RATH
A Relational Adaptive Tutoring Hypertext WWW–Environment Based on Knowledge Space Theory*

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Abstract

RATH is an adaptive tutoring WWW software prototype combining a mathematical model for the structure of hypertext with the theory of knowledge spaces from mathematical psychology. Using prerequisite relationships between different items in a domain of knowledge and using the knowledge about the pupil's current knowledge state RATH presents only those links in a hypertext document to the pupil which point to a document for which he/she fulfils all prerequisites and which he/she therefore should be able to understand.

In a first prototype course, this idea is applied to the field of elementary probability theory. This tiny course is based on prior research of the second author.

Thus, RATH is an important step in bringing psychological theories into a working tutorial system.

Keywords: Adaptive CAL software, Internet in Computer aided education, Mathematical hypertext model, Knowledge space theory

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1 Introduction

In the tradition of behavioural oriented psychology, Doignon and Falmagne (1985) have developed the theory of knowledge spaces as a basis for the efficient assessment of pupils' knowledge. Within this theoretical framework, a pupil's knowledge is described by his/her *knowledge state*, i. e. the set of *items* (or test problems) this pupil is able to solve. Utilizing the fact that there exist prerequisite relationships between different items in a domain of knowledge, the number of questions to be posed to a pupil to exactly determine his/her knowledge state can be reduced by drawing inferences from earlier answers.

Albert (1989) introduced the concept of *problem components* and *cognitive demands* into knowledge space theory. These cognitive demands may be used for, e. g., systematical problem construction. Held (1993) analyzed the demand structure for a number of problems in the field of elementary probability theory.

Influenced by knowledge space theory as well as the Dexter hypertext reference model (Halasz and Schwartz, 1990, 1994), the authors have developed a relational model for the structure of hypertext. This model can easily be combined with knowledge space theory and with relational algebra and database theory, and therefore facilitates the combination of efficient procedures from these fields (Albert, Held, & Hockemeyer, 1997; Albert and Hockemeyer 1997). These ideas have been implemented in a prototype system called `RATH` [1] (Hockemeyer, 1997).

As a domain of knowledge for a first `RATH` course we selected the field of elementary probability theory because (a) this field causes many problems for pupils and (b) we could access an analysis of the demand structure of this field by Held (1993).

In the following section, we introduce the theoretical models underlying `RATH`: Knowledge space theory (Doignon and Falmagne, 1985; Falmagne et al., 1990) and the relational model for the structure of hypertext (Albert et al., 1997). Section 3 contains an overview of Held's (1993) analysis of the domain of elementary probability theory used for constructing a simple course for this field. Finally, Section 4 describes the resulting system `RATH` itself.

2 Theoretical models underlying `RATH`

In the following, we give a short overview on the two mathematical models underlying `RATH`: Knowledge space theory and the relational hypertext model.

[1] `RATH` can be accessed in the WWW at the URL `http://wundt.kfunigraz.ac.at/rath/`.

2.1 Knowledge space theory

In knowledge space theory (Doignon and Falmagne, 1985; Falmagne et al., 1990), a domain of knowledge is characterized by a set Q of items, i. e. problems or questions which may be posed to a pupil. Each of these items can be considered to be answered either correctly or incorrectly by the pupil. His/her *knowledge state* is given by the subset $K \subseteq Q$ of items this pupil can answer correctly.

A teacher examining a pupil orally would not test each item but would start with an item of medium difficulty and afterwards select new items epending onccording to the pupil's previous answers. For this purpose, the teacher uses knowledge about prerequisite relationships between the items. In elementary probability theory, for example, a pupil who correctly solves a problem involving the computation of a Laplace probability will also be able to compute the numbers of possible or favourable events in a random experiment.

Such prerequisite relationships can drastically reduce the number of possible knowledge states for a given set of items. Doignon and Falmagne (1985) call the set of all possible knowledge states a *knowledge space*. Knowledge spaces contain the empty set $\emptyset$ and the complete set of items Q, and they are closed under union, i. e. for any two knowledge states K, K', their union $K \cup K'$ is a knowledge state, too. If a knowledge space is also closed under intersection, it can be represented by a *surmise relation*, i. e. a quasiorder on the set of items. If, additionally, equivalent items are grouped together then such a surmise relation also is a partial order. In the following, we will restrict ourselves to this case.

