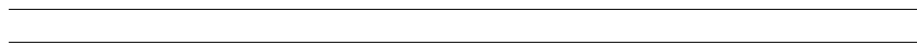# Supplementary Material for Chapter 3

Table 1: Comparison of the methods in dataset NLTCS. For each method and treewidth bound, the BIC score, the log-likelihood (LL), the learning time (Time) and the treewidth are shown. The optimal results are denoted in boldface.

Method	bound	BIC	LL	Time	Treewidth
hc-ET	3	**-100564.01**	**-100103.66**	0.23	3
hc-ET-poly	3	-100720.4	-100269.74	**0.2**	3
k-greedy	3	-102040.99	-101755.09	18.1	3
k-MAX	3	-101960.38	-101674.48	18.05	3
hc-ET	5	-99024.16	-98098.61	0.32	5
hc-ET-poly	5	-99024.16	-98098.61	**0.28**	5
k-greedy	5	**-98740.32**	**-97853.54**	18.29	5
k-MAX	5	-98794.59	-97907.81	18.39	5
hc-ET	7	-98756.78	-97676.16	**0.34**	7
hc-ET-poly	7	-98756.78	-97676.16	0.35	7
k-greedy	7	**-98536.68**	**-97611.13**	18.4	7
k-MAX	7	-98580.88	-97694.1	18.59	6

Table 2: Comparison of the methods in dataset MSNBC1. For each method and treewidth bound, the BIC score, the log-likelihood (LL), the learning time (Time) and the treewidth are shown. The optimal results are denoted in boldface.

Method	bound	BIC	LL	Time	Treewidth
hc-ET	3	**-1873370.83**	**-1872710.26**	1.91	3
hc-ET-poly	3	-1874104.64	-1873425.2	**1.87**	3
k-greedy	3	-1883854.01	-1883457.67	26.7	3
k-MAX	3	-1884927.39	-1884531.05	26.69	3
hc-ET	5	**-1851632.56**	**-1849449.55**	2.41	5
hc-ET-poly	5	-1859830.8	-1858088.17	**2.07**	5
k-greedy	5	-1867898.95	-1867150.31	26.91	5
k-MAX	5	-1868482.59	-1867733.95	26.82	4
hc-ET	7	**-1839616.67**	**-1834099.38**	2.78	7
hc-ET-poly	7	-1840347.39	-1834723.14	**2.26**	7
k-greedy	7	-1867689.47	-1866940.83	26.86	6
k-MAX	7	-1867594.36	-1866845.72	26.93	5

Table 3: Comparison of the methods in dataset KDDCup. For each method and treewidth bound, the BIC score, the log-likelihood (LL), the learning time (Time) and the treewidth are shown. The optimal results are denoted in boldface.

Method	bound	BIC	LL	Time	Treewidth
hc-ET	3	**-437380.55**	**-435093.42**	8.85	3
hc-ET-poly	3	-437390.84	-435103.71	**8.78**	3
k-greedy	3	-444347.69	-442901.6	71.62	3
k-MAX	3	-443621.52	-442223.83	71.82	3
hc-ET	5	**-433860.35**	**-429860.9**	**8.29**	5
hc-ET-poly	5	-433945.96	-429867.85	8.74	5
k-greedy	5	-444930.45	-443520.65	72.58	4
k-MAX	5	-443443.05	-441996.96	71.45	4
hc-ET	7	**-432665.72**	**-427686.07**	8.9	7
hc-ET-poly	7	-432862.24	-427737.38	**8.01**	7
k-greedy	7	-444687.69	-443265.79	72.5	4
k-MAX	7	-442516.69	-441058.49	71.59	4

Table 4: Comparison of the methods in dataset Plants. For each method and treewidth bound, the BIC score, the log-likelihood (LL), the learning time (Time) and the treewidth are shown. The optimal results are denoted in boldface.

Method	bound	BIC	LL	Time	Treewidth
hc-ET	3	**-246767.94**	**-244653.84**	1.86	3
hc-ET-poly	3	-246767.94	-244653.84	**1.83**	3
k-greedy	3	-260402.2	-259098.59	79.17	3
k-MAX	3	-258522.35	-257218.74	79.38	3
hc-ET	5	**-238869.23**	-234850.96	**2.3**	5
hc-ET-poly	5	-238943.78	**-234847.4**	2.46	5
k-greedy	5	-253393.2	-251391.4	79.11	5
k-MAX	5	-256918.65	-255575.97	79.21	4
hc-ET	7	**-234332.21**	**-228751.56**	3.3	7
hc-ET-poly	7	-234708.37	-229108.19	**2.5**	7
k-greedy	7	-253917.96	-251755.04	79.06	5
k-MAX	7	-254995.69	-253203.83	79.4	6

Table 5: Comparison of the methods in dataset Audio. For each method and treewidth bound, the BIC score, the log-likelihood (LL), the learning time (Time) and the treewidth are shown. The optimal results are denoted in boldface.

