

## Regular Languages

1. Draw out *DFAs* (not *NFAs*) for each of the following languages. For some of these, a small hint is provided. Your goal is to construct a *DFA* with as few states as possible (just like how we prefer to write succinct code when possible). For all of these, let  $\Sigma = \{a, b\}$ 
  - $\{w \mid w \text{ does not contain the substring } abba\}$  (\*Hint: Draw out the DFA for a simpler language that DOES contain *abba* and then try to change that machine slightly.)\*
  - $\{w \mid w \text{ contains BOTH the substrings } ab \text{ and } ba\}$
  - $\{w \mid w \in a^*b^*a^*\}$
  - $\{w \mid w \neq ab \wedge w \neq bb\}$
  - $\{w \mid w \in a^i w \mid i \in \mathbb{N}, w \in \{a, b\}^*, w \text{ contains at least } i \text{ a's}\}$  (\*Hint: This one LOOKS not regular but it actually is. Can you figure out why?\*)
2. Prove that regular languages are closed under *intersection*. Do this by starting with *DFAs* for two regular languages  $A$  and  $B$ , and describe how to construct a new *DFA* for  $A \cap B$
3. Prove that regular languages are closed under *complement*. Do this by starting with a *DFA* for a regular language  $A$ , and describe how to construct a new *DFA* for  $\bar{A}$ .
4. For any string  $w = w_1 w_2, \dots, w_n$ , let  $w^R$  be the reverse of string  $w$  (i.e.,  $w^R = w_n, \dots, w_2, w_1$ ). Prove that if a language  $A$  is regular, then the language  $A^R = \{w^R \mid w \in A\}$  is also regular.
5. Use the pumping lemma to show that the following languages are not regular OR argue that they are regular.
  - $A = \{0^* 0^n 1^n 1^* \mid n \geq 0\}$
  - $B = \{www \mid w \in \{0, 1\}^*\}$
6. Find and describe the error that exists in the following proof. The proof attempts to show that  $0^* 1^*$  is not regular, when in fact it is:

*Assume, for sake of contradiction, that  $0^* 1^*$  is regular. We select an element from this language that is greater than the pumping length  $p$ . We select  $0^p 1^p$ . In class, when proving that  $0^n 1^n$  was not regular, we showed that  $0^p 1^p$  cannot be pumped. Therefore,  $0^* 1^*$  is not regular.*