AEE343 Problem set #2

Problem #1

Determine the Mach number of a car moving in standard air at a speed of **(a)** 25 mph, **(b)** 55 mph, and **(c)** 100 mph.

The Mach number is the ratio of local velocity to speed of sound.

Thus

$$Ma = \frac{V}{c}$$

For standard air

$$c = \sqrt{RTk} = \sqrt{\left(1716 \frac{ft.lb}{slug.°R}\right)(519\ °R)(1.4)} = 1117 \frac{ft}{s}$$

or

$$c = \left(1117 \frac{ft}{s}\right) \frac{\left(3600 \frac{s}{hr}\right)}{\left(5280 \frac{ft}{mi}\right)} = 761.6 \text{ mph}$$

(a) For $V = 25$ mph

$$Ma = \frac{25 \text{ mph}}{761.6 \text{ mph}} = \underline{\underline{0.0328}}$$

(b) For $V = 55$ mph

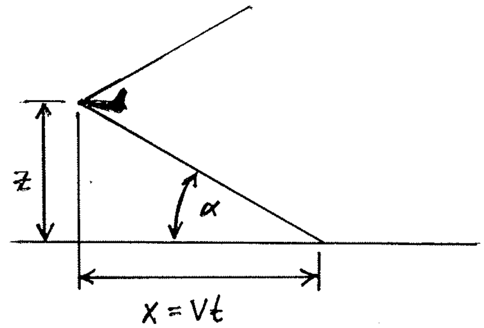
$$Ma = \frac{55 \text{ mph}}{761.6 \text{ mph}} = \underline{\underline{0.0722}}$$

(c) For $V = 100$ mph

$$Ma = \frac{100 \text{ mph}}{761.6 \text{ mph}} = \underline{\underline{0.131}}$$

Problem #2

At the seashore, you observe a high-speed aircraft moving overhead at an elevation of 10,000 ft. You hear the plane 8 s after it passes directly overhead. Using a nominal air temperature of 40 °F, estimate the Mach number and speed of the aircraft.



The Mach number is related to the angle α by Eq. 11.39. Thus

$$Ma = \frac{1}{\sin\alpha} = \frac{V}{c} \qquad (1)$$

Also

$$\tan\alpha = \frac{z}{Vt} \qquad (2)$$

Combining Eqs. 1 and 2 we obtain

$$\frac{\sin\alpha}{\cos\alpha} = \frac{z\,\sin\alpha}{c\;t}$$

or

$$\alpha = \cos^{-1}\left(\frac{c\,t}{z}\right)$$

Now

$$c = \sqrt{RTk} = \sqrt{\frac{\left(1716\,\frac{ft\cdot lb}{slug\cdot {}^\circ R}\right)(500\,{}^\circ R)(1.4)}{\left(1\,\frac{lb}{slug\cdot \frac{ft}{s^2}}\right)}} = 1096\,\frac{ft}{s}$$

Then

$$\alpha = \cos^{-1}\left[\frac{\left(1096\,\frac{ft}{s}\right)(8\,s)}{(10000\,ft)}\right] = 28.7^\circ$$

and

$$Ma = \frac{1}{\sin 28.7^\circ} = \underline{\underline{2.08}}$$

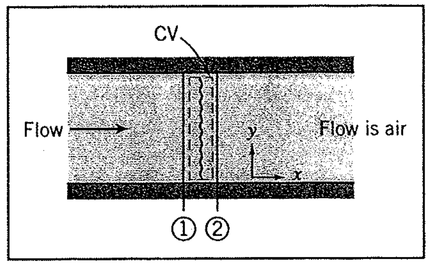
Further

$$V = (Ma)\,c = (2.08)\left(1096\,\frac{ft}{s}\right) = \underline{\underline{2280}}\,\frac{ft}{s}$$

Problem #3

GIVEN: Normal shock in a duct as shown:

$$T_1 = 5°\text{C}$$
$$p_1 = 65.0 \text{ kPa (abs)}$$
$$V_1 = 668 \text{ m/s}$$



FIND: (a) Properties at section ②.
(b) $s_2 - s_1$.
(c) Ts diagram.

