Contents


```
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% MAE 231 - HW6.2
% 02/25/15

clear all
close all
clc
```
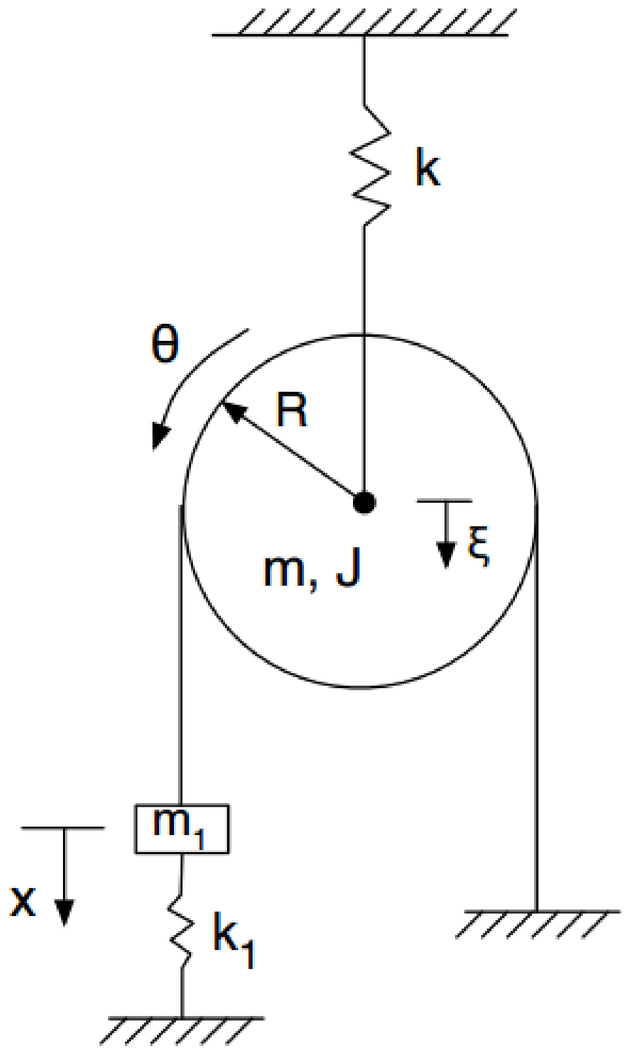

Problem 2:

For the system sketched on the next page, use the Euler-Lagrange equations to derive the (single) equation of motion in terms of one general coordinate x. Feel free to ignore the static deflection that it will cancel out as we've seen in class (and to not include Δ in the expression of the spring potential when you do). Some helpful reminders:

- $\text{Due to the pulley, } \xi = \frac{x}{2} \text{ and } x = 2R\theta$

- In terms of polar moment of inertia, consider the pulley as a disk (What is J in terms of m and R?)

For $m_1 = 1kg, m = 2kg, k = 2N/m, k_1 = 5N/m, R = 0.3m, x_0 = 0.1m$ and $v_0 = 0$, plot the response of this system. From the figure, estimate the natural frequency of the system. Separately, calculate ω_n from the coefficients in the equation of motion. How do they compare?



Find: Response of system, ω_n (Estimate/Calculate)

Known

$m_1, m, k, k_1, R, x_0, v_0$

```
mass1      = 1;   % kg
mass       = 2;   % kg
stiffness  = 2;   % N/m
stiffness1 = 5;   % N/m
radius     = 0.3; % m
xInitial   = 0.1; % m
vInitial   = 0;   % m/s
```

Calculations

$$\xi = \frac{x}{2},\ x = 2R\theta$$

$$T = \frac{1}{2}m_1\dot{x}^2 + \frac{1}{2}J\dot{\theta}^2 = \frac{1}{2}m_1\dot{x}^2 + \frac{1}{16}m\dot{x}^2$$

$$U = \frac{1}{2}k\xi^2 + \frac{1}{2}k_1x^2 = \frac{1}{8}kx^2 + \frac{1}{2}k_1x^2$$

$$L = T - U = \frac{1}{2}m_1\dot{x}^2 + \frac{1}{16}m\dot{x}^2 - \frac{1}{8}kx^2 - \frac{1}{2}k_1x^2$$

