

START

**Central China Normal University
Wuhan – China - June 27th - 2013**

**Workshop on
‘Knowledge & Competence/Skill
Spaces and their Applications’**

**Dietrich ALBERT & Reinhard SUCK &
Xiangen Hu**

Workshop on 'Knowledge and Competence/Skill Spaces and their Applications'

Introduction and Overview (Dietrich Albert)

Basic Approaches of Knowledge Representation - Knowledge Space Theory (KST): Examples, Basic Concepts and Axioms - Competence-Based KST (CbKST): Example, Basic Concepts - Generation and Validation of Knowledge and Competence Structures - Adaptive Knowledge Assessment - Applications and Projects (List)

Mathematics of Knowledge and Competence/Skill Spaces (Reinhard Suck)

Bases, Entailment Relation, Surmise Functions, Fringes - Skills and Competences: CbKST and Set Representations - Special topics: Learning Spaces, Meshing Spaces, Related Concepts

Applications, Demonstrations & Resources in Education (Dietrich Albert; Xiangen Hu)

Web-courses - Books and Documents – Bibliography – Software – (Adaptive, Personalised) eLearning Systems and Investigations – Other Applications

Final Remarks and Discussion

Introduction and Overview

Dietrich Alber t

Cognitive Science Section - CSS

Knowledge Technologies Institute - Graz University of Technology

Department of Psychology - University of Graz

Austria - Europe

QUESTIONS

Some knowledge questions for you

Where is AUSTRIA?

▪AUSTRIA flag



Where is AUSTRIA?

▪ **AUSTRIA is located**

of course in Europe –

because it doesn't have kangaroos

Where is AUSTRIA in EUROPE?

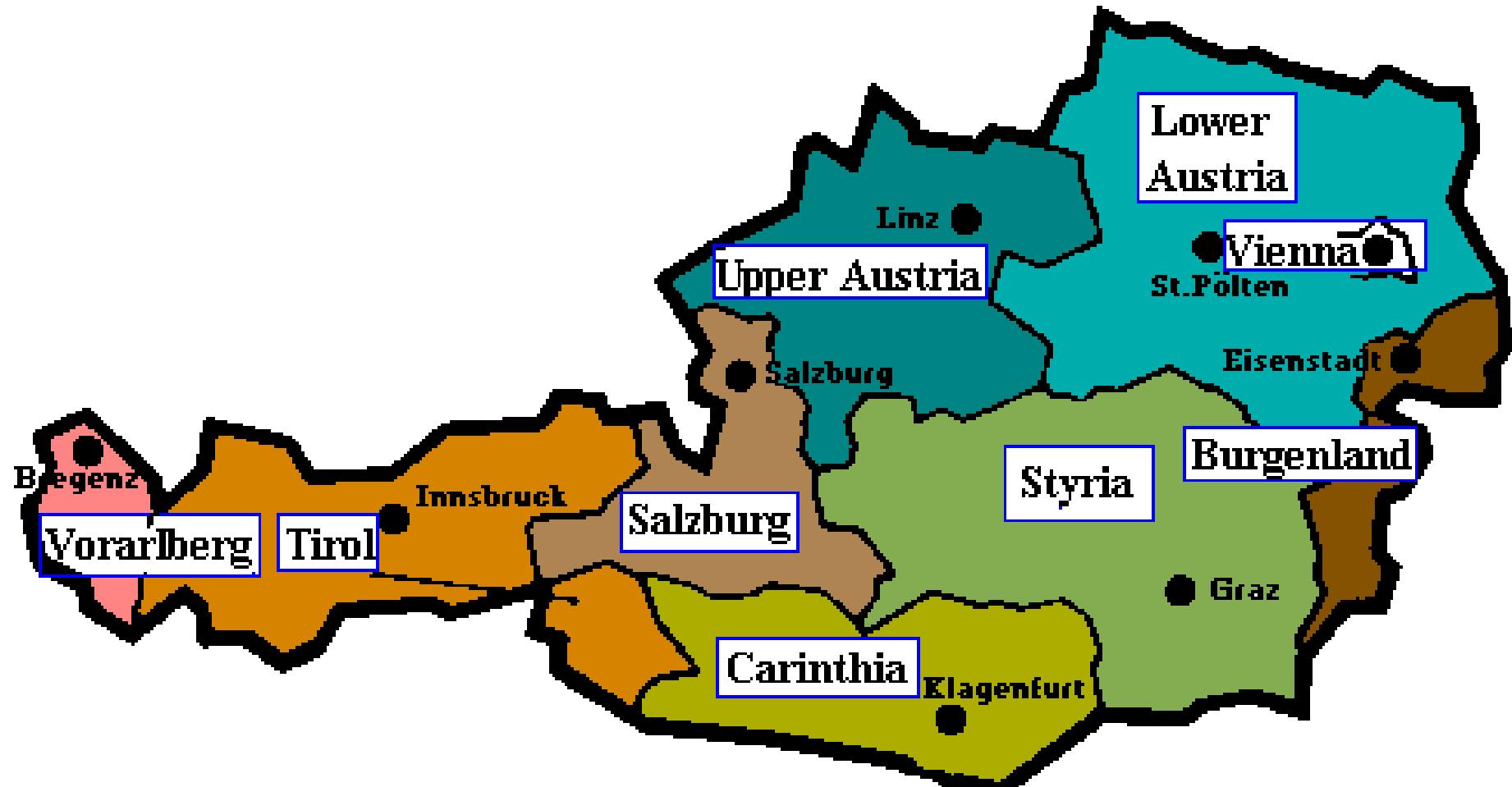


AUSTRIA flag



Where is STYRIA and It's Capital GRAZ?

Where is STYRIA and It's Capital GRAZ?



How does GRAZ look like?

How does GRAZ look like?



What's the Meaning of CSS?

What's the Meaning of CSS?

▪ Cognitive Science Section (CSS)

- Principal scientist: Prof. Dietrich Albert
- Interdisciplinary team of psychologists,
computer scientists, mathematicians

<http://kti.tugraz.at/about-kti/team/>

CSS has been founded in 1993 - since 2010 CSS is located at

- Knowledge Technologies Institute (KTI)

<http://kti.tugraz.at/>

- Head: Prof. Stefanie Lindstaedt
- Graz University of Technology (TUGraz)

http://portal.tugraz.at/portal/page/portal/TU_Graz

Our Experience & Expertise

- European-wide collaboration with universities and business partners from various disciplines



- Long term experience in international and national projects
 - Cooperative EU-Projects since 2001 in the 5th Framework Programme

Our R&D Projects since 2001

- ❖ weSPOT - Working Environment with Social and Personal Open Tools for Inquiry based Learning. (FP7 ICT STREP)
- ❖ INNOVRET - Innovative Online Vocational Training of Renewable Energy Technologies (FP7 – LLL Leonardo Da Vinci)
- ❖ RECOBIA - Reduction of the cognitive biases in intelligence analysis (FP7 SEC STREP)
- ❖ CULTURA - Cultivating Understanding and Research through Adaptive Learning (FP7 ICT STREP)
- ❖ ImREAL – Immersive Reflective Experience-based Adaptive Learning (FP7 ICT STREP)
- ❖ GaLA – Gaming and Learning Alliance (FP7 NoE)
- ❖ NEXT-TELL – Next Generation Teaching, Education and Learning for Life (FP7 ICT IP)
- ❖ ROLE - Responsive Open Learning Environments (FP7 ICT IP)
- ❖ TARGET – Transformative, Adaptive, Responsive and Engaging Environment (FP7 ICT IP)
- ❖ GRAPPLE – Generic Responsive Personalized Learning Environment (FP7 ICT STREP)
- ❖ 80Days - Around an inspiring virtual learning world in eighty days (FP7 ICT STREP)
- ❖ MedCAP - Competence Assessment for Spinal Anesthesia (FP7 – LLL Leonardo Da Vinci)
- ❖ Repeated Comprehensiosual Search (FWF)
- ❖ ELEKTRA - Enhanced Learning Experience and Knowledge Transfer (FP6 IST STREP)
- ❖ Graph Comprehension (FWF)
- ❖ Probabilistic Knowledge Space and Item Response Theories (FWF)
- ❖ ELeGI - European Learning Grid Infrastructure (FP6 IST IP)
- ❖ iCLASS - Intelligent Distributed Cognitive-based Open Learning System for Schools (FP6 IST IP)
- ❖ Kaleidoscope – TRAILS (FP6 IST NoE)
- ❖ LeGE-WG: Learning Grid of Excellence Working Group (FP5 Thematic Network)
- ❖ Efficient assessment of the organizational action (IHP Marie Curie Research Fellowship)
- ❖ EASEL: Educator Access to Services in the Electronic Landscape (FP5 IST IP)

Overview

- Introduction
- Knowledge Space Theory
 - Application: ALEKS
- Demand Component Approach
 - Application: RATH
- Competence-Performance Approach
 - Application: APeLS
- Ontology-based Skill Approach
 - Application: iClass

- **Introduction**
- Knowledge Space Theory
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General Remarks

- ICT in Education needs basic **and** applied research: Policy of the European Commission (EC)
- ICT in Education needs research and development not only in technology but also in pedagogy, psychology, cognitive sciences, and social sciences: ICT-Calls of the EC regarding eLearning and Technology enhanced Learning (TEL)
- The gap between basic and applied research at one hand and good practices and applications at the other hand is rather small: We want to demonstrate this by our talks.
- This is the first aim of our talks

General Remarks

- Second aim is to demonstrate that real interesting and relevant problems can be solved by using mathematics. Once solved, the solutions can be applied in different settings by re-interpreting the formal model
- Third aim is to demonstrate – in terms of Prof. Batchelder – the usefulness of starting at the lowest non trivial level of modelling. The used mathematics have not yet been available while psychology started in 19th century as a modern science. The founders took physics as a model and did not know about discrete mathematics. That was and is a pity.

General Remarks

- Last but not least, the third aim of our talks is even more important:
 - to interest YOU in our approaches and
 - to invite YOU to spend some time at Graz University of Technology for collaborative research and development

Classroom Teaching

Long Tradition



Is this the Future of Classroom Teaching?



Classroom Teaching Advantages

- Well known and integrated into the educational system
- High potential for modernisation
- Face-to-face interaction
 - Students peers are models
 - Learning strategies
 - Exchanging Metacognitions
 - Social aspects / Social skills
 - Motivational aspects (e.g. through competing)

Private/Home Teaching Even Longer Tradition



Private/Home Teaching Advantages

- Personalisation and individualisation
 - tailored
 - Instructions
 - Course content
 - Curriculum
 - Learning progress
 - Learning goal

Private/Home Teaching Advantages

- Intensive personal contact and tutor-learner interaction
 - Tutor supports learner in planning work
 - Discussion of course content
 - Tutor reacts on learners motivational and emotional states
- Flexibility
 - Individual times for learning
 - Individual places for learning
- etc.

Is Private/Home Teaching the Future of Education?

- Of course not this traditional type of private teaching - it is too expensive
- However, a modern type of private teaching may be realized and part of blended learning settings by

Todays Hi-Tech Class Room

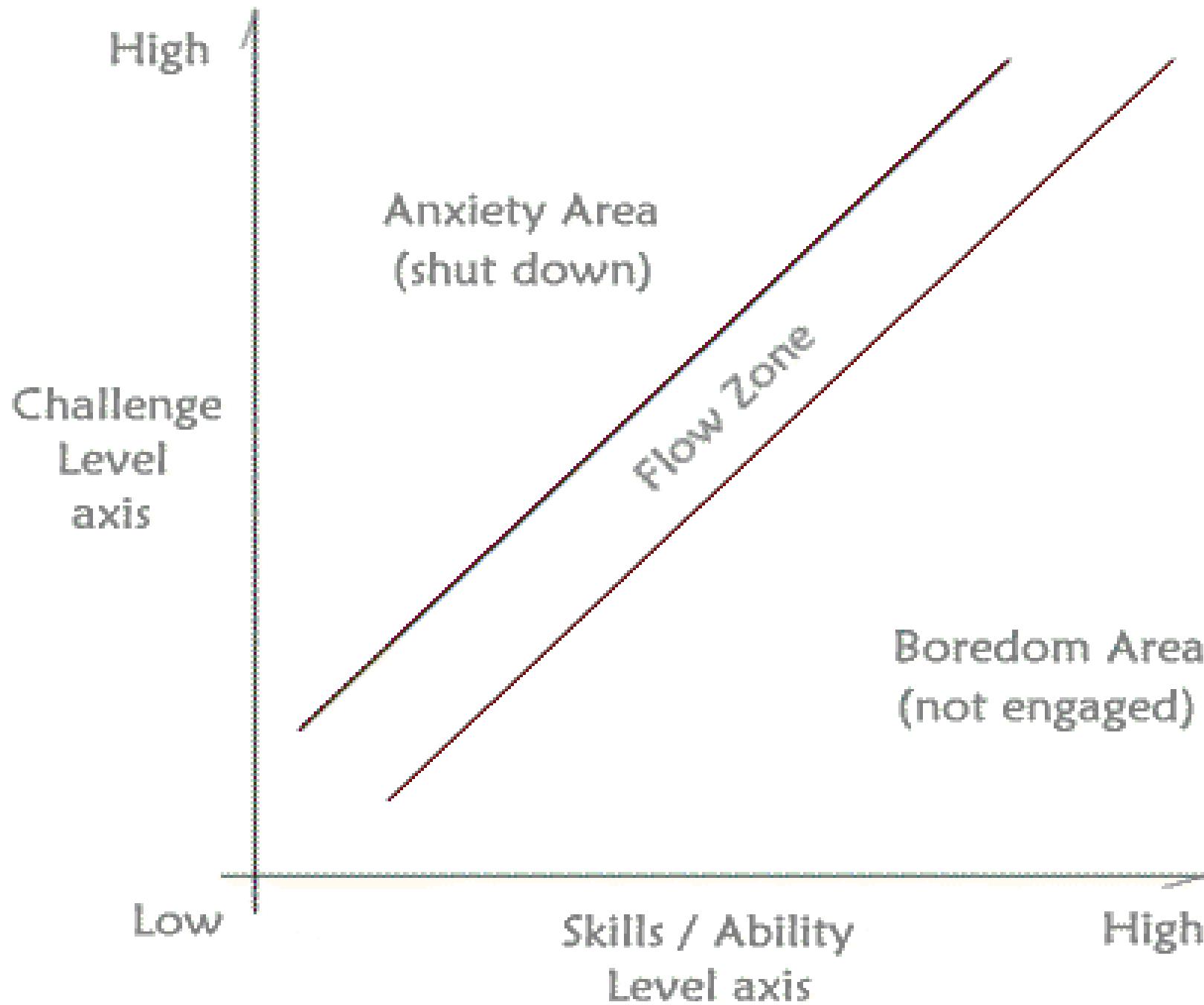


Main Questions of the Talk

- How to create adaptive, individualised eLearning systems based on psychology of learning which adapts to the individual student's knowledge, needs, decisions, self regulation skills ...
- On the other hand, how to guide the student to follow a strict curriculum given by educational authorities, like the teachers, the Ministry of Education etc.
- **The (Competence-based) Knowledge Space Theory [CbKST] is used for supporting personalised eLearning !**

- The American psychologist Mihaly Csikszentmihalyi is the discoverer of a mental state that he calls 'Flow'





A Simple, Wellknown Educational Principle

- The individual pupil/student has to be ready for what s/he is learning next
- Present the student with content and information for which s/he is ready for, that means
 - You have to know the prerequisite content and whether the student has knowledge about this prerequisite knowledge or not, and
 - choose and present content according to the students current knowledge

- This well known simple rule has a less simple theoretical consequence:
- A mathematical-psychological theory founded by Falmagne and Doignon in 1985, developed by them further and extended by others (e.g. Albert, Held, Hockemeyer, Korossy, Lukas,...): **The Knowledge Space Theory (KST)**
- KST and its CbKST-extensions are the basis for individualised eLearning – like private teaching

- Introduction
- **Knowledge Space Theory**
 - Application: ALEKS
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Knowledge Space Theory

Basic Concepts

- Basic Concepts of KST
 - Knowledge domain
 - Surmise/prerequisite/precedence relation
 - Knowledge state
 - Knowledge structure, knowledge space
 - Learning path
 - Learning goal

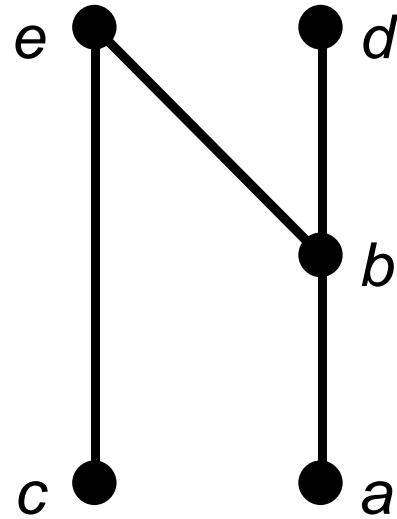
KST Basic Concepts

Knowledge Domain

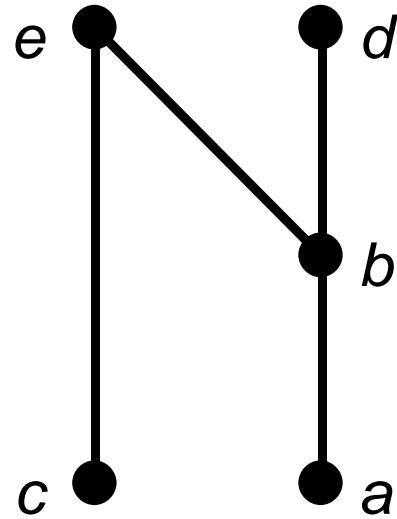
- In KST a knowledge domain is identified with a set of problems
 - Simple Example

a	$378 \times 605 = ?$
b	$58.7 \times 0.94 = ?$
c	$1/2 \times 5/6 = ?$
d	What is 30% of 34?
e	Gwendolyn is $\frac{3}{4}$ as old as Rebecca. Rebecca is $\frac{2}{5}$ as old as Edwin. Edwin is 20 years old. How old is Gwendolyn?

- Surmise/Prerequisite/Precedence Relation
 - defined on the knowledge domain Q of the Example



- Surmise/Prerequisite/Precedence Relation
 - defined on the knowledge domain Q of the Example

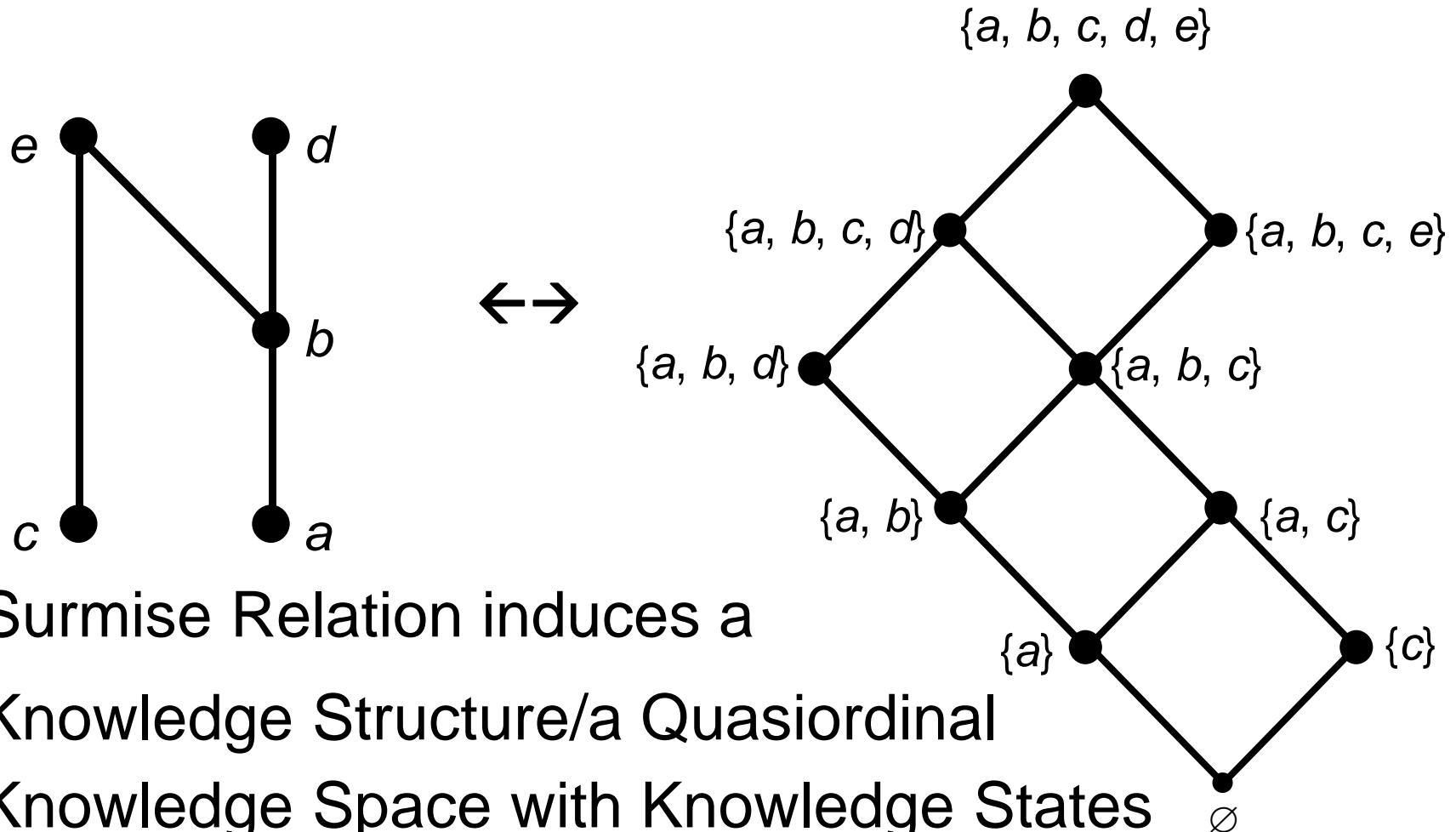


Attention please:

This is merely a simplified
visualisation of a Surmise
Relation $S = \{(a,b), (a,d), (a,e), (b,d), (b,e), (c,e)\} !!!$

