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Model of Volume-Delay Formula to Assess Travel Time Savings of Underground Tunnel Roads

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Abstract

This article presents the result of conducting a working model of the Volume to Delay Formula (VDF) that will be applied to analyze the economic feasibility of underground tunnel road projects in Seoul. Without this formula, highway engineers are unable to assess the travel time savings of the underground tunnel road projects, and this troubling situation deters a reasonable progress of this massive plan. To deal with this problem, the research carried out a field survey study in an existing mountain tunnel to capture the characteristics of tunnel traffic flow and conducted a follow-up study to calibrate operational parameters of this tunnel traffic flow. The Golden Section search method was adopted in this purpose and major findings revealed that a reliable VDF could be successfully developed based on the analysis of existing mountain tunnel flows. It was also found that traffic volume levels estimated with existing VDF's and the ones estimated with developed VDF's were much different: a more reasonable volume level was observed across all highway links. We thus concluded that the developed VDF's should be applied to assess travel time savings of the underground tunnel road projects in Seoul. The study results can be applied successfully to six lane tunnels, but more validation efforts are required for other types of tunnels.

Keywords: *underground road tunnels, economic feasibility, travel time savings, volume to delay formula, the Golden Section search*

1. Introduction

A total number of passenger car trips in Seoul reach approximately 8.3 million trips a day, the same level in Tokyo or Hong Kong. In contrast, Seoul's total road length in terms of lane-km is 8,148 km and this level is far below that of 11,700 km in Tokyo and 13,800 km in Hong Kong. It is also noted that the demographic data for the Seoul Metropolitan Area show that the population of 20,550,000 is living in an area of 5,076 km; This is most unique as it is the world's highest population density of 4,048 persons/km^2. Further, 65% of the available land in Seoul is covered by mountains which challenges future highway projects in Seoul. To overcome this problem, the city government applied various traffic engineering methods (Kim *et al.*, 2012) and recently began contemplating underground tunnel projects underneath Seoul's built-up land areas (Seoul, 2010; Seoul, 2012).

A challenging issue associated with Seoul's underground tunnel project is that this project requires a sizeable funding from the central government and, for highway projects with construction costs of more than US$ 50 million, the central government has a regulation that it provides the funding only when a highway project is economically feasible. Highway feasibilities are examined based on a national guideline with the title of "A Study on Standard Guidelines for Pre-feasibility Study on Road and Railway Projects (KDI, 2008)". For previous decades, the size of the highway budget from the South Korean government has increased substantially from approximately US $6 billion in 1998 to US $7.5 billion in 2008, and this national feasibility guideline has been applied without exception to all highway projects since 1998 (Seoul, 2012). Unfortunately, this national guideline is inapplicable to underground tunnel projects because these projects have appeared only recently, so the guideline lacks a procedure for assessing their economic benefits.

In a highway feasibility study, travel time savings is the most important form among road user benefits and is determined with the Volume to Delay Formula (VDF). Transportation engineers develop this formula by investigating relationships between travel time increase and vehicle volume for a proposed highway. At the time of this paper, neither the relationship nor study result with reference to the VDF for underground tunnels exists. This situation is troubling and we demonstrate how engineers can successfully deal with this problem.

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The research approach involved several tasks. First, the function and characteristics of existing VDF's were detailed. Second, an analysis method for calibrating operational parameters of tunnel traffic flow was adopted and field studies were carried out to investigate the local characteristics associated with these parameters. Due to the fact that no underground tunnel road is available in this nation, the field studies were carried out instead in an existing mountain tunnel in Seoul. Third, based on the result from the second task, a VDF was created and its contributing effects were examined by applying this VDF to Seoul's future road network.

2. Application of the Volume-Delay Function

The current VDF, as shown in Eq. (1), has two cost components including travel time cost and toll fare cost, if applicable (KDI, 2008).

$$T = T_0[1 + \alpha(V/C)^{\beta}] + (\text{Section distance} \times \text{Weight}) \quad (1)$$

Where,

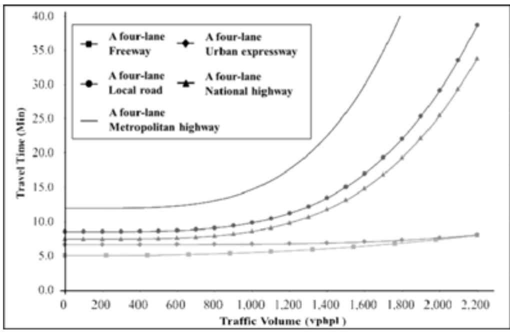
T = Link travel time (minutes)
T_0 = Link free flow travel time (minutes)
V = Link Traffic Volume (vehicles/hour)
C = Link Capacity (vehicles/hour)
α, β = Parameters
Weight = Factor to convert toll fares to travel time based on the value of travel time for each vehicle class (minute/km)

