Quantify the Road Link Performance and Capacity Using Deep Learning Models

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***Abstract*— The link performance and capacity are important quantitative features in road link performance assessment, and they play vital roles in many important transportation tasks, e.g., traffic assignment and dynamic routing. However, it remains a challenging task to quantify them, particularly in a dynamic traffic scenario requiring an accurate, fast, and dynamic output. This study proposes a tailored deep learning framework for the addressed problem, which combines important transport domain knowledge reflected by the Bureau of Public Road (BPR) link performance function. In specifics, the calibration of link performance function and the estimation of link travel time are combined in the proposed framework and realized by two neural network modules. Numerical experiments demonstrate the capability of the proposed framework to capture complex relationships between dynamic link capacity and various factors and show its value in estimating link travel time.**

***Index Terms*— Link performance function, link capacity, deep learning, BPR function, macroscopic and microscopic traffic modeling.**

I. Introduction

TRAVEL time and travel impedance on road links increase *per se* in a non-linear way as the traffic volume increases. Link performance functions, also termed as link capacity functions, road impedance functions, or volume delay functions in literature, are often employed to describe this relationship [1], [2]. In the field of traffic engineering, link performance functions are generally formulated in simple forms with analytical solutions, which are favorable as prerequisite analyses for traffic assignment and demand management [3]–[7]. The definition and calibration of link performance functions for a transport network largely determine the reliability of traffic assignment results and, accordingly, strategies of urban transportation planning and management [8].

Existing literature heavily focused on the applications of link performance functions rather than investigating the nature and improved mathematical formulations of link performance. For example, many traffic assignment studies managed to propose novel algorithms to obtain fast convergence based on different assignment theories (like deterministic user equilibrium and stochastic user equilibrium). While these researches have greatly contributed to the advances in traffic assignment [9]–[11], the direct use of link performance functions with recommended parameter values in their numerical examples seems to have a less solid ground. The appropriateness of link performance functions needs further discussion.

The general formulation of a link performance function is shown in (1).

$$t = f(t_0, x) = f\left(t_0, \frac{v}{c}\right) \tag{1}$$

where t_0 denotes the free-flow travel time of a link (when traffic flow equals zero), v is the traffic volume of the link, c is the capacity of the link, x is the ratio of the traffic volume and link capacity, and t is the estimated link travel time at volume-capacity ratio x.

The link performance function highly depends on three macroscopic traffic characteristics, including free-flow travel time, link volume, and link capacity. The free-flow travel time and link volume are directly measurable from field studies. However, the last characteristic, link capacity, lacks ground truth data and does not have a rigorous quantitative definition, and insufficient consensus has been reached on how the measurement of link capacity should be performed. Some engineering handbooks like *Highway Capacity Manual* (HCM) provide some guidelines on the estimation of link capacity in engineering practices. However, these approaches assume a simple additive and multiplicative relationship between capacity and traffic characteristics, which may not exactly reveal complex interactions and obtain relatively lower accuracy.

Adjustable parameters are introduced into link performance functions for modifying the function shape. For instance, the most widely used link performance function, the Bureau of Public Road (BPR) function, as defined in (2), has two parameters α and β to be calibrated.

$$t = t_0\left[1 + \alpha\left(\frac{v}{c}\right)^{\beta}\right] \tag{2}$$

These two adjustable parameters enable the BPR function to describe the performance of a variety of road links in a unified functional form. However, one major issue pertaining to the BPR function is the difficulty in finding a set of reliable parameters suitable for any location and time [12], because of the complexity of traffic flows and the spatio-temporal

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differences in various environments. Furthermore, the dynamics, uncertainties and unbalanced data in traffic flow also contribute to the difficulty of parameter calibration, even with advanced data-fitting techniques [13]. In practice, whereas some specific parameter values recommended by handbooks like HCM are widely applied, their universality was challenged by many practitioners [14]–[19].

Confronting limitations in the parameter calibration of link performance functions, this study leverages advanced data-driven approaches to enhance the estimation accuracy for link performance and capacity on the basis of the classical BPR function. To this end, we propose a deep-learning-based framework to dynamically calibrate link performance functions, considering its excellence in accuracy, scalability, and its ability to model nonlinear relationships. On the basis of the BPR function, we design two special end-to-end neural network modules for link capacity estimation and BPR parameter calibration. The two modules are then integrated into a synchronous training process, which takes travel time as its final output. In addition, the capacity estimation module takes many traffic-related feature inputs at both disaggregated and aggregated levels. The integration of neural network modules is trained with the loss function of travel time prediction accuracy, which can help to overcome the shortage in obtaining accurate capacity values. Finally, we also examine the choice of activation function, the pretraining method, and the data normalization method, in order to prepare a better neural network setting that can provide better performance. The objectives and contributions of this paper are summarized as follows:

- We propose a deep-learning-based framework to calibrate link performance functions. Taking advantage of the impressive performance of deep learning, the proposed framework is more accurate and generic for different traffic environments. Furthermore, the BPR function can be replaced by any other road impedance function forms if necessary.
- The proposed model can well capture the complex nonlinear relationship between various traffic-related features and the link capacity. Most end-to-end neural networks require true labels of link capacity, which is difficult to obtain. Instead, we introduce the BPR function at the end of the neural network and transform the problem of link capacity estimation into travel time estimation. In other words, the main target of the neural network training is to reduce the prediction error of mean travel time computed by the BPR function.
- We design a series of special neural network strategies for this proposed model, including the activation function, the pretraining method, and the data normalization method. These designs are highly based on the peculiarity of traffic problems and can significantly improve the stability and accuracy of the proposed framework. To the best of our knowledge, similar techniques are seldom used in previous transportation research relating to deep learning. It is promising that these improvements can inspire more thinking and discussions on neural networks for solving transportation problems.