Method	bound	BIC	LL	Time	Treewidth
hc-ET	3	**-632504.7**	**-629115.13**	3.51	3
hc-ET-poly	3	-632606.14	-629245.42	**3.23**	3
k-greedy	3	-644143.58	-642244.46	115.57	3
k-MAX	3	-642193.17	-640294.05	115.38	3
hc-ET	5	**-624876.99**	-618583.45	4.26	5
hc-ET-poly	5	-625034.14	**-618567.51**	**3.92**	5
k-greedy	5	-641798.45	-639553.16	115.24	5
k-MAX	5	-641040.58	-638833.76	115.64	4
hc-ET	7	**-622139.15**	**-614364.78**	4.75	7
hc-ET-poly	7	-622525.94	-614520.78	**4.53**	7
k-greedy	7	-641028.84	-638822.01	115.44	6
k-MAX	7	-640956.01	-638672.26	115.67	6

Table 6: Comparison of the methods in dataset Jester. For each method and treewidth bound, the BIC score, the log-likelihood (LL), the learning time (Time) and the treewidth are shown. The optimal results are denoted in boldface.

Method	bound	BIC	LL	Time	Treewidth
hc-ET	3	**-501424.72**	**-498251.63**	3.33	3
hc-ET-poly	3	-502030.94	-498912.49	**2.99**	3
k-greedy	3	-509177.79	-507388.66	115.66	3
k-MAX	3	-508016.94	-506218.71	115.35	3
hc-ET	5	-495804.83	-489945.77	4.14	5
hc-ET-poly	5	**-495693.17**	**-489779.49**	**3.73**	5
k-greedy	5	-506489.62	-503635.21	115.18	5
k-MAX	5	-505085.87	-502449.98	115.43	5
hc-ET	7	**-493840.62**	-487307.8	4.19	7
hc-ET-poly	7	-493921.69	**-487243.18**	**4.04**	7
k-greedy	7	-505869.35	-503087.78	115.61	6
k-MAX	7	-506471.26	-504108.52	115.54	6

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Table 7: Comparison of the methods in dataset Netflix. For each method and treewidth bound, the BIC score, the log-likelihood (LL), the learning time (Time) and the treewidth are shown. The optimal results are denoted in boldface.

Method	bound	BIC	LL	Time	Treewidth
hc-ET	3	**-878452.23**	**-875288.63**	3.67	3
hc-ET-poly	3	-879885.49	-876827.67	**3.21**	3
k-greedy	3	-888206.76	-886317.25	115.22	3
k-MAX	3	-886125.43	-884226.31	114.78	3
hc-ET	5	**-871824.91**	**-866464.1**	4.48	5
hc-ET-poly	5	-872236.11	-866740.68	**3.79**	5
k-greedy	5	-887052.49	-885153.37	115.63	4
k-MAX	5	-884913.08	-883013.96	115.34	5
hc-ET	7	-868321.3	-861518.12	4.96	7
hc-ET-poly	7	**-867815.53**	**-861012.35**	**4.5**	7
k-greedy	7	-885839.97	-883671.61	115.37	5
k-MAX	7	-885397.96	-883479.61	115.45	6

Table 8: Comparison of the methods in dataset Accidents. For each method and treewidth bound, the BIC score, the log-likelihood (LL), the learning time (Time) and the treewidth are shown. The optimal results are denoted in boldface.

Method	bound	BIC	LL	Time	Treewidth
hc-ET	3	**-375269.94**	**-372159.6**	3.65	3
hc-ET-poly	3	-375865.05	-372754.71	**3.59**	3
k-greedy	3	-395652.84	-393662.79	124.11	3
k-MAX	3	-393167.77	-391206.08	124.12	3
hc-ET	5	**-356310.64**	**-350335.77**	4.76	5
hc-ET-poly	5	-356745.43	-350449.13	**4.55**	5
k-greedy	5	-393473.5	-391497.63	124.31	4
k-MAX	5	-391053.62	-389063.57	124.47	4
hc-ET	7	**-350484.08**	**-342590.06**	5.14	7
hc-ET-poly	7	-351242.24	-342762.08	**4.82**	7
k-greedy	7	-393983.86	-391965.45	124.07	4
k-MAX	7	-391127.8	-389128.29	124.19	4

Table 9: Comparison of the methods in dataset Retail. For each method and treewidth bound, the BIC score, the log-likelihood (LL), the learning time (Time) and the treewidth are shown. The optimal results are denoted in boldface.

