# Regular Languages

1. Draw out *DFA*s (not *NFA*s) for each of the following languages. For some of these, a small hint is provided. Your goal is to construct a *DFA* with as few states as possible (just like how we prefer to write succinct code when possible). For all of these, let $\Sigma = \{a, b\}$

   - $\{w \mid w$ does not contain the substring $abba\}$ (**Hint: Draw out the DFA for a simpler language that DOES contain abba and then try to change that machine slightly.*)
   - $\{w \mid w$ contains BOTH the substrings ab and ba$\}$
   - $\{w \mid w \in a^*b^*a^*\}$
   - $\{w \mid w \neq ab \wedge w \neq bb\}$
   - $\{w \mid w \in a^i w' \mid i \in \mathbb{N}, w' \in \{a, b\}^*,$ w′ contains at least i a's (**Hint: This one LOOKS not regular but it actually is. Can you figure out why?*)

2. Prove that regular languages are closed under *intersection*. Do this by starting with *DFA*s for two regular languages $A$ and $B$, and describe how to construct a new *DFA* for $A \cap B$

3. Prove that regular languages are closed under *complement*. Do this by starting with a *DFA* for a regular language $A$, and describe how to construct a new *DFA* for $\bar{A}$.

4. For any string $w = w_1 w_2, ..., w_n$, let $w^R$ be the reverse of string $w$ (i.e., $w^R = w_n, ..., w_2, w_1$). Prove that if a language $A$ is regular, then the language $A^R = \{w^R \mid w \in A\}$ is also regular.

5. Use the pumping lemma to show that the following languages are not regular OR argue that they are regular.

   - $A = \{0^* 0^n 1^n 1^* \mid n \geq 0\}$
   - $B = \{www \mid w \in \{0, 1\}^*\}$

6. Find and describe the error that exists in the following proof. The proof attempts to show that $0^*1^*$ is not regular, when in fact it is:

   *Assume, for sake of contradiction, that $0^*1^*$ is regular. We select an element from this language that is greater than the pumping length $p$. We select $0^p 1^p$. In class, when proving that $0^n 1^n$ was not regular, we showed that $0^p 1^p$ cannot be pumped. Therefore, $0^*1^*$ is not regular.*