2) a) $\vec{x}$ & $\vec{b} \in \mathbb{R}^n$, $A \in \mathbb{R}^{n \times n}$

$f: \mathbb{R}^n \to \mathbb{R}$, $f(x) = \vec{b}^T \vec{x} + \vec{x}^T A \vec{x}$

$\boxed{\text{gradient}: \nabla_x f(x) = \vec{b} + (A + A^T)\vec{x}}$

$\to 2A\vec{x}$ if A is symmetric

$\boxed{\text{hessian}: H(x) = \nabla^2 f(x) = 2A}$

b) Taylor's Expansion @ $\vec{x} = 0$

1st order:

$f(x) = 0 + [\vec{b} + 2A\vec{x}]|_{x=0}\vec{x}$

$\boxed{f(x) = \vec{b}\vec{x}} \to$ 1st order approx.

2nd order:

$f(x) = 0 + [\vec{b} + 2A\vec{x}]|_{x=0}\vec{x} + \frac{1}{2}[2A]|_{x=0}\vec{x}^2$

$\boxed{f(x) = \vec{b}\vec{x} + A\vec{x}^2} \to$ 2nd order approx

$\boxed{\begin{array}{l}\text{1st order: not exact} \\ \text{2nd order: exact}\end{array}}$

c) A is positive definite if: $\boxed{\text{all eigenvalues } (\lambda\text{'s}) \text{ are positive}}$

d) A has full rank if: $\boxed{|A| \neq 0}$

e) $\exists y \in \mathbb{R}^n$ & $y \neq 0$ s.t. $A^T y = 0$ then for $A\vec{x} = \vec{b}$ to have a solution for $\vec{x}$:

1) $\vec{b}$ is in the column space of A
2) $\vec{y}$ is orthogonal to the columns of A
3) $\therefore$ $\boxed{\vec{y} \text{ is orthogonal to } \vec{b}}$