The remainder of the paper is organized as follows. Section II provides a brief review of existing research on link performance functions and points out the limitations of existing methods. The problem statement and the proposed methods can be found in Section III and Section IV. Then, the experiments based on simulation data and the corresponding results are provided in Section V. Finally, Section VI concludes the work in this paper and discusses further directions of improvement.

II. Related Work

A. Link Performance Function

Link performance functions express the travel time on a road link as a function of the traffic volume (see (1)). As an ideal input for traffic assignment procedure, a well-performed link performance function is usually required to satisfy the following conditions [20], [21].

First, high/unbiased goodness of fit is crucial, which means that the function well describes the relationship between the volume and the mean travel time in the link, ensuring the accuracy of the traffic assignment results [11], [22]. Second, the function $f(\cdot)$ is strictly positive and monotonically increasing with the increase of traffic volume, which ensures that the given link performance function is consistent with the prior knowledge about travel time [1], [23]. Also, a monotonically increasing function is the premise of the convergence of most traffic assignment algorithms [24]. Third, the function $f(\cdot)$ is differentiable everywhere within the feasible region [25]. This is because most existing traffic assignment algorithms are gradient-based, whether heuristic or non-heuristic. Last but not least, the evaluation procedure of the function $f(\cdot)$ should be relatively simple [26]. This can guarantee an acceptable computing time of traffic assignment algorithms, especially for large-scale network assignment tasks with a high precision requirement.

Numerous works can be appreciated about the link performance functions proposed by far [27]–[29]. One pioneering link performance function consists of two linear segments [30]. In the function, the capacity c'_P is defined as the practical capacity, where the travel time begins to increase rapidly with increasing traffic flow. Later, the two-piece linear function is further updated to a three-piece linear form by introducing the feasible capacity c'_S [31]. When the demand is over c'_S, the travel time increases rapidly as traffic flow increases. The piecewise linear function form describes the growing influence of unit volume on link travel time with the increase of traffic volume. While the linear relationships are simple and feasible for traffic assignment, it is difficult to determine the turning point in the function. Hence, a curvilinear link performance function without discontinuity would be preferable.

Currently, the most widely used link performance function is the BPR function developed by Bureau of Public Roads, as formulated in (2) [13]. Figure 1 shows the shape of several typical BPR functions with different parameter values. The BPR function possesses all the advantages mentioned above and has been the most popular link performance function since

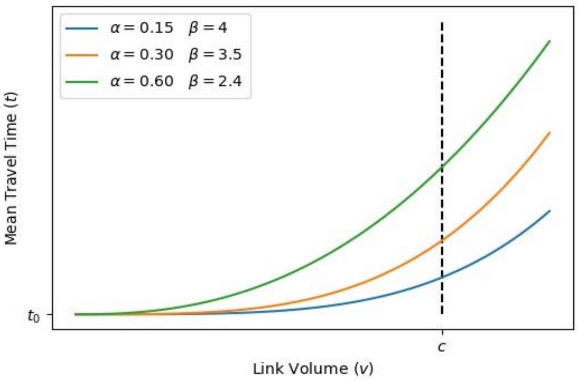

Fig. 1. BPR link performance function.

the 1960s [32]. Reasons for the evident popularity of the BPR function are twofold. First, the majority of theoretical studies on traffic assignment/analysis take the BPR function in their numerical examples in light of its elegant functional form [33]–[37]. Second, practitioners in different countries and regions kept calibrating and evaluating the BPR function with local data in the past decades, considering its reliable guidance to transportation planning and management [38]–[42].

In addition to a proper mathematical function form, the estimation of parameters is also crucial for a well-performing link performance function. In Section II.B, we will discuss existing methods for parameter estimation in the literature.

B. Estimation of Parameters in Link Performance Functions

The capacity of a road link, which is first simply compared to the size of a bucket, is a changing and evolved concept in the past few decades. Since 1950, HCM has been used as a standard guide in the United States and has spurred the development of similar manuals in other countries. In HCM 1950, practical capacity was defined as the maximum number of vehicles that can pass a given point on a roadway or lane during one hour without overly large traffic density. It may cause unreasonable delay, hazard, and restriction to the drivers' freedom to maneuver, under prevailing roadway and traffic conditions [14]. It was noted in the manual that the practical capacity is of primary interest to those striving to provide adequate highway facilities and the recommendation values of practical capacity were provided. Hence, most link performance functions proposed during this period take the practical capacity as the definition of the capacity in their formulations.

