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Estimation of Road Capacity and Free Flow Speed for Urban Roads Under Adverse Weather Conditions

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***Abstract*— This work aims to estimate changes in traffic characteristics of urban roads in dependence of adverse weather conditions like rain and snow. Investigated traffic characteristics are capacity and free flow speed which are elementary for describing the performance of traffic networks and setting up macroscopic traffic models. The methods are based on aggregated flow and speed measurements from local sensors. Results show a significant reduction of road capacity and free flow speed in dependence of intensity and type of precipitation.**

I. INTRODUCTION

Influences of weather are observable in every day's life. In the area of motorized individual traffic this influence is twofold. A changed infrastructure (e.g. due to rain or snow) could lead to a modified driving behavior and on the other hand traffic demand is influenced. The latter is because weather is influencing attractiveness of transportation modes (e.g. in terms of safety or comfort) and locations (e.g. less outdoor trips due to rain). This work aims to depict influences on traffic behavior in dependence of adverse weather conditions like rain and snow without considering influences on traffic demand.

The investigations described in this paper are based on data of roadside sensors and have been accomplished in the context of a co-founded research project with the purpose of adapting parameters of a macroscopic traffic model (VISUM, developed by PTV AG) in dependence of the prevailing weather condition in order to achieve a better representation of the traffic within the simulation. For macroscopic traffic modeling two parameters are of crucial importance: Free flow speed (FFS) and capacity of a road link. FFS is the mean velocity chosen by drivers under free flow conditions i.e. when drivers do not influence each other. On the other hand the capacity is the maximal possible number of cars passing an intersection (or road link) within a certain time interval. If demand is higher than capacity, congestion with low vehicle speeds will occur. Theoretically the traffic flow cannot exceed the capacity. The macroscopic traffic model used in this work cannot be set up without knowing these two properties for each road link of a network. Moreover these two parameters represent characteristic points in the fundamental diagram (1).

Beside macroscopic traffic simulation the parameters can be used for traffic information systems (e.g. for travel time estimations) and network planning.

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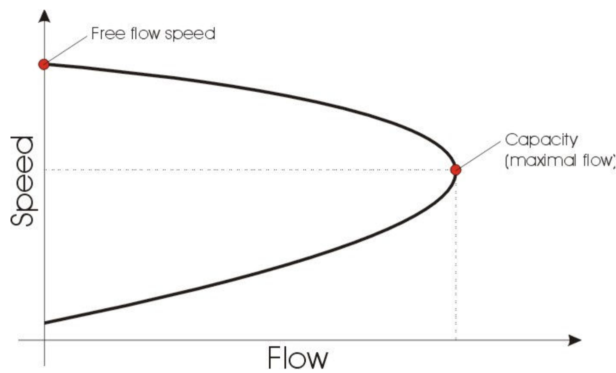

Fig. 1. Theoretical Fundamental Diagram with characteristic points representing Free Flow Speed and Capacity

II. STATE OF THE ART

Estimations of capacity of roads can be obtained from transportation guidelines like the Highway Capacity Manual (HCM) [1], where changes in traffic parameters due to adverse weather conditions are described. However the stated values are valid for highways only and require information about properties of the infrastructure (e.g. number and width of lanes). Moreover several authors compared their estimations with values proposed by the HCM and found discrepancies [15]. These discrepancies stem from differences in the customization of traffic participants to adverse weather conditions. Studies have shown that snowy road conditions in Alaska (USA) have less influence on the capacity than in Salt Lake City (USA) [19]. In the context of weather influences on traffic, the literature concentrates on the investigation of highways, where free flow speed has been analyzed in [9], [10], [11] and capacity in [3], [4], [6] and [12]. Others have investigated capacity reductions during adverse weather conditions as a result of changes in flow characteristics (e.g. slower speeds and changed maximal densities) [7], [8]. Research on capacity for urban roads is focused on the analysis of saturation flow, which is the number of passenger cars per hour of green time per lane [16], [17]. By combining the saturation flow with traffic signal settings, a maximum number of cars (i.e. a capacity) for a given road link can be stated. Estimation of saturation flow is challenging, because it can only be measured for oversaturated situations. E.g. the time period beginning from the green phase up to the moment where the queue, which formed during red phase, has dissolved can be called oversaturated. Thus the queue length in front of the intersection has to be observed constantly. Methods for capacity estimation based on fixed roadside sensors as used in this work, are described in [18]. One promising method, which has been applied in that work, has been further described and used in [13] and [14]. Again the focus of these works was on the investigation of highways. Moreover the method used in [18] has not been applied for different weather conditions.

