

“The worst ever conceived by a man of genius”
Hume’s probability argument in *A Treatise*

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The probability argument in Hume’s *A Treatise of Human Nature* (Section 1.4.1) has been widely criticized, with David Stove calling it “the worst [argument] ever conceived by a man of genius”. We explain that the argument is open to two interpretations: one that is in accordance with probability theory and one that is not. We surmise that Hume failed to distinguish between the two, and that this contributed to the confusion surrounding the argument.

KEYWORDS: diminution, Hume, infinity, iteration, probability, regression, *Treatise*

1. INTRODUCTION

In *A Treatise of Human Nature* David Hume presents an argument which purports to show that, if we rely purely on reason and ignore the sensitive part of our natures, then all our beliefs will be destroyed. In the literature, the argument has been given several names: Hume’s probability argument, the ‘probability reduces to nothing argument’, the ‘iterative probability argument’, ‘the reductio’, or the ‘regress argument’. The section in which the argument occurs (‘Of scepticism with regard to reason’, *Treatise* 1.4.1), has been deemed indispensable for grasping Hume’s theory of reason. In the words of William Morris:

If we ever are to understand Hume’s view of the role of reason,
... we should first figure out how to integrate ‘Of scepticism
with regard to reason’ into the picture (Morris, 1989, p. 58).

Yet the argument itself has been widely criticized. David Stove (1965, p. 174) went so far as calling it “the worst ever conceived by a man of genius”, while Robert Fogelin (1985, p. 16) and Mikael Karlsson (1990, p. 126) dubbed it simply “a morass”.

Recently the argument has attracted new attention through the work of Don Garrett, David Owen, Donald Ainslie and others, which has greatly improved and deepened our understanding of this “least understood” (Morris, 1989, p. 58) passage in Hume’s writings. Still there remain confusions. In this paper we surmise that they spring from Hume’s failure to distinguish between two readings of his argument, one that is in accordance with probability theory and one that is not.

In Section 2 we describe Hume’s argument in some detail. In Section 3 we discuss two different ways in which it has been analysed, a formal and an informal one, and we conclude that neither is satisfactory. In Section 4 we explain the precise sense in which the argument fails and in which it is correct.

2. THE PROBABILITY ARGUMENT IN *TREATISE* 1.4.1

Hume’s argument basically consists of three steps. The first encompasses the idea that all “knowledge degenerates into probability” (T 1.4.1.1).¹ By this Hume means that we can never know for sure whether a particular proposition is true. This applies not only to empirical propositions, but also to mathematical ones. In fact, Hume begins the section ‘Of scepticism with regard to reason’ with a reflection on the demonstrative sciences:

In all demonstrative sciences the rules are certain and infallible; but when we apply them, our fallible and uncertain faculties are very apt to depart from them, and fall into error. We must, therefore, in every reasoning form a new judgment, as a check or controul on our first judgment or belief; ... this means all knowledge degenerates into probability; and this probability is greater or less, according to our experience of the veracity or deceitfulness of our understanding, and according to the simplicity or intricacy of the question (T 1.4.1.1).

¹ *A Treatise of Human Nature*. References to this work are given by “T”, followed by four numbers, which indicate Book, Part, Section, and paragraph as in the volume edited by D.F. Norton and M.J. Norton, Oxford University Press, 2006 (first published 2000).

An example may help to understand this first step in the argument. Imagine that we have just performed by hand the addition of the first thousand natural numbers, and that we concluded:

A: The sum *S* is equal to 500500,

where *S* equals $1+2+3+ \dots +1000$. Hume's point is that we can never know that *A* is correct – there is always the possibility that we made a mistake. Of course, we can ask colleagues to do the addition, and if they arrive at the same result this will raise our confidence that *A* is true, but the salient point is that we can never be sure. The best we can do is to say that *A* is *probably* true. So our belief in *A* is gradual, and it leads to a belief in a new proposition, *B*:

B: *A* is probably true.

The second step is to apply this reasoning to *B* itself. For we cannot be certain that *B* is true either; the best we can say is that it is probably true, which leads to proposition *C*:

C: *B* is probably true.