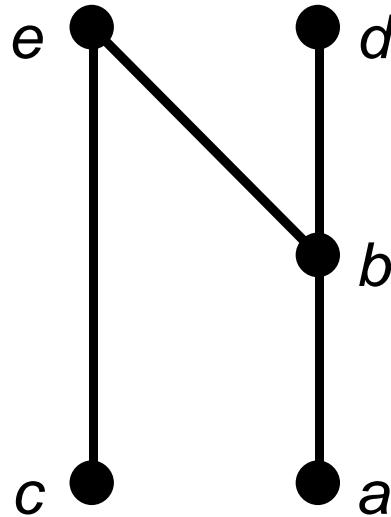
KST Basic Concepts

Knowledge Structure

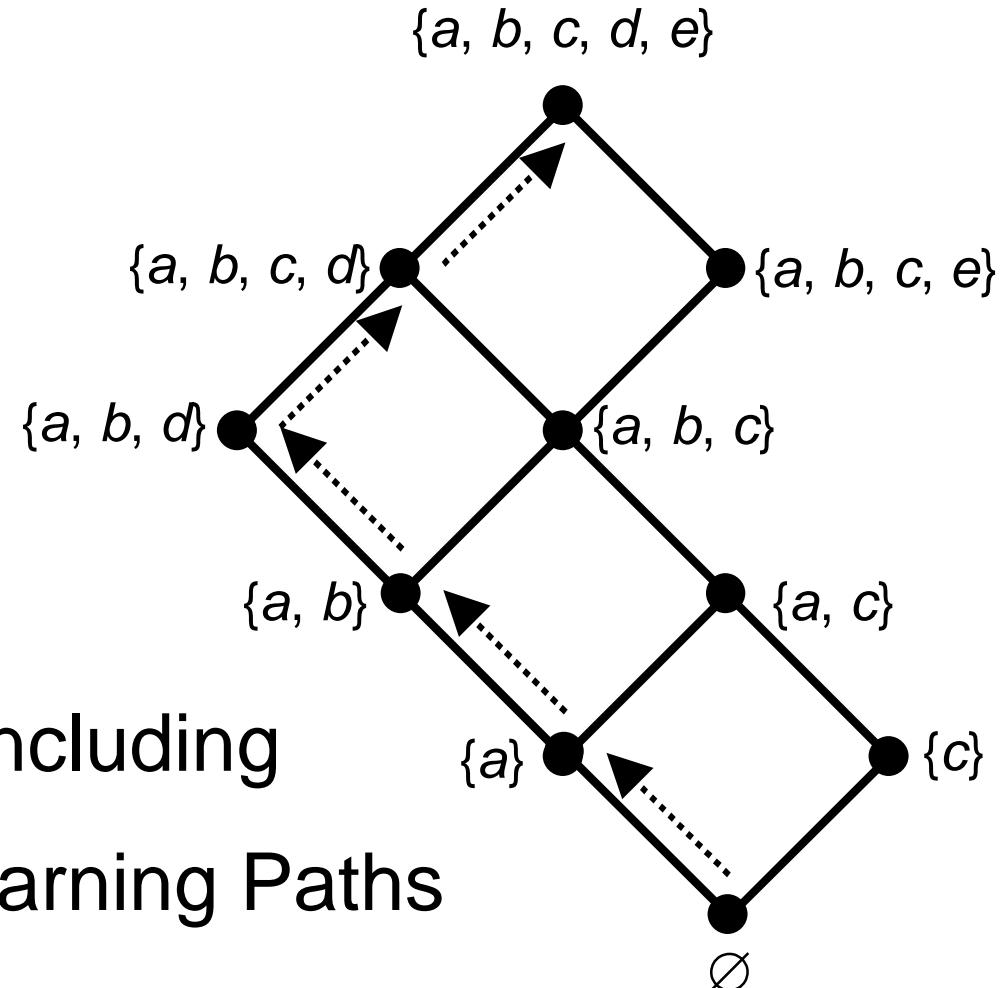


KST Basic Concepts

Learning Path and Goal



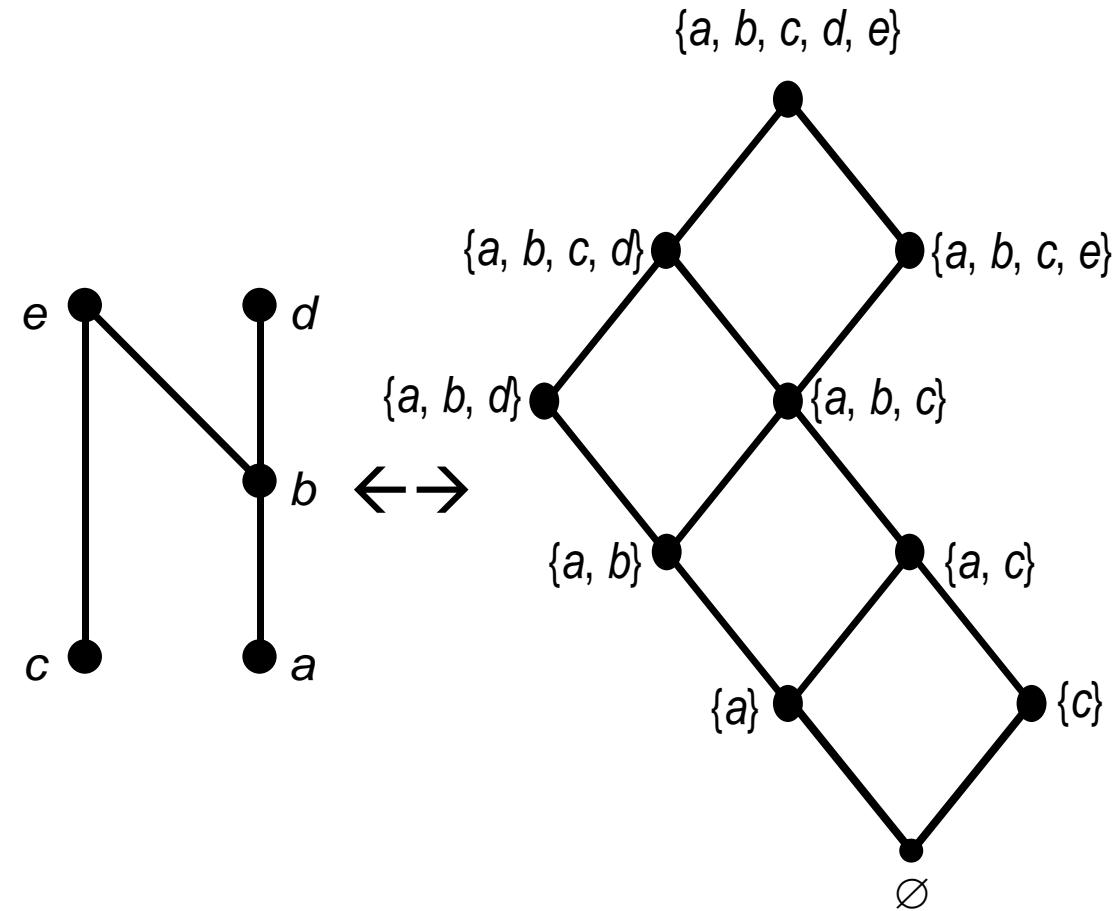
\leftrightarrow



- Knowledge structure including
- one of the possible Learning Paths

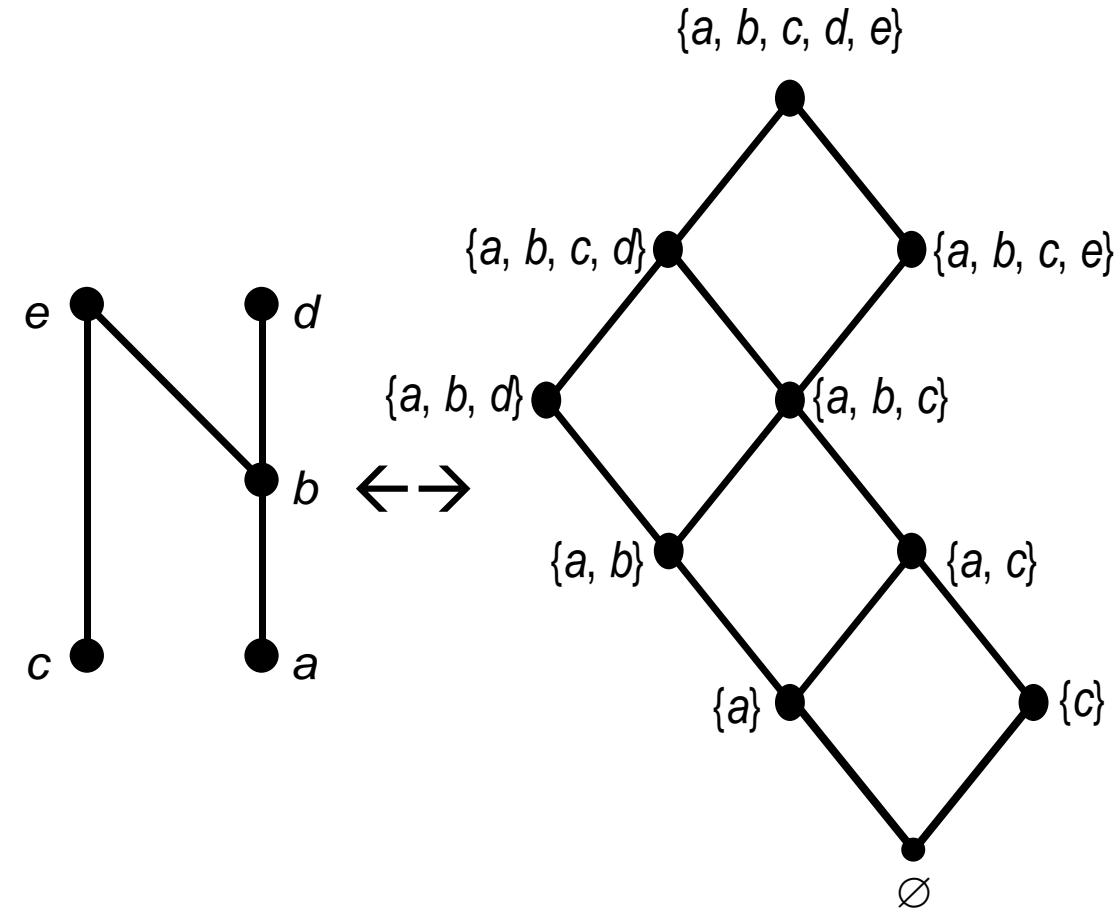
The Power of the Theory

- These two diagrams make understandable the power of the theory and
- enable to remember its main features



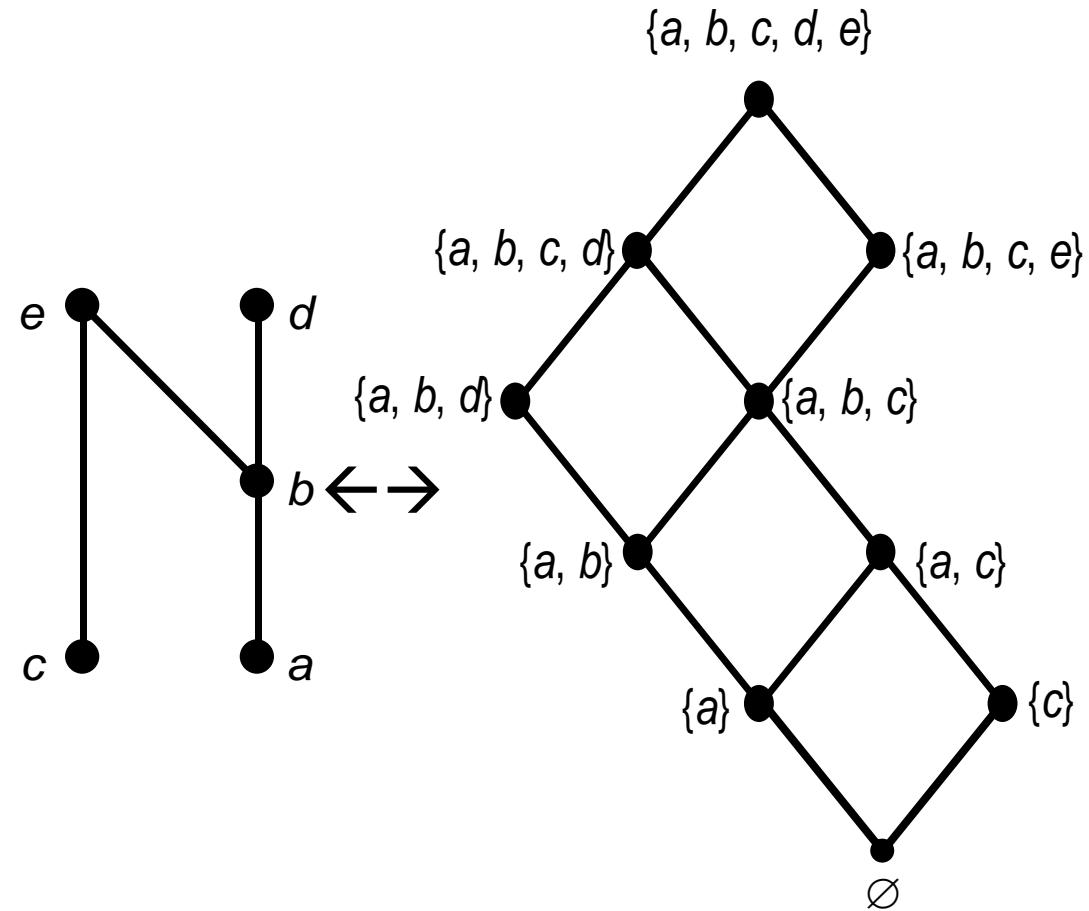
The Power of the Theory

- The content and its structure induces the curriculum and vice versa
- The content induces the knowledge states and their structure



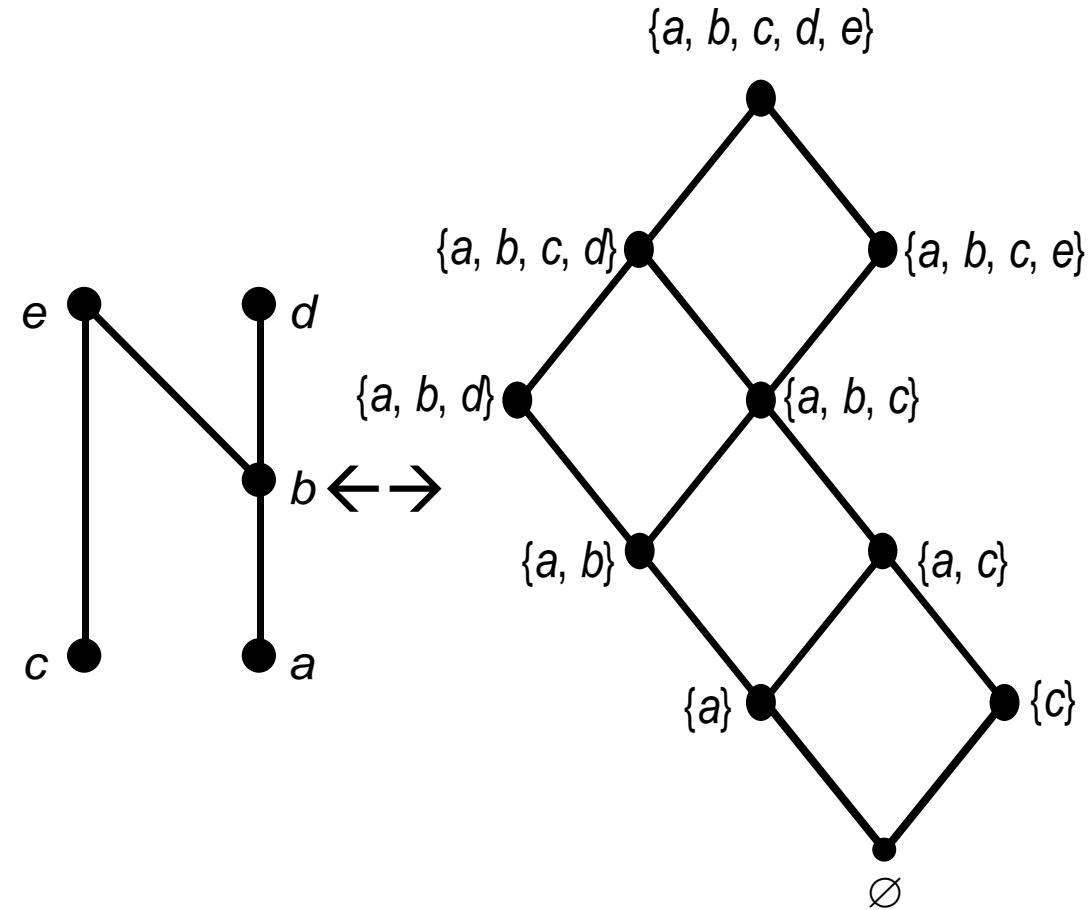
The Power of the Theory

- Personalised assessment for efficiently identifying a persons state of knowledge - like in an oral examination
 - by presenting only a subset of problems
- (Adaptive Testing!)



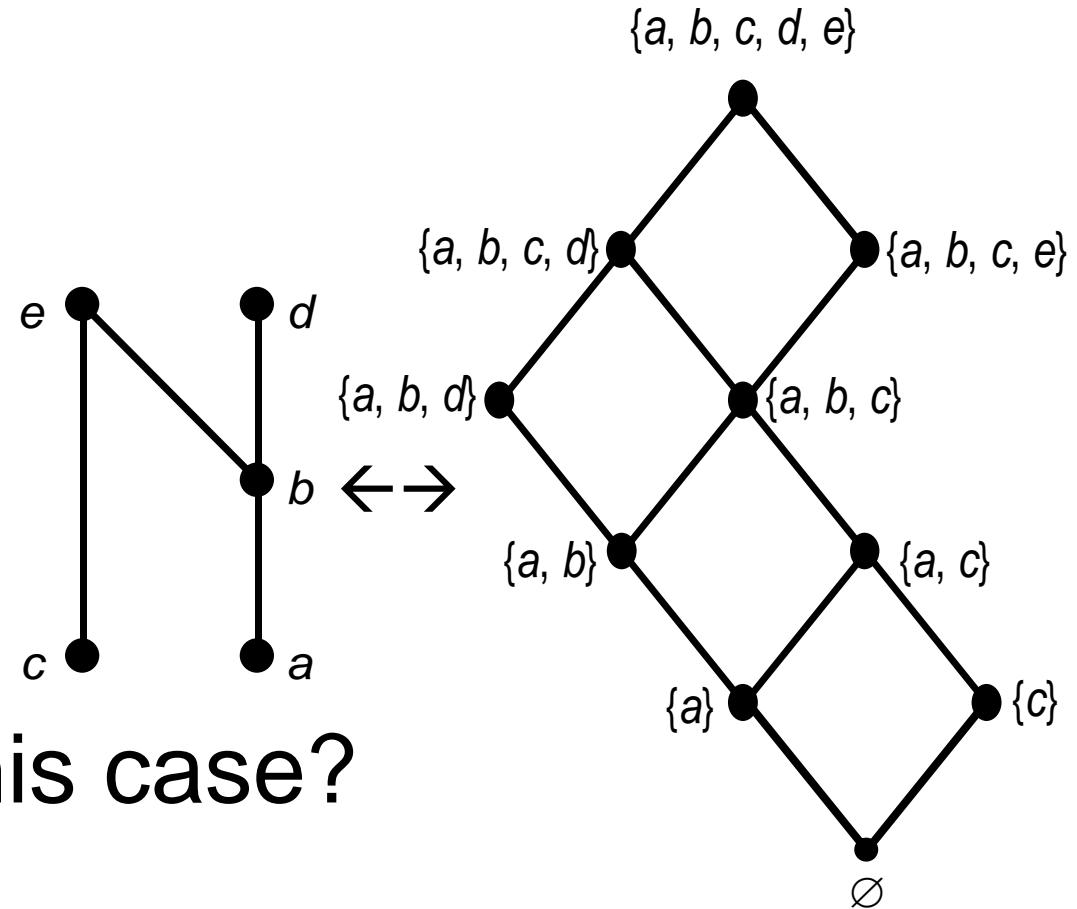
The Power of the Theory

- Individual starting state for learning depending on pre-knowledge
- Individual goal state for learning



The Power of the Theory

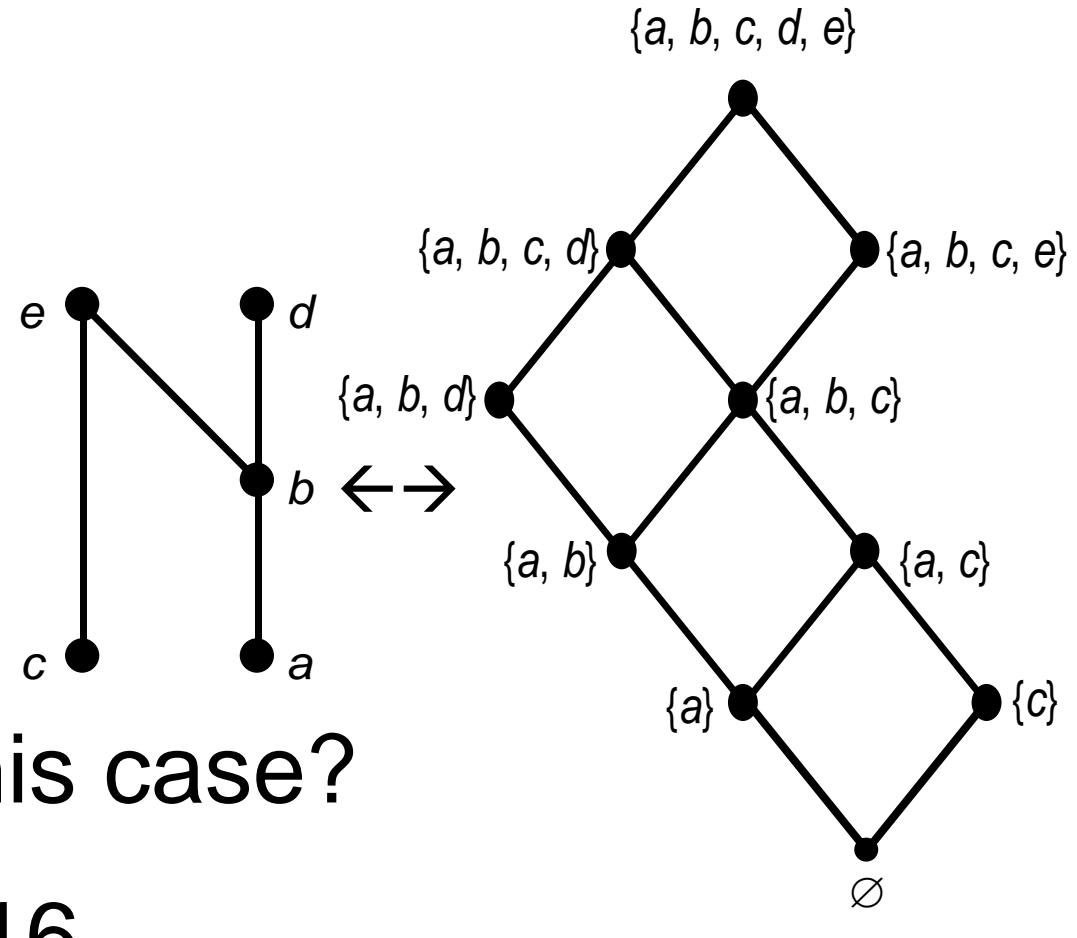
- Individual learning paths



- How many in this case?

