

$$Y = WN + B = W(f(HW \cdot X + HB)) + B$$

$$N_j = HW \cdot X_j + HB$$

$$\begin{aligned} \alpha \nabla_{HW} L &= \frac{\alpha}{N} \frac{\partial}{\partial HW} L = -\frac{\alpha}{N} \frac{\partial}{\partial HW} \sum_i D(S(WX_j + b), L_{ij}) = -\frac{\alpha}{N} \frac{\partial}{\partial HW} \sum_i \sum_j L_{ij} \ln(S_{ij}) \\ &= -\frac{\alpha}{N} \frac{\partial}{\partial HW} \sum_{i=2} \sum_{j=50} L_{ij} \ln\left(\frac{e^{y_{ij}}}{e^{y_{1j}} + \dots + e^{y_{ij}}}\right) = -\frac{\alpha}{N} \frac{\partial}{\partial HW} \sum_{i=2} \sum_{j=50} L_{ij} y_{ij} - L_{ij} \ln(e^{y_{1j}} + \dots + e^{y_{ij}}) \\ &= -\frac{\alpha}{N} \frac{\partial N}{\partial HW} \frac{\partial}{\partial N} \sum_{i=2} \sum_{j=50} L_{ij} (W_i N_j + B_i) - L_{ij} \ln(e^{y_{1j}} + \dots + e^{y_{ij}}) \\ &= -\frac{\alpha}{N} \frac{\partial N}{\partial HW} \frac{\partial}{\partial N} \sum_{j=50} L_{1j} (W_1 N_j + B_1) + \dots + L_{ij} (W_i N_j + B_i) - (L_{1j} + \dots + L_{ij}) \ln(e^{y_{1j}} + \dots + e^{y_{ij}}) \\ &= -\frac{\alpha}{N} \frac{\partial N}{\partial HW} \frac{\partial}{\partial N} [L_{11} (W_1 N_1 + B_1) + \dots + L_{i1} (W_i N_1 + B_i) + \dots + L_{1j} (W_1 N_j + B_1) + \dots + L_{ij} (W_i N_j + B_i) \\ &\quad - (L_{11} + \dots + L_{i1}) \ln(e^{y_{1j}} + \dots + e^{y_{ij}}) - \dots - (L_{1j} + \dots + L_{ij}) \ln(e^{y_{1j}} + \dots + e^{y_{ij}})] \\ &= -\frac{\alpha}{N} \frac{\partial N}{\partial HW} \frac{\partial}{\partial N} [L_{11} (W_1 N_1) + \dots + L_{i1} (W_i N_1) + \dots + L_{1j} (W_1 N_j) + \dots + L_{ij} (W_i N_j) \\ &\quad - (L_{11} + \dots + L_{i1}) \ln(e^{y_{11}} + \dots + e^{y_{i1}}) - \dots - (L_{1j} + \dots + L_{ij}) \ln(e^{y_{1j}} + \dots + e^{y_{ij}})] \\ &= -\frac{\alpha}{N} \frac{\partial N}{\partial HW} \left\{ \begin{bmatrix} L_{11} W_1 + \dots + L_{i1} W_i \\ \vdots \\ L_{1j} W_1 + \dots + L_{ij} W_i \end{bmatrix} - \begin{bmatrix} (L_{11} + \dots + L_{i1}) \frac{\frac{\partial}{\partial N_1} (e^{y_{11}} + \dots + e^{y_{i1}})}{e^{y_{11}} + \dots + e^{y_{i1}}} \\ \vdots \\ (L_{1j} + \dots + L_{ij}) \frac{\frac{\partial}{\partial N_2} (e^{y_{11}} + \dots + e^{y_{i1}})}{e^{y_{1j}} + \dots + e^{y_{ij}}} \end{bmatrix} \right\} \\ &= -\frac{\alpha}{N} \frac{\partial N}{\partial HW} \left\{ L^T \cdot W - \begin{bmatrix} \frac{1}{e^{y_{11}} + \dots + e^{y_{i1}}} [W_1 e^{y_{11}} + \dots + W_i e^{y_{i1}}] \\ \vdots \\ \frac{1}{e^{y_{1j}} + \dots + e^{y_{ij}}} [W_1 e^{y_{1j}} + \dots + W_i e^{y_{ij}}] \end{bmatrix} \right\} \\ &= -\frac{\alpha}{N} \frac{\partial (R(HWX + HB))}{\partial HW} \{L^T \cdot W - \begin{bmatrix} W_1 S(y_{11}) + \dots + W_i S(y_{i1}) \\ \vdots \\ W_1 S(y_{1j}) + \dots + W_i S(y_{ij}) \end{bmatrix}\} \\ &= -\frac{\alpha}{N} \left(\frac{\partial}{\partial HW} \begin{bmatrix} R(HW_1 X_1 + HB_1) & \dots & R(HW_1 X_j + HB_1) \\ \vdots & \ddots & \vdots \\ R(HW_k X_1 + HB_k) & \dots & R(HW_k X_j + HB_k) \end{bmatrix} \right) [L^T \cdot W - S^T \cdot W] \\ &= -\frac{\alpha}{N} \begin{bmatrix} R'(HW_1 X_1 + HB_1) X_1 & \dots & R'(HW_1 X_j + HB_1) X_j \\ \vdots & \ddots & \vdots \\ R'(HW_k X_1 + HB_k) X_1 & \dots & R'(HW_k X_j + HB_k) X_j \end{bmatrix} [L^T \cdot W - S^T \cdot W] = -\frac{\alpha}{N} [R'(HWX + HB) \cdot X] [(L^T - S^T) \cdot W] \end{aligned}$$

