

# 1.4 Multivariate Inheritance & Response to Selection



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# Thesis

- The statistical approach that we used for a single trait can be extended to multiple traits.
- The key statistical parameter that emerges is the  $G$ -matrix.
- The  $G$ -matrix affects the response of the multivariate mean to selection and drift.

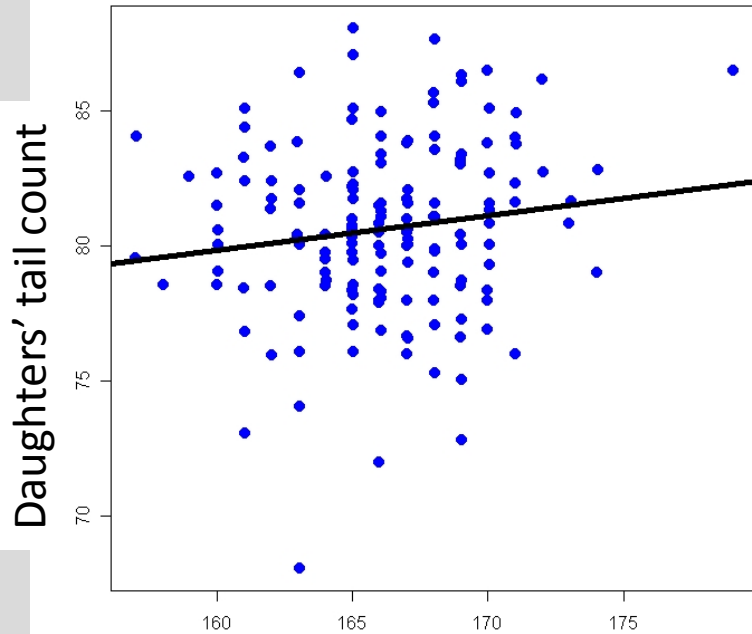
# Outline

1. Multivariate resemblance between parents and offspring is captured by the  $G$ -matrix.
2. Our model of inheritance is multivariate.
3. Some examples.
4. The  $G$ -matrix is affected by opposing forces.
5. The  $G$ -matrix affects the evolution of the multivariate mean.

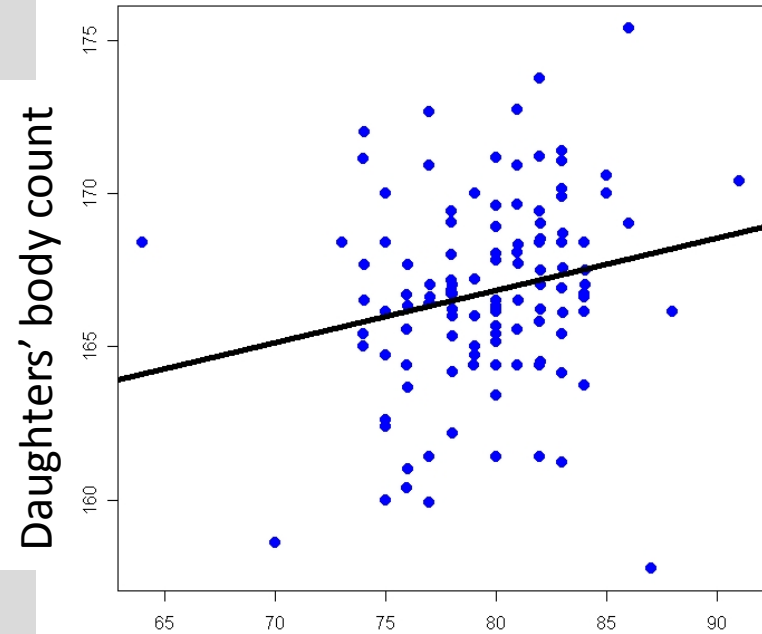
# 1. Multivariate resemblance

Traits can run together in families

[Animation 1](#)



