Effects of Uncertainty in Speed–Flow Curve Parameters on a Large-Scale Model

Case Study of the Danish National Model

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Uncertainty is inherent in transport models and prevents the use of a deterministic approach when traffic is modeled. Quantifying uncertainty thus becomes an indispensable step to produce a more informative and reliable output of transport models. In traffic assignment models, volume-delay functions express travel time as a function of traffic flows and the theoretical capacity of the modeled facility. The U.S. Bureau of Public Roads (BPR) formula is one of the most extensively applied volume delay functions in practice. This study investigated uncertainty in the BPR parameters. Initially, BPR parameters were estimated by analyzing observed traffic data related to the Danish highway network. Then, BPR parameter distributions were generated by using the resampling bootstrap technique. Finally, the generated parameter vectors were used to implement sensitivity tests on the four-stage Danish national transport model. The results clearly highlight the importance to modeling purposes of taking into account BPR formula parameter uncertainty, expressed as a distribution of values rather than assumed point values. Indeed, the model output demonstrates a noticeable sensitivity to parameter uncertainty. This aspect is evident particularly for stretches of the network with a high number of competing routes. Model sensitivity was also tested for BPR parameter uncertainty combined with link capacity uncertainty. The resultant increase in model sensitivity demonstrates even further the importance of implementing uncertainty analysis as part of a robust transport modeling process.

By modeling complex systems, transport models are subject to uncertainty that can affect all model components (i.e., context, model structure and methodology, inputs, and parameters) to finally propagate to the model output. The main consequence of this inherent uncertainty is that transport models do not provide reliable point estimates of modeled traffic flows and derived measures. Instead, modeled traffic flows are better expressed as a central estimate and an overall range of uncertainty margins articulated in regard to (output) values and the likelihood of occurrence (*1*). Uncertainty analysis relates to how uncertainty in each model component propagates to the model output and how to express the model output as a distribution, so reflecting the overall uncertainty present in the model.

The assignment algorithms of large-scale transport models often use static volume delay functions to express travel time as a function of traffic flow and the theoretical capacity of the modeled facility. However, travel time is not just a function of flow; it is in fact affected by a number of different factors, such as downstream bottlenecks and resulting spillback or less than ideal weather conditions, causing drivers to drive slower. Consequently, a problem arises whenever traffic data output of static models is used to feed cost–benefit analysis. In these cases, to produce valuable information, a necessary step is to address uncertainty in the volume delay functions by quantifying the sensitivity of the model output to the variability of the volume delay function components.

Volume delay functions can be divided into three main groups: hyperbolic, polynomial, and exponential (*2*). The U.S. Bureau of Public Roads (BPR) formula, belonging to the polynomial group and proposed in its original version in 1964, is one of the most extensively applied volume delay functions in practice (*3*). The BPR formula, given free-flow travel time, observed flow, and link capacity, uses parameters to represent different relationships between travel time and (modeled) flow-to-capacity ratios. Usually, values for the parameters are predefined according to assumptions and practice. However, as for any other model components, the BPR formula parameters have inherent uncertainty that originates from the modeler's ignorance of the true value of the parameters (epistemic uncertainty) and the stochastic behavior of the (true) parameters themselves (ontological uncertainty), which potentially vary by driver behavior, time of the day, weather conditions, and link characteristics.

An approach widely used in the transportation literature to quantify model uncertainty is to run model sensitivity tests by using distributions of input and parameters and output of stochastic sampling procedures. For that purpose, resampling techniques such as bootstrap have been used to generate model parameter distributions (*4*). Resampling approaches have a clear advantage compared with other sampling procedures. In fact, they do not require modelers' knowledge or assumptions about the shape of the parameter distributions, which becomes instead the output of the resampling methodology itself. Bootstrap has been implemented in many studies on transport uncertainty by Brundell-Freij (*5*), Hugosson (*6*), De Jong et al. (*7*), Matas et al. (*8*), and Petrik et al. (*9*). Bootstrap defines the parameter distributions by recalibrating the model parameters for a number of model samples, which are generated from the original sample by resampling with replacement.

