

Contents

- Problem 2:
- Known
- Conversions
- Calculations
- Plot
- Results

```
% Joel Lubinitsky - 02/11/15  
% MAE 321 - HW 4.2
```

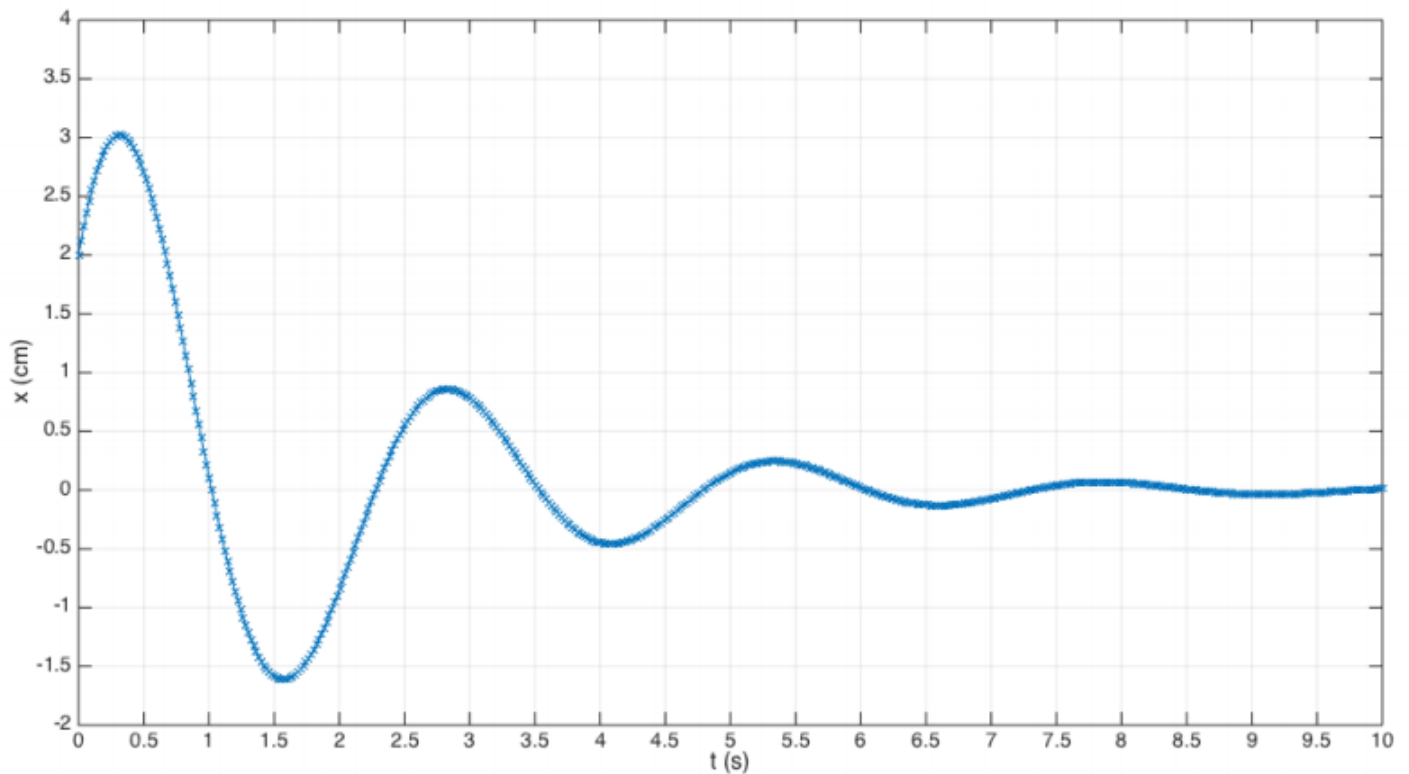
```
clear all  
close all  
clc
```

Problem 2:

The figure on the second page of this homework sheet shows the displacement of a vibrating spring-mass-damper. From the plot, determine the undamped natural frequency, the damping ratio, the damped natural frequency, the initial position, and the initial velocity. From these values, recreate the oscillation solution expression.