Mom's body count



Mom's tail count

## 2. A Model for Multivariate Resemblance

### a. A model for phenotypic value

phenotypic value

$$z = x + e = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

phenotypic mean

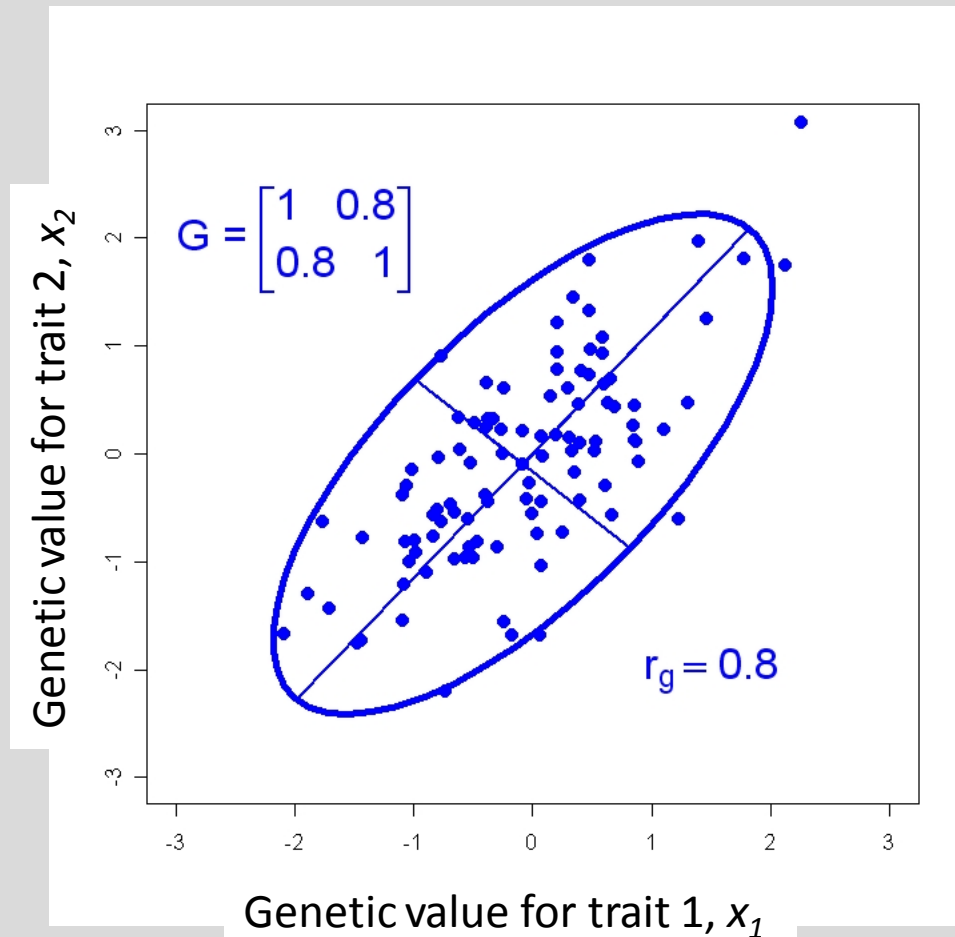
$$\bar{z} = \bar{x} + \bar{e} = \begin{bmatrix} \bar{z}_1 \\ \bar{z}_2 \end{bmatrix} = \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix} + \begin{bmatrix} \bar{e}_1 \\ \bar{e}_2 \end{bmatrix}$$

phenotypic var/covar

$$P = G + E = \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{12} & G_{22} \end{bmatrix} + \begin{bmatrix} E_{11} & E_{12} \\ E_{12} & E_{22} \end{bmatrix}$$

