# AEE 343 Compressible Flow
## Problem set 1

## Problem #1

(a) Recall that 1 atm $= 1.01 \times 10^5$ N/m$^2$.

$$\rho = \frac{p}{RT} = \frac{(20)(1.01 \times 10^5)}{(287)(300)} = 23.46 \text{ kg/m}^3$$

The total mass stored is then

$$M = \mathcal{V}\rho = (10)(23.46) = \boxed{234.6 \text{ kg}}$$

(b) $$c_v = \frac{R}{\gamma - 1} = \frac{287}{1.4 - 1} = 717.5 \text{ J/kg} \cdot \text{K} \qquad e = c_v T = (717.5)(300) = 2.153 \times 10^5 \text{ J/kg}$$

$$E = Me = (234.6)(2.153 \times 10^5) = \boxed{5.05 \times 10^7 \text{ J}}$$

(c) $p\mathcal{V} = MRT \qquad \mathcal{V} = \text{constant}, \quad M = \text{constant}$

$\therefore \quad \frac{P}{T} = \text{constant}$

The vessel has a constant volume; hence as the air temperature is increased, the pressure also increases. Let the subscripts 1 and 2 denote the conditions before and after heating, respectively. Then,

$$\frac{p_2}{p_1} = \frac{T_2}{T_1} = \frac{600}{300} = 2$$

we found that $c_v = 717.5$ J/kg $\cdot$ K. Thus,

$$c_p = c_v + R = 717.5 + 287 = 1004.5 \text{ J/kg} \cdot \text{K}$$

$$\begin{aligned} s_2 - s_1 &= c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} \\ &= 1004.5 \ln 2 - 287 \ln 2 = 497.3 \text{ J/kg} \cdot \text{K} \end{aligned}$$

the mass of air inside the vessel is 234.6 kg. Thus, the total entropy change is

$$S_2 - S_1 = M(s_2 - s_1) = (234.6)(497.3) = \boxed{1.167 \times 10^5 \text{ J/K}}$$

## Problem #2

$$c_p = \frac{\gamma R}{\gamma - 1} = \frac{(1.4)(1716)}{(0.4)} = 6006 \frac{\text{ft lb}}{\text{slug } ^\circ\text{R}}$$

$$h_o = h + \frac{V^2}{2} = c_p T + \frac{V^2}{2} = (6006)(480) + \frac{(1300)^2}{2} = \boxed{3.728 \times 10^6 \frac{\text{ft lb}}{\text{slug}}}$$

## Problem #3

Let $(h_o)_{res}$ = total enthalpy of the reservoir = $c_p\,(T_o)_{res}$

$(h_o)_e$ = total enthalpy at the exit = $c_p\,T_e + \frac{V_e^2}{2}$

For an adiabatic flow, $h_o$ = constant. Hence

$$(h_o)_{res} = (h_o)_e$$

$$c_p(T_o)_{res} = c_p\,T_e + \frac{V_e^2}{2}$$

$$V_e = \sqrt{2\ c_p\left[(T_o)_{res} - T_e\right]} = \sqrt{2(1004.5)(1000 - 600)} = \boxed{896.4 \text{ m/sec}}$$

Problem 4

An airfoil is in a freestream where $p_\infty = 0.61$ atm, $\rho_\infty = 0.61$ kg/m$^3$, and $V_\infty = 300$ m/s. At a point on the airfoil surface, the pressure is 0.5 atm. Assuming isentropic flow, calculate the velocity at that point. Calculate the percentage error obtained if the problem is *incorrectly* solved using the incompressible Bernoulli equation.

$$T_\infty = \frac{p_\infty}{\rho_\infty R} = \frac{(0.61)(1.01 \times 10^5)}{(0.61)(287)} = 351.9\,^\circ\mathrm{K}$$

$$\frac{T}{T_\infty} = \left(\frac{p}{p_\infty}\right)^{(\gamma-1)/\gamma} ;\ T = T_\infty \left(\frac{p}{p_\infty}\right)^{(\gamma-1)/\gamma} = 351.9\left(\frac{0.5}{0.61}\right)^{0.2857} = 332.4\,^\circ\mathrm{K}$$

Since the flow is isentropic, it is also adiabatic. Hence, $h_o$ = constant

$$h_\infty + \frac{V_\infty^2}{2} = h + \frac{V^2}{2}$$

$$V = \sqrt{2(h_\infty - h) + V_\infty^2} = \sqrt{2\ c_p (T_\infty - T) + V_\infty^2} = \sqrt{2(1004.5)(351.9 - 332.4) + (300)^2}$$

$$= \boxed{360\,\mathrm{m/sec}}$$

$$p_\infty + \rho\frac{V_\infty^2}{2} = p + \rho\frac{V^2}{2}$$

$$V = \sqrt{\frac{2(p_\infty - p)}{\rho} + V_\infty^{\ 2}} = \sqrt{\frac{2(1.01 \times 10^5)(0.61 - 0.5)}{0.61} + (300)^2} = 355 \quad \mathrm{m/sec}$$

$$\%\ \mathrm{error} = \left(\frac{360\text{-}355}{360}\right) \times 100 = \boxed{1.3\ \%}$$

Problem 5

Repeat Problem 4, considering a point on the airfoil surface where the pressure is 0.3 atm.

Solve similarly to problem 4 to obtain

$T = 289K$

$V = 464 \text{ m/s}$

Using the Bernoulli's we get,

$$V = \sqrt{\frac{2(1.01 \times 10^5)(0.61 - 0.3)}{0.61} + (300)^2} = 438 \text{ m/sec}$$

$$\% \text{ error} = \left(\frac{464 - 438}{464}\right) \times 100 = \boxed{5.6\ \%}$$