Method	bound	BIC	LL	Time	Treewidth
hc-ET	3	**-238981.6**	**-236961.47**	**4.17**	3
hc-ET-poly	3	-239014.4	-237039.27	4.67	3
k-greedy	3	-239334.04	-237668.93	150.38	3
k-MAX	3	-239155.71	-237505.6	150.21	2
hc-ET	5	**-238943.84**	**-236833.7**	**4.11**	5
hc-ET-poly	5	-238979.98	-236859.84	4.5	5
k-greedy	5	-239285.29	-237615.18	150.81	3
k-MAX	5	-239125.9	-237415.79	150.17	4
hc-ET	7	**-238921.91**	**-236751.77**	6.74	7
hc-ET-poly	7	-238975.69	-236865.55	**4.95**	7
k-greedy	7	-239288.11	-237592.99	150.81	3
k-MAX	7	-239129.03	-237378.91	150.37	4

Table 10: Comparison of the methods in dataset Pumsb-star. For each method and treewidth bound, the BIC score, the log-likelihood (LL), the learning time (Time) and the treewidth are shown. The optimal results are denoted in boldface.

Method	bound	BIC	LL	Time	Treewidth
hc-ET	3	-310965.19	-307439.55	5.91	3
hc-ET-poly	3	**-310451.38**	**-307029.3**	**5.32**	3
k-greedy	3	-325568.63	-322852.61	185	3
k-MAX	3	-325754.69	-323151.65	185.01	3
hc-ET	5	**-290489.19**	**-285268.98**	7.92	5
hc-ET-poly	5	-295712.63	-290299.43	**6.29**	5
k-greedy	5	-322307.63	-319605.74	185.69	4
k-MAX	5	-318995.93	-316374.06	185.7	5
hc-ET	7	**-285482.3**	**-278628.72**	7.77	7
hc-ET-poly	7	-289576.95	-283245.86	**6.73**	7
k-greedy	7	-321911.07	-319190.35	185.66	5
k-MAX	7	-316148.58	-313427.86	185.09	6

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Table 11: Comparison of the methods in dataset DNA. For each method and treewidth bound, the BIC score, the log-likelihood (LL), the learning time (Time) and the treewidth are shown. The optimal results are denoted in boldface.

Method	bound	BIC	LL	Time	Treewidth
hc-ET	3	**-130043.79**	**-127819.4**	4.27	3
hc-ET-poly	3	-130413.14	-128067.02	**3.56**	3
k-greedy	3	-131538.6	-129675.72	202.96	3
k-MAX	3	-130222.98	-128264.18	203.25	3
hc-ET	5	**-129998.66**	**-127619.33**	4.41	5
hc-ET-poly	5	-130310.55	-127728.34	**3.67**	5
k-greedy	5	-131426.57	-129438.26	203.17	3
k-MAX	5	-130198.45	-128217.52	203.07	3
hc-ET	7	**-129982.62**	**-127559.03**	4.52	7
hc-ET-poly	7	-130276.37	-127624.07	**4.05**	7
k-greedy	7	-131416.44	-129476.08	203.49	3
k-MAX	7	-130208.55	-128246.06	203.14	3

Table 12: Comparison of the methods in dataset Kosarek. For each method and treewidth bound, the BIC score, the log-likelihood (LL), the learning time (Time) and the treewidth are shown. The optimal results are denoted in boldface.

Method	bound	BIC	LL	Time	Treewidth
hc-ET	3	**-374477.15**	**-368420.5**	11.88	3
hc-ET-poly	3	-374662.98	-368481.35	**10.81**	3
k-greedy	3	-380494.2	-376796.68	212.81	3
k-MAX	3	-381097.15	-377462.12	212.49	2
hc-ET	5	**-372472.62**	**-365249.43**	12.65	5
hc-ET-poly	5	-372778.12	-365513.27	**11.38**	5
k-greedy	5	-379371.64	-375663.7	212.64	3
k-MAX	5	-379297.73	-375610.62	212.55	4
hc-ET	7	**-371673.19**	**-363616.76**	13.3	7
hc-ET-poly	7	-372142.91	-364190.63	**11.96**	7
k-greedy	7	-380726.25	-376997.48	212.67	4
k-MAX	7	-379468.39	-375781.29	212.64	4

Table 13: Comparison of the methods in dataset MSWeb. For each method and treewidth bound, the BIC score, the log-likelihood (LL), the learning time (Time) and the treewidth are shown. The optimal results are denoted in boldface.

Method	bound	BIC	LL	Time	Treewidth
hc-ET	3	**-294512.52**	-290221.53	16.84	3
hc-ET-poly	3	-294632.49	**-290197.44**	**14.1**	3
k-greedy	3	-298425.83	-295297.62	327.24	3
k-MAX	3	-297913.89	-294765.11	327.24	2
hc-ET	5	-293224.63	-288100.14	17.84	5
hc-ET-poly	5	**-292781.8**	**-287472.08**	**15.33**	5
k-greedy	5	-298510.13	-295258.44	327.1	4
k-MAX	5	-297806.14	-294647.07	326.47	4
hc-ET	7	**-291615.67**	**-285920.07**	19.53	7
hc-ET-poly	7	-292028.66	-286132.41	**17.4**	7
k-greedy	7	-298324.7	-295083.3	327.1	3
k-MAX	7	-297256.91	-293964.06	326.97	4

Table 14: Comparison of the methods in dataset Book. For each method and treewidth bound, the BIC score, the log-likelihood (LL), the learning time (Time) and the treewidth are shown. The optimal results are denoted in boldface.

