Département de gestion
des opérations et de la logistique

HEC MONTRÉAL

6-520-12A: PLANING AND CONTROL OF LOGISTICS SYSTEMS

Session 2: Forecasting

Multiplicative decomposition

Exercise: building a model in Excel

Set up: download the template workbook

Download the “E.Forecasting_exercises.xlsx” file from ZoneCours and open it in Excel. Select sheet “Decomposition”. Figure 1 shows the initial state of the model.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	Decomposition with s = 12													
2	Exploitable	Week	Demand	F(t)	Deseasonalized series			Detrented series		E(t)	Check Y(t)	Error	Abs. Err.	Sqrt. Error
3	Observation		Y(t)	S(t) x T(t)	12-MA	2x12-MA	T(t) - Regr.	Y(t)/T(t)	S(t)	Y(t)/(T(t)S(t))				
4		19	258											
5		20	231											
6		21	263											
7		22	308											
8		23	313											
9		24	293											
10	1	25	328								FALSE	-328,00	328,00	107584,00
11	2	26	349								FALSE	-349,00	349,00	121801,00
12	3	27	340								FALSE	-340,00	340,00	115600,00
13	4	28	309								FALSE	-309,00	309,00	95481,00
14	5	29	349								FALSE	-349,00	349,00	121801,00
15	6	30	366								FALSE	-366,00	366,00	133956,00
16	7	31	340								FALSE	-340,00	340,00	115600,00
17	8	32	302								FALSE	-302,00	302,00	91204,00
18	9	33	350								FALSE	-350,00	350,00	122500,00
19	10	34	362								FALSE	-362,00	362,00	131044,00
20	11	35	337								FALSE	-337,00	337,00	113569,00
21	12	36	326								FALSE	-326,00	326,00	106276,00
22	13	37	358								TRUE	-358,00	358,00	128164,00
23	14	38	359								TRUE	-359,00	359,00	128881,00
24	15	39	357								TRUE	-357,00	357,00	127449,00
25	16	40	341								TRUE	-341,00	341,00	116281,00
26	17	41	380								TRUE	-380,00	380,00	144400,00
27	18	42	404								TRUE	-404,00	404,00	163216,00
28	19	43	409								TRUE	-409,00	409,00	167281,00

	P	Q	R
2	Seasonal index		
3	Season	S(i)	Adj. s(i)
4	1		
5	2		
6	3		
7	4		
8	5		
9	6		
10	7		
11	8		
12	9		
13	10		
14	11		
15	12		
16		Sum	0

Causal | MovAvg | S.Exp.Smooth | Holt | Holt-Winters | Decomposition | Chart1 | Thail's U | Intervals | TS | +

Figure 1: the E.Forecasting_exercises.xlsx/Decomposition spreadsheet

Step 1: understanding the data

Our dataset contains demand information for 36 periods (weeks). The demand has a seasonal pattern (cycle) that repeats every 12 periods (weeks in our case). A brief description of the three columns holding the data follows.

A | Exploitable Observation: as the name suggests, this column contains an index for exploitable observations. Indeed, for a reason that you will understand later, the data for the first 6 and the last 6 periods is not exploitable for forecasting. This is the reason why the index starts at row 10 (week 25) and ends at row 33 (week 48).

B | Week: this column holds the identifier of the periods (weeks). The data on this column is for reference only, we will not use it on our computations.

C | Demand: as the name suggests, this column contains de observed demand **Y(t)** for each period (week). The blue and red colors help identifying observations belonging to the same demand cycle. Figure 2 shows a plot of the demand. For graphical convenience we use a line graph instead of an XY plot.

