Contents


```
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% AEE 342 - HW6: Incompressible Flow over Airfoils (2)
% 02/25/15

clear all
close all
clc
```

Governing Equations

NACA2412 mean camber line given by:

$$\frac{z}{c} = 0.250\left[0.800\frac{x}{c} - \frac{x}{c}^2\right] \; for \; 0 \leq \frac{x}{c} \leq 0.40$$

$$\frac{z}{c} = 0.111\left[0.200 + 0.800\frac{x}{c} - \frac{x}{c}^2\right] \; for \; 0.40 \leq \frac{x}{c} \leq 1.00$$

And chord distribution chosen to be:

$$x = \frac{c}{2}(1 - \cos\theta_0)$$

Vorticity distribution given by:

$$\gamma(\theta) = 2V_\infty\left(A_0\frac{1+\cos\theta}{\sin\theta} + \sum_{n=1}^{\infty} A_n \sin n\theta\right)$$

With coefficients:

$$A_0 = \alpha - \frac{1}{\pi}\int_0^\pi \frac{dz}{dx}d\theta_0$$

$$A_n = \frac{2}{\pi}\int_0^\pi \frac{dz}{dx}\cos n\theta_0 d\theta_0$$

Definitions/Conversions

```
alpha = 4.5;                % Degrees
m     = 2;                  % Percent Chord
```

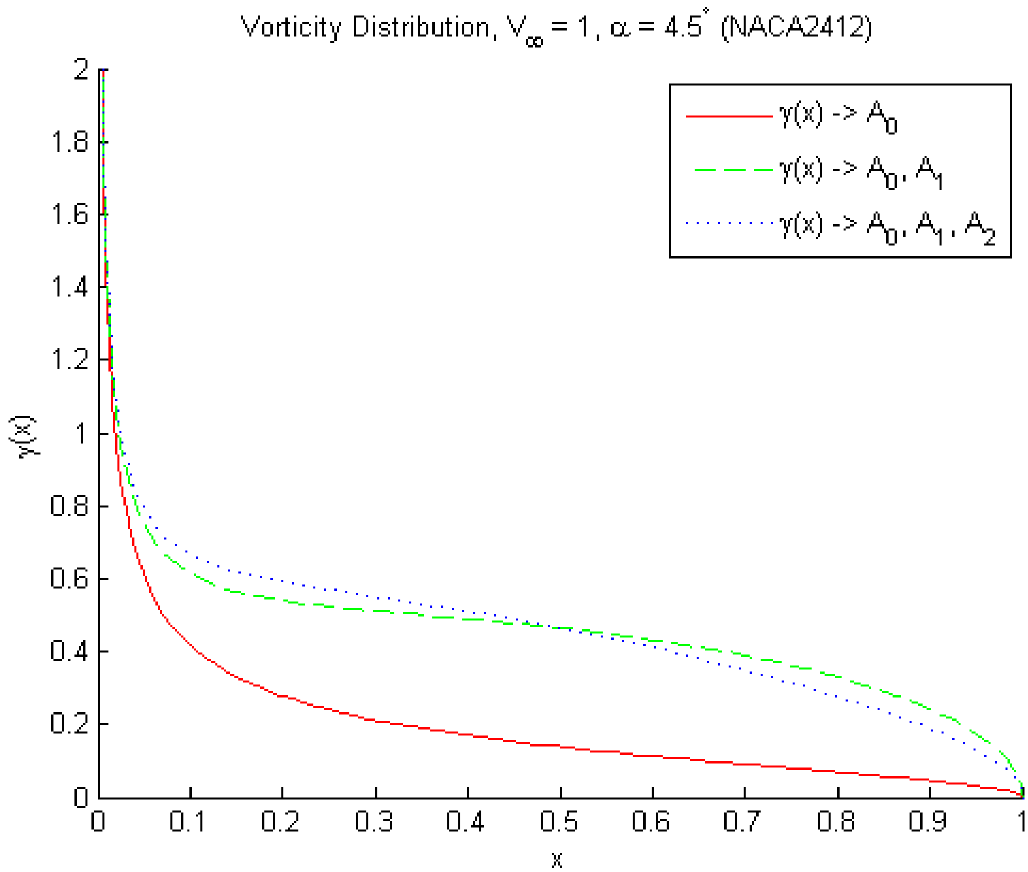
```
p     = 4;                    % Tenth Chord
tt    = 12;                   % Percent Chord

alpha = alpha * (pi / 180); % Radians
m     = m * 0.01;             % Fraction Chord
p     = p * 0.1;              % Fraction Chord
tt    = tt * 0.01;            % Fraction Chord
```

Calculations

```
theta    = linspace(0, pi, 100);
xCamber  = 0.5 * (1 - cos(theta));
indexP   = find(xCamber < p, 1, 'last');

dzdx1    = (1/5)  - (1/4) .* (1 - cos(theta(1          : indexP)));
dzdx2    = 0.0888 - 0.111 .* (1 - cos(theta(indexP + 1 : end)));
dzdx     = [dzdx1, dzdx2];
dzdxcos1 = dzdx .* cos(    theta);
dzdxcos2 = dzdx .* cos(2 * theta);

A0       = alpha - (1/pi) * trapz(theta, dzdx);
A1       =         (2/pi) * trapz(theta, dzdxcos1);
A2       =         (2/pi) * trapz(theta, dzdxcos2);

gamma0   = 2 .* (A0 .* ((1 + cos(theta)) ./ sin(theta)));
gamma1   = 2 .* (A0 .* ((1 + cos(theta)) ./ sin(theta))...
             + (A1 .* sin(theta)));
gamma2   = 2 .* (A0 .* ((1 + cos(theta)) ./ sin(theta))...
             + (A1 .* sin(theta)...
             + (A2 .* sin(2 .* theta))));
```

Plot

```
figure(1)
hold on
title('Vorticity Distribution, V_\infty = 1, \alpha = 4.5^\circ (NACA2412)')
xlabel('x')
ylabel('\gamma(x)')
axis([0 1 0 2])
plot(xCamber, gamma0, '-', 'color', [1 0 0])
plot(xCamber, gamma1, '--', 'color', [0 1 0])
plot(xCamber, gamma2, ':', 'color', [0 0 1])
legend('\gamma(x) -> A_0', '\gamma(x) -> A_0, A_1', '\gamma(x) -> A_0, A_1, A_2')
```



Results

The three solutions are similar in that they all have vertical asymptotes approaching infinity at x = 0 and all converge to zero at x = 1. This is consistent with theory because the Kutta Condition is satisfied at the trailing edge. The asymptote at the leading edge exists because of a singulariy at x = 0. The three solutions differ in their accuracy, since the solution using only A_0 gives the best fit using only sin/cos function. By introducing more leading-order terms, more periodic functions can be superimposed, allowing for a solution more capable of modelling the desired vorticity.

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