And so on. A more quantitative version of the second step yields:

B: $P(A) = x$

C: $P(B) = y$

D: $P(C) = z$

and so on, where *x*, *y*, *z* are values between 1 and 0. Thus the second step gives the argument the form of a regress of higher and higher order subjective probability judgements:

$P(P(P(A)=x)=y)=z \dots$

According to Hume this regress is vicious, because it will inevitably lead to the conclusion that the subjective probability or credence for the first statement, $P(A)$, is zero:

at last there remain nothing of the original probability, however great we may suppose it to have been, and however small the diminution by every new uncertainty. No finite object can subsist under a decrease repeated in infinitum; and even the vastest quantity, which can enter human imagination, must in this manner be reduc'd to nothing (T 1.4.1.6).

Together the two steps entail “a total extinction of belief and evidence” (T 1.4.1.6). This may be a welcome conclusion for “those sceptics, who hold that all is uncertain, and that our judgment is not in *any* thing possest of *any* measures of truth and falsehood” (T 1.4.1.7), but Hume hastens to say that he is no part of “that fantastic sect” (T 1.4.1.8). Although he declares that one “can find no error” in the above steps (ibid.), he recalls that we *do* have beliefs, both in philosophy and in daily life.

Then he takes the third step, which is to say that the former two steps reveal what would happen if reason were left to its own devices: reason would simply annihilate itself and all our beliefs would “terminate in total suspence of judgment” (ibid.). He concludes that reason is “deriv’d from nothing but custom” and that belief is “more properly an act of the sensitive, than of the cogitative part of our natures” (ibid.). Hume’s argument is therefore a regress argument, but it is also a reductio. It shows that he regress leads to an absurdity (namely that we do not have any beliefs or any knowledge), and the way out is to realise that we should not, and in fact do not, rely on reason alone.

3. FORMALIST AND ANTI-FORMALIST APPROACHES

Among the many disagreements that Hume’s argument has provoked, there is the controversy about whether or not formal tools, especially taken from probability theory, can be used to understand and evaluate the argument. Some think they can, and we will call them the ‘formalists’. Others, the ‘anti-formalists’, are strongly opposed to using probability theory. In this section we explain their positions further.

Formalists tend to point out that Hume talks about subjective degrees of probability and strongly suggests that these degrees can be measured. Moreover, formal probability theory was very much in vogue during Hume’s lifetime: Jacob Bernoulli’s *Ars Conjectandi* had just been published, and Hume was a contemporary of Thomas Bayes, whose famous essay on probability was posthumously published by Richard Price, Bayes’s literary executor and a friend of Hume. Doubtlessly Hume realized that formal probability theory existed, and that some of his contemporaries were making significant contributions to it.²

² According to Bernard Peach, Richard Price convinced Hume that some part of his reasoning was inconclusive (Peach, 1980). Since the regress argument in 1.4.1 is among the arguments from the *Treatise* that are not repeated in Hume’s later writings, David Raynor suggested that Price, perhaps in early conversations, convinced Hume that this piece of reasoning is incorrect (Raynor, 1981). Price explicitly criticizes Hume’s regress argument in *A Review of the Principal Questions in Morals* of 1787 (albeit not very successfully: Price

Among the formalists there are some well-known names: C.S. Peirce (1905), G.H. Von Wright (1941), W.V. Quine (1946/2008), and R. Popkin (1951). They all explicitly or implicitly assume that Hume's reasoning in the first two steps of his argument involves a simple multiplication of probabilities. This can be explained as follows.

According to Hume, we cannot be certain that A is correct because we cannot fully trust our calculational capabilities: there is always the possibility that we made a mistake. Now suppose we trust our calculational abilities only to at least 75%. So we believe

$$B: P(A) \geq \frac{3}{4}.$$

However, we are not sure of B either. Suppose we trust it also to a degree of at least 75%, so we have C :

$$C: P(B) \geq \frac{3}{4}.$$

The same goes for C , and D , and so on. The formalists then appear to assume that Hume reconstructed the unconditional probability of A as:

$$P(A) = P(A|B)P(B). \quad (1)$$

Formula (1) is of course incorrect, and in Section 4 we will identify this error as the heart of what goes wrong in Hume's argument. Here we restrict ourselves to the observation that none of the formalists criticized Hume for having used the wrong formula (1). If they have criticized Hume at all, then it is because Hume apparently assumed that a product of factors smaller than one always yields zero (we will shortly return to this).

