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Estimation of time delay functions for design of traffic systems

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ABSTRACT

Main aim of this research was to apply multiple approaches for the development of time delay functions on three highways in Bahrain, namely; Dry Dock Highway, Arad Highway and Zallaq Highway. Four equations were obtained from previous studies and two equations were, additionally, tailored for each of the three highways. The results were used to obtain two parameters that aid in design, optimum flow rate and level of service.

1. Introduction

The main objective of this research was to create a Time Delay Function (TDF) that estimates the effects of congestion on travel time, and aids in the future planning and design of road networks [1]. Congested roads and traffic delays can be alleviated with proper planning, design and analysis.

This research is expected to benefit the society since it provides valuable records for flow along with data and analysis of congestion, on different locations across the Kingdom. It also supports the governmental bodies by providing accurate functions for estimation of delays, which is linked with reduction in productivity and increase in wasting valuable resources.

The use of TDFs can help focus on the roads that are congested most of the time and analyse the cause of this problem, whether it is caused by people, type of roads or other facilities. TDFs aid in the design of highways since they provide results that can determine travel patterns and travel time [2]. In addition, they can also help in improving the flow of traffic by determining what kind of traffic facilities are suggested and designed. The use of TDFs can also help with the understanding of how delay is evident in the system, its patterns, and the management of this delay. This results in improvement of the roads, in addition to future planning and design by predicting network flows.

2. Theoretical Background and literature review

A TDF consists of the relationship between volume and delay on a road [3]. They are vital for trip-assignment since congestion directly affects the choice of route for the road user. The volume is taken into consideration as a ratio with the capacity on that road and the delay is measured by using travel time. These models accurately conclude results by using the free flow speed, and capacity. In addition, the functions include calibration parameters, which are executed for each specific scenario to make sure the results are realistic and accurate [4]. Different functions of time-delay range between a simple linear function to a complex formula.

The volume to capacity ratio helps in measuring the level of congestion on roads. To get the ratio, the units of volume and capacity must be the same. The ranges of V/C ratio have been categorized into four groups:

1. V/C ratio $>$ 1: Severe congestion
2. V/C ratio 0.5 to 0.74: Moderate congestion
3. V/C ratio 0.75 to 1.0: Heavy congestion
4. V/C ratio $<$ 0.5: Low or no congestion [5].

Interrupted flow is common in areas where roadways overlap and therefore cannot permit all movements for the drivers. That could be due to external factors like a traffic signal, a stop/yield sign and much more. These external factors are referred to as accesses, and they can be either controlled (usually for high-speed traffic) or uncontrolled. Both accesses manage the traffic flow through vehicle-vehicle interaction and vehicle roadway interaction that either stops the traffic or slows it down remarkably [6].

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2.1. Existing time delay functions

The general formula of TDFs is travel time, as the function of speed, and vehicle volume, as shown in Eq. (1) [7].

$$t(v) = t_0 * f\left(\frac{v}{c}\right) \tag{1}$$

The BPR function, Eq. (2), is the first prevailing time-delay function. It was developed in the year 1964 in the United Stated of America. Despite its limitations, it is still used today when the expected demand is low i.e., the volume is low [8].

$$T_{cur} = T_0\left(1 + \alpha\left(\frac{V}{C}\right)^{\beta}\right) \tag{2}$$

The INRETS formula, developed in France by the Institut National de REcherche sur les Transports et leur Sécurité (National Institute for Transport and Safety Research), Eq. (3) is beneficial since it speeds up convergence [7].

$$T_{cur} = T_0\left(\frac{1.1 - \alpha * \left(\frac{V}{C*c}\right)}{1.1 - \left(\frac{C}{C*c}\right)}\right) \text{ for } V < C*c \tag{3}$$

$$T_{cur} = T_0\left(\frac{1.1 - \alpha}{0.1} * \left(\frac{V}{C*c}\right)^2\right) \text{ for } V > C*c$$