200*50 400*50 2*50 2*200

i=50, j=2, k=200, m=400

$$\begin{aligned}
\alpha \nabla_{HW} L &= \alpha \frac{\partial}{\partial hw_{mk}} \sum_i D(S(WX_i + b), L_i) = -\frac{\alpha}{N} \frac{\partial}{\partial hw_{mk}} \sum_i \left[\sum_j L_{ij} \ln \left(\frac{e^{y_{ij}}}{\sum_j e^{y_j}} \right) \right] = -\frac{\alpha}{N} \frac{\partial}{\partial hw_{mk}} \sum_i \left[\sum_j L_{ij} y_{ij} - L_{ij} \ln \left(\sum_j e^{y_j} \right) \right] \\
&= -\frac{\alpha}{N} \frac{\partial}{\partial hw_{mk}} \sum_i \left[\sum_j L_{ij} \left(\sum_k (w_{jk} n_{ki}) + b_j \right) - L_{ij} \ln (e^{y_{1i}} + \dots + e^{y_{ji}}) \right] \\
&= -\frac{\alpha}{N} \frac{\partial}{\partial hw_{mk}} \sum_i \left[\sum_j L_{ij} \left(\sum_k \left(w_{jk} \cdot R \left(\sum_m (hw_{km} \cdot x_{mi}) + hb_k \right) \right) + b_j \right) - L_{ij} \ln (e^{y_{1i}} + \dots + e^{y_{ji}}) \right] \\
&= -\frac{\alpha}{N} \sum_i \left[\sum_j L_{ij} w_{jk} \cdot R' \left(\sum_m (hw_{km} \cdot x_{mi}) + hb_k \right) x_{mi} - L_{ij} \frac{(w_{1k} \cdot R' (\sum_m (hw_{km} \cdot x_{mi}) + hb_k) x_{mi}) \cdot e^{y_{1i}} + \dots}{e^{y_{1i}} + \dots + e^{y_{ji}}} \right] \\
&= -\frac{\alpha}{N} \sum_i \left[\sum_j L_{ij} \left(w_{jk} \cdot R' \left(\sum_m (hw_{km} \cdot x_{mi}) + hb_k \right) x_{mi} \right) - L_{ij} \left(w_{1k} \cdot R' \left(\sum_m (hw_{km} \cdot x_{mi}) + hb_k \right) x_{mi} \right) S(y_{1i}) + \dots \right. \\
&\quad \left. + \left(w_{jk} \cdot R' \left(\sum_m (hw_{km} \cdot x_{mi}) + hb_k \right) x_{mi} \right) S(y_{ji}) \right] \\
&= -\frac{\alpha}{N} \sum_i \left[\sum_j (L_{ij} - S_{ij}) \left(w_{jk} \cdot R' \left(\sum_m (hw_{km} \cdot x_{mi}) + hb_k \right) \cdot x_{mi} \right) \right]
\end{aligned}$$