Method	bound	BIC	LL	Time	Treewidth
hc-ET	3	**-316086.72**	**-306330.77**	54.93	3
hc-ET-poly	3	-316231.65	-306375.93	**51.79**	3
k-greedy	3	-322866.08	-315182.88	555.78	3
k-MAX	3	-321837.49	-314521.66	555.2	3
hc-ET	5	**-314838.9**	**-303622.51**	57.14	5
hc-ET-poly	5	-315114.45	-303947.96	**53.8**	5
k-greedy	5	-322722.76	-314876.28	555.45	5
k-MAX	5	-321998.8	-314964.18	555.83	4
hc-ET	7	**-314330.18**	**-302496.96**	**56.88**	7
hc-ET-poly	7	-314491.42	-302785.19	57.93	7
k-greedy	7	-322559.51	-314463.58	555.76	5
k-MAX	7	-322417.88	-315147.41	555.55	5

Table 15: Comparison of the methods in dataset EachMovie. For each method and treewidth bound, the BIC score, the log-likelihood (LL), the learning time (Time) and the treewidth are shown. The optimal results are denoted in boldface.

Method	bound	BIC	LL	Time	Treewidth
hc-ET	3	**-272188.53**	**-261805.97**	50.62	3
hc-ET-poly	3	-272249.62	-261867.06	**48.97**	3
k-greedy	3	-287208.45	-280260.09	555.47	3
k-MAX	3	-288065.81	-281201.63	556.03	3
hc-ET	5	**-266634.84**	**-253870.23**	55.2	5
hc-ET-poly	5	-266811.82	-253878.87	**54.01**	5
k-greedy	5	-286089.29	-279010.46	555.65	5
k-MAX	5	-286378.9	-279371.62	555.6	4
hc-ET	7	**-263913.56**	**-249726.45**	73.12	7
hc-ET-poly	7	-264513.74	-250507.59	**62.23**	7
k-greedy	7	-286622.81	-279501.9	555.68	5
k-MAX	7	-285553.88	-278395.09	556.14	5

Table 16: Comparison of the methods in dataset WebKB. For each method and treewidth bound, the BIC score, the log-likelihood (LL), the learning time (Time) and the treewidth are shown. The optimal results are denoted in boldface.

Method	bound	BIC	LL	Time	Treewidth
hc-ET	3	**-432570.33**	**-419598.91**	137.36	3
hc-ET-poly	3	-433097.84	-420035.13	**135.51**	3
k-greedy	3	-442320.34	-432444.92	925.36	3
k-MAX	3	-440476.27	-430708.02	925.13	3
hc-ET	5	-430802.8	-416323.07	146.07	5
hc-ET-poly	5	**-430752.25**	**-416145.51**	**143.83**	5
k-greedy	5	-442047.57	-431814.91	924.34	4
k-MAX	5	-440739.29	-431058.35	924.97	4
hc-ET	7	**-429859.82**	**-414463.21**	151.77	7
hc-ET-poly	7	-430104.61	-414842.95	**136.12**	7
k-greedy	7	-441716.24	-431566.94	925.42	5
k-MAX	7	-441129.84	-431972.84	924.65	5

Table 17: Comparison of the methods in dataset Reuters-521. For each method and treewidth bound, the BIC score, the log-likelihood (LL), the learning time (Time) and the treewidth are shown. The optimal results are denoted in boldface.

Method	bound	BIC	LL	Time	Treewidth
hc-ET	3	**-655489.71**	-637068.68	185.33	3
hc-ET-poly	3	-655642.7	**-636887.86**	**171.04**	3
k-greedy	3	-684036.94	-672344.81	986.01	3
k-MAX	3	-679243.6	-668241.05	985.87	2
hc-ET	5	**-647280.27**	-624932.58	201.39	5
hc-ET-poly	5	-647390.44	**-624537.65**	**197.69**	5
k-greedy	5	-683850.06	-671784.6	985.52	4
k-MAX	5	-678536.15	-667357.91	985.4	3
hc-ET	7	**-643830.27**	**-619198.62**	215.91	7
hc-ET-poly	7	-644939.56	-620690.03	**206.9**	7
k-greedy	7	-684131.61	-672052.97	985.43	3
k-MAX	7	-678946.09	-667759.07	986.13	4

Table 18: Comparison of the methods in dataset 20 NewsGroup. For each method and treewidth bound, the BIC score, the log-likelihood (LL), the learning time (Time) and the treewidth are shown. The optimal results are denoted in boldface.