## 2. A Model for Multivariate Resemblance

c. The  $G$ -matrix describes a cloud of genetic values



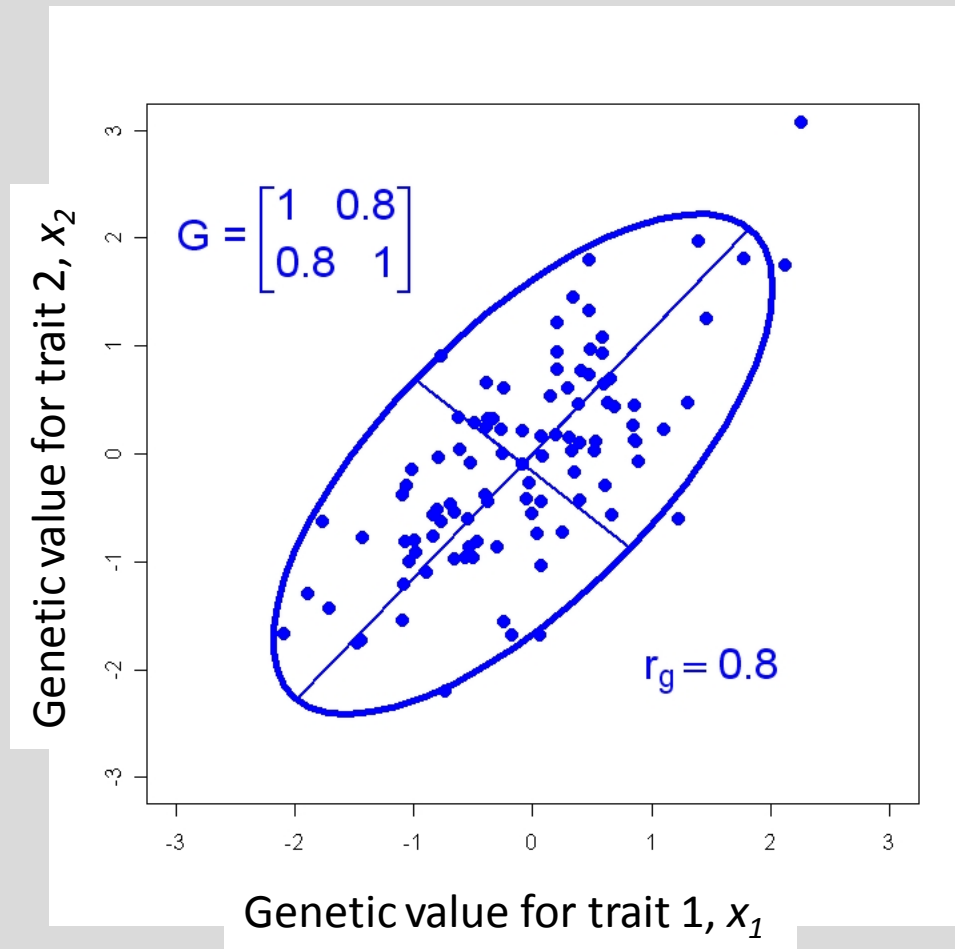
Causes of genetic covariance:

Pleiotropy

Linkage Disequilibrium

## 2. A Model for Multivariate Resemblance

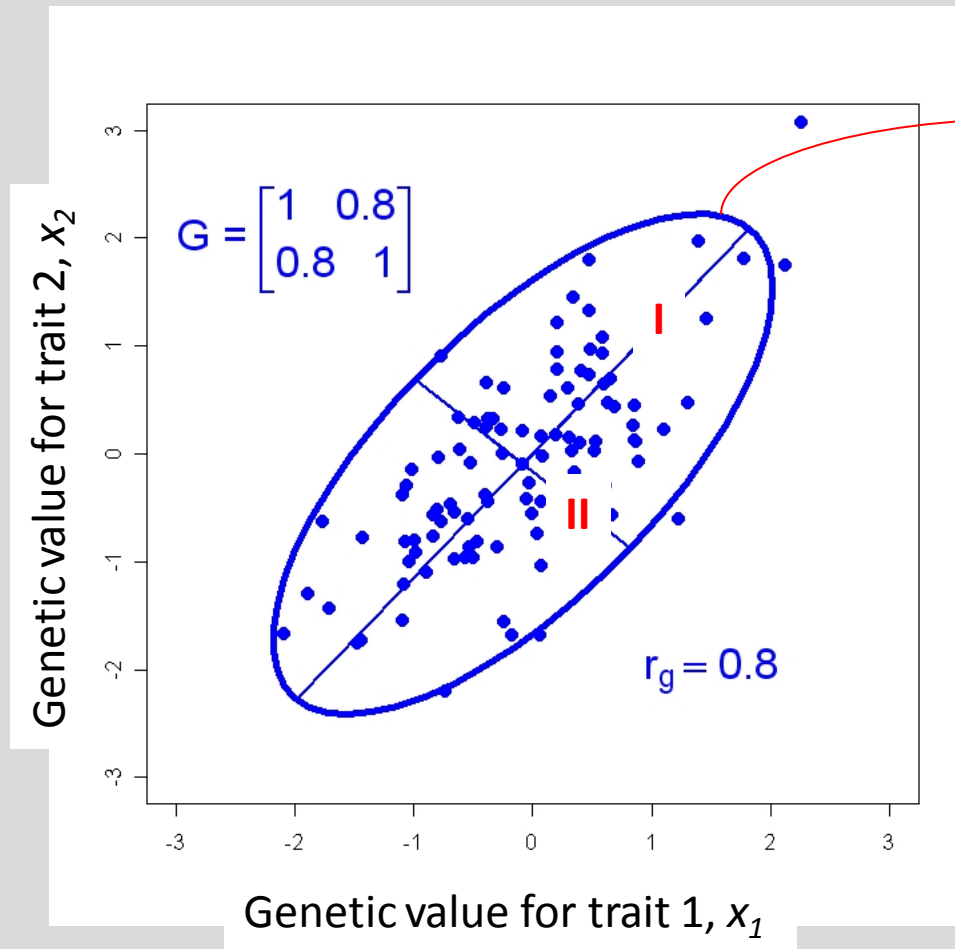
c. The  $G$ -matrix describes a cloud of genetic values