DAVIDSON, Eq. (4), is a function which is outlined by concepts of the queuing theory [9].

$$T_{cur} = T_0\left(1 + J\left(\frac{V}{C-V}\right)\right) \tag{4}$$

CONICAL, Eq. (5), was developed to conquer some of the complications of the BPR functions. Because these hyperbolical curves are the intersection of a two-dimensional planar surface and a three-dimensional cone, which is the reason for its name CONICAL [10].

$$T_{cur} = T_0\left(2 + \sqrt{\alpha^2\left(1 - \frac{V}{C}\right) + \beta^2}\right) - \alpha\left(1 - \frac{V}{C}\right) - \beta + V*\gamma, \text{ where } \alpha = 1.1,\ \beta = \frac{2\alpha - 1}{2\alpha - 2} \tag{5}$$

The equations consist of traffic characteristic and calibration values, as shown in Table 1.

Table 1
Equations parameters.

T_{cur}	Travel Time
T_0	Travel Time Free Flow
V	Volume at T_{cur}
C	Capacity
α, β, c, J γ	Calibration Parameters

3. Design and implementation

3.1. Methodology

The data employed for analysis was obtained from the Ministry of Works, Municipalities affairs, and Urban Planning (MUN). It was inspected thoroughly on three roads, namely, Arad Highway, Drydock Highway, and Zallaq Highway. These roads have similar characteristics, with respect to the geometric design and the traffic stream characteristics. Further field Data collection was performed to find the capacity and the flow was adjusted respectively.

The data which was attained from MUN covers 18-hour classified counts, speed data, and established values for the conversion factors from vehicles to passenger car unit (PCU). This count was taken on typical weekday in 2017. It was collected using Classified Automatic Traffic Counters (Metro Count/Pico Count). The data provided on the three roads was directional, classified count and speed for approximately 18-hours on each road, with 15-minute increments. Travel time was computed by dividing the speed data by the length of the segment on each road and converting it to seconds. the density was used as a dependent variable in the linear regression model. It can be found from dividing the adjusted flow by the speed. The density was found in pcu/km/ln.

Various data relationships were used to understand the data and ensure the appropriate implementation of data. The analysis on existing TDFs was done using MATLAB. The parameters were computed by calibrating the data, efficiency had to be checked and the curve fittings were created. Afterwards, two Linear Regression models were developed with the v/c ratio and density. The main criteria for the equations' validity are based on certain error calculations which include Root Mean Square Error (RMSE), Mean Absolute Error (MAE) and R^2. A sample of 20 % to 30 % of the data, from each highway, was set aside for error estimation. This sample was selected by selecting one 15-minute interval from each hour of the available study period, to avoid any biasness. The same sample was used for the validation of testing of all models developed in this study.

The analysis of the existing TDFs showed that the best results on the study locations were from the DAVIDSON equation. The DAVIDSON equation is linear, which is why the linear regression model was chosen for the available data. This choice ensured that the data would have a proper fit and low error values.

3.2. Existing TDFs

To begin with, MATLAB was used to find the parameters of the four main equations discussed previously. This was done as part of the calibration process for the present study. The purpose was to check the validity of the previous models on the dataset of the current study (Table 2).

For the BPR equation, alpha and beta were determined as 0.9289 and 0.8798, respectively for the combined data, with a very low error (MAE < 20 s). Afterwards, alpha and c had to be evaluated for the INRETS equation with values of 1 and 0.2219 for the combined road and again giving a very low error (MAE ~ 0). Moving on to Davidson, only one parameter had to be calibrated and that was J. It was found using specific equation and it was between the range of 0.038 and 0.1019. It was concluded that with calibration the combined data and Dry Dock's error was very high (MAE = 35 s for combined data and 38 s for Dry Dock highway), however TDFs used without calibration gave a reasonable error. Lastly, for the Conical equation, there was only one parameter to be found and that was gamma. After calibration, the average gamma was −0.1971. The errors were calculated, and the curve fitting was performed, and it was found that the error was too high (MAE = 100 – 120 s). Fig. 1-3 present a detailed comparison of different error terms calculated in this study. This shows that the conical equation is not fit for

Table 2
Existing TDFs calibration parameters.