Method	bound	BIC	LL	Time	Treewidth
hc-ET	3	-1634894.09	-1609534.54	312.12	3
hc-ET-poly	3	**-1634175.78**	**-1608699.59**	**306.66**	3
k-greedy	3	-1669236.13	-1653927.08	1005.39	3
k-MAX	3	-1672253.12	-1659995.62	1005.4	3
hc-ET	5	**-1626741.29**	**-1596295.85**	333.04	5
hc-ET-poly	5	-1627002.97	-1596748.82	**317.44**	5
k-greedy	5	-1669452.13	-1654185.08	1005.38	4
k-MAX	5	-1668015.39	-1653443.57	1005.28	3
hc-ET	7	**-1623159.23**	**-1590516.11**	329.1	7
hc-ET-poly	7	-1623722.53	-1590734.12	**324.95**	7
k-greedy	7	-1668760.34	-1653292.65	1005.85	4
k-MAX	7	-1666007.57	-1651239.78	1006.02	4

Table 19: Comparison of the methods in dataset Movie reviews. For each method and treewidth bound, the BIC score, the log-likelihood (LL), the learning time (Time) and the treewidth are shown. The optimal results are denoted in boldface.

Method	bound	BIC	LL	Time	Treewidth
hc-ET	3	**-555184.04**	**-545578.2**	110.86	3
hc-ET-poly	3	-555226.03	-545594.36	**108.18**	3
k-greedy	3	-558028.99	-550540.56	1109.62	2
k-MAX	3	-556930.56	-549224.49	1110.15	3
hc-ET	5	**-554903.9**	**-544748.42**	122.54	5
hc-ET-poly	5	-555004.21	-544789.71	**119.21**	5
k-greedy	5	-558014.82	-550393.6	1109.56	2
k-MAX	5	-556940.62	-549234.55	1109.76	3
hc-ET	7	**-554779.71**	**-544329.11**	**128.5**	7
hc-ET-poly	7	-554860.12	-544357.88	129.01	7
k-greedy	7	-557953.58	-550291.77	1109.36	3
k-MAX	7	-556829.32	-548953.56	1109.49	4

Table 20: Comparison of the methods in dataset BBC. For each method and treewidth bound, the BIC score, the log-likelihood (LL), the learning time (Time) and the treewidth are shown. The optimal results are denoted in boldface.

Method	bound	BIC	LL	Time	Treewidth
hc-ET	3	**-420183.48**	**-407831.93**	182.56	3
hc-ET-poly	3	-420516.04	-408153.36	**167.3**	3
k-greedy	3	-425765.37	-415814.38	1171.01	3
k-MAX	3	-424600.46	-415168.91	1171.26	3
hc-ET	5	**-419243.35**	**-405593.19**	192.51	5
hc-ET-poly	5	-419489.47	-405846.74	**185.63**	5
k-greedy	5	-425877.13	-415484.61	1171.05	4
k-MAX	5	-424331.07	-414584.14	1171.5	4
hc-ET	7	-418849.46	-404709.55	204.34	7
hc-ET-poly	7	**-418778.08**	**-404482.34**	**187.77**	7
k-greedy	7	-425554.13	-415246.95	1170.8	4
k-MAX	7	-424519.83	-415047.46	1171.65	5

Table 21: Comparison of the methods in dataset Voting. For each method and treewidth bound, the BIC score, the log-likelihood (LL), the learning time (Time) and the treewidth are shown. The optimal results are denoted in boldface.

Method	bound	BIC	LL	Time	Treewidth
hc-ET	3	-196837.41	-177772.96	770.3	3
hc-ET-poly	3	**-194965.93**	**-175546.4**	**753.33**	3
k-greedy	3	-892952.13	-887366.66	1876.83	3
k-MAX	3	-892952.13	-887366.66	1761.01	3
hc-ET	5	**-173592.3**	**-151193.61**	782.82	5
hc-ET-poly	5	-180262.93	-157661.85	**757.51**	5
k-greedy	5	-892952.13	-887366.66	1700.79	4
k-MAX	5	-892952.13	-887366.66	1751.33	4
hc-ET	7	-170704.44	-148554.31	757.61	7
hc-ET-poly	7	**-166606.55**	**-142716.51**	**736.15**	7
k-greedy	7	-892952.13	-887366.66	2936.37	4
k-MAX	7	-892952.13	-887366.66	1820.12	4

Table 22: Comparison of the methods in dataset Ad. For each method and treewidth bound, the BIC score, the log-likelihood (LL), the learning time (Time) and the treewidth are shown. The optimal results are denoted in boldface.

