Lab 1: Prelab

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1 Theory Problems

1.1 Spectrum of AM modulated signals

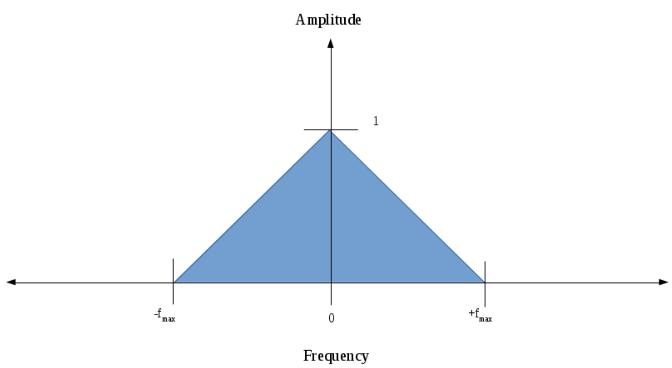

Figure 1: **A sketch of the frequency spectrum of the baseband signal $S(f)$.**

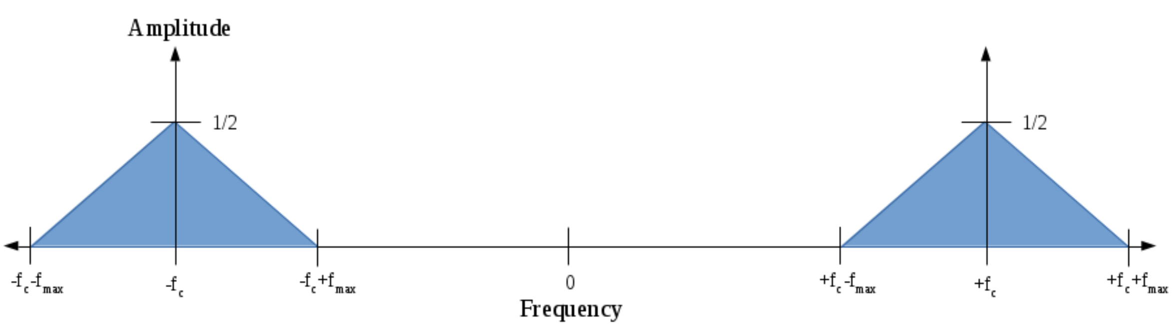

Figure 2: **A sketch of the frequency spectrum of the amplitude-modulated passband signal $\tilde{S}(f)$.**

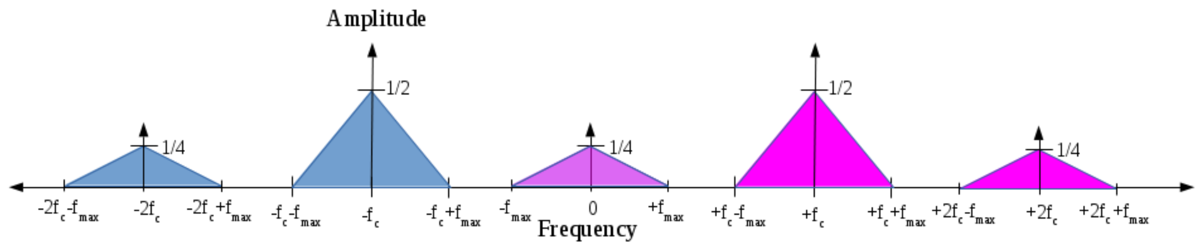

Figure 3: **A sketch of the frequency spectrum of the demodulated signal.**

1.2 Frequency demodulation errors

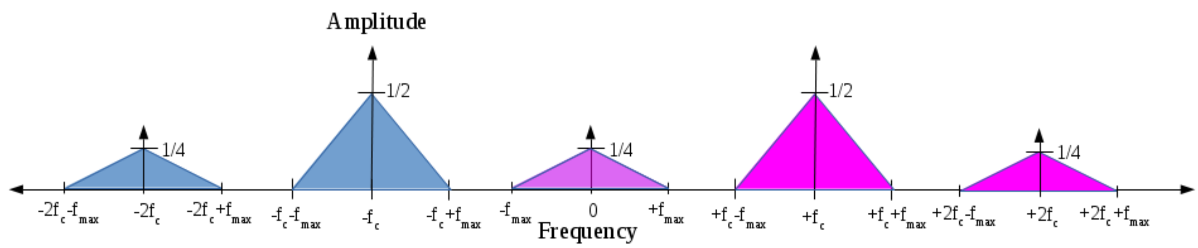

Figure 4: **A sketch of the frequency spectrum of the demodulated signal with frequency error $x = 0$.**

