Assignment 1

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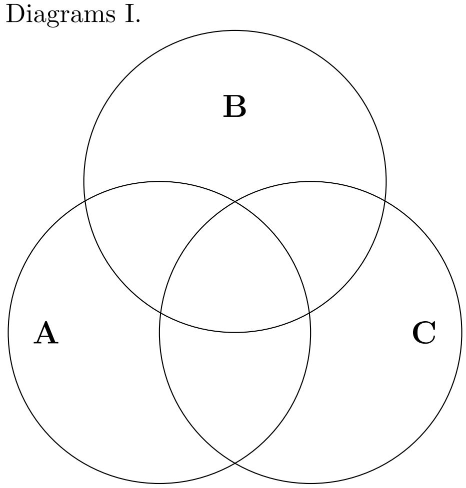
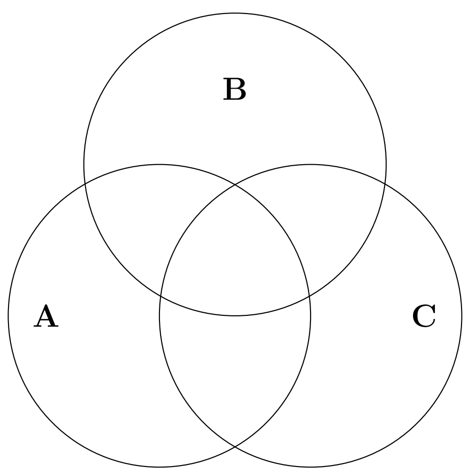
January 27, 2016

(I worked with Ellen Chen)

Problem 1. Venn Diagrams I.





Problem 2. *Venn Diagrams II* Give the simplest description you can of the following Venn diagrams.

Solution.

$$\textbf{A)}\ [A \cap (B \cup C)) \cup (B \setminus (B \cap C \cap \overline{A})]$$

$$\textbf{B)}\ [B \setminus (A \cup C)] \cup [A \cap B \cap C]$$

Problem 3. *Set Identities*

Solution. Proving using set identities.
A) Prove using double-inclusion the following :

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$

From the left hand-side :

$$\overline{A \cup B} = U \setminus (A \cup B)$$

$$= (U \setminus A) \cap (U \setminus B) = \overline{A} \cap \overline{B}$$

which is also a subset of the right hand-side.

From the right hand-side :

$$\overline{A} \cap \overline{B} = (U \setminus A) \cap (U \setminus B)$$

$$= U \setminus (A \cup B) = \overline{A \cup B}$$

which is also a subset of the left hand-side.

B) Prove using set identities the following :

$$(B \setminus A) \cup (C \setminus A) = (B \cup C) \setminus A$$

$$(B \cap \overline{A}) \cup (C \cap \overline{A}) = (B \cup C) \cap \overline{A}$$

(by set difference law)

$$(B \cup C) \cap \overline{A} = (B \cup C) \cap \overline{A}$$

(by distributivity)

Problem 4. *Truth Tables.* Use truth tables to determine which of the following statements are tautologies.

Solution. We need to find out whether the following logical expressions are tautologies or not. Let's write down the truth table for each of those.

Table 1: p $\oplus q = \overline{p} \vee \overline{q}$

p	q	p $\oplus q$	$\overline{p}$	$\overline{q}$	$\overline{p} \vee \overline{q}$
0	0	0	1	1	1
0	1	1	1	0	1
1	0	1	0	1	1
1	1	0	0	0	0

Since the 3rd column (left-hand side) and the 6th column (right-hand side) aren't the same, it follows that both sides of the equation are not logically equivalent. Therefore, **(A) is not a tautology.**

Table 2: $(\overline{q} \wedge \overline{(p \Rightarrow q)}) \equiv \overline{p}$

p	q	p $\Rightarrow q$	$\overline{p \Rightarrow q}$	$\overline{q}$	$\overline{q} \wedge \overline{(p \Rightarrow q)}$	$\overline{p}$
0	0	1	0	1	0	1
0	1	1	0	0	0	1
1	0	0	1	1	1	0
1	1	1	0	0	0	0

Since the 6th column (left-hand side) and the 7th column (right-hand side) aren't the same, it follows that both sides of the equation are not logically equivalent. Therefore, **(B) is not a tautology.**

Since the following logical expression is fairly long, it is useful to split both side of the equation into their own truth tables.

