Mathematical Concepts 2

Exercise 1: Matrix Calculus

Let $\mathbf{x}, \mathbf{c} \in \mathbb{R}^d, \mathbf{u}, \mathbf{v} : \mathbb{R}^d \to \mathbb{R}^d, \mathbf{u}, \mathbf{v} \in \mathcal{C}^2, \mathbf{Y} : \mathbb{R}^d \to \mathbb{R}^{d \times d}$

- (a) Compute $\frac{\partial \|\mathbf{x} \mathbf{c}\|_2^2}{\partial \mathbf{x}}$
- (b) Compute $\frac{\partial \|\mathbf{x} \mathbf{c}\|_2}{\partial \mathbf{x}}$
- (c) Compute $\frac{\partial \mathbf{u}^{\top} \mathbf{v}}{\partial \mathbf{x}}$
- (d) Show that $\frac{\partial \mathbf{u}^{\top} \mathbf{Y}}{\partial \mathbf{x}} = \begin{pmatrix} \mathbf{u}^{\top} \frac{\partial \mathbf{y}_{1}}{\partial \mathbf{x}} + \mathbf{y}_{1}^{\top} \frac{\partial \mathbf{u}}{\partial \mathbf{x}} \\ \vdots \\ \mathbf{u}^{\top} \frac{\partial \mathbf{y}_{d}}{\partial \mathbf{x}} + \mathbf{y}_{d}^{\top} \frac{\partial \mathbf{u}}{\partial \mathbf{x}} \end{pmatrix}$ where $\mathbf{y}_{i} : \mathbb{R}^{d} \to \mathbb{R}^{d}, i = 1, \dots, d$ are the column vectors of \mathbf{Y} .
- (e) Compute $\frac{\partial^2 \mathbf{u}^\top \mathbf{v}}{\partial \mathbf{x} \partial \mathbf{x}^\top}$ *Hint:* use c), d)

Exercise 2: Optimality in 1d

Let
$$f: [-1, 2] \to \mathbb{R}, x \mapsto \exp(x^3 - 2x^2)$$

- (a) Compute f'
- (b) Plot f and f' with R
- (c) Find all possible candidates x^* for local maxima and minima. $\mathit{Hint:}\ \,$ exp is a strictly monotone function.
- (d) Compute f''
- (e) Determine if the candidates are local maxima, minima or neither.
- (f) Find the global maximum and global minimum of f