Methods for estimating FFS reach from the definition of a level of service (LOS) to detailed analysis of variance of velocity [22]. In [21] free flow conditions are defined as situations where traffic flow is less than defined for LOS B according to HCM. This threshold is assumed to represent the highest possible traffic flow at which drivers are able to drive at their own chosen speed and are not restricted by other drivers. Another threshold for traffic flow is suggested in [23], where a traffic flow below 75 vehicles per hour and lane is assumed to represent traffic situations near to free flow conditions on highways.

Compared to existing studies in this work a method for estimating the distribution of capacity beside a mean (or median) value for urban roads has been performed. Furthermore the method for calculating free flow speeds is based not on a single threshold. Moreover data were filtered by several thresholds and weighted according to a predefined function. Both parameters have been analyzed under different adverse weather conditions.

III. DATA BASE

A. Traffic Data

The data used for the investigations were recorded from 35 fixed roadside sensors (loop sensors) on major roads (arterial and ring roads) in the approach of a signalized intersection in the urban area of Vienna (Austria). Speed limit is 50 km/h in the whole city Vienna, except urban motorways. Data were provided by the sensors in a one minute aggregation interval i.e. traffic flow was accumulated and vehicle speed was averaged. Both values have been captured for two vehicle types namely passenger cars and trucks. Single vehicle measurements were available from 10 additional locations. These sensors are able to determine point of time, speed and length of all vehicles passing the sensor location. However the aim of the project was to estimate a capacity and free flow speed based mainly on aggregated data. Single vehicle measurements have been used for establishing the method for FFS estimation based on the aggregated values.

The position of sensors is always approximately 250 meters before an intersection. Thus the spill back of vehicles at the intersection during the red phase should no be observable at the sensor location, unless traffic flow capacity has been reached.

B. Weather Data

Weather data were obtained from a weather sensor network in a time interval of 15 minutes. The weather situation in areas between sensors was interpolated by a weather model called ALADIN (Aire Limitee Adaptation dynamique Developpement International). It was not the purpose of this work to model weather data, but to use and rely on the data obtained from project partners.

From the beginning of 2006 up to mid 2009 both, weather and traffic data have been collected and were used for the investigations. To reduce the random variability of traffic flow the measurements have to be aggregated to a higher time interval. On the other hand the aggregation interval has to be as small as possible in order take into account transitions between traffic situation. Thus a trade off between random variability (for small intervals) and smoothing of different traffic situations (if the interval is to big) has to be found. Preliminary investigations resulted in a aggregation interval of 15 minutes. This is underlined by [23], where it is stated that traffic situations do not change significantly within 15 minutes. In [13] capacity has been estimated based on traffic data aggregated to 15 minute intervals. This time interval has been used in this work as well. To consider different types of vehicles the number of trucks was multiplied by a passenger car unit equivalent (PCU) before aggregation. Typically a value of 2 is used as PCU equivalent for trucks [1], meaning that a this type of vehicle is as much as two passenger car. The average velocity is calculated only based on passenger car velocities, ignoring speeds of trucks.

IV. ESTIMATION OF TRAFFIC PARAMETERS

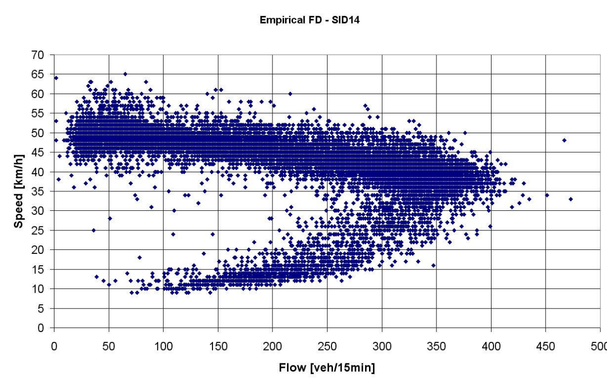

Fig. 2. Empirical fundamental diagram of a fixed traffic sensor

In figure 2 the speed and flow measurements of a fixed traffic sensor are visualized. It can be seen that free flow speed as well as capacity are not constant values, but vary within a certain distribution. The assumption is that a part of the variation can be explained by taking into account information about weather condition. But even if a part of the variations in the traffic parameters can be explained there will still remain a (hopefully only small) variation. This is because capacity and free flow speed are influenced by factors which are not measurable in this context (e.g. acceleration of cars or choice of safety distance due to mood of drivers). Thus the estimated parameters in this work are considered as stochastic variables and therefore will be described by a certain distribution.

