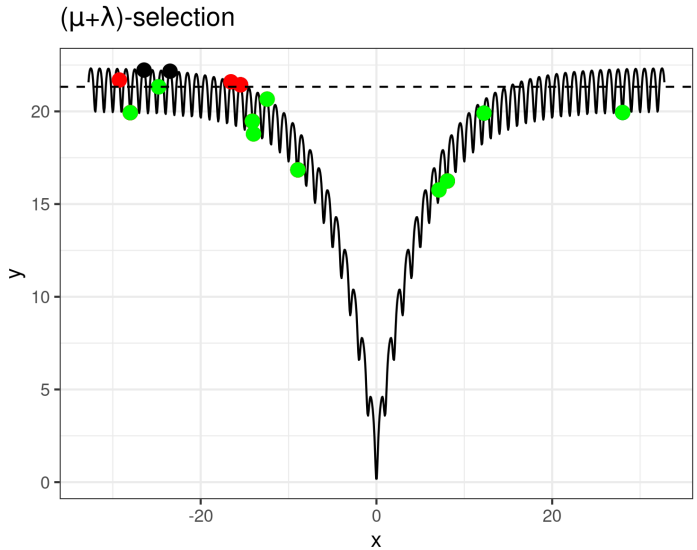
Optimization in Machine Learning

Evolutionary Algorithms - ES / Numerical Encodings



Learning goals

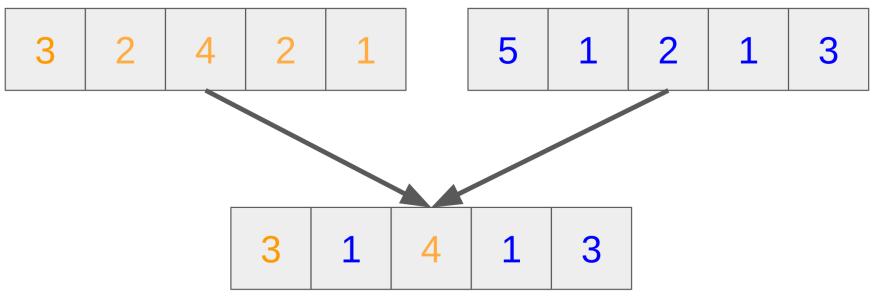
- Recombination
- Mutation
- A few simple examples

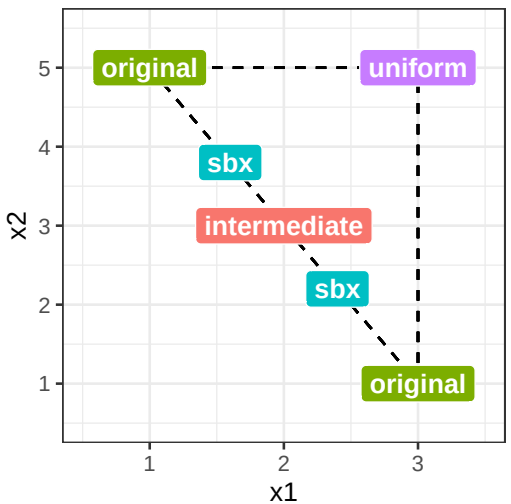
RECOMBINATION FOR NUMERIC

Options for recombination of two individuals $\boldsymbol{x}, \tilde{\boldsymbol{x}} \in \mathbb{R}^d$:

- **Uniform crossover**: choose gene j with probability p of 1st parent and probability $1 - p$ of 2nd parent.
- **Intermediate recombination**: new individual is created from the mean value of two parents $\frac{1}{2}(\boldsymbol{x} + \tilde{\boldsymbol{x}})$.
- **Simulated Binary Crossover (SBX)**: generate **two offspring**

$$\bar{\boldsymbol{x}} \pm \frac{1}{2}\beta(\tilde{\boldsymbol{x}} - \boldsymbol{x}), \bar{\boldsymbol{x}} = \frac{1}{2}(\boldsymbol{x} + \tilde{\boldsymbol{x}}), \beta \text{ randomly sampled}$$