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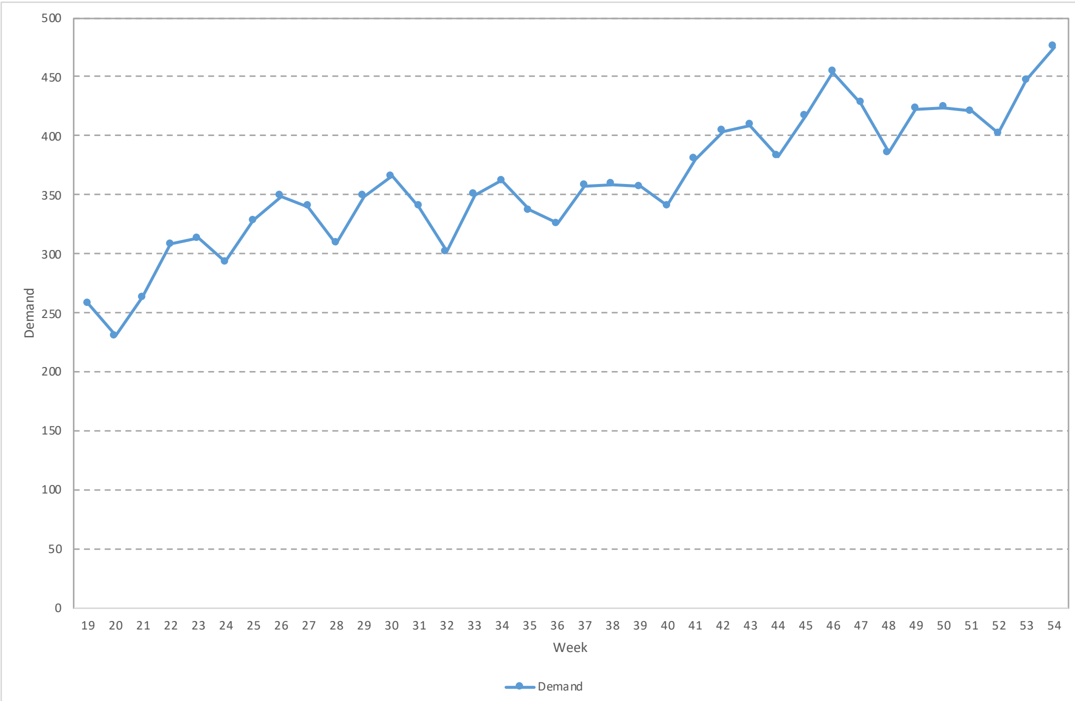

Figure 2: Demand observed in the past 36 weeks

Step 2: isolate the trend-cycle component of the demand (i.e., "deseasonalize" the series)

There are several techniques that can be applied to isolate the trend-cycle component of the demand. In our example, we will apply what is commonly known as moving average smoothing. In a nutshell, this technique consists in estimating the trend-cycle component at period at period t, by computing a moving average (MA) of order m (where m is equal to length of the demand cycle). More precisely, the trend-cycle component at period t, denoted, $\hat{T}(t)$ is computed as follows:

$$\hat{T}(t) = \frac{1}{m}\sum_{j=-k}^{k} Y(t+j)$$

where $m = 2k + 1$. This moving average is usually referred to as a m-MA. In our particular case, the demand cycle has 12 *seasons*. Therefore, we should use a m-MA with $m = 12$ and $k = 6.5$. The latter means that we need to center our MA around the "imaginary" season 6.5. To achieve this goal we will apply what is known as a 2×12-MA, that is, a MA of order 2 applied to a MA of order 12. A more thorough discussion on centered moving averages is out of the scope of this tutorial. We refer students interested on this topic to Chapter 6, Section 6.2 in [1].

First, we will compute in Column `D` a 12-MA centered in season 6. Go to Cell `D9` and insert the formula `=AVERAGE(C4:C15)` as shown in Figure 3a. Then, drag the formula down to cell `D34`. Next, we will compute in Column `E` a 2-MA centered in Season 7. Go to Cell E10 and insert the formula `=AVERAGE(D9:D10)` as shwon in Figure 3b. Then, drag the fomula down to Cell E34. What we obtain in Column `E` is $\hat{T}(t)$, a 12-MA centered in season 6.5. This "deseasonalized" series represents the isolated trend-cycle component of the demand. Figure 4 shows the original demand its trend-cycle component.

	A	B	C	D	E	F
1	Decomposition with s = 12					
2	Exploitable	Week	Demand	Deseasonalized series		F(t)
3	Observation		Y(t)	12-MA	2x12-MA	S(t) x T(t)
4		19	258			
5		20	231			
6		21	263			
7		22	308			
8		23	313			
9		24	293	=AVERAGE(C4:C15)		
10	1	25	328			
11	2	26	349			
12	3	27	340			
13	4	28	309			
14	5	29	349			
15	6	30	366			
16	7	31	340			
17	8	32	302			
18	9	33	350			
19	10	34	362			
20	11	35	337			
21	12	36	326			