It is generally understood that the first part of Eq. (1) is the amount of vehicle delay resulting from the increase of traffic volume and was originally developed by the Bureau of Public Roads (BPR) in the US (KDI, 2008). The second part of Eq. (1) is the amount of tolls to be paid by motorists. It is also understood that the VDF model is applicable only to a practical volume range and that a monotonically increasing curve should be used to describe the increasing pattern of travel time (Ortuzar *et al*., 1994; Choi *et al*., 2013; Park *et al*., 2011; Kim and Choi, 2013). In developing the VDF for local conditions and achieving reliable results, it is important for engineers to provide such highway geometric information as the highway function, the highway type, the capacity values according to different terrain types, and the number of travel lanes (Choi *et al.*, 2011). Other required variables for applying the VDF include α, β, and the free-flow speed. Once this becomes available, the VDF serves as a basis of deciding for travellers which travel paths to take within a proposed highway network. Fig. 1 demonstrates typical VDF's prescribed in the national guideline (KDI, 2008).

3. Methodology

The research conducts a working model of VDF's applicable to underground tunnel roads based on observing tunnel traffic flows, but, as mentioned in the previous section, no underground tunnel road exists in South Korea at this time. Hence, data collection is a major challenge and the research tries to cope with this problem by conducting field studies in a lengthy mountain tunnel operating in Seoul. Apparently, this mountain tunnel and the proposed underground tunnel road will have different flow characteristics, but the mountain tunnel is the most similar type and the research had no choice but to conduct field studies here. As running underneath Seoul's surface areas, drivers in both types of tunnels will experience highly limited vehicle manoeuvring capabilities. As a result, a stable traffic flow will be observed in both types of tunnel flows. However, to tell the difference, drivers in underground tunnels will tend to change speeds more sensitively according to tunnel geometric features such as lane width, shoulder width, and the length of a tunnel (Park *et al.*, 2011).

In preparation of conducting the field studies, there was a question of whether traffic flows inside tunnels would be different from the one observed at highways running in built-up areas. To answer this question, the One-Way ANOVA test was applied in the research. We then proceeded to calibrate the operational parameters of tunnel traffic flows that will be subsequently connected to a VDF for the underground tunnel project in Seoul. We considered that one-dimensional unconstrained optimization is usually applied for calibrating parameters within a given function (Steven *et al.*, 2002). We also considered that the Golden Section search represented this optimization technique and that highway engineers in South Korea applied this technique quite



Type	Speed (km/h)	α	β	Capacity (vphpl)
-■-	117	0.645	2.047	2,200
-◆-	90	0.15	4.0	2,000
-●-	70	0.15	4.0	1,000
-▲-	80	0.15	4.0	1,000
—	50	0.15	4.0	900

Fig. 1. Typical VDF's Prescribed in South Korean National Guideline

extensively (Kang, 2008). Hence, the research decided to apply this technique and the following explains the Golden Section search procedure (Kang, 2008).

[Step 1] Initialization

It was required to set up a range of the objective and the BPR function in the initialization step. The research applied the RMSE (Root Mean Square Error) as given in Eq. (2) that denotes a difference between the observed and the estimated traffic volumes. The research also applied Eq. (3) to satisfy the range of parameters as prescribed in the national feasibility guidelines (KDI, 2008).

$$Z(\alpha,\beta) = RMSE = \sqrt{\frac{(\bar{v}_a - v_a)^2}{N}} \tag{2}$$

Where, $\bar{v}_a$ = Estimated traffic volumes
v_a = Observed traffic volumes
N = Number of sample size

$$0.100 \le \alpha \le 1.000, 2.000 \le \beta \le 5.000 \tag{3}$$

[Step 2] The optimization of α_n, β_n

The research calculated α_{n+1} by applying the Gold Section search method with a fixed state from initial values, α_0, β_0, and then calculated β_{n+1} by using the Gold Section search method with a fixed state from α_{n+1} that was already calculated in the previous step. An initial range for the values of a to b is shown in Eq. (4) for the iteration n as shown in Fig. 2. The research then calculated the parameters for the iteration ($n+1$) by applying Eq. (5).

$$b_n - a_n \le 2\varepsilon \tag{4}$$

$$a_{n+1} \text{ or } b_{n+1} = 0.5(a_n + b_n) \tag{5}$$

[Step 3] The convergence

When the gap of parameters between iteration n and n+1 involves an allowable error range shown in Eqs. (6) and (7), the research then stops the iteration process. Fig. 3 demonstrates this process. The allowable error ranges were selected based on Eq. (3), i.e., 0.1 % of the range of parameters was adopted as the allowable error ranges.

