	4
TSEM-poly	4	30	-187987.27	**-49451.45**	0.93	**46.36**	4
SEM-kMAX	4	30	-198831.41	-53226.29	0.93	325.13	4
TSEM	4	50	-145842.62	-50513.93	0.93	54.16	4
TSEM-poly	4	50	**-145791.29**	**-50347.62**	**0.93**	**49.63**	4
SEM-kMAX	4	50	-162188.8	-59766.61	0.91	991.1	4
TSEM	4	70	**-100786.08**	-53760.34	0.91	**51.57**	4
TSEM-poly	4	70	-100811.13	**-53709.66**	**0.91**	53.98	4
SEM-kMAX	4	70	-132705.5	-84037.12	0.85	585.38	3
TSEM	5	30	**-184953.58**	-48682.05	0.93	96.67	5
TSEM-poly	5	30	-184987.76	**-48667**	**0.93**	**90.99**	5
SEM-kMAX	5	30	-197582.59	-53116.22	0.93	489.27	5
TSEM	5	50	-145618.57	-50464.98	0.93	**116.06**	5
TSEM-poly	5	50	**-145335.17**	**-50112.32**	**0.93**	121.64	5
SEM-kMAX	5	50	-161157.01	-59246.42	0.91	578.85	5
TSEM	5	70	**-99049.51**	**-52920.89**	**0.91**	**100.49**	5
TSEM-poly	5	70	-99136.55	-52960.25	0.91	107.34	5
SEM-kMAX	5	70	-130680.57	-85659.24	0.86	924.64	4

Table 16: Comparison of the methods in dataset Audio. For each method, treewidth bound (Bound) and percentage of missing values (Missing %), the BIC score in the training dataset (BIC), the log-likelihood in the test dataset (LL), the imputation accuracy (Acc), the learning time (Time) and the treewidth (Treewidth) are shown. The best results are denoted in boldface.

Method	Bound	Missing (%)	BIC	LL	Acc	Time	Treewidth
TSEM	2	30	**-460643.83**	**-129374.78**	**0.82**	17	2
TSEM-poly	2	30	-460764.65	-129453.69	0.82	**16.62**	2
SEM-kMAX	2	30	-481123.5	-136752.33	0.82	942.51	1
TSEM	2	50	**-335177.16**	**-130367.95**	**0.82**	18.31	2
TSEM-poly	2	50	-335233.26	-130399.37	0.82	**17.24**	2
SEM-kMAX	2	50	-362374.46	-144790.83	0.81	824.66	1
TSEM	2	70	**-206975.91**	-135097.43	0.81	36.46	2
TSEM-poly	2	70	-207236.58	**-134997.68**	**0.81**	**23.8**	2
SEM-kMAX	2	70	-246524.91	-168356.95	0.8	2852.42	1
TSEM	3	30	**-453385.66**	**-126813.61**	**0.83**	38.12	3
TSEM-poly	3	30	**-453385.66**	**-126813.61**	**0.83**	**36.65**	3
SEM-kMAX	3	30	-466779.07	-132098.46	0.82	589.86	3
TSEM	3	50	**-331538.17**	-128967.36	0.82	44.83	3
TSEM-poly	3	50	-331540.85	**-128960.33**	**0.82**	**34.72**	3
SEM-kMAX	3	50	-353548.76	-140830.05	0.81	711.63	3
TSEM	3	70	**-205018.98**	-133995.08	**0.81**	42.74	3
TSEM-poly	3	70	-205143.7	**-133738.76**	0.81	**40.48**	3
SEM-kMAX	3	70	-241482.75	-168945.71	0.8	1681.1	3
TSEM	4	30	**-450466.32**	**-125928.76**	0.83	77.44	4
TSEM-poly	4	30	-450820	-125969.38	**0.83**	**65.61**	4
SEM-kMAX	4	30	-467680.67	-132365.04	0.82	590.51	4
TSEM	4	50	**-328417.44**	**-127617.66**	0.82	76.53	4
TSEM-poly	4	50	-328500.48	-127654.63	**0.82**	**68.78**	4
SEM-kMAX	4	50	-356643.1	-142889.38	0.81	1193.02	4
TSEM	4	70	-204478.85	-134134.63	**0.81**	**62.01**	4
TSEM-poly	4	70	**-204392.32**	**-133783.98**	0.81	66.05	4
SEM-kMAX	4	70	-242527.09	-165371.68	0.8	1564.43	3
TSEM	5	30	**-447671.46**	**-124933.72**	**0.83**	156.28	5
TSEM-poly	5	30	-447757.93	-124976.58	0.83	**139.73**	5
SEM-kMAX	5	30	-466326.47	-131904.62	0.82	708.57	4
TSEM	5	50	**-327798.81**	-127282.34	0.82	144.83	5
TSEM-poly	5	50	-327840.44	**-127275.33**	**0.82**	**131.13**	5
SEM-kMAX	5	50	-353743.99	-141940.41	0.81	952.02	5
TSEM	5	70	-203485.91	-133852.33	0.81	113.39	5
TSEM-poly	5	70	**-203357.72**	**-133789.56**	**0.81**	**99.42**	5
SEM-kMAX	5	70	-241359.6	-165374.88	0.81	1321.83	4

Table 17: Comparison of the methods in dataset Jester. For each method, treewidth bound (Bound) and percentage of missing values (Missing %), the BIC score in the training dataset (BIC), the log-likelihood in the test dataset (LL), the imputation accuracy (Acc), the learning time (Time) and the treewidth (Treewidth) are shown. The best results are denoted in boldface.

Method	Bound	Missing (%)	BIC	LL	Acc	Time	Treewidth
TSEM	2	30	**-361520.29**	-232963.73	0.72	11.7	2
TSEM-poly	2	30	-361543.78	**-232938.71**	**0.72**	**11.24**	2
SEM-kMAX	2	30	-384493.45	-251544.51	0.69	1050.82	1
TSEM	2	50	**-262366.15**	-234966.54	0.71	14.05	2
TSEM-poly	2	50	-262367.66	**-234954.32**	**0.71**	**13.68**	2
SEM-kMAX	2	50	-303756.02	-281252.5	0.65	1522.44	1
TSEM	2	70	-161100.22	-244132.72	0.69	16.48	2
TSEM-poly	2	70	**-161065.08**	**-243975.98**	**0.69**	**16.29**	2
SEM-kMAX	2	70	-219523.46	-348226.56	0.64	2824.37	1
TSEM	3	30	-356693.01	-228805.94	0.73	22.77	3
TSEM-poly	3	30	**-356270.96**	**-228605.23**	**0.73**	**22.04**	3
SEM-kMAX	3	30	-372828.67	-243336.85	0.71	1170.73	3
TSEM	3	50	**-258473.58**	**-232042.56**	0.72	26.99	3
TSEM-poly	3	50	-258520.63	-232106.39	**0.72**	**25.01**	3
SEM-kMAX	3	50	-297222.34	-274118.91	0.67	1411.36	3
TSEM	3	70	**-159842.61**	**-243294.46**	**0.7**	28.72	3
TSEM-poly	3	70	-159880.9	-243332.88	0.7	**24.61**	3
SEM-kMAX	3	70	-215244.54	-341988.36	0.64	3551.63	3
TSEM	4	30	-354288.7	-227438.11	**0.73**	44.78	4
TSEM-poly	4	30	**-354268.42**	**-227429.65**	0.73	**44.39**	4
SEM-kMAX	4	30	-371628.11	-242249.96	0.71	703.76	4
TSEM	4	50	**-257797.55**	-231803.19	0.72	47.34	4
TSEM-poly	4	50	-257806.05	**-231769.76**	**0.72**	**44.72**	4
SEM-kMAX	4	50	-295685.33	-275333.78	0.67	1175.27	3
TSEM	4	70	**-159190.38**	**-242736.35**	**0.7**	38.9	4
TSEM-poly	4	70	-159196.18	-242775.69	0.7	**37.8**	4
SEM-kMAX	4	70	-217403.1	-339059.67	0.64	2009.39	3
TSEM	5	30	**-352866.67**	**-226522.6**	0.73	**71.39**	5
TSEM-poly	5	30	-353121.18	-226676.12	**0.73**	73.74	5
SEM-kMAX	5	30	-371184.5	-241526.08	0.71	702.92	5
TSEM	5	50	**-257025.03**	-231015.08	**0.72**	76.02	5
TSEM-poly	5	50	-257042.16	**-230964.88**	0.72	**66.83**	5
SEM-kMAX	5	50	-296478.43	-275374.72	0.67	1177.73	4
TSEM	5	70	**-158811.53**	**-242497.61**	**0.7**	56.37	5
TSEM-poly	5	70	**-158811.53**	**-242497.61**	**0.7**	**52.8**	5
SEM-kMAX	5	70	-216638.73	-340087.91	0.65	1298.17	3

Table 18: Comparison of the methods in dataset Netflix. For each method, treewidth bound (Bound) and percentage of missing values (Missing %), the BIC score in the training dataset (BIC), the log-likelihood in the test dataset (LL), the imputation accuracy (Acc), the learning time (Time) and the treewidth (Treewidth) are shown. The best results are denoted in boldface.

