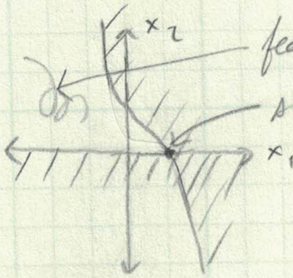
2) min $-x_1$
s.t. $g_1 = x_2 - (1-x_1)^3 \le 0$
$g_2 = x_2 \ge 0 \to -x_2 \le 0$



feasible domain

solution: $\boxed{x = \begin{bmatrix} 1 \\ 0 \end{bmatrix}}$

$$L = -x_1 + \mu_1 (x_2 - (1-x_1)^3) - \mu_2 x_2$$

$$\nabla_x L = \begin{bmatrix} -1 + 3\mu_1 (1-x_1)^2 \\ \mu_1 - \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

① if $x_2 - (1-x_1)^3 = 0$, $\mu_1 > 0$. if $x_2 - (1-x_1)^3 < 0$, $\mu_1 = 0$
② if $-x_2 = 0$, $\mu_2 > 0$, if $-x_2 < 0$, $\mu_2 = 0$

① $x_2 - (1-x_1)^3 = 0$, $\mu_1 > 0$, $-x_2 = 0$, $\mu_2 > 0$

$x = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\nabla_x L = \begin{bmatrix} -1 \\ \mu_1 - \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \to -1 = 0$? X not possible

② $x_2 - (1-x_1)^3 = 0$, $\mu_1 > 0$, $-x_2 < 0$, $\mu_2 = 0$

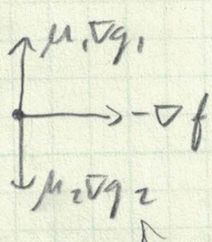
$\nabla_x L = \begin{bmatrix} \\ \mu_1 - 0 \end{bmatrix} = \begin{bmatrix} \\ 0 \end{bmatrix}$ $\mu_1 = 0$? X $\mu_1 > 0$

③ $x_2 - (1-x_1)^3 < 0$, $\mu_1 = 0$,
$\mu_2 = 0$ X $-1 = 0$ X

no possible scenario for the active constraints!
↳ ∴ cannot find solution based on optimality conditions
why?

$$\nabla f + \mu_1 \nabla g_1 + \mu_2 \nabla g_2 = 0$$

$$\begin{bmatrix} -1 \\ 0 \end{bmatrix} + \mu_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \mu_2 \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$



forces do not balance!

> The gradient of the objective func. does not exist in the space spanned by the linearly dependent gradients of the active constraints

↳ optimal solution is not regular so KKT conditions are not necessary or sufficient!