Context Free Languages

1. For this question, you will prove that context-free languages are NOT closed under intersection. Do this by showing the following:

   - **Part 1:** First, show that $A = \{a^m b^n c^n | m, n \geq 0\}$ is context-free by producing a context-free grammar that generates it.
   - **Part 2:** Do the same, but for language $B = \{a^n b^n c^m | m, n \geq 0\}$
   - **Part 3:** Lastly, find the intersection of these two sets and use the pumping lemma to show that the intersection language is not context-free.

2. The grammar below looks like a portion of a reasonable programming language. For this question, you need to first show that this grammar is ambiguous, then re-write the grammar to be unambiguous for the same language. *An ambiguous grammar is one in which there is at least one string that has two unique derivations.*

$$\begin{aligned} \text{STMT} &\rightarrow \text{ASSIGN}|\text{IF-THEN}|\text{IF-THEN-ELSE} \\ \text{IF-THEN} &\rightarrow \text{if condition then STMT} \\ \text{IF-THEN-ELSE} &\rightarrow \text{if condition then STMT else STMT} \\ \text{ASSIGN} &\rightarrow a := 1 \end{aligned}$$

3. Let us define a new operation using the $\diamond$ symbol as such: if A and B are languages, then $A \diamond B = \{xy | x \in A, y \in B, |x| = |y|\}$. Prove that if A and B are regular languages, then $A \diamond B$ must be a context-free language.