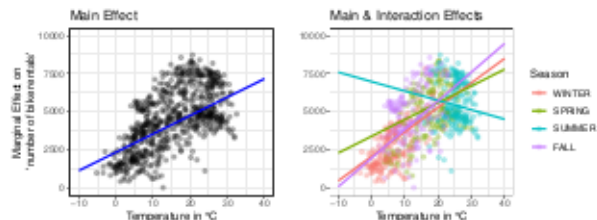
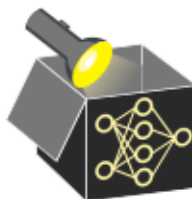


# EXAMPLE: INTERACTION EFFECT

**Example:** Interaction between temp and season will affect marginal effect of temp

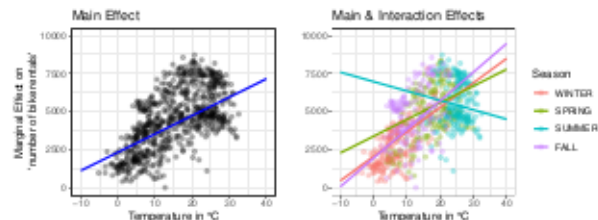


	Weights
(Intercept)	3453.9
seasonSPRING	1317.0
seasonSUMMER	4894.1
seasonFALL	-114.2
temp	160.5
hum	-37.6
windspeed	-61.9
days_since_2011	4.9
seasonSPRING:temp	-50.7
seasonSUMMER:temp	-222.0
seasonFALL:temp	27.2



# EXAMPLE: INTERACTION EFFECT

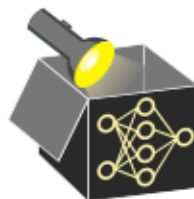
**Example:** Interaction between temp and season will affect marginal effect of temp



	Weights
(Intercept)	3453.9
seasonSPRING	1317.0
seasonSUMMER	4894.1
seasonFALL	-114.2
temp	160.5
hum	-37.6
windspeed	-61.9
days_since_2011	4.9
seasonSPRING:temp	-50.7
seasonSUMMER:temp	-222.0
seasonFALL:temp	27.2

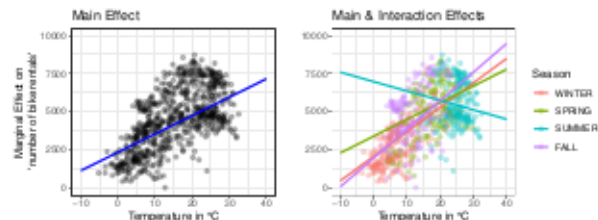
**Interpretation:** If temp increases by 1 °C, bike rentals

- increase by 160.5 in WINTER (reference)

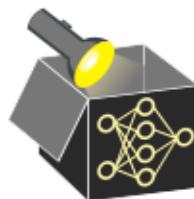


# EXAMPLE: INTERACTION EFFECT

**Example:** Interaction between temp and season will affect marginal effect of temp



	Weights
(Intercept)	3453.9
seasonSPRING	1317.0
seasonSUMMER	4894.1
seasonFALL	-114.2
temp	160.5
hum	-37.6
windspeed	-61.9
days_since_2011	4.9
seasonSPRING:temp	-50.7
seasonSUMMER:temp	-222.0
seasonFALL:temp	27.2

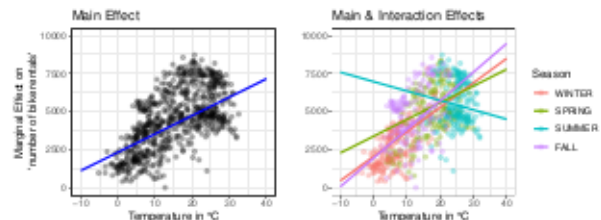


**Interpretation:** If temp increases by 1 °C, bike rentals

- increase by 160.5 in WINTER (reference)
- increase by 109.8 (= 160.5 - 50.7) in SPRING

# EXAMPLE: INTERACTION EFFECT

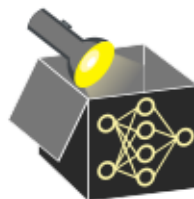
**Example:** Interaction between temp and season will affect marginal effect of temp



