Improvement on the BPR Link Performance Function and Classification Calibration in Parameter

CHEN Hong [1], ZENG Han [2], ZHOU Ji-biao and ZENG Rong

[1] Professor, Highway College, Chang'an University, Xi'an 710064; China PH 13700229619; FAX (029)82334886; email: chh@gl.chd.edu.cn
[2] Master, Grade Tow, Highway College, Chang'an University, Xi'an 710064; China, PH 15091855820; FAX (029)82334886; email: hanhan19880919@163.com

ABSTRACT

In order to make a scientific prediction of highway resistance, traffic density k instead of traffic q of BPR function is applied to calculate the new road resistance function. Three forms of the road resistance function form are studied including the traffic state of smooth to the traffic state of congested. The improved road resistance function models are obtained under the smooth, crowded and jam conditions, and the highway parameters are respectively graded demarcated. The result shows the improved BPR function model is suitable for highway of level 4 or above. The parameter value of the BPR improved model which calibrated by the measured data in highway is superior; value recommended by the U.S. federal highway bureau is more conform to China's highway traffic operation.

KEY WORDS

Traffic engineering; BPR function; least-squares linear regression model; capacity; parameter classification calibration

BPR FUNCTION

The mathematical expression of t, the BPR function of the U.S. federal highway bureau, is described as the fowling type (1). Use t/t_0 as ordinate, q/C as abscissa, and use statistical analysis software SPSS to drawing formula (1) and table (1).

$$t = t_0 \times [1+\alpha(q/c)^{\beta}] \tag{1}$$

Where:

t ——actual road traffic time, in second;

t_0 —— Time of passing the road is the vehicle unimpeded on the road, that free travel time, in second;

q —— Road traffic, in pcu/h;

c —— Actual road traffic capacity, in pcu/h;

α, β —— Calibration parameters of model, the U.S. federal highway bureau recommend uses α=0.15, $\beta = 4.0$

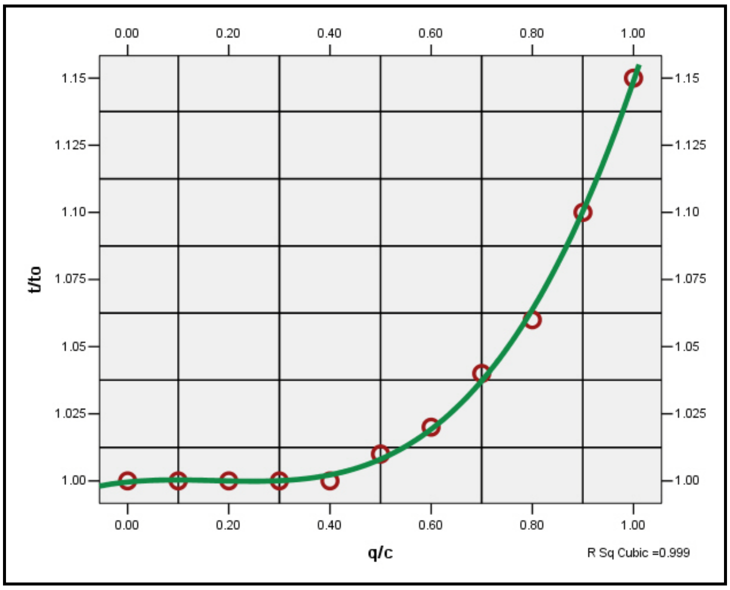

Figure 1. BPR function curve

Why BPR function is widely used? The reasons can be concluded as fowling:

(1) Road traffic is unrestricted by the road traffic capacity, there is no need to detect if there is feasible solution as traffic assignment;

(2) Table 1 shows that BPR function is the strictly monotone increasing function[7];

(3) Table 1 is drawing based on $\alpha = 0.15$, $\beta = 4.0$;

(4) Table 1 shows that when $q/c = 0, t = t_0$, which explains on the basis of the U.S. federal highway bureau recommendation $\alpha = 0.15$, $\beta = 4.0$, road travel time equal to the free-flow time. This speed is called free-flow speed; when $q/c = 1$, $t = 1.15t_0$, which explains on the basis of the U.S. federal highway bureau recommendation $\alpha = 0.15$, $\beta = 4.0$, road travel time is 1.15 times of the free-flow time. This speed is called critical speed. At the same time, BPR function plays weaker influence to the link flow. Using BPR function to make traffic assignment may increase the possibility of overflow.