Calibration		Highway			
Function		Dry Dock	Zallaq	Arad	Comb*
BPR	α	1.07	0.104	0.896	0.93
	β	0.97	−0.1	1.01	0.88
DAVIDSON	J	0.063	0.038	0.102	0.081
INRETS	α	2.05	5.75	4.78	1
	c	2.46	1.23	−0.64	0.22
CONICAL	γ	−0.38	−0.19	−0.067	−0.15

* Combined for all selected highways.

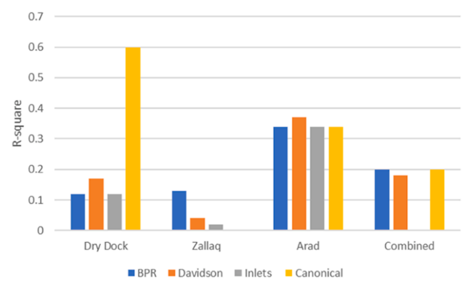

Fig. 1. R Squared Values for existing TDFs.

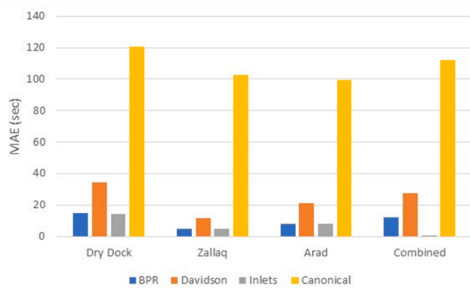

Fig. 2. MAE Values for existing TDFs.

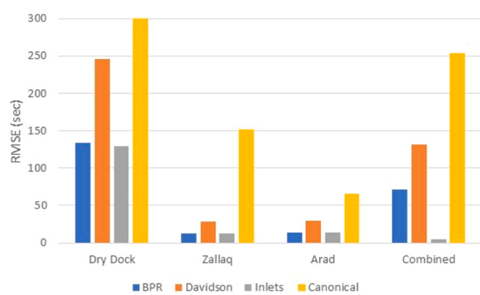

Fig. 3. RMSE Values for existing TDFs.

use on the available data.

All the functions had varying results in terms of fitting the data. Choosing the best function that suits the data for the roads here depends on its curve fitting and the errors. Davidson is the best choice, as it has the best curve fitting and the least MAE and RMSE with the highest R^2 among other functions. The errors were found the least for Davidson equation applied on Zallaq Highway, the errors were found to be; R^2 of 0.0216, MAE of 5.5717 and RMSE of 0.6536. Conical is the worse function because it has the highest errors, and it does not fit the data properly.

3.3. Linear regression model

The existing TDFs provided mediocre results, which made further action required. The development of the linear regression model allowed the customization of equations to fit the specific case presented in this research. Two equations were calibrated as the linear regression model, one with the v/c ratio as the independent variable and the other with density as the independent variable. With multiple trials, it was evident that calibration of the v/c ratio model to the data on each highway was the method that should be followed. Across the three highways and the different equations, this trial provided the best results, especially on Zallaq Highway. The sufficiency of this model was displayed in the curve fitting, high R^2 value, and low MAE and RSME. The errors were found to be 0.9156 for R^2, 8.4296 % MAE and 0.7728 RMSE (Table 3).

3.3.1. V/C Model

The overall results of the error (Figure) are reasonable and generally better than the existing equations. For Dry Dock highway, the error is the largest compared to the rest of the highways. The linear regression of v/c fits Zallaq highway the best, in terms of calibration. The error on this highway is the least amongst the other highways and even the combined data set. The combined data set has an error value that is reasonable, but slightly increased due to the values of Dry Dock Highway.

