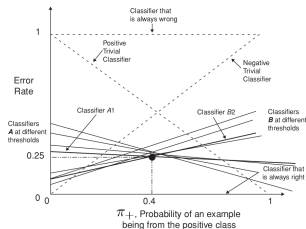


# Introduction to Machine Learning

## Imbalanced Learning: Cost Curves Part 1



### Learning goals

- Cost curves for misclassif error
- Duality between ROC points and cost lines
- CCs as envelopes over cost lines



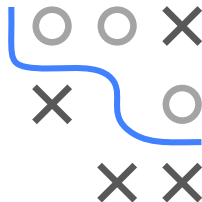
# COST CURVES

Simplifying assumption: equal misclassif costs, i.e.,  $cost_{FN} = cost_{FP}$

⇒ Expected misclassif cost reduces to misclassif error rate

With law of total prob, we write error rate as function of  $\pi_+$ :

$$\begin{aligned}\rho_{MCE}(\pi_+) &= (1 - \pi_+) \cdot \mathbb{P}(\hat{y} = 1|y = 0) + \pi_+ \cdot \mathbb{P}(\hat{y} = 0|y = 1) \\ &= (1 - \pi_+) \cdot FPR + \pi_+ \cdot FNR \\ &= (FNR - FPR) \cdot \pi_+ + FPR\end{aligned}$$



Confusion matrix

	True class	
	$y = 1$	$y = 0$
Pred. $\hat{y} = 1$	TP	FP
class $\hat{y} = 0$	FN	TN

Cost matrix

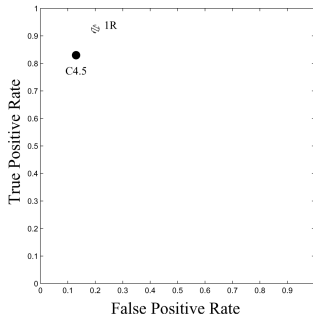
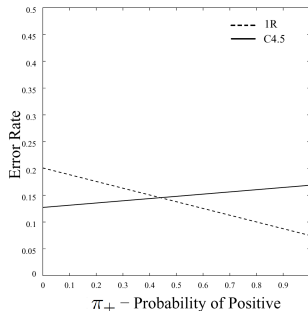
	True class	
	$y = 1$	$y = 0$
Pred. $\hat{y} = 1$	0	$cost_{FP}$
class $\hat{y} = 0$	$cost_{FN}$	0

# COST CURVES

- Cost line of a classifier with slope ( $FNR - FPR$ ) and intercept  $FPR$ :

$$\rho_{MCE}(\pi_+) = (FNR - FPR) \cdot \pi_+ + FPR$$

- Cost curves are point-line duals of ROC curves, i.e., a single classifier is represented by a point in the ROC space and by a line in cost space



Chris Drummond and Robert C. Holte (2006): Cost curves: An improved method for visualizing classifier performance. Machine Learning, 65, 95-130 ([URL](#)).

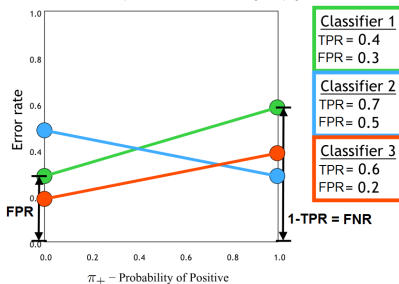
## COST LINES

Cost line of a classifier with slope ( $FNR - FPR$ ) and intercept  $FPR$ :

$$\rho_{MCE}(\pi_+) = (FNR - FPR) \cdot \pi_+ + FPR$$

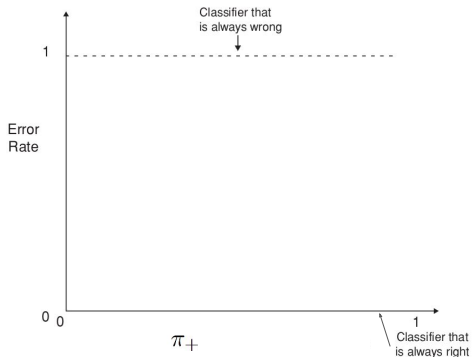
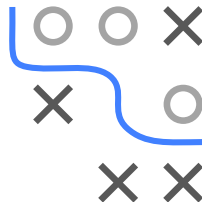
- Hard classifiers are points (TPR, FPR) in ROC space
- The cost line of a classifier connects  $(\pi_+, \rho_{MCE})$ -points at  $(0, FPR)$  and  $(1, 1 - TPR)$
- Classifier 3 always dominates classifier 1
- Classifier 3 is better than classifier 2 when  $\pi_+ < 0.7$

Cost lines plot different values of  $\pi_+$  vs.  $\rho_{MCE}(\pi_+)$



# COST LINES - EXAMPLE

- Horizontal dashed line: worst classifier (100% error rate for all  $\pi_+$ )  
 $\Rightarrow FNR = FPR = 1$
- x-axis: perfect classifier (0% error rate for all  $\pi_+$ )  $\Rightarrow FNR = FPR = 0$

