

AEE 343 Compressible Flow Materials covered in Exam 1

Textbook: Fundamentals of Aerodynamics, by J. D. Anderson, Jr., 5th edition.

Important: You are allowed to bring a 8.5" x 11" sheet with notes on both sides. Be sure to bring copies of Isentropic Flow and Normal Shock tables.

Basic compressible-flow concepts (chapter 7)

1. review of fluid dynamics and thermodynamics
2. definition of compressibility. When is the flow compressible?
3. definitions of total or stagnation conditions.
4. adiabatic flows and isentropic flows.
5. isentropic-flow relations (Appendix A) – use absolute temperature!

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2 \quad \frac{P_0}{p} = \left(1 + \frac{\gamma - 1}{2} M^2 \right)^{\frac{\gamma}{\gamma - 1}} \quad \frac{\rho_0}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2 \right)^{\frac{1}{\gamma - 1}}$$

Normal shock waves (chapter 8)

1. basic equations derived from 1D Euler equation (weak solution)
2. calculations of flow conditions across a normal shock (Appendix B)
3. Pitot tube calculations for incompressible flow, subsonic flow and supersonic flow. For $\gamma = 1.4$, highest possible reading for subsonic flow is $P_0/p = 1.893$ or $p/P_0 = 0.528$ (corresponding to $M = 1$). Results for supersonic flow tabulated in Appendix B.

Compressible flows in nozzles and diffusers (chapter 10)

1. what is quasi-1D flows?
2. what is the job of a nozzle, of a diffuser?
3. Area-Mach number relation $\frac{A}{A^*} = \sqrt{\frac{1}{M^2} \left[\frac{2}{\gamma + 1} \left(1 + \frac{\gamma - 1}{2} M^2 \right) \right]^{(\gamma + 1)/(\gamma - 1)}}$
4. Sonic conditions (set $M = 1$ in isentropic-flow relations).
5. Concept of choked flow: a choked duct (e.g. converging nozzle or converging-diverging nozzle) implies the Mach number is sonic at the throat, and the duct has reached its maximum mass flow rate capacity. The mass flow rate through a choked duct is a function (P_0, T_0, A_t, γ) of and can be shown to be

$$\dot{m} = \frac{P_0 A_t}{\sqrt{T_0}} \sqrt{\frac{\gamma}{R} \left(\frac{2}{\gamma + 1} \right)^{(\gamma + 1)/(\gamma - 1)}}$$

In the above expression, P_0 and T_0 are the stagnation pressure and stagnation temperature at the throat, respectively, and A_t denotes the throat area. Clearly, for given values of (A_t, γ) (e.g. fixed geometry and fluid), the choked mass flow rate depends on the local stagnation conditions (P_0, T_0). As a result, a choked nozzle implies $M_t = 1$ but does not imply that the mass flow rate is constant. Examples:

blow-down supersonic wind tunnel, some aircraft supersonic inlet diffusers operating at different flight Mach number and altitude.

6. converging/diverging nozzle flows – job of nozzle is to accelerate flow from subsonic to supersonic. What are the different flow regimes (condition 3, condition 5, and condition 6)? Use of the isentropic relations (or Appendix A) and the normal shock relations (or Appendix B) to calculate flows in a converging/diverging nozzle. For the situation where a normal shock resides in the diverging section, the problem is straight forward using Appendix B if the shock position is given and the exit pressure is to be calculated. For the situation where the exit pressure is given and the shock position is to be calculated, an iterative process is required (although a more straight forward technique is to use the method described in handout). Remember the following trend for a nozzle: increasing the back pressure moves the shock upstream (smaller A/A^*), and decreasing the back pressure moves the shock downstream (larger A/A^*). Note that for a nozzle, the condition at the throat is either subsonic or sonic.
7. Converging/diverging supersonic inlet – job here is to slow down a supersonic flow to subsonic. Note that for a supersonic inlet, a normal shock is not stable in the converging section and the condition at the throat can be supersonic or sonic.
8. design and operation of supersonic wind tunnel.