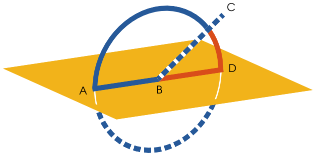
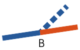


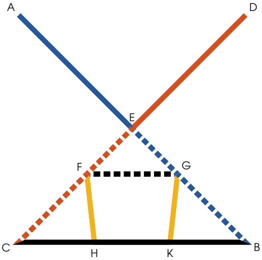

A PART *of a straight line cannot be in the plane of reference and a part in plane more elevated.*

For, if possible, let a part  of the straight line  be in the plane of reference  , and a part  be in a plane more elevated.

Then there is in the plane of reference  some straight line continuous with  in a straight line. Let it be  . Therefore  is a common segment of the two straight lines  and , which is impossible, since, if we describe a circle  with center  and radius , then the diameters cut off unequal circumferences  and  of the circle.

Therefore, a part of a straight line cannot be in the plane of reference and a part in plane more elevated.

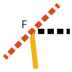
Q. E. D.

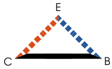


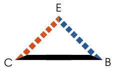
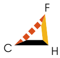
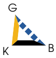

*If two straight lines cut one another, then they lie in one plane; and every triangle lies in one plane.*

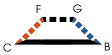
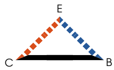
For let the two straight lines  and  cut one another at the point .

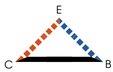
I say that  and  lie in one plane, and that every triangle lies in one plane.

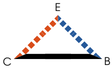
Take the points  and  at random on  and , join  and , and draw  and  across.

I say first that the triangle  lies in one plane.

For, if part of the triangle , either  or , is in the plane of reference, and the rest in another, then a part also of one of the straight lines  or  is in the plane of reference, and a part in another. (pr. 1)

But, if the part  of the triangle  is in the plane of reference, and the rest in another, then a part also of both the straight lines  and  is in the plane of reference and a part in another, which was proved absurd.

Therefore the triangle  lies in one plane. (pr. 1)

But, in whatever plane the triangle 

lies, each of the straight lines 

and 

also lies, and in whatever plane each of the straight lines 

and 

lies, 

and 

also lie.

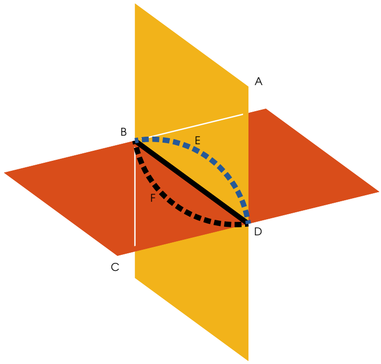
Therefore the straight lines 

and 

lie in one plane; and every triangle lies in one plane.

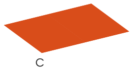
Therefore, if two straight lines cut one another, then they lie in one plane; and every triangle lies in one plane.

Q. E. D.




I*f two planes cut one another, then their intersection is a straight line.*

Let two planes 

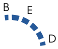
and 

cut one another, and let the line 

be their intersection.

I say that the line 

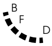
is a straight line.

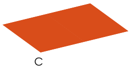
For, if not, join the straight line 

from 

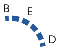
to 

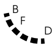
in the plane 

, and the straight line 

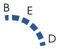
in the plane 

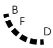
.

Then the two straight lines 

and 

have the same ends and clearly enclose an area, which is absurd.

Therefore 

and 

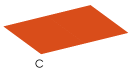
are not straight lines.

Similarly we can prove that neither is there any other straight line joined from 

to 

except 

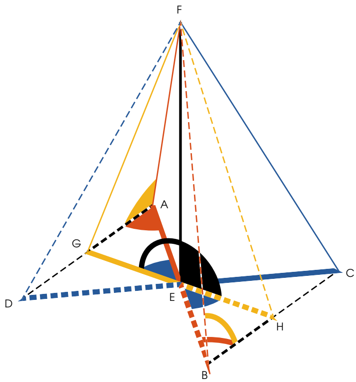
, the intersection of the planes 

and 

.

