Definition : The **substitution** of an expression for a (free) variable in a lambda expression is denoted by $E[v \to E_1]$ and is defined as follows:

1. $v[v \to E_1] = E_1$ for any variable v.
2. $x[v \to E_1] = x$ for any variable $x \neq v$.
3. $c[v \to E_1] = c$ for any constant c.
4. $(E_{rator}E_{rand})[v \to E_1] = ((E_{rator}[v \to E_1])(E_{rand}[v \to E_1]))$
5. $(\lambda v.\, E)[v \to E_1] = (\lambda v.\, E)$
6. $(\lambda x.\, E)[v \to E_1] = \lambda x.\, (E[v \to E_1])$ when $x \neq v$ and $x \notin FV(E_1)$.
7. $(\lambda x.\, E)[v \to E_1] = \lambda z.\, (E[x \to z][v \to E_1])$ when $x \neq v$, $x \in FV(E_1)$, $z \neq v$, and $z \notin FV(E\, E_1)$.

In part 7, the first substitution $E[x \to z]$ replaces the bound variable x that will capture the free xs in E_1 by an entirely new bound variable z. Then the intended substitution can be performed safely.

Definition : **α-reduction**

If v and w are variables and E is a lambda expression,

$\lambda v.\, E \Rightarrow_\alpha \lambda w.\, E[v \to w]$

provided that w does not occur at all in E, which makes the substitution $E[v \to w]$ safe. The equivalence of expressions under α-reduction is what makes part 7 of the definition of substitution correct.

Definition : **β-reduction**

If v is a variable and E and E_1 are lambda expressions,

$(\lambda v.\, E)E_1 \Rightarrow_\beta E[v \to E_1]$

provided that the substitution $E[v \to w]$ is carried out according to the rules for a safe substitution.

Definition : **η-reduction**

If v is a variable, E is a lambda expression (denoting a function), and v has no free occurrence in E,

$\lambda v.(E\ v) \Rightarrow_\eta E$.