MAE598 HW2

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1) $f(x_1,x_2) = 2x_1^2 - 4x_1x_2 + \frac{3}{2}x_2^2 + x_2$

$$g(x_1,x_2) = \nabla f = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 4x_1 - 4x_2 \\ -4x_1 + 3x_2 + 1 \end{bmatrix} \rightarrow 0 \text{ gradient } @ (1,1)$$

$$H(x_1,x_2) = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ -4 & 3 \end{bmatrix}$$

$f(1,1) = \frac{1}{2}$

$$f(x_1,x_2) = f(1,1) + \cancel{g(1,1)^T}^{\rightarrow 0} \cdot (\vec{x}-1) + \frac{1}{2}(\vec{x}-1)^T H(1,1)(\vec{x}-1)$$

$$f(x_1,x_2) = f(1,1) + \frac{1}{2}[(x_1-1),(x_2-1)] \begin{bmatrix} 4 & -4 \\ -4 & 3 \end{bmatrix} \begin{bmatrix} (x_1-1) \\ (x_2-1) \end{bmatrix}$$

$$f(x_1,x_2) - f(1,1) = \frac{1}{2}[4(x_1-1) - 4(x_2-1), -4(x_1-1) + 3(x_2-1)] \begin{bmatrix} (x_1-1) \\ (x_2-1) \end{bmatrix}$$

$$f(x_1,x_2) - f(1,1) = \frac{1}{2}[4(x_1-1)^2 - 4(x_1-1)(x_2-1) - 4(x_1-1)(x_2-1) + 3(x_2-1)^2]$$

$$f(x_1,x_2) - f(1,1) = \underbrace{2(x_1-1)^2 - 4(x_1-1)(x_2-1) + \frac{3}{2}(x_2-1)^2}_{[a(x_1-1) - b(x_2-1)] \cdot [c(x_1-1) - d(x_2-1)]} \qquad (1)$$

$$a \cdot c x_1^2 - 2ac x_1 + ac - ad x_1 x_2 + ad x_1 + ad x_2 - ad - bc x_1 x_2 + bc x_1 + bc x_2 - bc + bd x_2^2 - 2bd x_2 + bd \qquad (2)$$

expand (1): $2x_1^2 - 4x_1x_2 + \frac{3}{2}x_2^2 + x_2 - \frac{1}{2}$

x_1^2: $ac = 2$

x_1: $-2ac + ad + bc = 0$

x_1x_2: $-ad - bc = -4$

x_2: $ad + bc - 2bd = 1$

x_2^2: $bd = 3/2$

$-$: $ac - ad - bc + bd = -\frac{1}{2}$

Solve:

$a = 3$
$b = 3/2$
$c = 2/3$
$d = 1$

OR

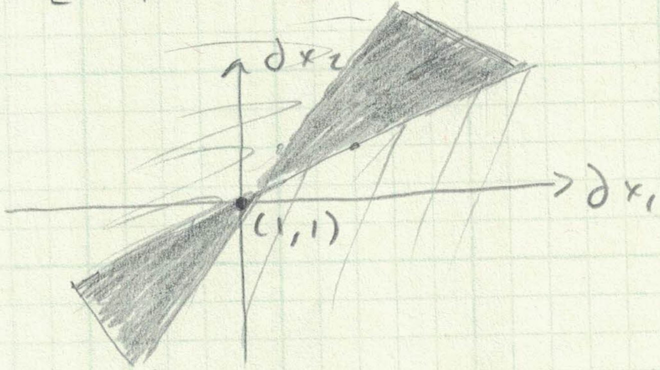
$$\boxed{\begin{aligned} a &= 1 \\ b &= 3/2 \\ c &= 2 \\ d &= 1 \end{aligned}}$$

use

→

1) $f(x_1,x_2)-f(1,1)=\left[(x_1-1)-\frac{3}{2}(x_2-1)\right]\cdot\left[2(x_1-1)-(x_2-1)\right]$

$$\left.\begin{array}{l}\text{if } \left[(x_1-1)-\frac{3}{2}(x_2-1)\right]>0 \\ \text{then } \left[2(x_1-1)-(x_2-1)\right]<0\end{array}\right\} \text{\& vice versa} \quad \begin{array}{l}\rightarrow \partial x_2 < \frac{2}{3}\partial x_1 \\ \rightarrow \partial x_2 > 2\partial x_1\end{array}$$



regions w/ lower function values

directions of downslopes: $\boxed{\begin{array}{l}\partial x_2 < \frac{2}{3}\partial x_1 \\ \partial x_2 > 2\partial x_1\end{array}}$ region 1

$\boxed{\begin{array}{l}\partial x_2 > \frac{2}{3}\partial x_1 \\ \partial x_2 < 2\partial x_1\end{array}}$ region 2