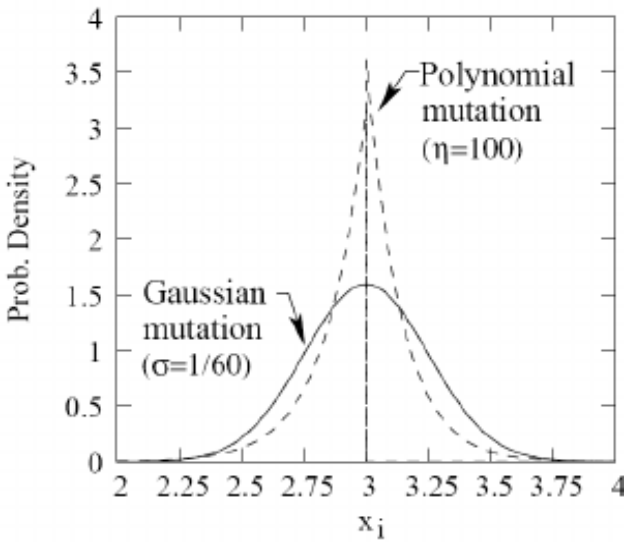




MUTATION FOR NUMERIC

Mutation: individuals are changed, for example for $\boldsymbol{x} \in \mathbb{R}^d$

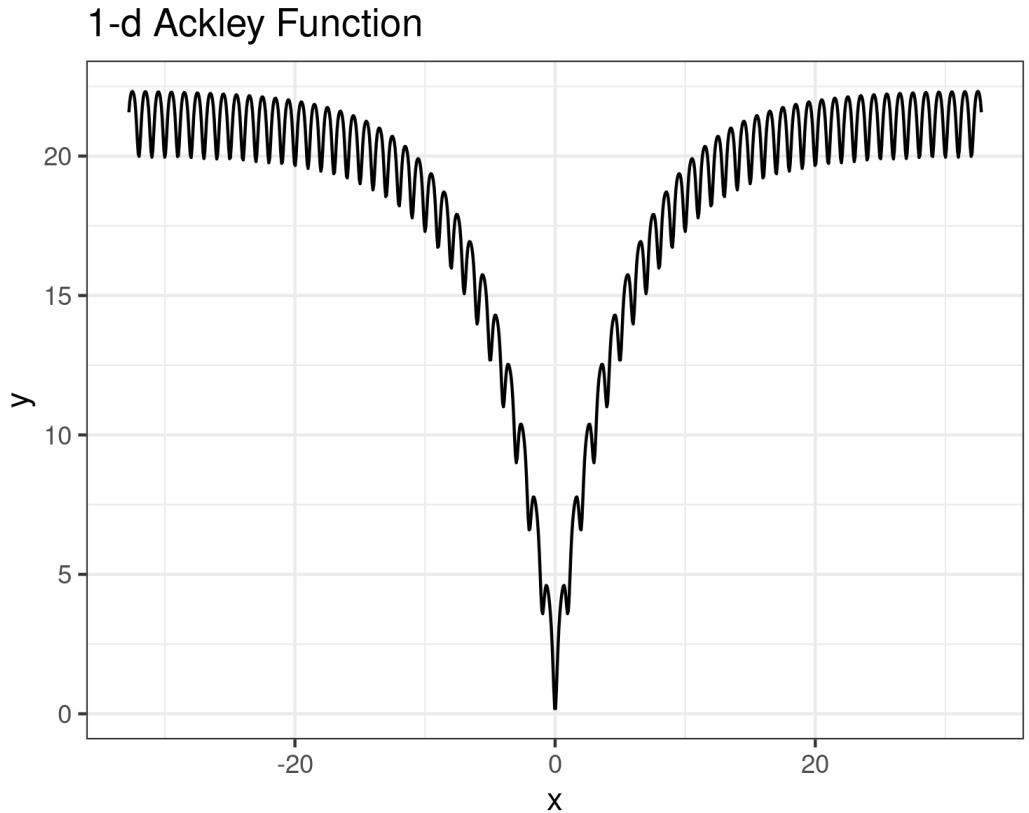
- **Uniform mutation:** choose a random gene $\boldsymbol{x}_j$ and replace it with a value uniformly distributed (within the feasible range).
- **Gauss mutation:** $\tilde{\boldsymbol{x}} = \boldsymbol{x} \pm \sigma \mathcal{N}(0, \boldsymbol{I})$
- **Polynomial mutation:** polynomial distribution instead of normal distribution



