Loop method for multiple pipelines

Example problem:

A portion of a municipal water distribution network is shown below:

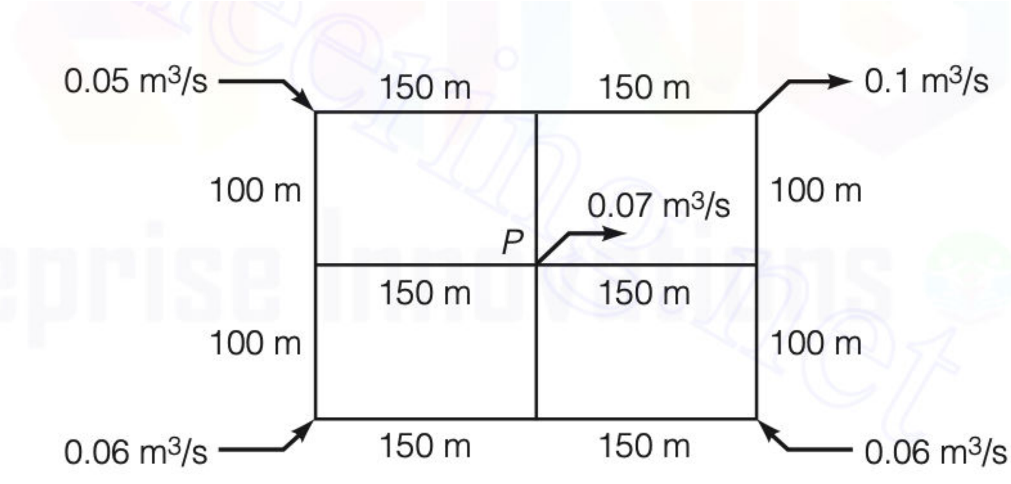


FIGURE 2.32: Four-loop pipe network

All pipes have diameters of 0.3m, and roughness coefficient k = 0.00026.

Use the Hardy Cross method to find the flow in each pipe.

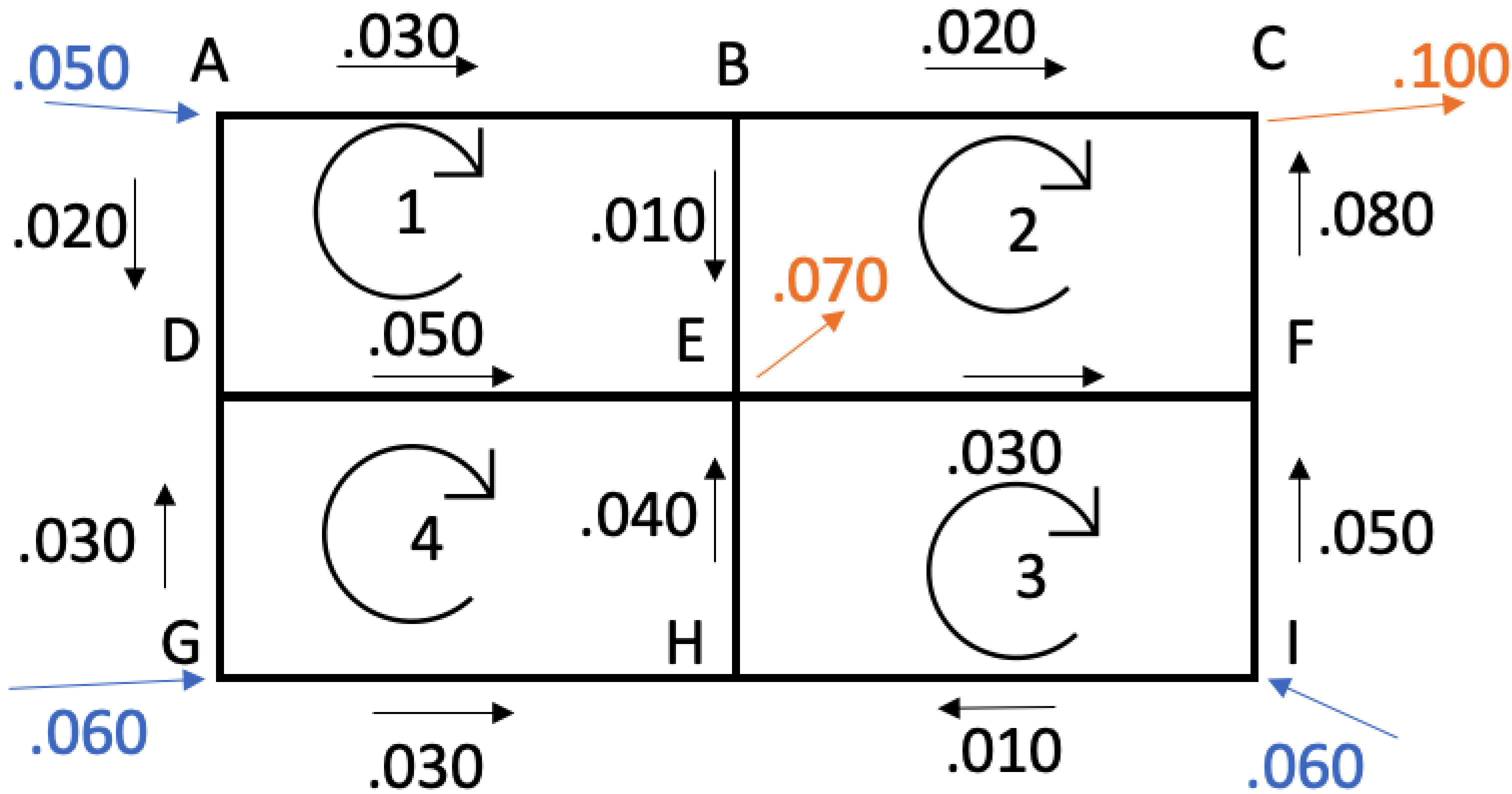
Steps to solve:

1) Make an initial guess for the flow distribution. This guess for flows must satisfy continuity (flow in must equal flow out at each junction).

2) Calculate the error in the estimated flows (ΔQ) for each loop. Add ΔQ to the flow estimate to get a new flow estimate.

3) Iterate until ΔQ is acceptably small (we converge on the actual flow rate).

Sketch out the problem and make initial guess for flow distribution



Here, we're going to store all of the pipe information in a table for convenience. In general, it would probably be easiest to create this table in a speadsheet and then import it into MATLAB, but in this example I'm creating the table directly to keep things self-contained.

The following code constructs a table with one row for each pipe, and columns for Q and L.

```
loop = [1;1;1;1;2;2;2;2;3;3;3;3;4;4;4];
pipe = ["AB"; "BE"; "ED"; "DA"; "BC"; "CF"; "FE"; "EB"; "EF"; "FI"; "IH";
"HE"; "DE"; "EH"; "HG"; "GD"];
Q = [0.030; 0.010; -0.050; -0.020; 0.020; -0.080; -0.030; -0.010; 0.030;
-0.050; 0.010; 0.040; 0.050; -0.040; -0.030; 0.030];
L = [150; 100; 150; 100; 150; 100; 150; 100; 150; 100; 150; 100; 150; 100;
150; 100];
pipes = table(loop, Q, L, 'RowNames', pipe)
```

pipes = 16×3 table

	loop	Q	L
1 AB	1	0.0300	150
2 BE	1	0.0100	100
3 ED	1	-0.0500	150
4 DA	1	-0.0200	100
5 BC	2	0.0200	150

	loop	Q	L
6 CF	2	-0.0800	100
7 FE	2	-0.0300	150
8 EB	2	-0.0100	100
9 EF	3	0.0300	150
10 FI	3	-0.0500	100
11 IH	3	0.0100	150
12 HE	3	0.0400	100
13 DE	4	0.0500	150
14 EH	4	-0.0400	100
15 HG	4	-0.0300	150
16 GD	4	0.0300	100

Head loss

Using Darcy-Weisbach:

$$h_L = rQ^n$$

Assuming all head losses are due to friction and the Darcy-Weisbach equation is used to calculate head losses:

$r = \frac{f\ L}{2g\ D\ A^2}$ and $n = 2$.

