Calculus Challenge Problems

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NO TEST MATERIAL ON THIS PAGE

1. The equation $\mathbf{y=e^{(.5x)}}$ and the point **(2,10)** are in the x y plane. What is the minimum distance between the point and curve?

2. The equation $\mathbf{y=x^2+1}$ and **its inverse** are in the x y plane. What is the minimum distance between the two curves?

3. Particle A's motion is defined by two parametric curves: $\mathbf{x=t}$ & $\mathbf{y=e^t}$ where **t** is seconds. Particle B is at the point **(0.5, 6)** when **t = 0**.

Particle B can travel at a maximum speed of **3 units/second** and does not have a specified path but must travel in a straight line.

(A) If particle B wants to intercept Particle A in as little time as possible and can leave at the same time Particle A leaves (t=0), what is the closest position to Particle A's start it can intercept Particle A? Give the x and y coordinate of this position.

(B) What is the time interval in which Particle B can intercept Particle A before Particle A begins to accelerate too quickly to catch?

4. The equation **$y=x^2$** and the polar equation **$r=5-4\cos\theta$** create an enclosed area with the x-axis in the first quadrant.

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(A) Determine the area.

(B) What is the maximum y value for **$y=x^2$** in polar form on the interval **$(0, 2\pi)$**?

5. $\mathbf{y=x^{1/2}}$ is bounded from $\mathbf{x=0}$ to $\mathbf{x=4}$. Each vertical cross section of this region is the area under the graph of $\mathbf{y=e^x}$ bounded by $\mathbf{x=0}$ and the y value of $\mathbf{y=x^{1/2}}$ at that cross section (think of the x bounds for $\mathbf{y=e^x}$ as the vertical distance of the curve $\mathbf{y=x^{1/2}}$ at that cross section). What is the <u>volume</u> of this region?

6. The polar curve $\mathbf{r = \sin\theta}$ is to be integrated from $\mathbf{(0, \pi)}$, where each cross section is an equilateral triangle that stands up perpendicularly to the polar plane with its base parallel to the plane. What is the length along the top edge of this shape? (Assume the top edge is formed by the tops of the equilateral triangles)

7. Integrate the following:

$$\int \frac{1}{x^2 \cdot \sqrt[3]{\left(x^3+1\right)^2}}dx$$

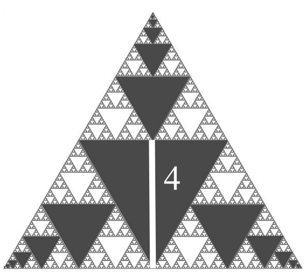
8. Integrate the following:

$$\int \frac{1}{\sqrt{x} + \sqrt[3]{x}}dx$$

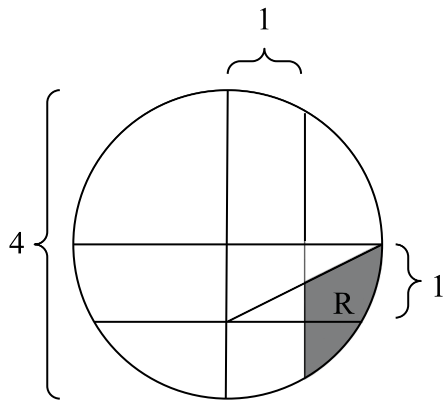
9. Determine the sum:

$$\sum_{n=1}^{\infty} \frac{4}{n^2 + 6n + 5}$$

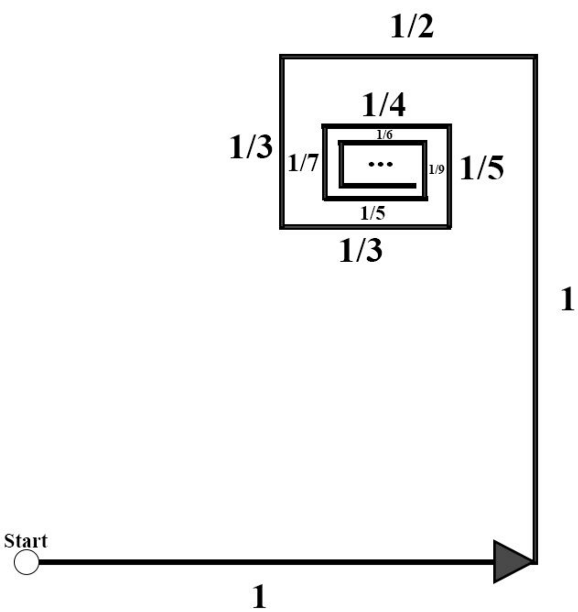
10. The figure below shows an equilateral triangle fractal. If the largest dark grey triangle has a height of 4, what is the total area of the infinite amount of dark grey triangles?



11. Determine the area of the shaded region of the circle:



12. If a particle starts at the origin and follows the infinite path shown below, how far is the particle from the start after travelling for infinite time (displacement)? How far as the particle travelled after travelling for infinite time?



$$1, 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{5}, \frac{1}{4}, \frac{1}{7}, \frac{1}{5}, \frac{1}{9}, \frac{1}{6}, \frac{1}{11}, \frac{1}{7}, \frac{1}{13}, \frac{1}{8}, \frac{1}{15} \cdots$$

13. The equations $\mathbf{y = -e^x}$ and $\mathbf{y = (x-2)^2 + 3}$ are in the x y plane. What is the minimum distance between the two curves?

14. There is a circular tower with radius **6** meters. A guard dog is tethered to this tower by a rope **3π** meters long. What is the total area that the guard dog can protect?

15. There is a wooden plank inclined **30°**. **10 feet** up the plane is a circular wheel with a **radius of 1**. If a dot is drawn on the part where the wheel is initially touching the plank, what is the maximum height of the dot as the wheel rolls down the plank?

Given: The parametric equations for a cycloid (the curve traced by a point on a circle as it rolls along a straight line without slipping) with radius r are: $x = r(\theta - \sin\theta)$ and $y = r(1 - \cos\theta)$

16. The equations $\mathbf{y=x^2}$ & $\mathbf{y=x}$ create an enclosed region. Find the volume of this region revolved around $\mathbf{y=x}$.

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