Source: K. Deb, Analysing mutation schemes for real-parameter genetic algorithms, 2014

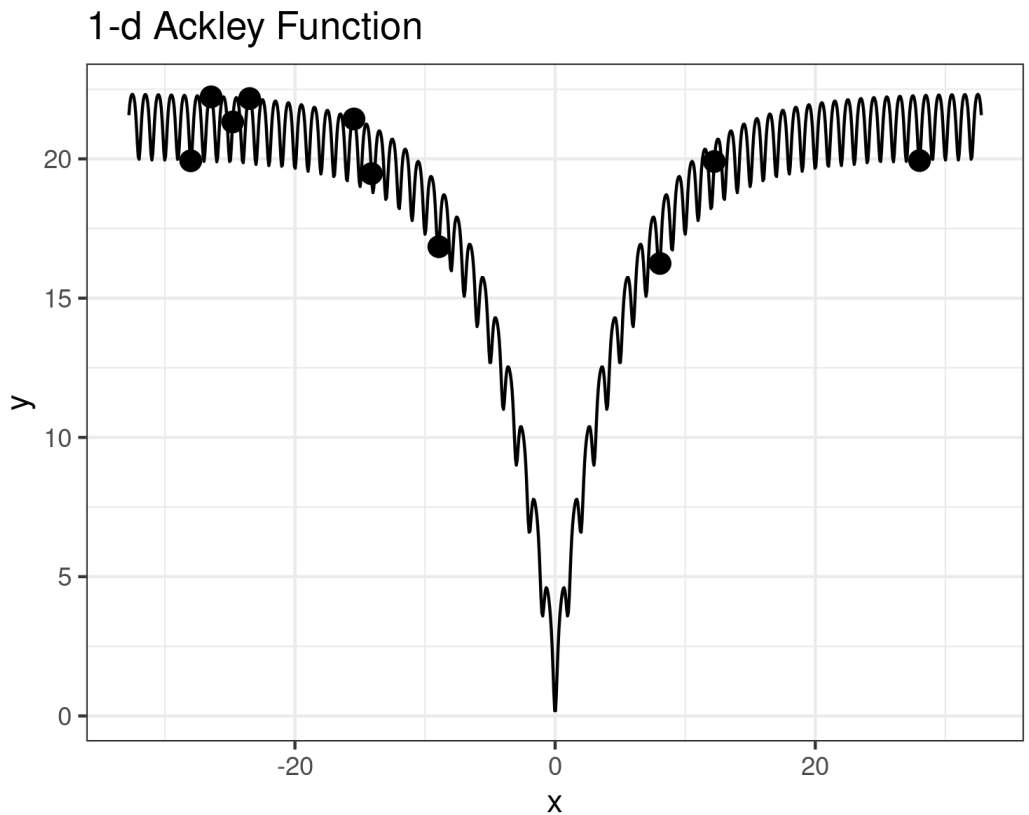
EXAMPLE OF AN EVOLUTIONARY ALGORITHM

In the following, a (simple) EA is shown on the 1-dim Ackley function, optimized on $[-30, 30]$. Usually, for the optimization of a function $f : \mathbb{R}^d \to \mathbb{R}$ individuals are coded as real vectors $\boldsymbol{x}_j \in \mathbb{R}^d$, so here we use simply one real number as a chromosome.