3a

	A	B	C	D	E	F
1	Decomposition with s = 12					
2	Exploitable	Week	Demand	Deseasonalized series		F(t)
3	Observation		Y(t)	12-MA	2x12-MA	S(t) x T(t)
4		19	258			
5		20	231			
6		21	263			
7		22	308			
8		23	313			
9		24	293	308,9		
10	1	25	328	315,8	=AVERAGE(D9:D10)	
11	2	26	349	321,7		
12	3	27	340	328,9		
13	4	28	309	333,4		
14	5	29	349	335,4		
15	6	30	366	338,2		
16	7	31	340	340,7		
17	8	32	302	341,5		
18	9	33	350	342,9		
19	10	34	362	345,6		
20	11	35	337	348,2		
21	12	36	326	351,3		

3b

Figure 3: Isolating the trend-cycle component of the demand

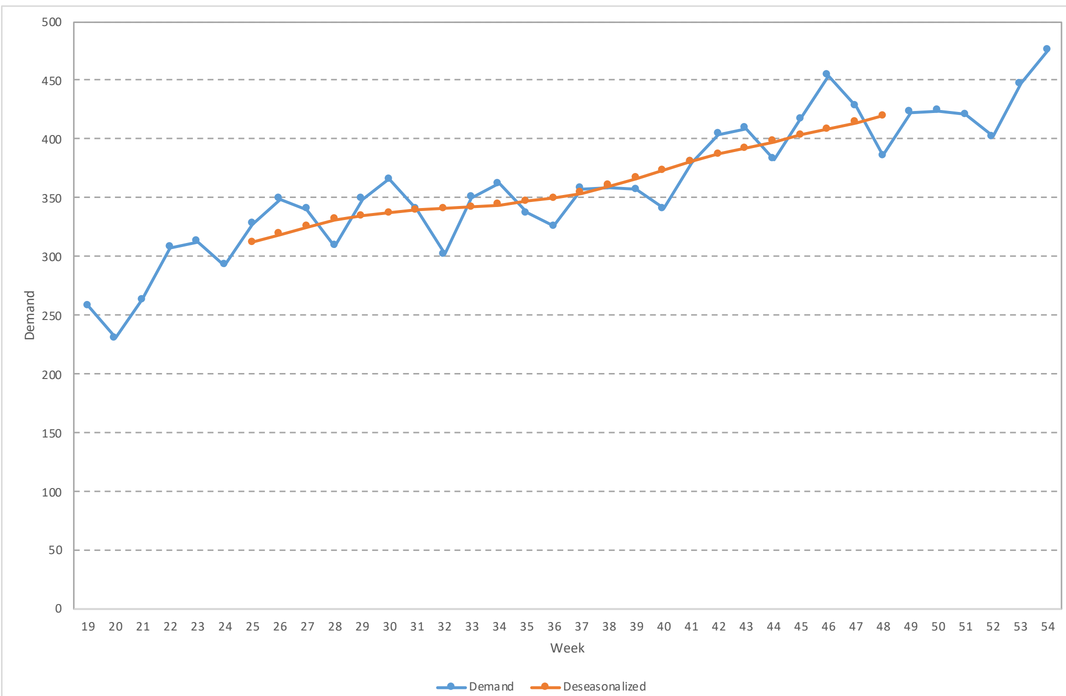

Figure 4: Demand and its isolated trend-cycle component $\hat{T}(t)$

Step 3: computing the seasonal indices

Now we are going to “detrend” the series so we can compute the seasonal indices (also known as seasonal factors or relatives). Since we are assuming a multiplicative model, the seasonal component of the demand at period *t* is equal to: $\hat{S}(t) = Y(t)/\hat{T}(t)$, where $\hat{T}(t)$ is the tread-cycle component we found in Step 2.

Go to Cell `F10` and insert the formula: `=C10/E10` as shown in Figure 5. Then, drag the formula down to cell `F34`.

	A	B	C	D	E	F	G	H
1	Decomposition with s = 12							
2	Exploitable	Week	Demand	Deseasonalized series		Detrented series		
3	Observation		Y(t)	12-MA	2x12-MA	Y(t)/2x12-MA	S(t)	
4		19	258					
5		20	231					
6		21	263					
7		22	308					
8		23	313					
9		24	293	308,9				
10	1	25	328	315,8	312,33	=C10/E10		
11	2	26	349	321,7	318,71			
12	3	27	340	328,9	325,29			
13	4	28	309	333,4	331,17			

Figure 5: “detrendeing” the series

To compute the seasonal indices, we average the seasonal component of the observed demand for similar seasons. For instance, week 19 and week 31 correspond to the same season (Season 1) in the demand cycle. The index for Season 1 is then be $\frac{\hat{S}(19)+\hat{S}(31)}{2} = \frac{1.05+1.01}{2}$, which is the average of the values in cells `F10` and `F22`.