Method	Bound	Missing (%)	BIC	LL	Acc	Time	Treewidth
TSEM	2	30	**-631430.3**	**-177859.52**	**0.69**	19.59	2
TSEM-poly	2	30	-631457.64	-177867.95	0.69	**16.14**	2
SEM-kMAX	2	30	-666874.81	-190648.66	0.67	1764.8	1
TSEM	2	50	-457429.78	-178352.06	0.68	**18.37**	2
TSEM-poly	2	50	**-457422.93**	**-178348.02**	**0.68**	20.54	2
SEM-kMAX	2	50	-534863.12	-213075.48	0.63	1062.93	1
TSEM	2	70	**-280727.57**	**-183234.01**	**0.65**	19.75	2
TSEM-poly	2	70	-280785.5	-183277.43	0.65	**19.16**	2
SEM-kMAX	2	70	-394841.17	-260689.63	0.62	352.56	1
TSEM	3	30	-626820.77	-176176.96	0.7	41.25	3
TSEM-poly	3	30	**-626487.12**	**-175954.48**	**0.7**	**35.71**	3
SEM-kMAX	3	30	-656737.95	-186648.06	0.68	825.78	3
TSEM	3	50	-455012.15	-177233.09	**0.68**	48.43	3
TSEM-poly	3	50	**-454988.1**	**-177209.42**	0.68	**33.74**	3
SEM-kMAX	3	50	-524030.86	-210310.9	0.65	1549.87	3
TSEM	3	70	**-279617.1**	**-182779.18**	0.66	39.94	3
TSEM-poly	3	70	-279643.4	-182786.43	**0.66**	**39.55**	3
SEM-kMAX	3	70	-392638.35	-256361.3	0.62	475.24	3
TSEM	4	30	-624278.29	-175323.86	0.7	64.51	4
TSEM-poly	4	30	**-623285.25**	**-175082.33**	**0.7**	**62.53**	4
SEM-kMAX	4	30	-656918.18	-187588.42	0.68	1305.45	4
TSEM	4	50	**-452728.68**	**-176311.27**	**0.69**	77.54	4
TSEM-poly	4	50	-452969.22	-176363.67	0.69	**66.59**	4
SEM-kMAX	4	50	-523917.78	-209297.35	0.65	1317.58	3
TSEM	4	70	-279236.3	**-182549.96**	0.66	63.48	4
TSEM-poly	4	70	**-279186.75**	-182561.27	**0.66**	**61.72**	4
SEM-kMAX	4	70	-392494.02	-255920.05	0.62	235.9	3
TSEM	5	30	-622144.16	**-174690.62**	0.7	114.97	5
TSEM-poly	5	30	**-622057.09**	-174895.34	**0.7**	**104.65**	5
SEM-kMAX	5	30	-656453.97	-187099.41	0.68	1062.77	4
TSEM	5	50	**-452563.2**	**-176200.98**	0.69	128.78	5
TSEM-poly	5	50	-452612.78	-176206.73	**0.69**	**123.12**	5
SEM-kMAX	5	50	-526339.44	-210627.46	0.65	1193.52	4
TSEM	5	70	**-278809.97**	**-182256.18**	**0.66**	105.4	5
TSEM-poly	5	70	**-278809.97**	**-182256.18**	**0.66**	**100.1**	5
SEM-kMAX	5	70	-392195.06	-255482.14	0.62	235.57	3

Table 19: Comparison of the methods in dataset Accidents. For each method, treewidth bound (Bound) and percentage of missing values (Missing %), the BIC score in the training dataset (BIC), the log-likelihood in the test dataset (LL), the imputation accuracy (Acc), the learning time (Time) and the treewidth (Treewidth) are shown. The best results are denoted in boldface.

Method	Bound	Missing (%)	BIC	LL	Acc	Time	Treewidth
TSEM	2	30	**-304659.98**	**-79228.11**	**0.9**	19.37	2
TSEM-poly	2	30	-304697.88	-79247.54	0.9	**19.14**	2
SEM-kMAX	2	30	-327830.56	-87588.51	0.88	757.06	1
TSEM	2	50	-234411.31	-79335.84	**0.88**	25.01	2
TSEM-poly	2	50	**-234377.02**	**-79297.38**	0.88	**19.98**	2
SEM-kMAX	2	50	-264435.79	-95306.43	0.86	1141.94	1
TSEM	2	70	**-153096.73**	**-83245.57**	**0.86**	28.02	2
TSEM-poly	2	70	-153378.57	-83635.21	0.86	**22.42**	2
SEM-kMAX	2	70	-196786.91	-116241.78	0.83	1658.79	1
TSEM	3	30	**-295171.44**	**-75044.69**	**0.91**	40.49	3
TSEM-poly	3	30	-295463.36	-75156.45	0.91	**34.11**	3
SEM-kMAX	3	30	-310361.01	-80887.97	0.9	886.16	3
TSEM	3	50	**-230221.95**	**-76175.9**	**0.89**	41.78	3
TSEM-poly	3	50	-230465.45	-76375.75	0.89	**35.12**	3
SEM-kMAX	3	50	-255275.84	-89130.64	0.87	1021.52	3
TSEM	3	70	**-152419.08**	**-82323.08**	**0.86**	39.6	3
TSEM-poly	3	70	-152707.75	-82767.87	0.86	**35.34**	3
SEM-kMAX	3	70	-191186.78	-115208.29	0.84	2317.37	2
TSEM	4	30	**-290188.48**	**-72768.81**	**0.91**	87.4	4
TSEM-poly	4	30	-290584.26	-72835.13	0.91	**72.53**	4
SEM-kMAX	4	30	-309718.48	-80357.57	0.9	762.25	3
TSEM	4	50	**-228333.75**	-74619.26	**0.89**	71.97	4
TSEM-poly	4	50	-228352.73	**-74586.59**	0.89	**59.29**	4
SEM-kMAX	4	50	-252696.44	-87818.37	0.87	1921.92	3
TSEM	4	70	**-152352.43**	**-82267.14**	0.86	40.89	4
TSEM-poly	4	70	-152536.15	-82781.16	**0.86**	**39.89**	4
SEM-kMAX	4	70	-193679.55	-116295.39	0.84	2578.48	3
TSEM	5	30	**-287975.43**	**-71295.47**	**0.91**	151.61	5
TSEM-poly	5	30	-288397.87	-71427.4	0.91	**132.07**	5
SEM-kMAX	5	30	-310062.15	-80665.34	0.9	758.25	4
TSEM	5	50	**-227617.25**	**-74326.65**	0.89	96.65	5
TSEM-poly	5	50	-227941.02	-74460.93	**0.89**	**79.52**	5
SEM-kMAX	5	50	-253342.98	-89590.05	0.87	2168.87	3
TSEM	5	70	**-152121.57**	**-82319.31**	**0.86**	63.36	5
TSEM-poly	5	70	-152398.41	-82723.02	0.86	**52.56**	5
SEM-kMAX	5	70	-194951.31	-116435.16	0.83	1927.8	3

Table 20: Comparison of the methods in dataset Retail. For each method, treewidth bound (Bound) and percentage of missing values (Missing %), the BIC score in the training dataset (BIC), the log-likelihood in the test dataset (LL), the imputation accuracy (Acc), the learning time (Time) and the treewidth (Treewidth) are shown. The best results are denoted in boldface.