Several ways for determining a knowledge space have been suggested: querying experts (see, e. g., Dowling and Kaluscha, 1995), analyzing students' data (Villano, 1991), and analysis and/or systematic construction of problems (Albert and Held, 1994). For the third method, cognitive demands for the problems are identified and partitioned into disjoint ordered subsets. Afterwards, existing problems can be analyzed with respect to these cognitive demands, or new problems can be systematically constructed by combining demands from the various subsets. The resulting set of problems may then be ordered according to the demands involved. Held (1993) has performed such a problem analysis and construction for the field of elementary probability theory.

Although many of the applications of knowledge space theory deal with mathematical domains of knowledge, there exist also applications to other fields like chess (Albert and Held, 1994; Albert et al., 1994), elementary reading and writing abilities or usage of the Autocad program (Dowling, 1991).

2.2 Relational Hypertext Model

The initial aim of knowledge space theory was the efficient assessment of pupils' knowledge (Doignon and Falmagne, 1985). In order to use the knowledge on prerequisite relationships between items and cognitive demands also for teaching, Albert et al. (1997) have developed a relational model for the structure of hypertext link structures which is based on the Dexter model of hypertext (Halasz and Schwartz, 1990, 1994). In the Dexter model, *components* (i. e. documents) of a hypertext consist of a *base component* (the unit of information in the component, e. g. a text or a picture), two sets of *source anchors* and *destination anchors* located on this base component, and some additional information. In the relational hypertext model, this has been formalized by defining a component c as a triple $c = (b, S_c, D_c)$ where b denotes a base component and S_c and D_c denote subsets of source and destination anchors respectively. A link l is then defined as a pair of pairs $l = ((c, s_c), (c', d_{c'}))$ where c and c' denote components and s_c and $d_{c'}$ denote a source and a destination anchor located on c and c' respectively. Such a construct formalizes a link from the source anchor s_c on component c to the destination anchor $d_{c'}$ on the component c'. By defining subsets P of anchors characterized by a specific property also links where these anchors are involved are distinguished: Given a set L of links on a set C of components and a subset $P \subseteq S$ of source anchors, a subset $L_P \subseteq L = \{l = ((c, s_c), (c', d_{c'})) \in L \mid s_c \in P\}$ of P*–links* can be defined. Examples for such properties are "is a prerequisite of" or "is an example of".

Based on these definitions, link relations between components can be defined directly: For two components $c, c' \in C$, we say $c \vdash c'$ if and only if there exists a link $l = ((c, s_c)(c', d_{c'})) \in L$ for some source anchor $s_c \in S_c$ and some destination anchor $d_{c'} \in D_{c'}$. The distinction of links with specific properties can also be transferred to this link relation between components defining a link relation $\vdash_P$ by setting $c \vdash_P c'$ if and only if there exists a link $l = ((c, s_c)(c', d_{c'})) \in L_P$ for some source anchor $s_c \in S_c \cap P$ and some destination anchor $d_{c'} \in D_{c'}$.

This relational formalization of hypertext link structures has an important practical advantage: Such relational information can easily be stored, processed and retrieved in a relational database system. Moreover, the relational formalization can almost directly be transferred into relation schema definitions of a relational database (for details see Albert et al., 1997).

3 The domain of elementary probability theory

There exist two reasons for choosing elementary probability theory as a domain of knowledge for the first prototype course for `RATH`. First, this is a field which causes many problems to pupils and, therefore, additional support for teaching is needed. Second, the structure of the knowledge in this field was

Table 1
Cognitive demands used in the course on elementary probability theory

0	Random experiments and elementary events
1	Laplace–probabilities
2	Number of possible events
3	Number of favourable events
4	Drawing multiple balls with a common property
5	Events containing elementary events with different properties
6	Addition of events and probabilities
7	Multiplication of events and probabilities
8	Different proportions of properties
9	Drawing without replacement
10	Generalized ("at least n balls) descriptions of events

the object of prior investigations (Held, 1993) whose results were reused in the course development. This research dealt with the sub domain of urn experiments.