To average the seasonal components we will use the secondary table in Range `P2:R16`. Go to Cell `Q4` and insert the formula `=AVERAGE(F10;F22)` as shown in Figure 6. Then, drag the formula down to cell `Q15`. Note that the values in Range `Q4:Q15` should add to 12 (the number of seasons in the demand cycle). In our case, however, the they add up to 12.08. To correct this issue, we adjust the seasonal index for each season as follows. Go to Cell `R4` and insert the following formula: `=Q4/AVERAGE($Q$4:$Q$15)`. Then, drag the formula down to Cell `R15`. Note that the sum in Cell `R16` now equals exactly 12.

		D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1																
2	and	Deseasonalized series		Detrented series	S(t)	T(t)	E(t)	Check Y(t)	F(t)	Error	Abs. Err.	Sqrt. Error		Seasonal index		
3	)	12-MA	2x12-MA	Y(t)/2x12-MA		Regr.	Y(t)/(T(t)S(t)		S(t) x T(t)					Season		
4	258													1	=AVERAGE(F10;F22)	
5	231													2		
6	263													3		
7	308													4		
8	313													5		
9	293	308,9												6		
10	328	315,8	312,33	1,05						-328,00	328,00	107584,00		7		
11	349	321,7	318,71	1,10						-349,00	349,00	121801,00		8		
12	340	328,9	325,29	1,05						-340,00	340,00	115600,00		9		
13	309	333,4	331,17	0,93						-309,00	309,00	95481,00		10		
14	349	335,4	334,42	1,04						-349,00	349,00	121801,00		11		
15	366	338,2	336,79	1,09						-366,00	366,00	133956,00		12		
16	340	340,7	339,42	1,00						-340,00	340,00	115600,00		Sum		0
17	302	341,5	341,08	0,89						-302,00	302,00	91204,00				
18	350	342,9	342,21	1,02						-350,00	350,00	122500,00				
19	362	345,6	344,25	1,05						-362,00	362,00	131044,00				
20	337	348,2	346,88	0,97						-337,00	337,00	113569,00				
21	326	351,3	349,75	0,93						-326,00	326,00	106276,00				
22	358	357,1	354,21	1,01						-358,00	358,00	128164,00				
23	359	363,8	360,46	1,00						-359,00	359,00	128881,00				

Figure 6: computing the seasonal indices

Now we are going to “import” the newly calculated season indices to our main table (model). Go to Cell `G10` and insert the following formula `=R4`. Then drag the formula down to Cell `G21`. The current state of the main table is shown in Figure 7. Repeat the operation starting at Cell `G22`. Note that observations belonging to the same season should have the same value in Column `G`. For instance, cells `G1` and `G13` should both have the same value (i.e., 1.05), so should cells `G2` and `G14` (i.e., 1,10).

Decomposition with s = 12							
Exploitable Observation	Week	Demand Y(t)	Deseasonalized series 12-MA	2x12-MA	Detrented series Y(t)/2x12-MA	S(t)	T(t) Regr.
	19	258					
	20	231					
	21	263					
	22	308					
	23	313					
	24	293	308,9				
1	25	328	315,8	312,33	1,05	1,02	
2	26	349	321,7	318,71	1,10	1,04	
3	27	340	328,9	325,29	1,05	1,00	
4	28	309	333,4	331,17	0,93	0,92	
5	29	349	335,4	334,42	1,04	1,01	
6	30	366	338,2	336,79	1,09	1,06	
7	31	340	340,7	339,42	1,00	1,02	
8	32	302	341,5	341,08	0,89	0,92	
9	33	350	342,9	342,21	1,02	1,02	
10	34	362	345,6	344,25	1,05	1,07	
11	35	337	348,2	346,88	0,97	1,00	
12	36	326	351,3	349,75	0,93	0,92	
13	37	358	357,1	354,21	1,01		
14	38	359	363,8	360,46	1,00		

Figure 7: computing the seasonal indices

Step 4: forecasting the trend-cycle component T(t)

We are going to forecast the trend-component of the demand using a linear regression built using the deseasonalized series (i.e., the trend-cycle component of the observed demand). There are several ways to run a linear regression in Excel (see for instance [2] for a tutorial). We are going to use the `SLOPE()` and `INTERSECT()` built-in functions. Remember that a linear model has the form $f(x) = bx + a$. The first function computes parameter b while the second computes parameter a. For both functions, the first parameter is the range holding the values of the independent variable x and the second parameter is the range holding the values of the dependent variable y.

