Regular Languages

1. Draw out *DFAs* (not *NFAs*) for each of the following languages. For some of these, a small hint is provided. Your goal is to construct a *DFA* with as few states as possible (just like how we prefer to write succinct code when possible). For all of these, let $\Sigma = \{a, b\}$

   - $\{w \mid w \text{ does not contain the substring } abba\}$ (**Hint: Draw out the DFA for a simpler language that DOES contain ab and then try to change that machine slightly.*)

   - $\{w \mid w \text{ contains BOTH the substrings ab and ba}\}$

   - $\{w \mid w \in a^*b^*a^*\}$

   - $\{w \mid w \neq ab \land w \neq bb\}$

   - $\{w \mid w \in a^i w \mid i \in \mathbb{N}, w \in \{a, b\}^*, \text{w contains at least i a's}$ (**Hint: This one LOOKS not regular but it actually is. Can you figure out why?*)

2. Prove that regular languages are closed under *intersection*. Do this by starting with *DFAs* for two regular languages A and B, and describe how to construct a new *DFA* for $A \cap B$

3. Prove that regular languages are closed under *complement*. Do this by starting with a *DFA* for a regular language A, and describe how to construct a new *DFA* for $\bar{A}$.

4. For any string $w = w_1 w_2, ..., w_n$, let w^R be the reverse of string w (i.e., $w^R = w_n, ..., w_2, w_1$). Prove that if a language A is regular, then the language $A^R = \{w^R \mid w \in A\}$ is also regular.

5. Use the pumping lemma to show that the following languages are not regular.

   - $A = \{0^*0^n1^n1^* \mid n \geq 0\}$

   - $B = \{w0^*011^*w \mid w \in \{0, 1\}^*\}$

6. Find and describe the error that exists in the following proof. The proof attempts to show that 0^*1^* is not regular, when in fact it is:

   Assume, for sake of contradiction, that 0^*1^* is regular. We select an element from this language that is greater than the pumping length p. We select 0^p1^p. In class, when proving that 0^n1^n was not regular, we showed that 0^p1^p cannot be pumped. Therefore, 0^*1^* is not regular.