SOLUTION:
First compute the remaining properties at section ①. For an ideal gas,

$$\rho_1 = \frac{p_1}{RT_1} = \frac{6.5\times10^4}{} \frac{\text{N}}{\text{m}^2} \times \frac{\text{kg}\cdot\text{K}}{287\,\text{N}\cdot\text{m}} \times \frac{1}{278\,\text{K}} = 0.815\,\text{kg/m}^3$$

$$c_1 = \sqrt{kRT_1} = \left[1.4 \times 287\,\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}} \times 278\,\text{K} \times \frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right]^{1/2} = 334 \text{ m/s}$$

Then

$$M_1 = \frac{V_1}{c_1} = \frac{668}{334} = 2.00, \text{ and (using isentropic stagnation relations, Eqs. 11.20b and 11.20a)}$$

$$T_{0_1} = T_1\left(1 + \frac{k-1}{2}M_1^2\right) = 278\,\text{K}\left[1 + 0.2(2.0)^2\right] = 500 \text{ K}$$

$$p_{0_1} = p_1\left(1 + \frac{k-1}{2}M_1^2\right)^{k/(k-1)} = 65.0\,\text{kPa}\left[1 + 0.2(2.0)^2\right]^{3.5} = 509 \text{ kPa (abs)}$$

From the normal-shock flow functions, Eqs. 12.41, at $M_1 = 2.0$,

M_1	M_2	p_{0_2}/p_{0_1}	T_2/T_1	p_2/p_1	V_2/V_1
2.00	0.5774	0.7209	1.687	4.500	0.3750

From these data

$$T_2 = 1.687T_1 = (1.687)278\text{ K} = 469\text{ K}$$ T_2

$$p_2 = 4.500p_1 = (4.500)65.0\text{ kPa} = 293\text{ kPa (abs)}$$ p_2

$$V_2 = 0.3750V_1 = (0.3750)668 \text{ m/s} = 251\text{ m/s}$$ V_2

For an ideal gas,

$$\rho_2 = \frac{p_2}{RT_2} = \frac{2.93 \times 10^5}{} \frac{\text{N}}{\text{m}^2} \times \frac{\text{kg} \cdot \text{K}}{287\ \text{N} \cdot \text{m}} \times \frac{1}{469\ \text{K}} = 2.18\ \text{kg/m}^3 \longleftarrow \underline{\rho_2}$$

Stagnation temperature is constant in adiabatic flow. Thus

$$T_{0_2} = T_{0_1} = 500\ \text{K} \longleftarrow \underline{T_{0_2}}$$

Using the property ratios for a normal shock, we obtain

$$p_{0_2} = p_{0_1} \frac{p_{0_2}}{p_{0_1}} = 509\ \text{kPa}\ (0.7209) = 367\ \text{kPa (abs)} \longleftarrow \underline{p_{0_2}}$$

For the change in entropy (Eq. 12.32g),

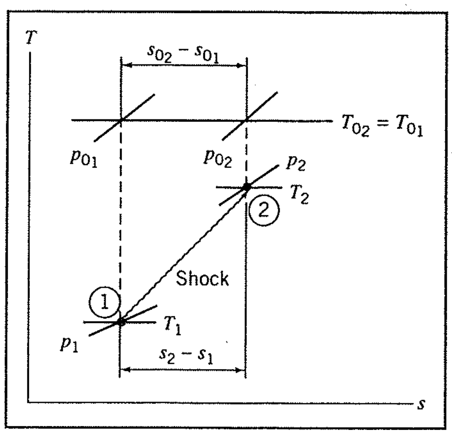
$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$$

But $s_{0_2} - s_{0_1} = s_2 - s_1$, so

$$s_{0_2} - s_{0_1} = s_2 - s_1 = c_p \ln \overbrace{\cancel{\frac{T_{0_2}}{T_{0_1}}}}^{=0} - R \ln \frac{p_{0_2}}{p_{0_1}} = -0.287 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \times \ln(0.7209)$$

$$s_2 - s_1 = 0.0939\ \text{kJ/(kg} \cdot \text{K)} \longleftarrow \underline{s_2 - s_1}$$

The Ts diagram is



This problem illustrates the use of the normal shock relations, Eqs. 12.41, for analyzing flow of an ideal gas through a normal shock.