However, the definition of practical capacity in the HCM 1950 is soon recognized to be subjective, since it strongly depends on individual judgment reasonable delay, safety, or restriction conditions. In the later edition of the manual, HCM 1965, the concept of practical capacity is thus abandoned [43]. Since then, the current widely used methods to estimate the link capacity have been preliminarily established. Since it is acknowledged that the capacity is defined under prevailing conditions, varying from location to location, it was impossible to tabulate simple values. Instead, the manual provided the values of capacity under ideal conditions (also refer to base conditions in the later editions). Meanwhile, the manual presented a series of capacity adjustment strategies for these links where base conditions do not exist [44], [45]. The later editions of the manual further specify the standard of base conditions, slightly modify the base capacity recommendation values, and continue updating the capacity adjustment methods. However, the framework of capacity estimation, calculating the capacity under base conditions and then adjusting the capacity to the prevailing condition, remains unchanged [46].

The capacity adjustment method in the HCM provides a standard, feasible, and relatively simple approach to estimate the capacities for various types of transport facilities. The given capacity estimation method plays a significant role in previous transport planning, design, and analysis. And predictably, it will still serve as an authoritative guide for capacity estimation for a long time in the future [47], [48].

However, from a theoretical point of view, the drawbacks of the current adjustment method in HCM are evident [49]. First, road facilities vary more or less in roadway conditions, traffic conditions, and other conditions. It is hardly possible to set an ideal road under base conditions and obtain its capacity. Hence, the determination of the base capacity is relatively subjective [50]. Second, more importantly, the majority of adjustment factors are calibrated individually. It means the influence factors of capacity are assumed to be uncorrelated and non-interacting. In these methods, the overall adjustment factor is simply obtained as the summation or multiplication of all independent adjustment factors [51]. The HCM method cannot model the interactions among various condition factors constrained by the nature of linear reduction and the estimated capacity is thus imprecise.

Therefore, a number of studies have been carried with the goal of more specific quantitative analysis of road capacity [52]. Chandra and Kumar [53] studied the effect of lane width on capacity under mixed traffic conditions. Van Goeverden *et al.* [54] explored the impact of road lighting on roadway capacity. Chandra [45] proposed the tailored capacity estimation procedure for two-lane roads with data collected at over 40 sections. Also, some studies analyzed the impact of adverse weather conditions on capacity [44], [55]. Nevertheless, these studies merely focused on one or two particular factors on roadway capacity, and they can hardly form an integrated capacity estimation procedure due to their inherent complexities.

Moreover, the estimated capacity influences the calibration of the link performance function. After determining the form of the link performance function and estimating the capacity of links, the link performance function will be calibrated with observed traffic data. The estimations of capacity and parameter calibration are commonly considered to be two independent procedures in the literature [39]. Due to the relatively simple function forms in common, the calibration procedure can usually be converted to a convex programming problem or even a linear programming problem, indicating a guaranteed solution to the global optimal (i.e., the best parameter set for the input traffic dataset). In this way, the

TABLE I
NOTATION LIST

Notation	Explanations
X	Link attributes of a road link affecting the traffic flow
E	External attributes of a road link affecting the traffic flow
v	Traffic volume of a segment in a certain time period
t_0	Free-flow travel time passing a road segment
t	Mean travel time of a segment in a certain time period
c	Capacity of a segment in a certain time period
t'	Estimated mean travel time
c'	Estimated capacity of a segment
α	Parameter of BPR function
β	Parameter of BPR function

calibration result of a link performance function is unique given fixed input data. Nevertheless, it should be noticed that the estimated capacity has great influence on the calibration result. For instance, the BPR engineers defined the capacity c as the practical capacity and suggested the parameter values to be 0.15 and 4 respectively. Later, Steenbrink [56] suggested a similar function with the only exception that the capacity is explained as the steady-state link capacity. Different from the recommended values by BPR, the parameters α and β were calibrated to be 2.62 and 5.

To sum up, current methods of link capacity estimation are mostly based on engineering manuals, where the relationships between capacity and a range of variables are oversimplified, in light of the convenience of calculation or the limitation of data and data fitting techniques. In this way, the link capacity is usually estimated with considerable bias. Moreover, the procedure of capacity estimation is independent of the calibration of link performance functions in practice. However, the roughly estimated link capacity is treated as a known input to the calibration procedure of link performance functions. This can significantly aggravate the goodness of fit of link performance functions and further influence the reliability of consequent traffic assignment and analysis results.

III. Problem Statement

Let us consider a road link in a certain condition to be estimated. The notations in this paper mainly follow the previous study [13], [39], which are summarized as follows:

The capacity and performance of a link are determined by link attributes X and external attributes E. Link attributes mainly include roadway attributes (e.g. number of lanes, lane widths, design speed, dedicated lanes), traffic attributes (e.g. vehicle type, directional and lane distribution, driver population), and control attributes (e.g. stop and yield signs, signal control strategy). External attributes are described as the dynamic factors of the link capacity, such as the weather condition, traffic accidents, and short-term working zones.