To the best of the authors' knowledge, no attempt has been made so far to estimate uncertainty in the BPR formula parameters from the analysis of observed data and to analyze its effect on traffic assignment results of large-scale models. For that purpose, observations of the Danish highway network were obtained from the Hastrid data set owned by the Danish Road Directorate. Nonlinear regression analyses were implemented to allow the calibration of the values of the BPR formula parameters simultaneously. Afterward, parameters

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Transportation Research Record: Journal of the Transportation Research Board, No. 2429, Transportation Research Board of the National Academies, Washington, D.C., 2014, pp. 30–37.
DOI: 10.3141/2429-04

were repeatedly calibrated on 10,000 bootstrap samples to generate parameter distributions. Finally, selected percentiles of the distributions were used to run sensitivity tests on the Danish national transport model [LandsTrafikModellen (LTM)]. In addition, a scenario investigating LTM sensitivity to BPR parameter uncertainty combined with link capacity uncertainty was tested. The link capacity uncertainty was quantified by creating vectors of capacity values through the implementation of Monte Carlo simulation.

The next section provides a description of the methodology applied to estimate the BPR parameter distributions, including a description of the data sets used for the research and the bootstrap sampling technique. After a brief description of the LTM, results from the sensitivity tests run are illustrated and discussed. Conclusions from this research are presented in the last section of this paper.

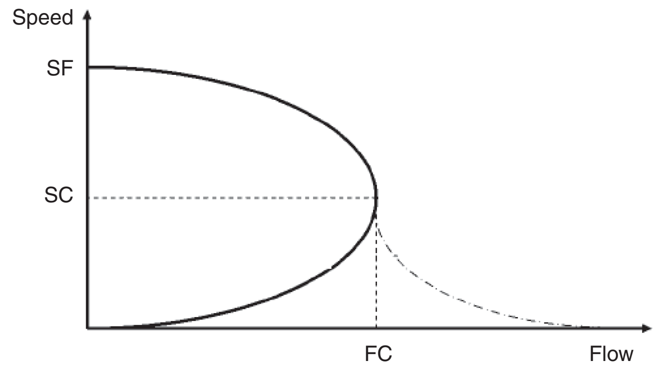

FIGURE 1 Assumed relationship between speed and traffic flow (SF = speed in free-flow conditions).

METHODOLOGY

Time–Flow Relationship: BPR Formula

In traffic assignment models a common way to describe the relationship between travel time and traffic flows is the BPR formula (*3*):

$$\text{TT}_r = \text{FFT}_r * \left\{ 1 + \alpha * \left[\frac{\text{flow}_r + \gamma(\text{flow}'_r)}{\text{capacity}_r} \right]^{\beta} \right\} \tag{1}$$

where

TT_r = congested travel time on link r (min),
FFT_r = free-flow time on link r (min),
flow_r = traffic volume on link r [vehicles per hour (vph)],
capacity_r = capacity of link r (vph),
flow'_r = traffic volume on opposite direction of link r (relevant only in case of nonseparated lanes) (vph), and
α, β, and γ = volume-delay parameters.

Specifically, α represents the ratio between free-flow speed and speed at capacity, β determines how steeply the curve bends once the capacity is reached, and γ captures the effect of speed reduction resulting from opposite traffic in roads with nonseparated lanes.

The BPR formula can be modified to express the relationship between speed (instead of congested time) and flow-to-capacity ratio, as illustrated by Nielsen and Jørgensen (*10*) and Fagnant and Kockelman (*11*):

$$S_r = \frac{\text{FF}S_r}{1 + \alpha * \left[\frac{\text{flow}_r + \gamma(\text{flow}'_r)}{\text{capacity}_r} \right]^{\beta}} \tag{2}$$

where S_r is the observed average speed on link r and FFS_r is the velocity in free-flow conditions on link r. The use of either the time–flow or the transformed speed–flow formulas is generally data driven, namely, it is dependent on the availability of data on either travel times or travel speeds. For example, the current study considers observations from a data set of travel speeds and therefore uses the transformed speed–flow formula for the calibration of the BPR parameters. The transformed formula implies an approximation. In fact, the speed is measured by local detectors, so it does not reflect precisely the link travel time, but rather is an expression of the overall link conditions. To the best of the authors' knowledge, no attempt has been made so far to quantify that discrepancy.

