Contents


```
% Joel Lubinitsky - 02/18/15
% MAE 321 - HW 5.3

clear all
close all
clc
```

Problem 3:

Use numerical integration to solve the system of Example 1.7.3 with a different set of physical parameters: $m = 1455\ kg,\ k = 2.5 * 10^5\ N/m,\ c = 3.75 * 10^3\ kg/s$ subject to the initial conditions $x_0 = 0$ and $v_0 = 0.01\ mm/s$. Compare your result using numerical integration to just plotting the analytical solution by plotting both on the same graph. Use different symbols to distinguish the two curves. You may use any of the numerical integration methods discussed in class or the learning objectives, but be clear about which you choose and why. Redesign the damper such that the oscillation dies out after $2\ s$. This should be accomplished by creating a Matlab program that varies c withing a reasonable range until the desired response is acquired. Present both the final response solution expression and a plot of the response.

Find: System Response, x (Numerical/Analytical); Design such that oscillation dies out after $2\ s$

Known

$m,\ k,\ c,\ x_0,\ v_0$

```
mass               = 1455;          % kg
stiffness          = 2.5 * 10 ^ 5;  % N/m
coefficientDamping = 3.75 * 10 ^ 3; % kg/s
xInitial           = 0;             % m
vInitial           = 0.01;          % mm/s
```

Conversions

```
vInitial = vInitial / 1000; % m/s
```

Calculations

$$\zeta = \frac{c}{c_{cr}} = \frac{c}{\sqrt{4km}}$$

$\zeta < 1$, so system is underdamped.

```
% Analytical Solution
coefficientDampingCritical = sqrt(4 * stiffness * mass);
ratioDamping               = coefficientDamping / coefficientDampingCritical
frequencyNatural           = sqrt(stiffness / mass);
frequencyNaturalDamped     = frequencyNatural * sqrt(1 - ratioDamping ^ 2);
stepTime                   = 0.01;
time                       = [0 : stepTime : 5];
xAnalytical                = xUnderdamped(xInitial, vInitial,...
                               ratioDamping, frequencyNatural,...
                               frequencyNaturalDamped, time);

% Numerical Solution: Runge-Kutta 4
    % Initialize Loop
T         = 5;
dt        = 0.001;
N         = T / dt;
vx        = zeros(N, 2);
vx(1, :) = [vInitial, xInitial];

    % Run Loop
for n = [1 : N - 1]
    vx(n + 1, :) = RK4SpringMassDamper(vx(n, 1), vx(n, 2), mass,...
                   coefficientDamping, stiffness, dt);
end

% Redesign Damper
coefficientDampingValues = [4 * 10 ^ 3 : 5 * 10 ^ 2 : 10 * 10 ^ 3];
xResponses               = zeros(length(time),...
                             length(coefficientDampingValues));
xResidualAllowed         = 1 * 10 ^ -8;

for n = [1 : length(coefficientDampingValues)]

    xResponses(:, n) = xUnderdamped(xInitial, vInitial,...
                      (coefficientDampingValues(n) /...
                       coefficientDampingCritical), frequencyNatural,...
                       frequencyNaturalDamped, time);

    if max(abs(xResponses(2 / stepTime : end, n))) < xResidualAllowed
        coefficientDampingSolution = coefficientDampingValues(n)
        indexSolution              = n;
        break
    end

end
```

```
ratioDamping =

    0.0983


coefficientDampingSolution =
```