The Power of the Theory

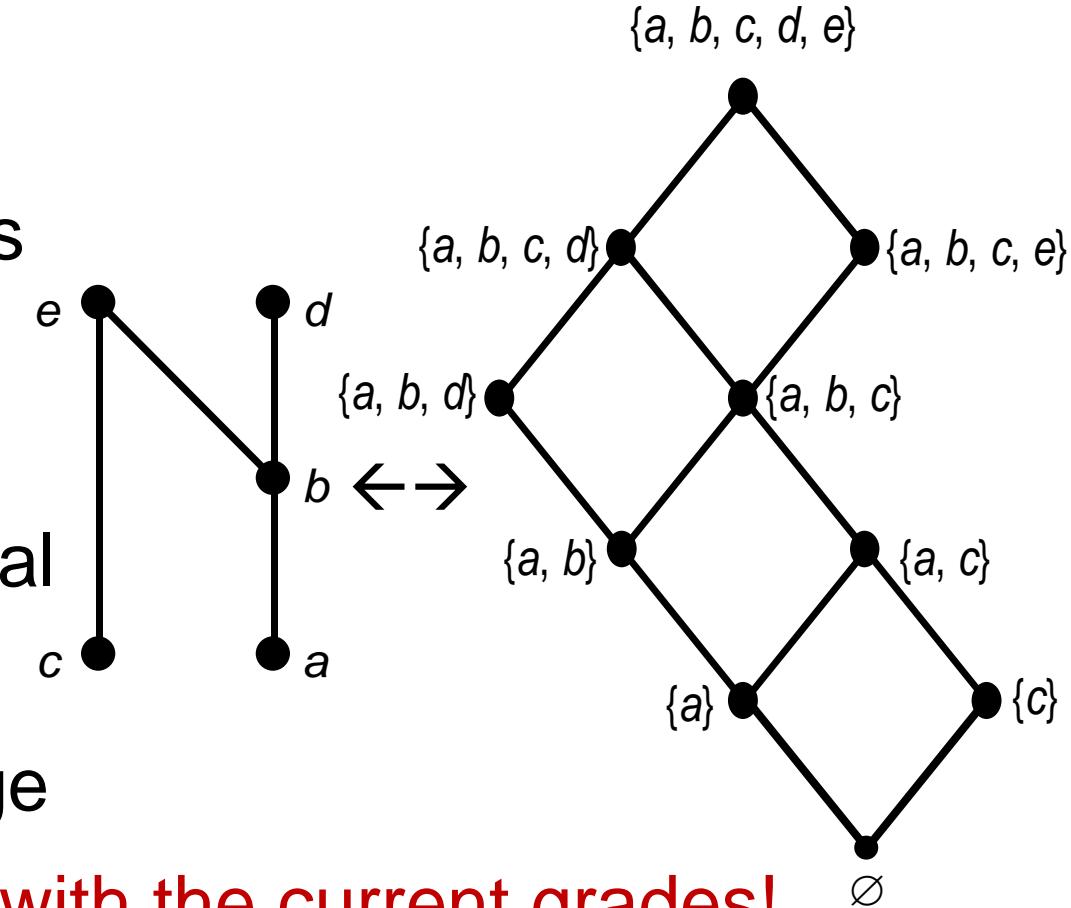
- Individual learning paths



- How many in this case?
- $2 \times 2 \times 2 \times 2 = 16$

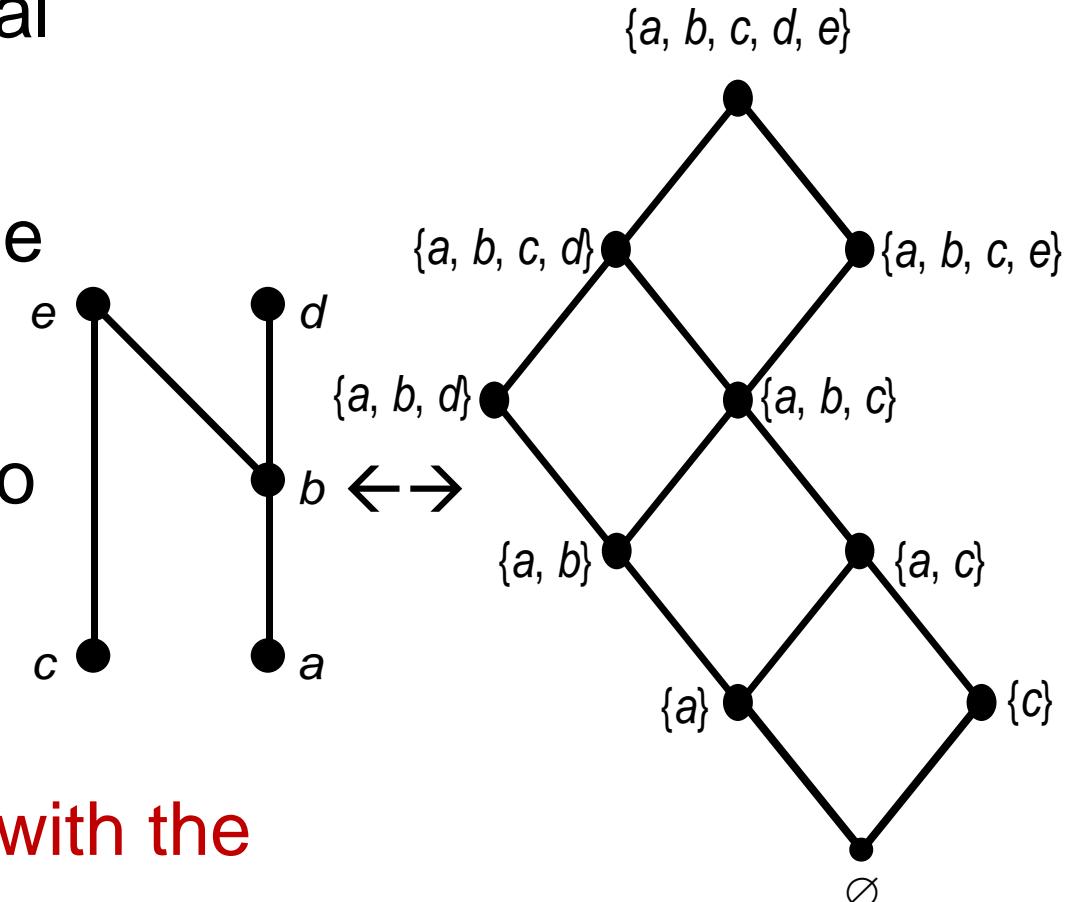
The Power of the Theory

- Detailed characterisation of the learners strengths and weaknesses in a given domain by
- Precise, non-numerical characterisation of the state of knowledge
- Please compare this with the current grades!



The Power of the Theory

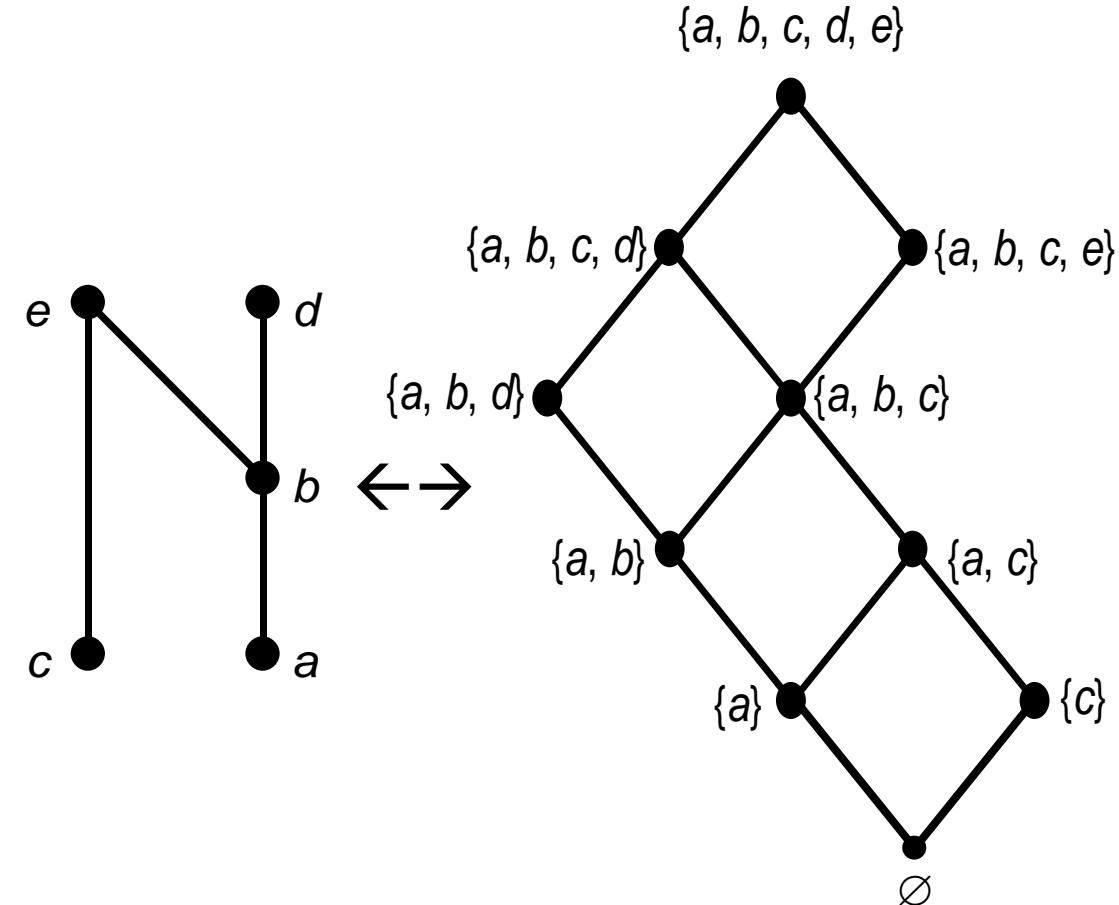
- Precise, non-numerical characterisation of the state of knowledge allows for **exactly** identifying what has to be teached next and to learned next



- Please compare this with the consequences of current grades like A,B...F !

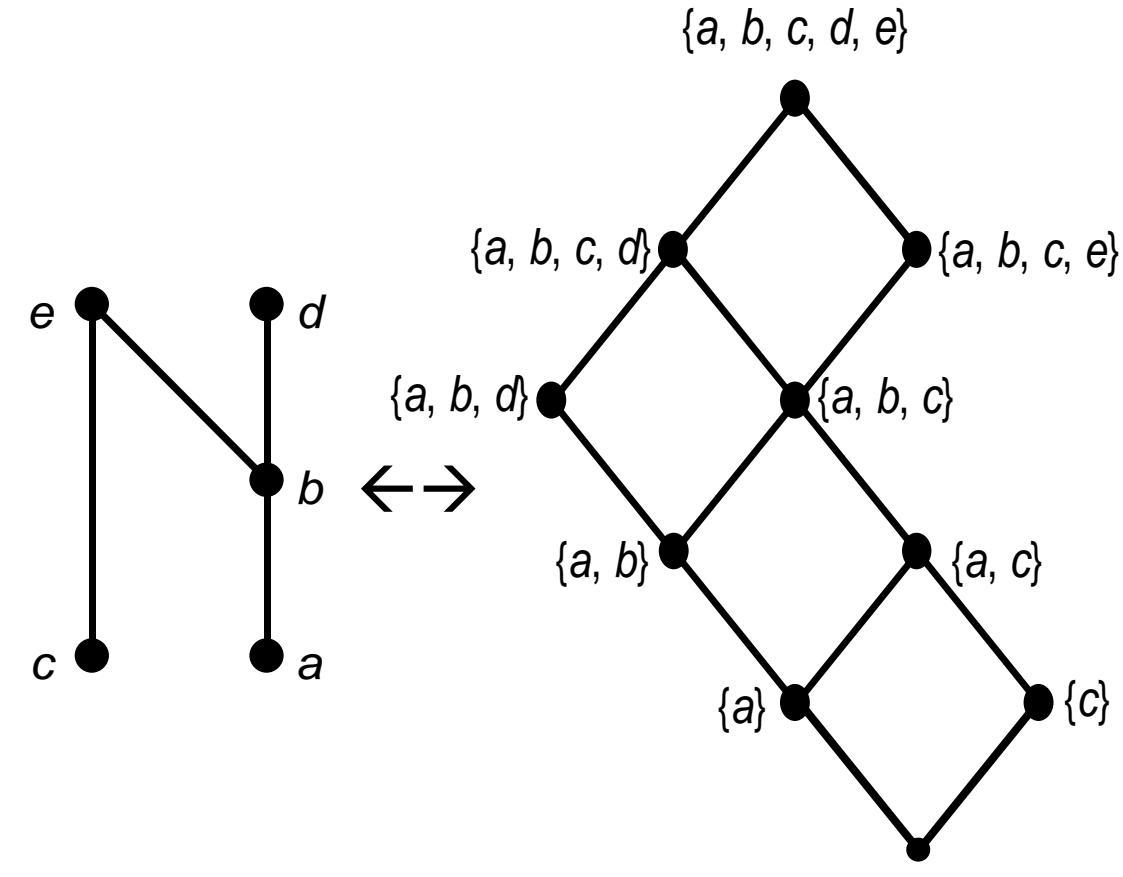
The Power of the Theory

- Access only to those learning objects which the student is ready to learn
- Student is neither overburdened nor underburdened - Challenge just OK
(Flow - Motivation!!)



The Power of the Theory

- Reasonable choices for navigation
- No strict order (boring)
- No total freedom (lost in hyperspace)



- Remark:
However, what about SRL, GBL and Open Learner Models???

Validating Knowledge Structures

- Knowledge structures, once they have been established, need to be validated before usage in adaptive assessment and personalised teaching and learning
 - for proving that the structure is empirically adequate
- Comparison between empirical data on the respective problems and the theoretically hypothesised structures via
 - solution frequencies – correspondence with/contradictions to the prerequisites captured by the prerequisite relation
 - investigating confirmations/violations in the empirical data w.r.t. problem pairs among which a prerequisite relationship is assumed
 - calculation of the minimal distances between the answer patterns and the respective knowledge structure

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Application: ALEKS

<http://www.aleks.com>

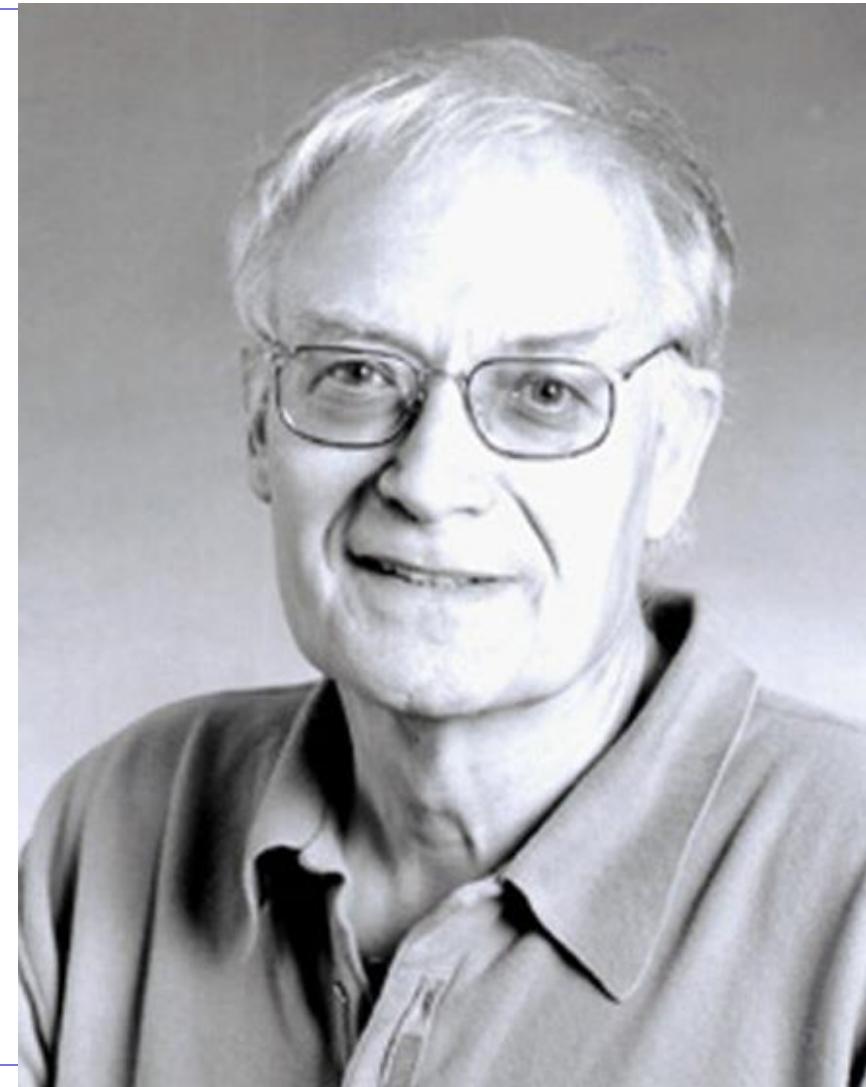
■ ALEKS

Adaptive LEarning with Knowledge Spaces

- Fully automated, multi-lingual, adaptive math tutor including explanations, practice, and feedback
- Assesses personalized which math concepts the student has mastered, which are shaky, and which are new but within reach
- Enables the student to work on those concepts the student is most ready to learn
- Closely interacts with the student, continuously updating its precise map of the student's knowledge state.

KST and ALEKS

ALEKS Corporation is a Delaware corporation formed in November 1996 by the Corporation's Chairman, Professor Jean-Claude Falmagne, an internationally recognized researcher in mathematical cognitive science, and his fellow researchers.



ssment and Learning, ...



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Initial Adaptive Assessment

Basic Idea

Simplifying assumption: Neither lucky guesses nor careless errors.

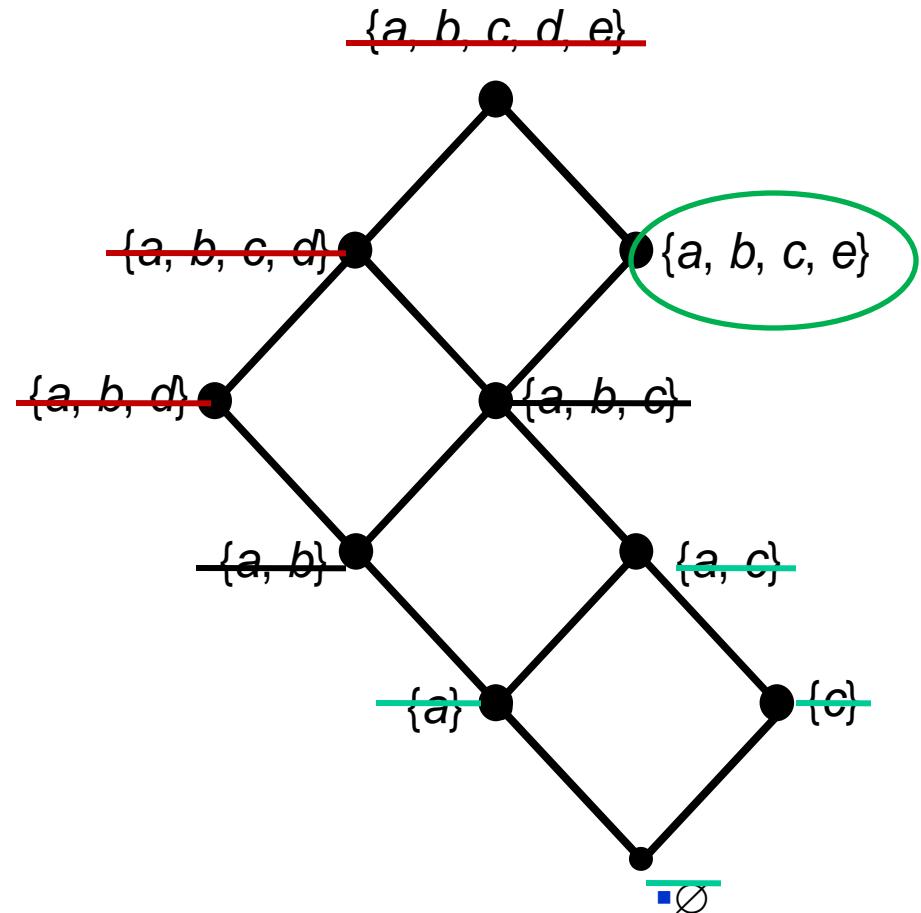
- Pose a problem that is contained in about half of the knowledge states.
- Eliminate/Leave all states that are in/consistent with the answer received.
- Continue until there is only one knowledge state left.