In general, criticisms have been moved to the BPR formula. As pointed out by Dowling et al., depending on the parameter values chosen, the BPR formula may be insensitive to volume changes until demand exceeds capacity, when the predicted speed drops abruptly (*12*). Nevertheless, other studies proved that with an appropriate choice of parameter values specific for road type, the BPR formula offers comparable or even better goodness of fit to observed data than do other volume delay functions (*13*).

Another drawback is that the BPR formula correctly models travel time only when the traffic flow is below capacity. In fact, when traffic flow reaches capacity (in Figure 1 the point corresponding to flow at capacity FC and the related speed at capacity SC), the curve representing the BPR formula takes the shape of the dotted curve on the right of FC. Instead, the observed traffic behavior is tendentially close to the pattern described by the bold line. To overcome that issue, it was suggested that the flow–capacity ratio be expressed in relation to density–density at the maximum flow ratio (*13*). With this approach in fact, the speed–flow observations assume an *s*-shape that it is possible to model.

Despite the criticism, in static assignment models the BPR formula is commonly used and accepted for practical reasons. Among others, with the BPR formula the speed–flow relationship curve is "continuous even beyond capacity and differentiable," as argued by Nielsen and Jørgensen (*10*).

Hastrid Data Set and Parameter Calibration

This study intended to calibrate the BPR formula parameters and therefore used information on the Danish highway network that was contained in the Hastrid data set, owned by the Danish Road Directorate. The Hastrid data set contains observations for vehicle flow and average speed by time intervals of 15 min. The data used in the present analysis were collected in September 2009 from three count stations located in the northeast part of Zealand. Two count stations were located on Highway M11, called *Holbækmotorvejen,* connecting Holbæk, in the northwest part of Zealand, with the southwest suburbs of Copenhagen. The third count station was located on Highway M16, called *Hillerødmotorvejen,* connecting Hillerød, in the north part of Zealand, with the northern suburbs of Copenhagen. Table 1 summarizes the main characteristics of the three sections where the count stations were located, and Figure 2 shows their geographic location on the highway network.

TABLE 1 Characteristics of Hastrid Data Set

Highway	Section	Section Length (km)	Capacity (vph)	Lanes	Observations
Holbæk (M11)	Taastrup–Fløng	1.460	4,200	3	1,141
Holbæk (M11)	Ringstedvej–Roskilde	0.953	3,400	2	1,582
Hillerød (M16)	Farum–Skovbrynet	3.701	4,200	2	1,229

To perform the parameter calibration, the 15-min data were transformed into hourly data by summing the 15-min vehicle flow observations and averaging the corresponding observed speeds. The flow-to-capacity ratio was calculated as density–density at a maximum flow ratio (*13*). The density of maximum flow was defined as 28 passenger cars per kilometer per lane, corresponding to the value suggested by the *Highway Capacity Manual* of 45 passenger cars per mile per lane (*14*). Finally, the free-flow speed was calculated for each section as corresponding to the average observed speed at density–density at a maximum flow ratio lower than 0.5.

However, this approach may result in curves with a long tail on the right-hand side (*15*). This result would imply the acceptance of relatively high speeds in situations over capacity, thus leading to an overestimation of the network accessibility. Thus, the density–density at the maximum flow ratio approach was partially modified to better model severe congested conditions. Accordingly, for the calibration the value X was used, calculated as

$$X = \frac{D}{D_{\max}} \qquad \text{if } \frac{D}{D_{\max}} < 1$$

$$X = 1 + 0.2 * \left(\frac{D}{D_{\max}}\right) \qquad \text{if } \frac{D}{D_{\max}} \geq 1 \tag{3}$$

where $D/D_{\max}$ is the density–density at the maximum flow ratio. As can be seen, for severe congested conditions, that is, $D/D_{\max} \geq 1$, the density–density at the maximum flow ratio values was reduced to avoid unreasonably high congested values.

The upper part of Figure 3 graphically shows the observed average speed plotted against X. Overall, the observed speed–flow relationship on the three links shows a trend consistent with what was theoretically expected. As can be seen, the majority of the observations cluster around the free-flow speed of approximately 110 km/h for low levels of congestion (corresponding to $X < 1$). Only a few observations unexpectedly register free-flow speed in congested

FIGURE 2 Location of sections on the Danish (Zealand) highway network.

