

VECTORS

A quantity that have both magnitude and direction.

$$\vec{A} = a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}$$

$$\vec{A} = \langle a_1, a_2, a_3 \rangle$$

$$|\vec{A}|$$

$$P \rightarrow Q$$

It is called

$$\vec{A} = \overrightarrow{P\phi}$$

$$P \rightarrow \phi$$

$$\vec{A} = 3i + 2j + k = (3, 2, 1)$$

$$\vec{A} = 3\hat{i} + 2\hat{j} + \hat{k} = (3, 2, 1)$$

$$|\vec{A}| = \sqrt{3^2 + 2^2 + 1^2} = \sqrt{14}$$

$$|\vec{B}| = \sqrt{3^2 + 2^2} = \sqrt{13}$$

$$|\vec{A}|^2 = |\vec{B}|^2 + 1$$

Note

$$\vec{A} = \langle a_1, a_2, a_3 \rangle = |\vec{A}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

Note

If

$$\vec{A} = \langle a_1, a_2, a_3 \rangle$$

then

$$|\vec{A}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

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Vector Addition

If

$$\vec{A} = \langle a_1, a_2, a_3 \rangle, \quad \vec{B} = \langle b_1, b_2, b_3 \rangle$$

then

$$\vec{A} + \vec{B} = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$$

(One geometric interpretation, one numerical interpretation.)

Multiplication of a Vector by a Scalar

For a scalar k ,

$$k\vec{A}$$

is a vector in the same direction as \vec{A} if $k > 0$, and in the opposite direction if $k < 0$.

Dot Product

Definition:

If

$$\vec{A} = \langle a_1, a_2, a_3 \rangle, \quad \vec{B} = \langle b_1, b_2, b_3 \rangle$$

then

$$\vec{A} \cdot \vec{B} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

(This is a scalar.)

$$= a_1^2 + a_2^2 + a_3^2$$

$$\vec{C} = \vec{A} - \vec{B}$$

$$|\vec{C}|^2 = \vec{C} \cdot \vec{C}$$

$$= \vec{A} \cdot \vec{A} - \vec{A} \cdot \vec{B} - \vec{B} \cdot \vec{A} + \vec{B} \cdot \vec{B}$$

$$= |\vec{A}|^2 + |\vec{B}|^2 - 2 \vec{A} \cdot \vec{B}$$

Application of Dot Product

1) Computing lengths and angles

Example:

Let

$$P = (1, 0, 0), \quad Q = (0, 1, 0), \quad R = (0, 0, 2).$$

$$\vec{PQ} \cdot \vec{PR} = |\vec{PQ}| |\vec{PR}| \cos \theta$$

Hence,

$$\cos \theta = \frac{\vec{PQ} \cdot \vec{PR}}{|\vec{PQ}| |\vec{PR}|}$$

Now,

$$\vec{PQ} = Q - P = (-1, 1, 0)$$

$$\vec{PR} = R - P = (-1, 0, 2)$$

Dot product

Applications of Dot Product

Sign of $\sin \theta$

$$\sin \theta(\vec{A} \cdot \vec{B})$$

$$> 0 \quad \text{if } \theta < 90^\circ$$

$$= 0 \quad \text{if } \theta = 90^\circ$$

$$< 0 \quad \text{if } \theta > 90^\circ$$

2) Detect Or thogonality

Example:

$$x + 2y + 3z = 0$$

(Equation of a plane)

Let

$$\vec{OP} = (x, y, z)$$

and

$$\vec{A} = \langle 1, 2, 3 \rangle$$

Then

$$\vec{OP} \cdot \vec{A} = 0 \iff \vec{OP} \perp \vec{A}$$