	Weights
(Intercept)	3453.9
seasonSPRING	1317.0
seasonSUMMER	4894.1
seasonFALL	-114.2
temp	160.5
hum	-37.6
windspeed	-61.9
days_since_2011	4.9
seasonSPRING:temp	-50.7
seasonSUMMER:temp	-222.0
seasonFALL:temp	27.2

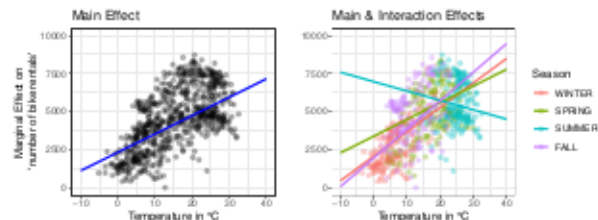
**Interpretation:** If temp increases by 1 °C, bike rentals

- increase by 160.5 in WINTER (reference)
- increase by 109.8 (= 160.5 - 50.7) in SPRING
- decrease by -61.5 (= 160.5 - 222) in SUMMER

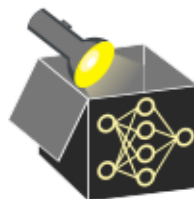


# EXAMPLE: INTERACTION EFFECT

**Example:** Interaction between **temp** and **season** will affect marginal effect of **temp**



Weights	
(Intercept)	3453.9
seasonSPRING	1317.0
seasonSUMMER	4894.1
seasonFALL	-114.2
temp	160.5
hum	-37.6
windspeed	-61.9
days_since_2011	4.9
seasonSPRING:temp	-50.7
seasonSUMMER:temp	-222.0
seasonFALL:temp	27.2

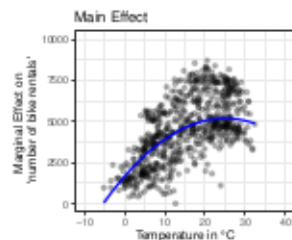


**Interpretation:** If **temp** increases by 1 °C, bike rentals

- increase by 160.5 in **WINTER** (reference)
- increase by 109.8 (= 160.5 - 50.7) in **SPRING**
- decrease by -61.5 (= 160.5 - 222) in **SUMMER**
- increase by 187.7 (= 160.5 + 27.2) in **FALL**

# EXAMPLE: QUADRATIC EFFECT

**Example:** Adding quadratic effect for **temp**

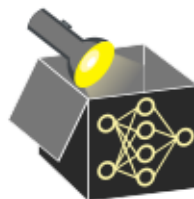


**Interpretation:** Not linear anymore!

- temp** depends on two weights:

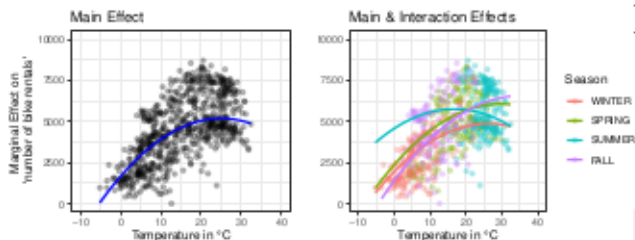
$$280.2 \cdot x_{temp} - 5.6 \cdot x_{temp}^2$$

Weights	
(Intercept)	3094.1
seasonSPRING	619.2
seasonSUMMER	284.6
seasonFALL	123.1
hum	-36.4
windspeed	-65.7
days_since_2011	4.7
temp	280.2
temp <sup>2</sup>	-5.6



# EXAMPLE: QUADRATIC EFFECT

**Example:** Adding quadratic effect for `temp` (left) and interaction with `season` (right)

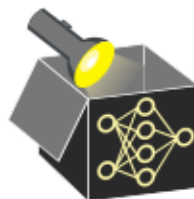


Weights	
(Intercept)	3802.1
seasonSPRING	-1345.1
seasonSUMMER	-6006.3
seasonFALL	-681.4
hum	-38.9
windspeed	-64.1
days_since_2011	4.8
temp	39.1
temp <sup>2</sup>	8.6
seasonSPRING:temp	407.4
seasonSPRING:temp <sup>2</sup>	-18.7
seasonSUMMER:temp	801.1
seasonSUMMER:temp <sup>2</sup>	-27.2
seasonFALL:temp	217.4
seasonFALL:temp <sup>2</sup>	-11.3

