Contents


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% AEE 343 - HW3.3
% 02/26/15

clear all
close all
clc
```

Problem 3, 4:

An ideal gas enters subsonically and flows isentropically through a choked converging-diverging duct having a circular cross section area A that varies with axial distance x from the throat according to the formula $A(x) = 0.1 + x^2$ where A is in ft^2 and x is in ft. Note that the throat is located at $x = 0$. For this flow situation, sketch the side view of the duct and graph the variation of Mach number, pressure ratio p/p_0, and temperature ratio T/T_0 through the duct from $x = -1\ ft$ to $x = +1\ ft$.

Carry out the same calculation as in problem 3 but for Helium, where $\gamma = 1.66$

Calculations

$$A(x) = 0.1 + x^2$$

$$y = \pm\sqrt{\frac{A(x)}{\pi}}$$

$$\left(\frac{A}{A^*}\right)^2 = \frac{1}{M^2}\left[\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{\gamma-1}}$$

$$x = \pm\sqrt{A - 0.1}$$

$$\frac{T_0}{T} = 1 + \frac{\gamma-1}{2}M^2$$

$$\frac{p_0}{p} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}$$

```
nPoints = 1001;
xDuct   = linspace(-1, 1, nPoints); % ft
area    = 0.1 + xDuct .^ 2;         % ft^2
radius  = sqrt(area ./ pi);         % ft
yUpper  =  radius;                  % ft
yLower  = -radius;                  % ft
```

```
% Air
gammaAir           = 1.4;
indexThroat        = find(xDuct == 0);
areaThroat         = area(indexThroat);                                    % ft^2
nMach              = linspace(0, 6, nPoints);
areaMachAir        = areaThroat .* sqrt((1 ./ (nMach .^ 2)) .*...          % ft^2
                     ((2 ./ (gammaAir + 1)) .* (1 + ((gammaAir - 1) ./ 2)...
                     .* nMach .^ 2)) .^ ((gammaAir + 1) ./ (gammaAir - 1)));
indexMachThroatAir = find(areaMachAir == min(areaMachAir));
xMach1Air          = -sqrt(areaMachAir(1 : indexMachThroatAir) - 0.1);
xMach2Air          =  sqrt(areaMachAir(indexMachThroatAir + 1 : end) - 0.1);
xMachAir           = [xMach1Air, xMach2Air];

p0_pAir            = (1 + ((gammaAir - 1) ./ 2) .* nMach .^ 2)...
                     .^ (gammaAir ./ (gammaAir - 1));
T0_TAir            = 1 + ((gammaAir - 1) ./ 2) .* nMach .^ 2;
p_p0Air            = 1 ./ p0_pAir;
T_T0Air            = 1 ./ T0_TAir;

% Helium
gammaHe            = 1.66;
areaMachHe         = areaThroat .* sqrt((1 ./ (nMach .^ 2)) .*...          % ft^2
                     ((2 ./ (gammaHe + 1)) .* (1 + ((gammaHe - 1) ./ 2)...
                     .* nMach .^ 2)) .^ ((gammaHe + 1) ./ (gammaHe - 1)));
indexMachThroatHe  = find(areaMachHe == min(areaMachHe));
xMach1He           = -sqrt(areaMachHe(1 : indexMachThroatHe) - 0.1);
xMach2He           =  sqrt(areaMachHe(indexMachThroatHe + 1 : end) - 0.1);
xMachHe            = [xMach1He, xMach2He];