$$r_g = G_{12} / \sqrt{G_{11}G_{22}}$$

## 2. A Model for Multivariate Resemblance

c. The  $G$ -matrix describes a cloud of genetic values



95% confidence ellipse

First principal component

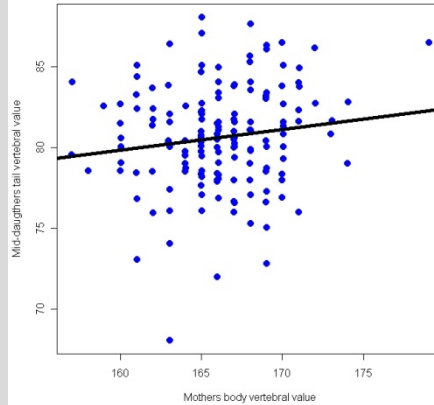
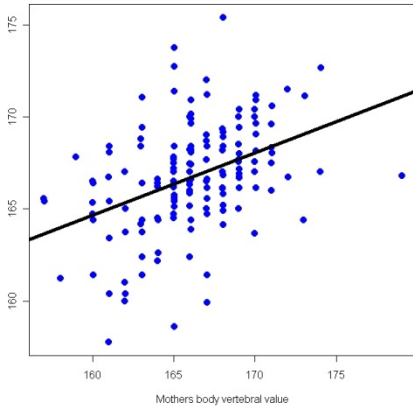
Second principal component



# 3. Some examples

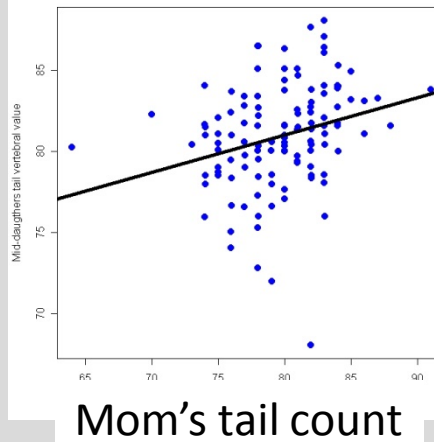
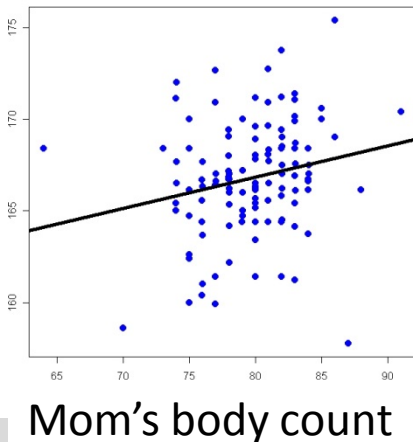
## a. Mother-daughter resemblance in vertebral counts in garter snakes

Daughters' body count



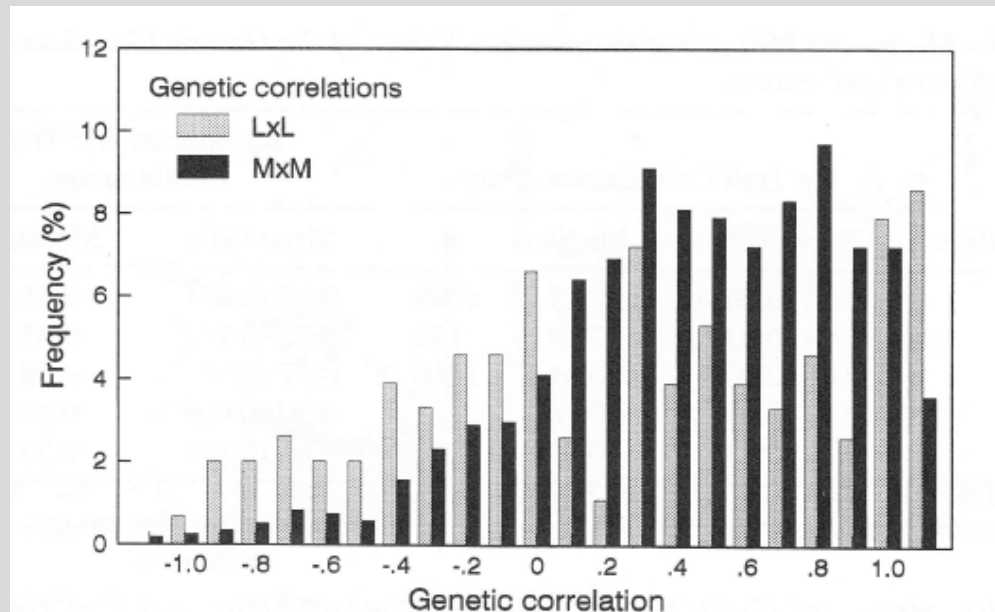
$$G = \begin{bmatrix} G_{11} & G_{12} \\ G_{12} & G_{22} \end{bmatrix} = \begin{bmatrix} 8.17 & 3.78 \\ 3.78 & 8.16 \end{bmatrix}$$