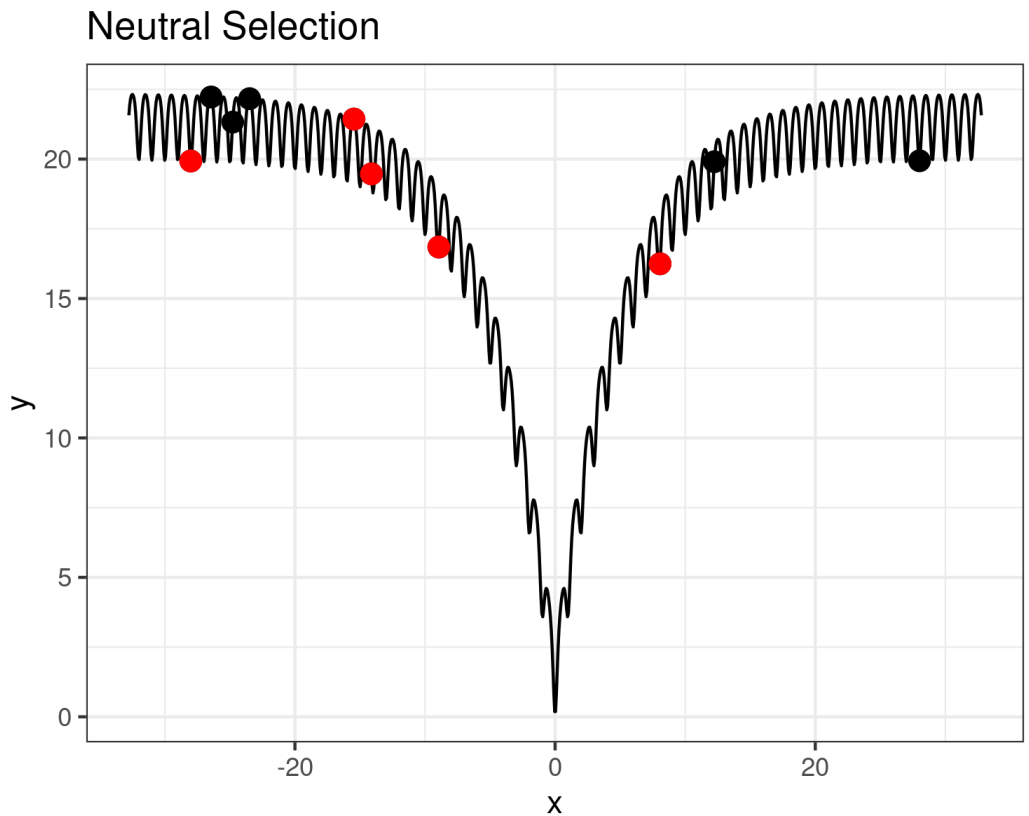
EXAMPLE OF AN EVOLUTIONARY ALGORITHM

Randomly init population with size $\mu = 10$.