To learn the relationship between these attributes and link capacity c, we aim to identify the mapping below using big traffic data and nonlinear data fitting models.

$$f : (X, E) \mapsto c \tag{3}$$

Links attributes and external attributes in the proposed model correspond to the inputs for capacity adjustment in the HCM, but two distinctive advantages should be mentioned. First, our model support larger dimensions and more types of attributes. More detailed and regional attributes can be fed into the model, rather than being limited to existing ones in the manual. Second, benefited from the nonlinear mapping of the neural network, unstructured data including pictures, videos, social network messages can be embedded and utilized in the model.

Further, we take the BPR function form as the prior knowledge (see (2)). As mentioned above, the BPR function is simple in terms of its functional form [29] and advantageous in data fitting [29]. As the most widely accepted link performance function form in both academia and industry [34], [37], [45], the BPR function is served as the proper basis of this study. In this way, the predicted mean travel time can be formulated as (4).

$$t' = t_0 \left[1 + \alpha \left(\frac{v}{f(X, E)} \right)^{\beta} \right] \tag{4}$$

To identify a proper link performance function, the objective is to minimize the gap between observed travel time and estimated travel time in all study road links. Let M denote all road links in the study area, the objective function is formulated below.

$$\min \sum_{i=1}^{M} (t_i' - t_i)^2 \tag{5}$$

Then, the mapping f, the BPR parameter α and β can be estimated by minimizing the prediction error of travel time. Differing from traditional methods, the mapping f is not predefined. Instead, it is defined as a complex nonlinear function and optimized in the programming solution process. In this paper, we model the mapping f using a neural network. While the optimal global solution of this problem is not guaranteed, we design an efficient deep-learning-based framework, which will be introduced in the next section, for searching the locally optimal solution.

IV. A Data-Driven Deep-Learning-Based Framework

A. Overview of the Framework

In the task of quantifying the capacity and performance of road links, a range of input features are helpful in incorporating the hidden and unobserved states of road links. Thus, deep learning is introduced to deal with high-dimensional inputs describing traffic characteristics at different spatial and temporal scales in this study. As one of the representative machine learning approaches, deep learning allows neural network models with multiple hidden layers to learn representations of data with multiple levels of abstraction [57]. In recent years, deep-learning-based methods have shown their advantages in processing high-dimension data and dramatically improved the state-of-the-art in a range of long-standing transport problems [58], [59]. For example, the accuracy of traffic flow prediction [60], [61], taxi demand prediction [62], and travel risk prediction [63] has been greatly improved by exploring spatial correlations, temporal dependencies, external conditions with neural networks.

In this study, a neural network module is used to approximate the complex functional relationship expressed as (3). Meanwhile, as aforementioned, the BPR function is employed to assist the analysis of intrinsically unknown capacity, which is taken to the right end of the deep learning structure. The BPR function relates link performance to capacity, thus yields only one output of link performance in terms of travel time. This greatly contributes to model training, as travel time is observable, and the related data is available in most urban transport systems [64]. The BPR function encodes our prior knowledge on link capacity in the training process of neural networks by limiting the intermediate output to a reasonable range.

Moreover, it should be noted that the calibration of the BPR function and the capacity estimation would be conducted synchronously to avoid propagation errors. To sum up, given the attributes of road links and their historical traffic data, the final output of this framework is the proper BPR functions of these links, including the estimated capacity as well as the calibrated function parameters.

B. Structure of the Model Enabling Back Propagation

In this subsection, we elaborate on the deep-learning-based framework for the quantification of link performance and capacity. The framework consists of three main modules, including a capacity estimation module, a parameter calibration module, and an error evaluation module, each of which is comprised of a more detailed computational graph and designed for different purposes. The forward and back propagation procedures will be taken to update the neural network parameters to approximate the complex functional relationship in (3) and optimize the link performance function parameters.

The capacity estimation module relates link attributes and external attributes to the link capacity. Multiple hidden layers are used to learn and represent the complex nonlinear relationship between them. Here, the structure of hidden layers is flexible according to the input data. For the inputs with spatio-temporal property (e.g. pictures and geographical data), advanced convolutional and recurrent structures should be further considered. However, for highly structured input features, several fully-connected layers would be suitable, as formulated below.

$$H_1 = g_1(X, E) = \sigma(W_1(X, E) + B_1) \tag{6}$$

$$H_t = g_t(H_{t-1}) = \sigma(W_t(H_{t-1}) + B_t), \forall t \in \{2, 3, \ldots, T\} \tag{7}$$

$$c' = g_o(H_T) = W_o(H_t) + B_o \tag{8}$$

where $\{1, 2, \ldots, t, \ldots T\}$ is the set of hidden layers, H_t are the nodes in the hidden layer t, W_t and B_t are the weight and bias parameters of the hidden layer t, W_o and B_o are the weight and bias parameters of the output layer of capacity estimation module, $\sigma(\cdot)$ is the activation function, and c' is the estimated capacity.

