Volume Delay Functions Based on Stochastic Capacity

Robert Neuhold and Martin Fellendorf

The standard highway assignment problem is solved by using a volume delay function (VDF). The primary link impedance is travel time, which increases with an increasing degree of saturation. VDFs contain link-specific input parameters, such as capacity and free-flow speed, as well as coefficients depending on VDF type. The coefficients either are taken from guidelines or are estimated from site-specific data. Because free-flow speed can be measured directly, mean values usually are used for particular links or link types. Capacity is not easy to determine because of its stochastic nature and large variations. VDFs for traffic assignment are presented that assume stochastically distributed capacity. These VDFs are suitable for two- and three-lane freeways in urban and non-urban areas. The stochastic capacity depends on the probability of traffic breakdowns and is determined with the use of the product-limit method and the Weibull distribution. The Bureau of Public Roads (BPR) function was chosen as a typical representative of a VDF for highway assignment models. Two models were applied. First, one coefficient was fixed, and then both BPR coefficients were estimated by regression analyses with the least squares method. The regression analyses provide suitable results at all measurement points for the model with one fixed coefficient; results differ when both BPR coefficients are estimated. The calibration of widely applied VDFs in travel demand models benefits from stochastic capacity analysis as applied in the field of traffic engineering. These results are unique to measurement points in Austria, but the method can be transferred to other countries where long-term data from freeway detectors are available.

Volume delay functions (VDFs) are an elementary part of travel demand models in transport planning. In some literature, VDFs are called link performance functions to capture the effect of traffic flow on link travel time (*1*). To compute link travel flows by solving the traffic assignment problem, required inputs include a graph representing the transportation network and origin–destination matrices as well as link-specific VDFs. Users are free to define customized VDFs, but the numerous travel demand software packages on the market offer little advice on the appropriate VDFs. In general, these models use VDFs with default settings for road characteristics, often from national standard guidelines. The VDFs for travel demand models require link-specific attributes, such as capacity and free-flow speed, and variable link flow to compute travel delay depending on the degree of saturation. Furthermore, one or more VDF coefficients are sourced from default values. In some studies, the coefficients are calibrated with empirical data by using regression analyses.

Institute of Highway Engineering and Transport Planning, Graz University of Technology, Rechbauerstrasse 12, 8010 Graz, Austria. Corresponding author: R. Neuhold, robert.neuhold@tugraz.at.

Transportation Research Record: Journal of the Transportation Research Board, No. 2421, Transportation Research Board of the National Academies, Washington, D.C., 2014, pp. 93–102.
DOI: 10.3141/2421-11

Common types of VDFs for travel demand models include the Bureau of Public Roads (BPR) function and the conical function by Spiess (*2*). Akcelik (*3*) and the French Institute of Science and Technology for Transport, Development, and Networks propose VDFs with different functions for under- and oversaturated cases. All functions strictly increase with volume, even when volume-to-capacity ratios are well beyond 1.0. Solving the standard traffic assignment problem is a mathematical requirement. These VDFs operate with link-specific, fixed-variable values for free-flow speed and capacity that typically are defined by road type. National guidelines such as the *Highway Capacity Manual* (HCM) provide fixed capacity values according to road characteristics such as number of lanes, free-flow speed, location, and gradient (*4*). Recent research in traffic engineering has identified capacity as a nondeterministic value (*5–9*). Capacity varies stochastically, influenced by environmental conditions (weather, traffic composition) as well as stochastic travel behavior.

Huntsinger and Rouphail develop locally calibrated VDFs with data from freeway detectors along I-40 in the Raleigh–Durham area of North Carolina, focusing on the estimation of demand beyond capacity (*10*). The VDFs in their research are fitted by bottleneck and queue analysis. The capacity is calculated as the average of the top 1% flow rate measured at the bottleneck. Huntsinger and Rouphail derive the demand-to-capacity ratio as the demand at capacity plus the queue under bottleneck conditions. Link travel time is calculated over the demand-to-capacity ratio for different fitted VDFs (e.g., BPR, conical, Akcelik, and a user-defined exponential function). Except for the conical function, Huntsinger and Rouphail present acceptable VDFs for travel demand models of that particular freeway.

In this paper, individual VDFs are calibrated with data from a detector on a local freeway with the assumption that the nature of link capacity is stochastic. Link capacity is distributed according to the probability of traffic breakdown. The product-limit method (PLM) is used to acquire link capacity values just before unstable traffic conditions occur. The VDF coefficients are continuously estimated by regression analysis. The VDFs are part of a larger traffic management system that continuously computes link travel times to estimate traffic states.

METHOD

The aim of this paper is to develop VDFs for measurement points on freeways that relate to breakdown probability. A capacity distribution is generated (instead of using a fixed input value) for the VDF on the basis of the breakdown probability of the particular freeway section. The data (traffic volume and speed) were obtained at 5-min intervals from measurement points located along a freeway. The

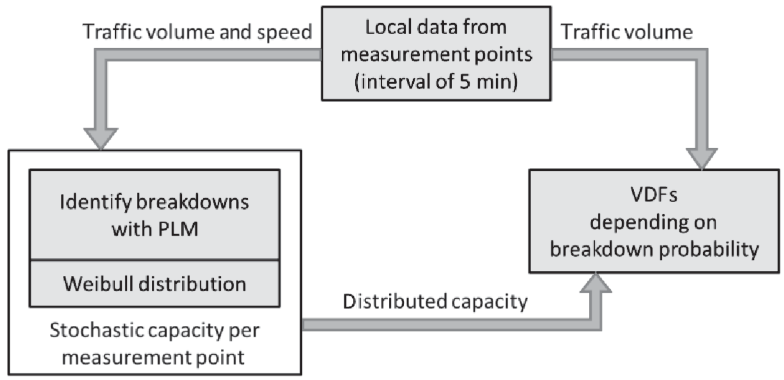

FIGURE 1 General method.

stochastic capacity is generated with a PLM according to a theory in natural science that Brilon et al. apply to analyze the traffic flow on freeways (*7*). The distributed capacity from the PLM is combined with observed traffic volumes at the local measurement points to establish the VDF (Figure 1).

Volume Delay Functions

VDFs are a common tool in transport planning for modeling travel time delays on road sections or traffic networks. Many types exist, such as BPR, conical (*2*), and Akcelik (*3*) functions. The VDF input parameters (e.g., free-flow travel time and capacity) must be determined, and VDF coefficients must be estimated (e.g., by regression analysis). The least squares method minimizes the sum of the squared error and commonly is used to estimate these coefficients (*11,* p. 731).

