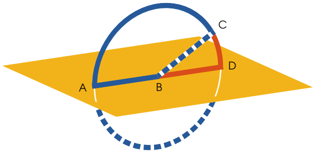
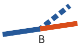
A PART *of a straight line cannot be in the plane of reference and a part in a plane more elevated.*



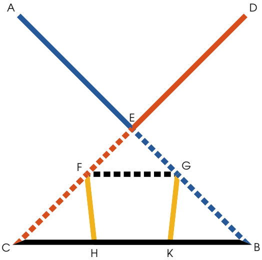
For, if possible, let a part  of the straight line  be in the plane of reference , and a part  be in a plane more elevated.

There then will be in  some straight line continuous with  in a straight line. Let it be .

Therefore  is a common segment of the two straight lines  and , which is impossible, inasmuch as, if we describe a circle  with centre  and radius , then the diameters will cut off unequal circumferences  and  of the circle.

Therefore, a part of a straight line cannot be in the plane of reference and a part in a plane more elevated.

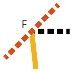
Q. E. D.

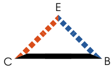


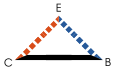
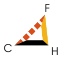
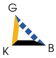
IF *two straight lines cut one another, they are in one plane, and every triangle is in one plane.*

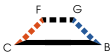
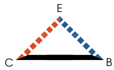
For let the two straight lines  and  cut one another at the point .

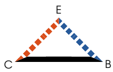
I say that  and  are in one plane, and every triangle is in one plane.

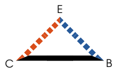
Take the points  and  at random on  and , let  and  be joined, and let  and  be drawn across.

I say first that the triangle  lies in one plane.

For, if part of the triangle , either  or , is in the plane of reference, and the rest in another, then a part also of one of the straight lines  or  is in the plane of reference, and a part in another.

But, if the part  of the triangle  is in the plane of reference, and the rest in another, then a part also of both the straight lines  and  is in the plane of reference and a part in another, which was proved absurd. (pr. 1)

Therefore the triangle  lies in one plane.

But, in whatever plane the triangle  is, each of the straight lines  and  also is, and in whatever plane each of the straight lines 

and 

is, 

and 

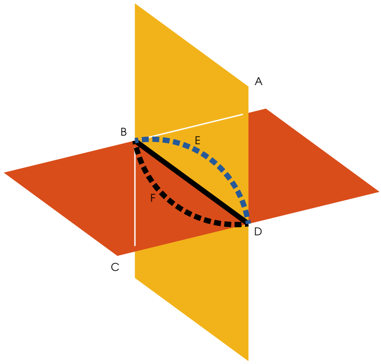
also are. (pr. 1)

Therefore the straight lines 

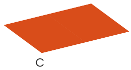
and 

are in one plane; and every triangle is in one plane.

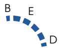
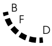
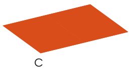
Q. E. D.

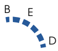
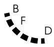


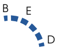
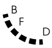
If two planes cut one another, their common section is a straight line.

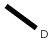
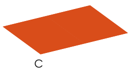
For let the two planes  and  cut one another, and let the line  be their common section.

I say that the line  is a straight line.

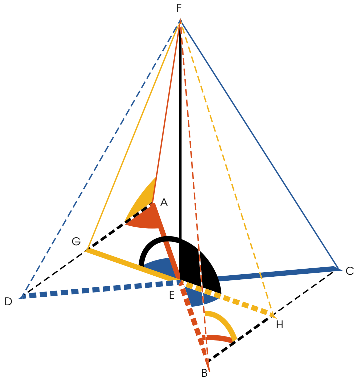
For, if not, join the straight line  from  to  in the plane , and the straight line  in the plane .

Then the two straight lines  and  have the same ends and clearly enclose an area, which is absurd.

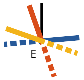
∴  and  are not straight lines.

Similarly we can prove that neither will there be any other straight line joined from  to  except , the common section of the planes  and .