A. *Estimation of Capacity*

For estimating capacity it is necessary that the prevailing capacity of a street has been reached several times during the observation period. Otherwise no conclusions can be drawn about the maximum possible number of vehicles passing the road link and the corresponding intersection. Moreover, if capacity of a road never has been reached, an analysis of it would be useless for traffic information, because it can be argued that congestion will never occur on the considered road link. Thus the first step was to determine situations where capacity has been reached. Therefore the time series of speed and traffic flow has been observed. As proposed in the literature ([13]), capacity has always been reached if measured speed falls below a certain threshold. I.e. if speed is low the "currently" measured traffic flow represents the capacity of a road link. The threshold for speed has been estimated by comparing with fundamental diagram (2), where maximal traffic flow is occurring at this speed value. Because in case of congestion, indicated by low speeds, the traffic flow reduces as well (as visible in the fundamental diagram, Figure 1), the traffic flow must not be taken from the same time interval. Instead the traffic flow of the previous time interval is interpreted as capacity value. It turned out, that if the maximum of the last two flow measurements before a speed drop is taken, a more robust estimation of capacity can be obtained (3). This is only valid for the aggregation interval used in this work (15 minutes). For greater intervals it may not be valid to take a traffic flow value as capacity long before the speed drops.

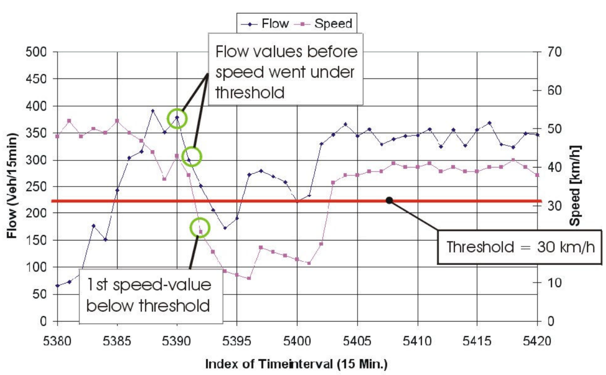

Fig. 3. Schematic representation of method for selecting capacity observations

By applying this method, all traffic flow observations were tagged whether the measurement represents a capacity observation or not. For situations with untagged traffic flow values the real capacity is unknown. It is only known, that real capacity is greater than the (untagged) traffic flow value. For estimating the distribution of capacity, the product limit method (PLM, also know as Kaplan Meier estimator) has been applied. This method was used in [13] for estimating capacity on freeways under different light conditions and is applied in this work for major urban roads under different weather conditions. The PLM is basically used for survival time analysis i.e. estimating a non-parametric survival function, where in our case a "death" is represented by a capacity observation (congestion occurred). Time on the other hand is replaced by traffic flow. In this manner the survival function gives an estimation about the probability of congestion for a given traffic flow. The main advantage of the method is to take into account "censored" data. These are observations where the real capacity is not observable. It is just known that capacity is above the current flow measurement because no congestion occurred (compare untagged traffic flow values). For applying the PLM all traffic flow observations have to be in ascending order. Some of them represent uncensored values (capacity observation detected by the method described above) and all others are censored values. Uncensored values are assigned with an additional index according to their occurrence in the series. E.g. The ascending series q_1, q_2, q_3, q_4, c_{51}, q_6, c_{72} contains seven traffic flow observations, where the 5^{th} and 7^{th} value is an uncensored observation denoted by c_{ij} (c for capacity). Censored observations are denoted by q_i. Thus i is the index in the series of all observations, whereas j is the (ascending) index just of uncensored observations The parameter free distribution of capacity is estimated by

$$S(c_{ij}) = \prod_{k=1}^{j} \frac{m_k - d_k}{m_k} \tag{1}$$

where m_k is the number of all observations (censored and uncensored) which are higher or equal than uncensored observation c_{ij}, d_k is the number of observations equal to c_{ij} and is usually one. A product is calculated of all probabilities for uncensored values, which represents the conjunction of probabilities for "surviving" all smaller and the current uncensored traffic flow measurement c_{ij}. $S(c_{ij})$ is the probability that real capacity is greater than the (uncensored) observation at index j. In figure 4 the probabilities estimated by Equation 1 are visualized (gray dots). The higher the traffic flow value, the lower the probability that capacity is higher than the corresponding traffic flow. Additionally the inverse $(1 - S(c_{ij}))$ of Equation 1 is presented in order to obtain a probability distribution (black curve). Thus this curve represents the probability that real capacity is lower or equal to the according traffic flow.

