

Context Free Languages

1. For this question, you will prove that context-free languages are NOT closed under intersection. Do this by showing the following:
 - **Part 1:** First, show that $A = \{a^m b^n c^n \mid m, n \geq 0\}$ producing a context-free grammar that generates it.
 - **Part 2:** Do the same, but for language $B = \{a^n b^n c^m \mid m, n \geq 0\}$
 - **Part 3:** Lastly, find the intersection of these two sets and use the pumping lemma to show that the intersection language is not regular.
2. The grammar below looks like a portion of a reasonable programming language. For this question, you need to first show that this grammar is ambiguous, then re-write the grammar to be unambiguous for the same language.

STMT \rightarrow ASSIGN|IF-THEN|IF-THEN-ELSE
IF-THEN \rightarrow if condition then STMT
IF-THEN-ELSE \rightarrow if condition then STMT else STMT
ASSIGN $\rightarrow a := 1$

3. Let us define a new operation using the \diamond symbol as such: if A and B are languages, then $A \diamond B = \{xy \mid x \in A, y \in B, |x| = |y|\}$. Prove that if A and B are regular languages, then $A \diamond B$ must be a context-free language.