The *Excel* workbook for this Example Problem is convenient for performing the calculations.

Problem #4

Just upstream of a normal shock in an ideal gas flow, Ma = 3.0, T = 600 °R, and p = 30 psia. Determine values of Ma, T_0, T, p_0, p, and V downstream of the shock if the gas is **(a)** air; **(b)** helium.

To determine Ma_y knowing Ma_x we use Eq. 11.149. Thus,

$$Ma_y = \sqrt{\frac{Ma_x^2 + \left(\frac{2}{k-1}\right)}{\left(\frac{2k}{k-1}\right) Ma_x^2 - 1}} \qquad (1)$$

or for air we use Table E.4 for Ma_y as a function of Ma_x.

To determine $T_{0,y}$ we use Eq. 11.56. Thus,

$$T_{0,y} = T_y \left[1 + \left(\frac{k-1}{2}\right) Ma_y^2 \right] \qquad (2)$$

or for air we use Table E.1 for $\frac{T_y}{T_{0,y}}$ as a function of Ma_y.

To obtain T_y we use Eq. 11.151. Thus,

$$T_y = T_x \left\{ \frac{\left[1 + \left(\frac{k-1}{2}\right) Ma_x^2\right]\left[2\left(\frac{k}{k-1}\right) Ma_x^2 - 1\right]}{\left[\frac{(k+1)^2}{2(k-1)}\right] Ma_x^2} \right\} \qquad (3)$$

or for air we use Table E.4 for $\frac{T_y}{T_x}$ as a function of Ma_x.

For $P_{0,y}$ we use Eq. 2 of Example 11.19 to get

$$P_{0,y} = P_x \left\{ \frac{\left[\left(\frac{k+1}{2}\right) Ma_x^2\right]^{\frac{k}{k-1}}}{\left[\left(\frac{2k}{k+1}\right) Ma_x^2 - \left(\frac{k-1}{k+1}\right)\right]^{\frac{1}{k-1}}} \right\} \qquad (4)$$

or for air we use Table E.4 for $\frac{P_{0,y}}{P_x}$ as a function of Ma_x.

For P_y we use Eq. 11.150 to obtain

$$P_y = P_x \left[\left(\frac{2k}{k+1}\right) Ma_x^2 - \left(\frac{k-1}{k+1}\right) \right] \qquad (5)$$

or for air we use Table E.4 for $\frac{P_y}{P_x}$ as a function of Ma_x.

For V_y we use

$$V_y = Ma_y \sqrt{R T_y k} \qquad (6)$$

(con't)

(con't)

(a) For air we read from Table E.4 for $Ma_x = 3.0$

$$Ma_y = \underline{\underline{0.47519}}$$

$$\frac{P_y}{P_x} = 10.333 \qquad (7)$$

$$\frac{T_y}{T_x} = 2.679 \qquad (8)$$

$$\frac{P_{0,y}}{P_x} = 12.061 \qquad (9)$$

and we obtain from Table E.1 for $Ma_y = 0.47519$ the closest values

$$Ma_y = 0.48$$

$$\frac{T_y}{T_{0,y}} = 0.95595 \qquad (10)$$

From Eq. 8 we get

$$T_y = (2.679)(600°R) = \underline{\underline{1610°R}}$$

and thus with Eq. 10

$$T_{0,y} = \frac{T_y}{0.95595} = \frac{1610°R}{0.95595} = \underline{\underline{1680°R}}$$

With Eq. 7 we obtain

$$P_y = (10.333)(30\,psia) = \underline{\underline{310}}\ psia$$

and Eq. 9 yields

$$P_{0,y} = (12.061)(30\,psia) = \underline{\underline{362}}\ psia$$

Then with Eq. 6 we obtain

$$V_y = (0.47519)\sqrt{\frac{\left(1716\ \frac{ft.lb}{slug.°R}\right)(1610°R)(1.4)}{\left(1\ \frac{lb}{slug.\frac{ft}{s^2}}\right)}} = \underline{\underline{935}}\ \frac{ft}{s}$$