Unlike the coefficients, the VDF input parameters for free-flow travel time (t_0) and capacity (c) can be measured. These functions assume that a fixed value is available for the capacity. Probably the most prevalent VDF is the BPR function:

$$t = t_0 * \left[1 + \alpha * \left(\frac{v}{c}\right)^{\beta}\right] \tag{1}$$

where

t = current travel time,
α and β = empirical coefficients to be estimated by empirical data, and
v = traffic volume.

Dividing Equation 1 by the free-flow travel time, t_0, yields the quotient of current and free travel time, so the following form of BPR can be achieved:

$$f(x) = 1 + \alpha * x^{\beta} \tag{2}$$

with

$$x = \frac{v}{c} \tag{3}$$

where x is the volume-to-capacity ratio and represents the degree of saturation. The coefficients α and β are estimated in different ways. The original BPR literature recommends 0.15 and 4.0 for α and β, respectively. Horowitz suggests approximating α from the free-flow speed and the speed at capacity, if x is calculated with ultimate capacity (*12*). The β coefficient can be determined by nonlinear regression. Horowitz optimizes α and β in the BPR function for six-lane freeways and four-lane rural divided highways according to the speed–volume relationship in the HCM—values very different from the original 0.15 and 4.0 (Table 1) (*4, 12*).

An alternative to the BPR function is the conical VDF developed by Spiess (*2*). The name reflects its geometric interpretation as hyperbolic conical sections. The form of this VDF is

$$f(x) = 2 + \sqrt{\alpha^2 * (1-x)^2 + \left(\frac{2\alpha - 1}{2\alpha - 2}\right)^2} - \alpha * (1-x) - \frac{2\alpha - 1}{2\alpha - 2} \tag{4}$$

Unlike the BPR function, the conical function contains only one coefficient to be estimated (α).

In 1966, Davidson proposed a general travel time formula for transport planning purposes based on queuing theory (*13*):

$$t = t_0 * \frac{1 + J_D * x}{1 - x} \tag{5}$$

where J_D is the delay parameter (or $1 - J_D$ is the quality of service parameter).

Akcelik presents a time-dependent form of the original Davidson function (*3*). The Akcelik function retains a flow period value (T_f) that represents the period of flow analysis (e.g., $T_f = 1$ h). Because the Akcelik function is time dependent, the ratio of current and free-

TABLE 1 Coefficients for BPR Functions (*12*)

Speed (mph)	α	β
Freeway		
70	0.88	9.8
60	0.83	5.5
50	0.56	3.6
Multilane		
70	1.00	5.4
60	0.83	2.7
50	0.71	2.1

flow travel time cannot be expressed without a time component. The form of the Akcelik function is

$$t = t_0 + 0.25 * T_f * \left(x - 1 + \sqrt{(x-1)^2 + \frac{8 * J_A * x}{c * T_f}} \right) \quad (6)$$

where J_A is a delay parameter that can be determined by nonlinear regression. The Akcelik function can be used for individual road links as well as for links leading to signal-controlled intersections.

All of the previously mentioned VDFs require a capacity value as a fixed input parameter. Brilon et al. (*7*), Schwietering and Steinauer (*14*), and others have provided empirical evidence that capacity may vary by about 20% between the lowest and highest maximum measured throughput before flow breakdown. Brilon et al. use a Weibull distribution to describe capacity as a stochastic parameter (*7*). In this paper, the VDF is combined with capacity distribution to obtain individual VDFs for transport planning on freeway networks.

Capacity Distribution

A practicable nonparametric method for estimating the capacity distribution is the PLM, which is used for lifetime data analysis but has been applied in several studies of capacity analysis (*7*). PLM estimates a life expectancy distribution for organisms, so traffic breakdown is treated like the moment of death. The PLM uses the analogy between capacity analysis and lifetime data analysis to estimate the capacity distribution. The distribution function of capacity is

$$F_C(v) = 1 - \prod_{i: v_i \le v} \frac{k_i - d_i}{k_i} \qquad i \in \{B\} \quad (7)$$

where

$F_C(v)$ = distribution function of capacity,
v_i = traffic volume in interval i,
k_i = number of intervals with a traffic volume $v \ge v_i$,
d_i = number of breakdowns at a volume of v_i, and
$\{B\}$ = set of breakdown intervals.

Each observed traffic volume v can be classified as one of four states:

- State F (free flow). Speed is faster than the critical speed at interval i and interval $i + 1$,
- State B (breakdown). Speed is faster than the critical speed at time interval i and slower than the critical speed at time interval $i + 1$,
- State C1 (congestion). Speed is slower than the critical speed at time interval *i,* and
- State C2 (growing congestion). Speed is slower than the critical speed at time interval i upstream and also at interval *i,* interval $i - 1$, or both at the downstream measurement point.

For States F and B, the PLM uses traffic volume to estimate breakdown probability. Each observed value of v is considered as its own class in the product of Equation 7; therefore, $d_i = 1$ in all cases. The product uses only traffic volumes belonging to the amount of $\{Z\}$. Equation 7 allows us to estimate a capacity distribution at a particular measurement point from maximum measured volumes and breakdowns. The volumes are taken over breakdown probability as empirical values and estimated with a suitable distribution function. The empirical values are noncontinuous, and breakdowns with high probabilities near 100% rarely are measured.

Because high traffic volumes are measured in some 1-min intervals, a wide span of estimated capacities can be observed. The empirical values are needed to estimate a continuous distribution function of the breakdown probability, even though breakdowns at high capacities rarely are measured. To identify a suitable mathematical function and its parameter values, the maximum likelihood estimate is applied. This method is common for the estimation of model parameters (*11,* p. 585). The maximum likelihood estimate (L) is described as

$$L = \prod_{i=1}^{n} f_C(v_i)^{\delta_i} * [1 - F_C(v_i)]^{1-\delta_i} \quad (8)$$

where

n = total number of objects,
$f_C(v_i)$ = probability density function for the capacity,
δ_i = logical value (1 in case of breakdown, 0 otherwise), and
$F_C(v_i)$ = distribution function for capacity.

Brilon et al. prove that the Weibull distribution is the most suitable continuous function for describing the stochastic capacity of freeway sections (*7*). The Weibull distribution has the following probability density (Equation 9) and distribution (Equation 10) functions:

$$f(x) = \frac{a}{b^a} * x^{a-1} * e^{-\left(\frac{x}{b}\right)^a} \quad (9)$$

$$F(x) = 1 - e^{-\left(\frac{x}{b}\right)^a} \quad (10)$$

where a is a form parameter and b is a scale parameter.

