Inference Cheatsheet

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1 Convergence in Probability

Def 1.1 (Convergence in Probability)**.** $Y_n \xrightarrow{p} c$ if for every $\epsilon > 0$ and $\delta > 0$, $\exists\ n_0(\epsilon, \delta)$ such that

$$P(|Y_n - c| > \epsilon) < \delta,\ \forall n > n_0(\epsilon, \delta)$$

Thm 1.1 (Chebyshev Inequality)**.** For random variable, Y, $a > 0$, and c,

$$P(|Y - c| \geq a) \leq \frac{\mathbf{E}(Y-c)^2}{a^2}$$

Def 1.2 (Markov Inequality)**.** If X is a non-negative random variable and $a > 0$ then

$$P(X \geq a) \leq \frac{\mathbf{E}X}{a}$$

Thm 1.2. If $\mathbf{E}(Y-c)^2 \to 0$, then $Y_n \xrightarrow{p} c$.

Thm 1.3. If $X_1, \ldots, X_n$ iid, $\mathbf{E}X_i = \mu$, $\mathbf{Var}X_i = \sigma^2 < \infty$, then

$$\bar{X} \xrightarrow{p} \mu$$

Thm 1.4. If $A_n \xrightarrow{p} a$ and $B_n \xrightarrow{p} b$, then

1. $A_n \pm B_n \xrightarrow{p} a \pm b$,
2. $A_n \cdot B_n \xrightarrow{p} a \cdot b$.

Thm 1.5. If $Y_n \xrightarrow{p} c$ and f is continuous at c, then $f(Y_n) \xrightarrow{p} f(c)$.

Def 1.3. A sequence of estimators δ_n of $g(\theta)$ is *consistent if*

$$\delta_n \xrightarrow{p} g(\theta)$$

Thm 1.6. If bias and variance of $\delta_n \to 0$ as $n \to \infty$, δ_n is consistent.

Def 1.4. $A_n = o_p(B_n)$ if $\frac{A_n}{B_n} \xrightarrow{p} 0$.

2 Convergence in Law

Def 2.1. Y_n is *bounded in probability* if for any $\epsilon > 0\ \exists K$ and $n_0(\epsilon)$ such that

$$P(|Y_n| > K) < \epsilon \text{ for all } n > n_0(\epsilon).$$

Def 2.2. Convergence in Law/Distribution $Y_n \xrightarrow{L} Y$ if $H_n(x) \to H(x)$ for all continuity points x.

Thm 2.1. If $Y_n \xrightarrow{L} H$ then Y_n is bounded in probability.

Thm 2.2. Slutsky's Theorem If $Y_n \xrightarrow{L} Y$, $A_n \xrightarrow{p} a$, and $B_n \xrightarrow{p} b$, then

$$A_n + B_n Y_n \xrightarrow{L} a + bY.$$

Cor 2.1. If $Y_n \xrightarrow{L} Y$, $R_n \xrightarrow{p} 0$, and $B_n \xrightarrow{p} 1$, then

$$Y_n + R_n \xrightarrow{L} Y$$
$$Y_n / B_n \xrightarrow{L} Y.$$

Thm 2.3. If $k_n(Y_n - c) \xrightarrow{L} H$ and $k_n \to \infty$ then $Y_n \xrightarrow{p} c$.

Def 2.3. $Y_n \xrightarrow{p} Y$ if $Y_n - Y \xrightarrow{p} 0$.

Thm 2.4. $Y_n \xrightarrow{p} Y \implies Y_n \xrightarrow{L} Y$

3 Central Limit Theorem

Thm 3.1. If $X_1 \ldots X_n$ iid, $\mathbf{E}X_i = \xi$, $\mathbf{Var}X_i = \sigma^2 < \infty$, then

$$\sqrt{n}(\bar{X} - \xi) \xrightarrow{L} N(0, \sigma^2).$$

Thm 3.2 (Berry-Esseen)**.** If $X_1 \ldots X_n$ iid $\sim F$ with finite 3rd moment, then $\exists C$ that is independent of F such that for all x,

$$|G_n(x) - \Phi| \leq \frac{C}{\sqrt{n}} \frac{\mathbf{E}|X_i - \xi|^3}{\sigma^3}$$

where $G_n(x)$ is the CDF of $\sqrt{n}(\bar{X} - \xi)/\sigma$.

Cor 3.1. Under the assumptions of Berry-Esseen's theorem,

$$G_n(x) \to \Phi(x) \text{ as } n \to \infty$$

for any sequence F_n with mean ξ_n and σ_n^2 for which

$$\frac{E_n|X_1 - \xi_n|^3}{\sigma_n^3} = o(\sqrt{n})$$

and in particular if it is bounded.

4 Delta Method

Thm 4.1. If $\sqrt{n}(T_n - \theta) \xrightarrow{L} N(0, \tau^2)$ and $f'(\theta) \neq 0$, then

$$\sqrt{n}(f(T_n) - f(\theta)) \xrightarrow{L} N(0, \tau^2[f'(\theta)]^2).$$

5 Miscellaneous

Thm 5.1 (Markov Inequality)**.** If X is a non-negative RV and $a > 0$ then $\mathbf{P}(X \geq a) \leq \frac{\mathbf{E}X}{a}$.

Thm 5.2 (Binomial p_n)**.** If $\frac{1}{n} = o_p(p_n)$ then

$$\frac{S_n - np_n}{\sqrt{np_nq_n}} \xrightarrow{L} N(0, 1).$$