Method	bound	BIC	LL	Time	Treewidth
hc-ET	3	-48294.73	-34840.99	296.2	3
hc-ET-poly	3	**-48293.89**	**-34836.24**	**295.86**	3
k-greedy	3	-53694.44	-41564.21	1718.25	3
k-MAX	3	-54823.92	-42853.76	1718.78	2
hc-ET	5	-47859.22	-34073.62	303.11	5
hc-ET-poly	5	**-47819.2**	**-33939.91**	**303.07**	5
k-greedy	5	-53773.53	-41678.44	1718.13	3
k-MAX	5	-54724.67	-42684.23	1718.22	3
hc-ET	7	-47642.94	-33611.38	314.47	7
hc-ET-poly	7	**-47640.44**	**-33483.95**	**312.07**	7
k-greedy	7	-53634.9	-41325.08	1717.81	3
k-MAX	7	-54716.17	-42644.5	1717.86	3

Table 23: Comparison of the methods in dataset Hailfinder. For each method and treewidth bound, the BIC score, the log-likelihood (LL), the learning time (Time) and the treewidth are shown. The optimal results are denoted in boldface.

Method	bound	BIC	LL	Time	Treewidth
hc-ET	3	**-252456.55**	**-246362.49**	0.41	3
hc-ET-poly	3	-252901.52	-246675.45	**0.39**	3
k-greedy	3	-254022.56	-248128.66	62.39	2
k-MAX	3	-252925.89	-246980.89	62.36	3
hc-ET	5	**-252456.55**	**-246362.49**	0.46	3
hc-ET-poly	5	-252901.52	-246675.45	**0.45**	3
k-greedy	5	-253664.48	-247770.58	62.45	2
k-MAX	5	-252828.2	-246930.05	62.19	2
hc-ET	7	**-252456.55**	**-246362.49**	0.51	3
hc-ET-poly	7	-252901.52	-246675.45	**0.39**	3
k-greedy	7	-253616.73	-247446.03	62.15	3
k-MAX	7	-252810.61	-246946.53	62.41	2

Table 24: Comparison of the methods in dataset Hepar II. For each method and treewidth bound, the BIC score, the log-likelihood (LL), the learning time (Time) and the treewidth are shown. The optimal results are denoted in boldface.

Method	bound	BIC	LL	Time	Treewidth
hc-ET	3	**-163976.66**	**-162716.12**	0.58	3
hc-ET-poly	3	-164173.13	-162882.78	**0.54**	3
k-greedy	3	-164302.68	-163063.43	79.96	3
k-MAX	3	-164091.3	-162932.96	79.97	3
hc-ET	5	**-164005.98**	**-162770.99**	0.61	3
hc-ET-poly	5	-164153.32	-162828.89	**0.54**	4
k-greedy	5	-164266.13	-163171.67	80.03	3
k-MAX	5	-164062.16	-162929.38	79.94	3
hc-ET	7	**-164005.98**	**-162770.99**	0.65	3
hc-ET-poly	7	-164153.32	-162828.89	**0.6**	4
k-greedy	7	-164210.69	-163005.51	80.21	3
k-MAX	7	-164030.28	-162880.45	80.03	3

Table 25: Comparison of the methods in dataset Win95pts. For each method and treewidth bound, the BIC score, the log-likelihood (LL), the learning time (Time) and the treewidth are shown. The optimal results are denoted in boldface.

Method	bound	BIC	LL	Time	Treewidth
hc-ET	3	**-49104.85**	**-47878.38**	0.86	3
hc-ET-poly	3	-49104.85	-47878.38	**0.78**	3
k-greedy	3	-51513.93	-50479.09	88.15	3
k-MAX	3	-50486.49	-49451.65	88.21	3
hc-ET	5	**-47602.03**	**-45894.33**	0.93	5
hc-ET-poly	5	-47646.73	-45939.03	**0.91**	5
k-greedy	5	-51005.58	-49915.38	88.14	4
k-MAX	5	-49829.5	-48688.2	88.29	3
hc-ET	7	**-47225.28**	**-45351.49**	1.07	7
hc-ET-poly	7	-47284.29	-45414.77	**0.88**	7
k-greedy	7	-50911.02	-49744.16	87.99	4
k-MAX	7	-49730.7	-48640.5	88.33	4

Table 26: Comparison of the methods in dataset Pathfinder. For each method and treewidth bound, the BIC score, the log-likelihood (LL), the learning time (Time) and the treewidth are shown. The optimal results are denoted in boldface.