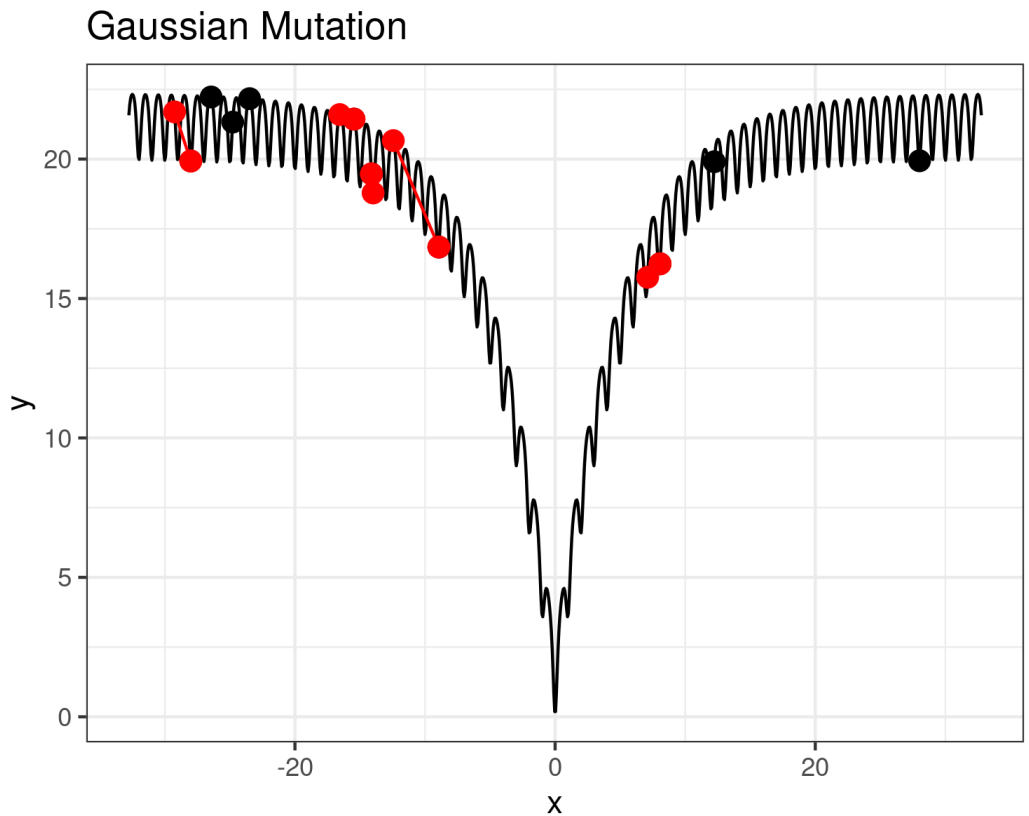
EXAMPLE 1: ACKLEY FUNCTION

We choose $\lambda = 5$ offspring by neutral selection (red individuals).