Daughters' tail count



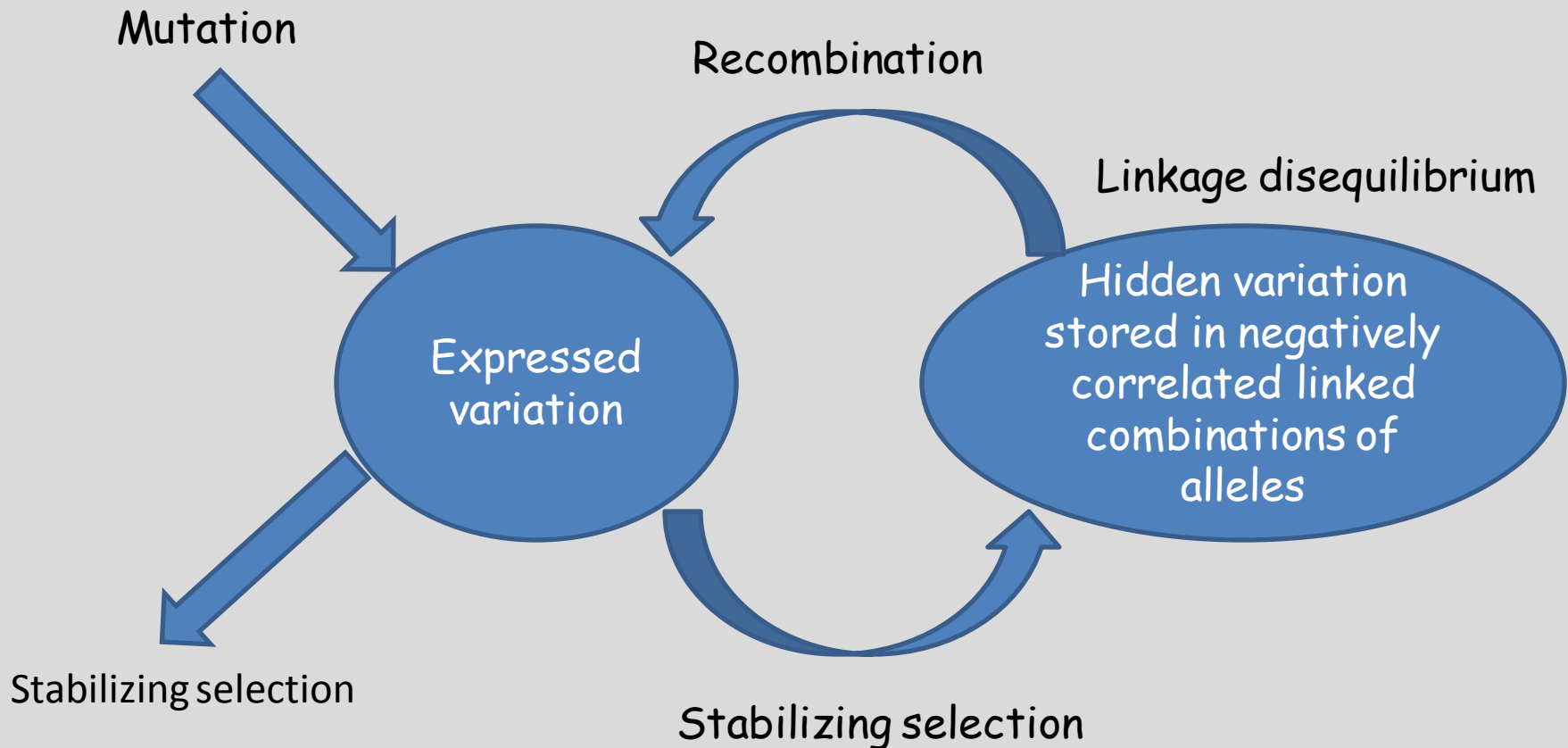
# 3. Some examples

## b. Prevalence of genetic correlation



# 4. Why don't we run out of additive genetic variance and covariance?

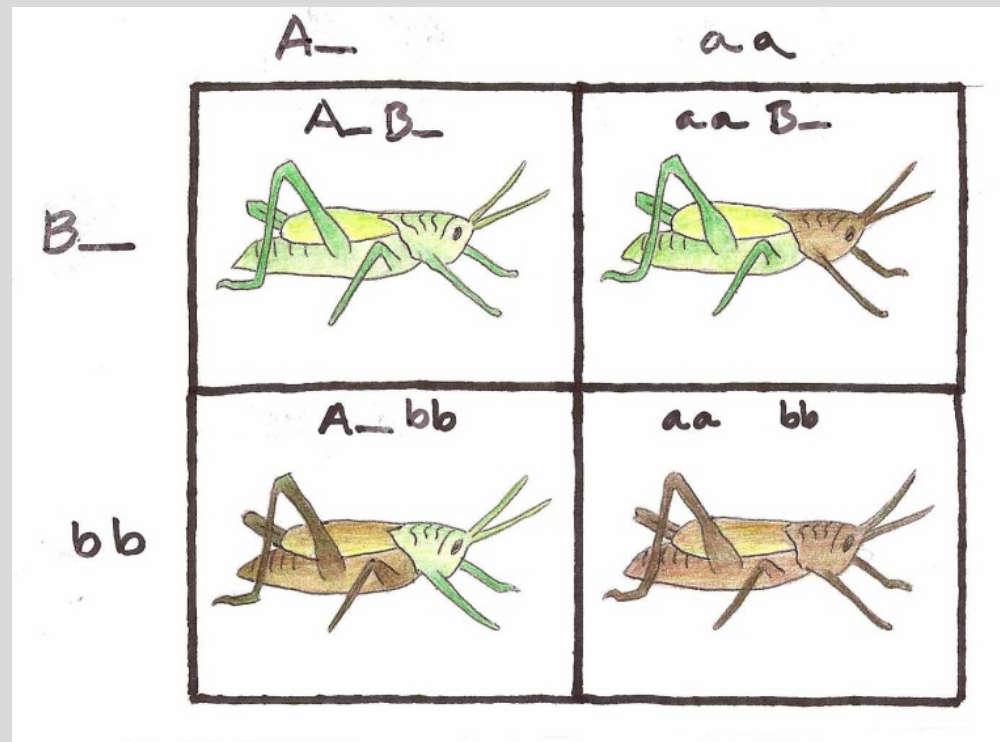