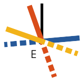
Q. E. D.

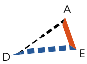
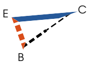
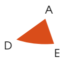
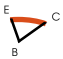


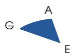
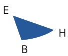
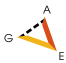
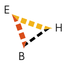
I*F a straight line be set up at right angles to two straight lines which cut one another, at their common point of section, it will also be at right angles to the plane passing through them.*

For let a straight line EF be set up at right angles to the two straight lines AB and CD at , the point at which the lines cut one another.

I say that EF is also at right angles to the plane passing through AB and CD.

Cut off AE, EB, CE, and ED equal to one another. Draw any straight line GH across through  at random. Join AD and CB, and join FA, FG, FD, FC, FH, and FB from a point  taken at random on EF. (pr. 2 ??)

Now, since the two straight lines AE and ED equal the two straight lines CE and EB and contain equal angles, therefore the base AD equals the base CB, and the triangle  equals the triangle , so that  = . (pr. ?? ??)

But  = , ∴  and  are two triangles which have two angles equal to two angles respectively, and one side equal to one side, namely that adjacent to the equal angles, that is to say, AE = EB. Therefore they also have the remaining sides equals to the remaining sides, that is, GE = EH, and AG = BH. (pr. ?? ??)

And, since AE = EB, while FE is common and at right angles, therefore the base FA equals the base FB.

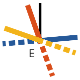
For the same reason, FC = FD. (pr. ??)

And, since AD = CB, and FA = FB, the two sides FA and AD equal the two sides FB and BC respectively, and the base FD was proved equal to the the base FC, therefore the angle FAD also equals the angle FBC. (pr. ??)

And since, again, AG was proved equal to BH, and further, FA = FB, the two sides FA and AG equal the two sides FB and BH, and the angle FAD was proved equal to the angle FBC, therefore the base FG = FH. (pr. ??)

Again, since GE was proved equal to EH, and EF is common, the two sides GE and EF equal the two sides HE and EF, and the base FG equals the base FH, ∴ GEF = FEH. (pr. ??)

Therefore each of the angles GEF and FEH is right.

∴ FE is at right angles to GH drawn at random through 

.

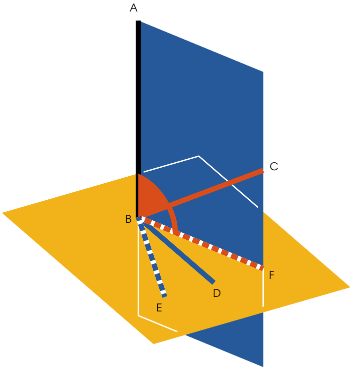
Similarly we can prove that FE also makes right angles with all the straight lines which meet it and are in the plane of reference.

But a straight line is at right angles to a plane when it makes right angles with all the straight lines which meet it and are in that same plane, therefore FE is at right angles to the plane of reference. (def. ??)

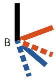
But the plane of reference is the plane through the straight lines AB and CD.

Therefore FE is at right angles to the plane through AB and CD.

Q. E. D.



If a straight line be set up at right angles to three straight lines which meet one another, at their common point of section, the three straight are in one plane.

For let a straight line AB be set up at right angles to the three straight lines BC, BD and BE at their point of meeting at B .

I say that BC, BD, and BE are in one plane.

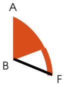
For suppose that they are not, but, if possible, let BD and BE be in the plane of reference  and BC in one more elevated. Produce the plane  through AB and BC. (pr. 3)

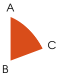
 intersects  in a straight line. Let the intersection be BF. Therefore the three straight lines AB, BC, and BF are in one plane , namely that drawn through AB and BC.

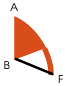
Now, since AB is at right angles to each of the straight lines BD and BE, therefore AB is also at right angles to the plane  through BD and BE. (pr. 4)

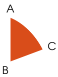
But the plane  through BD and BE is the plane of reference, therefore AB is at right angles to the plane of reference.