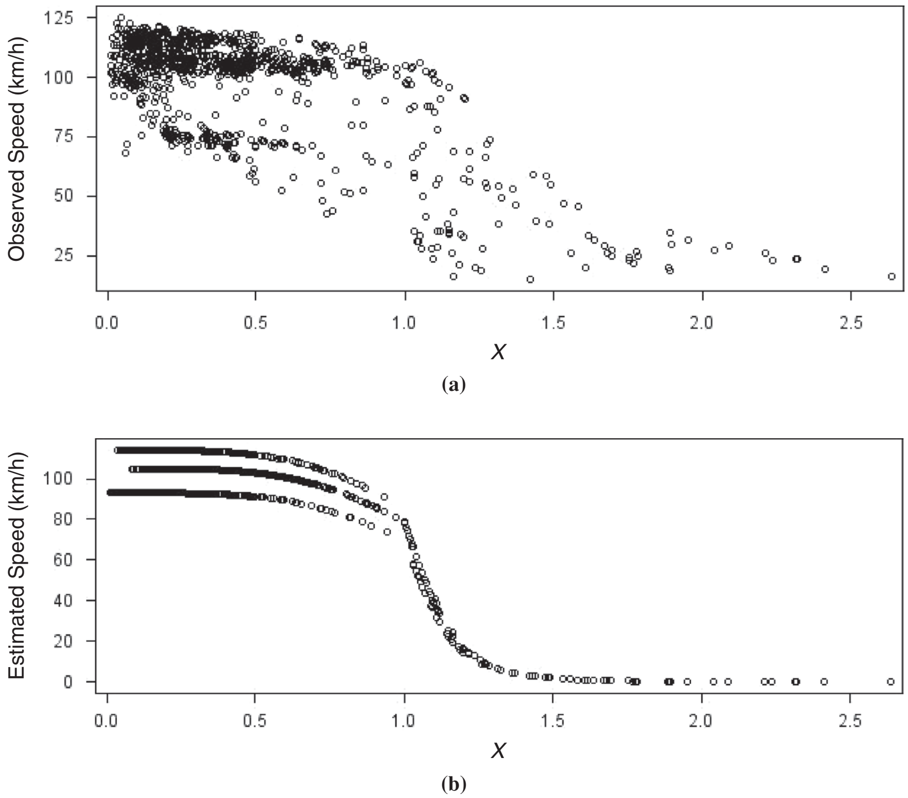

FIGURE 3 Speed plotted against density–density ratio (*X*): (*a*) observed and (*b*) estimated.

conditions also (corresponding to $X > 1$); these observations are probably the result of count mistakes. Besides, there is a cluster of observations corresponding to speeds of about 75 km/h for low levels of congestion. These observations are probably related to trucks in the inner lane, which have speed limits of 80 km/h (*10*).

The parameter calibration, implemented by using the statistical software SAS, resulted in $\alpha = 0.33$ and $\beta = 4.04$. With respect to the Danish road network, Hansen defined a range of values between 0.5 and 2 for α and between 1.4 and 11 for β (*15*). Thus, for validation purposes, vehicle speeds resulting from the BPR formula and the calibrated values of α and β were calculated and compared with observed average speeds through regression analysis and visual inspection. Results from the regression analysis were satisfactory ($R^2 = .9764$), as well as those from the visual inspection of the pattern of the speed estimated from the BPR formula, depicted in the bottom part of Figure 3.

Quantification of Uncertainty in BPR Formula Parameters

To produce BPR parameter distributions, the resampling technique bootstrap was used (*4*). The bootstrap method investigates the accuracy of an estimator θ based on the assumption of considering the original sample, originating θ, as the population. Bootstrap consists of a three-step procedure. First, from the original sample of n observations a number of samples are generated through (re)sampling with replacement. All bootstrap samples contain n observations as the original sample. The replacement approach guarantees that each observation in the original sample has a constant probability $1/n$ to be drawn; as a consequence the bootstrap samples have a high probability of differing from each other. Second, the estimator θ is calculated for each bootstrap sample. Third, the new θ values obtained are analyzed to infer the accuracy of the estimator by using some uncertainty measures such as variance or standard deviation.