1. Problem *b* solved

2. Problem *d* failed

3. Problem *e* solved

Result: $\{a, b, c, e\}$



Probabilistic Assessment

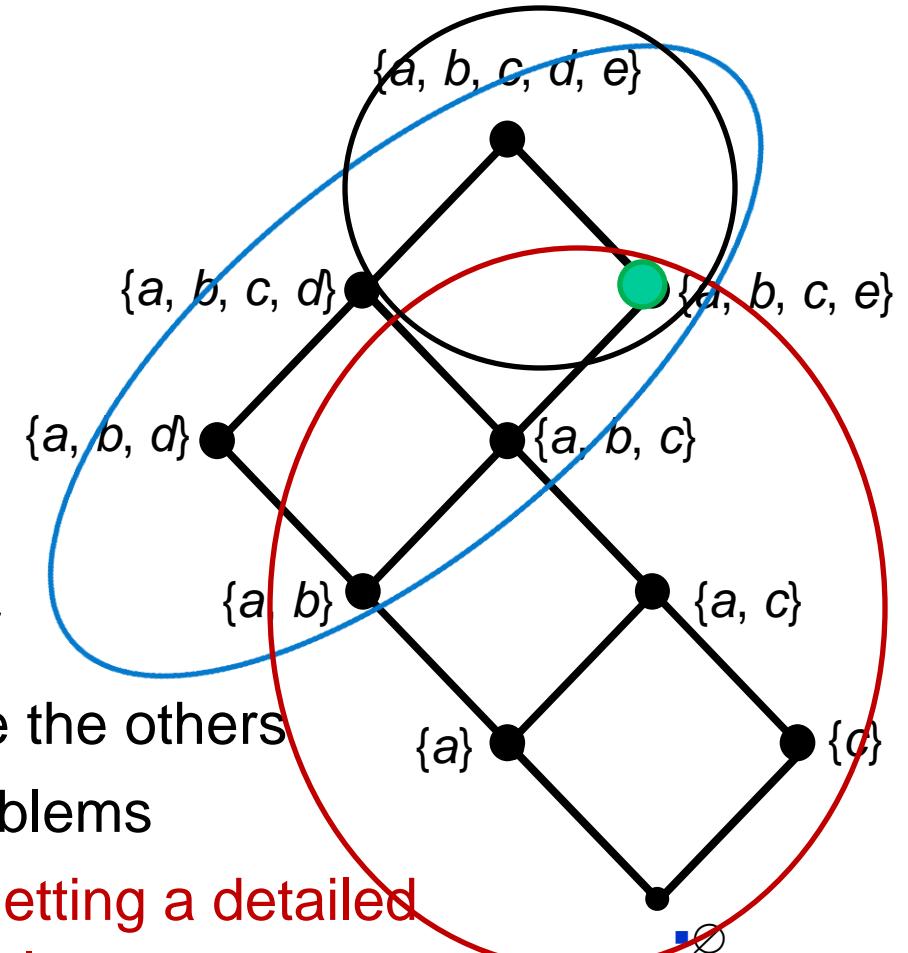
1. Problem *b* solved
2. Problem *d* failed
3. Problem *e* solved

Result: $\{a, b, c, e\}$

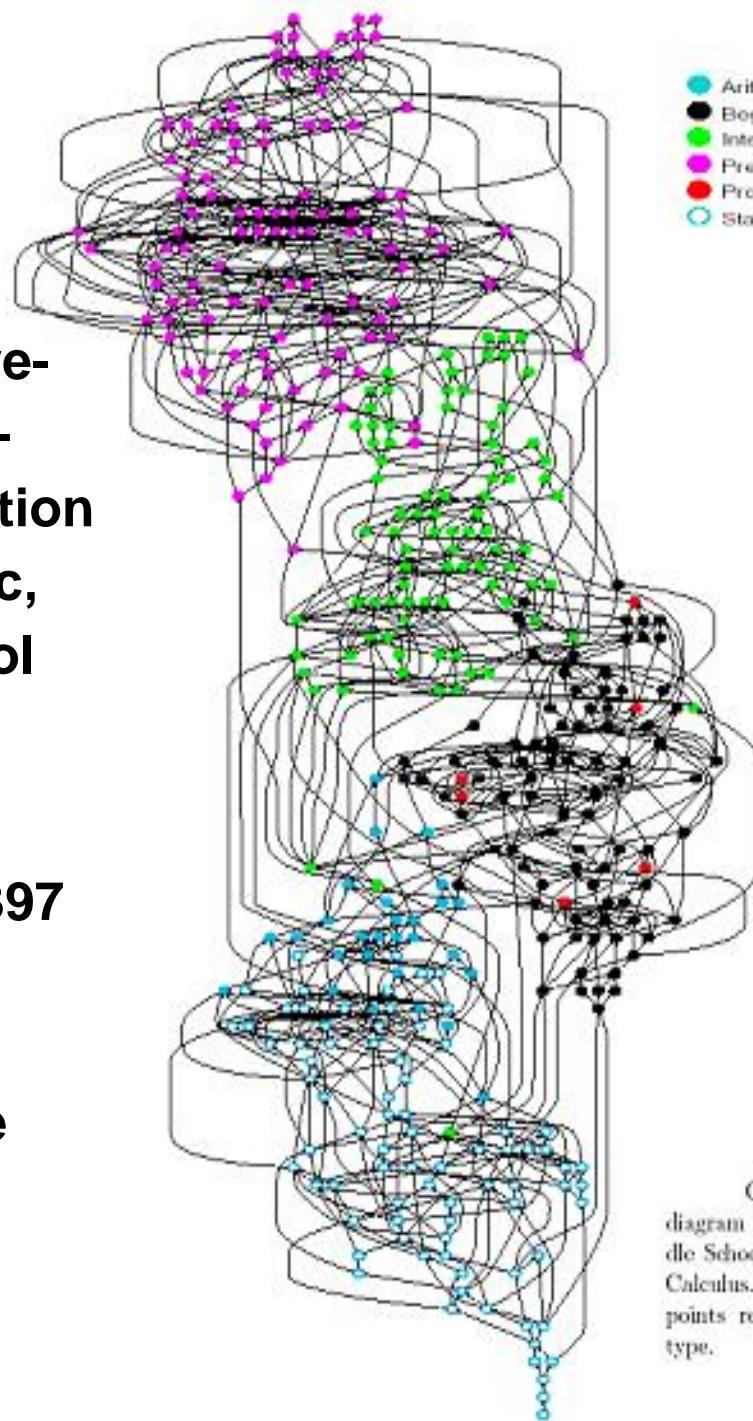
Principle: Increase likelihoods
of all the states which are in
accordance with the correct/
incorrect answer & decrease the others

Advantage: Only a subset of problems

has to be presented **for getting a detailed
knowledge profile - not only a score**



- **Combined pre-requisite/precedence relation for Arithmetic, Middle School Algebra, and Pre-Calculus**
- **Each of the 397 points represents a problem type**



Combined precedence diagram for Arithmetic, Middle School Algebra, and Pre-Calculus. Each of the 397 points represents a problem type.

Initial Adaptive Assessment

- The slide above is about the prerequisite relation used in ALEKS for a given domain
- The corresponding knowledge structure is extremely huge, however less than 2 to the power of 397 because of the constraints given by the prerequisite structure
- The books of Falmagne and Doignon describe the intelligent algorithms for computing

Initial Adaptive Assessment

ALEKS: Martin Kunibert - 2005-04-25 - Microsoft Internet Explorer

Datei Bearbeiten Ansicht Favoriten Extras ?

Exit English Options Print Report Dictionary Calculator Review Worksheet Message Help Quiz

ALEI My Pie Algebra

Please read this question and enter your answer in the box
(intermediate calculations should be done on paper).
When you are done, click on "Next."
If you don't know the answer, click on "I don't know." (Do not guess.)

Question #1

Write

$$\frac{5x}{2dk} + \frac{d^3 z^2}{8ck^2}$$

as a single rational expression.

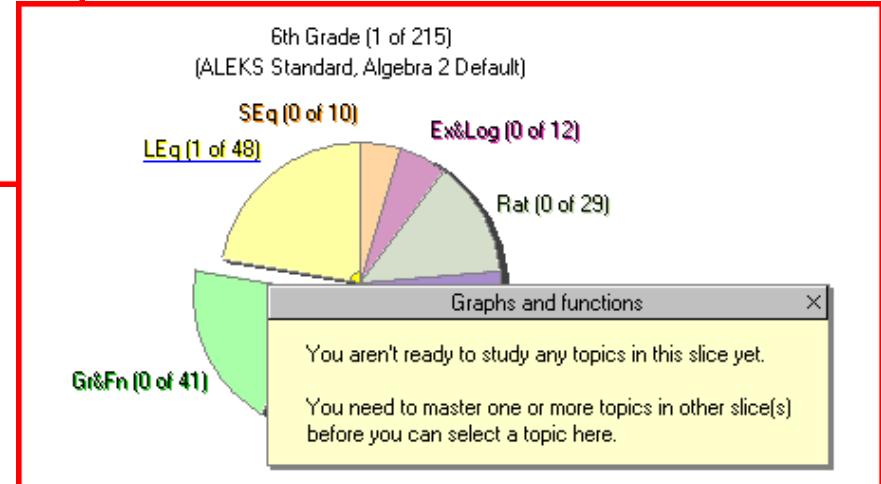
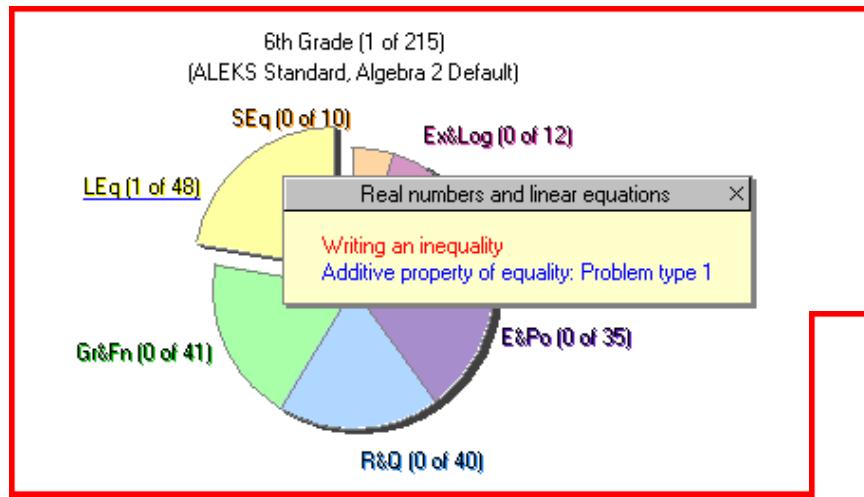
Clear Undo Help

Next >> I Don't Know

Applet aleksPack10.panel.PanelPage started

End of Initial Assessment

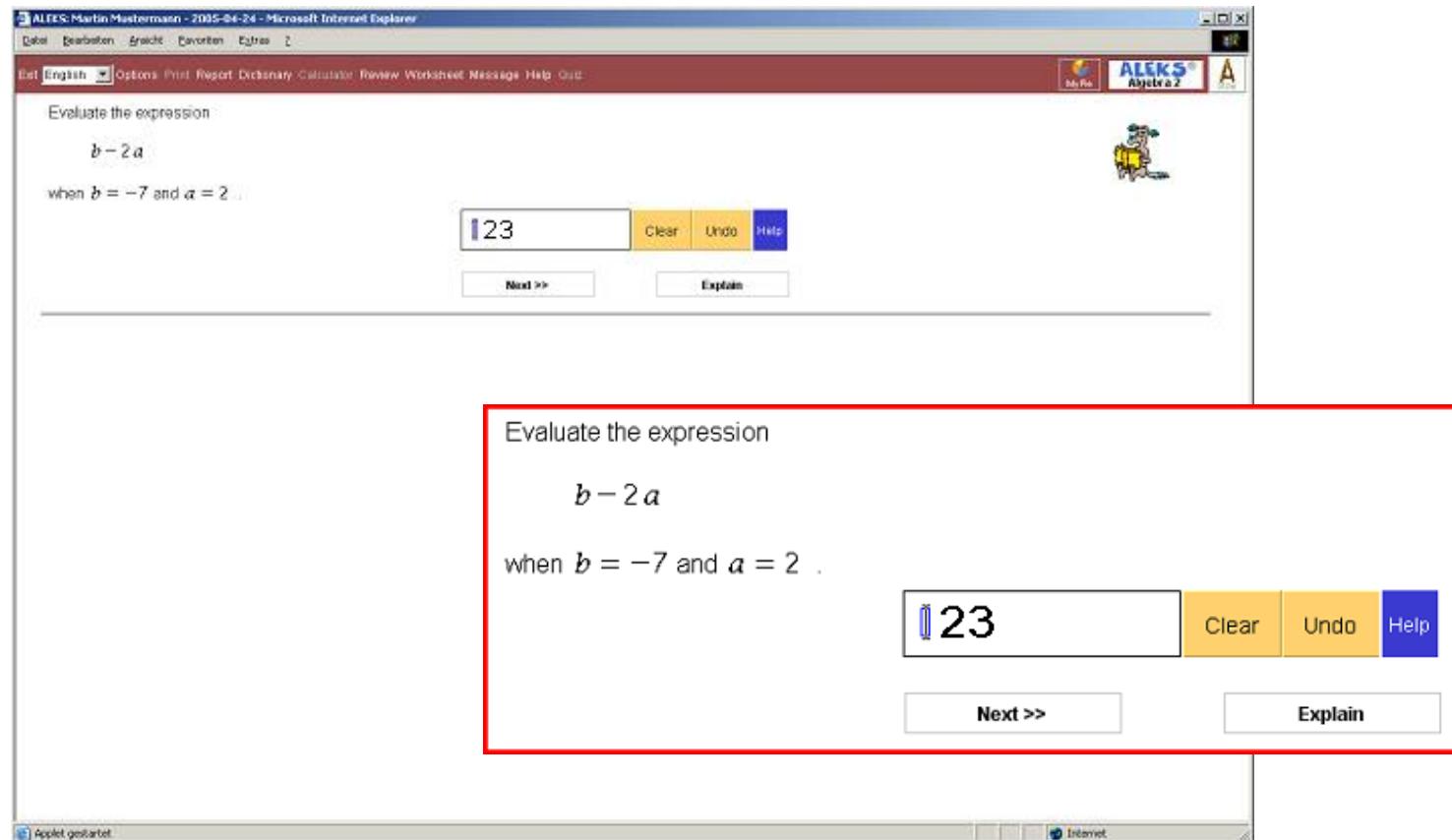
- ... details



ALEKS

Learning Phase

- Prototypical task or problem



The screenshot shows a Microsoft Internet Explorer window displaying the ALEKS Learning Phase. The title bar reads "ALEKS: Martin Meistermann - 2005-04-24 - Microsoft Internet Explorer". The menu bar includes "Datei", "Bearbeiten", "Ansicht", "Dokumenten", "Extras", and "Hilfe". The toolbar includes "Edit", "English", "Options", "Print", "Report", "Dictionary", "Calculator", "Review", "Worksheet", "Message", "Help", and "Quit". A cartoon character is visible in the top right corner.

The main content area displays a math problem:

Evaluate the expression
 $b - 2a$
when $b = -7$ and $a = 2$

An input field contains "123" with a cursor. Below it are buttons for "Clear", "Undo", and "Help". At the bottom are "Next >>" and "Explain" buttons.

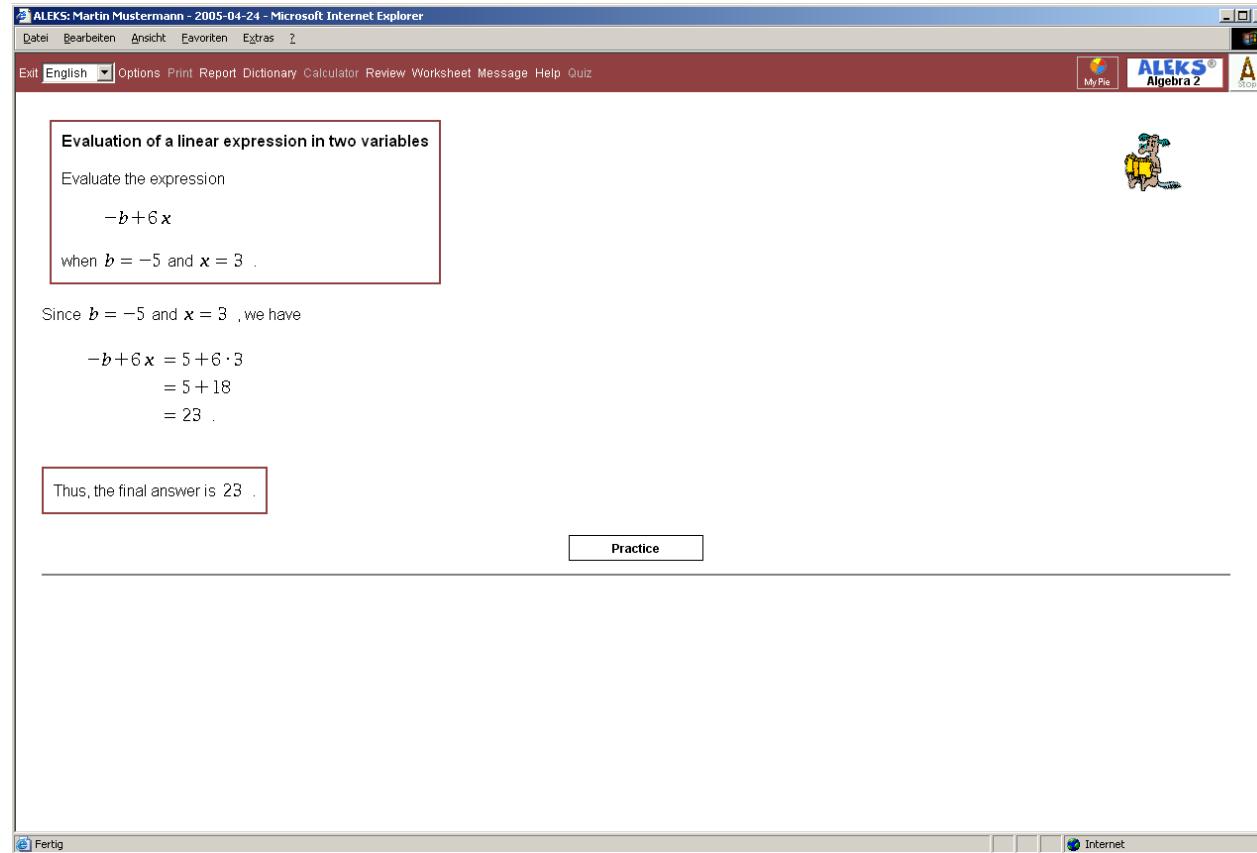
A red rectangular box highlights the input field and its surrounding area, specifically the expression and the variable values.

At the bottom of the browser window, the status bar shows "Applet gestartet." and "Internet".

ALEKS

Learning Phase

- Lesson → Explanation for a task



The screenshot shows a Microsoft Internet Explorer window titled "ALEKS: Martin Mustermann - 2005-04-24 - Microsoft Internet Explorer". The menu bar includes "Datei", "Bearbeiten", "Ansicht", "Favoriten", "Extras", "Help", and "Quiz". The toolbar includes "Exit", "English", "Options", "Print", "Report", "Dictionary", "Calculator", "Review", "Worksheet", "Message", and "Help". The status bar at the bottom shows "Fertig" and "Internet".

Evaluation of a linear expression in two variables

Evaluate the expression

$$-b + 6x$$

when $b = -5$ and $x = 3$.

Since $b = -5$ and $x = 3$, we have

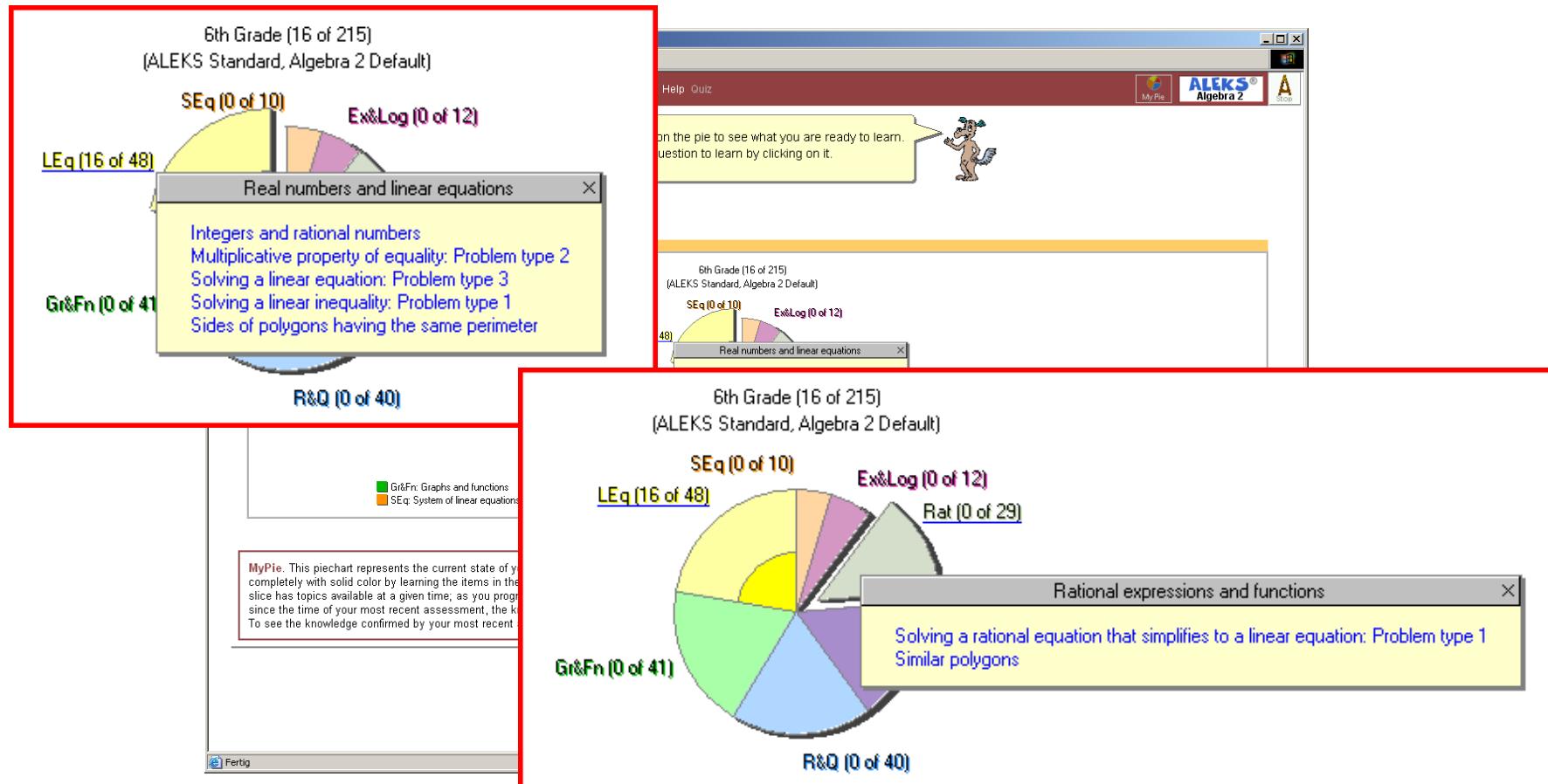
$$\begin{aligned} -b + 6x &= 5 + 6 \cdot 3 \\ &= 5 + 18 \\ &= 23. \end{aligned}$$

Thus, the final answer is 23.