Similarly, formalists assume that Hume sees the probability of B as $P(B|C)P(C)$, and if we insert the latter formula into the right hand side of (1), we obtain a new formula for $P(A)$:

$$P(A) = P(A|B)P(B|C)P(C). \quad (2)$$

By the same procedure, insertion of $P(C|D)P(D)$ for $P(C)$ in (2) gives us an even longer formula for $P(A)$, namely

$$P(A) = P(A|B)P(B|C)P(C|D)P(D). \quad (3)$$

argues that doubting one's doubt of A will make one believe A *more* because the higher order doubt cancels the doubt of a lower order).

If we repeat this procedure infinitely many times, then we evidently will end up with an infinite chain:

$$P(A) = P(A|B)P(B|C)P(C|D)P(D|E) \dots \quad (4)$$

in which the right hand side contains only conditional probabilities. In each of the four formulas above, $P(A)$ is a product of factors all less than one. Hence the longer the formula is, the smaller $P(A)$ will be, and in the limit that the chain goes to infinity, $P(A)$ will converge to zero. Thus Richard Popkin concludes:

Since [the] probabilities are smaller than 1, the product is smaller than either of them. ... This process of introducing new probabilities ... can go on ad infinitum, and thus, the probability that we could ever recognize ... that a particular piece of reasoning was correct, approached to zero (Popkin, 1951, p. 390).

Popkin does not seem to find anything wrong with this reasoning, and many formalists appear to have come to the same conclusion. Some formalists, however, have criticized Hume on the grounds that a multiplication of numbers smaller than one need not yield zero. A necessary condition for this to occur is that the higher order probabilities approach ever closer to one. Thus Quine (1946/2008) pointed out that in very special cases the product of numbers smaller than one might be positive, and he reproached Hume for having failed to see this.

Quine's criticism is however beside the point. For it is clear that Hume is not talking about these special cases. Hume is addressing the situation where a continual diminution takes place: he talks about something that in the end becomes nothing at all. It is simply irrelevant to explain, as Quine does, that Hume's reasoning in very exceptional circumstances has a non-zero outcome.

Anti-formalists such as David Owen (1999, 2004, 2015) and Don Garrett (2000, 2004, 2006, 2015) vehemently deny that the probability calculus can help us to understand Hume's argument. In their view, a formal rendering is not only useless, but actively blocks an understanding of what Hume was after. It can be noted that nowadays practically all the scholars who have studied Hume's probability argument adopt a more or less anti-formalist approach.

Anti-formalists do seem to agree with the formalists that, according to the formal calculus, $P(A)$ is computed as a multiplication that converges to zero in the limit. David Owen even calls this a "mathematical truism" (Owen, 2015, p. 114). However, he maintains that Hume cannot have had this alleged mathematical truism in mind.

The main reason is that such a truism has very unhumane consequences. After all, if in the limit $P(A)$ is zero, then in the limit $P(\neg A)$ is one. This would mean that we have certainty after all, and this goes against everything Hume is trying to do in T 1.4.1. In the well-chosen word of David Owen:

The point of Hume's argument is 'the total extinction of belief and evidence' ... It is a sceptical argument, not the argument of a negative dogmatist (Owen, 2015, p. 114).

Anti-formalists conclude that the word 'probability' in Hume's argument is not 'probability' as explicated in the calculus. It rather means 'force', 'vivacity' or 'retention' – all notions that cannot be explained in terms of formal probability theory.

In the next section we explain why we think this conclusion is too quick. We will argue that the standard formalist reading of Hume's argument, as explained above, is not in accordance with the probability calculus (although it may be in accordance with what Hume had in mind). If we reconstruct Hume's argument in a way that *is* in agreement with the calculus, then it becomes clear that there is indeed something that goes to zero, although it is different from what Hume may have meant (*cf.* Atkinson & Peijnenburg *forthcoming*).

4. NOT A PRODUCT, BUT A SUM

In this section, we argue for two claims. The first is that both formalists and anti-formalists are mistaken when they assume that, according to the calculus, $P(A)$ goes to zero in the limit. Not only is it not a mathematical truism, as Owen maintained, it is simply false. The reason why both factions made the mistake is that, as we will explain, both saw $P(A)$ as a product, whereas it is a sum. The second claim is that in a correct formal rendering of Hume's argument something goes to zero, but it may be something other than what Hume had envisaged.