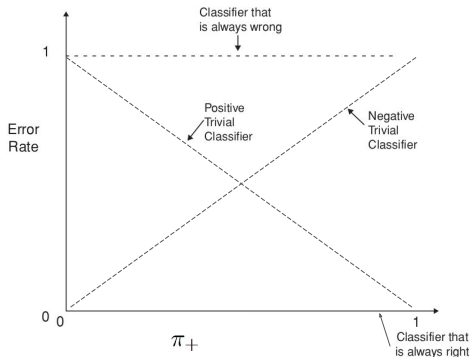
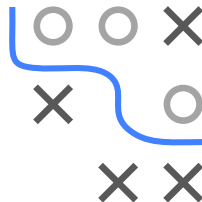


$$\rho_{MCE} = (FNR - FPR) \cdot \pi_+ + FPR$$

		Confusion matrix	
		True class	
Pred. class	$\hat{y} = 1$	TP	FP
	$\hat{y} = 0$	FN	TN

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- Dashed diagonal lines: trivial classifiers, i.e., ascending diagonal always predicts negative instances ( $\Rightarrow FNR = 1$  and  $FPR = 0$ ) and vice versa

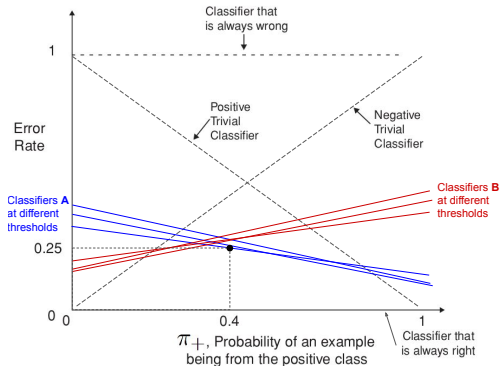
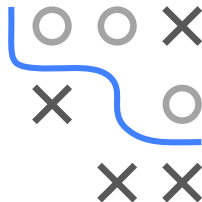


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# COST LINES - EXAMPLE

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- Dashed diagonal lines: trivial classifiers, i.e., ascending diagonal always predicts negative instances ( $\Rightarrow FNR = 1$  and  $FPR = 0$ ) and vice versa
- Descending/ascending bold lines: two families of classifiers *A* and *B* (represented by points in their respective ROC curves)



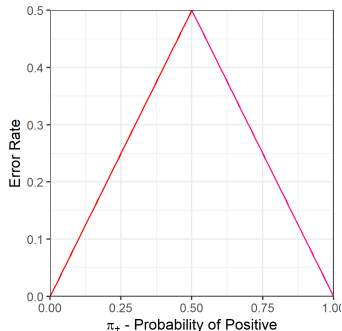
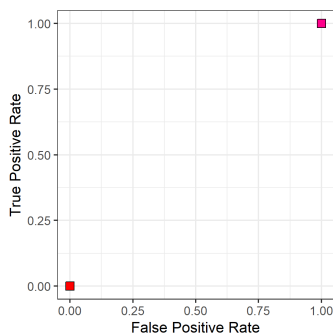
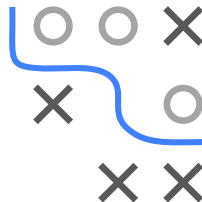
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		True class	
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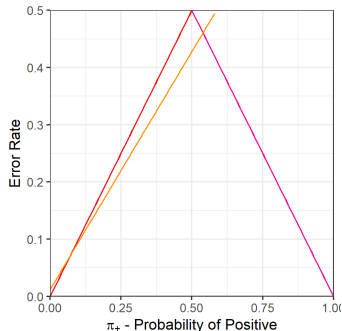
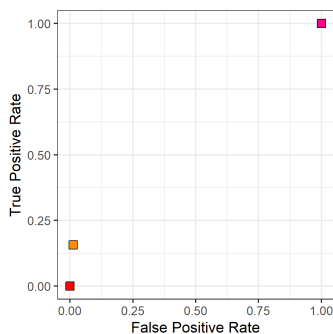
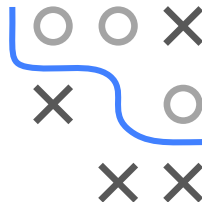
# VISUALIZE COST CURVE - LOWER ENVELOPE

- Left: ROC = TPR & FPR of a classifier for different prob thresholds
- Right: Corresponding cost lines
- Duality: For every ROC point we can construct the CC line, and vice versa.



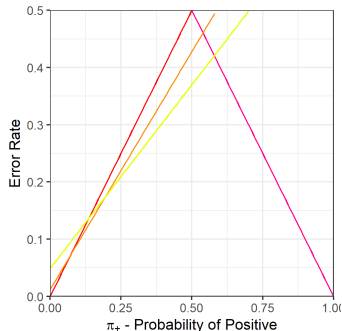
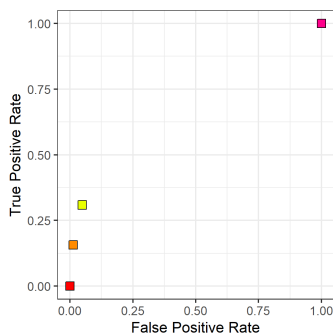
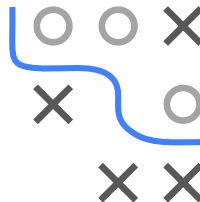
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# VISUALIZE COST CURVE - LOWER ENVELOPE

- Left: ROC = TPR & FPR of a classifier for different prob thresholds
- Right: Corresponding cost lines
- Duality: For every ROC point we can construct the CC line, and vice versa.
- **Cost curve** (right, black) is lower envelope of **cost lines**  
 $\triangleq$  pointwise minimum of error rate (as function of  $\pi_+$ )