Thus AB also makes right angles with all the straight lines which meet it and lie in the plane of reference. (def. ??)

But BF, which is the plane of reference, meets it, therefore the angle 

is right. And, by hypothesis, the angle 

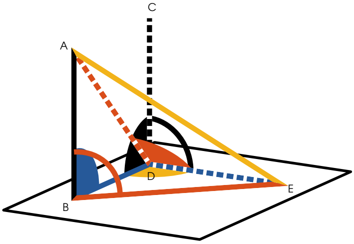
is also right, therefore the angle 

= 

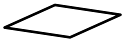
, and they lie in one plane, which is impossible.

Therefore the straight line BC is not in a more elevated plane. Therefore the three straight lines BC, BD, and BE are in one plane.

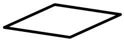
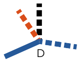
Q. E. D.

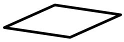


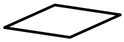
I*F two straight lines be at right angles to the same plane, the straight lines are parallel.*

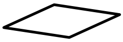
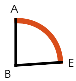
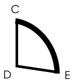
For let the two straight lines  and  be at right angles to the plane of reference .

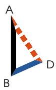
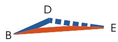
I say that  ∥ .

Let them meet the plane of reference  at the points  and .

Join the straight line . Draw  in  ⊥ , and make  = . (pr. ??, pr. ??)

Now, since  is at right angles to , it also makes right angles with all the straight lines which meet it and lie in the plane of reference. (def. ??)

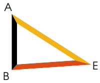
But each of the straight lines  and  lies in  and meets , therefore each of the angles  and  is right. For the same reason each of the angles  and  is also right.

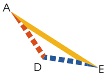
And since in  and   = , and  is common, therefore the two sides  and  equal the two sides  and . And they include right angles  and

, therefore the base 

= 

. (pr. ??)

And, since in 

and 

= 

while 

= 

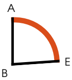
, the two sides 

and 

equal the two sides 

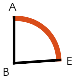
and 

, and 

is their common base, therefore the angle 

equals the angle 

. (pr. ??)

But the angle 

is right, therefore the angle 

is also right. Therefore 

is at right angles to 

.

But it is also at right angles to each of the straight lines 

and 

, therefore 

is set up at right angles to the three straight lines 

, 

, and 

at their intersection. Therefore the three straight lines 

, 

, and 

lie in one plane. (pr. 5)

But in whatever plane 

and 

lie, 

also lies, for every triangle lies in one plane. (pr. 2)

Therefore the straight lines 

, 

, and 

are in one plane. And each of the angles 

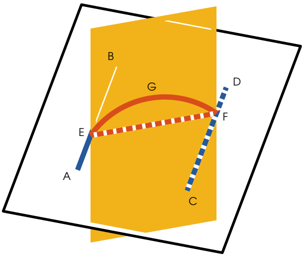
and 

is right, therefore 

is parallel to 

. (pr. ??)

Q. E. D.

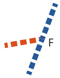


If two straight lines are parallel and points be taken at random on each of them, the straight line joining the points is in the same plane with the parallel straight lines.

Let 

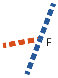
and 

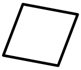
be two parallel straight lines, and let points 

and 

be taken at random on them respectively.

I say that the straight line joining the points 

and 

lies in the same plane  with the parallel straight lines.

For suppose it is not, but, if possible, let it be in a more elevated plane. Draw a plane  through 

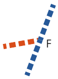
. Its intersection with the plane of reference is a straight line. Let it be 

. (pr. 3)

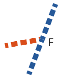
∴ the two straight lines 

and 

enclose an area, which is impossible. ∴ the straight line joined from 

to 

is not in a plane more elevated. ∴ the straight line joined from 

to 

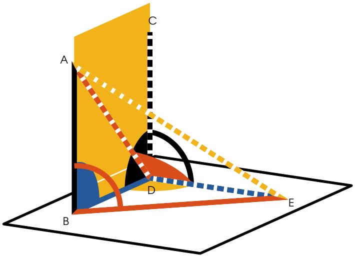
lies in the plane through the parallel straight lines 

and 

.