3.3.2. Density model

Unlike the error values from the v/c linear regression, the values for the density regression are higher, as shown in Figure .The errors are similar to those that were from the existing TDFs. The error values are not too bad for Dry Dock Highway, but the density model cannot be completely relied on for use, on this highway. On Arad Highway, the results convey the same conclusion as Dry Dock Highway. The results do not properly represent the data and it is not for use on Arad Highway. This is confirmed by the increased error. On Zallaq Highway, the error values are the worst among the three highways. The combined values show a fit that is not sufficient at all for use, in addition to the worst errors in this trial. Overall, the density model is better than the existing TDFs, in the calibration trial, but it is not ideal for use. Figs. 4–6 does show that the angular coefficient would be lower for the regression model. Building on this observation, there can be several possibilities for developing a customized model for the travel time, which is beyond the scope of the current study.

3.4. Application

Delay is a serious issue that must be addressed by the traffic engineer, since it is one of the aspects that is noticeable by the road users and it affects the efficiency of the entire system. The efficiency of the system starts off from the initial stages, or the design. Delay can be prevented by using specific methods in design. Multiple steps are used to forecast the demand in a model. The last step of this model is the traffic assignment [11]. Traffic assignment is used to estimate the routes the road-user will take to make a specific trip [12]. Since the decision to make a trip is a personal preference and cannot be exactly calculated, there are multiple methods of trip assignment. TDFs are considered one of the most important methods for the trip assignment model, under the capacity constrained approach [13]. Capacity constrained assignment takes the travel time and the demand into consideration, by ensuring that the road remains desirable in terms of the time taken to travel on it and the lack of congestion on it [14]. Due to its iterative nature, the concept of optimum flow rate is used for the capacity constrained assignment of TDFs. The iterations in capacity assignment models are repeated until the travel times converge to an equilibrium stage [15].

The use of TDFs for design is usually incorporated in Intelligent Transport Systems (ITS). ITS is used to automate the parameters used in the travel demand model by the automated improvement and development of TDFs, in addition to updating traffic and predicting travel time. The utilization of ITS on the capacity constrained assignment increases safety, convenience, and efficiency of the transportation system, starting from the design itself [16].

Table 3
Linear Regression Model Calibration Parameters.

Calibration		Roads			
Functions		Dry Dock	Zallaq	Arad	Comb
v/c	m	1.0762	0.9156	1.0046	1.0562
	b	1.0762	0.9156	1.0046	1.0562
Density	m	0.116	0.1812	0.0892	0.1149
	b	0.116	0.1812	0.0892	0.1149

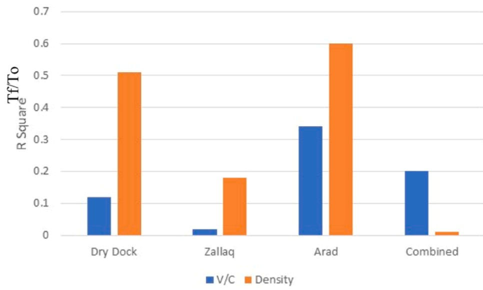

Fig. 4. R squared values for linear regression model.

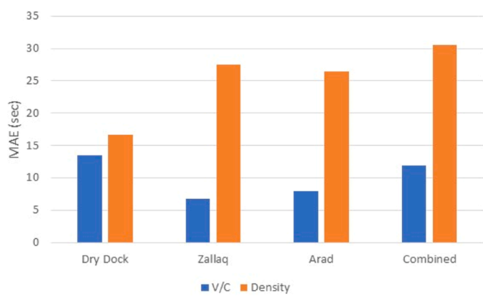

Fig. 5. MAE values for linear regression model.

