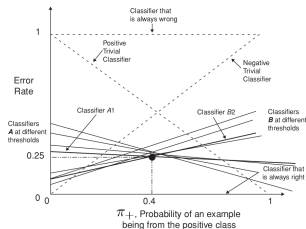


Introduction to Machine Learning

Imbalanced Learning: Cost Curves Part 1



Learning goals

- Cost curves for misclassif error
- Duality between ROC points and cost lines
- CCs as envelopes over cost lines

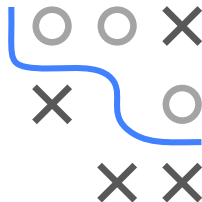
COST CURVES

Simplifying assumption: equal misclassif costs, i.e., $cost_{FN} = cost_{FP}$

⇒ Expected misclassif cost reduces to misclassif error rate

With law of total prob, we write error rate as function of π_+ :

$$\begin{aligned}\rho_{MCE}(\pi_+) &= (1 - \pi_+) \cdot \mathbb{P}(\hat{y} = 1|y = 0) + \pi_+ \cdot \mathbb{P}(\hat{y} = 0|y = 1) \\ &= (1 - \pi_+) \cdot FPR + \pi_+ \cdot FNR \\ &= (FNR - FPR) \cdot \pi_+ + FPR\end{aligned}$$



Confusion matrix

	True class	
	$y = 1$	$y = 0$
Pred. $\hat{y} = 1$	TP	FP
class $\hat{y} = 0$	FN	TN

Cost matrix

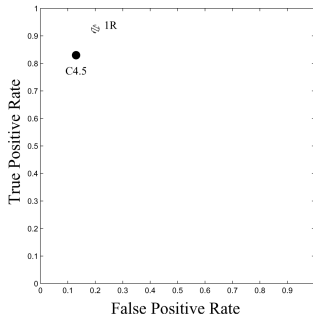
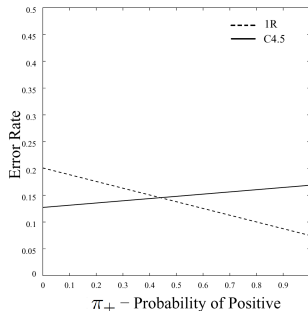
	True class	
	$y = 1$	$y = 0$
Pred. $\hat{y} = 1$	0	$cost_{FP}$
class $\hat{y} = 0$	$cost_{FN}$	0

COST CURVES

- Cost line of a classifier with slope ($FNR - FPR$) and intercept FPR :

$$\rho_{MCE}(\pi_+) = (FNR - FPR) \cdot \pi_+ + FPR$$

- Cost curves are point-line duals of ROC curves, i.e., a single classifier is represented by a point in the ROC space and by a line in cost space



Chris Drummond and Robert C. Holte (2006): Cost curves: An improved method for visualizing classifier performance. Machine Learning, 65, 95-130 ([URL](#)).

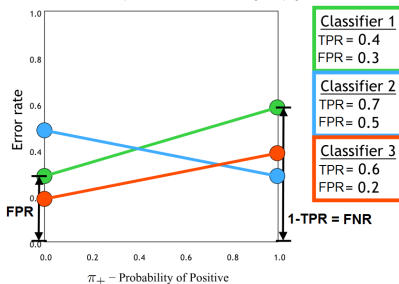
COST LINES

Cost line of a classifier with slope ($FNR - FPR$) and intercept FPR :