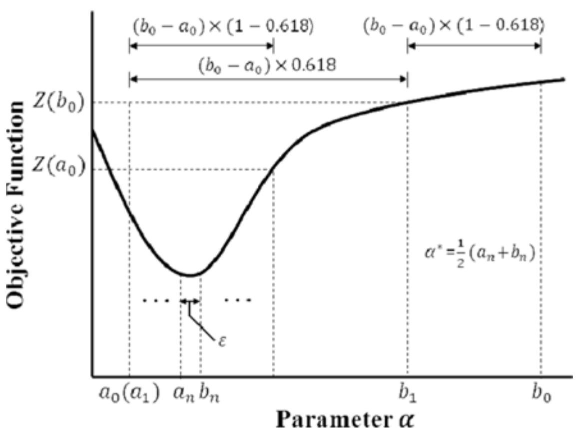


Fig. 2. Scheme of the Golden Section Method in Step 2 (Kumar and Sharma, 2010)

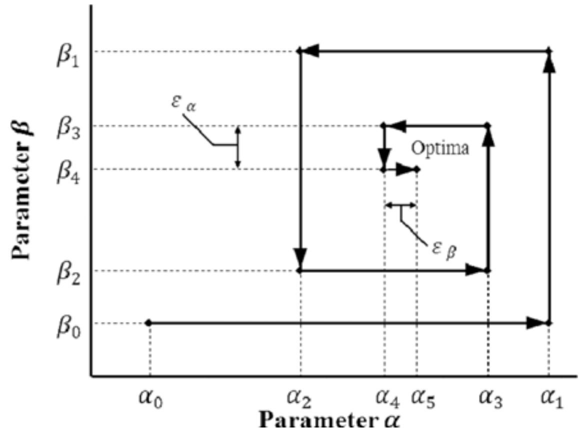


Fig. 3. Convergence of Parameters Applied in Step 3 (Kang, 2008)

$$|\alpha_n - \alpha_{n+1}| \le \varepsilon_\alpha, \quad \varepsilon_\alpha = 0.0009 \tag{6}$$

$$|\beta_n - \beta_{n+1}| \le \varepsilon_\beta, \quad \varepsilon_\beta = 0.003 \tag{7}$$

4. Field Study and Analysis

4.1 Site Selection and Traffic Flow Data

The research carried out a field study in a six lane lengthy mountain tunnel. This tunnel is slightly separated by an open 195 meter middle section and the separated tunnel sections have the names of Hongjimun tunnel and Jeongneung tunnel as depicted in Fig. 4. The design speed for the whole tunnel section is 80 km/h and the speed limit is 70 km/h. The cross section is a standard urban major arterial type with six lanes for both directions and 3.5 m of lane width.

The data collected included traffic volume and speed and they were measured with automatic traffic loop detectors. A total of eight automatic traffic loop detectors were located covering inside as well as outside tunnel areas as shown in Fig. 4. The data was collected with an interval of 15 minutes for four weeks ranging March 1 2012 to March 28 2012. In fact, Seoul is a mega-size city and its traffic flow pattern based on a four week study period will not be substantially different from the one based on ten weeks, so we decided to apply the four week base field data in further analysis. Table 1 is the summary of our field data.

Although it shows a relatively high value, our standard deviation actually lies within a reasonable range for the traffic data collected in Seoul area, which usually give a range of 300-600 based on hourly distributions (Kim, 2013; Seoul, 2010; Seoul, 2012). Based on the data collected in the field, the research conducted a traffic flow analysis and found the traffic flow characteristics, as shown in Fig. 5. A breakdown condition was found in this figure. It was also found that drivers generally slowed down on inside tunnel sections.