In MATLAB, we would write this as:

```
r = (f.*L./2.*g.*D.*A.^2)
```

Friction factor

Friction factor for rough pipe, turbulent flow:

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{k/D}{3.7}\right)$$

In MATLAB:

```
f = 1./((-2*log10(k./D./3.7)).^2)
```

Let's convert these to functions so they'll be more usable. In MATLAB, local function definitions have to be at the end of the file, so you'll find them at the very end.

Let's go ahead and define the other variables that were given in the problem. We were given k and D, and we're going to use n=2 for this problem.

```
k = 0.00026;
D = 0.3;
```

Now, we can calculate the friction factor and r for each pipe:

```
f = calc_f_turbulent(k, D)
```

```
f = 0.0190
```

Calculate r for each pipe:

```
pipes.r = calc_r(D, pipes.L, f)
```

```
pipes = 16×4 table
```

	loop	Q	L	r
1 AB	1	0.0300	150	96.7496
2 BE	1	0.0100	100	64.4998
3 ED	1	-0.0500	150	96.7496
4 DA	1	-0.0200	100	64.4998
5 BC	2	0.0200	150	96.7496
6 CF	2	-0.0800	100	64.4998
7 FE	2	-0.0300	150	96.7496
8 EB	2	-0.0100	100	64.4998
9 EF	3	0.0300	150	96.7496
10 FI	3	-0.0500	100	64.4998
11 IH	3	0.0100	150	96.7496
12 HE	3	0.0400	100	64.4998
13 DE	4	0.0500	150	96.7496
14 EH	4	-0.0400	100	64.4998
15 HG	4	-0.0300	150	96.7496
16 GD	4	0.0300	100	64.4998

Calculate the error in the estimated flows

Recall the Hardy Cross equation for the estimate error:

$$\Delta Q_i = -\frac{\sum_{j=1}^{\mathrm{NP}(i)} r_{\mathrm{ij}} Q_j |Q_j|^{n-1}}{\sum_{j=1}^{\mathrm{NP}(i)} \mathrm{n} r_{\mathrm{ij}} |Q_j|^{n-1}}$$

Again, we'll translate this into functions, which are found at the end of the file.

Next, we'll calculate the error for our initial estimate.

```
% Calculate error for initial estimate
 [delta_Q, pipes] = calc_loop(pipes, n)
```

```
Loop : 1
delta_Q: 0.00900
Loop : 2
delta_Q: 0.02158
Loop : 3
delta_Q: 0.00274
Loop : 4
delta_Q: -0.00068
delta_Q = 4×1
    0.0090
    0.0216
    0.0027
   -0.0007
pipes = 16×4 table
```

	loop	Q	L	r
1 AB	1	0.0390	150	96.7496
2 BE	1	-0.0026	100	64.4998
3 ED	1	-0.0403	150	96.7496
4 DA	1	-0.0110	100	64.4998
5 BC	2	0.0416	150	96.7496
6 CF	2	-0.0584	100	64.4998
7 FE	2	-0.0112	150	96.7496
8 EB	2	0.0026	100	64.4998
9 EF	3	0.0112	150	96.7496
10 FI	3	-0.0473	100	64.4998
11 IH	3	0.0127	150	96.7496
12 HE	3	0.0434	100	64.4998
13 DE	4	0.0403	150	96.7496

	loop	Q	L	r
14 EH	4	-0.0434	100	64.4998
15 HG	4	-0.0307	150	96.7496
16 GD	4	0.0293	100	64.4998