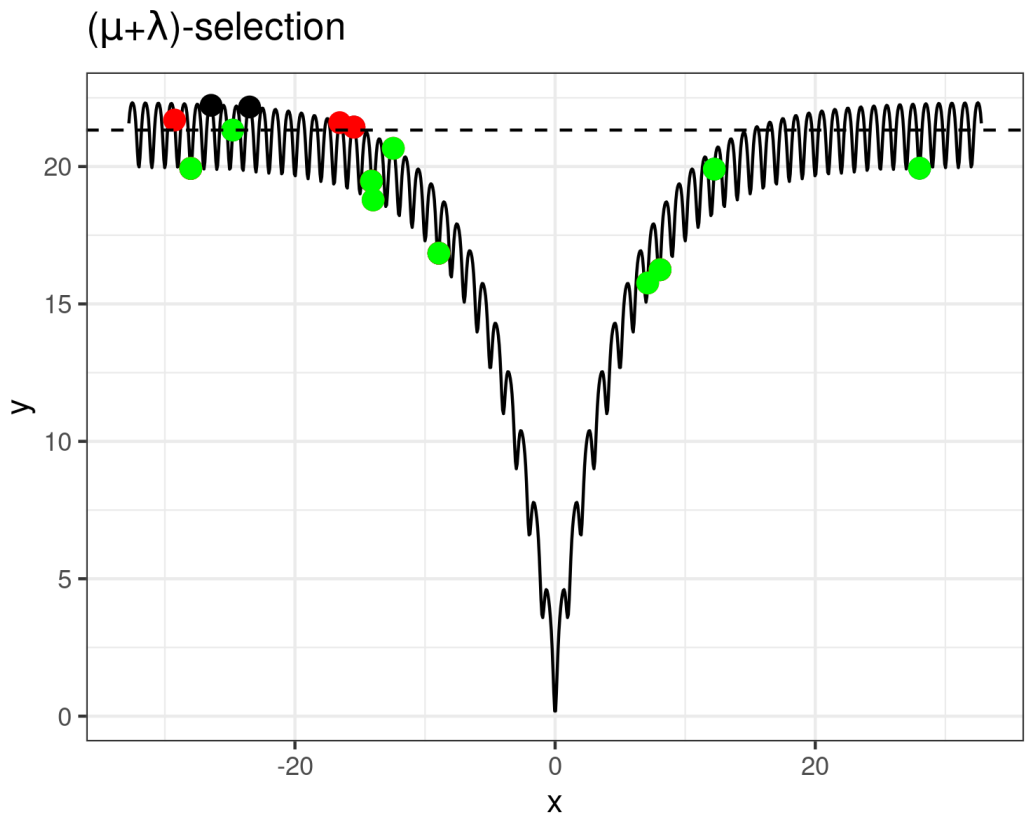
EXAMPLE 1: ACKLEY FUNCTION

Using a Gaussian mutation with $\sigma = 2$, without a recombination.



