Context Free Languages

1. For each of the languages below, provide a context-free grammar that generates it (Note that many of these are also regular languages). For all parts, $\Sigma = \{0, 1\}$:

   - All strings that contain at least three 1's
   - Strings of odd length that have 0 as middle symbol
   - Palindromes (i.e., $w = w^R$)

2. Draw PDAs for each of the languages in the previous exercise (note that you can draw a DFA / NFA if the language happens to be regular).

3. For this question, you will prove that context-free languages are NOT closed under intersection. Do this by showing the following:

   - **Part 1:** First, show that $A = \{a^m b^n c^n | m, n \geq 0\}$ is context-free by producing a context-free grammar that generates it.
   - **Part 2:** Do the same, but for language $B = \{a^n b^n c^m | m, n \geq 0\}$
   - **Part 3:** Lastly, find the intersection of these two sets and use the pumping lemma to show that the intersection language is not context-free.

4. Let us define a new operation using the $\diamond$ symbol as such: if A and B are languages, then $A \diamond B = \{xy | x \in A, y \in B, |x| = |y|\}$. Prove that if A and B are regular languages, then $A \diamond B$ must be a context-free language.

5. Are *Context-Free Grammars* closed under union, concatenation, and star? Explain at a high level why or why not. I do not need a formal proof here, but I want you to describe verbally how you would construct machines for each of these operations, and describe the primary differences between these constructions and the ones for regular languages.

6. Prove that the following language is *NOT* context-free: $\{wtw^R \mid w, t \in \{0, 1\}^* \wedge |w| = |t|\}$