## a. Mutation-Selection Balance



# 4. Why don't we run out of additive genetic covariance?

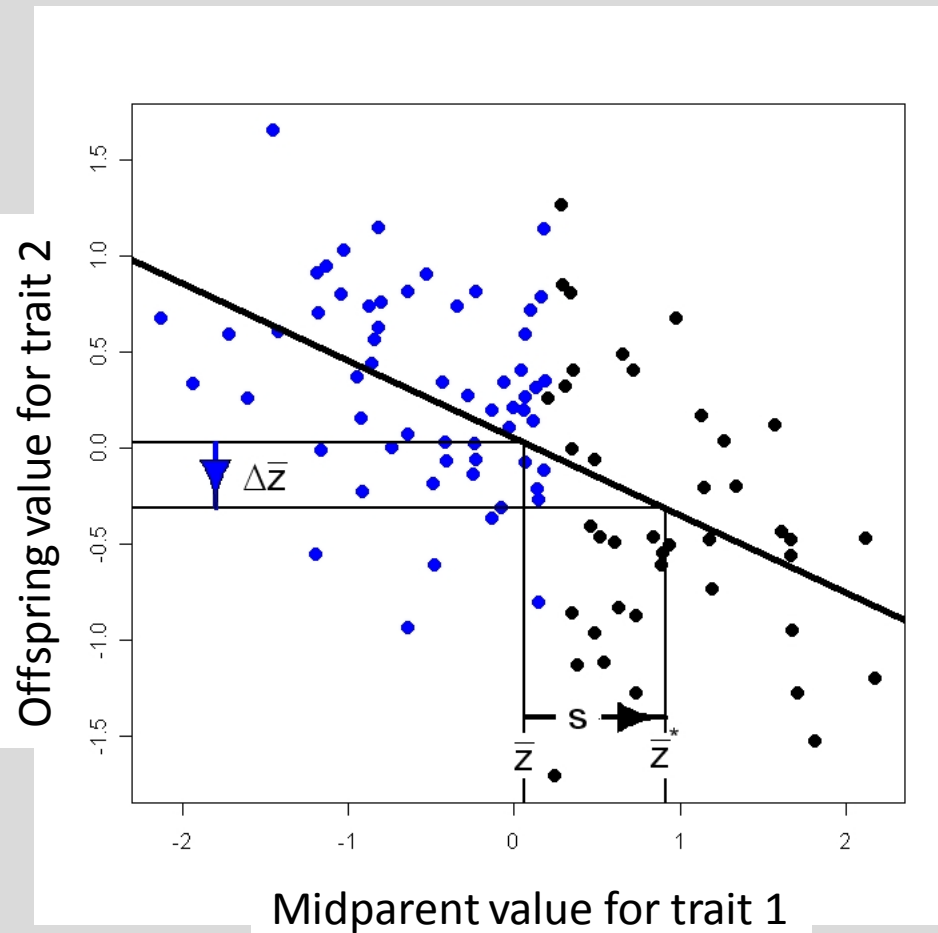
## b. Correlational selection

- one kind of multivariate stabilizing selection -  
can produce linkage disequilibrium



# 5. Changing the multivariate mean with directional selection

