Contents


```
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% MAE 321 - HW9.3
% 03/25/15

clear all
close all
clc
```

Problem 3:

Numerically integrate and plot the (full, not steady state) response of an underdamped system with m = 150 kg, k = 1500 N/m, and c = 100 kg/s. The system is subject to an applied force F(t) = 200 cos(10t) for the following set of initial conditions:

- $x_0 = 0.0\ m$ and $v_0 = 0.1\ m/s$
- $x_0 = 0.1\ m$ and $v_0 = 0.0\ m/s$
- $x_0 = 0.05\ m$ and $v_0 = 0.0\ m/s$
- $x_0 = 0.0\ m$ and $v_0 = 0.5\ m/s$

Known

$m,\ k,\ c,\ F(t),\ x_0,\ v_0$

```
mass              = 150;                  % kg
stiffness         = 1500;                 % N/m
coefficientDamping = 100;                 % kg/s
xInitial          = [0.0, 0.1, 0.05, 0.0]; % m
vInitial          = [0.1, 0.0, 0.0,  0.5]; % m/s
funForcing        = @(t) 200 .* cos(10 .* t);
```

Calculations

$$m\ddot{x} + c\dot{x} + kx = 200\cos 10t$$

Rewrite as system of first order ODEs:

$$\dot{v} = \frac{200}{m}\cos 10t - \frac{c}{m}v - \frac{k}{m}x$$

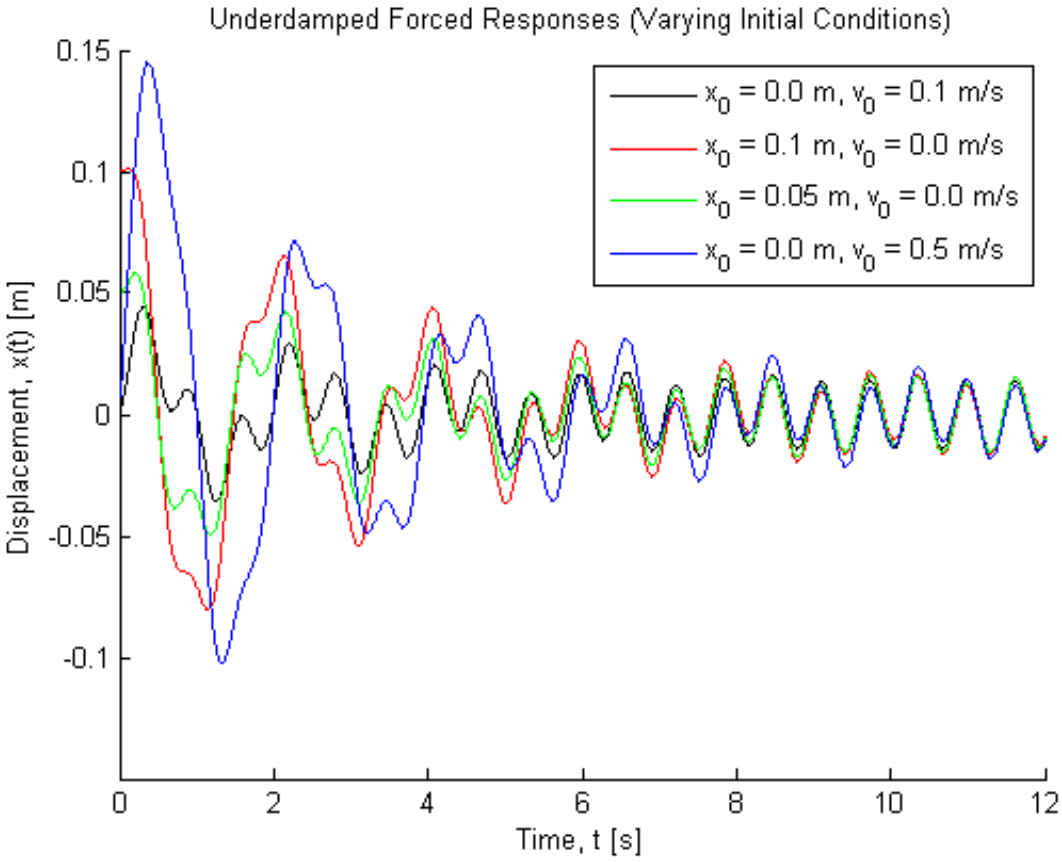
$\dot{x} = v$

Apply RK4:

```
% Initialize System
T  = 12;
dt = 0.001;
N  = T / dt;
times = linspace(0, T, N);
vx = zeros(N, 8);
vx(1, 2 : 2 : 8) = xInitial;
vx(1, 1 : 2 : 7) = vInitial;

% Run RK4 Integrator
for n = [1 : N - 1] % n step through time

    time = (n - 1) * dt;

    for i = [1, 3, 5, 7] % i step through responses

        vx(n + 1, i : i + 1) = RK4SpringMassDamperForced(vx(n, i), vx(n, i + 1), mass, ...
                               coefficientDamping, stiffness, dt, funForcing, time);

    end % i

end % n
```

Plots

```
figure(1)
hold on
title('Underdamped Forced Responses (Varying Initial Conditions)')
xlabel('Time, t [s]')
ylabel('Displacement, x(t) [m]')
plot(times, vx(:, 2), 'color', [0 0 0])
plot(times, vx(:, 4), 'color', [1 0 0])
plot(times, vx(:, 6), 'color', [0 1 0])
plot(times, vx(:, 8), 'color', [0 0 1])
axis([0 T -0.15 0.15])
legend('x_0 = 0.0 m, v_0 = 0.1 m/s', 'x_0 = 0.1 m, v_0 = 0.0 m/s', 'x_0 = 0.05 m, v_0 = 0.0 m/s', '
x_0 = 0.0 m, v_0 = 0.5 m/s')
```



Results

Responses with high initial displacements or velocities have transient parts with greater amplitudes. These responses take longer to settle into the steady state than those with smaller initial inputs.

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