Method	bound	BIC	LL	Time	Treewidth
hc-ET	3	**-152967.91**	**-134264.16**	**1.23**	3
hc-ET-poly	3	-152967.91	-134264.16	1.29	3
k-greedy	3	-154813.97	-138473.74	123.56	2
k-MAX	3	-153344.68	-136599.88	123.54	3
hc-ET	5	**-152502.51**	**-133168.48**	**1.27**	5
hc-ET-poly	5	-152502.51	-133168.48	1.38	5
k-greedy	5	-152857	-135481.92	123.4	3
k-MAX	5	-152835.72	-136005.74	123.37	4
hc-ET	7	-152394.25	**-132685.46**	1.33	6
hc-ET-poly	7	-152394.25	-132685.46	**1.32**	6
k-greedy	7	-153000.79	-136251.73	123.35	4
k-MAX	7	**-151644.88**	-135036.35	122.99	4

Table 27: Comparison of the methods in dataset Munin1. For each method and treewidth bound, the BIC score, the log-likelihood (LL), the learning time (Time) and the treewidth are shown. The optimal results are denoted in boldface.

Method	bound	BIC	LL	Time	Treewidth
hc-ET	3	-242078.42	-218174.91	3.02	3
hc-ET-poly	3	-242061.56	-218132.5	**2.88**	3
k-greedy	3	-247930.11	-224235.28	210.93	3
k-MAX	3	**-238690.27**	**-215076.35**	210.78	3
hc-ET	5	-240687.68	-215821.74	3.15	5
hc-ET-poly	5	-239897.04	-215652.85	**2.93**	5
k-greedy	5	-246169.33	-222887.58	211.11	3
k-MAX	5	**-237131.4**	**-212137.69**	210.96	3
hc-ET	7	-240032.23	-213722.62	3.18	7
hc-ET-poly	7	-240408.87	-215794.18	**2.96**	7
k-greedy	7	-245583.85	-219934.33	211.27	4
k-MAX	7	**-237527.05**	**-213508.57**	211.6	5

Table 28: Comparison of the methods in dataset Andes. For each method and treewidth bound, the BIC score, the log-likelihood (LL), the learning time (Time) and the treewidth are shown. The optimal results are denoted in boldface.

Method	bound	BIC	LL	Time	Treewidth
hc-ET	3	**-476123.08**	**-472882.28**	5.38	3
hc-ET-poly	3	-476814.54	-473394.88	**4.52**	3
k-greedy	3	-501412.92	-498670.38	247.01	3
k-MAX	3	-487883.54	-485085.64	246.82	3
hc-ET	5	**-471391.41**	**-467839.74**	5.85	5
hc-ET-poly	5	-473368.47	-468756.41	**5.13**	5
k-greedy	5	-499089.64	-496398.21	247.06	3
k-MAX	5	-487067.85	-484231.63	246.84	4
hc-ET	7	**-470382.5**	**-466617.9**	6.37	7
hc-ET-poly	7	-473030.82	-467788.49	**5.68**	7
k-greedy	7	-498219.53	-495506.8	247.11	4
k-MAX	7	-486900.38	-483987.5	247.52	4

Table 29: Comparison of the methods in dataset Diabetes. For each method and treewidth bound, the BIC score, the log-likelihood (LL), the learning time (Time) and the treewidth are shown. The optimal results are denoted in boldface.

Method	bound	BIC	LL	Time	Treewidth
hc-ET	3	-2028021.32	-1493375.81	15.77	3
hc-ET-poly	3	**-2024877.09**	**-1472980.01**	**14.59**	3
k-greedy	3	-2287664.52	-2059799.79	458.76	2
k-MAX	3	-2255056.28	-2030202.38	458.72	2
hc-ET	5	-2011238.14	-1433359.36	16.31	5
hc-ET-poly	5	**-2006252.09**	**-1406011.42**	**15.82**	5
k-greedy	5	-2269791.91	-2028542.41	458.98	2
k-MAX	5	-2251324.61	-2025704.16	458.89	2
hc-ET	7	-2011197.56	-1433318.78	16.31	7
hc-ET-poly	7	**-2005642.37**	**-1403085.02**	**16.09**	6
k-greedy	7	-2267757.61	-2026657.16	458.84	2
k-MAX	7	-2251653.81	-2026497.55	458.86	2

Table 30: Comparison of the methods in dataset Pigs. For each method and treewidth bound, the BIC score, the log-likelihood (LL), the learning time (Time) and the treewidth are shown. The optimal results are denoted in boldface.

Method	bound	BIC	LL	Time	Treewidth
hc-ET	3	**-1708154**	**-1684126.99**	19.78	3
hc-ET-poly	3	-1722816.72	-1695059.18	**19.33**	3
k-greedy	3	-1986608.9	-1974812.58	492.74	3
k-MAX	3	-1909862.01	-1897452.46	493.71	2
hc-ET	5	**-1689806.35**	**-1664740.25**	20.5	5
hc-ET-poly	5	-1706448.17	-1675420.04	**19.74**	5
k-greedy	5	-1984618.91	-1971068.06	493.66	3
k-MAX	5	-1909129.82	-1895919.65	493.47	2
hc-ET	7	**-1683045.13**	**-1657996.06**	20.8	7
hc-ET-poly	7	-1699864.6	-1667678.12	**20.32**	7
k-greedy	7	-1985979.31	-1973348.31	493.59	3
k-MAX	7	-1909879.05	-1897162.88	493.27	2

Table 31: Comparison of the methods in dataset Link. For each method and treewidth bound, the BIC score, the log-likelihood (LL), the learning time (Time) and the treewidth are shown. The optimal results are denoted in boldface.

