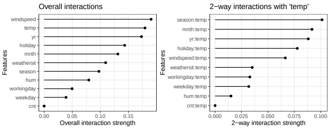
Interpretable Machine Learning

Friedman's H-Statistic

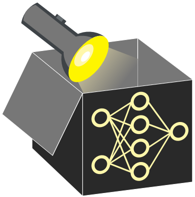


Learning goals

- Understand Friedman's H-statistic
- Measure 2-way interactions between pairs of features
- Measure a feature's overall interaction strength

IDEA

▸ Friedman and Popescu (2008)

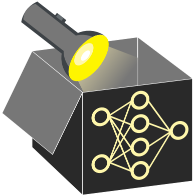


2-way interaction: If two features *j* and *k* do not interact, their mean-centered PD function is

$$\hat{f}_{jk,PD}(x_j, x_k) = \hat{f}_{j,PD}(x_j) + \hat{f}_{k,PD}(x_k)$$

- $\hat{f}_{jk,PD}(x_j, x_k)$: joint 2-dim PD function of feature *j* and *k*
- $\hat{f}_{j,PD}(x_j)$ and $\hat{f}_{k,PD}(x_k)$: 1-dim PD functions of single features *j* and *k*

IDEA



Overall interaction: If feature j does not interact with any other feature (denoted by index $-j$), the mean-centered prediction function can be decomposed by

$$\hat{f}(\mathbf{x}) = \hat{f}_{j,PD}(x_j) + \hat{f}_{-j,PD}(\mathbf{x}_{-j})$$

- $\hat{f}(\mathbf{x})$: mean-centered prediction function
- $\hat{f}_{j,PD}(x_j)$: 1-dim PD function of feature j
- $\hat{f}_{-j,PD}(\mathbf{x}_{-j})$: $(p-1)$-dim PD function of all p features except feature j

2-WAY INTERACTION STRENGTH

H-statistic measures interaction strength between feature *j* and *k* by

$$H^2_{jk} = \frac{\sum_{i=1}^n \left[\hat{f}_{jk,PD}(x_j^{(i)}, x_k^{(i)}) - \hat{f}_{j,PD}(x_j^{(i)}) - \hat{f}_{k,PD}(x_k^{(i)})\right]^2}{\sum_{i=1}^n \left[\hat{f}_{jk,PD}(x_j^{(i)}, x_k^{(i)})\right]^2}$$

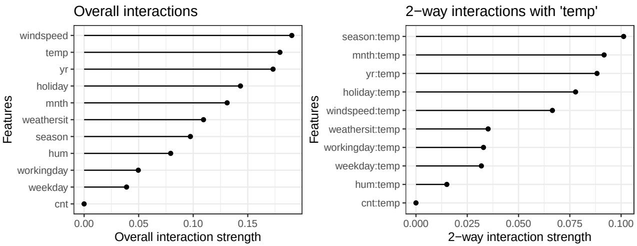
Note: The numerator is 0 if the two features x_j and x_k do not interact, i.e., $\hat{f}_{jk,PD}(x_j, x_k) - \hat{f}_{j,PD}(x_j) - \hat{f}_{k,PD}(x_k) = 0$.
$\Rightarrow$ The smaller the values of H^2_{jk}, the weaker the interaction between x_j and x_k.

OVERALL INTERACTION STRENGTH

Similarly, it is possible to measure whether a feature *j* interacts with any other feature (Overall interaction strength):

$$H_j^2 = \frac{\sum_{i=1}^{n}\left[\hat{f}(x^{(i)}) - \hat{f}_{j,PD}(x_j^{(i)}) - \hat{f}_{-j,PD}(x_{-j}^{(i)})\right]^2}{\sum_{i=1}^{n}\left[\hat{f}(x^{(i)})\right]^2}$$

Example: Inspect interactions of a random forest for the bike data


Overall interactions
Features
windspeed
temp
yr
holiday
mnth
weathersit
season
hum
workingday
weekday
cnt
Overall interaction strength
2−way interactions with 'temp'
Features
season:temp
mnth:temp
yr:temp
holiday:temp
windspeed:temp
weathersit:temp
workingday:temp
weekday:temp
hum:temp
cnt:temp
2−way interaction strength