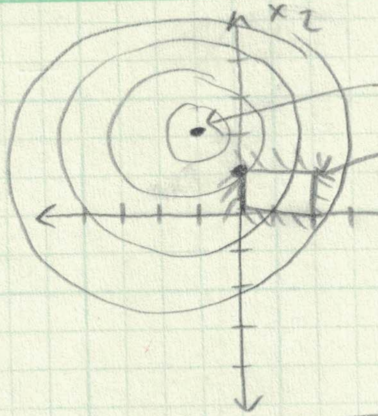
MAE598 HW4

1)



func. min

feasible space

$$\min_{x_1, x_2} \; (x_1+1)^2 + (x_2-2)^2$$

s.t. $g_3 = x_1 - 2 \le 0$

$g_4 = x_2 - 1 \le 0$

$g_1 = -x_1 \le 0$

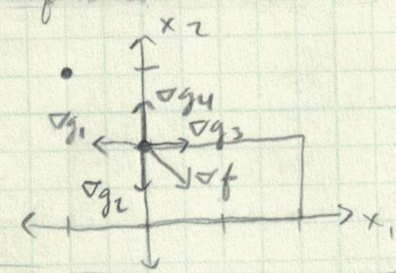
$g_2 = -x_2 \le 0$

$\therefore$ optimal: $\boxed{x = \begin{bmatrix} 0 \\ 1 \end{bmatrix}}$

$\nabla_x f = \begin{bmatrix} 2(x_1+1) \\ 2(x_2-2) \end{bmatrix}$, $\nabla_x g_1 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$, $\nabla_x g_2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$, $\nabla_x g_3 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\nabla_x g_4 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

directions of feasible descent at corners, ∇f's, & ∇g_i's

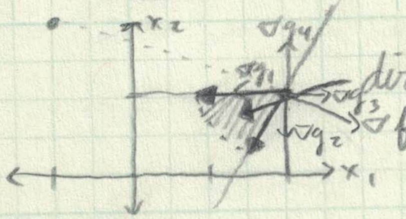
(0, 1)



$\nabla_x f(0,1) = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$

$\boxed{\text{no direction of feasible descent}}$

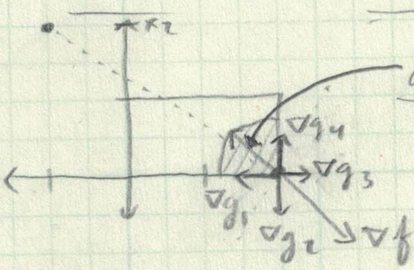
(2, 1)



direction of feasible descent

$\nabla_x f(2,1) = \begin{bmatrix} 6 \\ -2 \end{bmatrix}$

(2, 0)

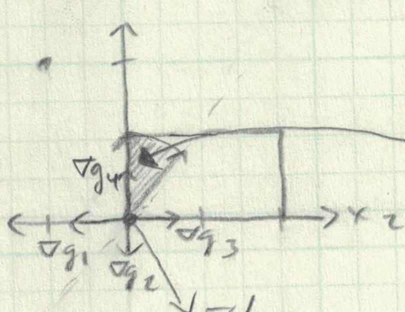


direction of feasible descent

$\nabla_x f(2,0) = \begin{bmatrix} 6 \\ -4 \end{bmatrix}$

(0, 0)

$\nabla_x f(0,0) = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$



direction of feasible descent

1) KKT conditions:

$$L = (x_1+1)^2 + (x_2-2)^2 \overbrace{-\mu_1 x_1}^{g_1} \overbrace{-\mu_2 x_2}^{g_2} \overbrace{+\mu_3(x_1-2)}^{g_3} \overbrace{+\mu_4(x_2-1)}^{g_4}$$

$$\nabla_x L = \begin{bmatrix} 2(x_1+1) - \mu_1 + \mu_3 \\ 2(x_2-2) - \mu_2 + \mu_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

if $-x_1 = 0,\ \mu_1 > 0$; if $-x_1 < 0,\ \mu_1 = 0$

$-x_2 = 0,\ \mu_2 > 0$; $-x_2 < 0,\ \mu_2 = 0$

$x_1 - 2 = 0,\ \mu_3 > 0$; $x_1 - 2 < 0,\ \mu_3 = 0$

$x_2 - 1 = 0,\ \mu_4 > 0$; $x_2 - 1 < 0,\ \mu_4 = 0$

verify solution w/ KKT conditions

$x = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \to$ active x_1: $x_1 = 0$, $x_2 = 1$

$\downarrow$ $\mu_1 > 0$, $\mu_3 = 0$; $\downarrow$ $\mu_2 = 0$, $\mu_4 > 0$

$$\nabla_x L \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 - \mu_1 \\ -2 + \mu_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{matrix} \to \mu_1 = 2 \\ \to \mu_4 = 2 \end{matrix}$$

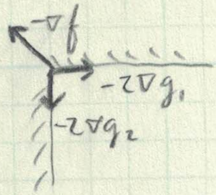
✓ (positive)
✓ (positive)

satisfies x & μ conditions, potential solution

check "balance of forces"

$$\nabla f + \mu_1 \nabla g_1 + \overset{\to 0}{\cancel{\mu_2}} \nabla g_2 + \overset{\to 0}{\cancel{\mu_3}} \nabla g_3 + \mu_4 \nabla g_4$$

$$\begin{bmatrix} 2 \\ -2 \end{bmatrix} + 2\begin{bmatrix} -1 \\ 0 \end{bmatrix} + 2\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$ ✓ "forces" balance



* only corner of feasible domain with no "external forces"