As frequency error is introduced, the alias baseband copies shift past each other. Because f_c dominates f_{max}, we assume f_c is at least several orders of magnitude larger than f_{max}.

The frequency error is now a magnitude larger than before, but is still relatively small because f_c is several order of magnitude larger than f_{max}.

Even as the aliased copies slide past each other at baseband with the introduction of frequency error, the Nyquist rate of the baseband signal is unaffected because f_{max} remains unchanged.

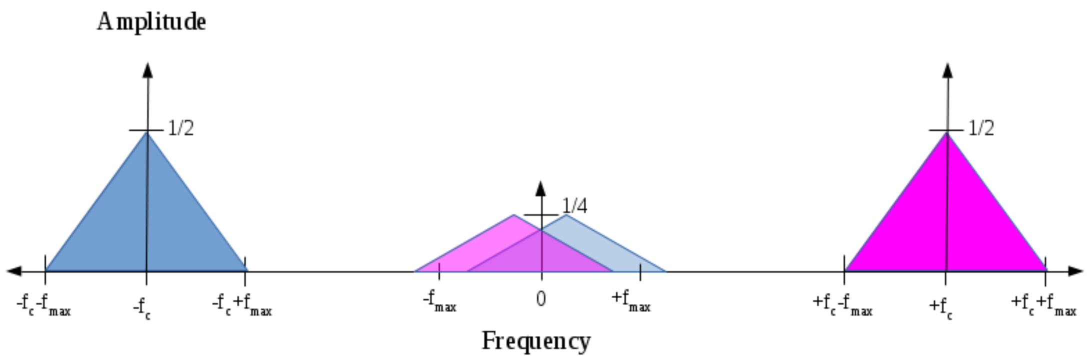

Figure 5: **A sketch of the frequency spectrum of the demodulated signal with frequency error $x = \frac{f_{max}}{10 f_c}$. Note that spectral copies at $\pm 2f_c$ exist but are not visible in the figure.**

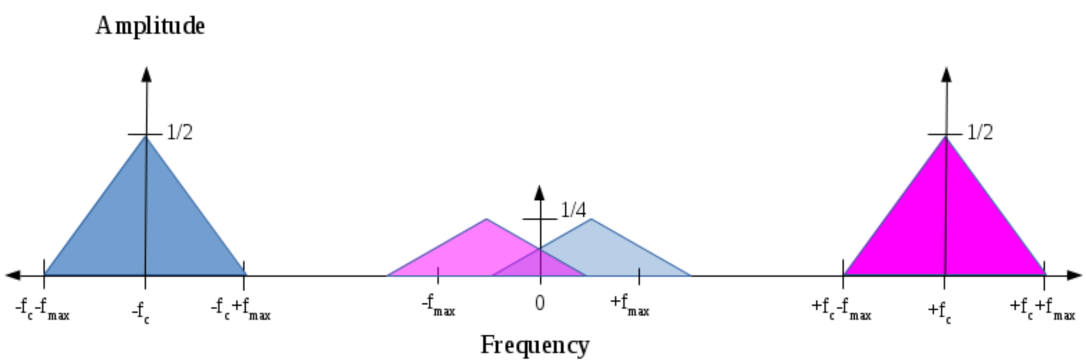

Figure 6: **A sketch of the frequency spectrum of the demodulated signal with frequency error $x = \frac{f_{max}}{f_c}$. Note that spectral copies at $\pm 2f_c$ exist but are not visible in the figure.**

1.3 Phase demodulation errors

An AM signal $\tilde{s}(t) = A\ cos(2\pi f_c t)$ where A is a constant is demodulated by $cos(2\pi f_c t + \phi)$ where ϕ represents a phase error.

