√3) $\max\ f = x_1 x_2 + x_2 x_3 + x_1 x_3$ ⎫ $\min\ -f$
s.t. $h = x_1 + x_2 + x_3 - 3 = 0$ ⎭ s.t. h

$L = -x_1 x_2 - x_2 x_3 - x_1 x_3 + \lambda(x_1 + x_2 + x_3 - 3)$

$$\nabla_x L = \begin{bmatrix} -x_2 - x_3 + \lambda \\ -x_1 - x_3 + \lambda \\ -x_2 - x_1 + \lambda \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\nabla_\lambda L = x_1 + x_2 + x_3 - 3 = 0$$

$$\boxed{\begin{aligned} x_1 &= 1 \\ x_2 &= 1 \\ x_3 &= 1 \\ \lambda &= 2 \end{aligned}}$$

$$L_{xx} = \begin{bmatrix} 0 & -1 & -1 \\ -1 & 0 & -1 \\ -1 & -1 & 0 \end{bmatrix} \to \text{not p.d.!!}$$

need to prove 2nd order sufficient condition (global min.)

$$d_x^T L_{xx} d_x = [dx_1\ dx_2\ dx_3] \begin{bmatrix} 0 & -1 & -1 \\ -1 & 0 & -1 \\ -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} dx_1 \\ dx_2 \\ dx_3 \end{bmatrix} > 0 ?$$

$$= -2dx_1 dx_2 - 2dx_1 dx_3 - 2dx_2 dx_3 \quad (1)$$

$$\frac{\partial h}{\partial x} \cdot d_x = 0 \to \begin{bmatrix} \frac{\partial h}{\partial x_1} & \frac{\partial h}{\partial x_2} & \frac{\partial h}{\partial x_3} \end{bmatrix} \begin{bmatrix} dx_1 \\ dx_2 \\ dx_3 \end{bmatrix} = 0$$

$$[1 \quad 1 \quad 1] \begin{bmatrix} dx_1 \\ dx_2 \\ dx_3 \end{bmatrix} = 0$$

$$dx_1 + dx_2 + dx_3 = 0$$

$$dx_1 = -dx_2 - dx_3$$

↳ sub in (1):

$$= -2((-dx_2 - dx_3)dx_2) + (-dx_2 - dx_3)dx_3 + dx_2 dx_3) + \tfrac{3}{4}dx_3^2)$$

$$= 2(dx_2^2 + dx_2 dx_3 + dx_3^2) = 2\left((dx_2 + \tfrac{1}{2}dx_3)^2 + \tfrac{3}{4}dx_3^2\right) > 0$$

if $dx_3 = 0$, $dx_2 = 0$, then $dx_1 = 0 \to$ NOT a perturbation

√ global min (2nd order sufficient condition met)