With Equations 9 and 10 inserted in Equation 8, the likelihood function becomes

$$L = \prod_{i=1}^{n} \left(a * b^{-a} * v_i^{a-1} * e^{-\left(\frac{v_i}{b}\right)^a} \right)^{\delta_i} * \left(e^{-\left(\frac{v_i}{b}\right)^a} \right)^{1-\delta_i} \quad (11)$$

In an empirical study, the optimal values for parameters a and b are obtained when the likelihood function reaches its maximum. Therefore, observed values of v_i and δ_i are needed as input to Equation 11. With the exact form of the Weibull distribution, any value of capacity can be chosen for a measurement point depending on the breakdown probability.

A suitable value to characterize the distribution function is the nominal capacity (C_{EV}), also known as the expected value of the Weibull distribution [$E(x)$]. The nominal capacity can be used to compare the distribution function with different fixed values for capacity (e.g., from the fundamental diagram) and has the following form:

$$E(x) = b * \Gamma * \left(1 + \frac{1}{a}\right) \quad (12)$$

where $\Gamma(x)$ is the gamma function at point x.

In this analysis, an observation period must be chosen and traffic breakdown defined. Short 1- or 5-min observation periods are reasonable to detect a breakdown between intervals. Zurlinden describes 5 min as a sensible, practical flow interval (*15*). In general, a breakdown is defined as the transition between stable and unstable traffic conditions. Traffic breakdowns caused by downstream

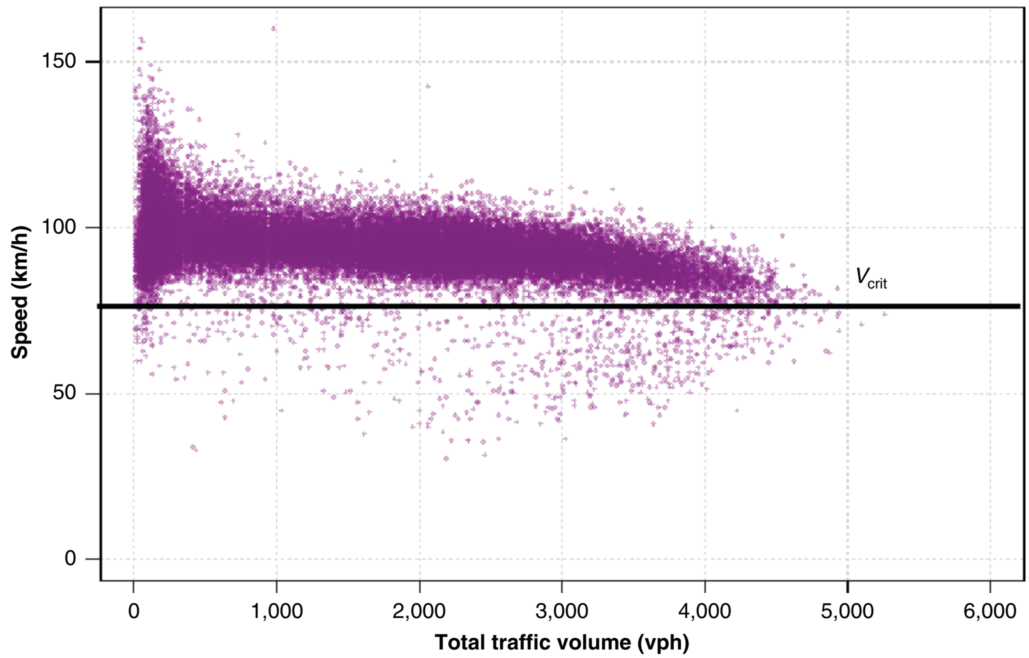

FIGURE 2 Critical speed in a speed–flow diagram separates stable and unstable traffic conditions for 6 months of measurements (January 1 to June 27, 2012) at 5-min intervals in a two-lane freeway section (MQ_A04_2_003.200). (vph = vehicles per hour.)

congestion are not considered. The most important parameter for separation in free-flow and congested traffic is critical speed. The critical speed ranges between 70 and 80 km/h (43.5 and 49.7 mph) depending on the measurement point and its speed–flow curve. Figure 2 is an example for one measurement point.

Two additional conditions are required to identify a breakdown that causes deceleration. First, a minimum speed difference between intervals before and at breakdown is necessary to consider the speed decrease caused by traffic breakdown. Furthermore, a minimum traffic flow should be defined to avoid the false detection of nighttime breakdowns that are caused by a high share of slow heavy-duty vehicles rather than high traffic volumes. Therefore, a speed difference of 10 km/h and a minimum traffic flow of 600 vehicles per lane per hour were set as additional conditions to identify breakdowns in the present empirical study.

PLM for VDFs

In travel demand models, highway traffic assignment requires a VDF, suitable parameters, and values for free-flow speed and link capacity. Scientific discussion continues on the appropriate VDFs discussed earlier. In practical travel demand models worldwide, the BPR function (Equation 1) probably is the most widely used type of VDF. Even though other VDFs could be selected, the BPR function was applied in this research because the existing Vienna travel demand model was calibrated without questioning BPR as the most appropriate VDF. Klieman et al. achieve good results with the BPR function in their analyses of VDF estimation for different road categories (*16*). The BPR function has two coefficients: α and β.

In this research, two models are applied to estimate the BPR parameter settings. In Model 1, coefficient α is defined as fixed value ($\alpha = 0.8$) and coefficient β was estimated. In Model 2, coefficients α and β are estimated. The coefficients were estimated by nonlinear regression analysis with the least squares method. Finally, different values for coefficients α and β are obtained for each measurement point at three capacities: nominal capacity (C_{EV}) and capacity at breakdown probabilities of 20% (C_{20}) and 80% (C_{80}).

The PLM combined with a continuous distribution function (e.g., Weibull) generates link capacities for each breakdown probability. Even though the link capacities in practical travel demand models are taken from guidelines, a vast amount of data (especially measured along freeway sections) is available for use nowadays. The results of the analyses with PLM are capacity distributions for each measurement point. If the distribution function is known, the capacity value can be determined from its breakdown probability. This research used three capacities (C_{EV}, C_{20}, and C_{80}) to estimate the VDF coefficients.