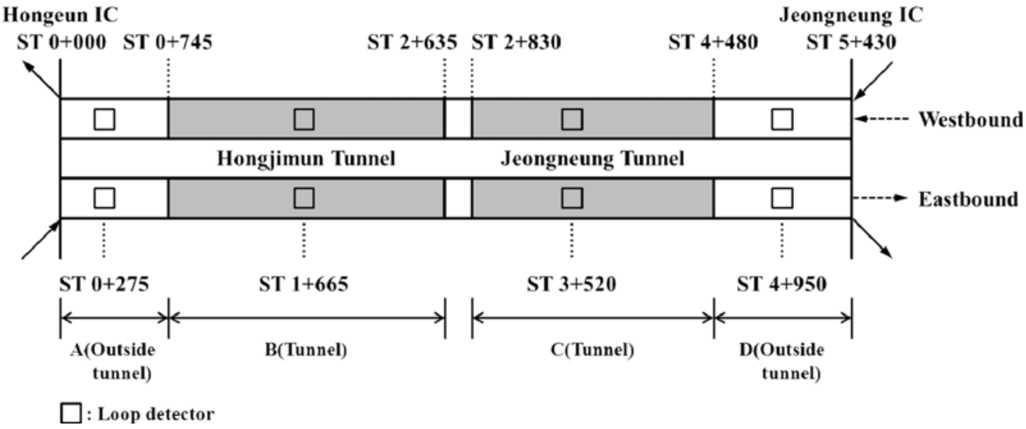


Fig. 4. Diagram of Field Study Sites and Deployment of Loop Detectors

Table 1. Summary of Data Collected in Field Study

		Min	Max	Mean	Std. Dev	No. of Samples
A (outside tunnel)	Volume (veh./h/lane)	214	1,808	1,212	400	1,344
	Running Speed (km/h)	17.0	111.7	74.0	29.8	
B (tunnel)	Volume (veh./h/lane)	180	1,782	1,191	441	1,344
	Running Speed (km/h)	1.6	121.1	74.5	30.2	
C (tunnel)	Volume (veh./h/lane)	240	1,819	1,220	418	1,344
	Running Speed (km/h)	1.2	120.4	87.5	17.6	
D (outside tunnel)	Volume (veh./h/lane)	200	1,867	1,175	447	1,344
	Running Speed (km/h)	51.0	122.1	99.3	10.2	

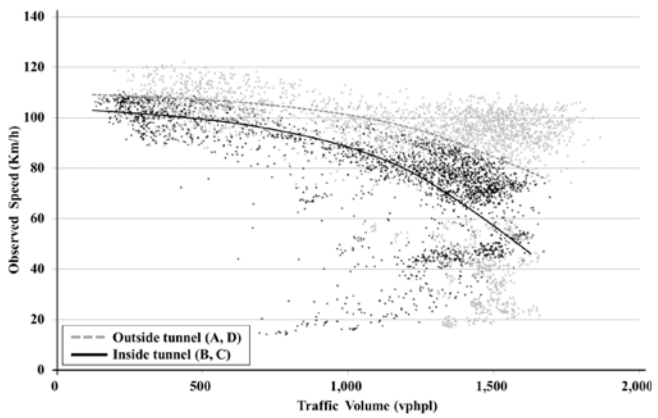


Fig. 5. Observed Traffic Volume and Speed at Mountain Tunnel Areas

Table 2. Statistical Summary of Kolmogorov-Smirnov Test

	Kolmogorov-Smirnov		
	Test values	Degrees of Freedom	Levels of Significance
A (outside tunnel)	0.225	1,343	0.000
B (tunnel)	0.133	1,343	0.003
C (tunnel)	0.169	1,343	0.000
D (outside tunnel)	0.173	1,343	0.000

Table 3. Statistical Summary of Levene Covariance Test

Test value of Levene Covariance	Degree of Freedom (1)	Degree of Freedom (2)	Level of Significance
3.342	3	5,373	0.020

4.2 Statistical Analyses

As described previously, we applied the One-Way ANOVA test to examine whether traffic flows observed inside tunnels would show a different pattern from the one for outside tunnels. This effort was necessary because we expected a unique flow pattern in underground tunnels and wanted to develop a VDF based on this unique flow pattern.

However, there were requirements to be satisfied before conducting the One-Way ANOVA test. First, the dependent variable is quantifiable. Second, the population of a sample shows the Normal Distribution. Third, there exists the covariance between variables. The first requirement is satisfied easily because the dependent variable in our field data is traffic volume, which is a quantity variable. To examine the second requirement, we applied the Kolmogorov-Smirnov test and found, as shown in Table 2, that the hourly volume followed the Normal Distribution at a confidence level of 95%. The third requirement was also satisfied as shown in Table 3 by conducting the Levene covariance test.