Method	Bound	Missing (%)	BIC	LL	Acc	Time	Treewidth
TSEM	2	30	**-169868.25**	**-48301.22**	0.98	24.41	2
TSEM-poly	2	30	-169874.55	-48301.91	**0.98**	**23.66**	2
SEM-kMAX	2	30	-172712.84	-49206.89	0.98	1394.49	1
TSEM	2	50	**-122790.9**	**-48448.96**	**0.98**	19.26	2
TSEM-poly	2	50	**-122790.9**	**-48448.96**	**0.98**	**19.12**	2
SEM-kMAX	2	50	-129813.63	-51396.77	0.98	775.25	1
TSEM	2	70	**-74412.98**	**-49066.38**	**0.98**	16.27	1
TSEM-poly	2	70	-74430.13	-49111.22	0.98	**11.69**	1
SEM-kMAX	2	70	-85320.93	-56653.42	0.98	463.28	1
TSEM	3	30	**-169843.08**	-48302.27	**0.98**	30.54	3
TSEM-poly	3	30	-169849.2	**-48302.23**	0.98	**25.7**	3
SEM-kMAX	3	30	-172567.68	-49101.78	0.98	1241.79	3
TSEM	3	50	**-122790.9**	**-48448.96**	**0.98**	19.65	2
TSEM-poly	3	50	**-122790.9**	**-48448.96**	**0.98**	**19.51**	2
SEM-kMAX	3	50	-129668.73	-51286.82	0.98	932.49	2
TSEM	3	70	**-74409.02**	**-49055.75**	**0.98**	16.43	1
TSEM-poly	3	70	-74430.13	-49111.22	0.98	**11.81**	1
SEM-kMAX	3	70	-85320.09	-56600.66	0.98	463.84	2
TSEM	4	30	**-169843.08**	-48302.27	**0.98**	29	3
TSEM-poly	4	30	-169849.2	**-48302.23**	0.98	**24.63**	3
SEM-kMAX	4	30	-172564.81	-49104.68	0.98	927.67	3
TSEM	4	50	**-122790.45**	-48450.77	**0.98**	20.03	2
TSEM-poly	4	50	-122790.9	**-48448.96**	**0.98**	**18.93**	2
SEM-kMAX	4	50	-129668.73	-51286.82	0.98	1087.33	2
TSEM	4	70	**-74430.13**	**-49111.22**	**0.98**	11.75	1
TSEM-poly	4	70	**-74430.13**	**-49111.22**	**0.98**	**11.62**	1
SEM-kMAX	4	70	-85318.95	-56605.31	0.98	463.16	2
TSEM	5	30	**-169843.08**	-48302.27	**0.98**	32.1	3
TSEM-poly	5	30	-169849.2	**-48302.23**	0.98	**26.2**	3
SEM-kMAX	5	30	-172553.23	-49103.39	0.98	1084.02	3
TSEM	5	50	**-122790.9**	**-48448.96**	**0.98**	20.79	2
TSEM-poly	5	50	**-122790.9**	**-48448.96**	**0.98**	**19.68**	2
SEM-kMAX	5	50	-129671.57	-51303.84	0.98	931.54	2
TSEM	5	70	**-74430.13**	**-49111.22**	**0.98**	12.43	1
TSEM-poly	5	70	**-74430.13**	**-49111.22**	**0.98**	**11.94**	1
SEM-kMAX	5	70	-85314.79	-56604.76	0.98	463.26	2

Table 21: Comparison of the methods in dataset Pumsb-star. For each method, treewidth bound (Bound) and percentage of missing values (Missing %), the BIC score in the training dataset (BIC), the log-likelihood in the test dataset (LL), the imputation accuracy (Acc), the learning time (Time) and the treewidth (Treewidth) are shown. The best results are denoted in boldface.

Method	Bound	Missing (%)	BIC	LL	Acc	Time	Treewidth
TSEM	2	30	**-271598.99**	**-68123.05**	**0.94**	52.1	2
TSEM-poly	2	30	-272846.45	-68558.05	0.94	**45.35**	2
SEM-kMAX	2	30	-309951.48	-79547.85	0.93	1317.65	1
TSEM	2	50	**-228570.57**	**-74505.22**	**0.93**	44.95	2
TSEM-poly	2	50	-229341.04	-74744.02	0.93	**43.19**	2
SEM-kMAX	2	50	-298565.28	-105334.76	0.86	1134.01	1
TSEM	2	70	**-154600.26**	**-76954.32**	**0.92**	62.81	2
TSEM-poly	2	70	-154827.75	-76959.66	0.92	**54.76**	2
SEM-kMAX	2	70	-264859.25	-158671.53	0.8	1708.91	1
TSEM	3	30	**-258182.38**	-63860.79	0.95	92.69	3
TSEM-poly	3	30	-258487.78	**-63858.72**	**0.95**	**84.97**	3
SEM-kMAX	3	30	-265757.76	-66885.87	0.94	943.77	3
TSEM	3	50	**-217595.88**	**-67994.36**	0.93	110.16	3
TSEM-poly	3	50	-218224.3	-68139.44	**0.93**	**94.62**	3
SEM-kMAX	3	50	-238593.85	-79674.05	0.91	1712.2	3
TSEM	3	70	**-149989.31**	**-71865.41**	**0.92**	124.32	3
TSEM-poly	3	70	-150541.53	-72055.33	0.92	**110.7**	3
SEM-kMAX	3	70	-229586.96	-133750.12	0.83	3648.26	3
TSEM	4	30	**-253669.44**	**-62165.71**	0.95	163.91	4
TSEM-poly	4	30	-254167.22	-62242.15	**0.95**	**149**	4
SEM-kMAX	4	30	-265893.74	-66836.01	0.95	943.99	4
TSEM	4	50	**-207579.23**	**-64454.47**	**0.94**	199.45	4
TSEM-poly	4	50	-208832.19	-64600.93	0.94	**171.97**	4
SEM-kMAX	4	50	-236432.53	-78331.4	0.91	1712.96	4
TSEM	4	70	**-146114.98**	**-70773.48**	0.92	192.58	4
TSEM-poly	4	70	-146705.55	-70866.95	**0.92**	**176.8**	4
SEM-kMAX	4	70	-244498.17	-141842.47	0.82	2879.61	3
TSEM	5	30	**-248851.52**	**-60894.82**	0.95	291.93	5
TSEM-poly	5	30	-249604.2	-60922.44	**0.95**	**251.03**	5
SEM-kMAX	5	30	-262542.09	-65915.98	0.95	1322.73	5
TSEM	5	50	**-204356.96**	**-62748.26**	**0.94**	395.65	5
TSEM-poly	5	50	-205446.02	-62820.25	0.94	**346.08**	5
SEM-kMAX	5	50	-228232.7	-75209.32	0.92	2663.02	4
TSEM	5	70	**-145874.19**	-70263.8	**0.92**	348.18	5
TSEM-poly	5	70	-146259.39	**-70255.63**	0.92	**310.26**	5
SEM-kMAX	5	70	-243068.2	-139673.44	0.82	2300.99	3

Table 22: Comparison of the methods in dataset DNA. For each method, treewidth bound (Bound) and percentage of missing values (Missing %), the BIC score in the training dataset (BIC), the log-likelihood in the test dataset (LL), the imputation accuracy (Acc), the learning time (Time) and the treewidth (Treewidth) are shown. The best results are denoted in boldface.

