Columbia University – Fall 2013

Machine Learning Homework #5

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1 Problem 1

Assume you are a contestant of a game show in which you are presented with three closed doors A, B, and C. Behind one of the doors is a car which will be yours if you choose the right door. After you have randomly (as you have no prior information) selected a door (say door A), the game host opens door B which has nothing inside, while keeping door A and C closed. The host then asks whether you want to change your selection from A to C. Should you change?

We should change. Assuming that the game is fair and the likelihood that the car is behind doors A,B,and C is equal, then $p(car = A) = p(car = B) = p(car = C) = \frac{1}{3}$. Therefore, without loss of generality if we choose door A, we have a $\frac{1}{3}$ chance of winning the car. However, the host then opens up one of the doors that is not the door that was chosen or the one that actually contains the car, WLOG let's assume it is door B.

The host then asks us if we would like to change our choice. Now there are two doors available to choose from (A and C), and if we were to choose randomly from this configuration we would have $p(car = A) = p(car = C) = \frac{1}{2}$. However, because the host is guaranteed not to choose the door that we first chose whether it had the car in it or not, the $p(car = chosen(A)) = \frac{1}{3}$ and does not change. Therefore, we should switch our pick to the other available door, because by switching we upgrade our probability of getting the car to $\frac{1}{2}$.

2 Problem 2

2.1 Problem 2a

We want to prove that $X \perp Y|Z$ if and only if the joint probability $p(x, y, z)$ can be expressed in the form of $a(x, z)b(y, z)$.

PROOF Assume that $X \perp Y|Z$, therefore we know that $p(x|y, z) = p(x, z)$ and $p(x|y) \neq p(x)$.

$$p(x, y, z) = p(y, z)p(x|y, z) \tag{1}$$
$$= p(y, z)p(x, z). \tag{2}$$

Therefore, we have $p(x, y, z)$ can be expressed in the form of $a(x, z)b(y, z)$.

2.2 Problem 2b

We want to prove or disprove with a counter example, that $X \perp Y|Z$ and $X \perp W|(Y, Z)$ implies that $X \perp (W, Y)|Z$. We want to use this knowledge to prove that $p(x|(w, y, z)) = p(x|z)$.

PROOF

$$p(x|(w, y, z)) = p(x|y, z), \textit{ by } X \perp W|(Y, Z) \tag{3}$$
$$= p(x|z), \textit{ by } X \perp Y|Z. \tag{4}$$

Thus, we have shown $X \perp (W, Y)|Z$.