The parameter calibration module is used for adjusting parameters of the link performance function. The inputs of this module are the initial values of parameters. For example, the recommended BPR function parameter values in the literature can be deemed as an appropriate initial solution. In this module, we set one individual parameter adjustment unit for every single parameter, where trainable weights will be multiplied to adjust the initial value of the parameter. For the BPR function, the parameter adjustment process is formulated as

$$f_\alpha(\alpha) = w_\alpha{}^* \alpha \tag{9}$$

$$f_\beta(\beta) = w_\beta{}^* \beta \tag{10}$$

where w_α and w_β are the two weight parameters in the parameter adjustment units.

Clearly, these two modules cannot be trained individually due to the lack of ground truth of capacity and link performance function parameters. Therefore, the BPR function formulation is introduced to the error evaluation module. These three modules are connected as shown in Figure 2.

With free-flow travel time, link volume, and the outputs of the capacity estimation module and the parameter calibration module being inputs, the error evaluation module can output the estimated mean travel time by applying the BPR function. The outputs are later compared with the observed mean travel time to calculate the loss. Since the mean travel time is an objective and observable value t, the loss function can be obtained as follows.

$$\epsilon = \text{Loss}\left(t', t\right) \tag{11}$$

We take mean square error (MSE) for error evaluation, the loss function can be formulated as:

$$\epsilon = \frac{1}{M} \sum_{i=1}^{M} \left(t' - t\right)^2 \tag{12}$$

Then, the computed MSE is propagated backward until each neural network parameter is updated by its contribution to the deviations of error. Note that MSE is equivalent to the goodness of fit of the model output link performance function as formulated in (5), and it would be back-propagated from the error evaluation module to the capacity estimation module and the parameter calibration module.

C. Special Techniques for the Neural Network

Because of the particularity of this task, some special designs are proposed to improve the model stability and accuracy. The three major neural network designs are introduced as follows.

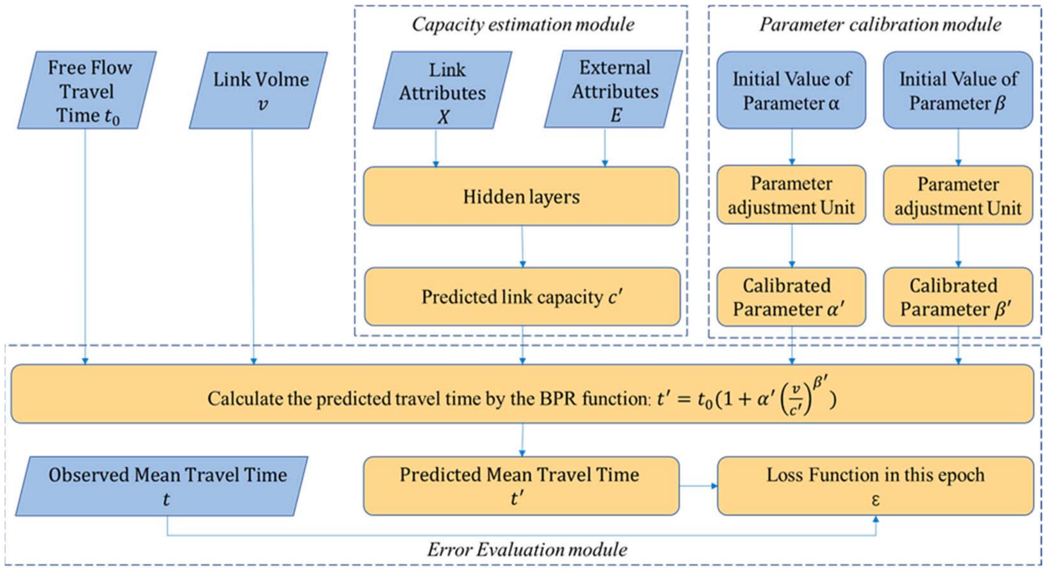

Fig. 2. Framework of the proposed deep-learning-based method.

1) Neural Network Pretraining: In the training process of the deep learning model, the neural network parameters, in the capacity estimation module and the parameter calibration module, are updated simultaneously. The initial parameters of neural networks play an important role in terms of convergence speed and accuracy. Appropriate initial values can greatly reduce the training time and avoid the model from falling into unsatisfactory local optimal solutions. In the pretraining process, a small number of temporary labels of link capacity are generated based on existing models and then used to train the capacity estimation module, which is a subnetwork in the framework. The parameters of the capacity estimation module will be saved as the initial parameters of the neural network.