a. Genetic covariance causes selection on one trait to affect a correlated trait



## 5. Changing the multivariate mean with directional selection

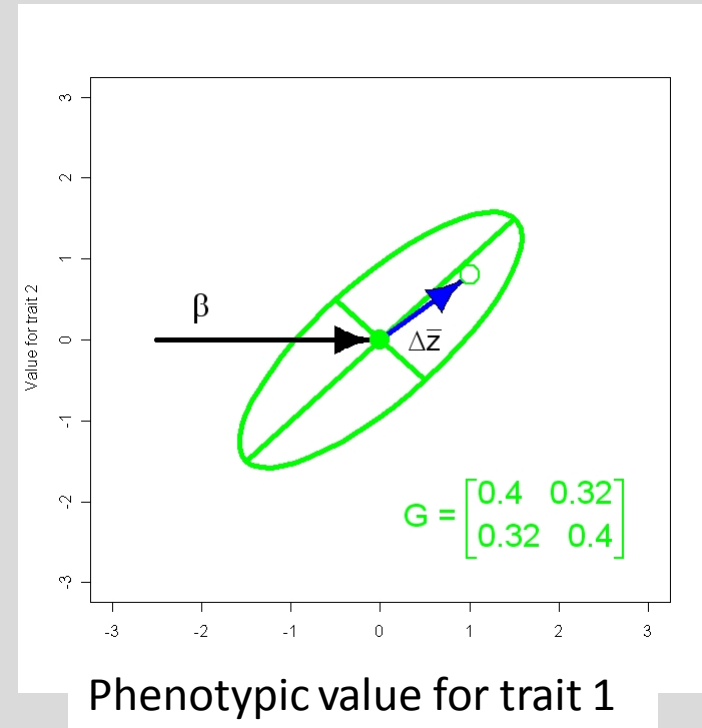
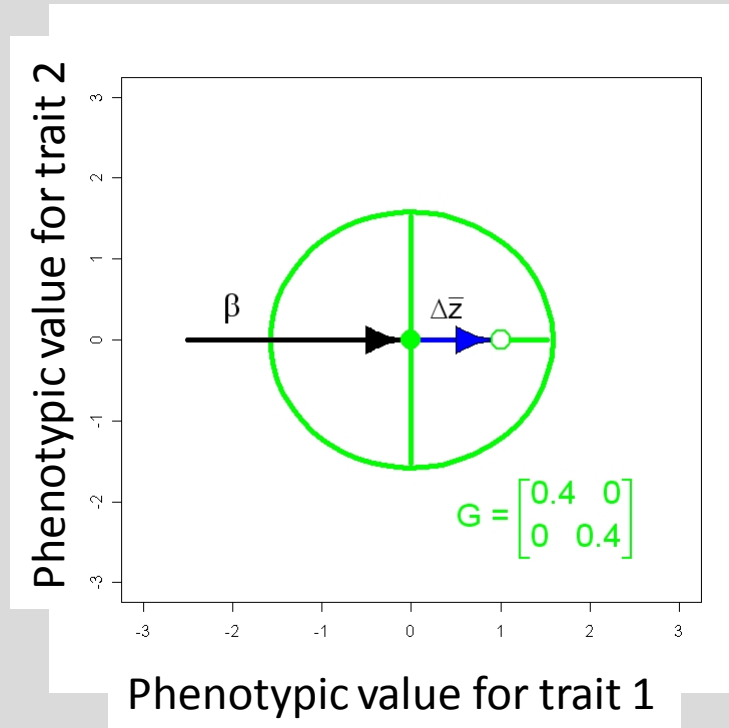
### a. Direct and correlated responses to selection