Therefore, if two planes cut one another, then their intersection is a straight line.

Q. E. D.




I*f a straight line is set up at right angles to two straight lines which cut one another at their common point of section, then it is also at right angles to the plane passing through them.*

For let a straight line EF be set up at right angles to the two straight lines AB and CD at E, the point at which the lines cut one another.

I say that EF is also at right angles to the plane passing through AB and CD. (pr. 2 ??)

Cut off AE, EB, CE, and ED equal to one another. Draw any straight line GH across through E at random. Join AD and CB, and join FA, FG, FD, FC, FH, and FB from a point F taken at random on EF. (pr. ?? ??)

Now, since the two straight lines AE and ED equal the two straight lines CE and EB and contain equal angles, therefore the base AD equals the base CB, and the triangle DAE equals the triangle ECB, so that DAE = EBC. (pr. ?? ??)

But GAE = EBH, ∴ GAE and EHB are two triangles which have two angles equal to two angles respectively, and one side equal to one side, namely that adjacent to the equal angles, that is to say, AE = EB. Therefore they also have the remaining sides equals to the remaining sides, that is, GE = EH, and AG = BH.

And, since AE = EB, while FE is common and at right angles, therefore the base FA equals the base FB. (pr. ??)

For the same reason, FC = FD. (pr. ??)

And, since AD = CB, and FA = FB, the two sides FA and AD equal the two sides FB and BC respectively, and the base FD was proved equal to the the base FC, therefore the angle 

also equals the angle 

. (pr. ??)

And since, again, AG was proved equal to BH, and further, FA = FB, the two sides FA and AG equal the two sides FB and BH, and the angle 

was proved equal to the angle 

, therefore the base FG = FH. (pr. ??)

Again, since GE was proved equal to EH, and EF is common, the two sides GE and EF equal the two sides HE and EF, and the base FG equals the base FH, ∴ 

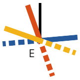
= 

.

Therefore each of the angles 

and 

is right.

∴ FE is at right angles to GH drawn at random through .


Similarly we can prove that FE also makes right angles with all the straight lines which meet it and are in the plane of reference. (def. ??)

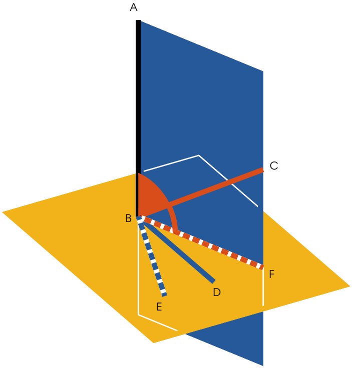
But a straight line is at right angles to a plane when it makes right angles with all the straight lines which meet it and are in that same plane, therefore FE is at right angles to the plane of reference.

But the plane of reference is the plane through the straight lines AB and CD.

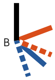
Therefore FE is at right angles to the plane through AB and CD.

Therefore if a straight line is set up at right angles to two straight lines which cut one another at their common point of section, then it is also at right angles to the plane passing through them.

Q. E. D.




*If a straight line is set up at right angles to three straight lines which meet one another at their common point of section, then the three straight lines lie in one plane.*

Let a straight line AB be set up at right angles to the three straight lines BC, BD, and BE at their intersection .

I say that BC, BD, and BE lie in one plane. (pr. 3)

For suppose that they do not, but, if possible, let BD, and BE lie in the plane of reference  and BC in one more elevated. Produce the plane  through AB and BC.

 intersects  in a straight line. Let the intersection be BF. Therefore the three straight lines AB, BC, and BF lie in one plane , namely that drawn through AB and BC. (pr. 4)

Now, since AB is at right angles to each of the straight lines BD and BE, therefore AB is also at right angles to the plane  through BD and BE.

But the plane  through 

and 

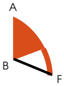
is the plane of reference, therefore 

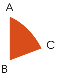
is at right angles to the plane of reference. (def. ??)

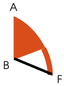
Thus 

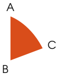
also makes right angles with all the straight lines which meet it and lie in the plane of reference.