Method	Bound	Missing (%)	BIC	LL	Acc	Time	Treewidth
TSEM	2	30	**-99245.57**	**-96190.11**	0.74	10.1	2
TSEM-poly	2	30	-99340.8	-96290.14	0.73	**8.8**	2
SEM-kMAX	2	30	-108826.11	-109733.38	**0.75**	1018.91	1
TSEM	2	50	-74714.27	**-96952.42**	0.7	10.17	2
TSEM-poly	2	50	**-74711.09**	-96958.07	0.7	**9.48**	2
SEM-kMAX	2	50	-86383.11	-121747.95	**0.75**	1018.45	1
TSEM	2	70	**-48899.4**	**-118793.32**	0.75	6.07	1
TSEM-poly	2	70	**-48899.4**	**-118793.32**	0.75	**6.02**	1
SEM-kMAX	2	70	-60902.05	-144968.78	**0.75**	1633.14	1
TSEM	3	30	**-99069.74**	**-95937.12**	0.74	12.13	3
TSEM-poly	3	30	-99182.51	-96049.22	0.74	**10.81**	3
SEM-kMAX	3	30	-104900.41	-103142.99	**0.75**	1222.61	3
TSEM	3	50	-74714.27	**-96952.42**	0.7	10.88	2
TSEM-poly	3	50	**-74710.41**	-96957.27	0.7	**10.4**	3
SEM-kMAX	3	50	-85162.46	-116955.46	**0.75**	2449.54	3
TSEM	3	70	**-48899.4**	**-118793.32**	0.75	6.92	1
TSEM-poly	3	70	**-48899.4**	**-118793.32**	0.75	**6.79**	1
SEM-kMAX	3	70	-60990.36	-143048.34	**0.75**	1225.2	2
TSEM	4	30	**-99040.09**	**-95918.5**	0.74	10.79	4
TSEM-poly	4	30	-99172.15	-96079.76	0.74	**9.01**	4
SEM-kMAX	4	30	-104987.23	-103169.64	**0.75**	1018.6	3
TSEM	4	50	-74714.27	**-96952.42**	0.7	9.48	2
TSEM-poly	4	50	**-74710.41**	-96957.27	0.7	**8.97**	3
SEM-kMAX	4	50	-85217.52	-117213.97	**0.75**	3262.4	3
TSEM	4	70	**-48899.4**	**-118793.32**	0.75	**6.43**	1
TSEM-poly	4	70	**-48899.4**	**-118793.32**	0.75	6.51	1
SEM-kMAX	4	70	-60993.33	-143042.28	**0.75**	1632.8	2
TSEM	5	30	**-99021.02**	**-95856.6**	0.74	16.46	5
TSEM-poly	5	30	-99147.7	-96048.67	0.74	**11.23**	5
SEM-kMAX	5	30	-104974.8	-103161.37	**0.75**	1630.15	2
TSEM	5	50	-74714.03	**-96952.69**	0.7	14.16	2
TSEM-poly	5	50	**-74710.41**	-96957.27	0.7	**9.93**	3
SEM-kMAX	5	50	-85176.07	-116953.46	**0.75**	2040.62	2
TSEM	5	70	**-48899.4**	**-118793.32**	0.75	8.9	1
TSEM-poly	5	70	**-48899.4**	**-118793.32**	0.75	**6.08**	1
SEM-kMAX	5	70	-60989.11	-143031.76	**0.75**	2246	2

Table 23: Comparison of the methods in dataset Kosarek. For each method, treewidth bound (Bound) and percentage of missing values (Missing %), the BIC score in the training dataset (BIC), the log-likelihood in the test dataset (LL), the imputation accuracy (Acc), the learning time (Time) and the treewidth (Treewidth) are shown. The best results are denoted in boldface.

Method	Bound	Missing (%)	BIC	LL	Acc	Time	Treewidth
TSEM	2	30	**-273769.05**	-75098.56	0.98	122.44	2
TSEM-poly	2	30	-273787.56	**-75090.71**	**0.98**	**118.53**	2
SEM-kMAX	2	30	-284817.59	-78738.18	0.98	1552.78	1
TSEM	2	50	**-200541.76**	**-76353.95**	0.98	139.24	2
TSEM-poly	2	50	-200633.66	-76394.05	**0.98**	**133.76**	2
SEM-kMAX	2	50	-214233.92	-82464.05	0.98	2243.19	1
TSEM	2	70	**-125027.24**	**-79356.72**	**0.98**	**130.82**	2
TSEM-poly	2	70	-125090.2	-79419.36	0.98	142.24	2
SEM-kMAX	2	70	-141910.06	-90836.32	0.98	2262.96	1
TSEM	3	30	**-271675.35**	-74502.16	0.98	208.33	3
TSEM-poly	3	30	-271775.84	**-74496.24**	**0.98**	**195.16**	3
SEM-kMAX	3	30	-276502.9	-76013.51	0.98	1118.23	2
TSEM	3	50	**-198966.24**	**-75710.35**	0.98	179.68	3
TSEM-poly	3	50	-199061.75	-75720.41	**0.98**	**175.41**	3
SEM-kMAX	3	50	-207625.29	-78926.24	0.98	1588.71	3
TSEM	3	70	-124678.71	**-79493.3**	0.98	161.45	3
TSEM-poly	3	70	**-124666.7**	-79599.12	0.98	**158.63**	3
SEM-kMAX	3	70	-138471.47	-88156.98	**0.98**	3912.78	2
TSEM	4	30	**-270270.23**	**-74149.21**	**0.98**	307.83	4
TSEM-poly	4	30	-270442.39	-74166.84	0.98	**269.58**	4
SEM-kMAX	4	30	-276047.59	-75922.1	0.98	1792.32	3
TSEM	4	50	**-197927.08**	**-75519.96**	0.98	239.64	4
TSEM-poly	4	50	-198124.76	-75559.46	0.98	**232.89**	4
SEM-kMAX	4	50	-207697.9	-79122.37	**0.98**	2273.6	3
TSEM	4	70	**-124317.47**	**-79448.19**	0.98	220.48	4
TSEM-poly	4	70	-124605.64	-79469.14	0.98	**165.66**	4
SEM-kMAX	4	70	-138809.48	-88752.11	**0.98**	1833.61	3
TSEM	5	30	**-269817.98**	-74037.64	0.98	487.05	5
TSEM-poly	5	30	-269916.58	**-74032.52**	**0.98**	**450.67**	5
SEM-kMAX	5	30	-275990.35	-75953.32	0.98	2017.9	3
TSEM	5	50	**-197746.1**	**-75574.59**	0.98	369.13	5
TSEM-poly	5	50	-197928.48	-75602.28	**0.98**	**349.54**	5
SEM-kMAX	5	50	-207848.97	-79420.35	0.98	1815.38	3
TSEM	5	70	-124360.83	-79308.96	**0.98**	**257.33**	5
TSEM-poly	5	70	**-124331.14**	**-79207.73**	0.98	262.92	5
SEM-kMAX	5	70	-139272.06	-89017.82	0.98	1606.22	3

Table 24: Comparison of the methods in dataset MSWeb. For each method, treewidth bound (Bound) and percentage of missing values (Missing %), the BIC score in the training dataset (BIC), the log-likelihood in the test dataset (LL), the imputation accuracy (Acc), the learning time (Time) and the treewidth (Treewidth) are shown. The best results are denoted in boldface.

Method	Bound	Missing (%)	BIC	LL	Acc	Time	Treewidth
TSEM	2	30	**-214351.24**	-50453.58	**0.99**	122.44	2
TSEM-poly	2	30	-214373.41	**-50442.93**	**0.99**	**119.14**	2
SEM-kMAX	2	30	-219184.39	-51877.43	0.99	2749.09	1
TSEM	2	50	**-157147.61**	**-51014.32**	**0.99**	131.27	2
TSEM-poly	2	50	-157205.05	-51061.33	0.99	**107.65**	2
SEM-kMAX	2	50	-165192.77	-53912.33	0.99	4158.45	1
TSEM	2	70	**-97020.65**	-52280.99	**0.99**	130.28	2
TSEM-poly	2	70	-97029.58	**-52276.1**	0.99	**90.02**	2
SEM-kMAX	2	70	-110577.04	-59488.73	0.99	2784.15	1
TSEM	3	30	**-213438.41**	**-50121.72**	0.99	151.22	3
TSEM-poly	3	30	-213649.14	-50166.72	**0.99**	**146.94**	3
SEM-kMAX	3	30	-218131.13	-51442.84	0.99	2409.71	2
TSEM	3	50	**-156681.88**	**-50790.46**	0.99	122.91	3
TSEM-poly	3	50	-156933.3	-51000.94	**0.99**	**117.65**	3
SEM-kMAX	3	50	-165168.6	-53845.87	0.99	2779.32	2
TSEM	3	70	**-97020.66**	-52280.98	**0.99**	131.47	2
TSEM-poly	3	70	-97029.58	**-52276.1**	0.99	**92.08**	2
SEM-kMAX	3	70	-110014.61	-58993.06	0.99	2446.49	2
TSEM	4	30	-213432.64	-50093.24	**0.99**	**182.61**	4
TSEM-poly	4	30	**-213319.27**	**-50041.32**	0.99	184.07	4
SEM-kMAX	4	30	-217994.22	-51380.75	0.99	3098.37	2
TSEM	4	50	**-156584.83**	**-50715.26**	**0.99**	140.37	4
TSEM-poly	4	50	-156680.51	-50847.74	0.99	**132.21**	4
SEM-kMAX	4	50	-164825.49	-53647.41	0.99	3827.62	3
TSEM	4	70	**-97029.58**	**-52276.1**	**0.99**	94.06	2
TSEM-poly	4	70	-97029.58	**-52276.1**	**0.99**	**93.75**	2
SEM-kMAX	4	70	-110262.5	-59191.04	0.99	3153.83	2
TSEM	5	30	**-213102.76**	-49971.7	0.99	279.47	5
TSEM-poly	5	30	-213133.38	**-49926.63**	**0.99**	**251.3**	5
SEM-kMAX	5	30	-217332.93	-51180.09	0.99	1711.77	4
TSEM	5	50	-156567.65	**-50741.35**	0.99	185.21	5
TSEM-poly	5	50	**-156567.57**	-50786.57	**0.99**	**153.82**	5
SEM-kMAX	5	50	-164756.3	-53550.75	0.99	3825.97	3
TSEM	5	70	**-97020.66**	-52280.98	**0.99**	137.02	2
TSEM-poly	5	70	-97029.58	**-52276.1**	0.99	**92.86**	2
SEM-kMAX	5	70	-110252.58	-59128.13	0.99	2796.06	2

Table 25: Comparison of the methods in dataset Book. For each method, treewidth bound (Bound) and percentage of missing values (Missing %), the BIC score in the training dataset (BIC), the log-likelihood in the test dataset (LL), the imputation accuracy (Acc), the learning time (Time) and the treewidth (Treewidth) are shown. The best results are denoted in boldface.