$$\Delta \bar{\mathbf{z}} = \mathbf{G} \mathbf{P}^{-1} \mathbf{s} = \mathbf{G} \boldsymbol{\beta}$$

$$\begin{bmatrix} \Delta \bar{z}_1 \\ \Delta \bar{z}_2 \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{12} & G_{22} \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} = \begin{bmatrix} G_{11}\beta_1 + G_{12}\beta_2 \\ G_{12}\beta_1 + G_{22}\beta_2 \end{bmatrix}$$

# 5. Changing the multivariate mean with directional selection

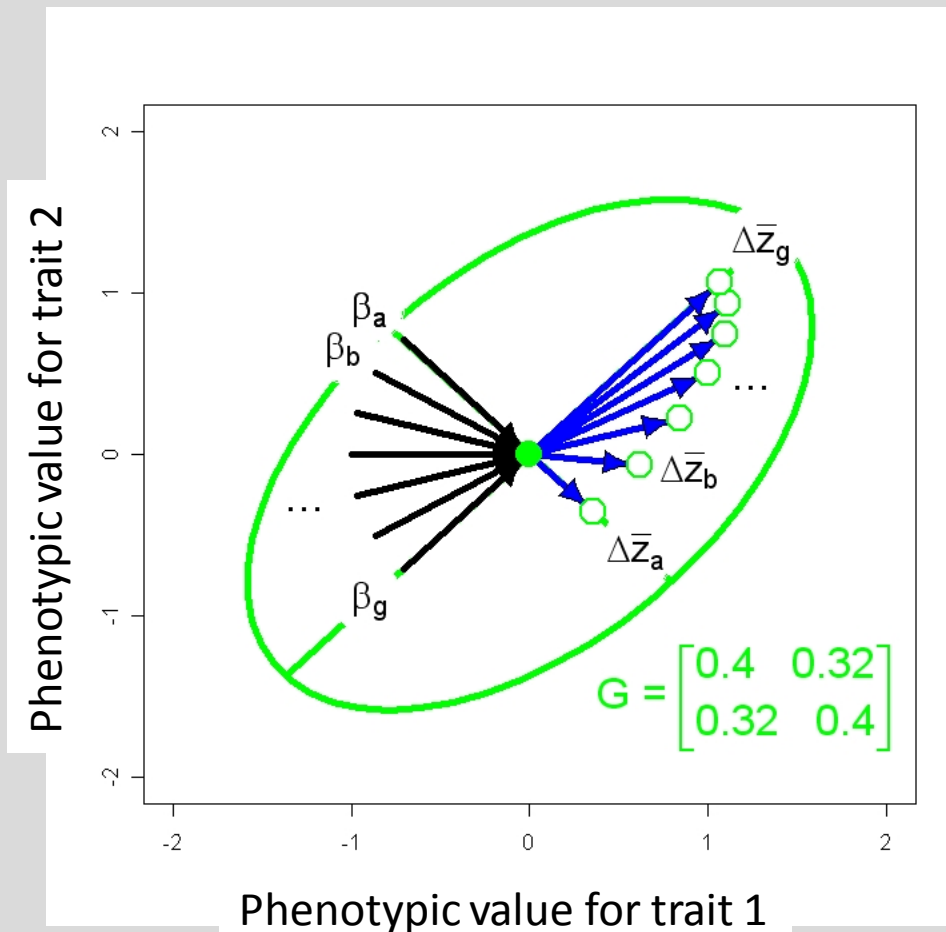
## b. Response to selection as a pool shot



[Animation 2](#)

# 5. Changing the multivariate mean with directional selection

## b. Response to selection as a pool shot, continued



[Animation 3](#)



# What have we learned?

1. The additive genetic variance-covariance matrix,  $G$ , is the key to understanding multivariate resemblance between parents and offspring.
2. Consequently, the  $G$ -matrix is also the key to modeling multivariate responses to selection.
3.  $G$  induces correlated responses to selection that may be non-intuitive.

# References

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