Table 3: $(\mathrm{p} \Rightarrow \overline{q}) \Leftrightarrow (r \Rightarrow (p \vee \overline{q}))$(left-hand side)

p	q	r	$\overline{q}$	$\mathrm{p} \Rightarrow \overline{q}$	$\mathrm{p} \vee \overline{q}$	$\mathrm{r} \Rightarrow (p \vee \overline{q})$	$(\mathrm{p} \Rightarrow \overline{q}) \Leftrightarrow (r \Rightarrow (p \vee \overline{q}))$
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	0	1	0	1	1
0	1	1	0	1	0	0	0
1	0	0	1	1	1	1	1
1	0	1	1	1	1	1	1
1	1	0	0	0	1	1	0
1	1	1	0	0	1	1	0

Table 4: $\mathrm{q} \vee(\overline{p} \wedge \overline{r})$ (right-hand side)

p	q	r	$\overline{p}$	$\overline{r}$	$\overline{p} \wedge \overline{r}$	$\mathrm{q} \vee(\overline{p} \wedge \overline{r})$
0	0	0	1	1	1	1
0	0	1	1	0	0	0
0	1	0	1	1	1	1
0	1	1	1	0	0	1
1	0	0	0	1	0	0
1	0	1	0	0	0	0
1	1	0	0	1	0	1
1	1	1	0	0	0	1

As we see in both truth table (*Table 3* and Table 4), both side of the expression are logically different. Hence, it follows that the expression **(C) is not a tautology.**

Problem 5. *Logic Rules.* Use the rules of logic to prove the following statements are tautologies.

Solution.

$$\text{A) } p \Rightarrow (q \vee \overline{q})$$

$$\overline{p} \vee (q \vee \overline{q})$$

(by definition)

$$\overline{p} \vee 1$$

(by complement)

$$= 1$$

(by identity)

B) $\overline{(p \oplus q)} \equiv (p \Leftrightarrow q)$

$$\overline{(p \lor q) \land \overline{(p \land q)}} \equiv (\overline{p} \land \overline{q}) \lor (p \land q)$$

(by definition)

$$\overline{(p \lor q) \land \overline{(p \land q)}} \equiv \overline{(p \lor q)} \lor (p \land q)$$

(by de Morgan's rules)

$$\overline{(p \lor q)} \lor \overline{\overline{(p \land q)}} \equiv \overline{(p \lor q)} \lor (p \land q)$$

(by de Morgan's rules)

$$\overline{(p \lor q)} \lor (p \land q) \equiv \overline{(p \lor q)} \lor (p \land q)$$

(by double negations)

$$= 1$$

C) $(p \Rightarrow q) \Rightarrow ((r \Rightarrow p) \Rightarrow (r \Rightarrow q))$

$$(p \Rightarrow q) \Rightarrow (r \Rightarrow (p \Rightarrow q))$$

(by implication distributivity)

$$(\overline{p} \lor q) \Rightarrow (r \Rightarrow (\overline{p} \lor q))$$

(by definition)

$$(\overline{p} \lor q) \Rightarrow (\overline{r} \lor (\overline{p} \lor q))$$

(by definition)

$$\overline{(\overline{p} \lor q)} \lor (\overline{r} \lor (\overline{p} \lor q))$$

(by definition)

$$\overline{(\overline{p} \lor q)} \lor ((\overline{p} \lor q) \lor \overline{r})$$

(commutation laws)

$$[\overline{(\overline{p} \lor q)} \lor (\overline{p} \lor q)] \lor \overline{r}$$

(associativity)

$$1 \lor \overline{r}$$

(identity)

$$= 1$$

Problem 6. *Circuits.* Show how NOR gates can be used to simulate OR, AND and NOT gates.

Solution. Let's draw the logic gate expression equivalent only using NOR gates.