Method	Bound	Missing (%)	BIC	LL	Acc	Time	Treewidth
TSEM	2	30	-226448.21	**-65626.7**	**0.98**	235.73	2
TSEM-poly	2	30	**-226417.91**	-65631.29	0.98	**212.44**	2
SEM-kMAX	2	30	-230669.21	-67552.26	0.98	6225.67	1
TSEM	2	50	**-164639.19**	-67405.66	0.98	205.73	2
TSEM-poly	2	50	-164666.43	**-67386.15**	**0.98**	**184.91**	2
SEM-kMAX	2	50	-169451.78	-70184.03	0.98	5679.33	1
TSEM	2	70	-100916.78	**-70584.33**	**0.98**	182.44	2
TSEM-poly	2	70	**-100861.44**	-70603.73	0.98	**180.82**	2
SEM-kMAX	2	70	-107127.8	-76088.25	0.98	5699.94	1
TSEM	3	30	**-224868.96**	-65409.91	0.98	276.99	3
TSEM-poly	3	30	-225230.15	**-65260.97**	**0.98**	**264.38**	3
SEM-kMAX	3	30	-228519.12	-66687.41	0.98	3973.83	3
TSEM	3	50	**-163563.67**	-67247.1	0.98	273.73	3
TSEM-poly	3	50	-163678.39	**-67245.15**	**0.98**	**233.35**	3
SEM-kMAX	3	50	-168090.82	-70107.36	0.98	5131.07	2
TSEM	3	70	**-100293.79**	**-70679.93**	**0.98**	227.9	3
TSEM-poly	3	70	-100366.52	-70709.77	0.98	**224.71**	3
SEM-kMAX	3	70	-107415.77	-76063.28	0.98	4560.08	2
TSEM	4	30	**-224307.87**	-65320.1	0.98	367.65	4
TSEM-poly	4	30	-224458.1	**-65137.47**	0.98	**336.78**	4
SEM-kMAX	4	30	-228131.42	-66180.43	**0.98**	3418.79	3
TSEM	4	50	**-162820.77**	**-67174.21**	**0.98**	324.35	4
TSEM-poly	4	50	-163085.11	-67200.82	0.98	**306.76**	4
SEM-kMAX	4	50	-168547.17	-69806	0.98	5129.5	3
TSEM	4	70	**-100037.36**	**-70833.55**	0.98	**284.06**	4
TSEM-poly	4	70	-100208.38	-70839.44	**0.98**	293.68	4
SEM-kMAX	4	70	-107025.4	-76131.6	0.98	5714.87	2
TSEM	5	30	-224002.87	-65235.87	0.98	476.11	5
TSEM-poly	5	30	**-223930.76**	**-65141.7**	0.98	**449.19**	5
SEM-kMAX	5	30	-228374.08	-66544.63	**0.98**	3975.92	3
TSEM	5	50	**-162726.81**	**-67131.93**	0.98	419.5	5
TSEM-poly	5	50	-163076.93	-67188.29	0.98	**367.07**	5
SEM-kMAX	5	50	-167748.72	-69079.22	**0.98**	3982.63	3
TSEM	5	70	-99989.25	-70846.24	0.98	323.67	5
TSEM-poly	5	70	**-99858.82**	**-70814.16**	0.98	**321.84**	5
SEM-kMAX	5	70	-107114.61	-75800.43	**0.98**	3985.69	2

Table 26: Comparison of the methods in dataset EachMovie. For each method, treewidth bound (Bound) and percentage of missing values (Missing %), the BIC score in the training dataset (BIC), the log-likelihood in the test dataset (LL), the imputation accuracy (Acc), the learning time (Time) and the treewidth (Treewidth) are shown. The best results are denoted in boldface.

Method	Bound	Missing (%)	BIC	LL	Acc	Time	Treewidth
TSEM	2	30	**-201472.9**	**-36303.39**	**0.96**	147.35	2
TSEM-poly	2	30	-201780.99	-36344.74	0.96	**138.75**	2
SEM-kMAX	2	30	-217258.13	-39550.34	0.95	2243.04	1
TSEM	2	50	**-149198.01**	**-37928.46**	**0.95**	129.66	2
TSEM-poly	2	50	-149204.45	-37928.76	**0.95**	**127.17**	2
SEM-kMAX	2	50	-164739.02	-42476.3	0.95	5624.64	1
TSEM	2	70	**-95282.89**	**-42141.09**	0.95	117.62	2
TSEM-poly	2	70	-95382.39	-42144.02	**0.95**	**116.94**	2
SEM-kMAX	2	70	-111185.43	-49087.16	0.94	4506.16	1
TSEM	3	30	-197174.66	-35332.01	0.96	205.62	3
TSEM-poly	3	30	**-196839.15**	**-35287.3**	**0.96**	**193.91**	3
SEM-kMAX	3	30	-207776.78	-38072.19	0.95	3943.25	2
TSEM	3	50	**-145790.85**	-37176.59	**0.95**	182.81	3
TSEM-poly	3	50	-145909.76	**-37167.25**	0.95	**170.71**	3
SEM-kMAX	3	50	-159258.37	-42008.24	0.95	5084.04	3
TSEM	3	70	-94064.25	-42293.57	0.95	158.8	3
TSEM-poly	3	70	**-93834.98**	**-42266.43**	**0.95**	**143.96**	3
SEM-kMAX	3	70	-108330.61	-47928.04	0.94	2827.82	3
TSEM	4	30	-194688.5	**-34827.64**	0.96	273.85	4
TSEM-poly	4	30	**-194510.54**	-34880.2	**0.96**	**257.57**	4
SEM-kMAX	4	30	-206794.53	-38221.12	0.95	4510.54	4
TSEM	4	50	-144419.13	-37216.96	0.95	230.79	4
TSEM-poly	4	50	**-144403.51**	**-37158.38**	**0.95**	**216.38**	4
SEM-kMAX	4	50	-159225.49	-41321.13	0.95	3385.35	4
TSEM	4	70	-92630.29	**-42060.63**	0.95	**184.5**	4
TSEM-poly	4	70	**-92579.83**	-42176.75	**0.95**	192.31	4
SEM-kMAX	4	70	-109480.86	-48778.99	0.94	4523.92	4
TSEM	5	30	-193600.48	-34923.47	0.96	382.65	5
TSEM-poly	5	30	**-193498.52**	**-34922.13**	**0.96**	**375.17**	5
SEM-kMAX	5	30	-206143.94	-37917.01	0.95	3380.93	5
TSEM	5	50	**-143270.36**	-36833.73	**0.95**	327.23	5
TSEM-poly	5	50	-143283.85	**-36786.53**	0.95	**327.16**	5
SEM-kMAX	5	50	-158017.83	-40953.58	0.95	2257.87	3
TSEM	5	70	**-92228.3**	**-41990.66**	**0.95**	251.4	5
TSEM-poly	5	70	-92388.95	-41999.49	0.95	**218.24**	5
SEM-kMAX	5	70	-108753.42	-48848.5	0.94	3962.91	4

Table 27: Comparison of the methods in dataset WebKB. For each method, treewidth bound (Bound) and percentage of missing values (Missing %), the BIC score in the training dataset (BIC), the log-likelihood in the test dataset (LL), the imputation accuracy (Acc), the learning time (Time) and the treewidth (Treewidth) are shown. The best results are denoted in boldface.

