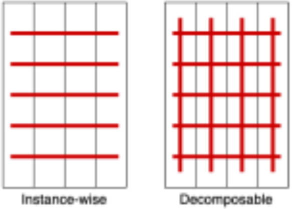
MULTIVARIATE LOSS FUNCTIONS



- We treat two categories: Decomposable and instance-wise

- L is decomposable over targets if

$$L(\mathbf{y}, f) = \frac{1}{l} \sum_{m=1}^{l} L_m(y_m, f(\mathbf{x})_m)$$

with single-target losses L_m.

- Example: *Squared error loss* (in multivariate regression):

$$L_{\text{MSE}}(\mathbf{y}, f) = \frac{1}{l} \sum_{m=1}^{l} (y_m - f(\mathbf{x})_m)^2 .$$

- Can also be used for cases with missing entries.

INSTANCE-WISE LOSSES

- *Hamming loss* averages over mistakes in single targets:

$$L_H(\mathbf{y}, \mathbf{h}) = \frac{1}{l} \sum_{m=1}^{l} \mathbb{1}_{[y_m \neq h_m(\mathbf{x})]},$$

where $h_m(\mathbf{x}) := [f(\mathbf{x})_m \geq c_m]$ is the threshold function for target m with threshold c_m.

- Hamming loss is identical to the average *0/1 loss* and is decomposable.
- The *subset 0/1 loss* checks for entire correctness and is not decomposable:

$$L_{0/1}(\mathbf{y}, \mathbf{h}) = \mathbb{1}_{[\mathbf{y} \neq \mathbf{h}]} = \max_m \mathbb{1}_{[y_m \neq h_m(\mathbf{x})]}$$

HAMMING VS. SUBSET 0/1 LOSS

- The risk minimizer for the Hamming loss is the *marginal mode*:

$$f^*(\mathbf{x})_m = \arg \max_{y_m \in \{0,1\}} \Pr(y_m \mid \mathbf{x}), \quad m = 1, \ldots, l,$$

while for the subset 0/1 loss it is the *joint mode*:

$$f^*(\mathbf{x}) = \arg \max_{\mathbf{y}} \Pr(\mathbf{y} \mid \mathbf{x}).$$

- Marginal mode vs. joint mode:

$\mathbf{y}$	$\Pr(\mathbf{y})$
0 0 0 0	0.30
0 1 1 1	0.17
1 0 1 1	0.18
1 1 0 1	0.17
1 1 1 0	0.18

Marginal mode: 1 1 1 1
Joint mode: 0 0 0 0