Held used four different problem components: the numerical ratio of differently coloured balls, the way of drawing, the specification of the asked event, and the wording of the problem. The last component was neglected in our course construction. For each of the components, there existed two or three possible values (*attributes*). Problems and demands were characterized by corresponding tuples of attributes. Based on orderings between the possible values for each of the components partial orders for problems and demands were derived.

Held used eighteen different problems, and identified thirteen cognitive demands necessary for the solution of the problems. Due to the neglection of the wording component, these numbers were reduced to six problems and ten demands (labelled from "1" to "10"). For the course, however, one introducing demand (labelled "0") on random experiments and elementary events was added. The list of the resulting eleven demands is shown in Table 1.

One example problem of the course is presetned below:

> *An urn contains five yellow and five black balls. Two balls are drawn from the urn successively. Drawing is performed with replacement. The drawn balls are yellow. Compute the probability of this event.*

For solving this problem, the demands 0 to 4 and 7 are required.

Based on the attribute specifications of problems and demands a partial order on the set of problems and demands could be identified[2]. The edges

[2]Such an order can be interpreted in a similar way as the concept lattices by Ganter

of the Hasse diagram of this order are the prerequisite links between the documents (lessons and exercises) in the RATH course[3].

4 The RATH system

The current status of RATH is that of a first prototype which has the purpose to show that the theoretically developed mathematical model for the structure of hypertext can be applied practically for obtaining an adaptive tutoring hypertext system.

Figure 1 shows the architecture of RATH. Figure 1 shows the architecture of RATH. The author of a course writes a *tutoring hypertext*, i. e. a hypertext containing e. g. lessons, examples, exercises which build the teaching material of the tutoring system (we do not provide an interface for this task since there already exist many HTML editors). The single components of this hypertext and the link structure are registered in a relational database. In a next step sub systems could be predefined, e. g. an algebra course from a large, general course in mathematics (this step is not yet available). When a pupil works with the system, components to be presented to the pupil are adapted according to the systems knowledge about the pupil. These components (lessons, examples, exercises, etc.) then are sent by the `http`–daemon (WWW server program) to the pupil's browser which presents the component to the user.

From a technical point of view, RATH consists of a number of small programs which access an Oracle database using the Oracle ProC programming interface. These programs are either run offline (e. g. the registry of components) or online (e. g. administration of pupils or selection and adaptation of components for pupils). *Online* here means that the programs are started by the `http` daemon via its CGI interface due to a user's selection in the web browser (see Hockemeyer 1997 for details)

During the implementation of the course on elementary probability theory the problem occurred that HTML is not well prepared for mathematical contents. Therefore we used LaTeX 2_ε for authoring the course and, afterwards, converted it to HTML using the LaTeX2HTML program. Finally, we manually added the prerequisite links between the documents. Figure 2 gives an idea of the resulting pages in the RATH course.

and Wille (1996; see also Wille, 1982). However, it does not fulfil their closure conditions which in general lead to a number of virtual, unlabelled nodes in the graph.

[3]Besides lessons and exercises, the RATH course also contains examples for the lessons (see also Figure 2 below). Examples in the RATH course contain a prerequisite link to the corresponding lesson.