Method	Bound	Missing (%)	BIC	LL	Acc	Time	Treewidth
TSEM	2	30	**-313135.36**	**-136975.95**	**0.94**	276.42	2
TSEM-poly	2	30	-313216.11	-137005.57	0.94	**258.17**	2
SEM-kMAX	2	30	-321044.02	-141763.25	0.94	5598.31	1
TSEM	2	50	-229473.08	-141497.17	**0.94**	259.8	2
TSEM-poly	2	50	**-229305.06**	**-141360.53**	0.94	**217.01**	2
SEM-kMAX	2	50	-240223.45	-149681.26	0.94	5606.57	1
TSEM	2	70	**-142628.55**	**-148300.78**	**0.94**	517.6	2
TSEM-poly	2	70	-142643.96	-148341.41	0.94	**507.89**	2
SEM-kMAX	2	70	-159068.94	-165577.51	0.94	7486.47	1
TSEM	3	30	**-311830.23**	-136404.59	**0.94**	330.81	3
TSEM-poly	3	30	-311959.85	**-136363.62**	0.94	**319.38**	3
SEM-kMAX	3	30	-317454.26	-140082.68	0.94	6564.99	3
TSEM	3	50	**-228115.97**	-140999.57	0.94	286.77	3
TSEM-poly	3	50	-228380.3	**-140936.24**	**0.94**	**265.68**	3
SEM-kMAX	3	50	-238637.51	-148847.26	0.94	8426.25	2
TSEM	3	70	**-142439.01**	**-148304.1**	0.94	561.26	3
TSEM-poly	3	70	-142456.3	-148333.49	**0.94**	**542.92**	3
SEM-kMAX	3	70	-158874.51	-166810.79	0.94	12174.29	2
TSEM	4	30	**-311342.82**	**-136265.14**	0.94	367.52	4
TSEM-poly	4	30	-311529.72	-136275.83	**0.94**	**346.1**	4
SEM-kMAX	4	30	-317443.63	-140113.78	0.94	6563.06	4
TSEM	4	50	**-228040.3**	**-140982.76**	**0.94**	**286.58**	4
TSEM-poly	4	50	-228062.23	-141031.1	0.94	296.24	4
SEM-kMAX	4	50	-238729.61	-149611.85	0.94	14982.18	3
TSEM	4	70	**-142155.54**	**-148511.35**	**0.94**	587.29	4
TSEM-poly	4	70	-142185.93	-148585.74	0.94	**547.32**	4
SEM-kMAX	4	70	-158987.83	-166944.61	0.94	10295.02	3
TSEM	5	30	**-310623.74**	-136085.17	**0.94**	450.81	5
TSEM-poly	5	30	-310803.05	**-136042.8**	0.94	**440.4**	5
SEM-kMAX	5	30	-317557.91	-140102.21	0.94	7500.96	3
TSEM	5	50	**-227346.79**	-141132.24	0.94	368.66	5
TSEM-poly	5	50	-227518.81	**-141035.34**	**0.94**	**366.94**	5
SEM-kMAX	5	50	-238083.12	-148280.65	0.94	7502.83	4
TSEM	5	70	-142193.59	**-148585**	**0.94**	**577.56**	5
TSEM-poly	5	70	**-142173.47**	-148673.31	0.94	581.94	5
SEM-kMAX	5	70	-157797.22	-167226.85	0.94	11238.21	3

Table 28: Comparison of the methods in dataset Reuters-521. For each method, treewidth bound (Bound) and percentage of missing values (Missing %), the BIC score in the training dataset (BIC), the log-likelihood in the test dataset (LL), the imputation accuracy (Acc), the learning time (Time) and the treewidth (Treewidth) are shown. The best results are denoted in boldface.

Method	Bound	Missing (%)	BIC	LL	Acc	Time	Treewidth
TSEM	2	30	**-477799.24**	-143209.31	**0.97**	606.15	2
TSEM-poly	2	30	-477883.53	**-143133.75**	0.97	**588.84**	2
SEM-kMAX	2	30	-492195.73	-147866.14	0.97	11102.6	1
TSEM	2	50	**-349064.23**	**-147658.47**	0.97	**506.49**	2
TSEM-poly	2	50	-349128.33	-147680.41	**0.97**	521.62	2
SEM-kMAX	2	50	-368446.42	-154279.32	0.97	11129.39	1
TSEM	2	70	**-219693.11**	-157322.07	**0.97**	623.64	2
TSEM-poly	2	70	-219718.44	**-157289.57**	0.97	**559.38**	2
SEM-kMAX	2	70	-241616.65	-171366.78	0.96	8101.74	1
TSEM	3	30	**-472378.79**	**-142010.34**	**0.97**	846.07	3
TSEM-poly	3	30	-472681.68	-142061.29	0.97	**816.24**	3
SEM-kMAX	3	30	-487029.74	-146694.79	0.97	8190.37	3
TSEM	3	50	**-345936.07**	-146932.52	0.97	**701.1**	3
TSEM-poly	3	50	-346016.61	**-146845.03**	**0.97**	715.94	3
SEM-kMAX	3	50	-363036.1	-151247.22	0.97	5116.47	3
TSEM	3	70	**-218343.82**	**-156467.61**	**0.97**	685.75	3
TSEM-poly	3	70	-218441.1	-156623.62	0.97	**679.49**	3
SEM-kMAX	3	70	-240666.28	-169889.87	0.96	8141.84	2
TSEM	4	30	**-469693.9**	**-141147.99**	0.97	1112.55	4
TSEM-poly	4	30	-470032.49	-141271.27	**0.97**	**1001.36**	4
SEM-kMAX	4	30	-487757.36	-146649.61	0.97	8189.66	3
TSEM	4	50	**-344751.07**	-146906.25	0.97	805.03	4
TSEM-poly	4	50	-344820.3	**-146878.51**	**0.97**	**756.76**	4
SEM-kMAX	4	50	-362478.15	-151460.78	0.97	8223.45	3
TSEM	4	70	**-217206.97**	**-156391.82**	0.97	754.63	4
TSEM-poly	4	70	-217446.64	-156435.44	**0.97**	**734.9**	4
SEM-kMAX	4	70	-240026.78	-169326.46	0.96	7116.36	2
TSEM	5	30	-472399.72	-142256.89	0.97	**1341.15**	5
TSEM-poly	5	30	**-468710.3**	**-140644.5**	**0.97**	1483.53	5
SEM-kMAX	5	30	-486366.75	-146214.15	0.97	5107.98	3
TSEM	5	50	-345230.77	-147232.68	0.97	**1080.91**	5
TSEM-poly	5	50	**-343825.23**	**-146304.08**	**0.97**	1148.09	5
SEM-kMAX	5	50	-361967.81	-151594.14	0.97	6141.43	3
TSEM	5	70	**-217148.95**	-156701.25	0.97	947.51	5
TSEM-poly	5	70	-217295.22	**-156674.08**	**0.97**	**858.02**	5
SEM-kMAX	5	70	-239935.56	-169293.43	0.96	7115.73	3

Table 29: Comparison of the methods in dataset 20 NewsGroup. For each method, treewidth bound (Bound) and percentage of missing values (Missing %), the BIC score in the training dataset (BIC), the log-likelihood in the test dataset (LL), the imputation accuracy (Acc), the learning time (Time) and the treewidth (Treewidth) are shown. The best results are denoted in boldface.

