Inference Cheatsheet

James Lee

1 Convergence in Probability

Def 1.1 (Convergence in Probability)**.** $Y_n \xrightarrow{p} c$ if for every $\epsilon > 0$ and $\delta > 0$, $\exists\, n_0(\epsilon, \delta)$ such that

$$P(|Y_n - c| > \epsilon) < \delta, \ \forall n > n_0(\epsilon, \delta)$$

Thm 1.1 (Chebyshev Inequality)**.** For random variable, Y, $a > 0$, and c,

$$P(|Y - c| \geq a) \leq \frac{\mathbf{E}(Y-c)^2}{a^2}$$

Def 1.2 (Markov Inequality)**.** If X is a non-negative random variable and $a > 0$ then

$$P(X \geq a) \leq \frac{\mathbf{E}X}{a}$$

Thm 1.2. If $\mathbf{E}(Y-c)^2 \to 0$, then $Y_n \xrightarrow{p} c$.

Thm 1.3. If $X_1, \ldots, X_n$ iid, $\mathbf{E}X_i = \mu$, $\mathbf{Var}X_i = \sigma^2 < \infty$, then

$$\bar{X} \xrightarrow{p} \mu$$

Thm 1.4. If $A_n \xrightarrow{p} a$ and $B_n \xrightarrow{p} b$, then

1. $A_n \pm B_n \xrightarrow{p} a \pm b$,
2. $A_n \cdot B_n \xrightarrow{p} a \cdot b$.

Thm 1.5. If $Y_n \xrightarrow{p} c$ and f is continuous at c, then $f(Y_n) \xrightarrow{p} f(c)$.

Def 1.3. A sequence of estimators δ_n of $g(\theta)$ is *consistent if*

$$\delta_n \xrightarrow{p} g(\theta)$$

Thm 1.6. If bias and variance of $\delta_n \to 0$ as $n \to \infty$, δ_n is consistent.

Def 1.4. $A_n = o_p(B_n)$ if $\frac{A_n}{B_n} \xrightarrow{p} 0$.

2 Convergence in Law

Def 2.1. Y_n is *bounded in probability* if for any $\epsilon > 0\ \exists K$ and $n_0(\epsilon)$ such that

$$P(|Y_n| > K) < \epsilon \text{ for all } n > n_0(\epsilon)$$

Def 2.2. Convergence in Law/Distribution $Y_n \xrightarrow{L} Y$ if $H_n(x) \to H(x)$ for all continuity points x.