2) Activation Function: In the link performance function, some parameters (e.g., capacity, α and β in the BPR function) have their strict feasible zones, greater than zero. In a typical neural network, the activation function of an output layer is mostly linear, which cannot ensure the output value positive. Hence, a special activation function named PeLU is designed and used in the output layer of both the capacity estimation module and parameter calibration module. PeLU is a monotonically increasing function that is differentiable everywhere, as formulated in (13) and shown in Figure 3.

$$\mathrm{PeLU}(x) = \begin{cases} e^x & x < 0 \\ x + 1 & x \geq 0 \end{cases} \tag{13}$$

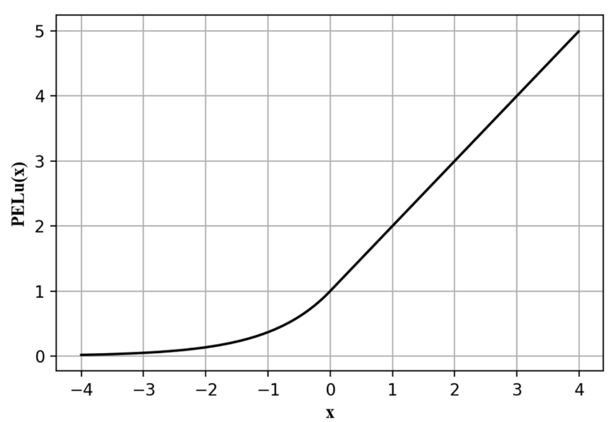

Fig. 3. PeLU activation function.

3) Data Normalization: Deep learning requires normalizing the input data, and the most widely used normalization methods are nonlinear transformations. The volume to capacity ratio can be affected and can disturb the travel time calculation in the error evaluation module. To avoid this, a simple but effective normalization for the volume is designed, as formulated in (14). The normalization method is also suitable for the capacity if the pretraining is performed.

$$V^* = \frac{V}{\max(V)} \tag{14}$$

where V is the original volume and V^* is the normalized volume.

V. Numerical Experiments

This paper focuses on the deep learning model in quantifying the capacity and performance of the road link. Due to the nature of the neural network, large amounts of data are needed for training and evaluating the proposed model. However, it is rather time-consuming and costly to collect attributes of a large number of road links and record their traffic flow conditions for a long period. Therefore, in this paper, data are collected using simulation in Vissim, considering that this paper focus on the models/methods rather than practical results. The road link attributes in reality can usually be obtained in simulation and their influences on traffic flow can also be reproduced.

Since the proposed model is flexible to the structure of inputs, even though the simulation data may be slightly different from those collected in reality, it may not influence the validation of model effectiveness. Two vital indices are

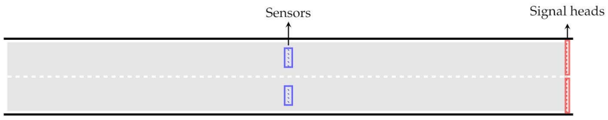

Fig. 4. Simulation of a two-lane road link.

used to evaluate the model performance. One is the accuracy of estimated link performance, which is equivalent to the goodness of fit of the link performance function, or, in other words, the mean square error of the travel time estimation. The other index is the rationality of the capacity estimation. Two measurements are taken as the estimation of roadway capacity; the first measurement is the maximum sustainable volume. In this paper, we take the 95th percentile traffic volume as the maximum sustainable volume (also termed as historical maximum sustainable volume). The second measurement is the historical maximum volume. The estimated capacity of links is compared to both the historical maximum volume and the historical maximum sustainable volume in the simulation.

A. Data Description

An urban road scenario is built in the microscopic traffic simulator Vissim, containing a basic road section and traffic signals downstream. 8,000 different road links under different conditions are built in the simulation, and their traffic operations are recorded every 30 minutes by virtual sensors, including volume and mean travel time. Figure 4 shows a simulated road link with two lanes. To reduce the effects of vehicle loading on link performance, all the simulation road links are created longer than 800m. Meanwhile, to make sure vehicles to adjust their speeds and distances before entering the study area, the data collection sensors are loaded far from the vehicle loading points. The 8000 simulation scenarios are randomly split into 70% train data set and 30% validation data set. Based on the model trained using train data, the validation data is employed to verify the model performance on travel time estimation and capacity estimation.

The road links in the simulation scenario differ in these attributes, as summarized in Table II, which are the inputs of the model (X and E) as well. Note that the listed car following (CF) model parameters are randomized from a large feasible range in the experiments, serving as the inputs of Vissim. They describe the driver behaviors in the simulation and should be replaced by other observable attributes in other studies with real data. Note that this study focuses on the methodological framework of the addressed topic rather than empirical outcomes, and thus the simulation data is qualified for the model development. For preprocessing techniques on data-driven transportation researches, the reader can refer to [65].

B. Results

Some hyper-parameters of the neural network in the experiments are set as follows. The hidden layers of the capacity estimation module are two fully connected layers with PeLU activation, which have 128 and 64 units respectively. The activation functions used in the output layers of the two modules are both PeLU. Some different settings of the depth and breadth of the model do not show significant performance improvement or degradation. The optimizer is Adam optimizer with a 0.0002 learning rate. Three different sets of initial values of link performance functions are Set 1 ($\alpha = 0.15$, $\beta = 4$), Set 2 ($\alpha = 2.4, \beta = 5$) and Set 3 ($\alpha = 4, \beta = 5$). For simplicity, we denote models with these three sets of initial values above as MS1, MS2, and MS3, respectively.