Method	bound	BIC	LL	Time	Treewidth
hc-ET	3	**-1378294.4**	**-1329299.24**	60.6	3
hc-ET-poly	3	-1385173.57	-1333295.34	**59.93**	3
k-greedy	3	-1700787.93	-1684745.79	801.6	2
k-MAX	3	-1671162.28	-1656342.36	801.61	2
hc-ET	5	**-1308633.7**	**-1250516.63**	72.07	5
hc-ET-poly	5	-1347437.22	-1268998.12	**68.14**	5
k-greedy	5	-1704996.32	-1689277.84	801.52	3
k-MAX	5	-1671973.51	-1657136.56	801.99	2
hc-ET	7	**-1317999.57**	**-1256735.4**	71.61	7
hc-ET-poly	7	-1342354.51	-1266696.28	**68.14**	7
k-greedy	7	-1701405.61	-1686206.68	801.64	4
k-MAX	7	-1668862.87	-1652773.89	801.44	3

Table 32: Comparison of the methods in dataset Munin2. For each method and treewidth bound, the BIC score, the log-likelihood (LL), the learning time (Time) and the treewidth are shown. The optimal results are denoted in boldface.

Method	bound	BIC	LL	Time	Treewidth
hc-ET	3	-890406.17	-815186.58	82.01	3
hc-ET-poly	3	**-888814.83**	**-813969.99**	**81.37**	3
k-greedy	3	-918996.42	-851995.92	1110.11	1
k-MAX	3	-895228.15	-826920.26	1109.86	1
hc-ET	5	-889744.86	-813392.48	86.03	5
hc-ET-poly	5	**-888120.04**	**-812610.86**	**84.84**	5
k-greedy	5	-917541.39	-850655.87	1109.97	2
k-MAX	5	-895165.56	-826304.06	1109.81	2
hc-ET	7	-889325.18	-813368.85	89.23	7
hc-ET-poly	7	**-888028.73**	**-812413.09**	**88.19**	7
k-greedy	7	-917967.95	-850660.84	1109.8	2
k-MAX	7	-895121.42	-826583.57	1110.36	2

Table 33: Comparison of the methods in dataset Munin3. For each method and treewidth bound, the BIC score, the log-likelihood (LL), the learning time (Time) and the treewidth are shown. The optimal results are denoted in boldface.

Method	bound	BIC	LL	Time	Treewidth
hc-ET	3	**-899697.23**	**-812996.46**	**101.26**	3
hc-ET-poly	3	-902608.41	-817777.16	104.22	3
k-greedy	3	-935572.28	-859420.05	1154.16	2
k-MAX	3	-907019.45	-828141.72	1153.48	1
hc-ET	5	**-900055.21**	-814508.52	118.65	5
hc-ET-poly	5	-901349.38	**-814048.15**	101.11	5
k-greedy	5	-936366.63	-860657.3	1153.47	1
k-MAX	5	-907083.81	-828184.8	1154.18	1
hc-ET	7	**-900143.76**	**-812442.22**	110.13	7
hc-ET-poly	7	-901272.35	-815069.84	**107.5**	7
k-greedy	7	-935965.42	-859438.44	1153.29	1
k-MAX	7	-906785.04	-827319.63	1153.43	1

Table 34: Comparison of the methods in dataset Munin4. For each method and treewidth bound, the BIC score, the log-likelihood (LL), the learning time (Time) and the treewidth are shown. The optimal results are denoted in boldface.

Method	bound	BIC	LL	Time	Treewidth
hc-ET	3	**-1019519.68**	**-922917.67**	**98.38**	3
hc-ET-poly	3	-1020366.85	-924139.6	103.24	3
k-greedy	3	-1064449.8	-984941.8	1145.14	2
k-MAX	3	-1034360.09	-949809.92	1145.25	1
hc-ET	5	**-1017959.65**	**-918913.21**	113.89	5
hc-ET-poly	5	-1018901.5	-920067.99	**101.42**	5
k-greedy	5	-1063607.6	-984516.94	1145.11	2
k-MAX	5	-1034455.62	-949935.26	1144.73	1
hc-ET	7	**-1017575.81**	**-916966.46**	107.18	7
hc-ET-poly	7	-1018219.08	-917213.68	**104.49**	7
k-greedy	7	-1061864.29	-980759.31	1144.69	2
k-MAX	7	-1034360.09	-949809.92	1145.51	1