(con't)

(con't)

(b) For helium we have with Eq. 1

$$Ma_y = \sqrt{\frac{(3.0)^2 + \left(\frac{2}{1.66-1}\right)}{\left[\frac{2(1.66)}{(1.66-1)}\right](3.0)^2 - 1}} = \underline{\underline{0.521}}$$

With Eq. 3 we obtain

$$T_y = (600\,°R)\left\{\frac{\left[1+\left(\frac{1.66-1}{2}\right)(3.0)^2\right]\left[2\left(\frac{1.66}{1.66-1}\right)(3.0)^2-1\right]}{\left[\frac{(1.66+1)^2}{2(1.66-1)}\right](3.0)^2}\right\} = \underline{\underline{2190\,°R}}$$

and with Eq. 2 we get

$$T_{0,y} = (2190\,°R)\left[1+\left(\frac{1.66-1}{2}\right)(0.521)^2\right] = \underline{\underline{2390\,°R}}$$

With Eq. 4 we have

$$p_{0,y} = (30\text{ psia})\frac{\left[\left(\frac{1.66+1}{2}\right)(3.0)^2\right]^{\left(\frac{1.66}{1.66-1}\right)}}{\left\{\left[\frac{2(1.66)}{1.66+1}\right](3.0)^2 - \left(\frac{1.66-1}{1.66+1}\right)\right\}^{\left(\frac{1}{1.66-1}\right)}} = \underline{\underline{409\text{ psia}}}$$

and with Eq. 5 we get

$$p_y = (30\text{ psia})\left[\frac{2(1.66)}{(1.66+1)}(3.0)^2 - \left(\frac{1.66-1}{1.66+1}\right)\right] = \underline{\underline{330\text{ psia}}}$$

With Eq. 6 we obtain

$$V_y = 0.521\sqrt{\frac{\left(1.242\times10^4\ \frac{ft\cdot lb}{slug\cdot °R}\right)(2190\,°R)(1.66)}{\left(1\ \frac{lb}{slug\cdot\frac{ft}{s^2}}\right)}} = \underline{\underline{3500\ \frac{ft}{s}}}$$

Problem #5

The Pitot tube on a supersonic aircraft cruising at an altitude of 30,000 ft senses a stagnation pressure of 12 psia. If the atmosphere is considered standard, determine the air speed and Mach number of the aircraft. A shock wave is present just upstream of the probe impact hole.

At 30,000 ft, we read from Table C.1 for standard atmosphere

$$T = -47.83°F = 412.2 \ °R$$

and

$$p = 4.373 \ psia$$

Thus,

$$\frac{p_{0,y}}{p_x} = \frac{12 \ psia}{4.373 \ psia} = 2.74$$

and with this value of $\frac{p_{0,y}}{p_x}$ we read for closest values in Table E.4

$$\frac{p_{0,y}}{p_x} = 2.7457$$

and

$$Ma_y = \underline{\underline{1.31}}$$

Thus,

$$V_x = Ma_x \sqrt{R T_x k} = 1.31 \sqrt{\frac{\left(1716 \ \frac{ft \cdot lb}{slug \cdot °R}\right)(412.2°R)(1.4)}{\left(1 \ \frac{lb}{slug \cdot \frac{ft}{s^2}}\right)}}$$

and

$$V_x = \underline{\underline{1304}} \ \frac{ft}{s}$$

Problem #6

In class, we derived the following relation from the energy equation

$$\frac{a^2}{\gamma-1} + \frac{u^2}{2} = \frac{\gamma+1}{2(\gamma-1)} a^{*2} \qquad (6a)$$

Divide Eq. (6a) by u^2

$$\frac{(1/M^2)}{\gamma-1} + \frac{1}{2} = \frac{\gamma+1}{2(\gamma-1)} \left(\frac{1}{M^*}\right)^2 \qquad (6b)$$

Solving for M^2 yield

$$M^2 = \frac{2}{(\gamma+1)/M^{*2} - (\gamma-1)}$$

Alternatively, solving for M^{*2} in Eq (6b) yield

$$M^{*2} = \frac{(\gamma+1) M^2}{2+(\gamma-1) M^2}$$