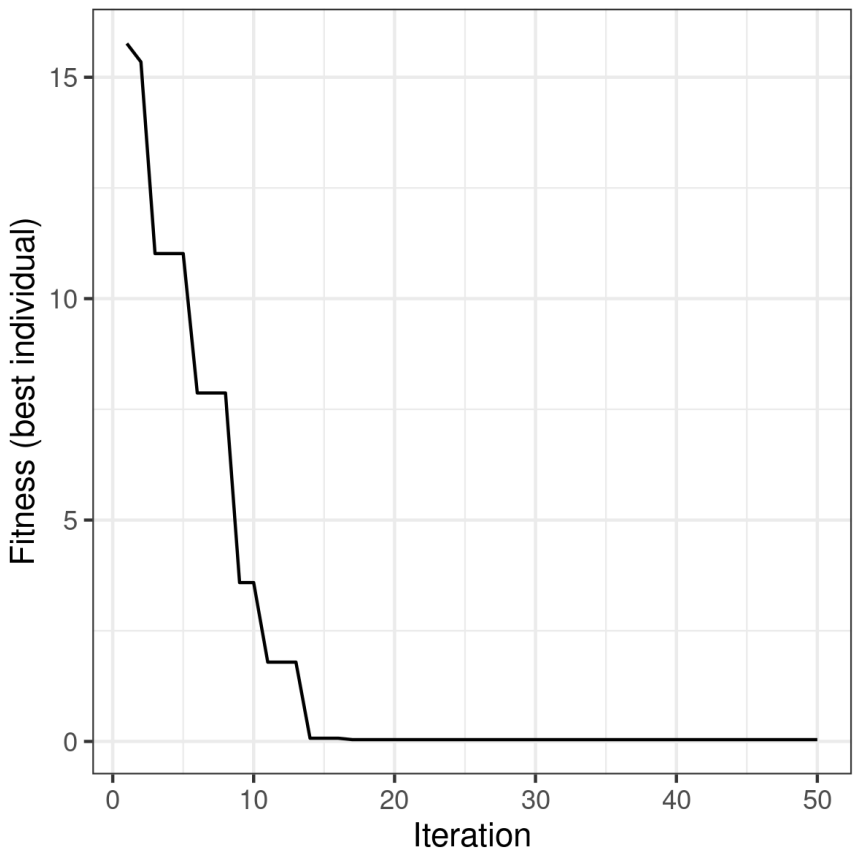
EXAMPLE 1: ACKLEY FUNCTION

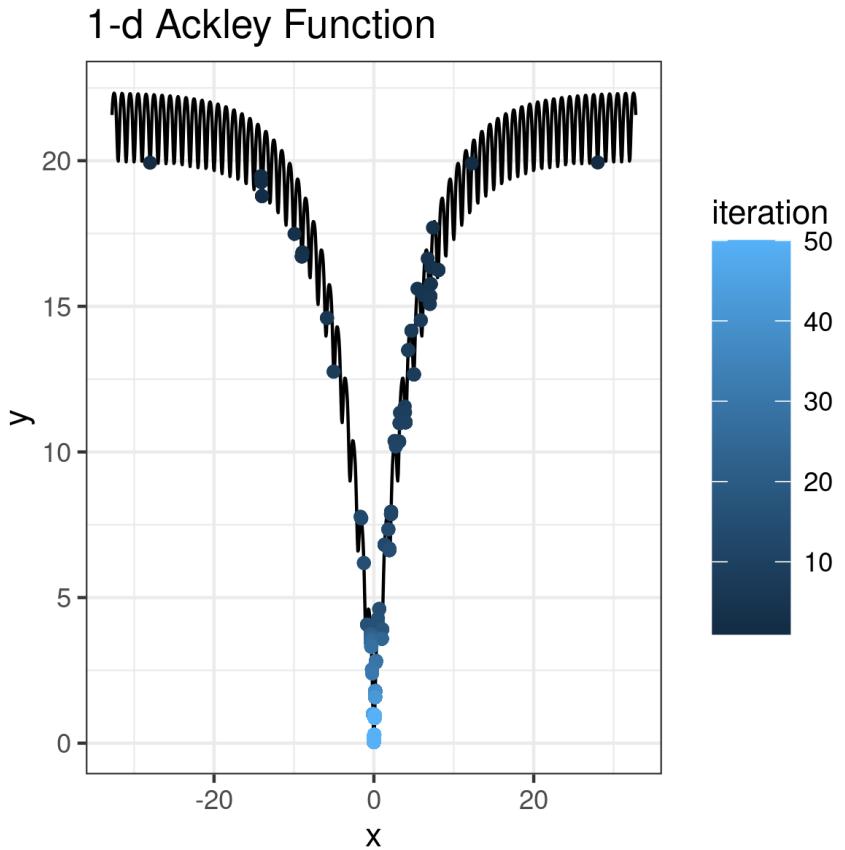
We use a $(\mu + \lambda)$ selection. The selected individuals are green.



EXAMPLE 1: ACKLEY FUNCTION

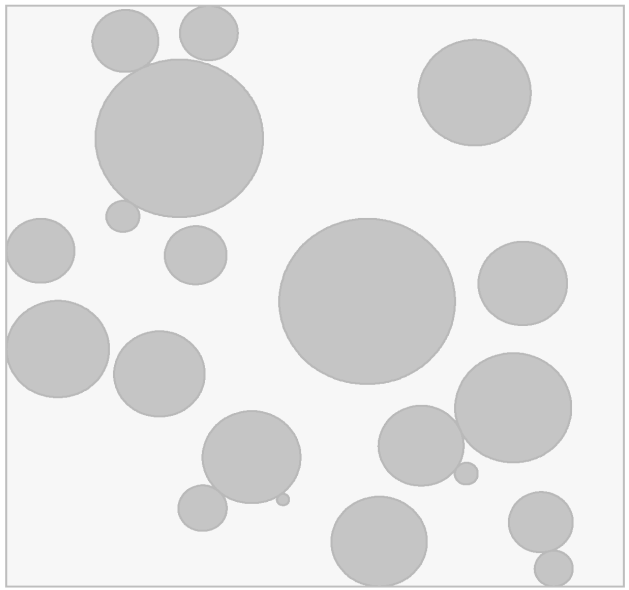
After 50 iterations:





EXAMPLE 2: GRID OF BALLS

Consider a grid in which n balls with random radius are placed.



Aim: Find the circle with the largest possible radius in the grid that does **not** intersect with the other existing circles.

- What is the fitness function?
- How is the population defined?

Implementation: `https://juliambr.shinyapps.io/balls/`

EXAMPLE 2: GRID OF BALLS

In our example, the chromosome of an individual is the center of a circle, so the chromosomes are encoded as 2-dimensional real vectors $\boldsymbol{x} = (x_1, x_2) \in \mathbb{R}^2$.

The population $P \subset \mathbb{R}^2$ is given as a set of circle centers.

The fitness function evaluates an individual $\boldsymbol{x} \in P$ based on the distance to the nearest neighboring gray circle k.

$$f(\boldsymbol{x}) = \min_{k \in \text{Grid}} \text{distance}(k, \boldsymbol{x}),$$

where the distance is defined as 0 if a circle center is within the radius of a circle of the grid.
This function is to be maximized: we are looking for the largest circle that does not touch any of the gray circles.