Method	Bound	Missing (%)	BIC	LL	Acc	Time	Treewidth
TSEM	2	30	**-1164613.48**	**-611127.07**	0.95	1066.94	2
TSEM-poly	2	30	-1164753.99	-611205.14	**0.95**	**991.25**	2
SEM-kMAX	2	30	-1196427.05	-631424	0.95	8319.02	1
TSEM	2	50	-841521.67	-622461.16	**0.95**	966.82	2
TSEM-poly	2	50	**-841082.91**	**-622148.33**	0.95	**903.17**	2
SEM-kMAX	2	50	-885464.98	-660095.73	0.95	11583.76	1
TSEM	2	70	-513907.5	-648585.21	0.95	1085.87	2
TSEM-poly	2	70	**-513891.28**	**-648325.36**	**0.95**	**1029.4**	2
SEM-kMAX	2	70	-569613.31	-718447.08	0.95	15902.42	1
TSEM	3	30	**-1157865.48**	-608008.68	**0.95**	1619.97	3
TSEM-poly	3	30	-1158177.86	**-607892.95**	0.95	**1524.52**	3
SEM-kMAX	3	30	-1185233.74	-625767.47	0.95	4304.63	3
TSEM	3	50	**-836036.77**	-620771.76	**0.95**	1322.66	3
TSEM-poly	3	50	-836612.61	**-620569.87**	0.95	**1251.5**	3
SEM-kMAX	3	50	-874486.85	-652410.16	0.95	5357.65	2
TSEM	3	70	**-511479.11**	-648487.54	0.95	1298.04	3
TSEM-poly	3	70	-511610.94	**-647934.25**	**0.95**	**1217.05**	3
SEM-kMAX	3	70	-565991.84	-715640.36	0.95	9562.11	3
TSEM	4	30	**-1154883.18**	**-606941.25**	0.95	2140.01	4
TSEM-poly	4	30	-1155166.33	-607316	**0.95**	**1934.09**	4
SEM-kMAX	4	30	-1185114.57	-623844.43	0.95	3173.34	3
TSEM	4	50	-834873.44	**-620519.76**	0.95	1737.2	4
TSEM-poly	4	50	**-834454.04**	-620612.76	**0.95**	**1531.32**	4
SEM-kMAX	4	50	-873676.96	-650151.77	0.95	4266.6	3
TSEM	4	70	**-510292.34**	-648539.6	0.95	1565.03	4
TSEM-poly	4	70	-510385.19	**-648325.34**	**0.95**	**1451.67**	4
SEM-kMAX	4	70	-565527.89	-714857.54	0.95	8509.51	2
TSEM	5	30	**-1152606.4**	**-606612.27**	**0.95**	3100.26	5
TSEM-poly	5	30	-1153067.9	-606619.49	0.95	**2856.11**	5
SEM-kMAX	5	30	-1182590.65	-625578.92	0.95	6486.73	3
TSEM	5	50	-833743.71	-620879.04	0.95	**2305.83**	5
TSEM-poly	5	50	**-832526.35**	**-619528.39**	**0.95**	2338.14	5
SEM-kMAX	5	50	-876593.32	-653617.73	0.95	5355.02	2
TSEM	5	70	**-509763.63**	-648710.37	**0.95**	**1901.37**	5
TSEM-poly	5	70	-509923.94	**-648428.57**	0.95	1990.88	5
SEM-kMAX	5	70	-565160.26	-711577.37	0.95	6339.79	2

Table 30: Comparison of the methods in dataset Movie reviews. For each method, treewidth bound (Bound) and percentage of missing values (Missing %), the BIC score in the training dataset (BIC), the log-likelihood in the test dataset (LL), the imputation accuracy (Acc), the learning time (Time) and the treewidth (Treewidth) are shown. The best results are denoted in boldface.

Method	Bound	Missing (%)	BIC	LL	Acc	Time	Treewidth
TSEM	2	30	-392663.79	**-85874.43**	0.87	250.79	2
TSEM-poly	2	30	**-392659.14**	-85931.44	0.87	**239.81**	2
SEM-kMAX	2	30	-400754.32	-87549.46	**0.87**	8937.15	1
TSEM	2	50	**-283662.38**	**-86292.09**	0.87	602.95	2
TSEM-poly	2	50	-283684.71	-86292.39	0.87	**586.46**	2
SEM-kMAX	2	50	-302160.6	-92376.09	**0.87**	13424.07	1
TSEM	2	70	**-173672.2**	**-86690.73**	**0.87**	370.06	1
TSEM-poly	2	70	**-173672.2**	**-86690.73**	**0.87**	**369.28**	1
SEM-kMAX	2	70	-202020.19	-103570.83	0.87	14549.96	1
TSEM	3	30	**-392496.33**	-85899.26	0.87	**268.81**	3
TSEM-poly	3	30	-392525.7	**-85888.89**	0.87	271.26	3
SEM-kMAX	3	30	-400865.46	-87382.08	**0.87**	5586.29	2
TSEM	3	50	**-283662.38**	**-86292.09**	0.87	632.35	2
TSEM-poly	3	50	-283684.71	-86292.39	0.87	**620.87**	2
SEM-kMAX	3	50	-302215.61	-92467.88	**0.87**	14547.8	2
TSEM	3	70	**-173672.2**	**-86690.73**	**0.87**	393.43	1
TSEM-poly	3	70	**-173672.2**	**-86690.73**	**0.87**	**393.16**	1
SEM-kMAX	3	70	-201814.94	-103459.65	0.87	16794.12	2
TSEM	4	30	**-392467.75**	**-85923.82**	0.87	**276.09**	4
TSEM-poly	4	30	-392479.64	-85927.02	0.87	281.1	4
SEM-kMAX	4	30	-400684.45	-87540.82	**0.87**	15651.98	3
TSEM	4	50	**-283650.98**	-86294.31	0.87	603.31	2
TSEM-poly	4	50	-283684.71	**-86292.39**	0.87	**587.12**	2
SEM-kMAX	4	50	-302428.7	-92388.04	**0.87**	14545.22	2
TSEM	4	70	**-173672.2**	**-86690.73**	**0.87**	374.76	1
TSEM-poly	4	70	**-173672.2**	**-86690.73**	**0.87**	**374.25**	1
SEM-kMAX	4	70	-201960.31	-103436.75	0.87	10070.48	2
TSEM	5	30	**-392366.1**	**-85902.04**	0.87	296.12	5
TSEM-poly	5	30	-392423.05	-85945.65	0.87	**274.3**	5
SEM-kMAX	5	30	-400724.1	-87449.71	**0.87**	7820.78	3
TSEM	5	50	**-283662.38**	**-86292.09**	0.87	613.14	2
TSEM-poly	5	50	-283684.71	-86292.39	0.87	**597.86**	2
SEM-kMAX	5	50	-301877.53	-92556.19	**0.87**	21261.99	2
TSEM	5	70	**-173672.2**	**-86690.73**	**0.87**	393.1	1
TSEM-poly	5	70	**-173672.2**	**-86690.73**	**0.87**	**378.98**	1
SEM-kMAX	5	70	-202315.34	-103636.8	0.87	6711.71	1

Table 31: Comparison of the methods in dataset BBC. For each method, treewidth bound (Bound) and percentage of missing values (Missing %), the BIC score in the training dataset (BIC), the log-likelihood in the test dataset (LL), the imputation accuracy (Acc), the learning time (Time) and the treewidth (Treewidth) are shown. The best results are denoted in boldface.

Method	Bound	Missing (%)	BIC	LL	Acc	Time	Treewidth
TSEM	2	30	**-299799.35**	**-86841.51**	0.92	**307.06**	2
TSEM-poly	2	30	-299900.4	-86856.47	0.92	307.87	2
SEM-kMAX	2	30	-305746.42	-89100.12	**0.92**	4717.21	1
TSEM	2	50	**-217882.53**	-89236.27	0.92	351.36	2
TSEM-poly	2	50	-217954.04	**-89192.44**	0.92	**345.73**	2
SEM-kMAX	2	50	-227908.42	-93918.46	**0.92**	10628.4	1
TSEM	2	70	**-135867.72**	-90784.29	0.92	554.76	2
TSEM-poly	2	70	-135867.82	**-90782.87**	**0.92**	**551.61**	2
SEM-kMAX	2	70	-149567.51	-103588.23	0.92	15365.97	1
TSEM	3	30	**-299158.41**	-86929	0.92	379.49	3
TSEM-poly	3	30	-299300.13	**-86916.02**	0.92	**347.72**	3
SEM-kMAX	3	30	-304860.43	-88952.72	**0.92**	10632.77	2
TSEM	3	50	**-217658.87**	**-89089.89**	**0.92**	386.31	3
TSEM-poly	3	50	-217728.39	-89117.09	0.92	**365.99**	3
SEM-kMAX	3	50	-228128.52	-94038.48	0.92	12996.18	2
TSEM	3	70	-135800.27	-90781.63	0.92	594.61	3
TSEM-poly	3	70	**-135793.3**	**-90761.76**	**0.92**	**571.05**	3
SEM-kMAX	3	70	-149648.05	-103630.28	0.92	14187.41	2
TSEM	4	30	**-298891.3**	**-86782.44**	0.92	409.64	4
TSEM-poly	4	30	-299117.69	-86895.03	0.92	**381.93**	4
SEM-kMAX	4	30	-304993.05	-89052.04	**0.92**	9453.44	3
TSEM	4	50	**-217403.72**	-89034.01	0.92	424.38	4
TSEM-poly	4	50	-217435.65	**-89005.55**	**0.92**	**416.33**	4
SEM-kMAX	4	50	-228111.6	-93966.25	0.92	9454.75	2
TSEM	4	70	**-135787.29**	-90769.1	0.92	594.3	3
TSEM-poly	4	70	-135793.3	**-90761.76**	**0.92**	**582.52**	3
SEM-kMAX	4	70	-149554.5	-103464.48	0.92	17738.53	2
TSEM	5	30	**-298796.27**	**-86774.32**	0.92	424.21	5
TSEM-poly	5	30	-298910.84	-86848.35	0.92	**415.65**	5
SEM-kMAX	5	30	-305158.1	-88819.35	**0.92**	5905.85	3
TSEM	5	50	**-217364.11**	**-89041.77**	0.92	**415.82**	5
TSEM-poly	5	50	-217406.32	-89043.86	0.92	416.21	5
SEM-kMAX	5	50	-228088.34	-93957.29	**0.92**	11814.46	3
TSEM	5	70	-135802.92	-90765.38	0.92	561.25	3
TSEM-poly	5	70	**-135793.3**	**-90761.76**	**0.92**	**551.32**	3
SEM-kMAX	5	70	-149865.98	-103749.94	0.92	15363.94	2