Let us start with the first claim. We have seen how the formalists reconstruct Hume's argument. If we have proposition A (in our example: 'The sum S is equal to 500500'), and we believe to at least 75% that A is true, then we have a new belief B : $P(A) \geq \frac{3}{4}$. Since we also trust B to a degree of at least 75%, we believe C : $P(B) \geq \frac{3}{4}$. And so on. As we have seen, formalists implicitly or explicitly assume:

$$P(A) = P(A|B)P(B). \quad (1)$$

But (1) is wrong. In determining the probability of A on the basis of B , we should also take into account what the probability of A is given that B is false. So rather than (1) we have

$$P(A) = P(A|B)P(B) + P(A|\neg B)P(\neg B). \quad (1')$$

which is not a product, but a sum. Of course, the same goes for $P(B)$, which is not given by $P(B|C)P(C)$ but by:

$$P(B|C)P(C) + P(B|\neg C)P(\neg C),$$

and similarly for $P(C)$, $P(D)$, et cetera. It is somewhat puzzling that none of the formalists have noticed this. Perhaps it is because in their lifetime the application of formal methods to philosophical problems was not as common as it is today (note that most of the formalists we mentioned wrote their works quite some time ago). Or perhaps the formalists were, like Quine, focussed on the fact that a multiplication of factors smaller than one may in exceptional circumstances yield a non-zero number, and consequently overlooked the fact that Hume made a much more fundamental mistake. Be that as it may, if we use the correct formulas for $P(A)$ and $P(B)$, then what we obtain as the new value for $P(A)$ is not, as the formalists thought,

$$P(A) = P(A|B)P(B|C)P(C), \quad (2)$$

which is a multiplication, but rather

$$P(A) = P(A|B)[P(B|C)P(C) + P(B|\neg C)P(\neg C)] + P(A|\neg B)[P(\neg B|C)P(C) + P(\neg B|\neg C)P(\neg C)] \quad (2')$$

which is a sum.

What happens if we repeat these transformations infinitely many times? The answer is: one still gets a sum rather than a product. Moreover, it can be proven that $P(A)$ converges to a unique and positive number, not zero. We will not stop to give the proof here, but readers who are interested can find it in (Atkinson & Peijnenburg, 2017).

This takes us to our second claim. If we iterate the correct formulas (1'), (2') et cetera infinitely many times, then it turns out that there is *something* that converges to zero. This is however not the probability of A , $P(A)$, but rather the influence exerted on $P(A)$ by the propositions in the chain. The further away a proposition is from A , the smaller is its contribution to $P(A)$, and in the limit this contribution vanishes completely. Again the proof can be found in (Atkinson & Peijnenburg, 2017). Here we restrict ourselves to giving an illustration.

Imagine again that we trust our calculational capabilities to at least 75% (nothing depends on the latter; our argument goes through with any exact or inexact number). Suppose further that our credence in A , given B , is 0.9:

$$P(A|B) = 0.9.$$

And let us suppose that the probability of A , given the falsity of B , is 0.5:

$$P(A|\neg B) = 0.5.$$

In the first instance we assume B to be true, so $P(B)=1$, and from (1') we find $P(A)=0.9$. Let us further assume that the numbers in the rest of the chain are the same (again, this assumption of uniformity is not essential):

$$\begin{aligned} P(B|C) &= 0.9 \text{ and } P(B|\neg C) = 0.5 \\ P(C|D) &= 0.9 \text{ and } P(C|\neg D) = 0.5, \end{aligned}$$

and so on. Now the first humean doubt assails us: we begin to doubt B after all, but we (provisionally) suppose at least C to be true, $P(C)=1$. This allows us to recalculate $P(B)$, which drops from 1 to 0.9; and as a consequence $P(A)$ drops from 0.9 to 0.86. Next we doubt C but believe D fully, and so on. This regress of doubting yields an infinite sequence of revisions of $P(A)$. A few steps are given in Table 1:

no. of propositions	1	2	3	5	10	∞
value of $P(A)$	0.9	0.86	0.84	0.83	0.833	5/6

Table 1. Decreasing higher-order probabilities of A

Two things attract our attention. First, in the limit the final value of $P(A)$ is not zero, but 5/6. Second, the further away a proposition is, the smaller is its contribution to that final value. $P(A)$ with only proposition B is 0.9, but when we also take C into account, then the probability is reduced to 0.86, which means that C contributes a (negative) correction is 0.04. With D , the value goes down still further to 0.844, so D contributes a correction of $0.86 - 0.844 = 0.016$. The combined effect of the sixth to the tenth orders, as can be seen from the table, produces a correction of less than two parts in a thousand.