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{x}}) - \frac{\partial L}{\partial x} = 0$$

$$(\frac{1}{8}m + m_1)\ddot{x} + (\frac{1}{4}k + k_1)x = 0$$

$$\ddot{x} + (\frac{\frac{1}{4}k + k_1}{\frac{1}{8}m + m_1})x = 0$$

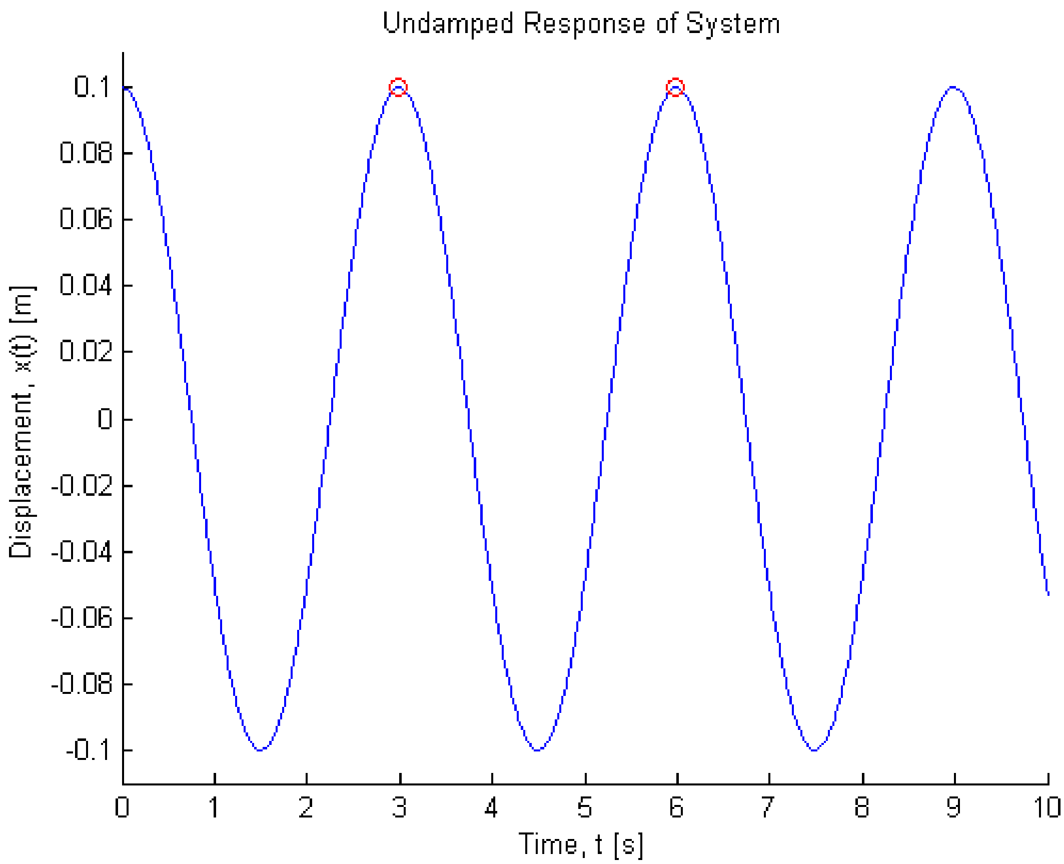
$$\omega_n = \sqrt{\frac{\frac{1}{4}k + k_1}{\frac{1}{8}m + m_1}}$$

```
frequencyNaturalEoM = sqrt((0.25 * stiffness + stiffness1) /...
                           (mass / 8 + mass1))
time                = linspace(0, 10, 1000);
x                   = xUndamped(xInitial, vInitial, frequencyNaturalEoM, time);
[peak1, indexPeak1] = max(x(200:400));
indexPeak1          = 199 + indexPeak1;
[peak2, indexPeak2] = max(x(500:700));
indexPeak2          = 499 + indexPeak2;
frequencyNaturalEst = (2 * pi) / (time(indexPeak2) - time(indexPeak1))
```

```
frequencyNaturalEoM =

    2.0976


frequencyNaturalEst =

    2.0993
```

Plot

```
figure(1)
hold on
axis([0 10 -0.11 0.11])
title('Undamped Response of System')
xlabel('Time, t [s]')
ylabel('Displacement, x(t) [m]')
plot(time, x)
plot(time(indexPeak1), x(indexPeak1), 'o', 'color', [1 0 0])
plot(time(indexPeak2), x(indexPeak2), 'o', 'color', [1 0 0])
```



Results

The natural frequency obtained from coefficients of the equation of motion is $\omega_n = 2.0976 \ rad/s$ and the natural frequency estimated from the plot of the response is $\omega_n = 2.0993 \ rad/s$. These numbers are very close, and theoretically should be the same. The difference in the values can be accounted for by error caused by a time step that is too large in the plot.

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