BPR FUNCTION IMPROVEMENT

The author improved BPR function on the basis of literature “Improvement Study on BPR Link Performance Function” (Wang, et al. 2009) and literature “Research of obstructing road function on Harbin urban roads” (Xia, et al., 2009), which based on the advantages of link flow being unrestricted of road traffic capacity and strictly monotone increasing, try to search a new function that is monotone increasing and increasing with degree of crowdedness, and make the link flow which is unrestricted by road traffic capacity as well.

Function improvement

On the basis of the relationship of three traffic flow parameters (Daniel, 1983; Ren, Liu & Rong, 2003), traffic volume q is increasing with traffic density k, when traffic density $k = k_m$, traffic volume q attains to the maximum traffic capacity c, that is: $q = c = v_f \cdot k_j / 4$. When traffic density $k > k_m$, traffic volume q is decreasing to zero with the increasing of traffic density k, that is: when $k = k_j$, traffic volume $q=0$. Since traffic flow states is a process from unimpeded to congestion, speed is decreased with the increasing of traffic congestion, traffic density and travel impedance as well. Due to travel impedance and traffic density increasing, traffic density k is used to replace traffic volume q of the BPR function in order to calculate the new road resistance function, that is: use the k/k_j to replace of q/c in BPR function, and use v/v_f to solve.

Greenshields came up with velocity- density linear relation model in 1943:

$$v = v_f(1-k/k_j) \tag{2}$$

The relationship of the three traffic flow parameters:

$$q = k \cdot v \tag{3}$$

On the basis of formula (2) and (3), get the relationship of flow and velocity:

$$q = k \cdot v_f(1-k/k_j) = k \cdot v_f - k^2 \cdot v_f / k_j \tag{4}$$

Differentiate k in formula (4):

$$dq/dk = v_f - 2k \cdot v_f / k_j \tag{5}$$

Make formula (5) of the left equal to zero, The formula will become: $dq/dk = v_f - 2k \cdot v_f / k_j = 0$;

It could be concluded that:

When $k = k_j/2$, the maximum traffic capacity:

$$q_{\max} = c = v_f \cdot k_j/4 \tag{6}$$

Combine the formula (4) and (6):

$$q/c = 4(k/k_j - k^2/k_j^2) = 4(v/v_f - v^2/v_f^2) \tag{7}$$

Combine the formula (1) and (7):

$$t/t_0 = 1 + \alpha\left[4\left(k/k_j - k^2/k_j^2\right)\right]^\beta \tag{8}$$

On the condition of traffic running smoothly, organize the formula (8):

$$t/t_0 = 1 + \alpha\left[(-4)\cdot\left(k/k_j - 1/2\right)^2 + 1\right]^\beta \quad (k/k_j \in (0{,}0.5)) \tag{9}$$

Make $z = -4\left(k/k_j - 1/2\right)^2 + 1$, use the statistical analysis software SPSS to draw the formula (1), as figure 2 shows:

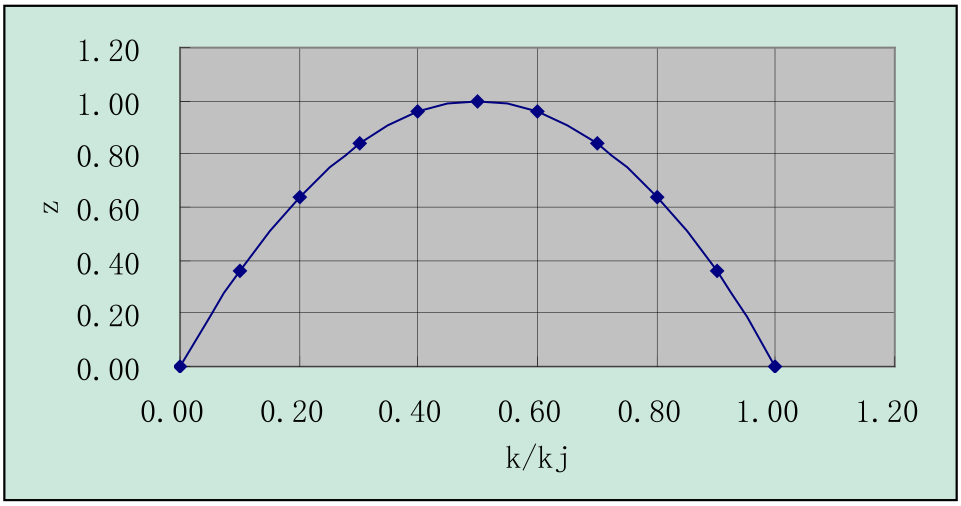

Figure 2. Curve of the function

On the condition of traffic congestion:

When $\dfrac{k}{k_j} \in (0.5{,}1)$, make symmetric function along the z=1, that is:

$$\frac{t}{t_0} = 1 + \alpha\left[4\left(\frac{k}{k_j} - \frac{1}{2}\right)^2 + 1\right]^\beta \qquad \frac{k}{k_j} \in (0.5{,}1) \quad \text{On the condition of congestion;}$$