Insert formula `=SLOPE($E$10:$E$33;$A$10:$A$33)*A10+INTERCEPT($E$10:$E$33;$A$10:$A$33)` into Cell `H10` as shown in Figure 8. Note that this formula simultaneously computes the linear model and predicts the trend-cycle component for the period in Cell `A10` (period 1 in this case). Next, drag the formula down to Cell `H33`. Figure 9 reflects the current state of the main table.

Optional remark: It is worth noting that our choice of the tool employed to run the linear regression may have an impact in the computational performance of the model. As a matter of fact, every time the spreadsheet is refreshed, Excel recomputes functions `SLOPE()` and `INTERSECT()` for every row. This basically means recomputing several times the same linear. In our example, this is not a perceivable issue, because the time series contains only a few tens of observations. However, if you are working with larger datasets, you may experience a degradation in the time needed to refresh the spreadsheet. A good alternative to solve this problem would be to run the linear regression only once on a separate sheet and "hard code" the value of the b and a coefficients in the formulas of Column `H`. For instance, in our case the linear model fitted to the deseasonalized series is: $f(x) = 4.43x + 279.67$. We could then write into Cell `H10` the formula `=4.43*A10+279.67` and drag the formula down to Cell `H33`. The reason why we favor our initial approach, is to have a model that is automatically updated if the demand data changes, rather than one that uses hard-coded values that must be recalculated "by hand" and then updated in the formulas. The latter is typically a source of errors, because users tend to forget to recompute and update those values.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	Decomposition with s = 12													
2	Exploitable	Week	Demand	Deseasonalized series		Detrented series	S(t)	T(t)	E(t)	Check Y(t)	F(t)	Error	Abs. Err.	Sqrt. Error
3	Observation		Y(t)	12-MA	2x12-MA	Y(t)/2x12-MA		Regr.	Y(t)/(T(t)S(t)		S(t) x T(t)			
4		19	258											
5		20	231											
6		21	263											
7		22	308											
8		23	313											
9		24	293	308,9										
10	1	25	328	315,8	312,33	1,05	1,02	=SLOPE($E$10:$E$33;$A$10:$A$33)*A10+INTERCEPT($E$10:$E$33;$A$10:$A$33)						
11	2	26	349	321,7	318,71	1,10	1,04					-349,00	349,00	121801,00
12	3	27	340	328,9	325,29	1,05	1,00					-340,00	340,00	115600,00
13	4	28	309	333,4	331,17	0,93	0,92					-309,00	309,00	95481,00
14	5	29	349	335,4	334,42	1,04	1,01					-349,00	349,00	121801,00
15	6	30	366	338,2	336,79	1,09	1,06					-366,00	366,00	133956,00
16	7	31	340	340,7	339,42	1,00	1,02					-340,00	340,00	115600,00
17	8	32	302	341,5	341,08	0,89	0,92					-302,00	302,00	91204,00
18	9	33	350	342,9	342,21	1,02	1,02					-350,00	350,00	122500,00
19	10	34	362	345,6	344,25	1,05	1,07					-362,00	362,00	131044,00
20	11	35	337	348,2	346,88	0,97	1,00					-337,00	337,00	113569,00
21	12	36	326	351,3	349,75	0,93	0,92					-326,00	326,00	106276,00
22	13	37	358	357,1	354,21	1,01	1,02					-358,00	358,00	128164,00
23	14	38	359	363,8	360,46	1,00	1,04					-359,00	359,00	128881,00
24	15	39	357	369,4	366,63	0,97	1,00					-357,00	357,00	127449,00
25	16	40	341	377,1	373,25	0,91	0,92					-341,00	341,00	116281,00
26	17	41	380	384,7	380,88	1,00	1,01					-380,00	380,00	144400,00
27	18	42	404	389,7	387,17	1,04	1,06					-404,00	404,00	163216,00
28	19	43	409	395,1	392,38	1,04	1,02					-409,00	409,00	167281,00
29	20	44	383	400,5	397,79	0,96	0,92					-383,00	383,00	146689,00
30	21	45	417	405,8	403,17	1,03	1,02					-417,00	417,00	173889,00
31	22	46	454	410,9	408,38	1,11	1,07					-454,00	454,00	206116,00
32	23	47	428	416,5	413,71	1,03	1,00					-428,00	428,00	183184,00
33	24	48	386	422,4	419,46	0,92	0,92					-386,00	386,00	148996,00