p0_pHe             = (1 + ((gammaHe - 1) ./ 2) .* nMach .^ 2)...
                    .^ (gammaHe ./ (gammaHe - 1));
T0_THe             = 1 + ((gammaHe - 1) ./ 2) .* nMach .^ 2;
p_p0He             = 1 ./ p0_pHe;
T_T0He             = 1 ./ T0_THe;
```

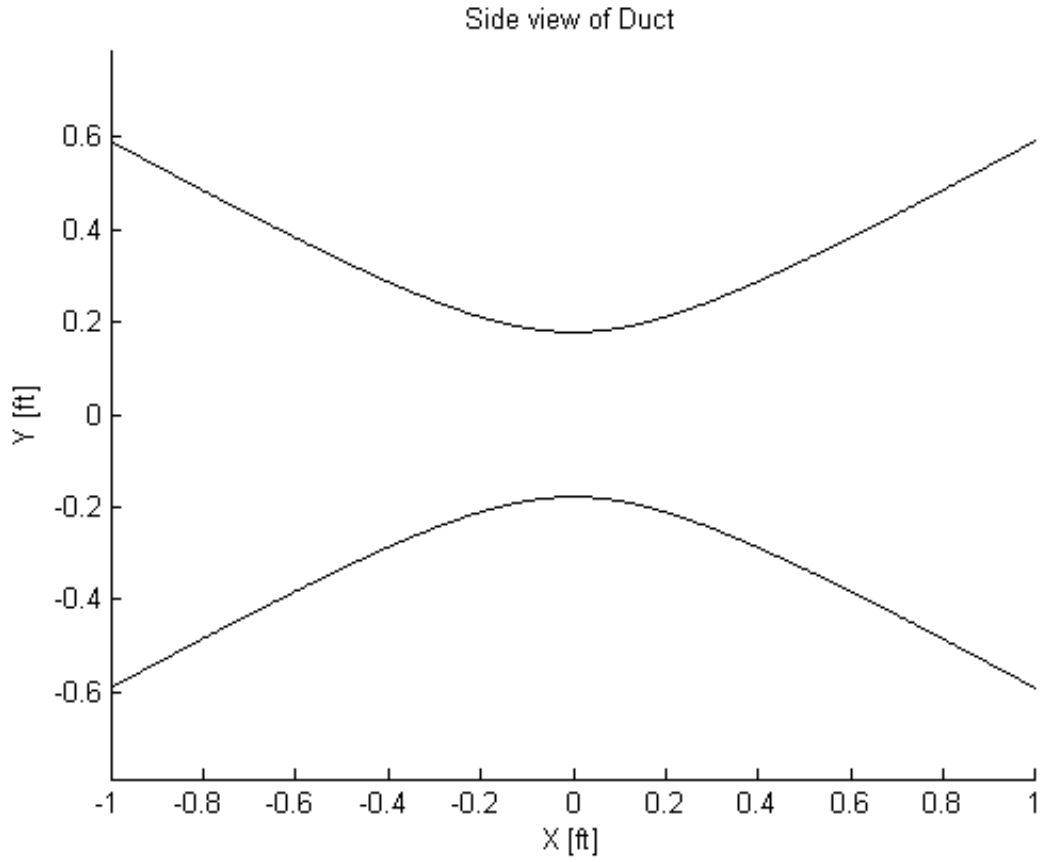
Plots

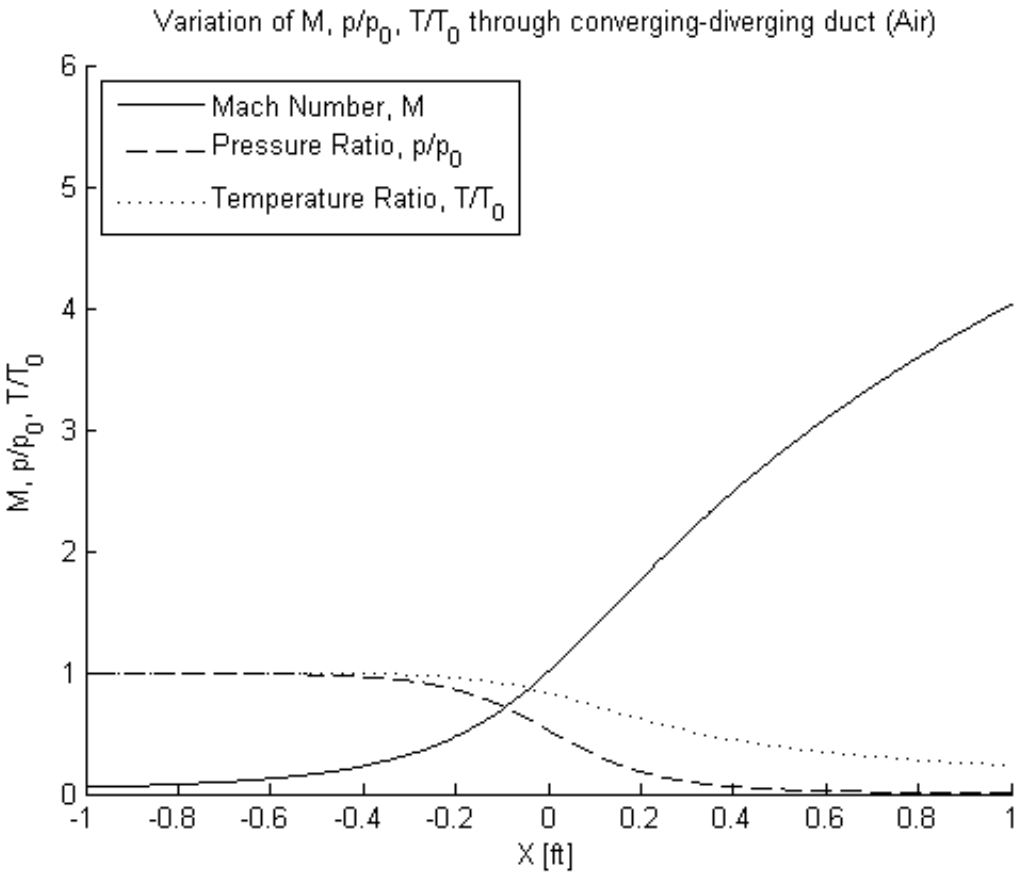
```
figure(1)
hold on
axis equal
title('Side view of Duct')
xlabel('X [ft]')
ylabel('Y [ft]')
plot(xDuct, yUpper, 'color', [0 0 0])
plot(xDuct, yLower, 'color', [0 0 0])