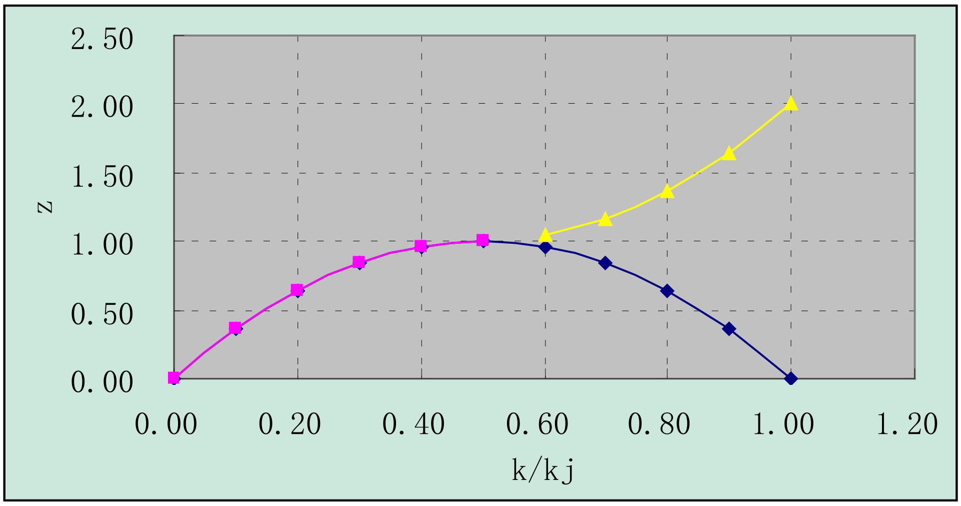

Figure 3. The curve by turning over the second half lower part

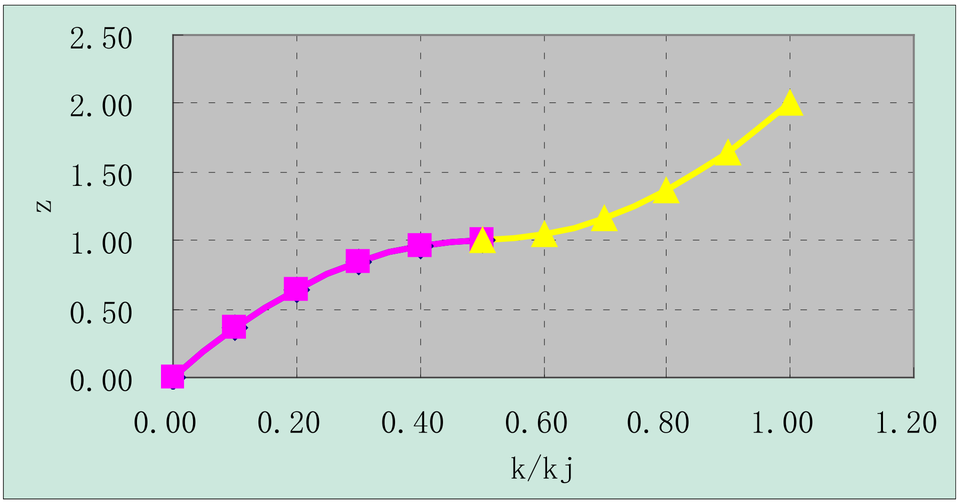

Figure 4. The improvement function

On the condition of traffic congestion:

$$t/t_0 = 1+\alpha \qquad\qquad k/k_j = 1$$

So get the function after improvement:

$$\begin{cases} t/t_0 = 1+\alpha\left[-4\left(k/k_j - 1/2\right)^2 + 1\right]^{\beta}, & (k/k_j \in (0,0.5)) \\ t/t_0 = 1+\alpha\left[4\left(k/k_j - 1/2\right)^2 + 1\right]^{\beta}, & (k/k_j \in (0.5,1.0)) \\ t/t_0 = 1+\alpha, & (k/k_j \geq 1.0) \end{cases} \quad (9)$$

Parameter calibration

Use the least square method to calibrate the related parameters. There is specific method as follow:

Suppose that a road length is l, then $t=l/v$, $t_0=l/v_f$, combine two equations to get:

$$t/t_0 = v_f/v \quad (10)$$

In that way, by putting the equations (1), (7), (10) together, BPR function can be converted as:

$$\ln(v_f/v - 1) = \ln\alpha + \beta\ln\left[4(v/v_f - v^2/v_f^2)\right] \quad (11)$$

Make $Y = \ln(v_f/v - 1)$, $a = \ln\alpha$, $b = \beta$, $X = \ln\left[4(v/v_f - v^2/v_f^2)\right]$, it can be concluded that:

$$Y = a + bX \quad (12)$$

It can be seen from formula (12) that BPR function is transformed as linear model, taking advantage of the least square linear regression model to solve α, β.

Construct function L:

$$L = \sum_{i=1}^{n}[y_i - (a + bx_i)]^2 \quad (13)$$

Derivate the tow sides of equation (13):

$$\partial L/\partial a = -\sum_{i=1}^{n} 2[y_i - (a + bx_i)] = 0 \quad (14)$$

$$\partial L / \partial b = -\sum_{i=1}^{n} 2x_i \cdot [y_i - (a + bx_i)] = 0 \tag{15}$$

Solve equation (14) and (15):

$$a = [\sum_{i=1}^{n} x_i y_i - (1/n)(\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i)] / [(\sum_{i=1}^{n} x_i^2) - (1/n)(\sum_{i=1}^{n} x_i)^2] \tag{16}$$

$$b = (1/n)\sum_{i=1}^{n} y_i - (a/n)\sum_{i=1}^{n} x_i \tag{17}$$

$$R^2 = [n\sum_{i=1}^{n} x_i y_i - n(\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i)] / \{\sqrt{[n\sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i)^2][\sum_{i=1}^{n} y_i^2 - (\sum_{i=1}^{n} y_i)^2]}\} \tag{18}$$