RESULTS

The analyses are based on data generated by automatic traffic counters located along the A4 freeway in Austria. The recorded data include traffic flows and speeds for all vehicles in the first 6 months of 2012, aggregated to 5-min intervals. The method was applied to data from several detector locations. Four measurement points that differ by direction, section, traffic characteristics, speed limit, and number of lanes are studied (Figure 3). All locations are near the urban area of Vienna. The critical speed was defined individually from the speed–flow diagrams for each measurement point considered as well as the downstream measurement point. Data obtained between 8 p.m. and 6 a.m. were excluded from these analyses to avoid incorrect breakdowns caused by slow vehicles during intervals with low traffic volumes. In Austria, trucks have a general

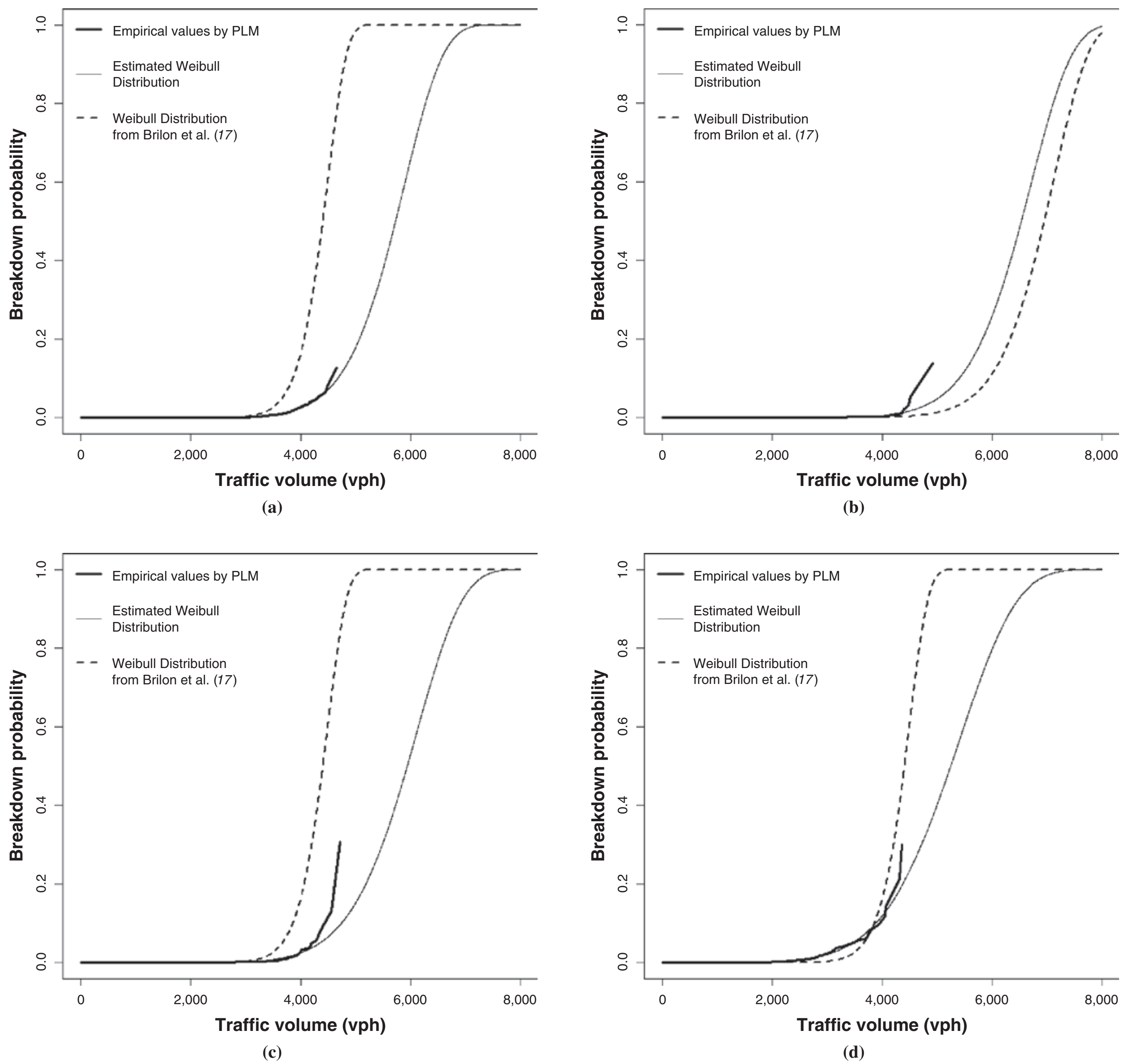

FIGURE 3 Capacity distributions for four measurement points using PLM and Weibull function (5-min intervals, January to June 2012, 06:00 to 20:00): (*a*) MQ_A04_1_000.875, (*b*) MQ_A04_1_011.597, (*c*) MQ_A04_2_003.200, and (*d*) MQ_A04_2_001.455.

speed limit of 80 km/h during the day and 60 km/h during night. Because the proportion of trucks is high in night traffic, nighttime data could distort the results. The critical speeds in Table 2 were identified from Figure 2.

Capacity Distribution

First, the capacity distributions for the four measurement points are determined with the use of the PLM and the Weibull function (Figure 3). Up to a traffic volume of about 4,200 vph, the Weibull function (green curve) approximates the PLM estimates (blue curve). The breakdown probability at 4,200 vph is less than 15% and differs across the measurement points. The PLM curve is steeper than the Weibull function for traffic volumes greater than 4,200 vph. Because fewer breakdowns are identified at traffic volumes greater than 4,200 vph, the curve of PLM estimations ends approximately at that point, where a breakdown probability of about 15% can be noted at the measurement points in the direction from Vienna (Figure 3, *a* and *b*) and about 30% at the measurement points in the direction to Vienna (Figure 3, *c* and *d*).

With the exact form of the capacity distribution (Weibull function) known, the capacity values of C_{20}, C_{EV}, and C_{80} are selected at each measurement point (Table 3). As expected, the highest capacities are measured at Measurement Point B, which is the only three-lane section. There, the capacity ranges from 4,381 to 5,829 vph for

TABLE 2 Characteristics of Observed Measurement Points on Freeway A4 Near Vienna

Measurement Point	Direction	Number of Lanes	Spatial Area	Speed Limit (km/h)	Critical Speed (km/h)
A (MQ_A04_1_000.875)	From Vienna	2	Urban, near Prater interchange	80	70
B (MQ_A04_1_011.597)	From Vienna	3	Suburban, homogenous section	130	80
C (MQ_A04_2_003.200)	To Vienna	2	Urban, homogenous section	100	75
D (MQ_A04_2_001.455)	To Vienna	2	Urban, near Prater interchange	100	75

C_{20}, 5,179 to 6,447 vph for C_{EV}, and 6,011 to 7,111 vph for C_{80}. The number of breakdowns differs between 15 (Measurement Point B with three lanes) and 96 (Table 3).

