Original Optimization Problem:

$$\min_{\vec{x}} d^2 = (x_1+1)^2 + x_2^2 + (x_3-1)^2$$

$$\text{s.t. } x_1 + 2x_2 + 3x_3 = 1$$

Unconstrained Optimization Problem:

$$\min_{x_2, x_3} 5x_2^2 + 12x_2x_3 - 8x_2 + 10x_3^2 - 14x_3 + 5$$

Analytical Solution:

$$\vec{x} = \begin{pmatrix} -\frac{15}{14} \\ -\frac{1}{7} \\ \frac{11}{14} \end{pmatrix} \approx \begin{pmatrix} -1.0714285714285714 \\ -0.14285714285714285 \\ 0.7857142857142857 \end{pmatrix}$$

Gradient Descent Solution of Unconstrained Optimization Problem:

Table 1:
Gradient Descent Initial Points, Corresponding Solutions, and Number of Iterations

Initial Point, $\begin{pmatrix} x_{2,0} \\ x_{3,0} \end{pmatrix}$	Solution, $\begin{pmatrix} x_2 \\ x_3 \end{pmatrix}$	# Iterations
$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -0.14285676 \\ 0.78571402 \end{pmatrix}$	84
$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$	$\begin{pmatrix} -0.14285675 \\ 0.78571404 \end{pmatrix}$	86
$\begin{pmatrix} 100 \\ -100 \end{pmatrix}$	$\begin{pmatrix} -0.14285683 \\ 0.78571408 \end{pmatrix}$	114

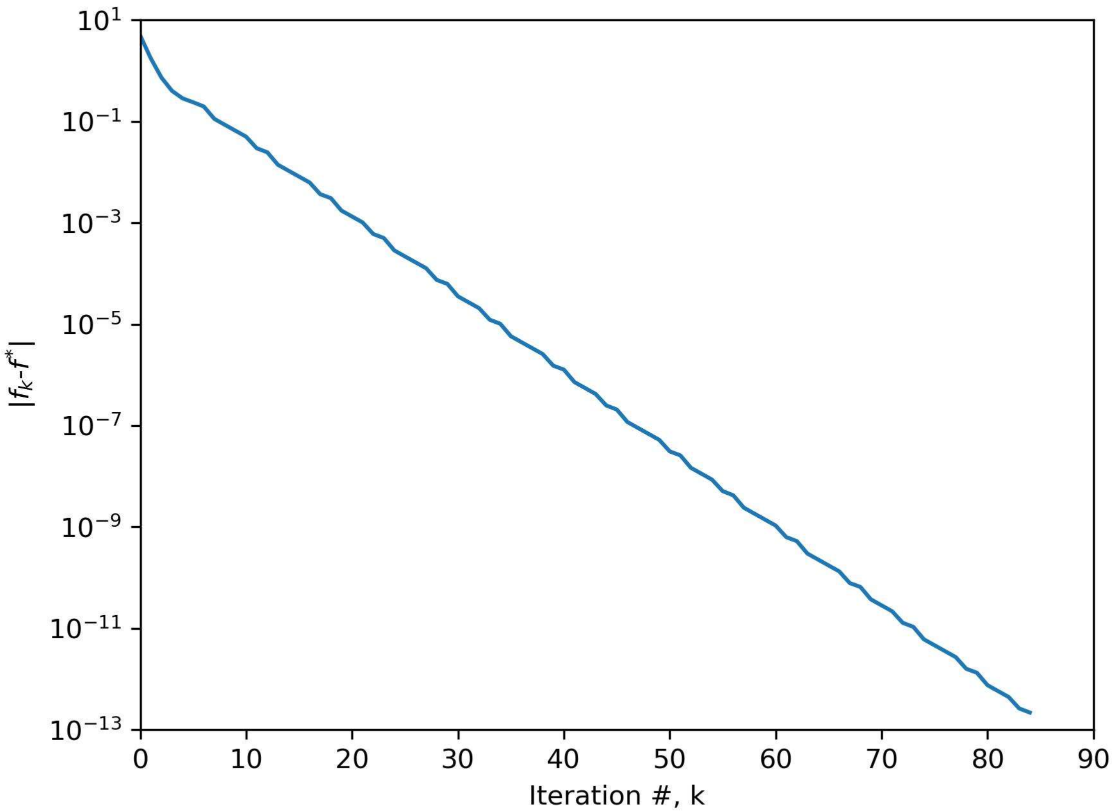

Figure 1: Gradient descent convergence of error vs. iteration number for initial guess $\begin{pmatrix} x_{2,0} \\ x_{3,0} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Newton's Algorithm Solution of Unconstrained Optimization Problem:

