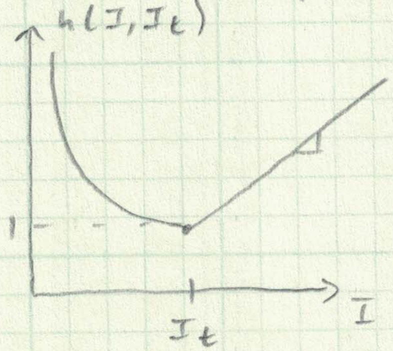
4) $\min_{p} \max_{k} \{ h(a_k^T p, I_t) \}$

s.t. $0 \le p_i \le p_{max}$ → convex (linear) w.r.t. p

$h(I, I_t) = \begin{cases} I_t / I & I \le I_t \\ I / I_t & I \ge I_t \end{cases}$ $\boxed{\text{convex domain}}$

a) show problem is convex

first show $h(a_k^T p, I_t)$ is convex w.r.t. I



$\frac{\partial h}{\partial p} = \frac{dh}{dI} \cdot \frac{\partial (a^T p)}{\partial p} = h' a$

$\frac{\partial^2 h}{\partial p^2} = \frac{\partial^2 h}{\partial I^2} \cdot \frac{\partial (a^T p)}{\partial p} a^T = h'' a a^T$ ← H is a matrix

$h'' \ge 0$ → h is convex w.r.t. I

$\left.\begin{array}{l} I < I_t \to h'' > 0 \\ I > I_t \to h'' = 0 \end{array}\right\}$ h'' (Hessian) is p.s.d.

since $h(I, I_t)$ is convex w.r.t. I

$h(a^T p, I_t)$ is convex w.r.t. p → $\max\{h(a^T p, I_t)$ is convex

$\boxed{\text{convex objective}}$

$\boxed{\therefore \text{ problem is convex since objective \& domain are convex}}$

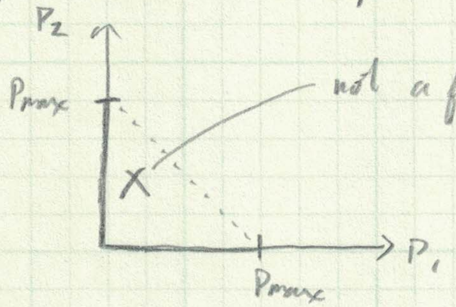
4) b) if $C_n^{10} \begin{cases} P_1 + \dots + P_{10} \le P^* \\ P_2 + \dots + P_{11} \le P^* \\ \vdots \end{cases}$

$$[1, \dots 1, 0, \dots, 0] \begin{bmatrix} P_1 \\ \vdots \\ P_N \end{bmatrix}$$

linear constraint,
linear objective,
strictly convex
↓
$\boxed{\text{unique solution}}$

c) no more than 10 lamps switched on?

simpler case: 2 lamps P_1 & P_2 but no more than one can be on



not a feasible region
↓
not a convex set
↓
expanding to 10 lamps
will not be a convex set
↓
constraint is not convex
↓
$\boxed{\text{may or may not have unique sol.}}$