After obtaining a empirical probability distribution by

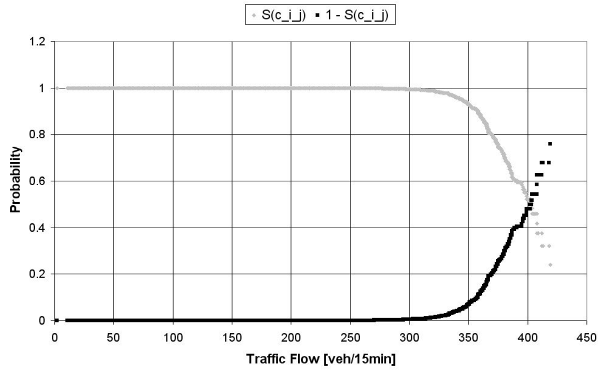

Fig. 4. Estimated survival function ($S(c_{ij})$) and probability distribution ($1 - S(c_{ij})$)

applying the PLM a theoretical distribution was fitted. This was necessary because as visible in figure 4 the distribution is truncated over a certain traffic flow. This is because the highest traffic flow observations occurred during the test period were censored values. In the literature a Weibull distribution defined by

$$F(x) = 1 - e^{(-\frac{x}{\beta})^{\alpha}} \quad (2)$$

is recommended to represent capacity distribution [13]. α and β represent the two parameters of the distribution also called "shape" (α) "and scale" (β) and x is in our case the uncensored traffic flow measurement. Fitting was performed, by varying the parameters of the Weibull function in order to minimize the sum of squared deviations between empirical and theoretical distribution. For minimizing the goal function of the problem

$$f(\alpha, \beta) = \sum_{j=1}^{N} ([1 - S(c_{ij})] - [1 - e^{(-\frac{c_{ij}}{\beta})^{\alpha}}])^2 \rightarrow min \quad (3)$$

a Newton-type algorithm was used [24]. Both, the estimation of the survival function by PLM and the fitting to the Weibull distribution has been performed for two types and several amounts of precipitation for each location. In table I the results for one location are presented. The unit for rain is mm/h, whereas for snow it is mm/2h measured as equivalent of water.

Probability distributions for capacity corresponding to the parameters of table I are presented in figure 5. As expected the distribution for dry weather situations (no rain or snow) represents highest capacity values. A reduction of the scale-parameter in dependence of the amount of precipitation is visible. On the other hand dependence between weather

TABLE I

ESTIMATED PARAMETERS OF WEIBULL DISTRIBUTION

Weather					Capacity	
Cat.	Prec.	Range $From(>)$	$To(\leq)$	Unit	**Alpha** [-]	**Beta** [veh/ 15min]
1	No Prec.	-1	0	-	18.44	402.2
2	Rain	0	0.25	mm/h	17.02	390.9
3	Rain	0.25	1	mm/h	19.69	382.1
4	Rain	1	$+\infty$	mm/h	14.67	373.1
5	Snow	0.2	0.8	mm/2h	20.58	380.8
6	Snow	0.8	$+\infty$	mm/2h	17.29	379.8

situation and shape-parameter is not significant compared to the scale of the distribution. In order to obtain a single value for the capacity a percentile value (e.g. median value) from the distribution can be derived.

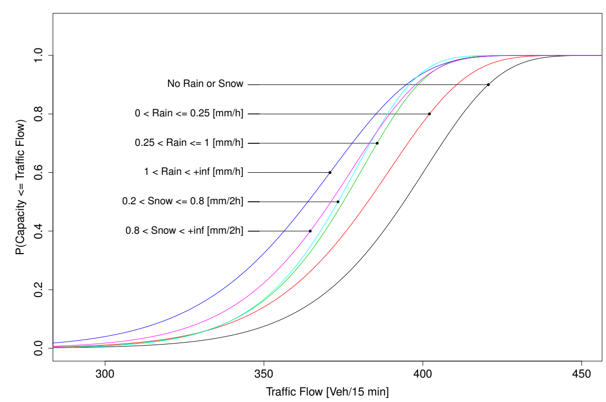

Fig. 5. Estimated distribution of capacity for different weather categories

V. ESTIMATING OF FREE FLOW SPEED

For estimating FFS the challenge is to determine free flow situations for road links where the sensors are located. The definition of free flow situation can be found in transportation guidelines for traffic engineering as:

> Free-flow speed on roads is defined as the average speed at which cars would move if drivers were not influenced by other vehicles.

