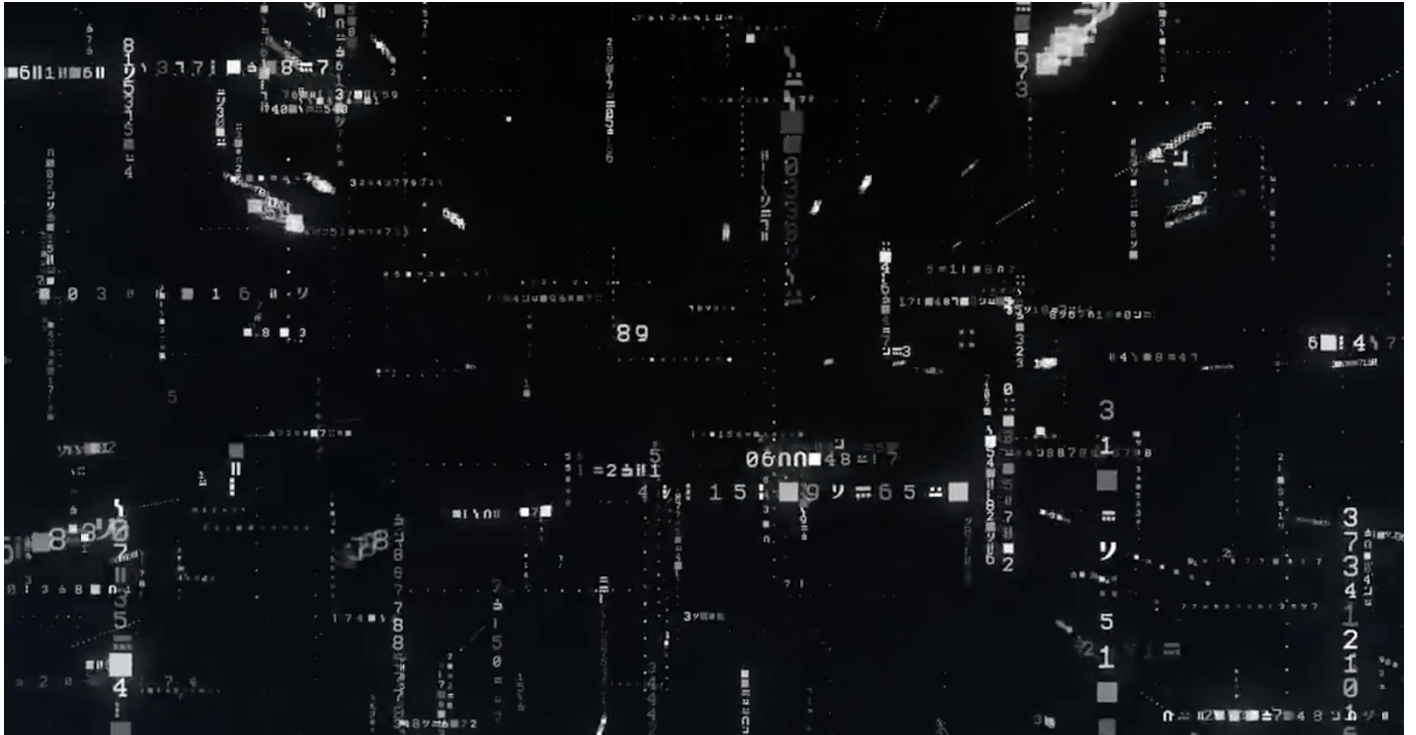




ÉCOLE  
POLYTECHNIQUE  
DE BRUXELLES



# Information theory

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Année académique : 2021-2022



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# Chapter 1

## Introduction

### 1.1 Fundamental goals, questions, and operations

#### 1.1.1 Highest achievable data compression

#### 1.1.2 Highest achievable data send rate through channel

### 1.2 Overview of the course

### 1.3 First theorem : source coding theorem

#### 1.3.1 Definition of entropy

#### 1.3.2 Example

#### 1.3.3 Block coding

### 1.4 Second theorem : channel coding theorem

#### 1.4.1 Definition of capacity

#### 1.4.2 Example





# Chapter 2

## Overview of Shannon's entropy

### 2.1 Probabilities prerequisite

### 2.2 Entropy

#### 2.2.1 Units

#### 2.2.2 Examples

#### 2.2.3 Remarks

#### 2.2.4 Link with uncertainty : axiomatic approach

#### 2.2.5 Interpretation of $H$

Special case : Bernoulli variable

Concavity of  $H$

### 2.3 Joint entropy

### 2.4 Conditional entropy

#### 2.4.1 Definition

#### 2.4.2 Chain rule

#### 2.4.3 Examples

#### 2.4.4 Illustration : entropy Venn diagrams, link with mutual entropy

### 2.5 Mutual entropy

#### 2.5.1 Formal definition of mutual entropy

### 2.6 Relative entropy

#### 2.6.1 Definition

#### 2.6.2 Conditional relative entropy

#### 2.6.3 Chain rule for relative entropy