One restriction to the use of bootstrap is that it can be implemented only for variables that are the output of calibration processes and only when the sample is available. Thus, it cannot be applied to variables observed, assumed, or imported. In addition, it is important to be aware that the bootstrap method has two downsides. First, there is no rule defining the correct number of bootstrap samples to generate although the number should be large and, in theory, tendentially infinite. Second, the results are constrained by the quality of the original sample, given that the bootstrap samples do not increase the amount of information there contained.

By using as the original sample the sample used for the parameter calibration, 9,999 bootstrap samples were created and the calibration process was repeatedly implemented for each of them. The resultant parameter statistics are summarized in Table 2. The coefficients of

TABLE 2 Bootstrap Parameter Statistics

Parameter	Estimate	SD	Min.	Max.	CV
Alpha	0.335	0.030	0.216	0.462	0.090
Beta	4.070	0.254	3.238	5.373	0.062

NOTE: SD = standard deviation; min. = minimum; max. = maximum.

TABLE 3 Bootstrap Distribution Percentiles

	Values, by Percentile										
Parameter	1	10	20	30	40	50	60	70	80	90	99
Alpha	0.27	0.30	0.31	0.32	0.33	0.33	0.34	0.35	0.36	0.37	0.41
Beta	3.55	3.76	3.86	3.93	3.99	4.04	4.12	4.18	4.27	4.40	4.77

variation (CV) are also reported and from this point on are used as a measure of uncertainty. Table 3 shows selected percentiles of the distribution. The sensitivity tests on the LTM were run on the basis of these values rather than for all 10,000 parameter values (9,999 from the bootstrap samples plus one from the original calibration) because of the LTM model's extremely long run times. Finally, Figure 4 graphically shows the resultant distributions for α and β.

Link Capacity Uncertainty

Although this study focuses on BPR parameter uncertainty, the other variables of the BPR formula, namely FFT_r (or FFS_r), $flow_r$, and $capacity_r$, potentially have inherent uncertainty. A comprehensive analysis of model uncertainty should also include the assessment of model sensitivity to the uncertainty of these variables. However, with respect to LTM, FFT_r is based on legal speed limits, and $flow_r$ depends on trip generation processes; thus only uncertainty inherent in link capacity has been investigated.

As previously highlighted, bootstrap can be applied only to calibrated variables. Thus, Monte Carlo simulation has been implemented to quantify link capacity uncertainty. Triangular distributions were used to avoid illogical sampling results, such as negative or too high capacity values. The limits of the triangular distributions were defined as ±25% of the capacity link value provided in the LTM network description. The resultant vector values were used in combination with BPR parameter values resulting from the bootstrap procedure to run sensitivity tests on the LTM model. In this way it was possible to analyze the combined effect of the two uncertainty sources (i.e., BPR parameters and link capacity) on the model. As for the bootstrap vectors, only selected percentiles from the Monte Carlo simulations were used to run the sensitivity tests.

CASE STUDY

The LTM

The LTM is meant to establish a unified reference model for transport policy analysis and project evaluation in Denmark (*16*). The model relies on two main data sources: the Danish travel survey, namely, *Transportvane Undersøgelsen* (a national survey ongoing from 1992 that contains travel information from about 1,000 individuals per month), and the Danish national register, which provides socioeconomic information for the entire Danish population. The model zone system is based on four different aggregation levels going from