In the end, the feeling of being influenced by other drivers or not is, within a certain range for traffic flow, subjective. Therefore it is necessary to define properties describing free flow conditions. The method for the definition of free flow conditions depends on whether traffic measurements are aggregated or not. For single vehicle measurements the time gaps between consecutive vehicles can be used. To detect uninfluenced cars a threshold for the gap has to be defined.

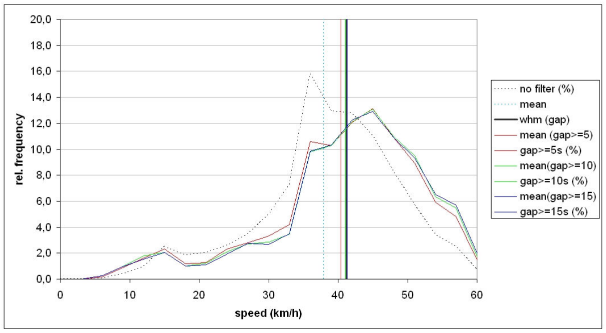

Fig. 6. Speed distribution of single vehicle measurements filtered by different time gaps (5s, 10s, 15s) with mean values and weighted harmonic mean (WHM)

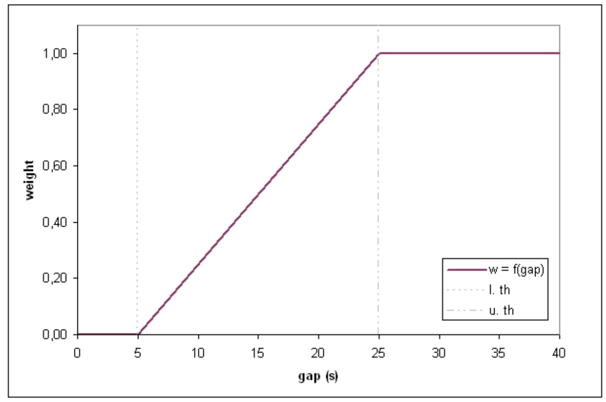

Fig. 7. Weight as a function of gap

Figure 6 represents the relation between the time gap used for filtering uninfluenced vehicles and the resulting speed distribution. By filtering measurements with very low gap values (less then five seconds) a displacement to higher velocities can be recognized. Increasing the threshold for the time gap beyond five seconds reduces the amount of data for estimating FFS radically but has less effects on the distribution of velocities. Free flow situations under adverse weather conditions would be rare if the threshold is too high. Thus the objective is not to select a static threshold, but to define a fuzzy transition between influenced and uninfluenced driving in order to maximize the number of samples. The knowledge about the influence of gap on estimating a free flow speed establishes a method where gaps are used as weights in the calculation of a weighted harmonic mean (WHM). In figure 6 it is visible, that the WHM is close to the mean value achieved by filtering with a high and constant ($\geq$ 10 seconds) threshold. But the advantage of a WHM is a higher number of samples and thus a more reliable estimation of FFS during adverse weather conditions.

$$v_{freeflow} = \frac{\sum_{i=1}^{n} f(gap_i)}{\sum_{i=1}^{n} \frac{f(gap_i)}{v_i}} \tag{4}$$

For calculating a weighted harmonic mean (4) out of measured speed values v_i the weights are defined as a function of the gap ($f(gap_i)$).

As represented in 7 the weight is defined as a function of time gap between two consecutive vehicles. With the constant slope between five and 25 seconds all measurements beginning from the lower value are considered. This approach enables to cover the area of subjective perception of drivers of being influenced or not. In our project only some of the sensors were able to measure single vehicle data. Because of the amount of data and communication costs a data aggregation is implemented already at sensor site. Referring to [23] an aggregation of traffic measurements to 15 min intervals was established. If data are aggregated, it is not possible to calculate time gaps between consecutive vehicles. Instead free flow traffic situations are estimated by considering traffic flow observations. In [23] this situation is supposed to be considered when the intensity of traffic is less than 200 cars/hour. However within one time interval only the mean velocity of all vehicles and the aggregated number of vehicles is known. Without knowledge about the distribution of vehicles within a time interval, free flow situations cannot be determined clearly. This is because during very low traffic the vehicles can occur as bulk, where only the leading car is not influenced. Derived from the method for FFS estimation based on single measurements, for aggregated data a weighted harmonic mean has been used as well. Again the weights applied in the calculation of the WHM are inferred from (aggregated) traffic flow measurements.