Practice

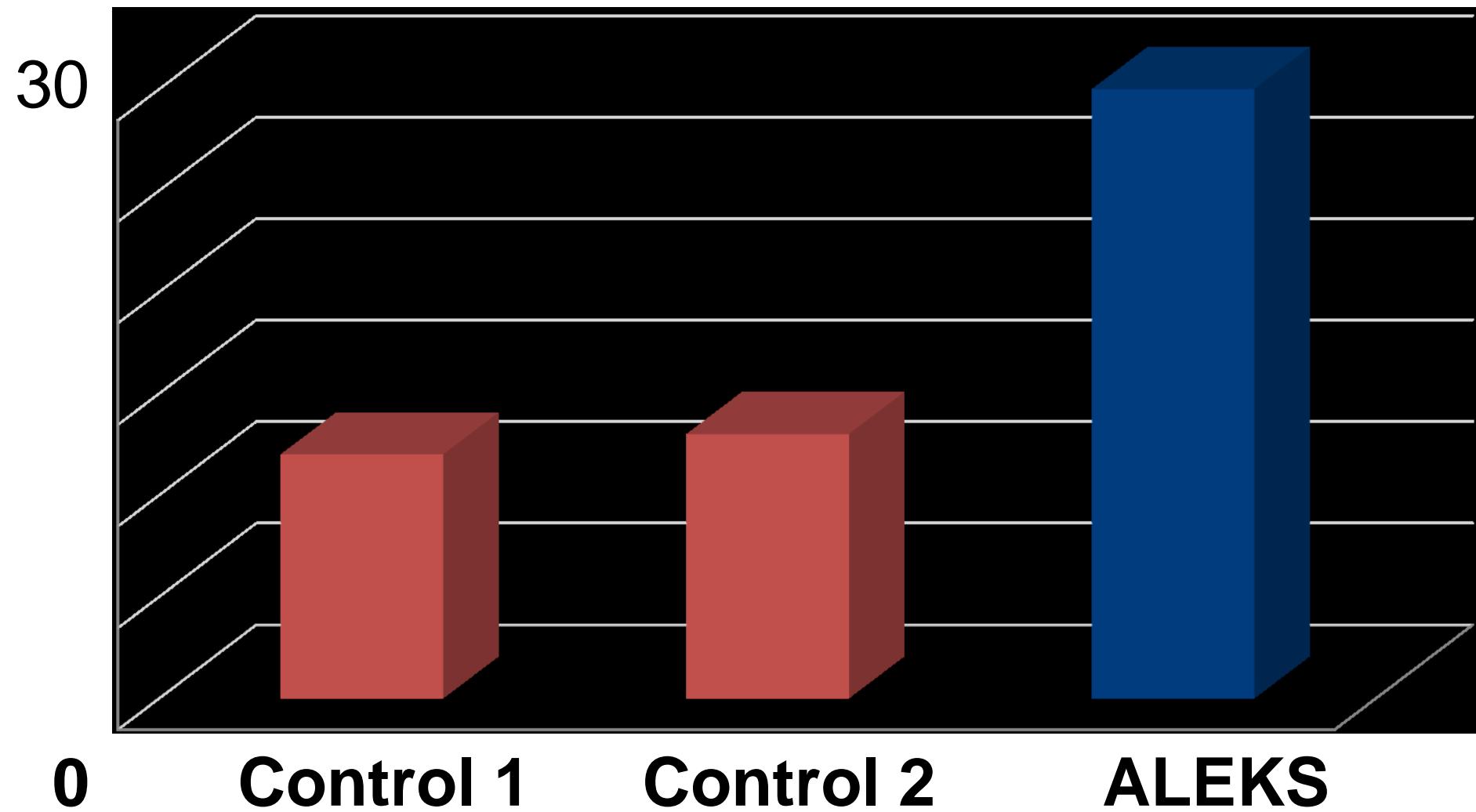
Learning Phase

- Overview of learning progress



Increase in Math CST score

CST: California Standard Test



- Introduction
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Elementary Probability

Example Problem

Example of one PROBLEM

An urn contains three red and three blue balls

Two balls are drawn successively

Drawing is performed with replacement

The drawn balls are red

Compute the probability of this event

Six problems, labeled A, B, C, ...F of this kind are used

Components and their Attributes

Example of one PROBLEM-COMPONENT with three ATTRIBUTES:

method of drawing

with three attributes:

(1) drawing one ball

(2) drawing multiple balls with replacement, and

(3) drawing multiple balls without replacement

Demands and Types of Lessons

The DEMANDS

0. Definition of random experiments and elementary events
1. Knowledge that, in general, Laplace probabilities are computed as the ratio between the number of favourable events and the number of possible events
2. Ability to determine the number of possible events
3. Ability to determine the number of favourable events if one ball is drawn
4. Ability to determine a favourable event if one ball is drawn, or if the sample for which the probability has to be computed consists of equally coloured balls
5. Knowledge that if an outcome like “exact/at least n balls are of colour x” is asked for, all possible sequences of drawing are favourable events

Demands and Types of Lessons

6. Knowledge that probabilities are added for two disjoint events A and B
7. Knowledge that probabilities are multiplied for two events A and B that are (stochastically) independent
8. Knowledge that the probability of drawing a ball of a specific colour is not equal to 0.5 if there are different numbers of balls of different colours in the urn
9. Knowledge that drawing without replacement reduces both, the total number of balls in the urn as well as the number of balls that have the same colour as the drawn ball
10. Knowledge that drawing at least a number of certain balls includes the - not explicitly stated - results of drawing more balls of the certain kind

Demand-based Component Approach

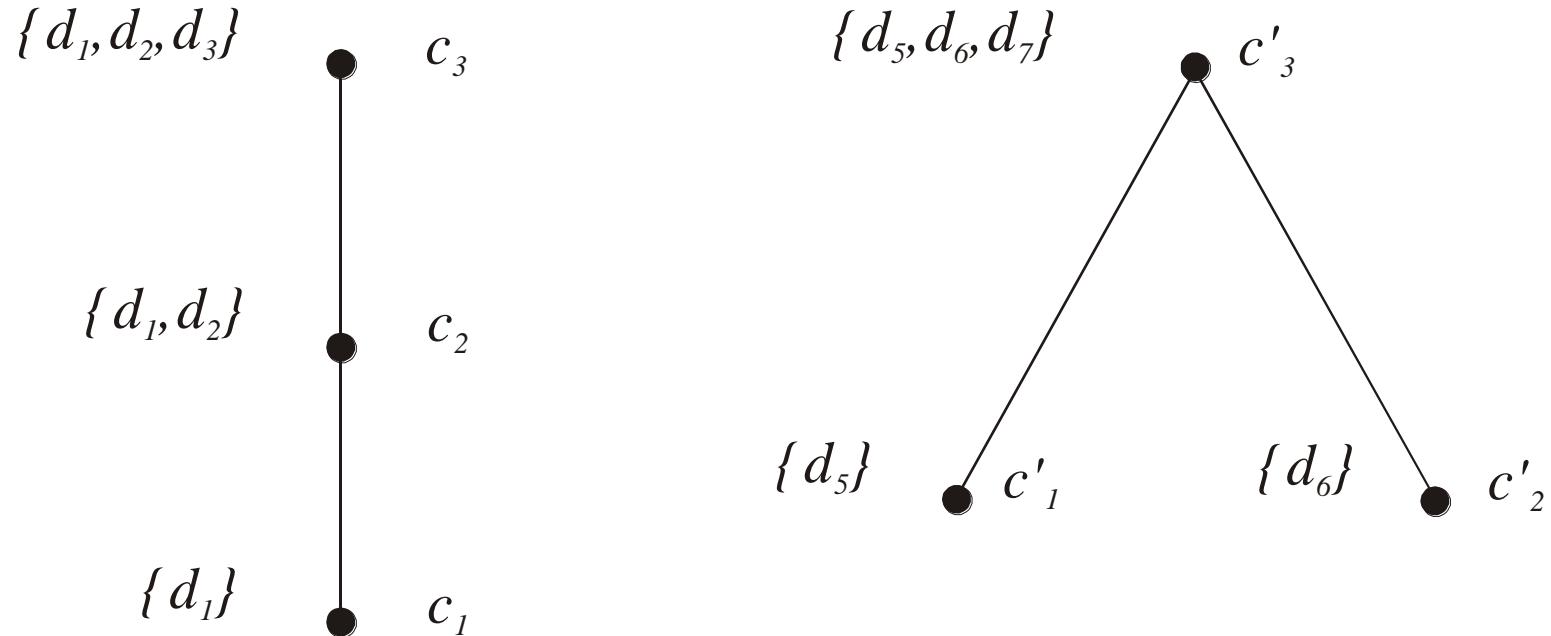
Where these demands come from?

- Problems of a knowledge domain are analysed/constructed with respect to the cognitive demands they pose to the learner
- The cognitive demands are considered
 - to constitute elementary attributes assigned to components for characterising and ordering problems
 - to establish the lessons presented to the learner

Ordering Attributes by Set Inclusion

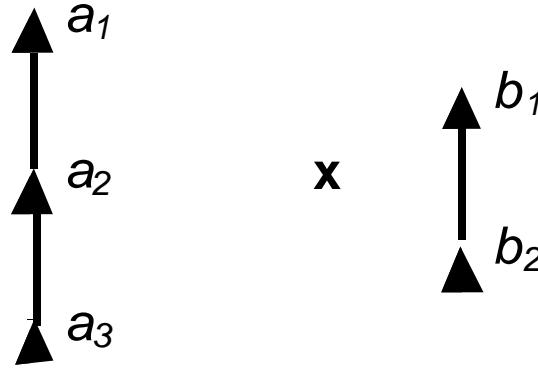
How to use the demands for ordering attributes?

- Demand-induced difficulty structures for attributes:

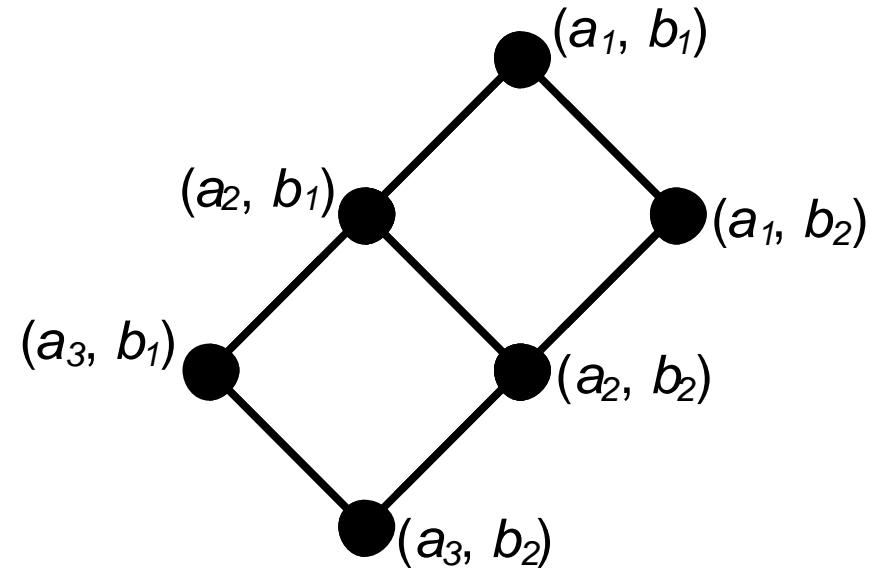


Surmise Relation via Component-wise ordering

- Principle of component-wise ordering
 - Cartesian product of problem components → problem types
 - Dominance rule applied on attribute tuples (problem types)
→ surmise relation



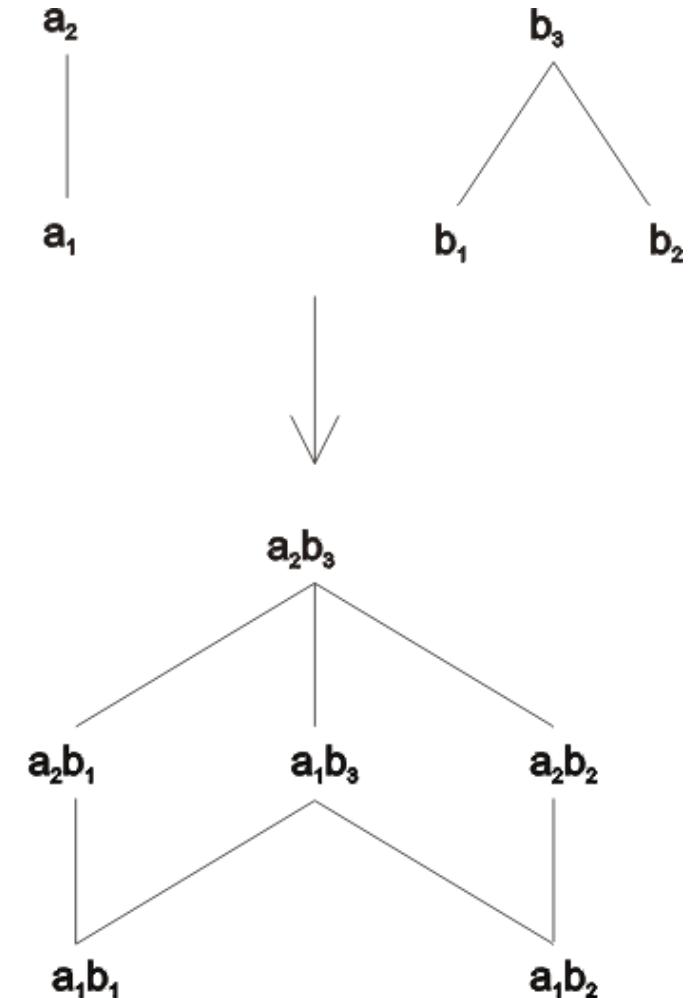
Attribute orders



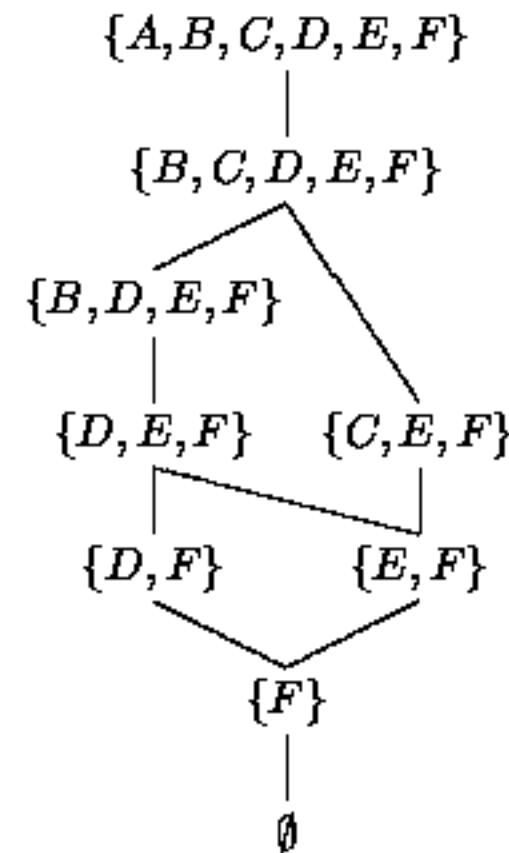
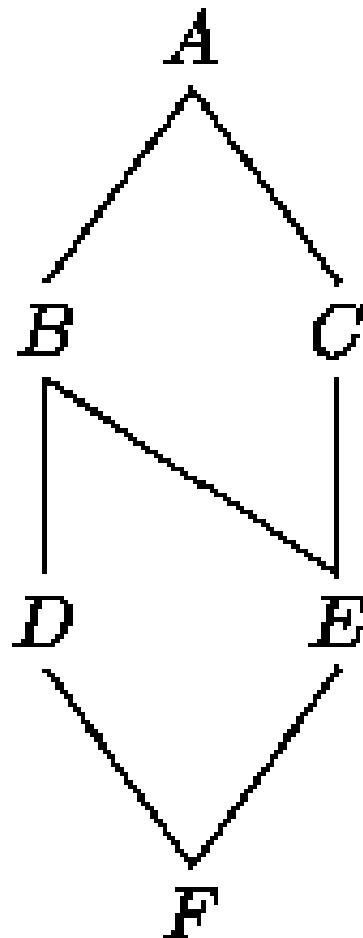
Surmise relation

In other words

- Difficulty structures for attributes
- Surmise structure for problems



Problem Structure and Knowledge Space



Demand, Lesson and Skill Structure

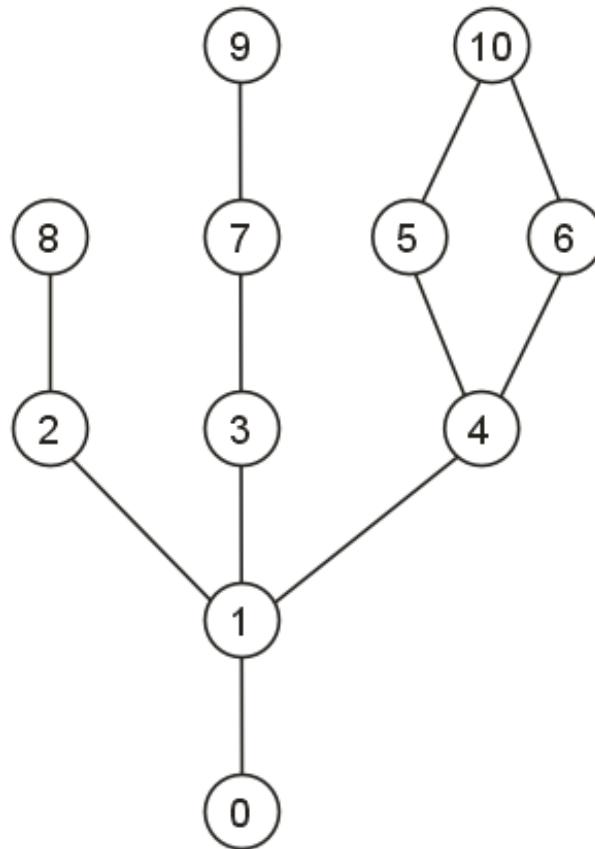
Demand assignments for attributes of *way of drawing*

Attribute	Demands
a1	1, 2
a2	1, 2, 3
a3	1, 2, 3, 4

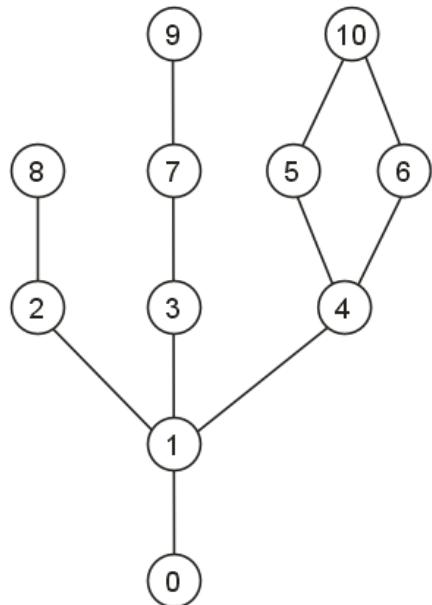
Attribute assignments for Skills/Lessons/Dem.