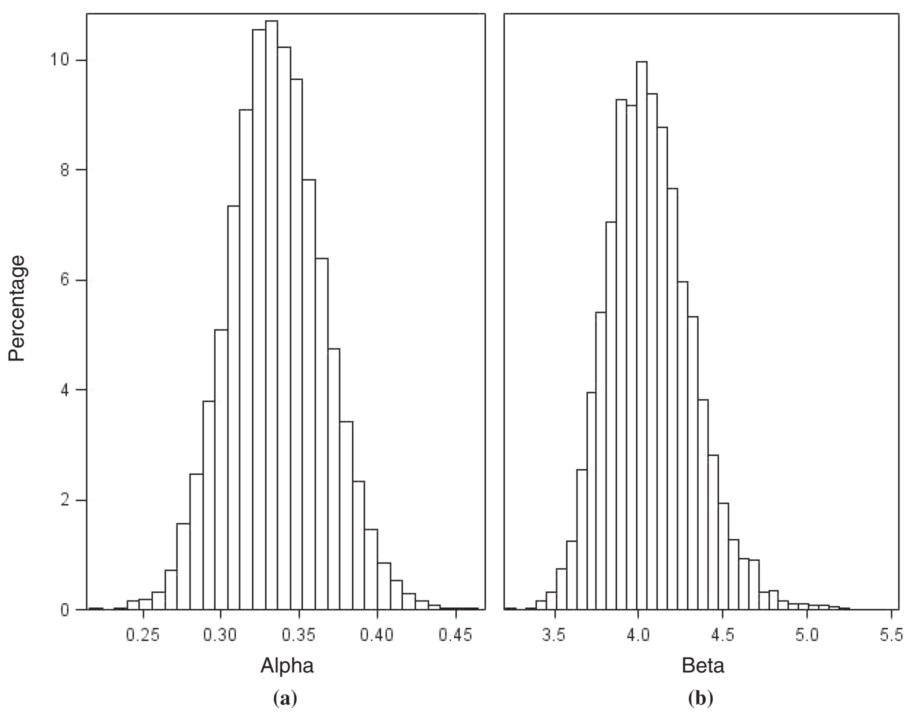

FIGURE 4 Distributions: (a) alpha and (b) beta.

the more disaggregated up to the more aggregated: Level 3 (regional level, 3,670 zones), Level 2 (national level, 907 zones), Level 1 (strategic level, 176 zones), and Level 0 (municipality level, 98 zones).

Figure 5 graphically describes the model framework, which is based on four stages for the passenger demand model and three stages for the freight demand model. At the initial stage, the model assumptions exogenous to the model are defined, specifically population, employment, and the road and transit networks. In the second stage, the model consists of two parallel segments, the passenger demand model and the freight demand model. Both of these models feed the assignment model that defines the route choice equilibrium. The equilibrium solution in turn provides feedback to the demand models.

As can be seen, the passenger demand model is divided into two sequential models: the strategic model, which defines strategic choices, and the passenger model, which delineates transport-related choices. The models are linked in a random utility framework. At the upper level, the strategic model defines the prerequisites for the passenger model. The passenger model then provides information to the assignment model, which in turn sends feedback in regard to accessibility measures to the strategic and passenger models.

This study focuses on the passenger road assignment model. The model is tour based, and the model structure can be divided into two main submodels modeling the primary tour activity of the day and the intermediate stop activities (conditional on the primary activity). A limitation is imposed so that a tour can consist of a maximum of four trips (i.e., home–stop, stop–main destination, main destination–stop, and stop–home) and only two tours are allowed per individual per day.

In more detail, the passenger road assignment model is a link-based model solved by the method of successive averages to reach stochastic user equilibrium. The chosen route to travel by mode k between origin zone i and destination zone j is the route that minimizes the cost of traveling, calculated at the link level as

$$C_{ijkr} = \omega_l L_{ijkr} + \omega_{tf}\mathrm{FFT}_{ijkr} + \omega_{tc}\mathrm{TC}_{ijkr} + \omega_c c_c + \varepsilon_{ijkr}$$

where

C_{ijkr} = cost of traveling by mode k from zone i to zone j using link r [Danish kroner (DKK)],
L_{ijkr} = length of link r by mode k from zone i to zone j (km),
FFT_{ijkr} = free-flow travel time (min),
TC_{ijkr} = extra travel time as a result of congestion (min),
c_c = monetary cost of traveling (varying according to mode and purpose) (DKK),
ε_{ijkr} = vector of residuals, and
ωs = parameters associated with respective variable.

The relationship between travel time and traffic flows is based on the BPR formula.