[(L11 –S11) w1k + (L12 – S12) w2k] xm1 Rk1 + [(L21 –S21)w1k +(L22- S22) w2k] xm2 Rk2 +...+ [(L50,1 –S50,1)w1k +(L50,2- S50,2) w2k] xm50 Rk50

$$\begin{aligned}
&[w1k \quad w2k] \begin{bmatrix} (L11 - S11)xm1 Rk1 + (L21 - S21)xm2 Rk2 + \dots \\ (L12 - S12)xm1 Rk1 + (L22 - S22)xm2 Rk2 + \dots \end{bmatrix} \\
&= [w1k \quad w2k] \begin{bmatrix} (L11 - S11) & (L21 - S21) & \dots \\ (L12 - S12) & (L22 - S22) & \dots \end{bmatrix} [Rk1xm1 \quad Rk2xm2 \quad \dots \quad Rk50xm50]^T \\
&= [w1k \quad w2k] (L - S) [Rk1xm1 \quad Rk2xm2 \quad \dots \quad Rk50xm50]^T \\
&= [w1k \quad w2k] (L - S) ([xm1 \quad xm2 \quad \dots \quad xm50]^T * [R'k1 \quad R'k2 \quad \dots \quad R'k50]^T)
\end{aligned}$$

$$\begin{aligned}
\alpha \nabla_{HB} L &= \alpha \frac{\partial}{\partial hb_k} \sum_i D(S(WX_i + b), L_i) = -\frac{\alpha}{N} \frac{\partial}{\partial hb_k} \sum_i \left[\sum_j L_{ij} \ln \left(\frac{e^{y_{ij}}}{\sum_j e^{y_j}} \right) \right] = -\frac{\alpha}{N} \frac{\partial}{\partial hb_k} \sum_i \left[\sum_j L_{ij} y_{ij} - L_{ij} \ln \left(\sum_j e^{y_j} \right) \right] \\
&= -\frac{\alpha}{N} \frac{\partial}{\partial hb_k} \sum_i \left[\sum_j L_{ij} \left(\sum_k (w_{jk} n_{ki}) + b_j \right) - L_{ij} \ln (e^{\sum_k (w_{1k} n_{ki}) + b_1} + \dots + e^{\sum_k (w_{jk} n_{ki}) + b_j}) \right] \\
&= -\frac{\alpha}{N} \frac{\partial}{\partial hb_k} \sum_i \left[\sum_j L_{ij} \left(\sum_k \left(w_{jk} \cdot R \left(\sum_m (hw_{km} \cdot x_{mi}) + hb_k \right) \right) + b_j \right) - L_{ij} \ln (e^{y_{1i}} + \dots + e^{y_{ji}}) \right] \\
&= -\frac{\alpha}{N} \sum_i \left[\sum_j L_{ij} \left(w_{jk} \cdot R' \left(\sum_m (hw_{km} \cdot x_{mi}) + hb_k \right) \right) - L_{ij} \frac{(w_{jk} \cdot R' (\sum_m (hw_{km} \cdot x_{mi}) + hb_k)) \cdot e^{y_{1i}} + \dots}{e^{y_{1i}} + \dots + e^{y_{ji}}} \right] \\
&= -\frac{\alpha}{N} \sum_i \left[\sum_j L_{ij} \left(w_{jk} \cdot R' \left(\sum_m (hw_{km} \cdot x_{mi}) + hb_k \right) \right) - L_{ij} \frac{(w_{jk} \cdot R' (\sum_m (hw_{km} \cdot x_{mi}) + hb_k)) \cdot e^{y_{1i}} + \dots}{e^{y_{1i}} + \dots + e^{y_{ji}}} \right] \\
&= -\frac{\alpha}{N} \sum_i \left[\sum_j L_{ij} \left(w_{jk} \cdot R' \left(\sum_m (hw_{km} \cdot x_{mi}) + hb_k \right) \right) - L_{ij} \frac{(w_{1k} \cdot R' (\sum_m (hw_{km} \cdot x_{mi}) + hb_k)) \cdot e^{y_{1i}} + \dots}{e^{y_{1i}} + \dots + e^{y_{ji}}} \right]
\end{aligned}$$

