5) $c^*(y) = \max_x \{xy - c(x)\}$

show this is convex w.r.t. y

convex func.: $f(x)$ is convex if & only if

$$f(\lambda x_1 + (1-\lambda)x_2) \leq \lambda f(x_1) + (1-\lambda) f(x_2)$$

$$\forall x_1, x_2 \in X \;, \forall \lambda \in [0, 1]$$

$$\frac{dc^*(y)}{dy} = x$$

$$\frac{d^2c^*(y)}{dy^2} = 0$$

$c^*(y)$ is linear w.r.t. y

$\therefore$ $c^*(y)$ is convex w.r.t. y