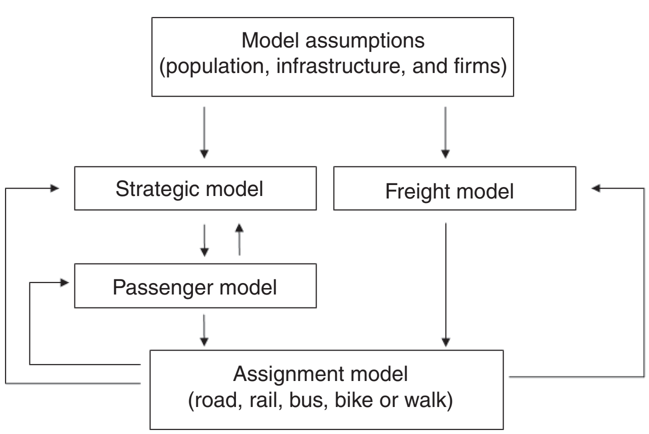

FIGURE 5 LTM framework.

Results and Discussion

The results from the sensitivity test runs on the LTM traffic assignment are summarized in Tables 4 and 5. The upper part of the tables (Scenario 1) shows results for model sensitivity to BPR parameter uncertainty. The bottom part (Scenario 2) illustrates instead results for model sensitivity to BPR parameter uncertainty and link capacity uncertainty combined.

Table 4 shows the links' average CV referring to vehicle kilometer (veh km) and average speed for both the entire network and the highway links only. As can be seen, the mean CV values for vehicle kilometers and average speed are low, a reflection of low model sensitivity to the BPR parameters uncertainty. However, uncertainty was quantified only for parameters α and β referring to highways links, which amount to approximately 5% of the network. Besides, the parameter uncertainty resulting from the bootstrap approach was high neither for α (CV 0.09) nor for β (CV 0.054). As expected, the combined effect of BPR parameters uncertainty and links capacity uncertainty (Scenario 2) increases the model uncertainty for both the overall network and the highways links.

TABLE 4 Vehicle Kilometers and Average Speed CV Statistics

	All Links CV		Highway Links CV	
Statistic	veh km	Avg. Speed	veh km	Avg. Speed
Scenario 1				
Min.	0	0	0	0
Max.	0.931	0.055	0.052	0.055
Mean	0.011	0.000	0.003	0.001
SD	0.026	0.001	0.003	0.003
Scenario 2				
Min.	0	0	0	0
Max.	1.360	0.070	0.111	0.070
Mean	0.015	0.001	0.012	0.007
SD	0.029	0.003	0.010	0.009

NOTE: Avg. = average.

TABLE 5 Network Travel Time

Time	Mean (h)	SD (h)	CV
Scenario 1			
Free	17,727,618	18,012	0.001
Congested	935,988	9,738	0.010
Scenario 2			
Free	17,461,650	30,483	0.001
Congested	961,328	192,646	0.200

The mean vehicle kilometer for highway links is lower than that for all links, despite the fact that uncertainty was represented only in highway links. That result comes as no surprise. In fact, for highway links the traffic demand can be assumed to be less elastic to changes in travel time (defined by the BPR formula) as compared with journeys using an urban or local network. This assumption is primarily due to a lower number of competitive routes, which characterizes journeys on highway facilities. Nevertheless, as a result of the differences in capacity, a small percentage variation in the traffic demand for highway links may easily result in a high variation for the links of the competitive routes that absorb the diverted traffic. This effect explains why the CV values for highway links are lower than those of the overall network. With respect to average speed, the model appears to be insensitive. The reason can probably be traced to the lower congestion levels that characterize the overall network.

Table 5 shows the total network travel time, divided into free and congested travel time. As can be seen, the corresponding CV for free and congested times is very low. This result is consistent and reflects the low variability resulting from the analysis of the average speed. However, links capacity uncertainty has a high impact on congested time uncertainty, which increases from 0.01 to 0.2.

Although overall the model showed low sensitivity to BPR parameter variation, the traffic demand for some links revealed instead high elasticity, resulting in a maximum mean vehicle kilometer CV of 0.931 and 1.360 for Scenario 1 and Scenario 2, respectively. Thus, for analysis of differences in the network, the data set was divided into three groups including links with vehicle kilometer CV lower than 0.1 (Group 1), between 0.1 and 0.5 (Group 2), and higher than 0.5 (Group 3). Statistics referring to the three groups are shown in Table 6.