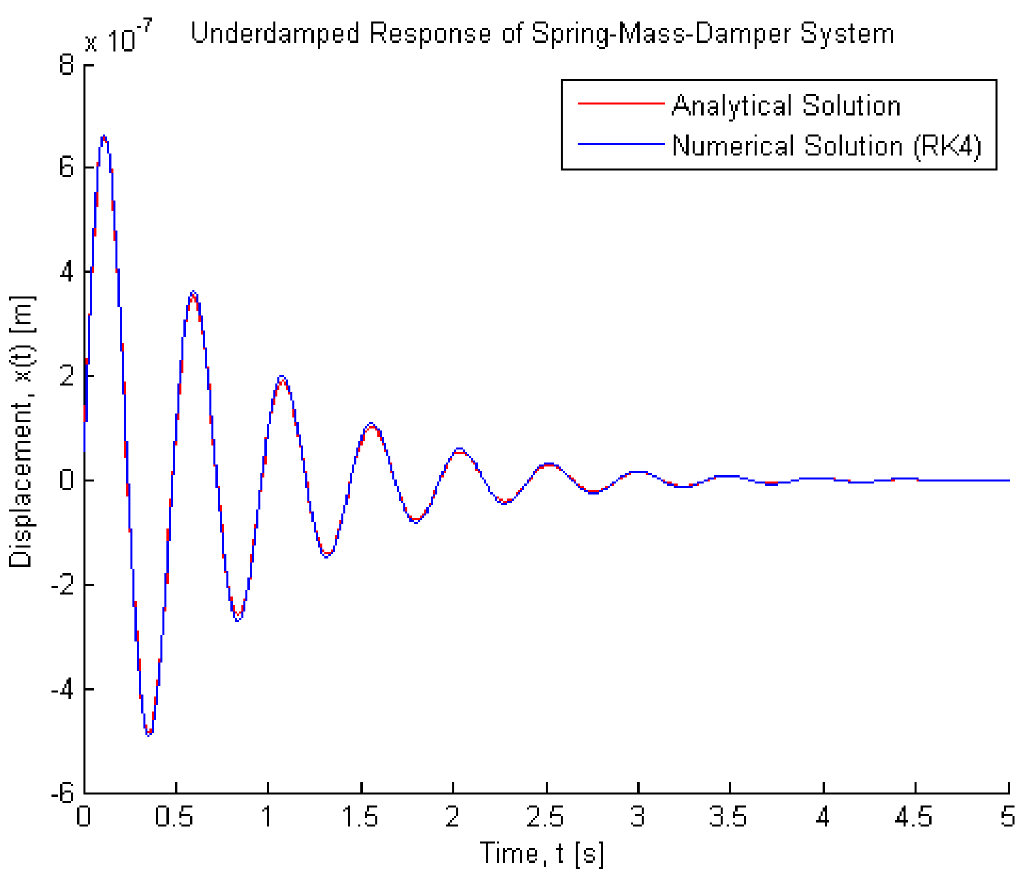
```
6500
```

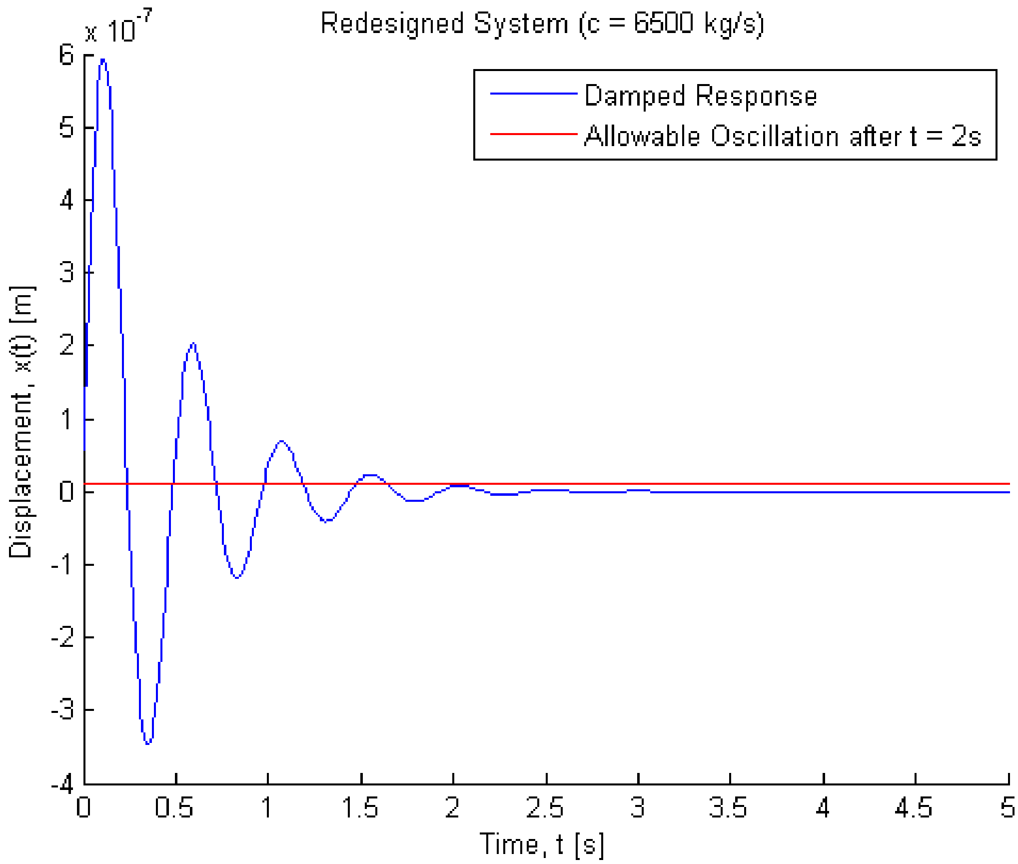
Plots

Analytical/Numerical Solutions of Response

```
figure(1)
hold on
title('Underdamped Response of Spring-Mass-Damper System')
xlabel('Time, t [s]')
ylabel('Displacement, x(t) [m]')
plot(time, xAnalytical, 'color', [1 0 0])
plot(linspace(0, T, N), vx(:, 2), 'color', [0 0 1])
legend('Analytical Solution', 'Numerical Solution (RK4)', 'location', 'northeast')

%Redesigned System
figure(2)
hold on
title('Redesigned System (c = 6500 kg/s)')
xlabel('Time, t [s]')
ylabel('Displacement, x(t) [m]')
plot(time, xResponses(:, n), 'color', [0 0 1])
plot(time, xResidualAllowed, 'color', [1 0 0])
legend('Damped Response', 'Allowable Oscillation after t = 2s','location', 'northeast')
```





Results

At a sufficiently small time step $dt = 0.001$, the numerical and analytical responses are nearly indistinguishable. The damping coefficient required so that the response dies out after $2\ s$ is $c = 6500\ kg/s$.

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