After confirming that the test requirements were satisfied, we proceeded to carry out the One-Way ANOVA test. In order to conduct this test, we divided the observed traffic flow levels into two groups, i.e., inside and outside of tunnels, and the test result, as listed in Table 4 revealed that traffic flows for these two

Table 4. Statistical Summary of the One-Way ANOVA Test

	Degree of Freedom	Mean of the Sum of Squares	F-value	Significance Level
Between Groups	3	6,104,187.4	28.483	0.000
Within Groups	5,373	214,312.5		
Sum	5,376			

locations were significantly different. Thus, we concluded that tunnel flow data should be separated from the data collected on general urban highways to develop a VDF for underground tunnel roads. Further, we applied a bimodality test to our data and the result showed that the value of bimodality coefficient was 0.35, a far less value than 5/9 (0.56). This result indicates that our data are not from a bimodal distribution, so we decided to apply the Normal Distribution.

We next applied Tukey's HSD (Honestly Significant Different) test to find that further divided traffic flows within these two groups would be significantly different from each other. As a result, we found, as described in Table 5, that the outside tunnel traffic flows observed at A and D were indeed different from inside tunnel flows, which were observed at B and C. Meanwhile, we also found that traffic flows at A and D were the same type of flow, and that B and C were the same type of flow. These findings all led to the conclusion that we could consider the data collected in two different tunnel sections in Fig. 4 as a data set collected from one tunnel section.

4.3 VDF For Underground Tunnel Roads

There are many methods for calibrating the parameters in the VDF, and we found that they usually apply fundamental relationships between traffic volume and speed observed in a given highway (Kang, 1996; Kim *et al.*, 2003). For instance, Kang *et al.* calibrated the parameters by transferring traffic volume, speed, and occupancy rates observed in regular highways to the BPR model and they realized that the resulting VDF produced reliable output (Kang, 1996; Kim *et al.*, 2003). For freeways, they also found that the VDF produced even more reliable output (Kang *et al.*, 1998). Therefore, we were confident that a VDF for underground tunnel roads should be developed based on traffic flow analysis.

It is noted that there are two operational parameters to describe the traffic flow of the underground tunnel roads. Thus, to find the parameter values, we applied free-flow speeds with a range of 90-120 km/h, as well as a capacity value of 2,200 vehicles per hour per lane. We selected these values with reference to South Korean study results, which found that capacity values for all highways with a speed range of 90-120 km/h would be 2,200 vehicles per hour per lane constantly (Kang, 1996; Kang *et al.*, 1998; Kim *et al.*, 2003; KDI, 2008). Upon completion of our work, we developed the VDF for underground tunnels as shown in Table 6 and Fig. 6. There were several interesting findings. First, based on the Root Mean Square Error (RMSE) associated with existing and developed VDF, the VDF developed in the research are more accurate, i.e. it involves less RMSE's over the whole range of vehicle speeds, as well as the number of lanes. In addition, it is expected that the best model performance will occur when engineers apply the VDF for speeds of 110 km/h. It is also expected, by looking into Fig. 6, that the delay growth at relatively high volume levels is quite substantial. This delay growth seems to result from the characteristics of the underground tunnel, where drivers slow down greatly when they perceive the presence of other vehicles. Obviously, the dark, closed, and lengthy environment in underground tunnels must have a strong impact on this delay growth. Finally, it is noticed in Fig. 6 that travel time savings of a highway project that can be assessed more realistically when engineers apply this result, because travel time increases sensitively with traffic volume.

Last, Eq. (8) is the finalized model of the VDF that engineers can apply to assess the travel time savings for future underground tunnel roads in Seoul.

$$T = \frac{L}{110} \times \left[1 + 0.894 \times \left(\frac{V}{2{,}200}\right)^{2.003}\right] \quad (R = 0.954) \tag{8}$$

Table 5. Statistical Summary of Tukey's HSD Test

	Between Groups	Difference of the Mean of Traffic Volumes	Standard Error	Significance Level	95% Confidence Level	
					Lower Limit	Upper Limit
A (outside tunnel)	B	200.726	25.255	0.000	135.80	265.65
	C	168.094	25.255	0.000	103.17	233.02
	D	49.482	25.255	0.204	-15.44	114.40
B (tunnel)	A	-200.726	25.255	0.000	-265.65	-135.80
	C	-32.632	25.255	0.568	-97.55	32.29
	D	-151.224	25.255	0.000	-216.17	-86.32
C (tunnel)	A	-168.094	25.255	0.000	-233.02	-103.17
	B	-32.632	25.255	0.568	-32.29	97.55
	D	-118.612	25.255	0.000	-183.53	-53.69
D (outside tunnel)	A	-49.482	25.255	0.204	-114.40	15.44
	B	151.244	25.255	0.002	86.32	216.17
	C	118.612	25.255	0.000	53.69	183.53