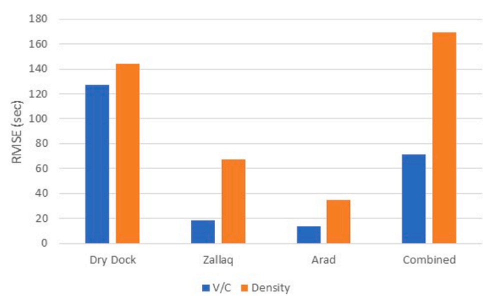

Fig. 6. RMSE values for linear regression model.

The calibration of TDFs provides ease for the engineer when it comes to data collection. Due to constraints such as time, funding, or labour, data collection of each parameter might be challenging. TDFs can aid in this issue since they provide a relationship between the different parameters. Therefore, the data collection might be recorded for one of the parameters and the rest can be found by solving the TDFs, without the need of extensive field data collection.

Level of service (LOS) denotes the operating conditions of a lane or a roadway under a given volume. This is based on different aspects regarding quality measures within a traffic stream such as speed, travel time, freedom to maneuver, comfort and safety. The level of service is represented with letter-grade scale A to F, A having the highest operating quality and F the worst [5]. The LOS was calculated for both the peak and the optimum conditions. For the peak condition, the maximum flow within the peak hour was used. This implementation of these concepts is shown in the proceeding sub-sections. The site conditions, used for the LOS calculations, are shown in Table 4. The PCU conversion factors are taken from MUN database and provided in Table 5.

3.4.1. *Optimum flow rate*

The first design application was to find and utilize the optimum flow rate, from the conducted study by using excel. Due to the nature of linear equations, the developed equations could not be used to find it, instead the DAVIDSON equation was used, since it yielded the best results from the existing time delay functions. The calibrated parameter, capacity, T_0, and V were plugged into the DAVIDSON equation to find T_f. The values for V were taken from 0 to the capacity of the highway at a constant interval of 50. Afterwards, the flow was plotted against the calculated T_f, as shown in Table 6. The optimum flow rate is at equilibrium; therefore, it is when the graph started to increase rapidly. After the optimum flowrate was found for each highway, its value was plugged into the (v/c) linear regression model to find the value of T_f at this point (Table 6). The (v/c) linear regression model yielded the best results in the entire study; therefore, it was used to make sure T_f was accurate. From this relationship, the optimum speed was computed for each highway, which will be later used to determine the LOS, in the next section.

Table 4
Highway characteristics.

	Dry Dock	Arad	Zallaq
Posted Speed (km/hr)	100	100	100
Number of Lanes	3	3	3
Lane Width (m)	3.7	3.7	3.6
Shoulder Width (m)	2.5	0	3
Median Width (m)	8.5	11.2	7.5
Segment Length (km)	2.38	0.468	0.913
PHF	0.91	0.96	0.87
% of Heavy vehicles	27	19	13

Table 5
PCU factors.

Vehicle Type	PCU Factor
Motorcycle	0.35
Private Cars	1
Taxis	1
Passenger Pickups	1.5
Private Pickups	1.5
Microbuses	1.5
Scheduled Buses	2
Non-Scheduled Buses	2
Freight Pickups	1.5
LGVs	1.75
Medium Trucks: HGVs 2–3 axles	2
Heavy Trucks: HGVs > 3 axles	2.5

3.4.2. *Level of service*

Determination of LOS is the next procedure of application, where the linear regression model (v/c) was once again used. The first step was to find the flow by using the travel time that was provided by the authorities. Afterwards, the maximum (v/c) was determined to find the LOS. The LOS was found using the traditional method of Highway Capacity Manual [17]. The peak hour on Dry Dock Highway is from 17:30 to 18:30, 7:00 to 8:00 on Arad Highway, and 13:00 to 14:00 on Zallaq Highway. Peak flow on Dry Dock Highway was 1332 pcu/hr/ln which yielded travel time of 207 s, it was 1900 pcu/hr/ln with 45 second travel time on Arad Highway, and 776 pcu/hr/ln with 57 s travel time on Zallaq Highway. These values were used to determine the peak (v/c) which is 0.59 on Dry Dock Highway, 0.80 on Arad Highway, and 0.38 on Zallaq Highway. According to level of service definitions, it could be determined that the LOS is A on Dry Dock Highway and Zallaq Highway. On the other hand, Arad Highway has LOS D. To confirm this, the values were checked and compared to the optimum flow rate (v/c), Table . The

Table 6
Optimum parameters.