But 

, which is the plane of reference, meets it, therefore the angle 

is right. And, by hypothesis, the angle 

is also right, therefore the angle 

= 

, and they lie in one plane, which is impossible.

Therefore the straight line 

is not in a more elevated plane. Therefore the three straight lines 

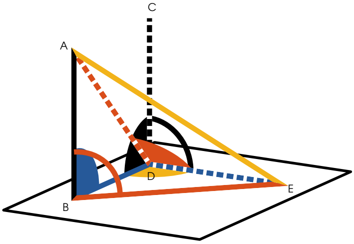
, 

, and 

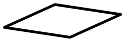
lie in one plane.

Therefore, if a straight line is set up at right angles to three straight lines which meet one another at their common point of section, then the three straight lines lie in one plane.

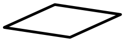
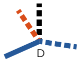
Q. E. D.

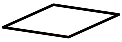


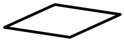

*If two straight lines are at right angles to the same plane, then the straight lines are parallel.*

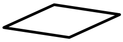
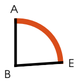
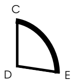
Let the two straight lines  and  be at right angles to the plane of reference .

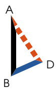
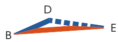
I say that  ∥ .

Let them meet the plane of reference  at the points  and . (pr. ??, pr. ??)

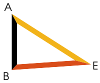
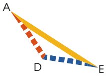
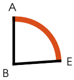
Join the straight line . Draw  in  ⊥ , and make  = . (def. ??)

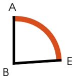
Now, since  is at right angles to , it also makes right angles with all the straight lines which meet it and lie in the plane of reference.

But each of the straight lines  and  lies in  and meets , therefore each of the angles  and  is right. For the same reason each of the angles  and  is also right. (pr. ??)

And since in  and   = , and  is common, therefore the two sides  and  equal the two sides  and . And they include right angles  and

, therefore the base AD = BE. (pr. ??)

And, since in  and  AB = DE while AD = BE, the two sides AB and BE equal the two sides ED and DA, and AE is their common base, therefore the angle  equals the angle .

But the angle  is right, therefore the angle  is also right. Therefore ED is at right angles to DA. (pr. 5)

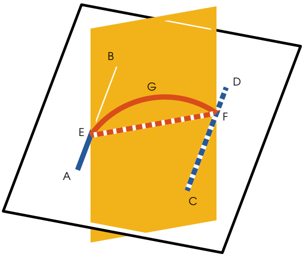
But it is also at right angles to each of the straight lines BD and DC, therefore ED is set up at right angles to the three straight lines BD, DA, and DC at their intersection. Therefore the three straight lines BD, DA, and DC lie in one plane. (pr. 2)

But in whatever plane DB and DA lie, AB also lies, for every triangle lies in one plane.

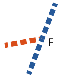
Therefore the straight lines AB, BD, and DC lie in one plane. And each of the angles  and  is right, therefore AB is parallel to CD. (pr. ??)

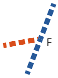
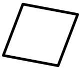
Therefore, if two straight lines are at right angles to the same plane, then the straight lines are parallel.

Q. E. D.

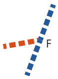
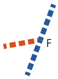



IF *two straight lines are parallel and points are taken at random on each of them, then the straight line joining the points is in the same plane with the parallel straight lines.*

Let  and  be two parallel straight lines, and let points  and  be taken at random on them respectively.

I say that the straight line joining the points  and  lies in the same plane  with the parallel straight lines. (pr. 3)

For suppose it is not, but, if possible, let it be in a more elevated plane. Draw a plane  through . Its intersection with the plane of reference is a straight line. Let it be .

∴ the two straight lines  and  enclose an area, which is impossible. ∴ the straight line joined from  to  is not in a plane more elevated. ∴ the straight line joined from  to  lies in the plane through the parallel straight lines  and .

∴, if two straight lines are parallel and points are taken at random on each of them, then the straight line joining the points is in the same plane with the parallel straight lines.

Q. E. D.