TABLE II
ATTRIBUTE EXPLANATION IN VISSIM

Attributes	Meanings
Lane number	Number of lanes per side (1 to 4)
Heavy vehicle ratio	Proportion of heavy vehicles (0 to 1)
Left-green ratio	Ratio of left-turn green time in a signal cycle
Straight-green ratio	Ratio of straight green time in a signal cycle
CF parameter 1	Multiplicative factor of desired safety distance in Wiedemann 74 car following model
CF parameter 2	Additive factor of desired safety distance in Wiedemann 74 car following model
CF parameter 3	Average standstill distance in Wiedemann 74 car following model

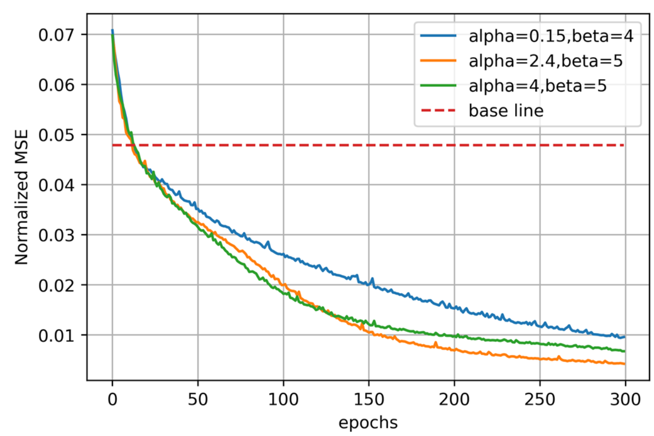

Fig. 5. Training process of different initial parameter values.

The baseline link performance function in the experiment is determined by the traditional adjustment method, where the link capacity is a pre-determined value rather than a trainable variable in the proposed model. With the normalized MSE as the metric of travel time prediction errors, the training process is shown in Figure 5.

It can be observed that the deep-learning-based models catch up with the baseline in terms of training error after 20 epochs of training, and the models are insensitive to the

TABLE III
CONDITIONS AND ESTIMATED CAPACITY IN THE TEST CASES

	Case 1	**Case 2**	**Case 3**	**Case 4**	**Case 5**
Conditions					
Lane number	1	1	2	3	4
Straight-green ratio	0.088	0.233	0.233	0.233	0.233
Left-green ratio	—	—	0.116	0.116	0.116
Heavy vehicle ratio	0.1	0.1	0.2	0.222	0.111
CF parameter 1	3.5	2	2.6	1.9	3.4
CF parameter 2	1	1.5	3.1	2.4	4
CF parameter 3	1.5	2	3.7	3	1.5
Capacity					
Historical maximum volume (pc/h)	246-304	480-600	448-560	660-860	800-1160
Historical maximum sustainable volume (pc/h)	240-284	464-588	440-512	580-796	768-1060
Adjustment method estimated capacity (pc/h)	316	390	338	1114	1577
Deep learning method estimated capacity (pc/h)	288	408	480	644	840
Percentage gap1[a]					
Adjustment method estimated capacity	14.91%	27.78%	23.02%	46.58%	60.92%
Proposed method estimated capacity	4.70%	24.44%	4.76%	15.26%	14.26%
Percentage gap2[b]					
Adjustment method estimated capacity	20.61%	25.86%	29.00%	61.92%	72.54%
Proposed method estimated capacity	10.00%	22.43%	0.84%	6.40%	8.10%

a: Percentage gap1 is calculated based on the historical maximum volume.
b: Percentage gap2 is calculated based on the historical maximum sustainable volume.

initial values of α and β. After around 300 epochs of training, the models with different initial parameters reach convergence, and the output road impedance functions all have superior goodness of fit to the baseline functions.

The performance of the trained model on travel time estimation is validated based on the validation data set. Fig. 6 represents the model performance in terms of how the estimated link travel time fit the recorded link travel time in simulation (i.e., the real link travel time in Fig. 6). Fig. 6 indicates that the estimated link travel time is close to the simulated value. The normalized MSEs of MS1, MS2, and MS3 are 0.030, 0.050 and 0.047 respectively. The normalized MSE of the conventional method is 0.219. Hence, our model outperforms the conventional model in the validation set and has a stronger generalization ability.

Table III shows the prevailing conditions of five test scenarios and their capacity estimated by different methods. Compared to the conventional adjustment methods, the link capacity estimated by our model is closer to both the historical maximum sustainable volume and the historical maximum volume in general. Specifically, when the number of lanes gets larger, the performance of conventional method gets worse. However, our model performs even better. It to some extent indicates that the proposed deep learning method is potential to be used to estimate and evaluate link capacity, especially for links with multiple lanes.

Moreover, the gap between actual operation data and simulation data is considered. The traffic volume can be corrupted by noises due to the limitation of the sensors. Sensitivity experiments are thus conducted. Two different degrees of volume noises, which are uniformly distributed between 0 and 10 percentage error and between 10 and 20 percentage error respectively, are introduced to the model inputs. It should be mentioned that all segments for validation are not included in the train dataset. The results show that the models trained with noisy data have a slight drop in performance compared to the control group trained with accurate simulation data, but the goodness of fit is still better compared to the baseline.