Table 32: Comparison of the methods in dataset Voting. For each method, treewidth bound (Bound) and percentage of missing values (Missing %), the BIC score in the training dataset (BIC), the log-likelihood in the test dataset (LL), the imputation accuracy (Acc), the learning time (Time) and the treewidth (Treewidth) are shown. The best results are denoted in boldface.

Method	Bound	Missing (%)	BIC	LL	Acc	Time	Treewidth
TSEM	2	30	**-175438.05**	-69270.96	**0.96**	1773.1	2
TSEM-poly	2	30	-175701.44	**-68833.32**	0.96	**1706.65**	2
SEM-kMAX	2	30	-643895.13	-223082.26	0.72	8983.05	1
TSEM	2	50	**-137995.81**	-76425.96	**0.95**	2127.23	2
TSEM-poly	2	50	-138283.38	**-76360.33**	0.95	**2067.38**	2
SEM-kMAX	2	50	-513045.22	-224817.66	0.72	10339.07	1
TSEM	2	70	**-94193.1**	**-94060**	**0.95**	2186.19	2
TSEM-poly	2	70	-95692.38	-94382.9	0.95	**2099.07**	2
SEM-kMAX	2	70	-165462.08	-155884.41	0.83	16890.74	1
TSEM	3	30	**-169371.96**	-66380	**0.96**	2176.98	3
TSEM-poly	3	30	-170051.8	**-66179.32**	0.96	**2082.31**	3
SEM-kMAX	3	30	-643804.91	-223012.33	0.72	10152.38	2
TSEM	3	50	**-133527.29**	-74741.6	**0.95**	2391.6	3
TSEM-poly	3	50	-134184.07	**-74339.16**	0.95	**2304.89**	3
SEM-kMAX	3	50	-513003.86	-224716.19	0.72	13286.74	2
TSEM	3	70	-92382.05	-93525.82	0.95	2467.8	3
TSEM-poly	3	70	**-92168.49**	**-92606.25**	**0.95**	**2377.79**	3
SEM-kMAX	3	70	-175416.86	-169045.2	0.82	55834.15	3
TSEM	4	30	**-166301.72**	-65536.75	**0.96**	2459.08	4
TSEM-poly	4	30	-167664.87	**-65377.44**	0.96	**2348.13**	4
SEM-kMAX	4	30	-644008.43	-223040	0.72	21242.52	2
TSEM	4	50	**-131472.29**	-74061.95	**0.96**	2884.45	4
TSEM-poly	4	50	-132205.28	**-73894.98**	0.96	**2774.12**	4
SEM-kMAX	4	50	-513285.05	-224742.89	0.72	15426.74	2
TSEM	4	70	**-89999.14**	-91955.12	0.95	**2618.68**	4
TSEM-poly	4	70	-90820.18	**-91769.58**	**0.95**	2680.56	4
SEM-kMAX	4	70	-179603.78	-167189.89	0.82	59194.93	3
TSEM	5	30	**-163862.78**	-64430.43	0.96	3182.15	5
TSEM-poly	5	30	-165346.64	**-64153.24**	**0.96**	**3003.01**	5
SEM-kMAX	5	30	-643802.75	-223015.8	0.72	10785.32	2
TSEM	5	50	**-130602.22**	-73605.42	**0.96**	3953.49	5
TSEM-poly	5	50	-131497.64	**-73434.96**	0.96	**3808.87**	5
SEM-kMAX	5	50	-513006.18	-224721.38	0.72	10205.63	2
TSEM	5	70	**-89505.06**	**-91668.41**	**0.95**	3456.38	5
TSEM-poly	5	70	-89972.49	-91892	0.95	**3258.57**	5
SEM-kMAX	5	70	-179527.24	-161377.16	0.81	45751.83	4

Table 33: Comparison of the methods in dataset Ad. For each method, treewidth bound (Bound) and percentage of missing values (Missing %), the BIC score in the training dataset (BIC), the log-likelihood in the test dataset (LL), the imputation accuracy (Acc), the learning time (Time) and the treewidth (Treewidth) are shown. The best results are denoted in boldface.

Method	Bound	Missing (%)	BIC	LL	Acc	Time	Treewidth
TSEM	2	30	**-44824.58**	**-8740.92**	**1**	881.01	2
TSEM-poly	2	30	-45185.39	-8759.14	1	**835.33**	2
SEM-kMAX	2	30	-50685.1	-10848.2	1	6953.52	1
TSEM	2	50	**-40471.17**	**-10148.64**	**1**	849.86	2
TSEM-poly	2	50	-40750.75	-10183.81	1	**806.33**	2
SEM-kMAX	2	50	-45541.54	-12022.5	1	6959.04	1
TSEM	2	70	**-33150.28**	**-14123.67**	**1**	722.12	2
TSEM-poly	2	70	-33304.91	-14134.72	1	**687.76**	2
SEM-kMAX	2	70	-37828.11	-16409.05	0.99	6966.9	1
TSEM	3	30	**-44350.98**	-8581.86	1	981.36	3
TSEM-poly	3	30	-44665.81	**-8554.83**	**1**	**933.31**	3
SEM-kMAX	3	30	-50529.19	-10770.78	1	6955.64	2
TSEM	3	50	**-40218.09**	**-10034.99**	**1**	983.74	3
TSEM-poly	3	50	-40553.58	-10077.71	1	**930.62**	3
SEM-kMAX	3	50	-45792.73	-12199.11	1	6964.75	2
TSEM	3	70	**-33111.49**	**-14132.71**	**1**	763.52	3
TSEM-poly	3	70	-33233.53	-14143.55	1	**755.92**	3
SEM-kMAX	3	70	-37688.43	-16860.77	0.99	20949.99	2
TSEM	4	30	**-44227.97**	**-8530.22**	**1**	1029.8	4
TSEM-poly	4	30	-44611.99	-8555.85	1	**974.47**	4
SEM-kMAX	4	30	-50577.91	-10817.43	1	6958.58	2
TSEM	4	50	**-40109.7**	**-10028.44**	**1**	1035.42	4
TSEM-poly	4	50	-40429.81	-10050.85	1	**959.64**	4
SEM-kMAX	4	50	-45790.33	-12094.5	1	6964.77	2
TSEM	4	70	**-33099**	**-14150.34**	**1**	785.69	4
TSEM-poly	4	70	-33215.4	-14158.65	1	**777.45**	4
SEM-kMAX	4	70	-37881.25	-17054.97	0.99	19208.18	2
TSEM	5	30	**-44178.99**	**-8473.3**	**1**	1038.56	5
TSEM-poly	5	30	-44488.43	-8512.83	1	**974.93**	5
SEM-kMAX	5	30	-50484.58	-10789.13	1	6956.23	2
TSEM	5	50	**-40018.88**	**-9941.79**	**1**	993.59	5
TSEM-poly	5	50	-40321.77	-10001.26	1	**928.99**	5
SEM-kMAX	5	50	-45267.73	-11815.66	1	10451.75	2
TSEM	5	70	**-33066.81**	-14146.88	**1**	766.48	5
TSEM-poly	5	70	-33172.99	**-14069.74**	1	**743.14**	5
SEM-kMAX	5	70	-37741.13	-16616.48	0.99	12208.21	2