Turing Machines and Decidability

1. Give implementation level descriptions for Turing Machines that decides the following two languages.

   - $\{w|w$ contains an equal number of 0s and 1s$\}$
   - $\{w|w$ contains an twice as many 0s as 1s$\}$

2. Let $INF_{DFA} = \{A|$A is a DFA and L(A) is infinite$\}$. Show that INF_{DFA} is decidable.

3. A *2-PDA* is a pushdown automata that has two stacks instead of just one. Prove that a 2-PDA is more powerful than a traditional PDA with one stack. *Hint: Show how to simulate a Turing Machine's tape with the 2-PDA.*

4. A *Turing Machine with doubly-infinite tape* is an ordinary Turing Machine, except that the tape is infinitely indexed in both the left and right direction. Prove that this machine is equivalent in computational power to a traditional Turing Machine.

5. For this question, you will do five separate proofs. Prove that the class of *Decidable Languages* is closed under *union*, *concatenation*, *star*, *complement*, and *intersection*.

6. Prove the following claim: Let C be a language. Prove that C is Turing-recognizable if and only if a decidable language D exists such that $C = \{x|\exists y((x, y) \in D)\}$

7. Consider the following decision problem: "Given a Turing Machine *M* and input *w*, does *M* ever write a 1 to the 10th cell of its tape?". Show, via reduction, that this problem is *undecidable*.