To compare the results of capacity distribution with a different but similar research the Weibull distribution of an analysis by Brilon et al. is incorporated in Figure 3 (*17*). In their analysis, Brilon et al. measure speed–flow data at 5-min intervals for a full year (2000) on the A1 freeway (two lanes) and the A3 freeway (three lanes) near the city of Cologne, Germany. They estimate the Weibull distribution for the two-lane (Figure 3, *a*, *c*, and *d*) and three-lane (Figure 3*b*) sections. The analysis of Brilon et al. provides slightly higher capacities in three lanes and lower capacities with less variation at the two-lane measurement points than the present research on Austrian motorways. Measurements for this study were taken on the A4 freeway, which has various speed limits (80, 100, and 130 km/h), whereas the A1 freeway in Cologne had no speed limit at that time. Differences in freeway speeds and a time lag of more than a decade may better explain the higher capacities of the two-lane section in Austria than that in Germany. An insufficient number of breakdowns in the two-lane sections at high capacities leads to high variation in the Weibull distribution results in the present study. On the three-lane section, the Weibull distribution matches capacity values and variations much better. The values in Austria are slightly lower in general, which may be a result of closely spaced entries and exits near the measurement point. This finding agrees with other research that identifies the lowest lane-specific capacities on three-lane (rather than two- or four-lane) sections because of low use of the rightmost lane.

TABLE 3 Estimated Capacities, Number of Breakdowns, and Weibull Parameters for the Four Measurement Points

Parameter	Measurement Point A	B	C	D
Estimated capacity (vph)				
C_{20}	5,073	5,829	5,192	4,381
C_{EV}	5,652	6,447	5,860	5,179
C_{80}	6,272	7,111	6,571	6,011
Number of breakdowns	96 (933)[a]	15 (834)[a]	59 (933)[a]	83 (933)[a]
Weibull parameter *a*	9.3 (14.1)[a]	9.9 (12.1)[a]	8.4 (14.1)[a]	6.2 (14.1)[a]
Weibull parameter *b*	5,960 (4,532)[a]	6,779 (7,170)[a]	6,208 (4,532)[a]	5,570 (4,532)[a]

[a]Values of Brilon et al. (*17*).

Volume Delay Functions

PLM-based capacities (C_{20}, C_{EV}, and C_{80}) are used for subsequent analyses to estimate VDFs. The BPR function was processed as a VDF (Equation 1), in which values for coefficients α and β had to be determined. Two approaches were tested, with a fixed α-value in Model 1 and a variable setting in Model 2:

Model 1. α is equal to 0.8, and β is estimated by nonlinear regression analysis.

Model 2. α and β are estimated by nonlinear regression analysis.

The free travel time t_0, defined as the ratio of the length of the freeway section to the free-flow speed, also is required for the BPR function (Equation 1); the link was normalized to 1 kilometer link length. According to the Austrian guidelines for road alignment, the free-flow speed was the 85% fractile of all detected speed values (v_{85}). The v_{85} parameter is essential for most road design issues in Austria (minimum curve radius, gradient, sag and crest radius, and so forth). The free-flow speed v_0 was determined for each measurement point and ranges from 81 km/h for Measurement Point A to 121 km/h for Measurement Point B (Table 4).

The regression analyses provide different values for coefficients α and β at each measurement point and for each capacity value (Table 4). Table 4 also includes the standard deviation (σ) and the *t*- and *p*-values for each estimated coefficient. If coefficient α is set to a fixed value (e.g., $\alpha = 0.8$ in Model 1), then the highest values for coefficient β (maximum $\beta = 6.83$) are observed at Measurement Point A, which has the lowest free-flow speed (81 km/h) and is located in the direction from Vienna near the Prater interchange. It leads to steep curves of the BPR function (Figure 4*a*). For Measurement Points B, C, and D, the value of coefficient β in the case of a fixed coefficient α (Model 1) ranges from 1.73 to 4.36, depending on the capacity value (C_{20}, C_{EV}, C_{80}). The standard deviations for β in Model 1 decrease with increasing breakdown probability and capacity. In contrast, the standard deviations for α in Model 2 increase with decreasing capacity.

Figure 4 presents the BPR functions of Model 1 plotted for each measurement point with the estimated coefficients of Table 4. Considering the curve for the capacity C_{20} (blue line), the doubled free-flow travel time $2 \cdot t_0$ can be observed at traffic volumes of about 4,800 to 5,600 vph for the two-lane sections (Figure 4, *a*, *c*, and *d*) and nearly 6,300 vph for the three-lane section (Figure 4*b*). It is important to note that the breakdown probability increases by decreasing

TABLE 4 BPR Coefficients α and β and Standard Deviation (σ) for Four Measurement Points, for Each Model and Capacity Value

	Measurement Point							
	A		B		C		D	
Capacity	α	β	α	β	α	β	α	β
Model 1								
C_{20}	0.8 (fix)	**6.83**	0.8 (fix)	**3.03**	0.8 (fix)	**4.36**	0.8 (fix)	**2.68**
σ	—	*0.03548*	—	*0.03372*	—	*0.02433*	—	*0.01191*
t-value	—	*192.6*	—	*89.9*	—	*179.3*	—	*224.8*
p-value	—	*<2e–16*	—	*<2e–16*	—	*<2e–16*	—	*<2e–16*
C_{EV}	0.8 (fix)	**4.63**	0.8 (fix)	**2.57**	0.8 (fix)	**3.29**	0.8 (fix)	**2.07**
σ	—	*0.02009*	—	*0.02728*	—	*0.01606*	—	*0.00769*
t-value	—	*230.3*	—	*94.2*	—	*205.1*	—	*268.9*
p-value	—	*<2e–16*	—	*<2e–16*	—	*<2e–16*	—	*<2e–16*
C_{80}	0.8 (fix)	**3.62**	0.8 (fix)	**2.25**	0.8 (fix)	**2.73**	0.8 (fix)	**1.73**
σ	—	*0.01459*	—	*0.02324*	—	*0.01247*	—	*0.00596*
t-value	—	*248.1*	—	*96.9*	—	*218.5*	—	*290.9*
p-value	—	*<2e–16*	—	*<2e–16*	—	*<2e–16*	—	*<2e–16*
Model 2								
C_{20}	**0.26**	**2.44**	**0.68**	**2.73**	**0.27**	**1.85**	**0.33**	**1.25**
σ	*0.00401*	*0.04346*	*0.03200*	*0.08380*	*0.00553*	*0.03620*	*0.00379*	*0.01554*
t-value	*63.7*	*56.1*	*21.1*	*32.5*	*49.2*	*51.1*	*87.9*	*80.6*
p-value	*<2e–16*	*<2e–16*	*<2e–16*	*<2e–16*	*<2 e–16*	*<2e–16*	*<2e–16*	*<2e–16*
C_{EV}	**0.33**	**2.44**	**0.89**	**2.73**	**0.34**	**1.85**	**0.41**	**1.25**
σ	*0.00664*	*0.04346*	*0.04912*	*0.08379*	*0.00829*	*0.03620*	*0.00562*	*0.01554*
t-value	*50.1*	*56.1*	*18.1*	*32.5*	*41.1*	*51.1*	*80.0*	*80.6*
p-value	*<2e–16*	*<2e–16*	*<2e–16*	*<2e–16*	*<2 e–16*	*<2e–16*	*<2e–16*	*<2e–16*
C_{80}	**0.43**	**2.44**	**1.16**	**2.73**	**0.42**	**1.85**	**0.49**	**1.25**
σ	*0.01038*	*0.04346*	*0.07325*	*0.08380*	*0.01189*	*0.03620*	*0.00784*	*0.01554*
t-value	*41.3*	*56.1*	*15.9*	*32.5*	*35.4*	*51.1*	*63.1*	*80.6*
p-value	*<2e–16*	*<2e–16*	*<2e–16*	*<2e–16*	*<2 e–16*	*<2e–16*	*<2e–16*	*<2e–16*

NOTE: v_0 (km/h): Measurement Point A = 81; Measurement Point B = 121; Measurement Point C = 95; Measurement Point D = 98. — = no value. Figures in boldface represent estimated coefficients (main results from regression analysis). Figures in italics represent statistical parameters (additional results from the regression analysis).