Unknown: ω_n , ζ , ω_d , x_0 , v_0

Known



```
xPeak1 = 3;    % cm  
xPeak2 = 0.8;  % cm  
  
tPeak1 = 0.3;  % s  
tPeak2 = 2.8;  % s  
  
xInitial = 2;  % cm
```

```

xStep    = 2.5; % cm

tInitial = 0;   % s
tStep    = 0.08; % s

```

Conversions

```

xPeak1 = xPeak1 / 100; % m
xPeak2 = xPeak2 / 100; % m

xInitial = xInitial / 100 % m
xStep    = xStep / 100;   % m

```

```

xInitial =

    0.0200

```

Calculations

$$\delta = \ln \frac{x_1}{x_2}$$

$$\zeta = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}}$$

$$\omega_n = \frac{\delta}{\zeta T}$$

$$\omega_d = \frac{2\pi}{T}$$

$$v_0 = \frac{\Delta x}{\Delta t}$$

```

period          = tPeak2 - tPeak1;
delta           = log(xPeak1 / xPeak2);
ratioDamping    = delta / sqrt(4 * pi ^ 2 + delta ^ 2)
frequencyNatural = delta / (ratioDamping * period)
frequencyNaturalDamped = (2 * pi) / period
vInitial        = (xStep - xInitial) / (tStep - tInitial)
time            = [0 : 0.01 : 10];
x               = xUnderdamped(xInitial, vInitial, ratioDamping, ...
                                frequencyNatural, frequencyNaturalDamped, time);

```

```

ratioDamping =

    0.2059

```

```

frequencyNatural =

    2.5683

```

```

frequencyNaturalDamped =

    2.5133

```

```

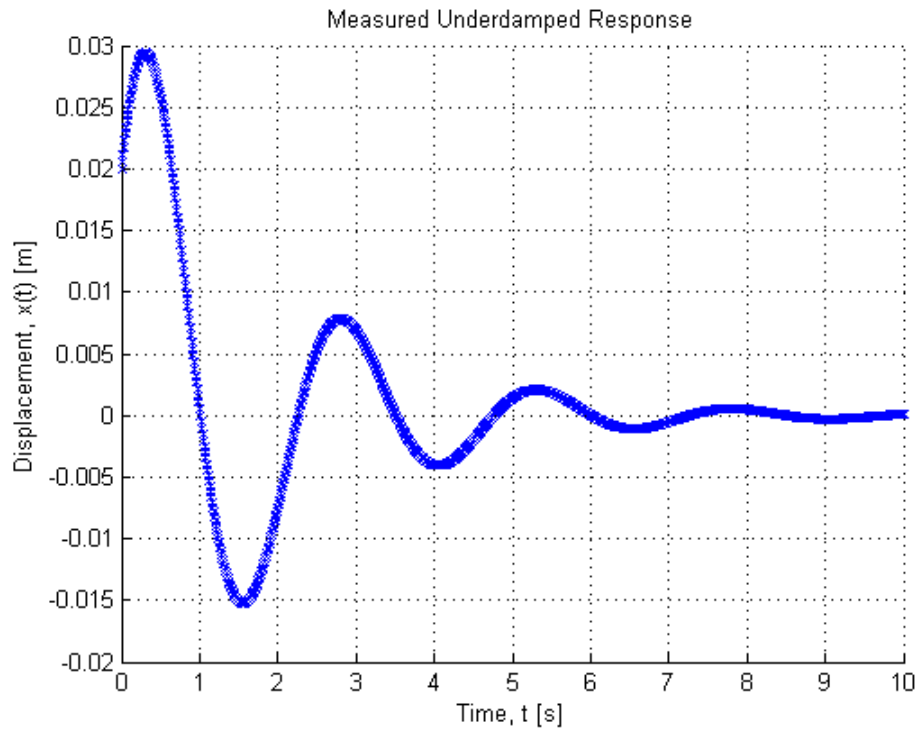
vInitial =

```

0.0625

Plot

```
figure(1)
hold on
grid on
title('Measured Underdamped Response')
xlabel('Time, t [s]')
ylabel('Displacement, x(t) [m]')
plot(time, x, '-x')
```



Results

The given system has an undamped natural frequency $\omega_n = 2.57 \text{ rad/s}$, damping ratio $\zeta = 0.206$, damped natural frequency $\omega_d = 2.51 \text{ rad/s}$, initial position $x_0 = 0.02 \text{ m}$, and initial velocity $v_0 = 0.0625 \text{ m/s}$.