

High-Dimensional Variable Selection for Competing Risks with Cooperative Penalized Regression «CooPeR»

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- ▶ **Main goal:** Fit cause-specific model for event 1 *using shared information* from event 2

Building Blocks

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 - ▶ Idea: Some features predictive for event 1 will also be predictive for event 2

Foundation: Elastic Net

The elastic net objective function with some negative log-likelihood term:

$$\operatorname{argmin}_{\beta} \quad \text{NLL}(\beta) + \lambda \sum_{j=1}^p \left(\alpha |\beta_j| + \frac{1 - \alpha}{2} \beta_j^2 \right)$$

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- ▶ $0 \Rightarrow$ only ℓ_2 penalty (ridge)

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→ Need different approach

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Two Applications

1. Assign features to K groups w/ separate penalization weights
 2. Adjust penalization weights within group
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Feature Weighting: Groups

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Interesting when e.g. $p \gg 1000$ w/ 50 clinical + 5000 gene expression features

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- ▶ X_2, X_3 : More important \rightarrow weaker penalization
- ▶ X_4 : “Irrelevant” \rightarrow stronger penalization

Feature-Weighting: New Objective Function

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$$w_j(\theta) = \frac{\sum_{l=1}^p \exp(\mathbf{z}_l^T \theta)}{p \exp(\mathbf{z}_j^T \theta)}$$

- **Penalization weight** of β_j based on corresponding value in \mathbf{Z} and parameter $\theta \in \mathbb{R}^{K \times 1}$

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 - ▶ Higher score \rightarrow lower w_j , feature is “more important”

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Dubbed “Cooperative Penalized (Cox) Regression” (CooPeR)

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- ▶ Compare CooPeR, penalized Cox (glmnet), RSF (rsfrc), CoxBoost

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- ▶ Block 4 (**Cor. Noise**): 500 variables, $\rho \approx 0.32$

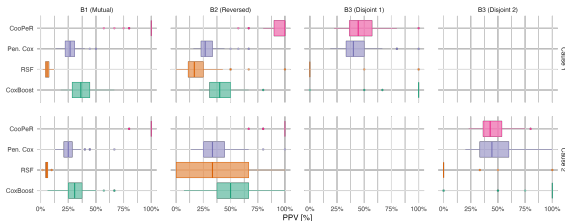
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- ▶ Remaining variables: Uncorrelated noise

Positive Predictive Value

Detection of true effects: PPV

Simulation setting with approx. equal proportions of cause 1 and 2

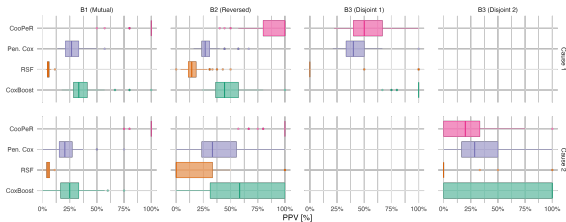


$$\text{PPV} = \frac{\text{TP}}{\text{TP} + \text{FP}}$$

Positive Predictive Value $PPV = \frac{TP}{TP+FP}$

Detection of true effects: PPV

Simulation setting with cause 2 less prevalent

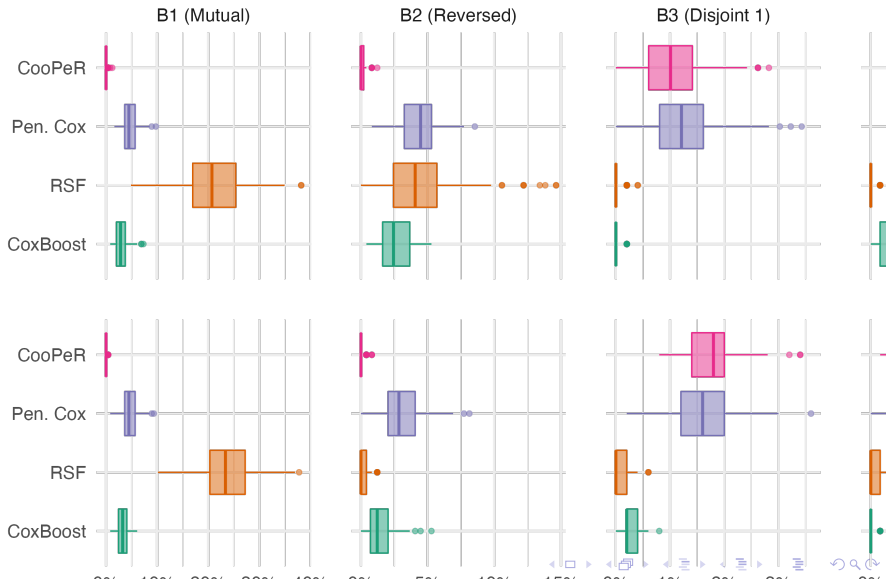


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False Positive Rate

Susceptibility to noise: FPR

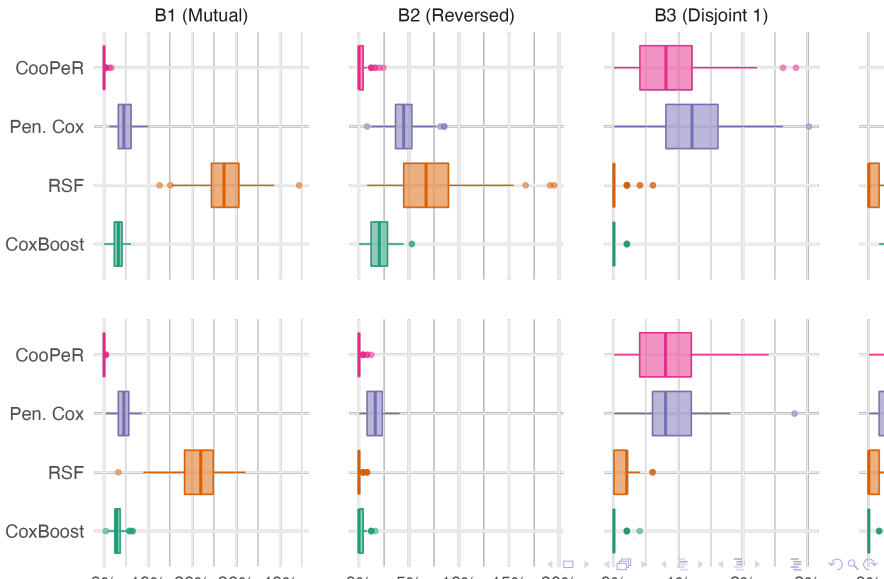
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- ▶ Idea:
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- ▶ Tried bladder cancer data (Dyrskjød et al. (2005)), did not go well

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- ▶ What about $k > 2$ events? No trivial generalization

Thanks for listening!

References

- Binder, Harald, Arthur Allignol, Martin Schumacher, and Jan Beyersmann. 2009. "Boosting for High-Dimensional Time-to-Event Data with Competing Risks." *Bioinformatics* 25 (7): 890–96. <https://doi.org/10.1093/bioinformatics/btp088>.
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