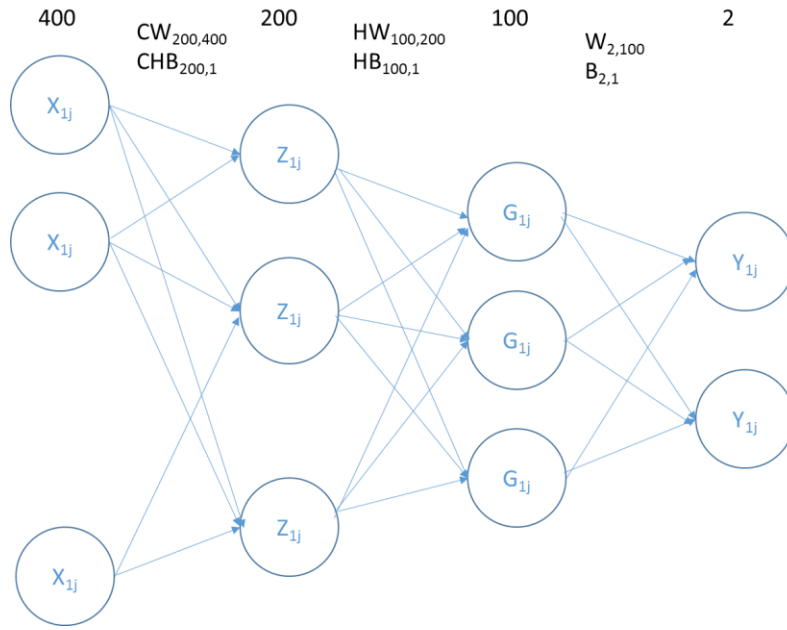
$$\begin{aligned}
&= -\frac{\alpha}{N} \sum_i \left[\sum_j L_{ij} (w_{jk} \cdot R' \left(\sum_m (hw_{km} \cdot x_{mi}) + hb_k \right)) - L_{ij} \left(w_{1k} \cdot R' \left(\sum_m (hw_{km} \cdot x_{mi}) + hb_k \right) \right) S(y_{1i}) + \dots \right. \\
&\quad \left. + \left(w_{jk} \cdot R' \left(\sum_m (hw_{km} \cdot x_{mi}) + hb_k \right) \right) S(y_{ji}) \right] \\
&= -\frac{\alpha}{N} \sum_i \left[\sum_j (L_{ij} - S_{ij}) (w_{jk} \cdot R' \left(\sum_m (hw_{km} \cdot x_{mi}) + hb_k \right)) \right]
\end{aligned}$$

$$\begin{aligned}
&= w1k((L11-S11) Rk1+(L21- S21) Rk2+...(L50,1- S50,1) Rk50)+ w2k((L12-S12) Rk1+(L22-S22) Rk2+...(L50,2-S50,2) Rk50) \\
&=w1k[(L11-S11) (L21- S21)...(L50,1- S50,1)][Rk1 Rk2...Rk50]^T+ w2k[(L12-S12) (L22-S22) ...(L50,2-S50,2)] [Rk1 Rk2...Rk50]^T
\end{aligned}$$

$$\begin{bmatrix} w1k & w2k \end{bmatrix} \begin{bmatrix} [(L11 - S11)(L21 - S21) \dots (L50,1 - S50,1)][Rk1 Rk2 \dots Rk50]^T \\ [(L12 - S12)(L22 - S22) \dots (L50,2 - S50,2)] [Rk1 Rk2 \dots Rk50]^T \end{bmatrix}$$

