Regular Languages

1. Draw out *DFA*s (not *NFA*s) for each of the following languages. For some of these, a small hint is provided. Your goal is to construct a *DFA* with as few states as possible (just like how we prefer to write succinct code when possible). For all of these, let $\Sigma = \{a, b\}$

- $\{w \mid w$ does not contain the substring $ab\}$ (**Hint: Draw out the DFA for a simpler language that DOES contain ab and then try to change that machine slightly.*)
- $\{w \mid w$ contains neither the substrings ab nor ba$\}$
- $\{w \mid w \notin a^*b^*\}$
- $\{w \mid w \neq a \wedge w \neq b\}$
- $\{w \mid w$ does not contain a pair of a's that are separated by an odd number of characters$\}$ (**Hint: This one should have exactly 5 states*)

2. Prove that regular languages are closed under *intersection*. Do this by starting with *DFA*s for two regular languages A and B, and describe how to construct a new *DFA* for $A \cap B$

3. Prove that regular languages are closed under *complement*. Do this by starting with a *DFA* for a regular language A, and describe how to construct a new *DFA* for $\bar{A}$.

4. For any string $w = w_1w_2, ..., w_n$, let w^R be the reverse of string w (i.e., $w^R = w_n, ..., w_2, w_1$). Prove that if a language A is regular, then the language $A^R = \{w^R \mid w \in A\}$ is also regular.

5. Use the pumping lemma to show that the following languages are not regular.

- $A = \{0^n1^n2^n \mid n \geq 0\}$
- $B = \{www \mid w \in \{a, b\}^*\}$

6. Find and describe the error that exists in the following proof. The proof attempts to show that 0^*1^* is not regular, when in fact it is:

Assume, for sake of contradiction, that 0^*1^* is regular. We select an element from this language that is greater than the pumping length p. We select 0^p1^p. In class, when proving that 0^n1^n was not regular, we showed that 0^p1^p cannot be pumped. Therefore, 0^*1^* is not regular.