Skills (Lessons/ Demands)	Attributes
1	a1, a2, a3
2	a1, a2, a3
3	a2, a3
4	a3

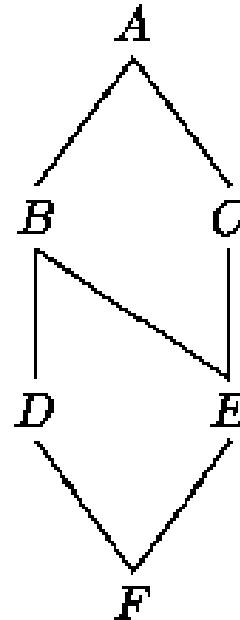
Demand and Lesson Structure



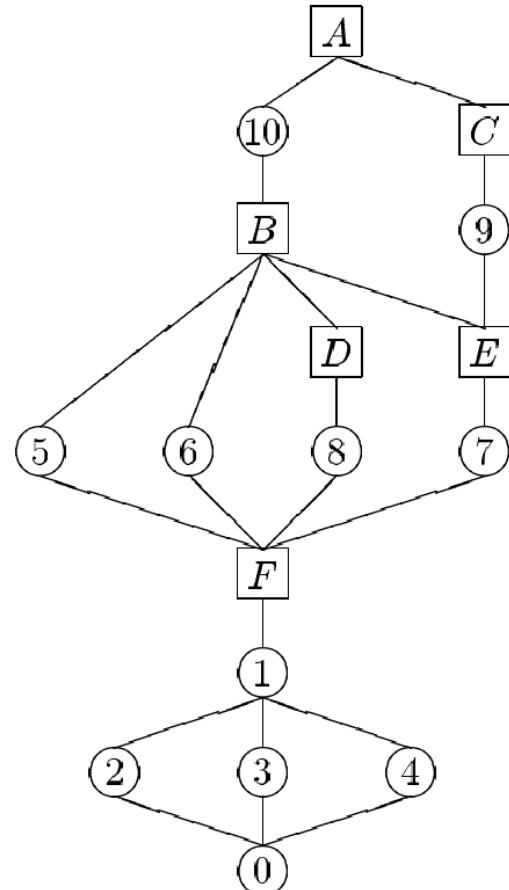
Combined Surmise Structure of Lessons and Problems



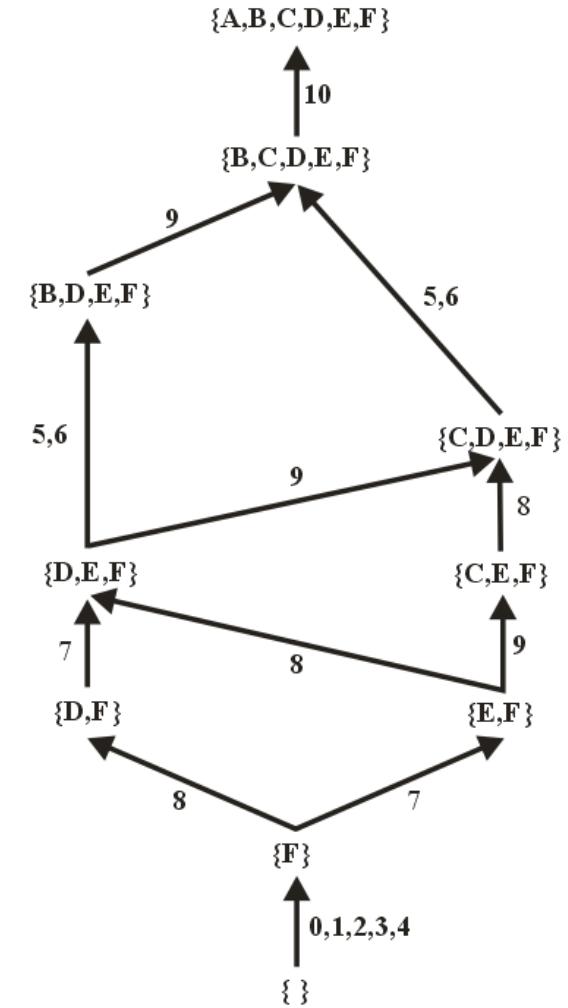
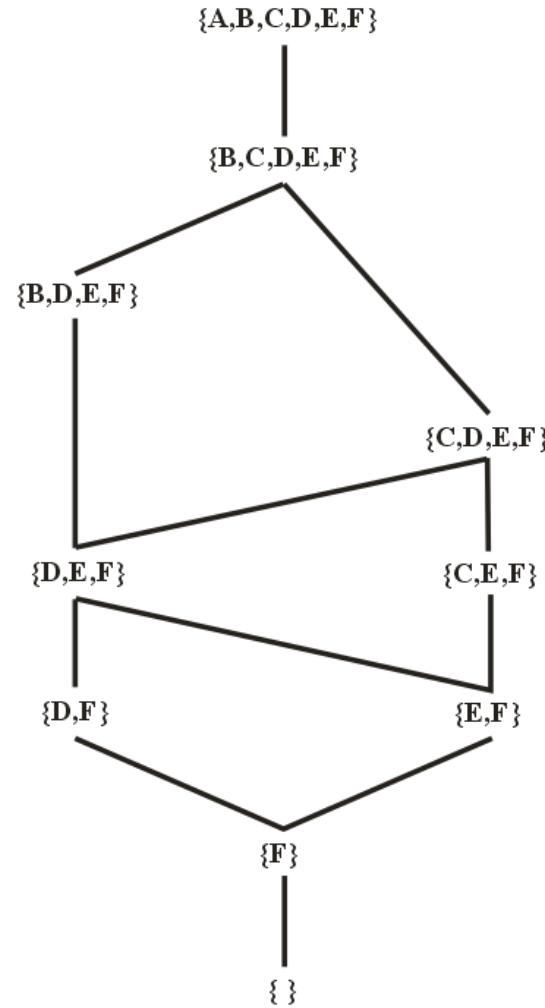
&



→



Induced Knowledge Space with Lessons: Didactic



- Introduction
- Knowledge Space Theory
 - Application: ALEKS
- **Demand Component Approach**
 - **Application: RATH**
- Competence-Performance Approach
 - Application: APeLS
- Ontology-based Skill Approach
 - Application: iClass

Application: RATH

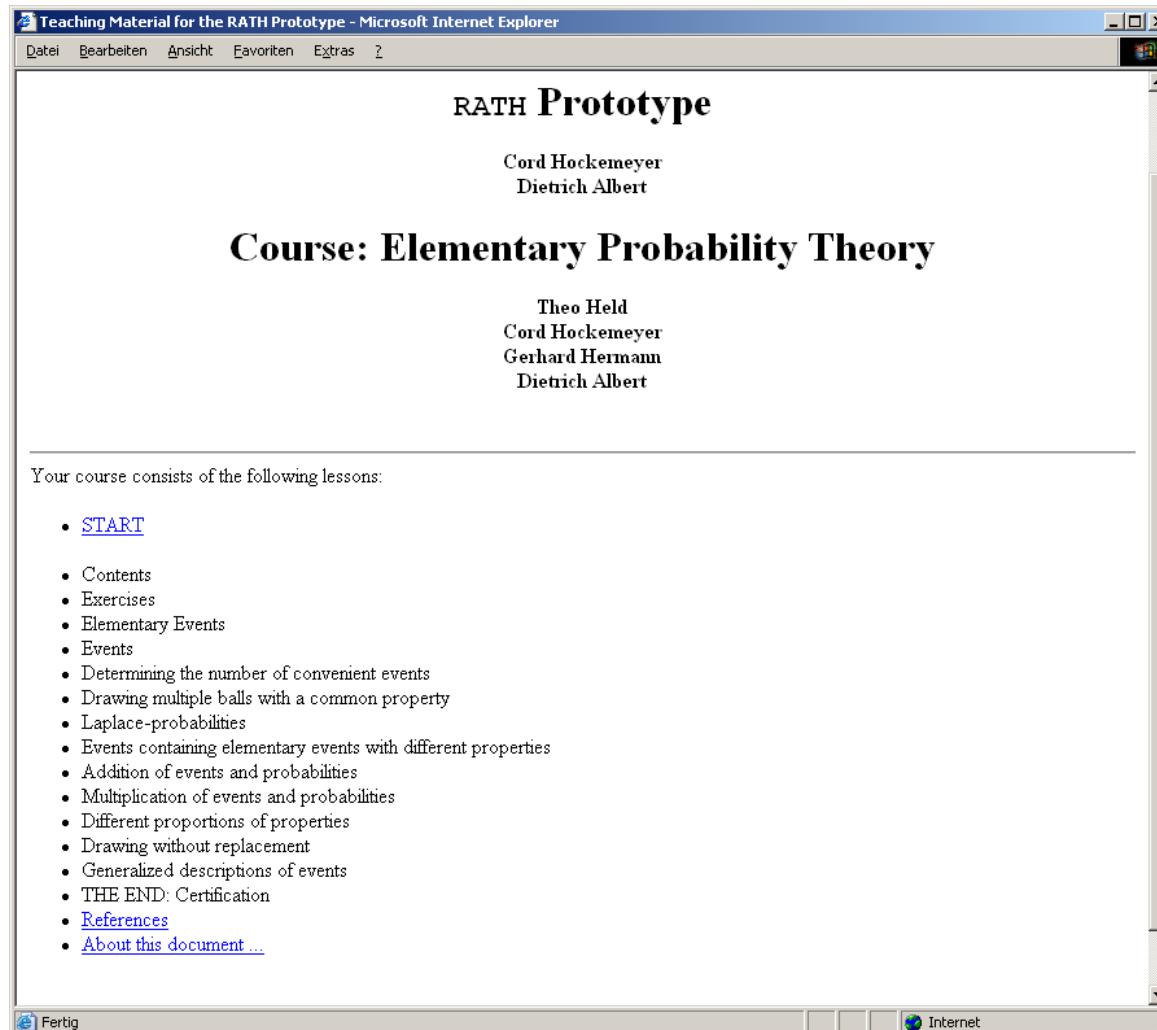
<http://css.uni-graz.at/rath>

■ RATH

Relational Adaptive Tutoring Hypertext

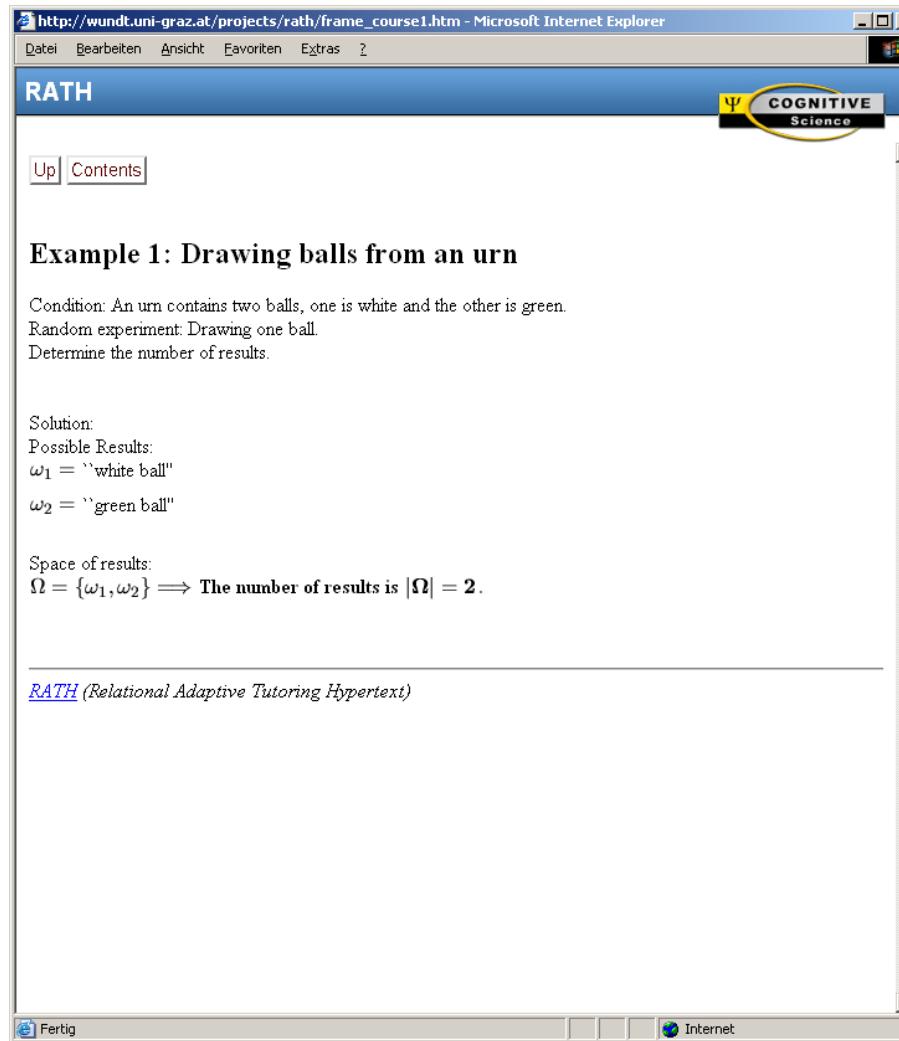
- Adaptive course on elementary probabilistic theory
 - Lessons, exercises, tests
 - Personalized learning paths: efficient selection of appropriate learning objects
- Based on three sources
 - A mathematical model of Hypertext
 - The Relational Database Theory
 - A correspondence between a mathematical hypertext model and Knowledge Space Theory

RATH Starting Page



The screenshot shows a Microsoft Internet Explorer window displaying the "Teaching Material for the RATH Prototype". The title bar reads "Teaching Material for the RATH Prototype - Microsoft Internet Explorer". The menu bar includes "Datei", "Bearbeiten", "Ansicht", "Favoriten", "Extras", and a question mark icon. The main content area features a large blue header "RATH Prototype". Below it, two names are listed: "Cord Hockemeyer" and "Dietrich Albert". A section titled "Course: Elementary Probability Theory" follows, listing course members: "Theo Held", "Cord Hockemeyer", "Gerhard Hermann", and "Dietrich Albert". A horizontal line separates this from the lesson list. The list includes: "START", "Contents", "Exercises", "Elementary Events", "Events", "Determining the number of convenient events", "Drawing multiple balls with a common property", "Laplace-probabilities", "Events containing elementary events with different properties", "Addition of events and probabilities", "Multiplication of events and probabilities", "Different proportions of properties", "Drawing without replacement", "Generalized descriptions of events", "THE END: Certification", "References", and "About this document ...". The status bar at the bottom shows "Fertig" and "Internet".

RATH Part of a Lesson



The screenshot shows a Microsoft Internet Explorer window displaying a RATH (Relational Adaptive Tutoring Hypertext) lesson. The URL in the address bar is http://wundt.uni-graz.at/projects/rath/frame_course1.htm. The page title is "RATH". In the top right corner, there is a logo for "COGNITIVE Science" featuring a Greek letter Psi (Ψ) inside a yellow circle. The main content area contains the following text:

Example 1: Drawing balls from an urn

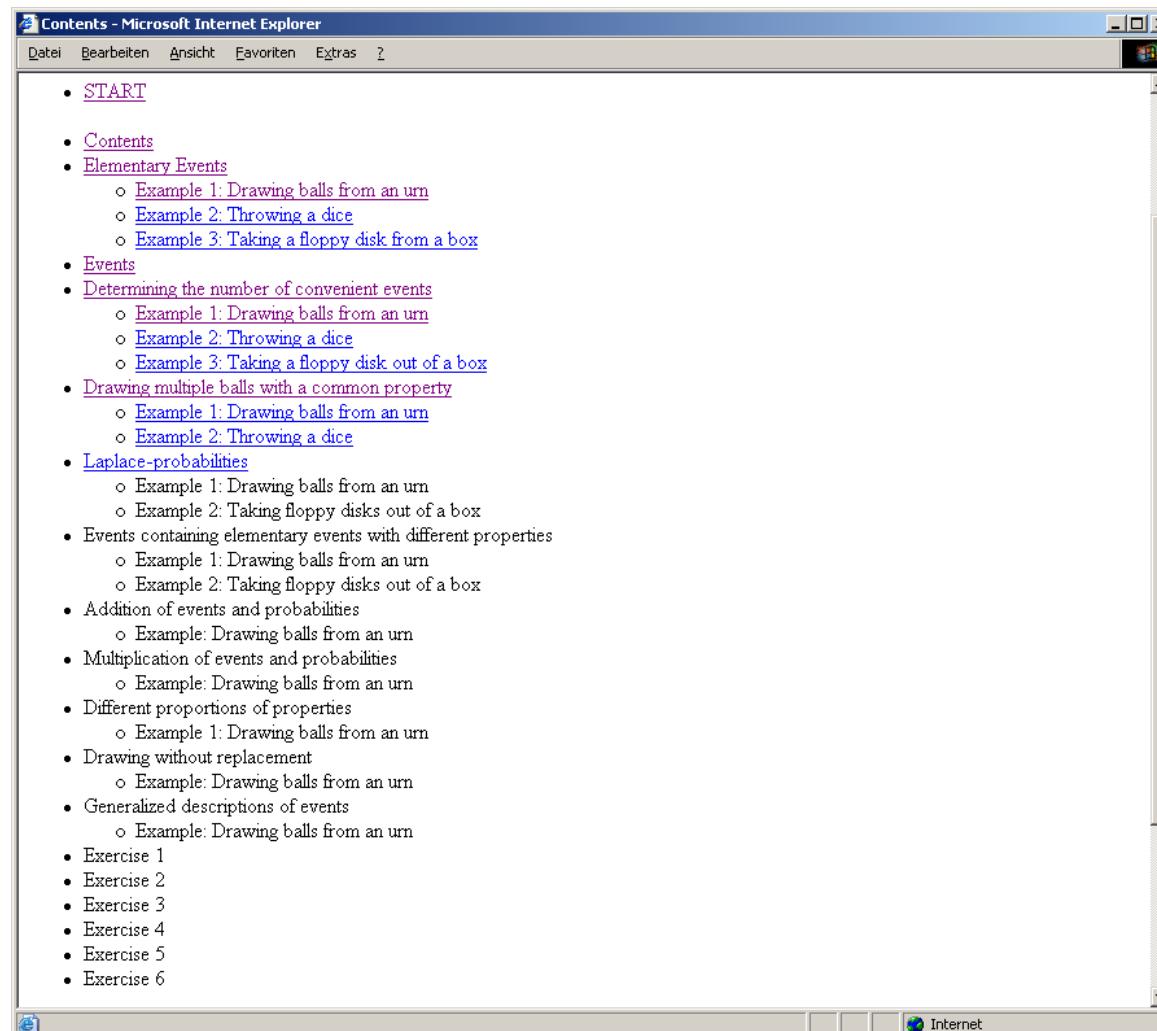
Condition: An urn contains two balls, one is white and the other is green.
Random experiment: Drawing one ball.
Determine the number of results.

Solution:
Possible Results:
 ω_1 = "white ball"
 ω_2 = "green ball"

Space of results:
 $\Omega = \{\omega_1, \omega_2\} \implies$ The number of results is $|\Omega| = 2$.

RATH (Relational Adaptive Tutoring Hypertext)

RATH: Access to other Lessons



RATH: Part of a Lesson

http://wundt.uni-graz.at/projects/rath/frame_course1.htm - Microsoft Internet Explorer

Datei Bearbeiten Ansicht Favoriten Extras ?

RATH

Ψ COGNITIVE Science

Up Contents

Laplace-probabilities

Theorem by Laplace:
If Ω is a finite set of elementary events with an equally distributed probability then the probability $P(A)$ for an arbitrary event $A \subseteq \Omega$ can be computed as the ratio

$$P(A) = \frac{|A|}{|\Omega|} = \frac{\text{Number of convenient elementary events}}{\text{Total number of elementary events}}$$

Random experiments which fulfil both conditions

1. Ω is a finite set of elementary events $\omega: \Omega = \{\omega_1, \omega_2, \dots, \omega_n\}$ with $n < \infty$.
2. All n elementary events $\{\omega_1\}, \{\omega_2\}, \dots, \{\omega_n\}$ are equally probable, i.e.
 $P(\{\omega_1\}) = \dots = P(\{\omega_n\}) = p$.

are called *Laplace experiments*.
$$\#P(\{\omega_1\})=\\[\text{dots}=P(\{\omega_n\})=p\$]$$

- [Example 1: Drawing balls from an urn](#)
- [Example 2: Taking floppy disks out of a box](#)

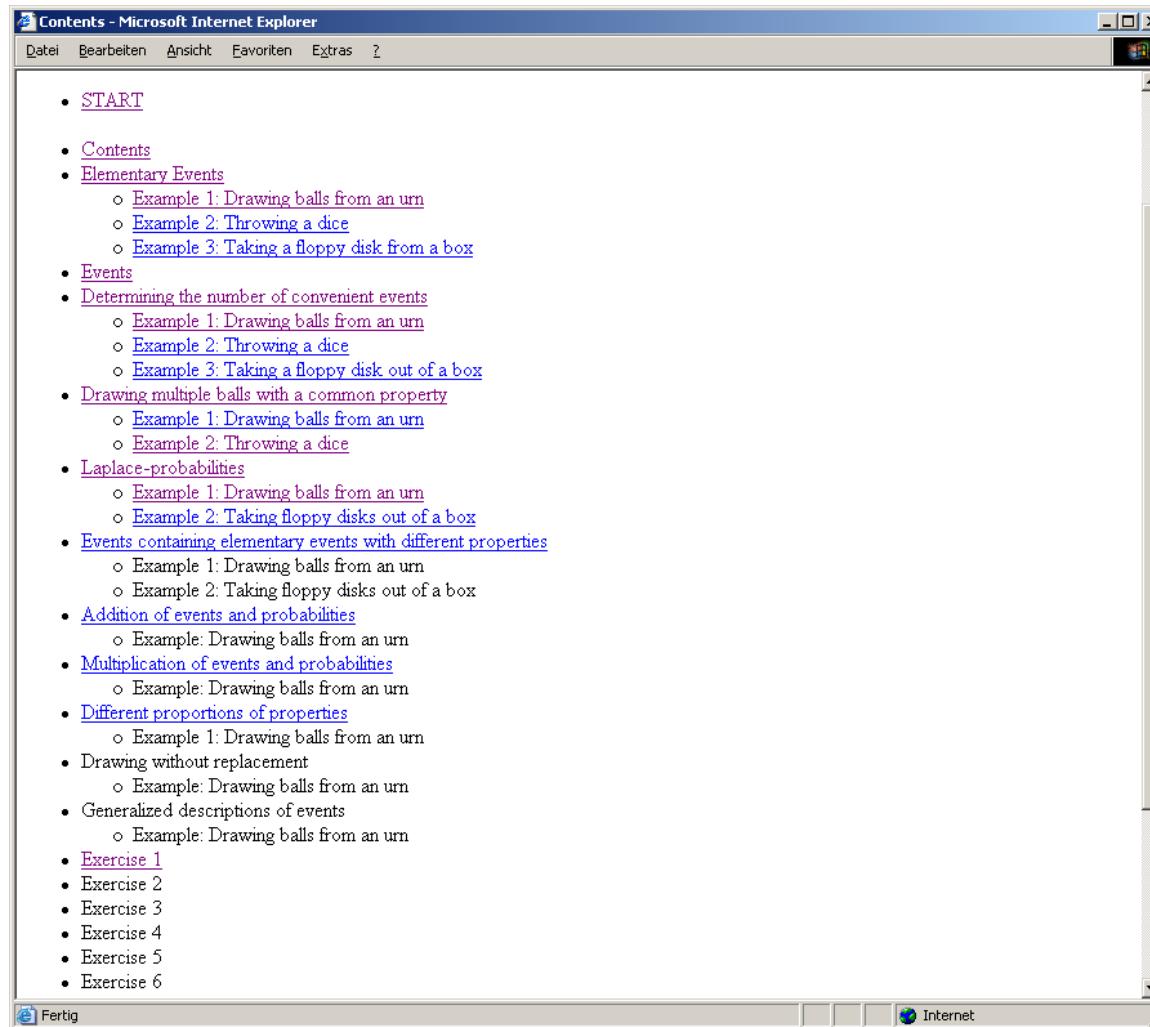
RATH (Relational Adaptive Tutoring Hypertext)

Fertig Internet

RATH: Access to Exercise



RATH Access to other Lessons



The screenshot shows a Microsoft Internet Explorer window with a blue title bar containing the text "Contents - Microsoft Internet Explorer". Below the title bar is a menu bar with the following items: Datei, Bearbeiten, Ansicht, Favoriten, Extras, and a question mark icon. The main content area displays a hierarchical navigation tree:

- [START](#)
- [Contents](#)
- [Elementary Events](#)
 - [Example 1: Drawing balls from an urn](#)
 - [Example 2: Throwing a dice](#)
 - [Example 3: Taking a floppy disk from a box](#)
- [Events](#)
- [Determining the number of convenient events](#)
 - [Example 1: Drawing balls from an urn](#)
 - [Example 2: Throwing a dice](#)
 - [Example 3: Taking a floppy disk out of a box](#)
- [Drawing multiple balls with a common property](#)
 - [Example 1: Drawing balls from an urn](#)
 - [Example 2: Throwing a dice](#)
- [Laplace-probabilities](#)
 - [Example 1: Drawing balls from an urn](#)
 - [Example 2: Taking floppy disks out of a box](#)
- [Events containing elementary events with different properties](#)
 - Example 1: Drawing balls from an urn
 - Example 2: Taking floppy disks out of a box
- [Addition of events and probabilities](#)
 - Example: Drawing balls from an urn
- [Multiplication of events and probabilities](#)
 - Example: Drawing balls from an urn
- [Different proportions of properties](#)
 - Example 1: Drawing balls from an urn
- [Drawing without replacement](#)
 - Example: Drawing balls from an urn
- [Generalized descriptions of events](#)
 - Example: Drawing balls from an urn
- [Exercise 1](#)
- [Exercise 2](#)
- [Exercise 3](#)
- [Exercise 4](#)
- [Exercise 5](#)
- [Exercise 6](#)

