ME598

2) a) $x_1 + 2x_2 + 3x_3 = 1$

find the point nearest to $(-1, 0, 1)^T$

is this convex?

objective:

$$\min_{\vec{x}} d^2 = (x_1+1)^2 + x_2^2 + (x_3-1)^2 \qquad s.t.\ x_1 + 2x_2 + 3x_3 = 1$$

expand: $f_1 = x_1^2 + 2x_1 + x_2^2 + x_3^2 - 2x_3 + 2$

$$\nabla_x f_1 = \begin{bmatrix} \partial x_1 \\ \partial x_2 \\ \partial x_3 \end{bmatrix} = \begin{bmatrix} 2x_1 + 2 \\ 2x_2 \\ 2x_3 - 2 \end{bmatrix}$$

$$\nabla_x^2 f_1 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \qquad \lambda = 2, 2, 2 \rightarrow p.d.$$

$\downarrow$

objective is convex

constraint: $x_1 + 2x_2 + 3x_3 = 1$

hyper plane $\rightarrow$ convex

$\boxed{\therefore \text{ yes this is a convex problem: convex objective with convex constraints}}$

solve it! convert to unconstrained problem:

$\downarrow$

take care of constraint in objective function

$$x_1 = 1 - 2x_2 - 3x_3$$

$$f_2 = (1 - 2x_2 - 3x_3)^2 + 2(1 - 2x_2 - 3x_3) + x_2^2 + x_3^2 - 2x_3 + 2$$

$$f_2 = 5x_2^2 + 12x_2x_3 - 8x_2 + 10x_3^2 - 14x_3 + 5$$

$\min_{x_2, x_3} f_2 \leftarrow$ unconstrained objective!

$$\nabla_x f_2 = \begin{bmatrix} 10x_2 + 12x_3 - 8 \\ 12x_2 + 20x_3 - 14 \end{bmatrix}$$

2) $\nabla^2_x f_2 = \begin{bmatrix} 10 & 12 \\ 12 & 20 \end{bmatrix}$ $\lambda = 2, 28 \rightarrow$ positive definite $\rightarrow$ convex problem

$\rightarrow$ solution will be a minimum

where does $\nabla_x f_2 = 0$?

$$\left.\begin{aligned} 10x_2 + 12x_3 - 8 &= 0 \\ 12x_2 + 20x_3 - 14 &= 0 \end{aligned}\right\} \quad \begin{aligned} x_2 &= -1/7 \\ x_3 &= 11/14 \end{aligned}$$

$\downarrow$

$$x_1 = 1 - 2\left(-\frac{1}{7}\right) - 3\left(\frac{11}{14}\right)$$

$$x_1 = -\frac{15}{14}$$

$$\boxed{\tilde{x} = \left(-\frac{15}{14}, -\frac{1}{7}, \frac{11}{14}\right)}$$