Figure 8: forecasting the trend-cycle component

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	Decomposition with s = 12													
2	Exploitable	Week	Demand	Deseasonalized series		Detrented series	S(t)	T(t)	E(t)	Check Y(t)	F(t)	Error	Abs. Err.	Sqrt. Error
3	Observation		Y(t)	12-MA	2x12-MA	Y(t)/2x12-MA		Regr.	Y(t)/(T(t)S(t)		S(t) x T(t)			
4		19	258											
5		20	231											
6		21	263											
7		22	308											
8		23	313											
9		24	293	308,9										
10	1	25	328	315,8	312,33	1,05	1,02	311			318,0	-9,97	9,97	99,44
11	2	26	349	321,7	318,71	1,10	1,04	315			327,3	-21,72	21,72	471,73
12	3	27	340	328,9	325,29	1,05	1,00	320			320,4	-19,55	19,55	382,22
13	4	28	309	333,4	331,17	0,93	0,92	324			297,2	-11,83	11,83	140,03
14	5	29	349	335,4	334,42	1,04	1,01	328			333,0	-16,02	16,02	256,56
15	6	30	366	338,2	336,79	1,09	1,06	333			352,2	-13,83	13,83	191,22
16	7	31	340	340,7	339,42	1,00	1,02	337			342,4	2,43	2,43	5,92
17	8	32	302	341,5	341,08	0,89	0,92	342			313,7	11,69	11,69	136,68
18	9	33	350	342,9	342,21	1,02	1,02	346			353,7	3,67	3,67	13,44
19	10	34	362	345,6	344,25	1,05	1,07	351			376,7	14,69	14,69	215,70
20	11	35	337	348,2	346,88	0,97	1,00	355			353,7	16,73	16,73	279,83
21	12	36	326	351,3	349,75	0,93	0,92	359			330,7	4,70	4,70	22,04
22	13	37	358	357,1	354,21	1,01	1,02	364			372,5	14,46	14,46	209,12
23	14	38	359	363,8	360,46	1,00	1,04	368			382,5	23,51	23,51	552,70
24	15	39	357	369,4	366,63	0,97	1,00	373			373,8	16,78	16,78	281,43
25	16	40	341	377,1	373,25	0,91	0,92	377			345,9	4,94	4,94	24,42
26	17	41	380	384,7	380,88	1,00	1,01	382			386,9	6,90	6,90	47,60
27	18	42	404	389,7	387,17	1,04	1,06	386			408,4	4,44	4,44	19,68
28	19	43	409	395,1	392,38	1,04	1,02	390			396,4	-12,58	12,58	158,15
29	20	44	383	400,5	397,79	0,96	0,92	395			362,5	-20,49	20,49	419,93
30	21	45	417	405,8	403,17	1,03	1,02	399			408,0	-9,00	9,00	81,02
31	22	46	454	410,9	408,38	1,11	1,07	404			433,8	-20,17	20,17	407,03
32	23	47	428	416,5	413,71	1,03	1,00	408			406,7	-21,29	21,29	453,09
33	24	48	386	422,4	419,46	0,92	0,92	413			379,6	-6,38	6,38	40,70

Figure 9: state of the table after forecasting the trend-cycle component

Step 5: forecasting

In theory we need to forecast not only the trend-cycle component but also the seasonal component. However, as Hyndman and Athanasopoulos point out (Section 6.8, Chapter 6 in [1]) “it is usually assumed that the seasonal component is unchanging, or changing extremely slowly” so we can forecast it using the naïve method. The latter simply means using the last available observed value (the values estimated in Step 3).