Calibration results

On the basis of BPR function, calibrate the alpha and beta, the results are shown as follows:

Table 1. The results of parameter calibration after BPR function improvement

Highway grade	design speed	a	b	alpha	beta	determinative coefficient R-Sq
freeway	*vf=120*	-0.93	1.1669	0.3934	1.17	0.9957
	vf=100	-1.04	1.1265	0.3551	1.13	0.9978
	vf=80	-0.97	1.1682	0.3782	1.17	0.9963
first-class highway	*vf=100*	-0.90	1.2066	0.4048	1.21	0.9928
	vf=80	-0.87	1.2293	0.4192	1.23	0.9936
	vf=60	-1.00	1.1514	0.3691	1.15	0.9975
second-class highway	*vf=80*	-0.71	1.3167	0.4924	1.32	0.9788
	vf=60	-0.78	1.2700	0.4593	1.27	0.9834
third-class highway	*vf=40*	-0.86	1.2366	0.4226	1.24	0.9897
	vf=30	-0.75	1.3398	0.4725	1.34	0.9864
fourth-class highway	*vf=20*	-	-	-	-	-

It can be found from results that it is well fitted in different speed under freeway, first class highway, second class highway and third class highway. But at forth class highway, it is unsatisfactory by fitting. So author believes that BPR function is well used above forth class highway. At the same time, the reason why BPR function does not fit the forth class highway is that forth class highway is more influenced by the non-motor vehicle.

CONCLUSION

It is mentioned that recommended values α, β in BPR function, which is only used in the United State. It should be modified in using in China or other countries.

It is used the relationship of traffic volume and traffic density to replace the traffic volume of BPR function to calculate the road resistance. This is the real formula improvement. It is kept that the function is monotone increasing function after improvement so that the function can truly reflect the road resistance of travel congestion. The United State has given recommended values, but China has not. There are 650 cities in China; there are different parameters in different cities. The author suggest to construct a more perfect travel data acquisition system and rich database, then give a scope of parameters to every city, in order to make it more accurate in directing the phase IV in traffic demand forecast, and be well used in urban traffic planning and design.

ACKNOWLEDGEMENT

The study is supported by the Project of Regional Traffic State, Chang'an University (No.2171-2008843).

REFERENCES

JTG B01-2003, Highway technical standard JTG B01-2003[S].Beijing: Standards Press of China, 2003.

Quan Jin-hua, Research of city road resistance function madel [D]. Nanjing: Master thesis of Southeast University, 2008.

Ren Fu-tian, Liu Xiao-ming, Rong jian. Traffic engineering [M] . Beijing China Communications Press,2003.

Shi Feng，Li Zhi-chun. Combined model and algorithm of expanding network capacity and road pricing[J]. China Journal of Highway and Transport, 2003, 14(2)：90-94.

[US] Written by Daniel. L. Epicureans rove, Matthew. J. Sue, Jiang Huang translation. Traffic flow theory[M]. Beijing: China Communications Press, 1983.

Wang Rong-yan, Research of the best path algorithm on dynamic road impedance function of UTFGS [D].Xi’an: Master thesis of Chang’an University, 2008.

Wang Shu-shen, et al. Deduction of Link Performance Function and Its Regression Analysis[J] . Journal of Highway and Transportation Research and Development, 2006, 23(4): 107-110.

Wang Su-xin ， et al . Improvement Study on BPR Link Performance Function[J].Journal of Wuhan University of Technology, 2009, 33(3):446-449.

Wang Yuan-qing，Zhou Wei，et al. Theory and Application Study of the Road Traffic Impedance Function[J]. Journal of Highway and Transportation Research and Development, 2004, 21(9): 82-85.

Xia Zheng-hao，Bai Lu-tao，Zhou Ji-biao，et al. Research of obstructing road function on Harbin urban roads[J] . Journal of Heilongjiang Institute of Technology, 2009, 23(3): 39-44.