∴, if two straight lines are parallel and points are taken at random on each of them, then the straight line joining the points is in the same plane with the parallel straight lines.

Q. E. D.



If two straight lines be parallel, and one of them be at right angles to any plane, then the remaining one will also be at right angles to the same plane.

Let AB and CD be two parallel straight lines, and let one of them, AB, be at right angles to the plane of reference.

I say that the remaining one, CD, is also at right angles to the same plane.

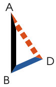
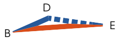
Let AB and CD meet the plane of reference at the points B and D. Join BD. Then AB, CD, and BD lie in one plane. (pr. 7)

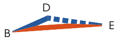
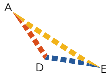
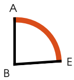
Draw DE in ⊥ BD, make DE = AB, and join BE, AE, and AD. (pr. ??, pr. ??)

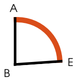
Now, since AB ⊥ , therefore AB is also at right angles to all the straight lines which meet it and lie in . Therefore each of the angles ABD and ABE is right. (def. ??)

And, since the straight line BD falls on the parallels AB and CD, ∴ ABD + CDB = .

But the angle ABD is right, therefore the angle CDB is also right. ∴ CD ⊥ BD. (pr. ??)

And since in  and  AB = DE, and BD is common, the two sides AB and BD equal the two sides ED and DB, and  = , for each is right, therefore the base AD = BE. (pr. ??)

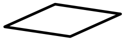
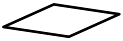
And since in  and  AB = DE, and BE = AD, the two sides AB and BE equal the two sides ED and DA respectively, and AE is their common base, ∴  = . (pr. ??)

But the angle  is right, therefore the angle  is also right. ∴ ED ⊥ AD. But it is also ⊥ DB. ∴ ED is also at right angles to the plane through BD and DA. (pr. 4)

∴ ED also makes right angles with all the straight lines which meet it and lie in the plane through BD and DA. But DC lies in the plane  through BD and DA inasmuch as AB and BD lie in the plane  through BD and

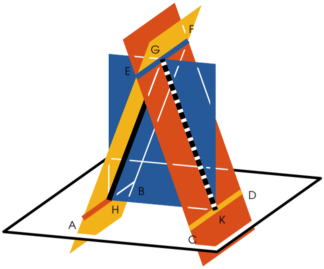
DA, and DC also lies in the plane  BD in which AB and BD lie.

∴ ED ⊥ DC, so that CD ⊥ DE. But CD ⊥ BD. ∴ CD is set up at right angles to the two straight lines DE and DB so that CD is also at right angles to the plane through DE and DB. (pr. 4)

But the plane through DE and DB is the plane of reference , ∴ CD ⊥ .

Therefore, if two straight lines are parallel, and one of them is at right angles to any plane, then the remaining one is also at right angles to the same

Q. E. D.



STRAIGHT *lines which are parallel to the same straight line and are not in the same plane with it are also parallel to one another.*

For let each of the straight lines AB, CD be parallel to EF, not being in the same plane  with it. I say that AB ∥ CD.

For let a point G be taken at random on EF, and from it let there be drawn GH, in the plane  through EF, AB, at right angles to EF, and GK in the plane  through FE, CD again at right angles to EF.

Now, since EF is at right angles to each of the straight lines GH, GK, ∴ EF ⊥ to the plane HK through GH, GK. (pr. 4)

And EF ∥ AB, ∴ AB ⊥ HK. (pr. 8)

For the same reason CD ⊥ HK. ∴ each of the straight lines AB, CD ⊥ HK.

But, if two straight lines be at right angles to the same plane, the straight lines are parallel (pr. 6).

Therefore AB ∥ CD.

Q. E. D.