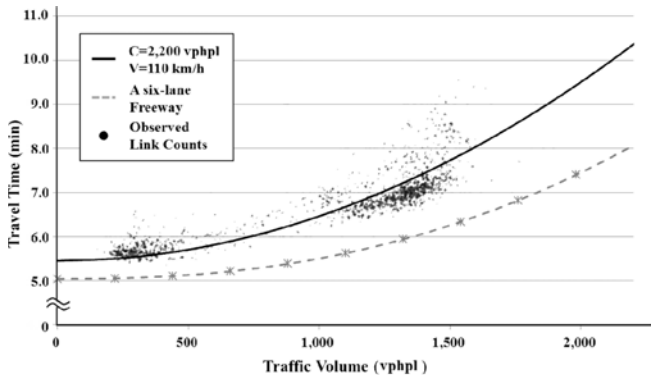


Fig. 6. Relationship between Travel Time and Volume for Developed VDF's

Table 6. Parameters for the Developed VDF's and Existing VDF's

		Speed (km/h)	Capacity (vehicles/ hour/lane)	α	β	RMSE (Root Mean Square Error)
Developed VDF's		120	2,200	1.000	2.000	18.030
		110	2,200	0.894	2.003	6.837
		100	2,200	1.000	2.000	6.915
		90	2,200	0.666	2.455	9.629
Existing VDF's	A six-lane Freeway	119	2,200	0.601	2.378	24.968
	A four-lane Freeway	117	2,200	0.645	2.047	20.556

Where, T = Link travel time (min)
L = Link distance (km)
V = Link travel volume (Vehicles/hour)

4.4 Validation

We also validated developed VDF's based on a separate data analysis. The Korean Highway Corporation, which is the central authority for freeway operations, was consulted for the data for this validation and they provided a data set from freeway tunnel sections. The data included traffic volume and speed data for two freeway tunnels including Suam tunnel and Masung tunnel. Suam tunnel is a six lane 1.3 km long freeway tunnel with a 100 km/h speed limit, while Masung tunnel is a four lane 1.5 km long freeway tunnel with the same speed limit. We were also informed that this data base was created with traffic detector records. To be compatible with the field data applied in our survey in Seoul, a same time period with reference to our data collected in Seoul's mountain tunnel section was used as a guide for this validation data.

Table 7 provides the analysis results. As expected, the VDF developed in our study produces better results in terms of Root Mean Square Errors, which are 6.892 for Suam tunnel and 7.754 for Masung tunnel. Improved results can also be found in Fig. 7 and Fig. 8.