$$\rho_{MCE}(\pi_+) = (FNR - FPR) \cdot \pi_+ + FPR$$

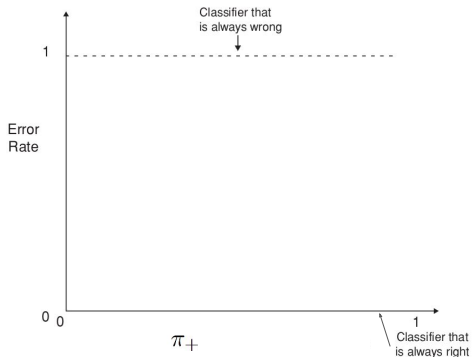
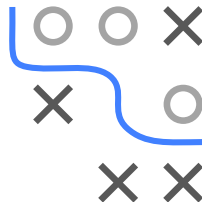
- Hard classifiers are points (TPR, FPR) in ROC space
- The cost line of a classifier connects (π_+, ρ_{MCE}) -points at $(0, FPR)$ and $(1, 1 - TPR)$
- Classifier 3 always dominates classifier 1
- Classifier 3 is better than classifier 2 when $\pi_+ < 0.7$

Cost lines plot different values of π_+ vs. $\rho_{MCE}(\pi_+)$



COST LINES - EXAMPLE

- Horizontal dashed line: worst classifier (100% error rate for all π_+)
 $\Rightarrow FNR = FPR = 1$
- x-axis: perfect classifier (0% error rate for all π_+) $\Rightarrow FNR = FPR = 0$



$$\rho_{MCE} = (FNR - FPR) \cdot \pi_+ + FPR$$

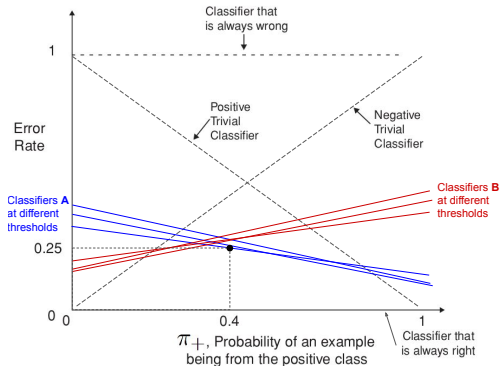
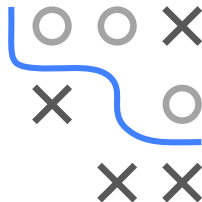
		Confusion matrix	
		True class	
Pred. class	$\hat{y} = 1$	TP	FP
	$\hat{y} = 0$	FN	TN

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- The graph illustrates the error rate of three classifiers as a function of π_+ (the proportion of positive examples in the training set). The x-axis is labeled π_+ and ranges from 0 to 1. The y-axis is labeled 'Error Rate' and ranges from 0 to 1. The three classifiers are represented by dashed lines: a horizontal line at error rate 1 (labeled 'Classifier that is always wrong'), a diagonal line from (0, 1) to (1, 0) (labeled 'Positive Trivial Classifier'), and a diagonal line from (0, 0) to (1, 1) (labeled 'Negative Trivial Classifier'). The lines intersect at $\pi_+ = 0.5$ and error rate 0.5.

		Confusion matrix	
		True class	
		$y = 1$	$y = 0$
Pred. class	$\hat{y} = 1$	TP	FP
	$\hat{y} = 0$	FN	TN

COST LINES - EXAMPLE

- Horizontal dashed line: worst classifier (100% error rate for all π_+)
 $\Rightarrow FNR = FPR = 1$
- x-axis: perfect classifier (0% error rate for all π_+) $\Rightarrow FNR = FPR = 0$
- Dashed diagonal lines: trivial classifiers, i.e., ascending diagonal always predicts negative instances ($\Rightarrow FNR = 1$ and $FPR = 0$) and vice versa
- Descending/ascending bold lines: two families of classifiers *A* and *B* (represented by points in their respective ROC curves)