The detection of free flow situations is more accurate if time gaps were used, thus results from this method were used as benchmark to adjust the weights for estimating a WHM from aggregated traffic flow. The weights used for aggregated values in dependence of the ratio between flow and capacity are visualized in Figure 8.

Figure 9 shows the characteristics of free flow speed under different adverse weather conditions. Method ”gap” (blue bars) uses the original single vehicle measurements, method ”Flow” (red bars) was established for aggregated data at same detector location. The FFS estimated from aggregated values conforms to estimations obtained by the method ”gap”. Both methods exhibit a reduction of FFS in dependence of the intensity of precipitation. As expected FFS decreases within

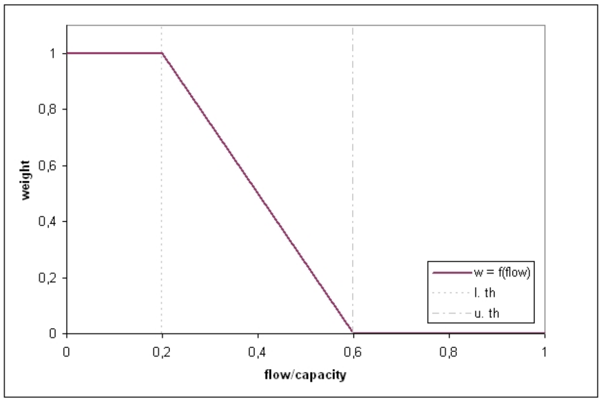

Fig. 8. Weight as a function of flow

the categories for the same type of precipitation (e.g. rain - categories 2,3,4 or snow - categories 5,6). The reduction due to heavy rain is higher than for light snow.

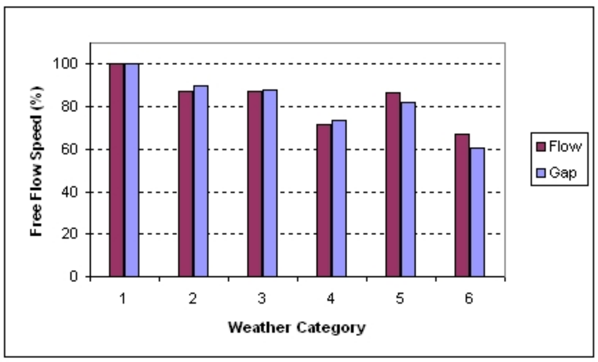

Fig. 9. Reduction of Free Flow Speed based on "Gap" and "Flow" for different weather categories. Values are related to a FFS of 42 km/h (100%)

VI. CONCLUSIONS AND FUTURE WORKS

Two methods for estimating capacity and free flow speed under adverse weather conditions on urban roads have been presented in this work. The product limit method has been applied for estimating the distribution of capacity while a WHM was used for estimating a FFS. Both methods are applicable on aggregated traffic measurements, if the aggregation interval does not exceed 15 minutes. The estimated capacity on urban roads is mainly dependent on the ratio between duration of a green phase and cycle length at the signalized intersection on the end of the link. Thus the method is only valid for constant signal phases. Moreover estimated capacity values are valid only for 15 Minute interval. At higher aggregation intervals capacity would be lower. In contrary it would be lower if aggregation interval is lower as well. Weights for the harmonic mean value results from a function with traffic flow as input value. The purpose of the function is to represent the fuzzy definition of free flow traffic. Both capacity and free flow speed are significantly reduced during adverse weather conditions. Moreover within the most adverse category for rain the reduction of capacity is higher compared to both snow categories. Results have been use inside a macroscopic traffic simulation (VISUM), where capacity and FFS of each links are required parameters.

Both methods have been applied on further sensor locations providing aggregated data. Reductions of capacity and FFS in dependence of type and amount of precipitation was qualitatively equal for each location, whereas the exact values were different and depended on the capacity and FFS under clear conditions. E.g. at locations with high FFS during clear weather conditions a stronger reduction during adverse weather conditions was observed. Thus a clustering of road links according to traffic parameters under clear condition would be useful in order to estimate changes for links, where no traffic information is available.

VII. ACKNOWLEDGEMENTS

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