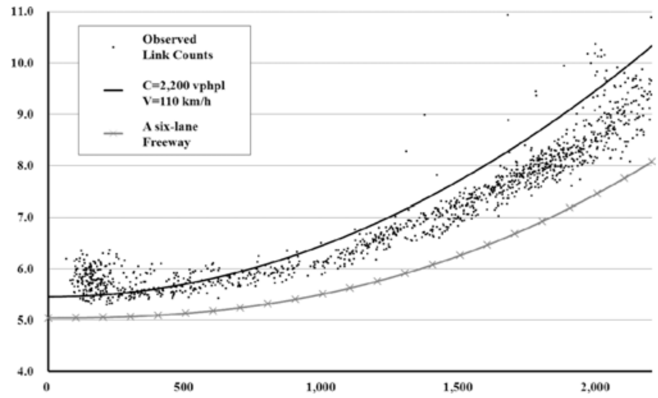


Fig. 7. Validation Result with Data from Suam Tunnel

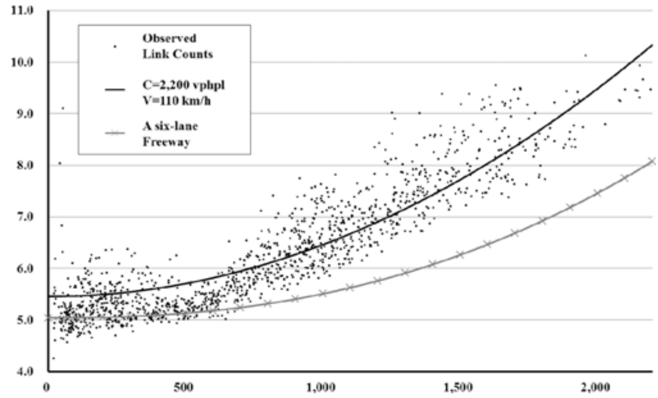


Fig. 8. Validation Result with Data from Masung Tunnel

5. Summary and Discussions

We conducted a working model of VDF's for underground tunnel roads and wanted to demonstrate the contribution of applying the developed VDF's on the basis of traffic volume levels generated by the existing VDF and the VDF developed in this study. A comprehensive feasibility study for the proposed underground tunnel road project in Seoul was performed and published last year (Seoul, 2012). The proposed network consists of three north-south and three east-west underground tunnel corridors as shown in Fig. 9. Each underground tunnel corridor includes a few underground interchange facilities, creating forty nine links, and this tunnel network covers a total length of 135.4 km (Seoul, 2012). With this feasibility study data being available, we considered the traffic volume forecast published in the study as the volume based on the existing VDF's. Obviously,

Table 7. Validation of Developed VDF's with Data Collected on Other Tunnel Areas

	Speed Limit (km/h)	Number of Lanes	Average Volume (veh./hour)	Capacity (veh./hour/lane)	Root Mean Square Error for Existing VDF	Root Mean Square Error for Developed VDF
Suam Tunnel	100	6	968	2,200	15.611	6.892
Masung Tunnel	100	4	320	2,200	14.124	7.754

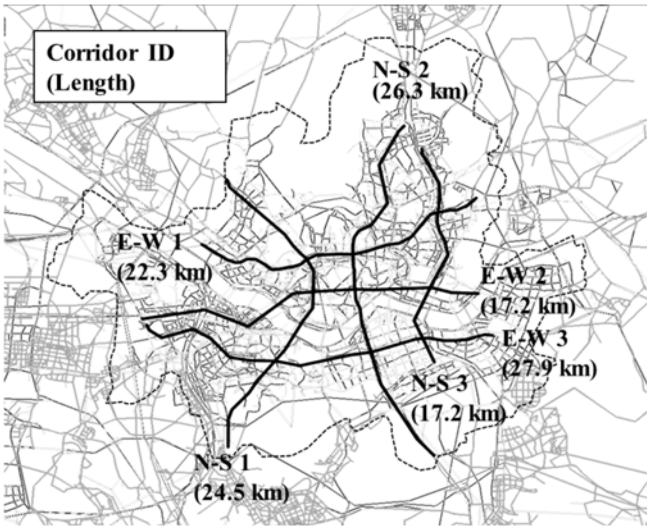


Fig. 9. Seoul's Proposed Underground Tunnel Road Project

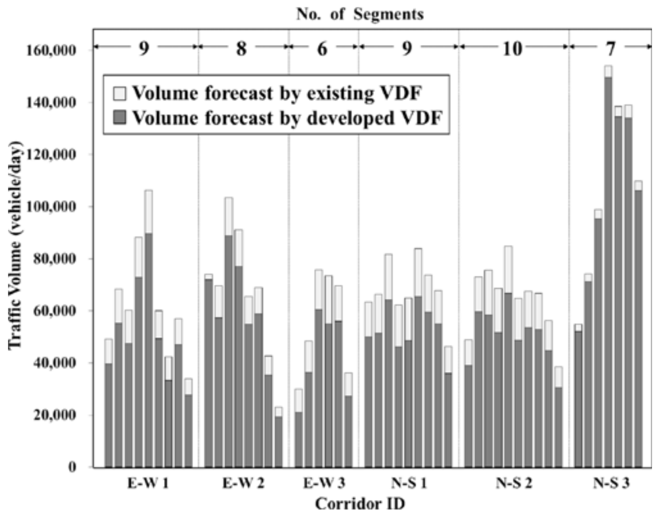


Fig. 10. Comparison of Traffic Volumes by Existing VDF's and Developed VDF's

we estimated the traffic volume based on the developed VDF's by applying our research efforts. Fig. 10 is the outcome of this comparison.

We found that volume levels resulting from the existing VDF's and the developed VDF's were quite different: traffic volume based on the developed VDF's involved a consistent pattern of volume reduction across all the subject corridors. It has been argued among South Korean engineers that the underground tunnel projects and its master plan stated by the Seoul Metropolitan Government failed to provide a reliable future traffic volume forecast. They also blamed this failure on reaching an unrealistically high economic feasibility of this project, in spite of massive funding. Therefore, it is clear that the developed VDF's will produce a more reasonable result.

It was also observed that the smaller the travel volume goes, the greater the reduction becomes, implying that the developed VDF will create reduced amounts of travel time savings for low to moderate traffic volume levels. This result also fits conventional traffic flow theory, where drivers start to experience vehicle delay when they reach a relatively high level of traffic volume.

