Overview of Joint and Conditional Probabilities
Comp Phylo Spring 2015

The following statements about joint and conditional probabilities are always true:

$$(1) \quad P(A,B) = P(A|B)P(B) = P(B|A)P(A)$$

$$(2) \quad P(A|B) = \frac{P(A,B)}{P(B)} = \frac{P(B|A)P(A)}{P(B)}$$

$$(3) \quad P(B|A) = \frac{P(A,B)}{P(A)} = \frac{P(A|B)P(B)}{P(A)}$$

If A and B are independent, then:

$$(4) \quad P(A,B) = P(A)P(B)$$

$$(5) \quad P(A|B) = P(A)$$

$$(6) \quad P(B|A) = P(B)$$

To convince yourself, here are some statements about the joint probability of rain (R) or sun (S) on two consecutive days. In each table, rows give probabilities for Day 1 and columns give probabilities for Day 2. Numbers inside the table are joint probabilities of rain and sun for both days [e.g., $P(R,S)$]. Numbers on the outside of the table are the marginal probabilities of rain and sun [e.g., $P(R)$]].

The weather on the two days is independent:

		Day 2 R	Day 2 S	
Day 1	R	0.16	0.24	0.4
	S	0.24	0.36	0.6
		0.4	0.6	

The weather on the two days is dependent:

		Day 2 R	Day 2 S	
Day 1	R	0.3	0.1	0.4
	S	0.1	0.5	0.6
		0.4	0.6	

For each of these tables, try verifying that equations (1) – (3) apply to both tables, while equations (4) – (6) only apply to the first table.