4) $\min_{p} \max_{k} \{ h(a_k^T p, I_t) \}$

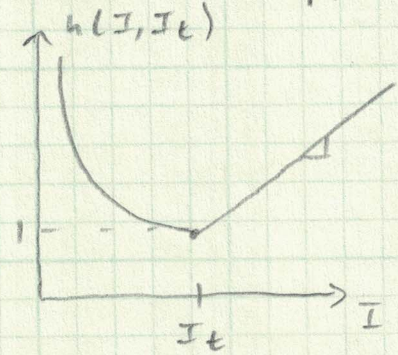
s.t. $0 \le p_i \le p_{max}$ $\longrightarrow$ convex (linear) w.r.t. p

$$h(I, I_t) = \begin{cases} I_t / I & I \le I_t \\ I / I_t & I \ge I_t \end{cases}$$

$\downarrow$ $\boxed{\text{convex domain}}$

a) show problem is convex

first show $h(a_k^T p, I_t)$ is convex w.r.t. I



$$\frac{\partial h}{\partial p} = \frac{dh}{dI} \cdot \frac{\partial (a^T p)}{\partial p} = h' a$$

$$\frac{\partial^2 h}{\partial p^2} = \frac{\partial^2 h}{\partial I^2} \cdot \frac{\partial (a^T p)}{\partial p} a^T = h'' a a^T$$ $\leftarrow$ H is a matrix

$\underline{h'' \ge 0} \rightarrow h$ is convex w.r.t. I

$\left. \begin{array}{l} I < I_t \rightarrow h'' > 0 \\ I > I_t \rightarrow h'' = 0 \end{array} \right\}$ h'' (Hessian) is p.s.d.

since $h(I, I_t)$ is convex w.r.t. I

$h(a^T p, I_t)$ is convex w.r.t. p ✓

$\downarrow$ $\boxed{\text{convex objective}}$

$\boxed{\therefore \text{ problem is convex since objective \& domain are convex}}$

4) b) if $\underbrace{\sum_{i=1}^{10} p_i \leq p^*}$ will there be a unique solution?

linear (convex) w.r.t. p

$\downarrow$

convex domain + convex objective

$\downarrow$

$\boxed{\text{unique solution}}$

c) if $\sum_{i=1}^{10} p_i > 0$

simpler case: 2 lamps p_1 & p_2 but no more than one can be on

$p_1 > 0$ or $p_2 > 0$

$\boxed{\text{no unique solution}}$