RATH Access to other Exercises



The screenshot shows a Microsoft Internet Explorer window displaying a list of exercises under the heading "RATH". The page is titled "frame_course1.htm" and includes a "COGNITIVE Science" logo. The menu bar at the top includes "Datei", "Bearbeiten", "Ansicht", "Favoriten", "Extras", and "?". The main content area lists various exercises:

- [START](#)
- [Contents](#)
- [Elementary Events](#)
 - [Example 1: Drawing balls from an urn](#)
 - [Example 2: Throwing a dice](#)
 - [Example 3: Taking a floppy disk from a box](#)
- [Events](#)
- [Determining the number of convenient events](#)
 - [Example 1: Drawing balls from an urn](#)
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- [Generalized descriptions of events](#)
 - [Example: Drawing balls from an urn](#)
- [Exercise 1](#)
- [Exercise 2](#)
- [Exercise 3](#)
- [Exercise 4](#)
- [Exercise 5](#)
- [Exercise 6](#)
- [THE END: Certification](#)

- Introduction
- Knowledge Space Theory
 - Application: ALEKS
- Demand Component Approach
 - Application: RATH
- **Competence-Performance Approach**
 - Application: APeLS
- Ontology-based Skill Approach
 - Application: iClass

Extending KST: Competence-based Approach

- Knowledge Space Theory in its original formalisation is purely behaviouristic
 - focus on observable behaviour
 - The underlying competences and skills have to be taken into account
- Competence/skill-based extensions of Knowledge Space Theory: CbKST

Competence-based KST (CbKST)

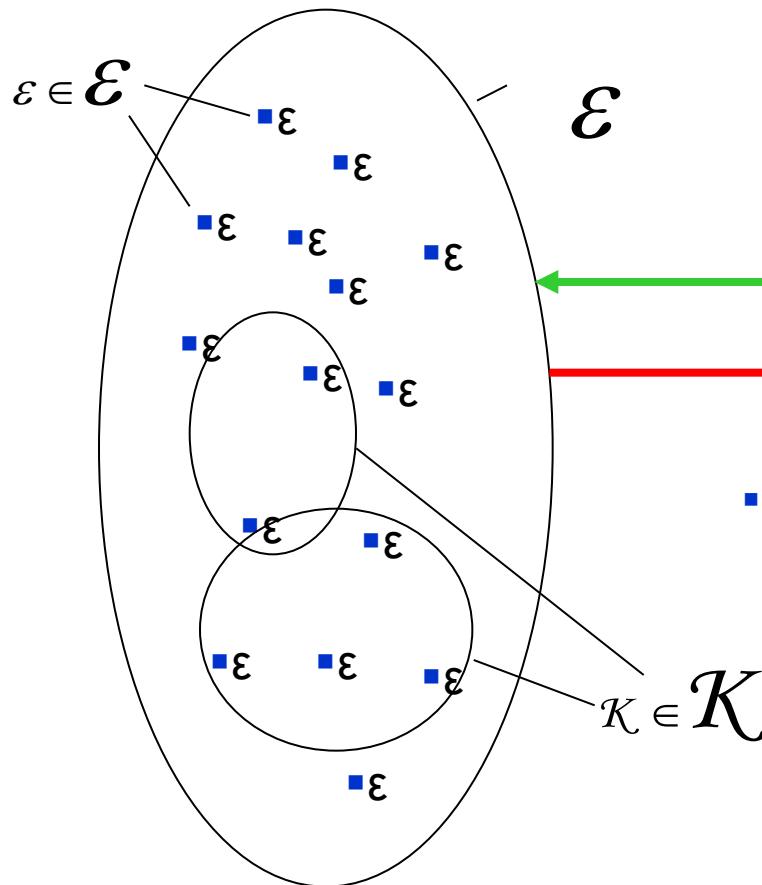
- Competence-based Knowledge Space Theory (CbKST)
 - Incorporates underlying skills and competencies
 - provides information for teaching
 - Includes explicit learning objects
 - explains transfer of knowledge
 - explains creating new knowledge
 - etc.

CbKST - Competence Performance Approach

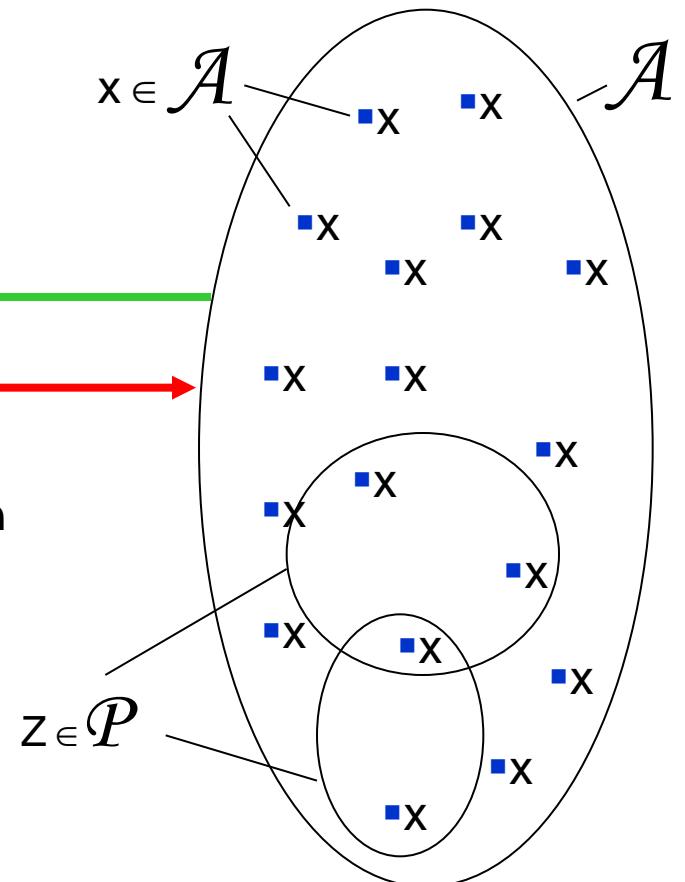
- Modelling knowledge through
 - (latent) competencies and
 - (observable) behaviour and performances
- Interpretation function (skill function)
 - Assigning to each item/problem the subset of competencies necessary to solve the item
- Representation function (problem function)
 - Assigning to each competence state the subset of items/problems solvable in this state

Competence Performance Approach

Competence-structure $(\mathcal{E}, \mathcal{K})$



Performance-structure $(\mathcal{A}, \mathcal{P})$



▪ Interpretation

$$k: \mathcal{A} \rightarrow \wp(\mathcal{K})$$

▪ Representation

$$p: \mathcal{K} \rightarrow \wp(\mathcal{A})$$

Competence Performance Approach

- Interpretation Function k : to each item x (of A) a subset k_x of Competence States is assigned in which the item is solvable
- Representation Function p : to each Competence State e (out of K) a set $p(e)$ of items solvable in this state is assigned

Competence Performance Approach

- Interpretation/skill and representation/problem function induce competence and performance spaces/structures
- Both concepts are equivalent, i.e. given one function the other is uniquely determined
- The assignment of competences/skills puts constraints on the possible knowledge states and thus defines a knowledge structure
- Additionally, by a theoretically derived competence structure the number of competence and knowledge/performance states can be reduced

Competence Performance Approach

- Assessing a learner's competence state by Problem-based skill assessment
 - Step 1: adaptive assessment of the knowledge respectively performance state
 - Step 2: mapping to corresponding competence state

Example: Material and Structure

Geometry problems

- adapted from Korossy (1993, 1996)
- based on a Competence Performance Modeling
- e.g.



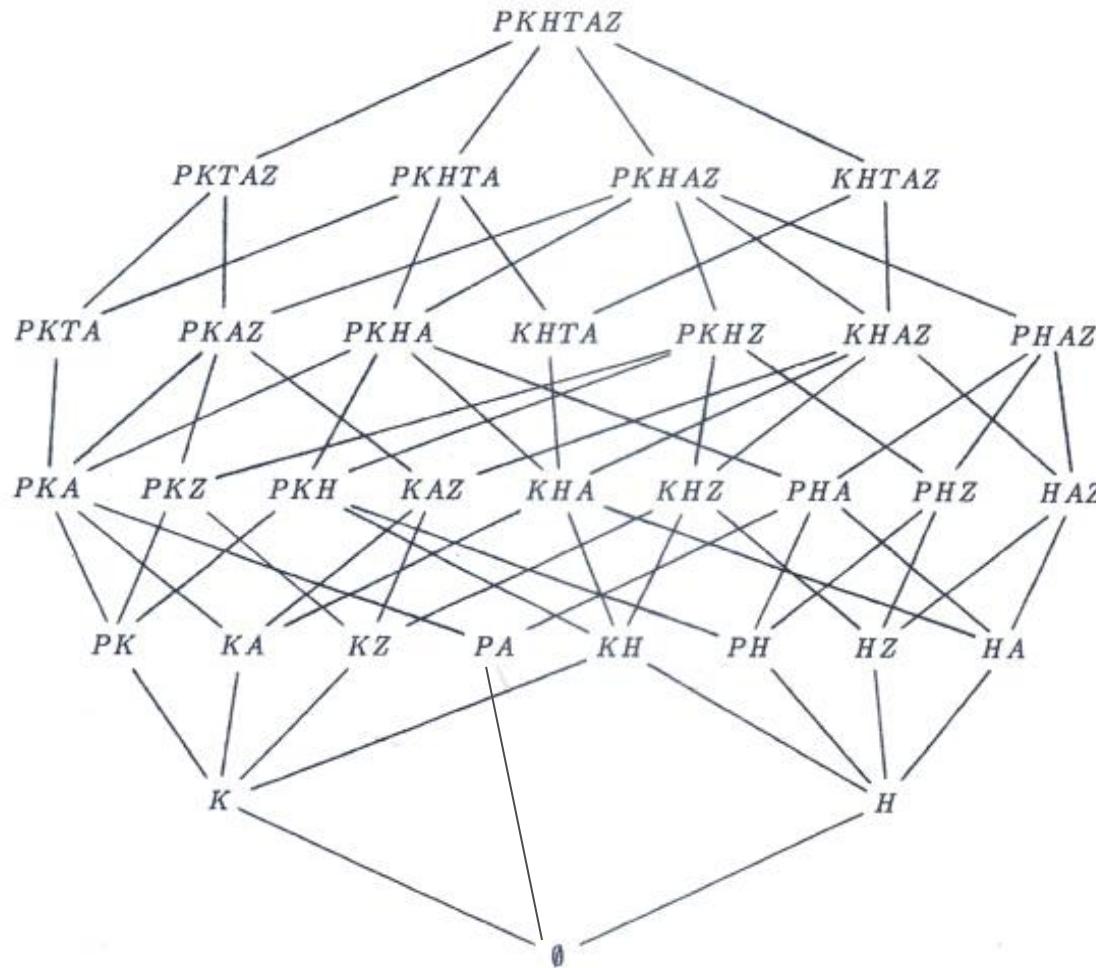
Example: Material and Structure

- Elementary Competencies:

<i>abbreviation</i>	<i>domain-specific meaning</i>
P	knowledge of the Theorem of Pythagoras
K	knowledge of the Kathetensatzes
H	knowledge of the Euclid's altitude theorem
A	knowledge about calculating the area of a right-angled triangle
Z	knowledge of constructing a square with the same area as a given rectangle
T	knowledge of properties of tangents on circles

- definition of dependencies between the Elementary Competencies
- resulting Competence Structure:
32 Competence States

Competence Structure

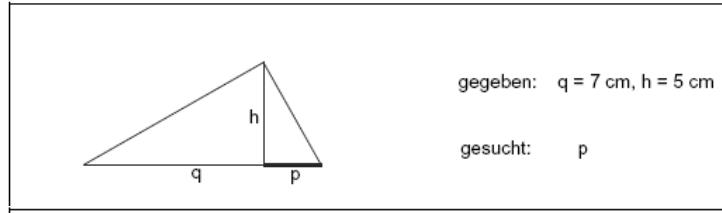


Material and Structure

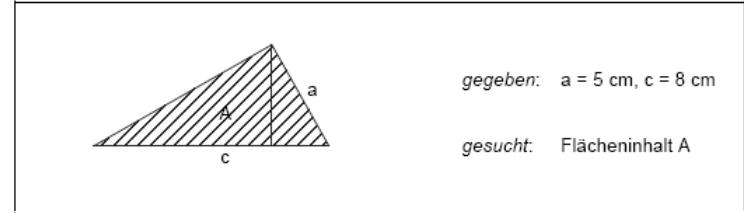
- In accordance with Competence Structure:
construction of 10 geometry problems
- solution analysis: identification of possible
solution ways applying the Elementary
Competencies

Material and Structure

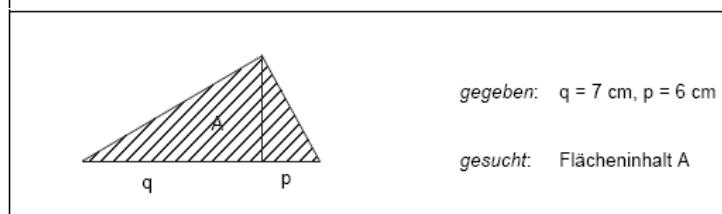
a)



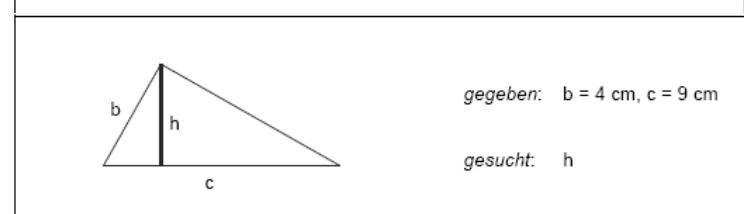
f)



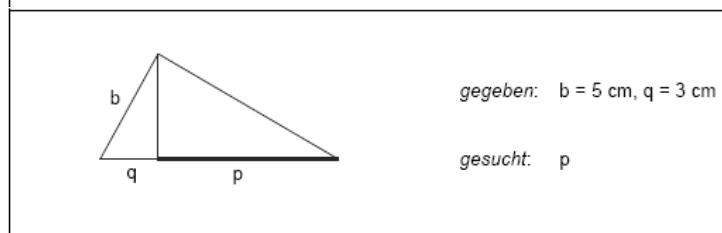
b)



g)



c)



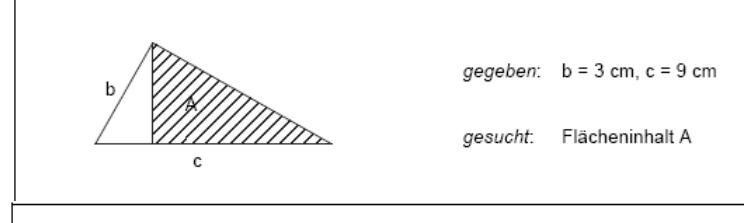
h)



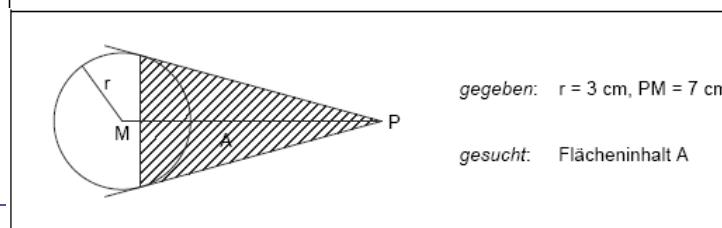
d)

gegeben: Rechteck
Verwandle zeichnerisch in ein flächeninhaltsgleiches Quadrat, d.h. nur durch Benutzung von Geodreieck, Zirkel, Bleistift und ohne Durchführung von Berechnungen!

i)



e)



j)

gegeben: Rechteck, Quadrat (unten)
Konstruiere ein Quadrat mit dem gleichen Flächeninhalt wie das gegebene Rechteck und Quadrat zusammen haben. D.h. nur durch Benutzung von Geodreieck, Zirkel, Bleistift und ohne Durchführung jeglicher Berechnungen!

- Interpretation Function:

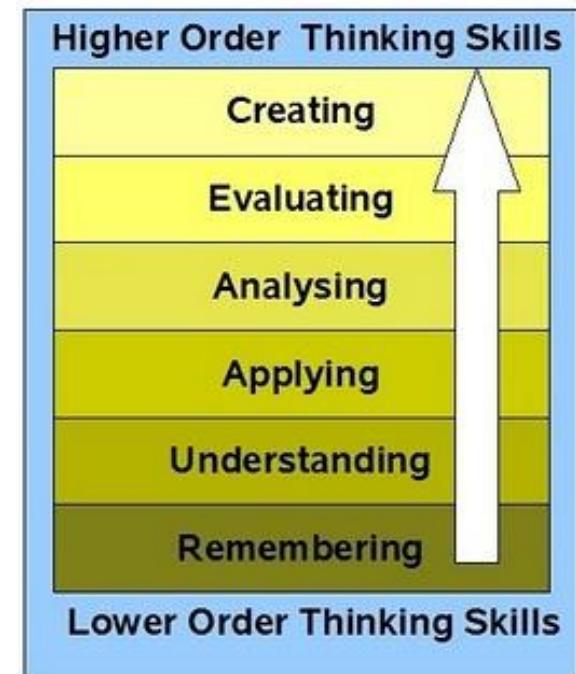
<i>a</i>	$\{H, PK\}$
<i>b</i>	$\{HA, KA\}$
<i>c</i>	$\{K, PH\}$
<i>d</i>	$\{KZ, HZ\}$
<i>e</i>	$\{PKTA, KHTA\}$
<i>f</i>	$\{PA, KHA\}$
<i>g</i>	$\{PK, KH, PA\}$
<i>h</i>	$\{PHA, PKA\}$
<i>i</i>	$\{KHA, PKA\}$
<i>j</i>	$\{PKZ, PHZ\}$

- Representation Function (extract):

\emptyset	\emptyset	KHA	$abcfgi$
K	c	PHA	$abcfg$
H	a	$PKZ, PKHZ$	$acd gj$
KA	bc	PHZ	$acd j$
KZ	cd	KHZ	$acd g$
PK, KH, PKH	acg	$KHAZ$	$abcd fgi$
$PKA, PKHA$	$abcfghi$	$PKTAZ, PKHTAZ$	$abcdefhij$
$KHTAZ$	$abcdefgi$

CbKST: iClass Skill Definition

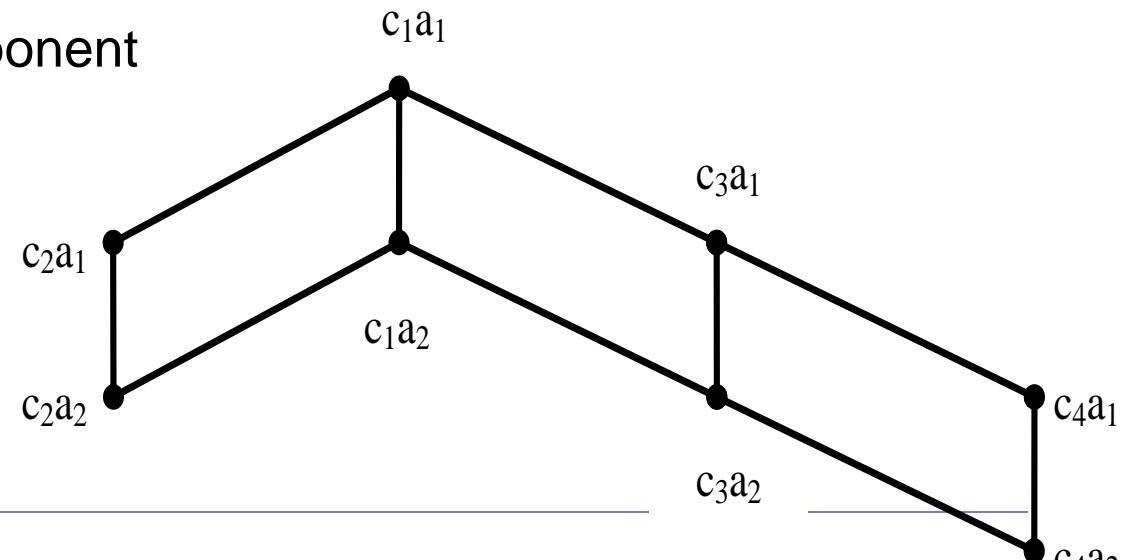
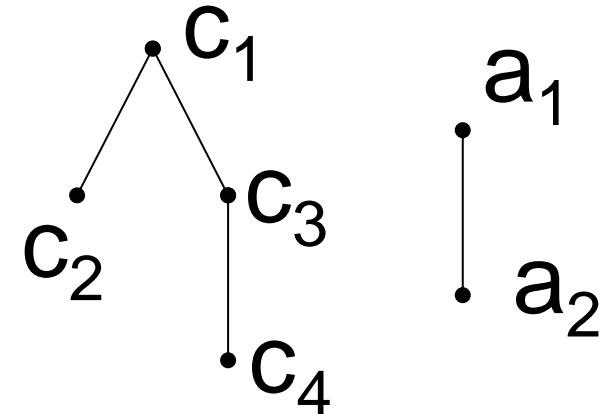
- skill components
 - declarative component:
concept(s) (e.g. Pythagorean Theorem)
 - procedural component:
action verb (e.g. state, apply)
 - may be associated with Bloom's revised taxonomy of educational objectives:
 - levels of cognitive processing



(e.g. Heller, Steiner, Hockemeyer, & Albert, 2006; Anderson et al., 2001)

CbKST: iClass Skill Definition

- concepts with their hierarchical structure
 - e.g. 'Theorem of Pythagoras' is prerequisite for 'Altitude Theorem' corresponding to curriculum
 - order on the action verbs
 - e.g.: 'state' is prerequisite for 'apply'
 - the product of these two component orderings results in a surmise relation on the skills
- e.g. skill c_2a_2 is a prerequisite to the skills c_2a_1 , c_1a_2 , an



CbKST: Personalised Learning Paths

- Once the competence state of a learner has been determined a personalised learning path may be selected
 - based on skill assignments to learning objects
- Deciding upon next learning object, given a certain competence state
 - referring to learning path of the competence structure a suitable learning object is selected, featuring
 - required skills that the learner has already available
 - taught skills that correspond to next step in learning path

- Introduction
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- Ontology-based Skill Approach
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Application: APeLS

<http://css.unigraz.at/demos/apels>

- **APeLS**

Advanced **P**ersonalised **L**earning **S**ystem (TCD)

<http://css.uni-graz.at/demos/apels>

User name: admin pass word: atwundt

- Competence learning structures

- Set of learning objects and a set of assigned competencies: **Taught vs. required competencies**
 - Competence structures only implicitly defined (by metadata)

Newtonian Mechanics: Examples of Kinematics: Two Exercises

Exerc. 5. A body moves vertically away from earth, according to the law $s = 19,6 t - 4,9 t^2$. Demonstrate that it has only the half velocity after rising 14,7 m.