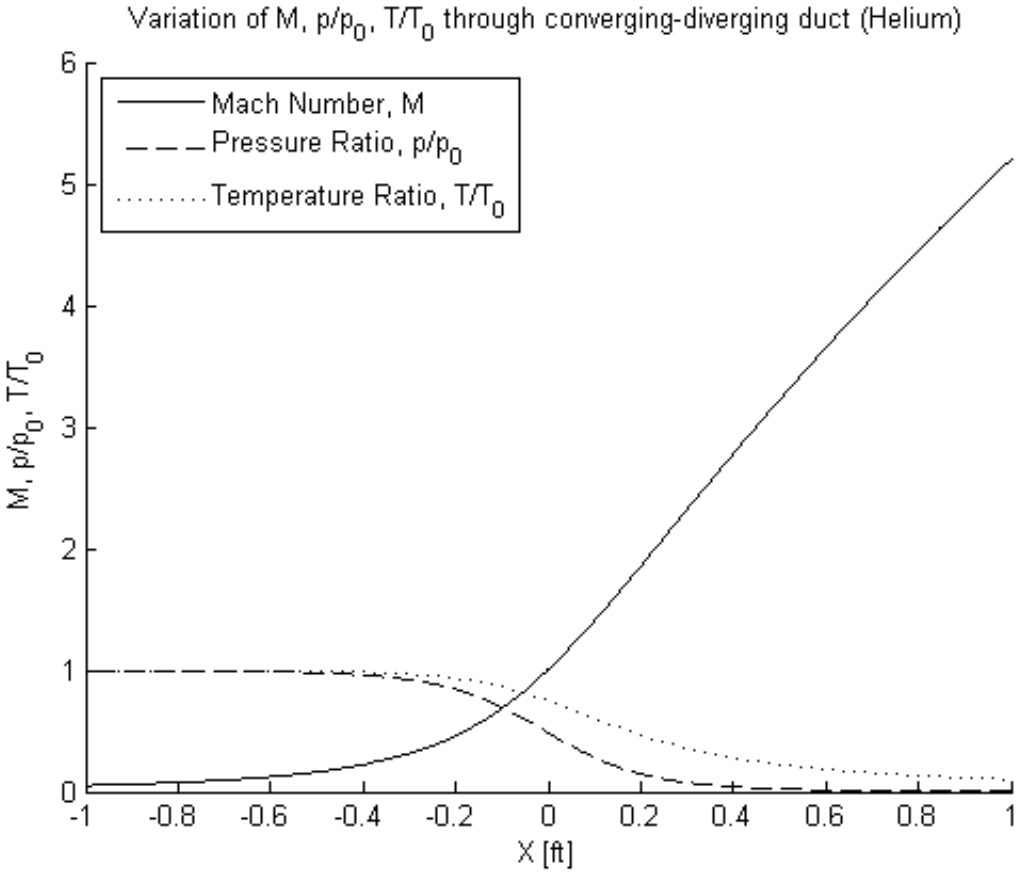
figure(2)
hold on
title('Variation of M, p/p_0, T/T_0 through converging-diverging duct (Air)')
xlabel('X [ft]')
ylabel('M, p/p_0, T/T_0')
axis([-1 1 0 6])
plot(xMachAir, nMach, '-', 'color', [0 0 0])
plot(xMachAir, p_p0Air, '--', 'color', [0 0 0])
plot(xMachAir, T_T0Air,  ':', 'color', [0 0 0])
legend('Mach Number, M', 'Pressure Ratio, p/p_0',...
```

```
       'Temperature Ratio, T/T_0', 'location', 'northwest')

figure(3)
hold on
title('Variation of M, p/p_0, T/T_0 through converging-diverging duct (Helium)')
xlabel('X [ft]')
ylabel('M, p/p_0, T/T_0')
axis([-1 1 0 6])
plot(xMachHe, nMach, '-', 'color', [0 0 0])
plot(xMachHe, p_p0He, '--', 'color', [0 0 0])
plot(xMachHe, T_T0He,  ':', 'color', [0 0 0])
legend('Mach Number, M', 'Pressure Ratio, p/p_0',...
       'Temperature Ratio, T/T_0', 'location', 'northwest')
```




Variation of M, p/p0, T/T0 through converging-diverging duct (Air)
Mach Number, M
Pressure Ratio, p/p0
Temperature Ratio, T/T0
M, p/p0, T/T0
X [ft]
0
1
2
3
4
5
6
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1


Variation of M, p/p0, T/T0 through converging-diverging duct (Helium)
Mach Number, M
Pressure Ratio, p/p0
Temperature Ratio, T/T0
M, p/p0, T/T0
X [ft]
0
1
2
3
4
5
6
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1

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