An expression for the demodulated signal $d(t, \phi)$ as a function of the phase error ϕ is given by:

$$d(t, \phi) = A\ cos(2\pi f_c t) cos(2\pi f_c t + \phi) \tag{1}$$

The period of the carrier f_c is given by $T = \frac{1}{f_c}$.

If the demodulated signal $d(t, \phi)$ is integrated over a time period T that is many times the period of the carrier (i.e., $N\ T$, where $N >= 2$), the value of the integral without phase error ($\phi = 0$) is given by:

$$\begin{aligned} M_0 &= \int_0^{\frac{2}{f}} A\ cos(2\pi f_c t) cos(2\pi f_c t) \\ &= \frac{1}{f} \end{aligned} \tag{2}$$

The value of the integral with phase error ($\phi \neq 0$) over the same period is given by:

$$\begin{aligned} M_1 &= \int_0^{\frac{2}{f}} A\ cos(2\pi f_c t) cos(2\pi f_c t + \phi) \\ &= \frac{cos(\phi)}{f} \end{aligned} \tag{3}$$

The maximum phase error ϕ that can be tolerated for the demodulated signal to ensure the amplitude is within ten percent of the amplitude without a phase error is given by:

$$\begin{aligned} \frac{M_0}{M_1} &\leq 10 \\ sec(\phi) &\leq 10 \\ |\phi| \leq &\ \ sec^{-1}(10) \\ |\phi| \leq &\ \ 0.4706 \end{aligned} \tag{4}$$

1.4 Agilent tutorial on Spectrum Analyzers

The resolution bandwidth required to resolve signals separated by 50 kHz that differ by 40 dB in power is given by:

$$-40\ log_{10}\left(\left(\frac{50000}{\frac{B}{2\sqrt{\sqrt[4]{2}-1}}}\right)^2 - 1\right) = -40\ dB \tag{5}$$

Solving for the resolution bandwidth $B \approx 13115\ Hz$.

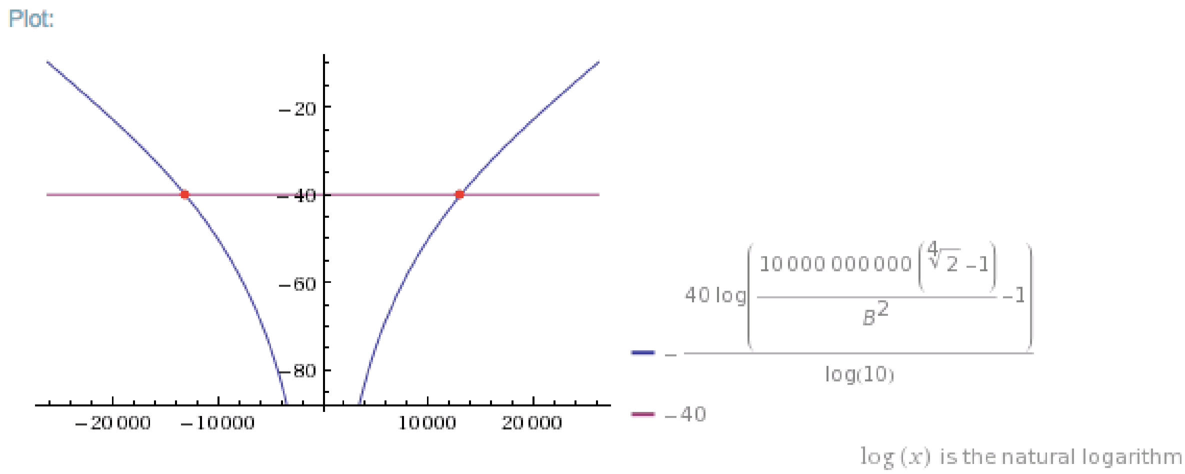

Figure 7: **Calculating the resolution bandwidth that meets design specifications.**

2 Matlab/Simulink Simulations