Exerc. 6. During the upwards motion of an freight elevator ($v_0 = 0,8 \text{ m/s}$) the cable breaks. Which velocity does the cabin have, when the catching mechanism takes action 25 cm after begin of the free fall? Which lag is acting when the cabin comes to stand after

Kinematics Skills

- p₁ To be able to transform and apply formula. $\Delta s = v \cdot \Delta t$
- p₂ To be able to state and apply the definition of acceleration.
- p₃ To be able to perform computations concerning the free fall.
- P₄ To be able to transform and apply formulae for steadily accelerated motion with initial velocity zero.
- p₅ To be able to transform and apply formulae for steadily accelerated motion with initial velocity unequal zero.
- p₆ To be able to state and apply the connection between braking distance, initial velocity and braking lag. $\Delta s = \frac{v_0^2}{2a}$
- p₇ To be able to compose motions by vector addition, with at least one of the motions being unequal.

APeLS Initial Assessment

Mechanics Course Pre-Test - Microsoft Internet Explorer

Datei Bearbeiten Ansicht Favoriten Extras 2

Table of Contents Content

Mechanics Course Pre-Test

y = x ^ (10 - x), What is y when x = 3?

21
 27

The "slope" of a graph of y vs x is ...

The value of y squared divided by the value of x squared
 The change in y divided by the corresponding change in x

If V is a particular vector, then |V| symbolizes ...

Its magnitude
 Its angle

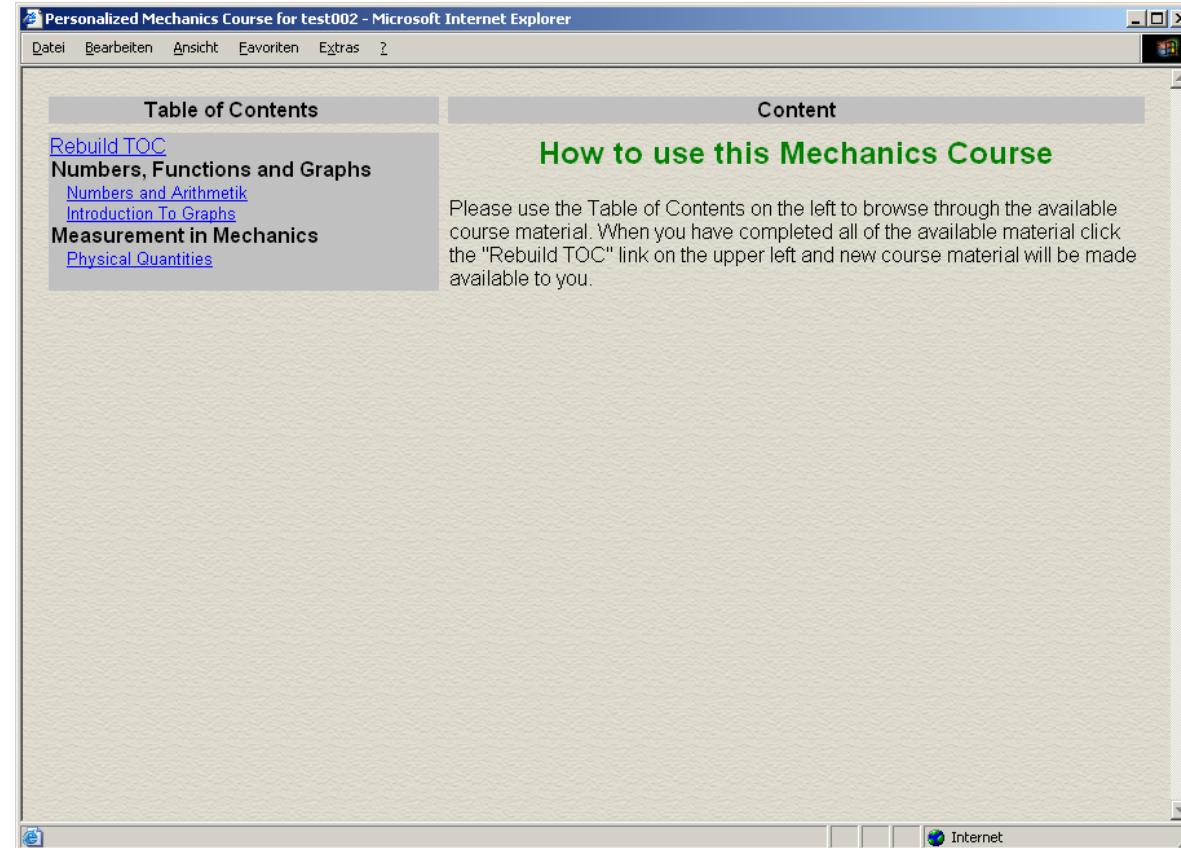
On the standard Real Number Line the left-hand side of zero is ...

Positive
 Negative
 There is no left-hand side

Anfrage senden Zurücksetzen

Fertig Internet

APeLS Start of the Course



The screenshot shows a Microsoft Internet Explorer window titled "Personalized Mechanics Course for test002 - Microsoft Internet Explorer". The menu bar includes "Datei", "Bearbeiten", "Ansicht", "Favoriten", "Extras", and "?". The main interface is divided into two sections: "Table of Contents" on the left and "Content" on the right.

Table of Contents:

- [Rebuild TOC](#)
- Numbers, Functions and Graphs**
 - [Numbers and Arithmetik](#)
 - [Introduction To Graphs](#)
- Measurement in Mechanics**
 - [Physical Quantities](#)

Content:

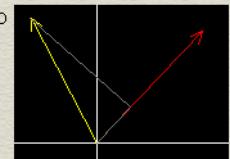
How to use this Mechanics Course

Please use the Table of Contents on the left to browse through the available course material. When you have completed all of the available material click the "Rebuild TOC" link on the upper left and new course material will be made available to you.

- Learning progress (learning object va4)

Personalized Mechanics Course for test002 - Microsoft Internet Explorer

Datei Bearbeiten Ansicht Favoriten Extras ?

Table of Contents	Content
Rebuild TOC Numbers, Functions and Graphs Numbers and Arithmetic Introduction To Theory of Functions Introduction To Graphs Rate of Change Linear Rate of Change Non-Linear ROC and Derivatives Some Important Derivatives Vector Arithmetic Geometric Vector Addition Algebraic Vector Addition and Multiplication Perpendicular Vector Components Unit Vectors and Vector Subtraction Statics - Modelling Forces Modelling forces - particles Modelling forces - weight Modelling forces - normal reaction Modelling forces - Tension Torques - Extended and rigid bodies Torques- Combining forces Measurement in Mechanics Physical Quantities Particles - Objects without Size Reference of Frame 1D Reference of Frame 2D Motion in One Dimension Displacement Velocity	<h2>Multiplication of Vectors I - The Dot Product</h2> <p>Having added, subtracted, multiplied by a scalar and resolved vectors, we may as well finish the story with the multiplication of one vector by another. We will look at two ways to multiply vectors, both of which have physical significance. First we will consider the scalar product of two vectors. It is called the scalar product because the result of the multiplication is a scalar quantity. This is distinctly different from the scalar multiplication which we already covered. Sorry about the use of the word scalar in both cases. In fact that may be why an alternate name for this kind of multiplication has come into use. It is also called "the dot product" after the dot symbol which distinguishes this kind of multiplication. The dot product of two vectors V_1 and V_2 is written $V_1 \cdot V_2$ where the "\cdot" between the vectors is the scalar product operator.</p> <p>To get the dot product of two vectors, multiply the two lengths together and then multiply by the cosine of the angle between them. This may be thought of as the product of the length of one vector times the component of the other vector in the direction of the first. Run the Dot Product display to see some examples.</p>  <p>An alternative way to calculate the dot product is to write each vector in terms of its components and multiply them as you multiply any polynomials in algebra. For example suppose we had the vectors</p> $V_1 = 4\hat{i} + 3\hat{j} \text{ and } V_2 = 4\hat{i} + 0\hat{j}$ <p>The dot product would be:</p> $(4\hat{i} + 3\hat{j}) \cdot (4\hat{i} + 0\hat{j}) = 4 \cdot 4 \cdot \hat{i} \cdot \hat{i} + 4 \cdot 0 \cdot \hat{i} \cdot \hat{j} + 3 \cdot 4 \cdot \hat{j} \cdot \hat{i} + 3 \cdot 0 \cdot \hat{j} \cdot \hat{j}$

Internet

■ TOC of learning objects

[Rebuild TOC](#)

Numbers, Functions and Graphs

[Numbers and Arithmetic](#)
[Introduction To Theory of Functions](#)
[Introduction To Graphs](#)

Rate of Change

[Linear Rate of Change](#)
[Non-Linear ROC and Derivatives](#)
[Some Important Derivatives](#)

Vector Arithmetic

[Geometric Vector Addition](#)
[Algebraic Vector Addition and Multiplic. ...](#)
[Perpendicular Vector Components](#)
[Unit Vectors and Vector Subtraction](#)
[Multiplication Vectors I - The Dot Product](#)
[Multiplication Vectors II - The Cross Product](#)

Statics - Modelling Forces

[Modelling forces - particles](#)
[Modelling forces - weight](#)
[Modelling forces - normal reaction](#)
[Modelling forces - Tension](#)
[Modelling forces - Friction](#)
[Two or more particles - Newton's third law](#)
[Two or more particles - Pulleys](#)
[Torques - Extended and rigid bodies](#)
[Torques- turning effect of a force](#)
[Torques- Combining forces](#)
[Torques - Slipping or tipping](#)

Measurement in Mechanics

[Physical Quantities](#)
[Particles - Objects without Size](#)
[Reference of Frame 1D](#)

[Reference of Frame 2D](#)
[Reference of Frame 3D](#)

Motion in One Dimension

[Displacement](#)
[Velocity](#)
[Acceleration](#)

Constant Acceleration in One Dimension

[Constant Acceleration](#)
[Displacement and Acceleration](#)
[Motion 1D - A Typical Problem I](#)

Motion in Two Dimensions

[Velocity and Vector Arithmetic I](#)
[Velocity and Vector Arithmetic II](#)
[Acceleration and Vector Arithmetic](#)
[Projektile Motion](#)

Newton's Laws of Motion

[Motion and Forces - An Introduction](#)
[Newton's First and Second Law of Motion](#)
[Applications for N. 1. and 2. Law of M. I](#)
[Applications for N. 1. and 2. Law of M. II](#)
[Newton's Third Law of Motion](#)

Circular Motion

[Circular Motion and Central Force](#)
[Curved Motion](#)

Work and Energy

[Work](#)
[Kinetic Energy](#)
[Work and Kinetic Energy - Training Problem](#)
[Work of Varying Force - The Spring](#)

Potential Energy and Fields

[Potential vs. Kinetic Energy](#)
[Conservative Systems](#)
[Dynamical Systems from Two Perspectives](#)
[Potential Fields](#)

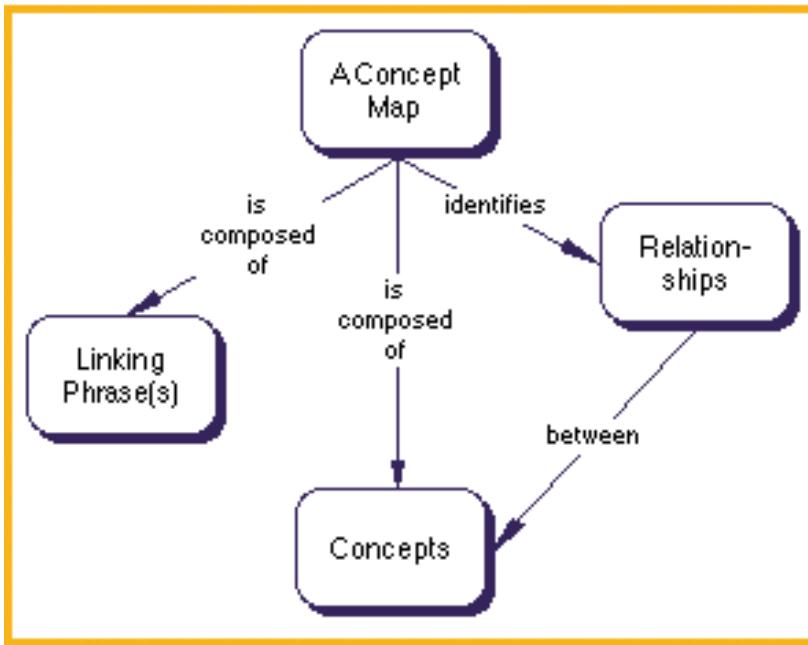
- Introduction
- Knowledge Space Theory
 - Application: ALEKS
- Demand Component Approach
 - Application: RATH
- Competence-Performance Approach
 - Application: APeLS
- **Ontology-based Skill Approach and**
 - **Application: iClass**

Ontology-based Skill Approach and iClass-Application

- **iClass:** intelligent distributed **Cognitive**-based open learning **system** for **school**
- <http://css.uni-graz.at/projects/iclass/iclass.php>
- The iClass System incorporates main features of the above mentioned knowledge space based systems. However, iClass goes beyond these systems with regard to the needs of the stakeholders (students, teachers, parents, educational authorities).

Concept Maps

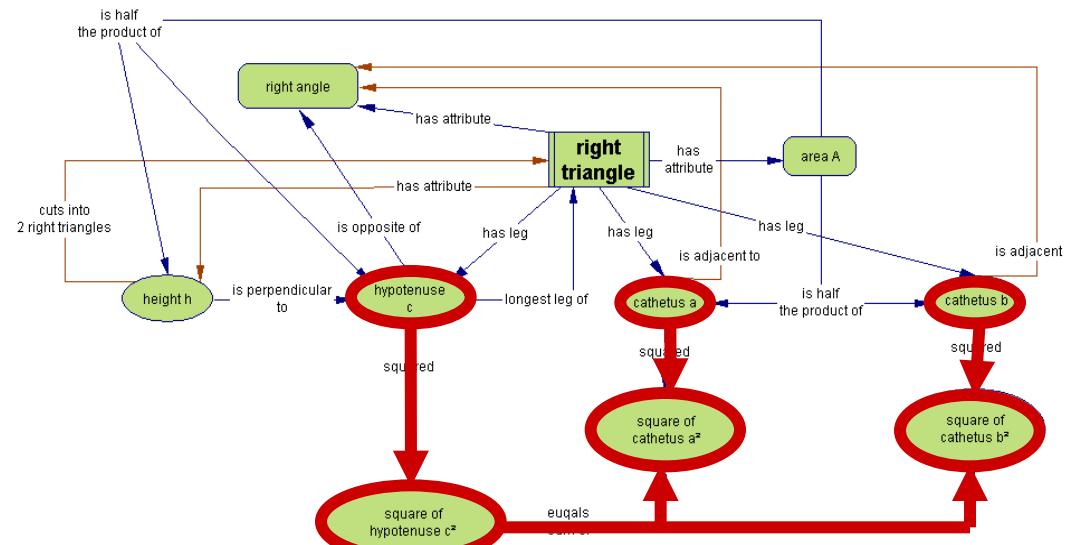
- Concept map = a directed graph := a set of propositions
- Proposition connects two related concepts



- *A Concept Map is composed of Linking Phrases.*
- *A Concept Map is composed of Concepts.*
- *A Concept Map identifies Relationships.*
- *Relationships are between Concepts.*

Deriving Skills from Domain Ontologies

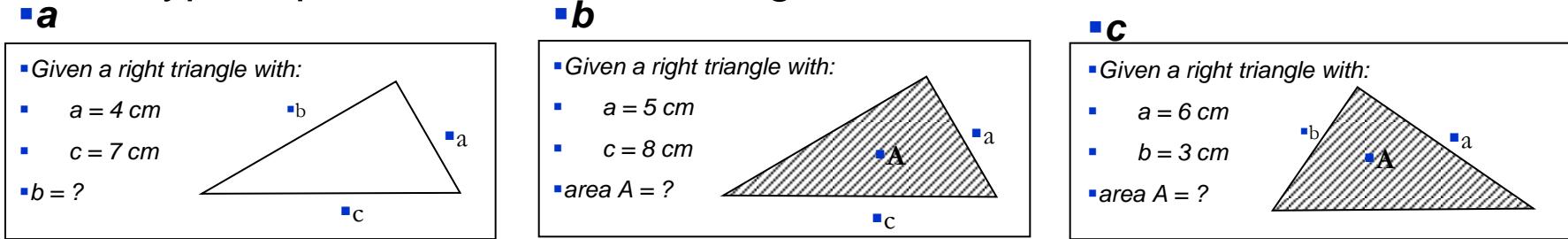
- Competence = subset of propositions of expert concept map
- Example: Geometry of right triangles



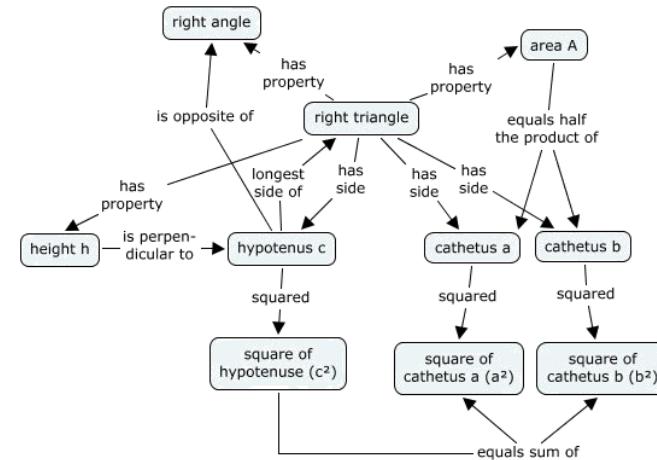
- Competence ,Knowing the Theorem of Pythagoras'

An Application Example

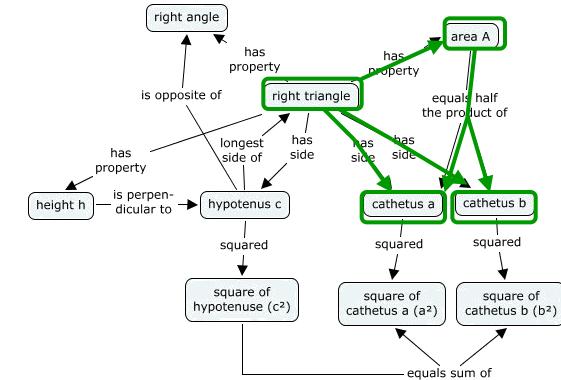
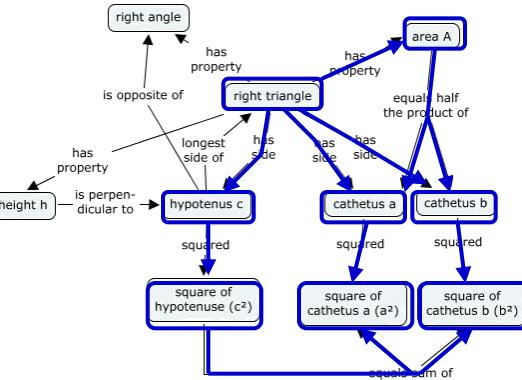
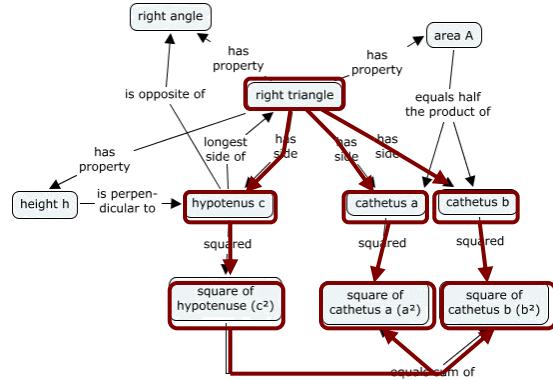
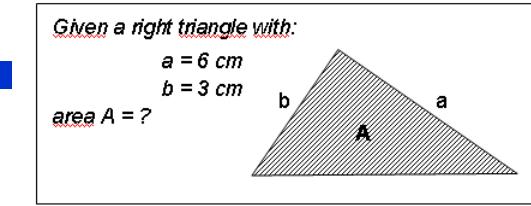
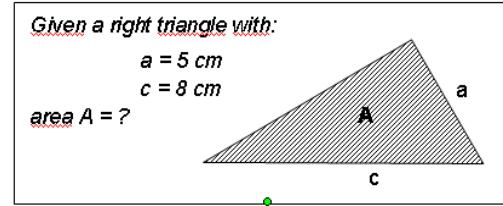
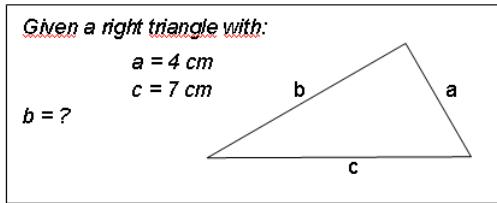
- deriving dependencies between problems from concept maps
 - typical problems of a knowledge domain



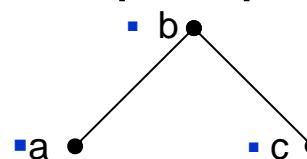
- concept map representing the semantic structure of the domain



An Application Example



- the representations of both, a and c, are subsets of the concept map of problem b



Advantages

- By using ontologies (concept maps, proposition sets) the content, the curriculum, the learner and the needs of the other stakeholder can be described and structured by a common, interrelated framework
- The relationship with e.g. conceptual graphs will in the future allow to generate semi- automatically the involved structures on the basis of digital information in test books, official documents etc.

Handing over

- In this first motivating part some basics of KST and CbKST have been illustrated
- In the third part we will come back to more recent applications like SRL, GBL, to resources etc.
- In the following, the mathematical part, some of the already mentioned and extended concepts will be defined and explained in more detail by **Reinhard SUCK**

END OF THE FIRST PART