Table 2:
Newton's Algorithm Initial Points, Corresponding Solutions, and Number of Iterations

Initial Point, $\begin{pmatrix} x_{2,0} \\ x_{3,0} \end{pmatrix}$	Solution, $\begin{pmatrix} x_2 \\ x_3 \end{pmatrix}$	# Iterations
$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -0.14285714 \\ 0.78571429 \end{pmatrix}$	1
$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$	$\begin{pmatrix} -0.14285714 \\ 0.78571429 \end{pmatrix}$	1
$\begin{pmatrix} 100 \\ -100 \end{pmatrix}$	$\begin{pmatrix} -0.14285714 \\ 0.78571429 \end{pmatrix}$	1

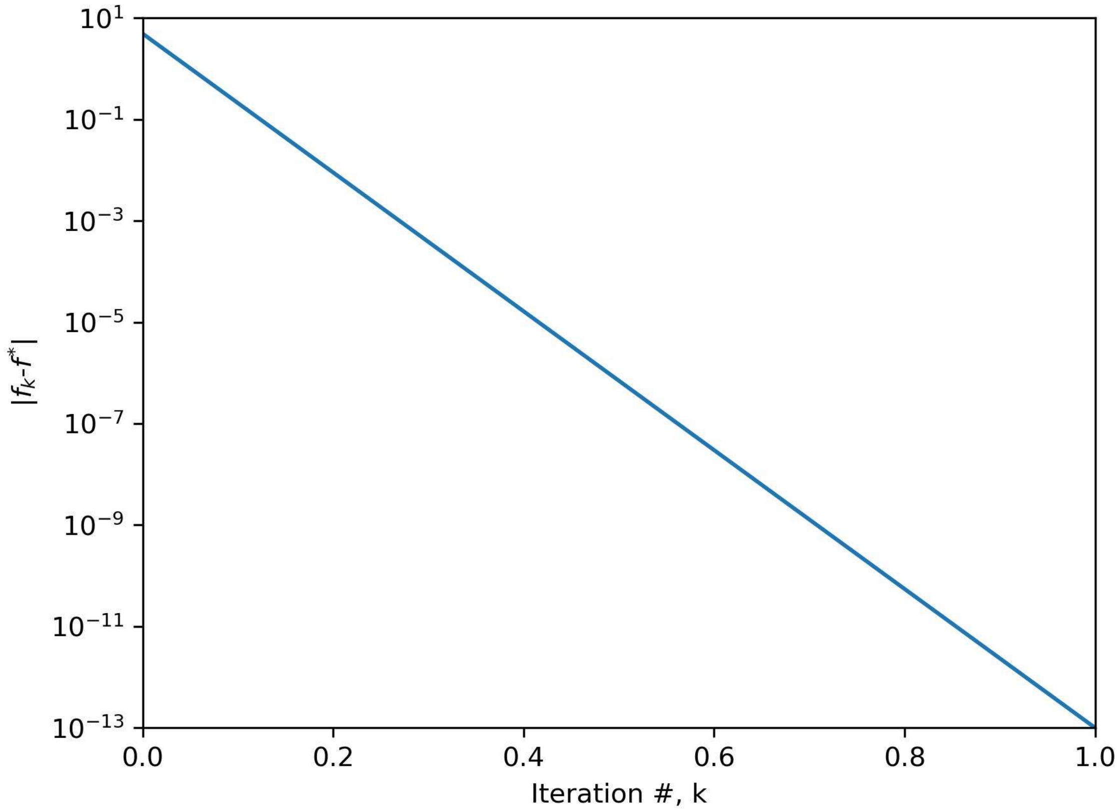

Figure 2: Newton's algorithm convergence of error vs. iteration number for initial guess $\binom{x_{2,0}}{x_{3,0}} = \binom{0}{0}$