Optimum	Dry Dock	Arad	Zallaq	
V	1800	1800	1700	pcu/hr/ln
v/c	0.875	0.827	0.754	
tf	234	85	39	Sec
Speed	37	20	84	km/hr

(v/c) ratios on Dry Dock Highway and Zallaq Highway, at optimum flow rates are 0.875 and 0.827, respectively, which is higher than the peak flow rate, confirming the accuracy of the deduced LOS, since the optimum flow rate provides a LOS D on Dry Dock and Arad Highway and C on Zallaq Highway. On the other hand, on Arad Highway, the optimum flow rate has v/c of 0.754 which is lower than the peak flow rate, and it is an indicator of need to apply travel demand management strategies that should be applied to achieve this and improve the LOS on the road [5] (Table 7).

Table 7
LOS comparison.

	Optimum LOS	Peak LOS
Dry Dock Highway	A	D
Arad Highway	D	D
Zallaq Highway	A	C

3.5. Comparison of models

Statistical analysis was used by employing two-way Analysis of Variance (ANOVA) for comparing different error terms for different highways and combined data. The analysis included the results from the calibrated as well as the regression models. Following null hypotheses were checked with this comparison.

"There is no significant difference between the error terms given by a model for different cases".

"There is no significant difference between the error terms given by different models for the same case".

The results of the ANOVA are summarized in Table 8. All hypotheses were checked at significance level of 5 %.

The ANOVA results show that the model parameters were generally similar in predicting the travel for different cases. Moreover, the comparison between the models also showed similarity in their predicting performance.

Table 8
ANOVA results.

Parameter	Comparison for cases		Comparison for models	
	F-value (*p*-value)	Result*	F-value (*p*-value)	Result*
R square	7.33 (0.00)	Rejected	1.35 (0.30)	Accepted
MAE (sec)	2.86 (0.07)	Accepted	165 (0.00)	Rejected
RMSE (sec)	1.79 (0.19)	Accepted	1.49 (2.90)	Accepted

* Null hypothesis acceptance.

4. Conclusions and recommendations

This research covers the analysis of existing TDFs and their development on specific study locations. TDFs are employed in capacity constrained assignment models. The existing TDFs had high errors, with the CONICAL equation being the worst one and the DAVIDSON equation with the least errors. It can be concluded that the cause of these errors was the nature of the facilities, the lack of detailed studies for some of the equations, and the complexity of the equation. Since the results of the existing TDFs were not sufficient, an additional step was administered with the development of two linear regression models, one using the v/c ratio and the other using the density. The DAVIDSON equation was a linear equation which was giving the best results from the existing model; therefore, the linear regression model was chosen. The developed models seem to provide much better results than the existing ones, with decreased errors and better curve fitting. The analysis shows that the v/c ratio model was better and more accurate than the density model. However, the statistical comparison between the calibrated models and the regression model showed that the difference in performance of different models and for different highways was not significant. The analysis was taken further by applying it to two design applications, the optimum flow rate, and the level of service.

The calibration of the developed functions on the study locations, might be beneficial on more roads, with different characteristics, in Bahrain. Future studies should include the analysis on different road segments, such as uninterrupted flow facilities and intersections, and on different movements of the traffic stream. Additionally, developing a non-linear model might be a step forward towards getting more accurate results in various locations and under different circumstances.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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