As can be seen, the majority of the links show a modest or null sensitivity, consistent with results for the overall model. Only a few links, included in the third group, show instead a very high sensitivity, but because of their low number at least part of them are considered outliers. More interesting for modeling purposes are the links included in the second group. Most of them (about 200 in both scenarios) should be no cause for concern, given that they represent international Danish traffic and the relatively high variability is probably the result of the low number of observations in absolute values. However, the rest, for a total of 107 (Scenario 1) and 241 (Scenario 2) links, refer mainly to short, middistance road types (*hovedvej* and *trafikvej*) potentially hosting commuting traffic. As a consequence, the assessment of projects planned for implementation in the areas of the network where they are located can be highly affected by their inherent uncertainty. In fact, in the case of changes in the network that result, for example, from structural changes or transport policy, the high sensitivity they demonstrate may cause the traffic to divert from the originally modeled routes. In areas characterized by a dense network, and therefore many competitive routes, these changes or policies can easily cause a shock wave throughout the surrounding network.

TABLE 6 Vehicle Kilometer CV by Groups

Statistic	1[a]	2[b]	3[c]
Scenario 1			
Min.	0	0.100	0.501
Max.	0.099	0.494	0.931
Mean	0.009	0.189	0.573
SD	0.010	0.089	0.110
Scenario 2			
Min.	0	0.100	0.507
Max.	0.099	0.481	1.360
Mean	0.013	0.178	0.859
SD	0.013	0.088	0.392

[a]Scenario 1: $n = 33{,}385$; Scenario 2: $n = 33{,}265$.
[b]Scenario 1: $n = 307$; Scenario 2: $n = 442$.
[c]Scenario 1: $n = 25$; Scenario 2: $n = 10$.

CONCLUSIONS

This paper describes the results of a study carried out to test the LTM sensitivity to BPR parameter (α and β) uncertainty. BPR parameter uncertainty was quantified by using the bootstrap resampling approach. The speed and flow data used to calibrate the BPR parameters, and, successively, to implement the bootstrap analysis, refer to three highway links that are part of the Danish road network. Model sensitivity to link capacity uncertainty, combined with BPR parameter uncertainty, was also tested. The model outputs analyzed were (*a*) vehicle kilometer and average speed at the link level and (*b*) travel resistance at the network level.

The results confirm the importance of uncertainty analysis as a decision tool for transportation projects. In fact, although the LTM as a whole proved to be quite inelastic to the variability in the BPR formula parameters, some links showed high elasticity. Any assessment of projects potentially affecting traffic flow on those links should then take into consideration this elasticity and integrate uncertainty analysis in the decision process.

In more detail, the results clearly highlight the importance for modeling purposes of taking into account BPR formula parameter uncertainty, expressed as a distribution of values, rather than as assumed point values. The increasing amount of traffic data available nowadays, as a result of the diffusion and improvements of technology, allows in fact the estimation of specific traffic delay formula parameters for different facilities and projects. This opportunity to produce more reliable modeled traffic results should not be missed. In addition, when combined with uncertainty analysis, it may produce the necessary information required to increase the quality of the decision process and to develop robust or adaptive plans.

Limitations of this study and avenues for further research should be acknowledged. First, a possible limitation relates to the limited number of count stations providing the traffic data on which the analysis is based. Further research could use a higher number of count stations, with a wider geographic distribution, to calibrate parameter values more representative for the overall network. Nonetheless, the results clearly underline the importance of taking into account parameter uncertainty, and their essence would likely not change but rather improve from additional data. Second, further analysis including urban and rural facilities parameters uncertainty would provide a more comprehensive picture of the topic, including the possibility of developing a class reference approach for uncertainty analyses of that kind. Last, because of the characteristics of the LTM and the scope of the study, the analysis presented in this paper did not quantify the effects on the model output deriving from uncertainty in the BPR formula variables free-flow speed and link flows. Further research could investigate these issues, depending on the model tested and the objectives of the analysis.

ACKNOWLEDGMENTS

The authors gratefully acknowledge the financial support of the Danish Council for Strategic Research for the project Uncertainties in Transport Project Evaluation and are thankful for the insightful and valuable comments of four anonymous reviewers.

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The Transportation Demand Forecasting Committee peer-reviewed this paper.