Then, we need to iterate until the error is acceptable.

```
tolerance = 0.000001;
iteration = 1;
while mean(delta_Q) > tolerance
    fprintf("Iteration: %0.0d", iteration)
 [delta_Q, pipes] = calc_loop(pipes, n);
 iteration = iteration + 1;
end
```

```
Iteration: 1
Loop : 1
delta_Q: 0.00107
Loop : 2
delta_Q: 0.00361
Loop : 3
delta_Q: 0.00008
Loop : 4
delta_Q: 0.00038
Iteration: 2
Loop : 1
delta_Q: 0.00026
Loop : 2
delta_Q: 0.00002
Loop : 3
delta_Q: 0.00014
Loop : 4
delta_Q: 0.00012
Iteration: 3
Loop : 1
delta_Q: 0.00005
Loop : 2
delta_Q: 0.00001
Loop : 3
delta_Q: 0.00004
Loop : 4
delta_Q: 0.00003
Iteration: 4
Loop : 1
delta_Q: 0.00001
Loop : 2
delta_Q: 0.00000
Loop : 3
delta_Q: 0.00001
Loop : 4
delta_Q: 0.00001
Iteration: 5
Loop : 1
delta_Q: 0.00000
Loop : 2
delta_Q: 0.00000
```

```
Loop : 3
delta_Q: 0.00000
Loop : 4
delta_Q: 0.00000
Iteration: 6
Loop : 1
delta_Q: 0.00000
Loop : 2
delta_Q: 0.00000
Loop : 3
delta_Q: 0.00000
Loop : 4
delta_Q: 0.00000
```

```
pipes
```

pipes = 16×4 table

	loop	Q	L	r
1 AB	1	0.0404	150	96.7496
2 BE	1	-0.0048	100	64.4998
3 ED	1	-0.0394	150	96.7496
4 DA	1	-0.0096	100	64.4998
5 BC	2	0.0452	150	96.7496
6 CF	2	-0.0548	100	64.4998
7 FE	2	-0.0078	150	96.7496
8 EB	2	0.0048	100	64.4998
9 EF	3	0.0078	150	96.7496
10 FI	3	-0.0470	100	64.4998
11 IH	3	0.0130	150	96.7496
12 HE	3	0.0432	100	64.4998
13 DE	4	0.0394	150	96.7496
14 EH	4	-0.0432	100	64.4998
15 HG	4	-0.0301	150	96.7496
16 GD	4	0.0299	100	64.4998

```
function f = calc_f_turbulent(k, D)
    f= 1./((-2*log10(k./D./3.7)).^2);
end

function r = calc_r(D, L, f)
%r=fL/(2gDA^2)
    g=9.81;
```

```
    A=0.25*pi*D.^2;
    r=f.*L./(2.*g.*D.*A.^2);
end

function [num] = calc_delta_Q_num(r, Q, n)
    num = r.*Q.*abs(Q).^(n-1);
end

function [denom] = calc_delta_Q_denom(r, Q, n)
    denom = n.*r.*abs(Q).^(n-1);
end


function [delta_Q, pipes] = calc_loop(pipes, n)
    n_loops = length(unique(pipes.loop));
    delta_Q = ones(n_loops,1);

    shared_pipes = intersect(pipes.Row, reverse(pipes.Row));

    for i=1:n_loops
        loop_pipes = pipes(pipes.loop==i, :);
        loop_pipes.num = calc_delta_Q_num(loop_pipes.r, loop_pipes.Q, n);
        loop_pipes.denom = calc_delta_Q_denom(loop_pipes.r, loop_pipes.Q,
n);
        delta_Q(i) = -1* sum(loop_pipes.num) / sum(loop_pipes.denom);

        fprintf("Loop : %0.0f\n", i)
        fprintf("delta_Q: %0.5f\n", delta_Q(i))

        % Update the flow estimates
        pipes{pipes.loop==i, "Q"} = loop_pipes.Q + delta_Q(i);

        loop_shared_pipes = intersect(loop_pipes.Row, shared_pipes);
        for j=1:length(loop_shared_pipes)
            shared_pipe = loop_shared_pipes{j};
            shared_pair = reverse(loop_shared_pipes{j});
            pipes{shared_pair, "Q"} = -pipes{shared_pipe, "Q"};
        end
    end
end
```