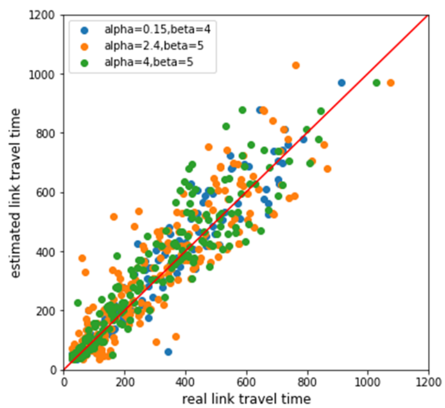

Fig. 6. Validation of the model performance on link travel time estimation.

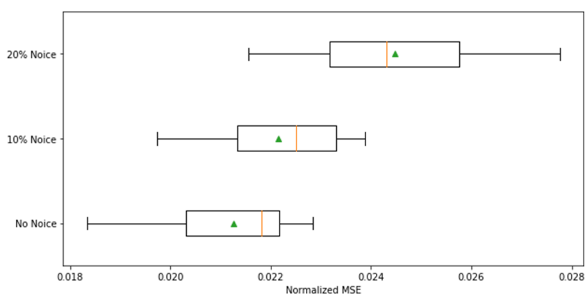

Fig. 7. Model performance with different levels of volume noise.

Figure 6 shows the prediction errors of twenty experiments with random noises. The proposed method is superior in quantifying link performance in terms of link travel time, even when noises are introduced (considering the errors of the baseline are over 0.075 in this experiment). Table III shows the experiment results of prediction errors in different sets.

TABLE IV

EXPERIMENT RESULTS OF PREDICTION ERRORS (NORMALIZED MSE)

	MS1	MS2	MS3	Baseline
No noise introduced	0.019	0.015	0.015	0.079
10% Volume Noise	0.022	0.016	0.015	0.079
20% Volume Noise	0.026	0.021	0.017	0.080

VI. CONCLUSION

Link performance functions have long been the primary element in traffic assignment modeling and have provided the foundations for subsequent policymaking and management practices. Numerous studies assume the appropriateness of the current link performance function, but quantifying the capacity and performance of a road link remains a challenge due to the diversity of road conditions and urban driving behaviors. To overcome the limitations of existing methods to determine link performance functions, we proposed a deep-learning-based framework in this paper. In this framework, the capacity estimation module relates various features of traffic flow and road geometries to capacity, and the parameter calibration module is used to adjust the parameters of the link performance function. With the BPR function as the prior knowledge, the deep-learning-based model can be trained and evaluated with observable travel time. A series of techniques are also proposed to improve the accuracy and robustness of the neural network model.

The experiment based on the simulation data shows that the proposed method can produce link performance functions with better link travel time prediction, even for the cases with considerable noises in volume data. Meanwhile, although the final output of this model is a link performance function, aiming at travel time estimation, the trained neural networks do show the capability to accurately infer the link capacity based on the features of traffic flow and road geometries, as shown in Table IV. we need to point out that, apart from the capacity, some other parameters in the link performance function are also well adjusted. Thus, the capacity produced by the model provides the idea of capacity estimation in a supervised learning way.

As an initial step of the addressed topic, this paper mainly focuses on the methodological framework and discussions, and thus only simulation data are used. When implementing the proposed model on real world, a significant concern is the availability of data. Although deep learning is known to be flexible to the input dimensions, it is still a challenging task to explore how the absence of these missing data will impact the model performance. Meanwhile, the road facilities in real world significantly vary in geometry (e.g., roundabouts and variable lanes) and property (i.e., interrupted and uninterrupted). The proposed model is expected to be a general solution to these roads, while more meticulous experiments are necessary.

From the theoretical point of view, improvements can also be made in two areas: the functional form of link performance functions and the use of multi-source data. Regarding the functional form, the BPR function is used in this paper because of its wide recognition. More different forms of link performance functions can be tested [7], and they may have their own advantages in different traffic scenarios. Aside from the functional form, the model can be trained with richer input data. For instance, considering the impressive progress in computer vision recently, the model may be trained with picture or design drawings. In this way, artificial intelligence in the future may accurately provide human engineers with the capacity of a road link by analyzing aerial images or even three-dimensional models in advanced roadway design software. To this end, a larger scale of traffic and roadway data collection, the continuous improvement in machine learning algorithms, and the combination with prior knowledge are all needed. In terms of application, validating and implementing of the proposed framework under different traffic conditions (e.g., a road segment containing a roundabout or road segments in different countries) and in practical transportation modelling problems offer an interesting avenue in further study.

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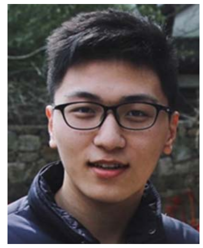

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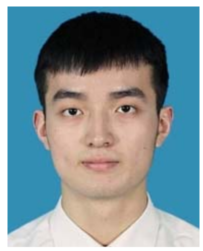

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