Joel Lubinitsky, Michael Iati, Evan Hyde
MAE 321

Project 1 Milestone:

Derive and represent with some text, equations, and a sketch, the equation of motion and the forcing function.

Equation of Motion:

General Form

$$m_{eff}\ddot{x} + c\dot{x} + kx$$

The system is modelled with a single degree of freedom. The direction of deflection is chosen to be parallel to the longitudinal axis of the nacelle. Each blade is modelled as a cantilevered beam with all forces acting at the center. This decision is a simplification made by the assumption that the force of wind will be distributed evenly along the length of the blade and that this can be represented as a single point-force of equivalent magnitude. The displacement, x, is taken at this point as well in order to simplify calculations. The deflection at any point on the blade can then be determined using the derivative of the deflection equation, shown in Appendix 1.

Three terms must be determined in order to derive the corresponding equation of motion; effective mass, damping coefficient, and stiffness. These values are determined as shown below.

Effective mass:

The effective mass must be recalculated for a vibrating beam in which the displacement is taken at the center, or $L/2$, when L is the length of the beam. Knowing that the beam is fixed on one end and free on the other (cantilevered), the effective mass is determined to be

$$m_{eff} = \frac{61}{35}m$$

(See Appendix 1 for derivation).

Where m is the total mass of a single blade.

Damping Coefficient:

Since the turbine blade will not vibrate indefinitely, there must be some internal damping. This value may be determined iteratively. The system's response may be modelled for a range of damping coefficients in order to determine a value that best satisfies the expected response. That is, the response that will reasonably perform within a factor of safety under normal and maximum operating conditions.

Stiffness:

The stiffness of the blade can be determined from elementary beam theory with forces and displacement taken at the center. This will be a function of beam geometry and material properties. The material of the blade is a constructed from a combination of carbon, wood, glass, and epoxy[1]. The modulus of elasticity, E, for this compound is not known, so a value must be assumed based on known values for each of these materials. The area moment of inertia, I, is assumed to be for an ellipse matching the approximate dimensions of the blade's airfoil shape. The effective stiffness is determined to be

$$k = \frac{24EI}{L^3}$$

(See Appendix 1 for derivation).

Where I for an ellipse is known to be

$$I = \frac{\pi}{4} ab^3$$

Here, a and b are the largest and smallest dimensions of the ellipse, respectively.

Forcing Function:

General Form

$$F_0 \cos(\omega t)$$

The system is assumed to be forced solely by periodic fluctuations in wind velocity. The change in velocity is of particular interest because steady wind is assumed to induce a constant deflection in the blade, whereas periodic gusts will cause the blade to oscillate. However, the steady velocity should not be ignored because the forcing is proportional to the square of velocity in the drag equation, so static deflection will not cancel out these terms. This equation is as follows

$$F = \frac{C_D \rho_\infty A V^2}{2}$$

Where C_D is the drag coefficient of the blade, $\rho_\infty$is the freestream density, A is the blade area, and V is the freestream velocity. Since the velocity term in this equation is periodic, the forcing function does not have to appear in the general form shown above. In fact, this equation for drag force is the forcing function itself, rewritten as

$$F(t) = \frac{C_D \rho_\infty A V(t)^2}{2}$$

Velocity, V, can be rewritten as a function of time, varying about a certain steady velocity V_0 with a particular gust velocity V_g. This can be represented as

$$V(t) = V_0 + V_g \cos(\omega t)$$

Where ω is the frequency of gusts. Altogether, the forcing function is

$$F(t) = \frac{C_D \rho_\infty A}{2}\left(V_0 + V_g \cos(\omega t)\right)^2$$

Values for V_0 and V_g are selected from common values published by the national weather service. The system will be modelled in a variety of conditions, but two regimes of generally calm and gusty weather are defined. The national weather service reports gusts when peak wind speed is at least 16 knots, and wind variation is at least 9 knots. The period of these gusts is typically less than 20 seconds[2]. This will be the boundary used to define the difference between calm and gusty conditions. Here, the variation of 9 knots corresponds to $2V_g$, 16 knots is V_{max}, and the forcing frequency, ω, can be determined from the period of 20 seconds.

In conclusion, the equation governing the vibrational motion of the turbine blade is

$$\frac{61}{35} m\ddot{x} + c\dot{x} + \frac{24EI}{L^3} x = \frac{C_D \rho_\infty A}{2}\left(V_0 + V_g \cos(\omega t)\right)^2$$

References

[1]http://mn.gov/commerce/energyfacilities/documents/18884/General%20Specifications%20V82-1.65%20MW%20MK%20II.pdf

[2]http://graphical.weather.gov/definitions/defineWindGust.html