$$\begin{bmatrix} w1k & w2k \end{bmatrix} (L - S) [Rk1 Rk2 \dots Rk50]^T$$

2*200 200*50 50*2



i=50, j=2,k=200,m=400, q=100

$$\begin{aligned}
\alpha \nabla_{cw} L &= -\frac{\alpha}{N} \frac{\partial}{\partial cw_{mk}} \sum_i \left[\sum_j L_{ij} \ln \left(\frac{e^{y_{ij}}}{\sum_j e^{y_j}} \right) \right] = -\frac{\alpha}{N} \frac{\partial}{\partial cw_{mk}} \sum_i \left[\sum_j L_{ij} y_{ij} - L_{ij} \ln \left(\sum_j e^{y_j} \right) \right] \\
&= -\frac{\alpha}{N} \frac{\partial}{\partial cw_{mk}} \sum_i \left[\sum_j L_{ij} \left(\sum_q (w_{jq} g_{qi}) + b_j \right) - L_{ij} \ln (e^{y_{1i}} + \dots + e^{y_{ji}}) \right] \\
&= -\frac{\alpha}{N} \frac{\partial}{\partial cw_{mk}} \sum_i \left[\sum_j L_{ij} \left(\sum_q \left(w_{jq} \cdot R' \left(\sum_k (hw_{qk} z_{ki}) + hb_q \right) \right) + b_j \right) - L_{ij} \ln (e^{y_{1i}} + \dots + e^{y_{ji}}) \right] \\
&= -\frac{\alpha}{N} \frac{\partial}{\partial cw_{mk}} \sum_i \left[\sum_j L_{ij} \left(\sum_q \left(w_{jq} \cdot R' \left(\sum_k \left(hw_{qk} \cdot R' \left(\sum_m (cw_{km} x_{mi}) + cb_k \right) \right) + hb_q \right) \right) + b_j \right) - L_{ij} \ln (e^{y_{1i}} + \dots + e^{y_{ji}}) \right] \\
&= -\frac{\alpha}{N} \sum_i \left[\sum_j L_{ij} \left(\sum_q \left(w_{jq} \cdot R' \left(\sum_k \left(hw_{qk} \cdot R' \left(\sum_m (cw_{km} x_{mi}) + cb_k \right) \right) + hb_q \right) \cdot hw_{qk} \cdot R' \left(\sum_m (cw_{km} x_{mi}) + cb_k \right) \cdot x_{mi} \right) \right. \right. \\
&\quad \left. \left. - L_{ij} \frac{(\quad) e^{y_{1i}} + \dots + (\quad) e^{y_{ji}}}{(e^{y_{1i}} + \dots + e^{y_{ji}})} \right] \right]
\end{aligned}$$

$$\begin{aligned}
&= -\frac{\alpha}{N} \sum_i \left[\sum_j L_{ij} \left(\sum_q \left(w_{jq} \cdot R' \left(\sum_k \left(hw_{qk} \cdot R \left(\sum_m (cw_{km} x_{mi}) + cb_k \right) \right) + hb_q \right) \cdot hw_{qk} \cdot R' \left(\sum_m (cw_{km} x_{mi}) + cb_k \right) \cdot x_{mi} \right) \right) - L_{ij} ((\quad) S(y_{1i}) \right. \\
&\quad \left. + \cdots (\quad) S(y_{ji}) \right] \\
&= -\frac{\alpha}{N} \sum_i \left[\sum_j L_{ij} \left(\sum_q \left(w_{jq} \cdot R' \left(\sum_k (hw_{qk} \cdot z_{ki}) + hb_q \right) \cdot hw_{qk} \cdot R' \left(\sum_m (cw_{km} x_{mi}) + cb_k \right) \cdot x_{mi} \right) \right) - L_{ij} ((\quad) S(y_{1i}) + \cdots (\quad) S(y_{ji}) \right] \\
&= -\frac{\alpha}{N} \sum_i \left[\sum_j (L_{ij} - S_{ij}) \left(\sum_q \left(w_{jq} \cdot R' \left(\sum_k (hw_{qk} \cdot z_{ki}) + hb_q \right) \cdot hw_{qk} \cdot R' \left(\sum_m (cw_{km} x_{mi}) + cb_k \right) \cdot x_{mi} \right) \right) \right]
\end{aligned}$$