Recall that in the multiplicative decomposition model, the forecast is given by $F(t) = T(t) \times S(t)$. Therefore, we only need to multiply the values in Column `G` and `H` to obtain our forecast. Go to Cell `I10` and insert the following formula: `=H10*G10`. Drag the formula down to Cell `J33`. Figure 10 shows the forecast along with the observed demand, the isolated trend-cycle component (i.e., deseasonalized series), and the forecasted trend-cycle component.

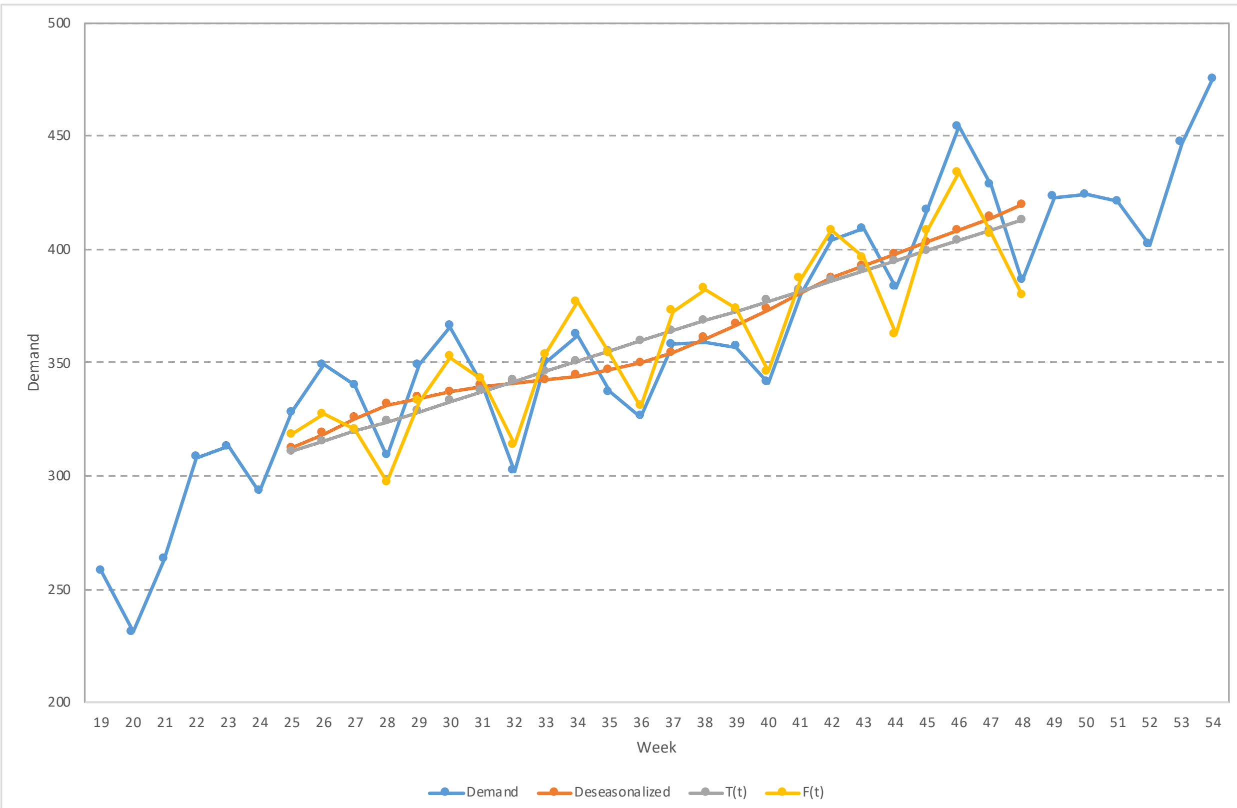

Figure 10: Demand, Isolated trend-cycle component, forecasted trend-cycle component, and forecast

Step 6: estimating the error E(t) component

Recall that the objective of a forecasting method is to predict the systematic component of a demand (see Step 5) and estimate the random component. Recall also that in the multiplicative decomposition model, the random or error component is given by $E(t) = \frac{Y(t)}{T(t) \times S(t)}$. We can then estimate the random component of the future demand by dividing the observed demand by our forecast.

Go to Cell `I10` and insert the formula `=C10/(H10*G10)`. Drag the formula down to Cell `I33`. Figure 11 shows the final state of the model. Column `L` computes, for the forecast, the single error, Column `M` the absolute error, and Column `N` the squared error.

