Linear algebra for AI & ML

$$Ax = b$$

$$\begin{bmatrix} - & a_i^T & - \end{bmatrix} \begin{bmatrix} \vdots \\ x \\ \vdots \end{bmatrix} = b_i$$

$$\min_{x \in \mathbb{R}^n} \|b - Ax\|_2^2$$

$$(b_i - a_i^T x)^2$$

weighted LS

$$\min_{x \in \mathbb{R}^n} \sum_{i=1}^{m} w_i (b_i - \tilde{a}_i^T x)^2 = \min_{x \in \mathbb{R}^n} \sum_{i=1}^{m} w_i r_i^2$$

$w_i > 0$

for $i = 1, 2, \ldots, m$

LS data fitting.

$x^T \in \mathbb{R}^p$

$\{(\cancel{x_i}, y_i)\}_{i=1}^{N}$

$f : \mathbb{R}^p \to \mathbb{R}$

$f(x) = y$

f: unknown functional relationship

To approximate this functional relationship

by $\tilde{\hat{f}}$

$\tilde{f}(x_i) \approx \hat{y}_i \quad \forall i = 1, 2, \dots, N$

choose basis f's $f_1, f_2, \dots, f_m$

$$\tilde{f} = \alpha_1 f_1 + \alpha_2 f_2 + \dots + \alpha_m f_m$$

$$\hat{y}_i = \tilde{f}(x_i)$$

$$\hat{y}_i = \alpha_1 f_1(x_i) + \alpha_2 f_2(x_i) + \dots + \alpha_m f_m(x_i)$$

$$\forall\, i = 1, 2, \dots, N$$

compare y_i with $\hat{y}_i$

$$\min_{\alpha_1, \alpha_2, \dots, \alpha_m} \sum_{i=1}^{N} (y_i - \hat{y}_i)^2$$

$x_i \in \mathbb{R}^p$, $f_i \in \mathbb{R}^p \to \mathbb{R}$

$$\begin{bmatrix} f_1(x_i) & f_2(x_i) & \dots & f_m(x_i) \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_m \end{bmatrix} = \hat{y}_i$$