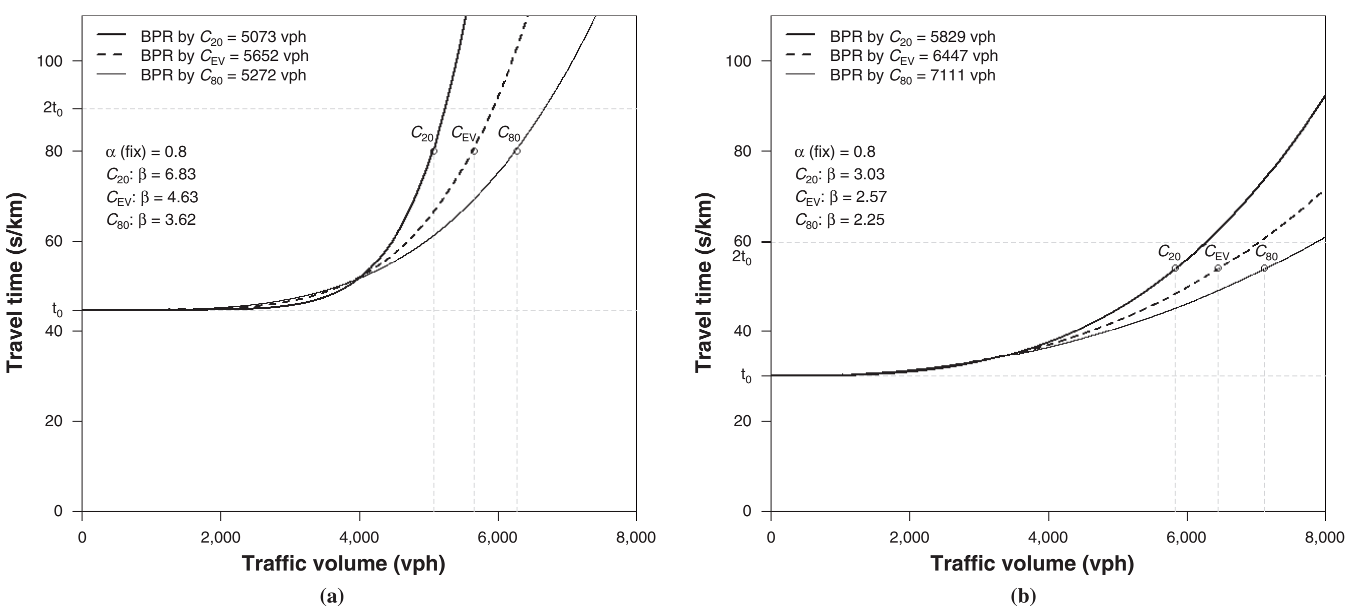

FIGURE 4 BPR volume delay functions with coefficient α = 0.8 (Model 1) as a result of regression analysis for different capacities at four measurement points on the A4 freeway near Vienna (5-min intervals, January to June 2012, 06:00 to 20:00): (*a*) MQ_A04_1_000.875 and (*b*) MQ_A04_1_011.597.

(continued on next page)

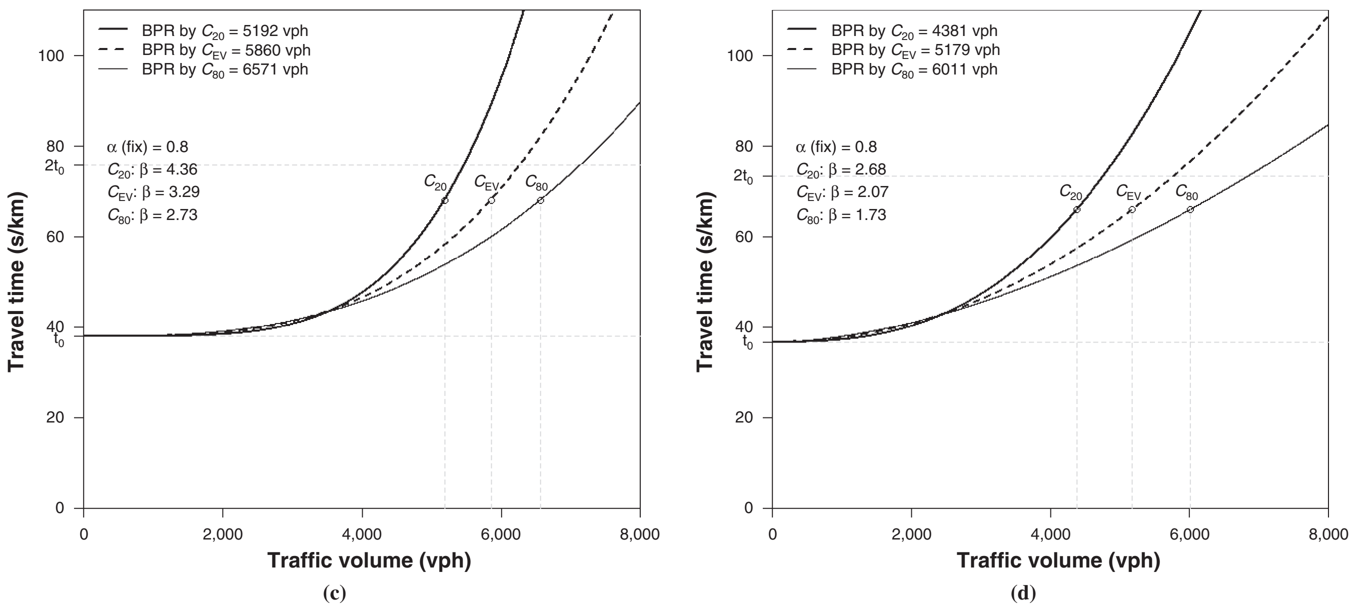

FIGURE 4 *(continued)* BPR volume delay functions with coefficient $\alpha = 0.8$ (Model 1) as a result of regression analysis for different capacities at four measurement points on the A4 freeway near Vienna (5-min intervals, January to June 2012, 06:00 to 20:00): (*c*) MQ_A04_2_003.200 and (*d*) MQ_A04_2_001.455.

the BPR coefficient β. A high value of β causes a steep gradient of the curve, which is demonstrated in Figure 4a with $\beta = 6.83$ for the capacity C_{20}. For each measurement point, the VDFs intersect at a point in the diagram. Traffic volumes below the intersection will lead to lower travel times with capacity C_{20}, rather than the nominal capacity C_{EV}.