In Table 1 the probabilities of A decrease, but they could actually increase. Whether they decrease or increase depends on the values of the conditional and unconditional probabilities. Suppose we set $P(A|B)$, $P(B|C)$, and so on, equal to 0.8, and $P(A|\neg B)$, $P(B|\neg C)$, and so on, equal to

0.3, while the values of the unconditional probabilities are 0.5, rather than 1 (the latter reflects the idea that we initially think we might be just as well right as wrong about *B*, *C*, *D*, et cetera). That leads to Table 2:

no. of propositions	1	2	3	5	10	∞
value of $P(A)$	0.5	0.57	0.58	0.59	0.599	$3/5$

Table 2: Increasing higher-order probabilities of *A*

As in Table 1, the value of $P(A)$ is a well-defined number, namely $3/5$. However, in Table 2 the probability goes up rather than down as the number of doubttings increases. Yet the contribution of the higher orders to the final value of the probability of *A* once again decreases. Further it can be proved that this final value, after an infinite number of doubttings, does not depend at all on whether we set the unconditional probabilities equal to a half or to one: it is a function solely of the conditional probabilities.

Both tables illustrate that the probability of the original belief not only fails to go to zero, but generally approaches a positive number that is unique and well-defined; and this is what usually happens (the only situation in which this does not happen is when the – nonuniform – conditional probabilities in the chain rapidly approach 1, that is, when they are close to material implications). Moreover, the tables show that there is something that invariably diminishes as the chain of doubttings increases, namely the effect of higher-order doubttings on the unconditional probability of *A*. The further away a proposition is from *A*, that is the more intermediate doubttings there are, the smaller is its influence on the final value of $P(A)$.

The tables reveal that the approach to the limit can be rather rapid. This should remove any feeling of uneasiness that one might have about drawing conclusions from reasoning that goes on forever. In line with Hume's claim that the diminution already occurs in a finite sequence of doubttings, the tables tell us that we do not need to go all the way to infinity in order to see the effect that we have been talking about: a few steps suffice to indicate that the significance of the higher orders diminishes as their number increases. This is because a few steps are enough to suggest that a regress of higher and higher-order probabilities converges to a non-zero value.³

³ Of course, we need a mathematical proof to demonstrate that what we are actually observing is a firm fact rather than a fluctuation. But this has been provided.

5. CONCLUSION

Anti-formalists have protested that the standard formalist reading of Hume's probability argument in *Treatise* 1.4.1 turns Hume into a negative dogmatist – and they are right. What they appear to have missed, however, is that the standard formalist reading implicitly accuses Hume of having made an elementary formal mistake. That reading is based on a faulty formula for $P(A)$ and wrongly presupposes that the higher and higher order doubts form a multiplication rather than a sum. It thus takes Hume as claiming that the credence or subjective probability in A decreases to zero as the chain of doubts lengthens, and such a claim violates the probability calculus.

In this paper we have investigated what happens if we rectify the formal mistake. If we reconstruct the chain of Humean doubts in a way that agrees with the probability calculus, then we discover that indeed something goes to zero. What decreases is however not the credence in A , nor is it the force or vigour of that credence. Rather it is the contribution to that credence of the successive doubts in the chain. The further away a doubt is from A , the smaller is its contribution, and in the limit the latter peters out completely.

There are thus two formal interpretations of Hume's probability argument, a valid and an invalid one. Which of these interpretations did Hume have in mind? In 'Of scepticism with regard to reason' Hume appears to go back and forth between them: most expressions point to the invalid interpretation, a few indicate that he was thinking of the valid one. We are therefore drawn to the conclusion that Hume failed to distinguish between the two. This conclusion appears to be supported by David Owen's analysis of Hume's argument. Owen, an outspoken anti-formalist, has paraphrased Hume's argument as follows:

As the number of intermediate ideas increases and the chain of reasoning becomes longer, the relationship between the ideas at each end of the chain of ideas becomes more indirect and the certainty of the conclusion is lessened (Owen, 2015, 120).

If Owen is right, then Hume failed to distinguish between a valid and an invalid version of his argument. For probability theory teaches us that the first part of Owen's sentence hits the mark, but the second part is false. It is indeed the case that, as the chain becomes longer, the relationship between the ideas at each end of the chain becomes less direct. It is however not so that the certainty of the conclusion is lessened. No matter how long the chain is, the conclusion can still be almost certain, and moreover be believed with great force and vivacity.

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