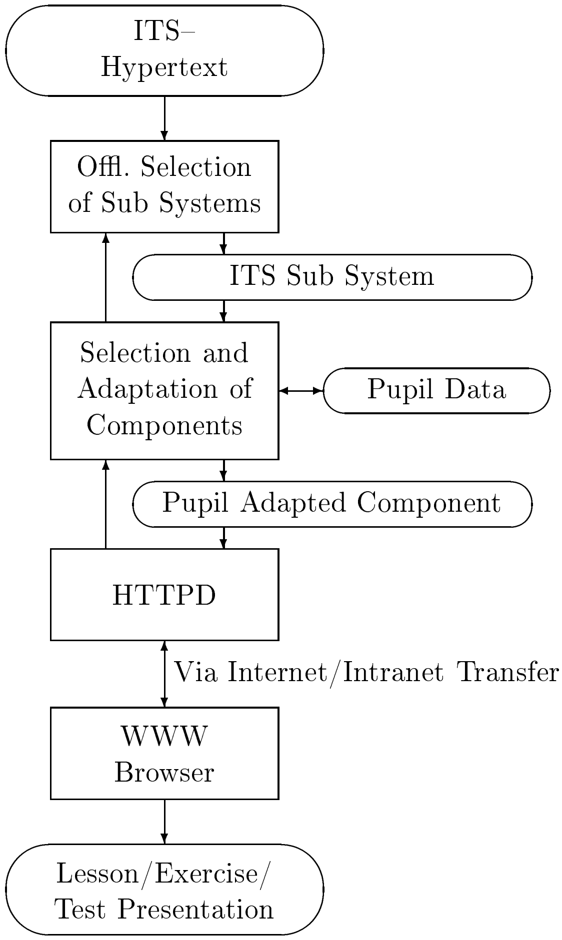

Figure 1. Architecture of RATH from an application–oriented point of view

5 Discussion and Further Research

The mathematical model for the structure of hypertext introduced in Section 2.2 was developed with two goals in mind: (i) The relational structure facilitates adapting hypertexts to users' needs and preferences. This holds especially with regard to the use of relational database systems, which allow a direct transfer of the hypertext model's formalization to relational database table specifications. (ii) Due to the fact that all concepts are mathematically well defined, it is possible to combine the hypertext model with mathematical models from application fields like knowledge space theory in the case of tutoring systems. The RATH system presented here shows that the hypertext model is not only a theoretical idea but can also be implemented and used in practice.

Nevertheless, RATH is a prototype system and there is still much to do:

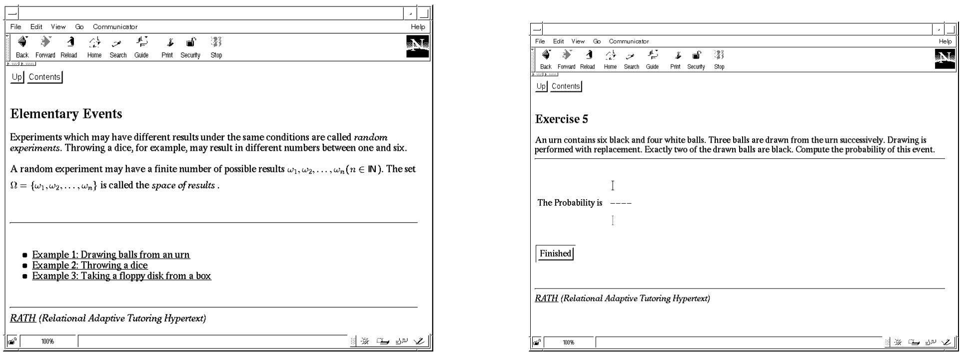

Figure 2. Example screenshots of a RATH lesson and of an exercise

One important point is the introduction of an initial assessment of a pupil's knowledge. Currently, RATH assumes that each pupil starts without any knowledge at all. It is important to extend RATH by an assessment module using assessment procedures from knowledge space theory (see, e. g., Doignon 1994; Dowling, Hockemeyer & Ludwig, 1996). A second, important task is to improve the analysis of pupils' answers to the exercises. At the moment, the pupils only receive a short message stating whether an answer was correct or wrong. This should be improved in order to give a more helpful feedback.

Regarding the original goal of obtaining a system with high adaptability it is important to realize the idea of preselecting sub hypertexts as already mentioned in Section 4.

Another topic which must not be forgoten is the evaluation of the system. Here, we have to distinguish two parts: the evaluation of the course and the evaluation of the underlying system. For the course, there exists the evaluation of its structure (Held, 1993) but no evaluation of the lessons' quality. Regarding the underlying system itself, a systematic evaluation of stability and performance has not been performed yet. A demonstratin at an ERASMUS seminar with about 20 participants using RATH simultaneously showed no problems.

Summarizing we can say that — in its current state — RATH is an important step in bringing psychological theories into a working tutorial system.

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