Decomposition with s = 12

	Exploitable Observation	Week	Demand Y(t)	Deseasonalized series 12-MA	Deseasonalized series 2x12-MA	Detrented series Y(t)/2x12-MA	S(t)	T(t) Regr.	E(t) Y(t)/(T(t)S(t))	F(t) S(t) x T(t)	Check Y(t)	Error	Abs. Err.	Sqrt. Error
4		19	258											
5		20	231											
6		21	263											
7		22	308											
8		23	313											
9		24	293	308,9										
10	1	25	328	315,8	312,33	1,05	1,02	311	1,03	318,0	TRUE	-9,97	9,97	99,44
11	2	26	349	321,7	318,71	1,10	1,04	315	1,07	327,3	TRUE	-21,72	21,72	471,73
12	3	27	340	328,9	325,29	1,05	1,00	320	1,06	320,4	TRUE	-19,55	19,55	382,22
13	4	28	309	333,4	331,17	0,93	0,92	324	1,04	297,2	TRUE	-11,83	11,83	140,03
14	5	29	349	335,4	334,42	1,04	1,01	328	1,05	333,0	TRUE	-16,02	16,02	256,56
15	6	30	366	338,2	336,79	1,09	1,06	333	1,04	352,2	TRUE	-13,83	13,83	191,22
16	7	31	340	340,7	339,42	1,00	1,02	337	0,99	342,4	TRUE	2,43	2,43	5,92
17	8	32	302	341,5	341,08	0,89	0,92	342	0,96	313,7	TRUE	11,69	11,69	136,68
18	9	33	350	342,9	342,21	1,02	1,02	346	0,99	353,7	TRUE	3,67	3,67	13,44
19	10	34	362	345,6	344,25	1,05	1,07	351	0,96	376,7	TRUE	14,69	14,69	215,70
20	11	35	337	348,2	346,88	0,97	1,00	355	0,95	353,7	TRUE	16,73	16,73	279,83
21	12	36	326	351,3	349,75	0,93	0,92	359	0,99	330,7	TRUE	4,70	4,70	22,04
22	13	37	358	357,1	354,21	1,01	1,02	364	0,96	372,5	TRUE	14,46	14,46	209,12
23	14	38	359	363,8	360,46	1,00	1,04	368	0,94	382,5	TRUE	23,51	23,51	552,70
24	15	39	357	369,4	366,63	0,97	1,00	373	0,96	373,8	TRUE	16,78	16,78	281,43
25	16	40	341	377,1	373,25	0,91	0,92	377	0,99	345,9	TRUE	4,94	4,94	24,42
26	17	41	380	384,7	380,88	1,00	1,01	382	0,98	386,9	TRUE	6,90	6,90	47,60
27	18	42	404	389,7	387,17	1,04	1,06	386	0,99	408,4	TRUE	4,44	4,44	19,68
28	19	43	409	395,1	392,38	1,04	1,02	390	1,03	396,4	TRUE	-12,58	12,58	158,15
29	20	44	383	400,5	397,79	0,96	0,92	395	1,06	362,5	TRUE	-20,49	20,49	419,93
30	21	45	417	405,8	403,17	1,03	1,02	399	1,02	408,0	TRUE	-9,00	9,00	81,02
31	22	46	454	410,9	408,38	1,11	1,07	404	1,05	433,8	TRUE	-20,17	20,17	407,03
32	23	47	428	416,5	413,71	1,03	1,00	408	1,05	406,7	TRUE	-21,29	21,29	453,09
33	24	48	386	422,4	419,46	0,92	0,92	413	1,02	379,6	TRUE	-6,38	6,38	40,70
34		49	423											
35		50	424											

Seasonal index		
Season	S(i)	Adj. s(i)
1	1,03	1,023604
2	1,05	1,038572
3	1,01	1,002792
4	0,92	0,917213
5	1,02	1,013892
6	1,07	1,058044
7	1,02	1,015272
8	0,92	0,917991
9	1,03	1,021725
10	1,08	1,074475
11	1,00	0,996392
12	0,93	0,920027
	Sum	12

Figure 11: Demand, Isolated trend-cycle component, forecasted trend-cycle component, and forecast

References:

[1] Rob J Hyndman and George Athanasopoulos. Forecasting: Principles and Practice 2nd Edition. OTexts. Available as open content at: https://otexts.com/fpp2/.

[2] https://www.ablebits.com/office-addins-blog/2018/08/01/linear-regression-analysis-excel/. Last Accessed: 01/19/2019