Optimization

Multivariate Roots

Learning goals

- LEARNING GOAL 1
- LEARNING GOAL 2

OVERDETERMINED SYSTEMS OF EQUATIONS

When solving an overdetermined system of equations, i.e.

$$f(\mathbf{x}) = 0$$

with $f : \mathbb{R}^d \to \mathbb{R}^m$ with $m > n$ (more equations than unknowns), there is usually no solution for the corresponding equation.

However, in order to find an approximate solution, we interpret the problem of root search as an **optimization problem**:

$$\min_{\mathbf{x}} \|f(\mathbf{x})\|_2^2$$

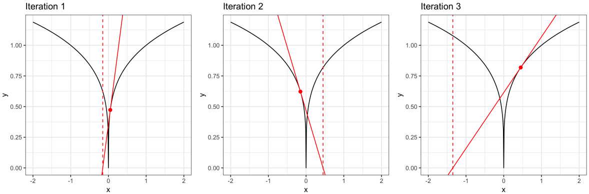
To solve this, general optimization techniques or special methods for (nonlinear) **least squares problems** (see Gauss-Newton algorithm later in this chapter) can be used.

CONVERGENCE OF NEWTON'S METHOD

Under certain conditions, Newton's method converges **quadratically**. Otherwise, the procedure may converge slowly or not at all.

Possible reasons:

- Stationary point is hit: Method terminates because $\nabla f(\mathbf{x}) = 0$, although root has not yet been found.
- Bad starting point: Starting point must be close enough to root.
- "Overshot": Function not sufficiently smooth.



Newton's method for $f(x) = |x|^{1/4}$ and starting point $x = 0.05$. Although the starting point is close to the root, we move further away from it in each step.