**Interpretation:** Not linear anymore!

- `temp` depends on multiple weights due to `season`:

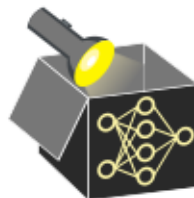
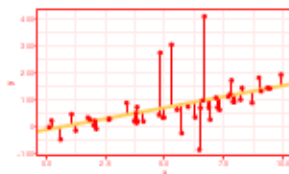
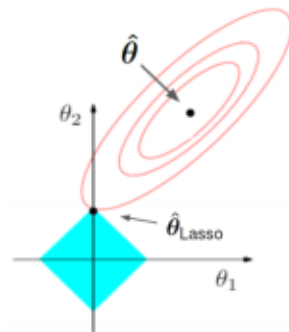
$$\begin{aligned} &\leadsto \text{WINTER: } 39.1 \cdot x_{\text{temp}} + 8.6 \cdot x_{\text{temp}}^2 \\ &\leadsto \text{SPRING: } (39.1 + 407.4) \cdot x_{\text{temp}} + (8.6 - 18.7) \cdot x_{\text{temp}}^2 \\ &\leadsto \text{SUMMER: } (39.1 + 801.1) \cdot x_{\text{temp}} + (8.6 - 27.2) \cdot x_{\text{temp}}^2 \\ &\leadsto \text{FALL: } (39.1 + 217.4) \cdot x_{\text{temp}} + (8.6 - 11.3) \cdot x_{\text{temp}}^2 \end{aligned}$$



# REGULARIZATION VIA LASSO ► Tibshirani (1996)

- LASSO adds an  $L_1$ -norm penalization term ( $\lambda \|\theta\|_1$ ) to least squares optimization problem
  - ↪ Shrinks some feature weights to zero (feature selection)
  - ↪ Sparser models (fewer features): more interpretable
- Penalization parameter  $\lambda$  must be chosen (e.g., by CV)

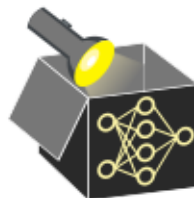
$$\min_{\theta} \left( \underbrace{\frac{1}{n} \sum_{i=1}^n (y^{(i)} - \mathbf{x}^{(i)\top} \theta)^2}_{\text{Least square estimate for LM}} + \lambda \|\theta\|_1 \right)$$





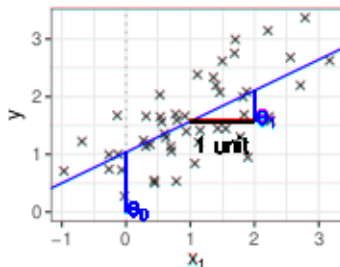
# REGULARIZATION VIA LASSO

► Tibshirani (1996)



**Example** (interpretation of weights analogous to LM):

- LASSO with main effects and interaction **temp with season**
- $\lambda$  is chosen  $\rightsquigarrow$  6 selected features ( $\neq 0$ )
- **LASSO shrinks weights of single categories** separately (due to dummy encoding)  
 $\rightsquigarrow$  No feature selection of whole categorical features (only w.r.t. category levels)  
 $\rightsquigarrow$  **Solution: group LASSO** ► Yuan and Lin (2006)



	Weights
(Intercept)	3135.2
seasonSPRING	767.4
seasonSUMMER	0.0
<b>seasonFALL</b>	<b>0.0</b>
<b>temp</b>	<b>116.7</b>
<b>hum</b>	<b>-28.9</b>
windspeed	-50.5
days_since_2011	4.8
<b>seasonSPRING:temp</b>	<b>0.0</b>
<b>seasonSUMMER:temp</b>	<b>0.0</b>
<b>seasonFALL:temp</b>	<b>30.2</b>