$$\begin{aligned}
\alpha \nabla_{CB} L &= \alpha \frac{\partial}{\partial cb_k} \sum_i D(S(WX_i + b), L_i) = -\frac{\alpha}{N} \frac{\partial}{\partial cb_k} \sum_i \left[\sum_j L_{ij} \ln \left(\frac{e^{y_{ij}}}{\sum_j e^{y_{ij}}} \right) \right] = -\frac{\alpha}{N} \frac{\partial}{\partial cb_k} \sum_i \left[\sum_j L_{ij} y_{ij} - L_{ij} \ln \left(\sum_j e^{y_{ij}} \right) \right] \\
&= -\frac{\alpha}{N} \frac{\partial}{\partial cb_k} \sum_i \left[\sum_j L_{ij} \left(\sum_q (w_{jq} g_{qi}) + b_j \right) - L_{ij} \ln(e^{y_{1i}} + \cdots + e^{y_{ji}}) \right] \\
&= -\frac{\alpha}{N} \frac{\partial}{\partial cb_k} \sum_i \left[\sum_j L_{ij} \left(\sum_q \left(w_{jq} \cdot R \left(\sum_k (hw_{qk} z_{ki}) + hb_q \right) \right) + b_j \right) - L_{ij} \ln(e^{y_{1i}} + \cdots + e^{y_{ji}}) \right] \\
&= -\frac{\alpha}{N} \frac{\partial}{\partial cb_k} \sum_i \left[\sum_j L_{ij} \left(\sum_q \left(w_{jq} \cdot R \left(\sum_k \left(hw_{qk} \cdot R \left(\sum_m (cw_{km} x_{mi}) + cb_k \right) \right) + hb_q \right) \right) + b_j \right) - L_{ij} \ln(e^{y_{1i}} + \cdots + e^{y_{ji}}) \right] \\
&= -\frac{\alpha}{N} \sum_i \left[\sum_j L_{ij} \left(\sum_q \left(w_{jq} \cdot R' \left(\sum_k \left(hw_{qk} \cdot R \left(\sum_m (cw_{km} x_{mi}) + cb_k \right) \right) \right) + hb_q \right) \cdot hw_{qk} \cdot R' \left(\sum_m (cw_{km} x_{mi}) + cb_k \right) \right) \right. \\
&\quad \left. - L_{ij} \frac{(\quad) e^{y_{1i}} + \cdots + (\quad) e^{y_{ji}}}{(e^{y_{1i}} + \cdots + e^{y_{ji}})} \right] \\
&= -\frac{\alpha}{N} \sum_i \left[\sum_j L_{ij} \left(\sum_q \left(w_{jq} \cdot R' \left(\sum_k \left(hw_{qk} \cdot z_{ki} \right) + hb_q \right) \cdot hw_{qk} \cdot R' \left(\sum_m (cw_{km} x_{mi}) + cb_k \right) \right) \right) \right. \\
&\quad \left. - L_{ij} \left(\sum_q \left(w_{1q} \cdot R' \left(\sum_k (hw_{qk} \cdot z_{ki}) + hb_q \right) \cdot hw_{qk} \cdot R' \left(\sum_m (cw_{km} x_{mi}) + cb_k \right) \right) S(y_{1i}) + \cdots + (\dots) S(y_{ji}) \right) \right] \\
&= -\frac{\alpha}{N} \sum_i \left[\sum_j (L_{ij} - S_{ij}) \left(\sum_q \left(w_{jq} \cdot R' \left(\sum_k \left(hw_{qk} \cdot z_{ki} \right) + hb_q \right) \cdot hw_{qk} \cdot R' \left(\sum_m (cw_{km} x_{mi}) + cb_k \right) \right) \right) \right]
\end{aligned}$$

$$\begin{aligned}
&(\text{L11-S11}) [\text{w11 Rh}'_{11} \text{hwk},1 + \dots + \text{w1},100 \text{Rh}'_{1,100} \text{hwk},100] \text{RC}'\text{k1} \\
&+ (\text{L12-S12}) [\text{w21 Rh}'_{11} \text{hwk}1 + \dots + \text{w2},100 \text{Rh}'_{1,100} \text{hwk},100] \text{RC}'\text{k1} \\
&+ \dots \\
&+ (\text{L50,1-S50,1}) [\text{w11 Rh}'_{1,50} \text{hwk},1 + \dots + \text{w1},100 \text{Rh}'_{50,100} \text{hwk},100] \text{RC}'\text{k},50 \\
&+ (\text{L50,2-S50,2}) [\text{w21 Rh}'_{1,50} \text{hwk},1 + \dots + \text{w2},100 \text{Rh}'_{50,100} \text{hwk},100] \text{RC}'\text{k},50 \\
&=
\end{aligned}$$

$$\left(\begin{bmatrix} \text{hwk},1 & \dots & \text{hwk},100 \end{bmatrix} \begin{bmatrix} \text{w1},1 & \text{w1},100 \\ \vdots & \vdots \\ \text{w2},1 & \text{w2},100 \end{bmatrix} \cdot \begin{bmatrix} \text{L11} - \text{S11} & \dots & \text{L50,1} - \text{S50,1} \\ \text{L12} - \text{S12} & \dots & \text{L50,2} - \text{S50,2} \end{bmatrix} * \left(\begin{bmatrix} \text{Rh}'_{1,1} & \dots & \text{Rh}'_{50,1} \\ \vdots & \ddots & \vdots \\ \text{Rh}'_{1,100} & \dots & \text{Rh}'_{50,100} \end{bmatrix} \right) \right) \begin{bmatrix} \text{RC}'\text{k1} \\ \vdots \\ \text{RC}'\text{k},50 \end{bmatrix}$$