The results of Model 2 (coefficients α and β estimated by regression analysis) are remarkable because of their equal values for coefficient β, despite different capacities. Estimation with nonlinear regression gives α values that are significantly lower than the fixed values in Model 1 for the urban Measurement Points A, C, and D.

Finally, the BPR functions of the four measurement points are compared in Figure 5 by using the capacity C_{20} for Model 1 (coefficient $\alpha = 0.8$; Figure 5*a*) and Model 2 (estimated coefficient α; Figure 5*b*). Except at Measurement Point B, the curves differ between Models 1 and 2. The BPR functions of Measurement Points A, C, and D have flat curves in Model 2 but steep curves in Model 1.

Discussion of Results

The method for the determination of capacity or capacity distribution used in this research (PLM with Weibull distribution) leads to higher capacity values than the static capacities stated in guidelines such as HCM 2010 and its German equivalent, *Handbuch für die Bemessung von Straßenverkehrsanlagen* (*18*). The nominal capacity C_{EV} of about 5,800 vph (5-min interval) is too high for a two-

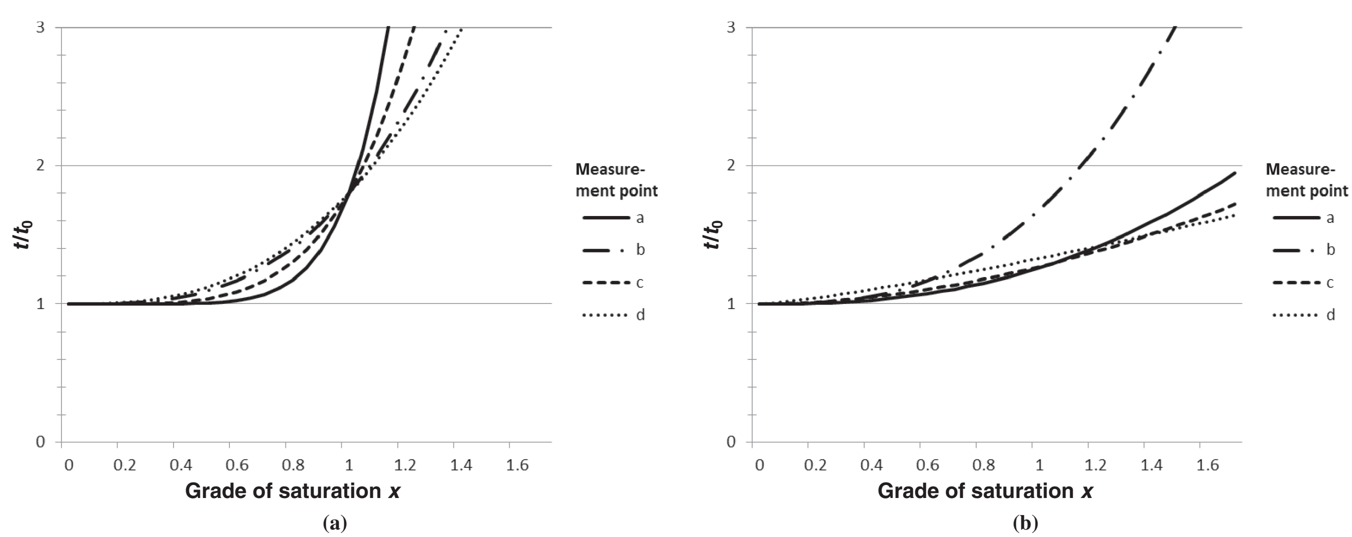

FIGURE 5 BPR functions with (*a*) a fixed coefficient $\alpha = 0.8$ (Model 1) and (*b*) a variable coefficient α using C_{20} (Model 2) for the Measurement Points A through D.

lane freeway section and does not fit with observed traffic volumes. In contrast are the calculated capacities at a breakdown probability of 20% (C_{20}), which deal approximately with estimated capacities from other research [e.g., base capacities for freeways in HCM (*4,* p. 10-6)]. However, to compare the results of capacities the different intervals for the applied traffic data must be considered. Capacities based on 5-min intervals leads to higher values than on 60-min intervals and to lower values than on 1 min intervals. This analysis based on 5-min intervals provides capacities of 4,400 to 5,200 vph for 2-lane, and 5,800 for three-lane freeway sections (Table 3).

Furthermore, the estimation with PLM without using a distribution function showed capacities of a breakdown probability of less than 30% for the four investigated measurement points (Figure 3) caused by a rather low number of traffic breakdowns in total in combination with a wide flow range of measured breakdowns. Generally, capacity estimation based on PLM works only with a sufficient number of traffic breakdowns, so saturated traffic conditions are required for this method. In this research, a minimum of 15 and a maximum of 96 intervals of breakdown were observed for the measurement points. Compared with the total number of intervals (about 30,000 intervals of 5 min for one-half of a year), these numbers of breakdowns are low but sufficient for the application of the PLM.

When the estimated capacities for VDFs are used, the regression analysis yields variable results for the coefficients of the BPR function depending on the adopted settings (Models 1 and 2). Model 2, with estimated coefficients α and β, provides feasible results for the suburban three-lane Measurement Point B. The remaining measurement points offer at least a degree of saturation of 1.6 at twice free-flow travel time in contraction with other research (Figure 5b). Generally, the results of Model 2 do not fit well with the analysis of Horowitz (Table 1) (*12*). In contrast the result of Model 1 with a fixed coefficient α match with the analyses of Horowitz (*12*). The estimated coefficients in Model 2 are higher for coefficient α, and lower for coefficient β in comparison with the default values for the coefficients α (0.15) and β (4.00) or the results of Huntsinger and Rouphail ($\alpha = 0.17$, $\beta = 4.50$) (*10*). However, this method was applied to a different data set. When comparing guidelines on freeway capacity in Germany and Austria with the U.S.-based HCM, the different form of speed over volume is obvious.

The model with a fixed coefficient α (Model 1) provides practical values for coefficient β (Table 4). The urban Measurement Point A, near the Prater interchange, has the lowest free-flow speed ($v_0 = 81$ km/h) and includes the steepest curve. Also in Model 1, the ratio of current to free travel time (t/t_0) is lower than that of the other three measurement points (Measurement Points B, C, and D) at a grade of saturation less than 1.0 and higher at a grade of saturation more than 1.0.