6. Conclusions

The research tried to conduct a work model of the VDF's for underground tunnel projects in Seoul. No VDF for the underground tunnel projects is available now, and, with this situation, highway engineers are unable to determine the economic feasibility of proposed underground tunnel projects. The study findings are as follows:

The VDF's applicable to the underground tunnel projects can be successfully developed with the data collected in existing mountain tunnels.

Traffic volume levels estimated by two methods; the existing VDF's and the VDF's developed in this study, showed that they were quite different: traffic volume based on the developed VDF's produce more realistic results across all the traffic corridors.

We concluded that the study result would be readily applicable to six lane underground tunnel roads and help engineers provide reliable assessments of economic feasibility for future underground tunnel projects. However, more validation efforts are required when applying our result to other types of underground tunnels.

Acknowledgements

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References

Choi, J., Kim, S. Y., Heo, T. Y., and Lee, J. H. (2011). "Safety effects of highway terrain types in vehicle crash model of major rural roads." *KSCE Journal of Civil Engineering*, Vol. 15, No. 2, pp. 405-412.

Choi, J., Tay, R., and Kim, S. Y. (2013). "Effects of changing highway design speed." *Journal of Advanced Transportation*, Vol. 47, No.2, pp. 239-246.

Kang, H. I. (1996). *Development of classified congestion functions and impact analysis on traffic assignment*, PhD Thesis, Seoul National University, Korea.

Kang, H. I. and Park, C. H. (1998). "Development of classified congestion functions." *Journal of Korean Society of Transportation*, Vol. 16, No. 2, pp. 117-131.

Kang, M. G. (2008). *A volume-delay function parameter estimation for traffic assignment: In the case of a national expressway and highway*, PhD Thesis, Chonnam National University, Korea.

KDI (2008). *A study on standard guidelines for pre-feasibility study on road and railway projects*, 5th Edition, Korea Development Institute, Seoul, Korea.

Kim, B. K., Kang, W. E., Baik, N. C., and Han, D. C. (2003). "A study on the travel impedance functions by functional classification of national highway." *Proceedings of the KSCE Annual Conference*, pp. 158-163.

Kim, D. (2013). “Freeway automatic incident detection using learning models-backpropagation, SVM and fuzzyartmap.” *International Journal of Urban Sciences*, Vol. 17, No. 1, pp. 109-116.

Kim, S. Y. and Choi, J. (2013). “Effects of preceding geometric conditions on operating speed consistency of multilane highways.” *Canadian Journal of Civil Engineering*, Vol. 40. No. 6, pp. 528-536.

Kim, S. Y., Choi, J., Kim, M. K., and Kim, S. G. (2012). “Determination of accident modification factors for the median bus lanes on urban arterials.” *International Journal of Urban Sciences*, Vol. 16, No.1, pp. 99-113.

Klieman, L., Zhang, W., Bernardin, V., and Livshits, V. (2011). “Estimation and comparison of volume delay functions for arterials and freeway HOV and general purpose lanes.” *Transportation Research Board Annual Meeting 2011*, Paper No. 11-0242.

Kumar, A. and Sharma, S. (2010). *Introduction to optimization*, Institute of Transportation Engineers.

Ortuzar, J. de D. and Wilumsen L. G. (1994). *Modeling transport*, Wiley.

Park, J. T., Lee, S. B., and Kim, T. H. (2011). “Qualitative factor analysis on speed reduction of drivers in expressway tunnel sections.” *Journal of the Korean Society of Road Engineers*, Vol. 13, No. 4, pp. 151-158.

Penchina, C. M. (2009). “Minimal revenue tolls: Price for networks with BPR cost function.” *The Open Transportation Journal*, Vol. 3, pp. 87-92.

Seoul (2010). *Master plan of the underground tunnel project in Seoul*, Korea.

Seoul (2012). *Master plan for the development of eco-friendly underground tunnel project and establishment of transportation management strategies in Seoul*, Korea.

Steven, C. C. and Raymond, P. C. (2002). *Numerical method for engineers,* 4th Edition, McGraw-Hill, pp. 337-362.

Taylor, A. L. (2010). *Applying transportation forecasting to the atlanta area public transportation system*, PhD Thesis, Georgia Southern University, USA.

Yun, S. S. and Yun, D. S. (2004). “Development of BPR functions with track traffic impacts for network assignment.” *Journal of Korean Society of Transportation*, Vol. 22, No. 4, pp. 117-134.