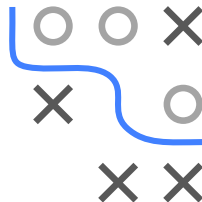
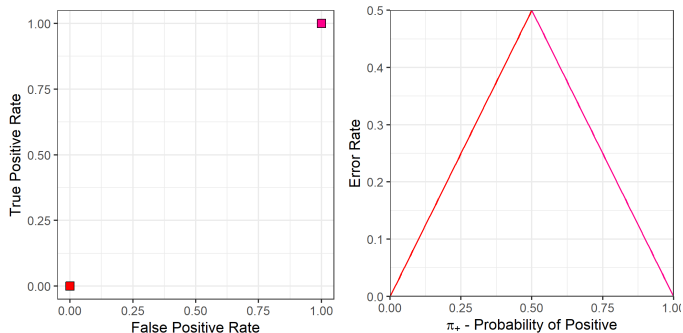


$$\rho_{MCE} = (FNR - FPR) \cdot \pi_+ + FPR$$

		Confusion matrix	
		True class	
Pred. class	$\hat{y} = 1$	$y = 1$	$y = 0$
	$\hat{y} = 0$	TP	FP
		FN	TN

VISUALIZE COST CURVE - LOWER ENVELOPE

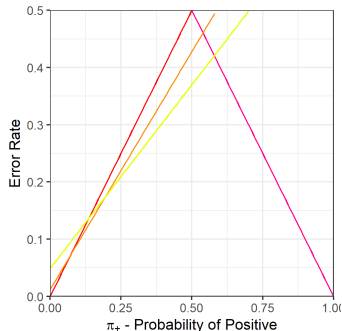
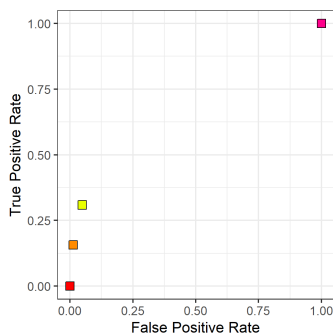
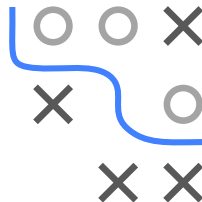
- Left: ROC = TPR & FPR of a classifier for different prob thresholds
- Right: Corresponding cost lines
- Duality: For every ROC point we can construct the CC line, and vice versa.



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- The left plot is a True Positive Rate (TPR) vs. False Positive Rate (FPR) plot. The x-axis is labeled 'False Positive Rate' and ranges from 0.00 to 1.00. The y-axis is labeled 'True Positive Rate' and ranges from 0.00 to 1.00. A red square is at (0.00, 0.00), an orange square is at approximately (0.02, 0.15), and a magenta square is at (1.00, 1.00).
- The right plot is an Error Rate vs. π_+ - Probability of Positive plot. The x-axis is labeled ' π_+ - Probability of Positive' and ranges from 0.00 to 1.00. The y-axis is labeled 'Error Rate' and ranges from 0.0 to 0.5. A red line starts at (0.00, 0.00) and peaks at (0.50, 0.50). A magenta line starts at (0.00, 0.00) and ends at (1.00, 0.00). An orange line starts at (0.00, 0.00) and ends at (0.50, 0.50).

VISUALIZE COST CURVE - LOWER ENVELOPE

- Left: ROC = TPR & FPR of a classifier for different prob thresholds
- Right: Corresponding cost lines
- Duality: For every ROC point we can construct the CC line, and vice versa.



[illegible]

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- The figure consists of two plots. The left plot is a True Positive Rate (TPR) vs. False Positive Rate (FPR) curve, showing a black curve with colored square markers (red, orange, yellow, green, cyan, blue, purple, magenta) representing different threshold values. The right plot shows the Error Rate (ER) vs. the Probability of Positive (π_+). It features a black curve representing the overall error rate, which is the sum of the false negative rate (FNR) and the false positive rate (FPR). The FNR is shown as a red curve, and the FPR is shown as a blue curve. The error rate curve is the sum of these two, and it is shown as a black curve. The plot also includes several other colored lines (green, cyan, magenta, orange, yellow, light blue) representing the error rate for different threshold values.