CONCLUSIONS

This paper applies stochastically distributed freeway capacities to define VDFs for transport planning purposes. In these analyses, the BPR function was applied, and two coefficients (α and β) and two input parameters (t_0 and c) were calibrated. Several β values are provided for the BPR function which coefficient α is fixed (Model 1) or estimated (Model 2). Various results for the coefficients are combined with the stochastic value of capacity. The parameter t_0 is link specific and defined by road type. Variations of the BPR coefficients and input parameters reveal differences in the identification of breakdown probability (Figure 4). Coefficients α and β are adjusted with the least squares method, and different capacity values (C_{20}, C_{EV}, and C_{80}) are calculated with the use of the PLM and the Weibull distribution. PLM in combination with the Weibull function provide a proper method for achieving a capacity distribution, provided that the local road link under consideration offers intervals of saturated traffic flow.

The regression analyses with a fixed coefficient α (Model 1) offer suitable curves for the BPR function; however, in the analysis with an estimated coefficient α (Model 2), plausible results occur only at the suburban three-lane Measurement Point B. The capacities, especially C_{EV} and C_{80}, are higher than in HCM and other national guidelines because data are from 5-min counts rather than hourly averages. Additional tests indicate that, according to the PLM, the capacity probability should be set not higher than 40% to define a traffic breakdown. C_{20} is a proper assumption for the capacity in VDF. The least squares method (to calibrate coefficients) and PLM (to define a stochastic capacity value) can be applied in real time. This approach that combines VDFs with stochastic capacity also can be applied to other types of VDFs, such as conical or Akcelik functions. With the method presented, important parameters of travel demand models such as free-flow speed and link capacity can be determined from measurement data, which is now more available than in the past. Therefore, this VDF can be used in travel demand models to estimate traffic state. The results of this research are unique to measurement points in Austria, but the method can be transferred to other countries where long-term data from freeway detectors are available.

ACKNOWLEDGMENTS

The authors thank the Austrian freeway operator ASFINAG for providing traffic data from the local measurement points. Julia Vogel contributed to this research by scripting a data analysis tool using R software for statistical computing.

REFERENCES

1. Sheffi, Y. *Urban Transportation Networks. Equilibrium Analysis with Mathematical Programming Methods.* Prentice-Hall, Englewood Cliffs, N.J., 1985.
2. Spiess, H. Conical Volume-Delay Functions. Technical Note. *Transportation Science,* Vol. 24, No. 2, 1990, pp. 153–158.
3. Akcelik, R. Travel Time Functions for Transport Planning Purposes: Davidson's Function, Its Time-Dependent Form and an Alternative Travel Time Function. *Australian Road Research,* Vol. 21, No. 3, 1991, pp. 49–59. (Minor revisions: December 2000)
4. *Highway Capacity Manual 2010.* Transportation Research Board of the National Academies, Washington, D.C., 2010.
5. Jia, A., X. Zhou, M. Li, N. M. Rouphail, and B. M. Williams. Incorporating Stochastic Road Capacity into Day-to-Day Traffic Simulation and Traveler Learning Framework: Model Development and Case Study. In *Transportation Research Record: Journal of the Transportation Research Board, No. 2254,* Transportation Research Board of the National Academies, Washington, D.C., 2011, pp. 112–121.
6. Zhou, X., N. M. Rouphail, and M. Li. Analytical Models for Quantifying Travel Time Variability Based on Stochastic Capacity and Demand Distributions. Presented at 90th Annual Meeting of the Transportation Research Board, Washington, D.C., 2011.
7. Brilon, W., J. Geistefeldt, and M. Regler. Reliability of Freeway Traffic Flow: A Stochastic Concept of Capacity. *Proc., 16th International*

Symposium on Transportation and Traffic Theory, College Park, Md., 2005, pp. 125–144.
8. Elefteriadou, L., and P. Lertworawanich. Defining, Measuring, and Estimating Freeway Capacity. Presented at 82nd Annual Meeting of the Transportation Research Board, Washington, D.C., 2003.
9. Lorenz, M. R., and L. Elefteriadou. Defining Freeway Capacity as Function of Breakdown Probability. In *Transportation Research Record: Journal of the Transportation Research Board, No. 1776,* TRB, National Research Council, Washington, D.C., 2001, pp. 43–51.
10. Huntsinger, L. F., and N. M. Rouphail. Bottleneck and Queuing Analysis: Calibrating Volume–Delay Functions of Travel Demand Models. In *Transportation Research Record: Journal of the Transportation Research Board, No. 2255,* Transportation Research Board of the National Academies, Washington, D.C., 2011, pp. 117–124.
11. Tan, P., M. Steinbach, and V. Kumar. *Introduction to Data Mining.* Pearson Education, Boston, Mass., 2006.
12. Horowitz, A. J. *Delay/Volume Relations for Travel Forecasting Based upon the 1985 Highway Capacity Manual.* Department of Civil Engineering and Mechanics, University of Wisconsin, Milwaukee, 1991.
13. Davidson, K. B. A Flow–Travel Time Relationship for Use in Transportation Planning. *Proc., 3rd ARRB Conference,* Vol. 3, No. 1, 1966, pp. 183–194.
14. Schwietering, C., and R. Steinauer. Method for Traffic Management Applications on Freeways Based on the Probability of Breakdown. *Proc., 2011 IEEE Forum on Integrated and Sustainable Transportation System,* Vienna, Austria, 2011, pp. 201–206.
15. Zurlinden, H. *Schriftenreihe/Lehrstuhl für Verkehrswesen, Ruhr-Universität Bochum, Vol. 26. Ganzjahresanalyse des Verkehrsflusses auf Straßen.* Lehrstuhl für Verkehrswesen, Ruhr-Universität Bochum, Germany, 2003.
16. Klieman, L., W. Zhang, V. L. Bernardin, Jr., and V. Livshits. Estimation and Comparison of Volume-Delay Functions for Arterials and Freeway HOV and General Purpose Lanes. Presented at 90th Annual Meeting of the Transportation Research Board, Washington, D.C., 2011.
17. Brilon, W., J. Geistefeldt, and H. Zurlinden. Implementing the Concept of Reliability for Highway Capacity Analysis. Presented at 86th Annual Meeting of the Transportation Research Board, Washington, D.C., 2007.
18. *Handbuch für die Bemessung von Straßenverkehrsanlagen.* Forschungs gesellschaft für Straßen- und Verkehrswesen, Cologne, Germany, 2009.

The Traffic Flow Theory and Characteristics Committee peer-reviewed this paper.