Linear Algebra for AI & ML
Assignment 1

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Given :- $\text{avg}(x) = \frac{1}{n} 1_n^T x$

$$\text{std}(x) = \frac{\| x - \text{avg}(x) 1_n \|_2}{\sqrt{n}}$$

(a) $\text{avg}(\alpha x + \beta 1_n)$

$$= \frac{1}{n} 1_n^T (\alpha x + \beta 1_n)$$

$$= \alpha \left(\frac{1_n^T x}{n} \right) + \beta \left(\frac{\overset{n}{\cancel{1_n^T 1_n}}}{\cancel{n}} \right) \quad (\text{As } 1_n^T 1_n = n)$$

$\boxed{\therefore \text{avg}(\alpha x + \beta 1_n) = \alpha \, \text{avg}(x) + \beta}$ (Proved)

(b) $\text{std}(\alpha x + \beta 1_n)$

$$= \frac{\| \alpha x + \beta 1_n - \text{avg}(\alpha x + \beta 1_n) 1_n \|_2}{\sqrt{n}}$$

$$= \frac{\| \alpha x + \cancel{\beta 1_n} - (\alpha \, \text{avg}(x) + \cancel{\beta}) 1_n \|_2}{\sqrt{n}}$$

$$= \frac{\alpha \| x - \text{avg}(x) \cdot 1_n \|_2}{\sqrt{n}} = \boxed{\alpha \, \text{std}(x)} \text{ (Proved)}$$

(c) $$\left(\frac{\text{std}(x)}{a} \right)^2 = \frac{\| x - \text{avg}(x) 1_n \|_2^2}{n a^2}$$

$$= \frac{\sum_{i=1}^{n} (x_i - \text{avg}(x))^2}{n a^2}.$$

WLOG, assume that last 'K' entries in x are such that $|x_i - \text{avg}(x)| \geq a$

$$\therefore \frac{(\text{std}(x))^2}{a^2} = \frac{\sum_{i=1}^{n} (x_i - \text{avg}(x))^2}{n a^2} \geq \sum_{i=n-K}^{n} (x_i - \text{avg}(x))^2 + \frac{K}{n} \frac{\cancel{a^2}}{\cancel{a^2}}$$

$$\boxed{\left(\frac{\text{std}(x)}{a} \right)^2 \geq \frac{K}{n}} \text{ (Proved)}$$

(Q)

Given:- $\|X\|_W = \sqrt{\sum_{i=1}^{n} W_i x_i^2}$

Condition 1 :- $\|X\|_W \geq 0$ and $\|X\|_W = 0 \Leftrightarrow X = 0$:-

$\|X\|_W = \sqrt{\underbrace{\sum_{i=1}^{n} W_i x_i^2}}$ = +ve sqr root of a +ve no. ≥ 0. (Proved)

($W_i > 0$ and $x_i^2 \geq 0$) $\Rightarrow$ $\boxed{+ve}$

Also, $\|X\|_W = \sqrt{\sum_{i=1}^{n} W_i x_i^2} = 0$

$\Rightarrow \sum_{i=1}^{n} W_i x_i^2 = 0$ (where, $W_i > 0$, $x_i^2 \geq 0$)

This is only possible when $x_i = 0 \ \forall i = 1, 2, \ldots, n$

OR, $\boxed{X = 0}$ (Proved).

Condition 2 :- $\|KX\|_W = |K| \, \|X\|_W$:-

$\|KX\|_W = \sqrt{\sum_{i=1}^{n} W_i K^2 x_i^2}$

$= \sqrt{(K^2) \sum_{i=1}^{n} W_i x_i^2}$

$= |K| \sqrt{\sum_{i=1}^{n} W_i x_i^2}$

$\boxed{\therefore \|KX\|_W = |K| \, \|X\|_W}$ (Proved)

Condition 3 :- $\|X+Y\|_W \leq \|X\|_W + \|Y\|_W$:-

RHS $\geq$ LHS :- $\sqrt{\sum_{i=1}^{n} W_i x_i^2} + \sqrt{\sum_{i=1}^{n} W_i y_i^2} \geq \sqrt{\sum_{i=1}^{n} W_i (x_i + y_i)^2}$

Squaring both sides, we get

$\cancel{\sum_{i=1}^{n} W_i x_i^2} + \cancel{\sum_{i=1}^{n} W_i y_i^2} + \cancel{2} \left(\sqrt{\left(\sum_{i=1}^{n} W_i x_i^2\right)\left(\sum_{i=1}^{n} W_i y_i^2\right)} \right)$

$\geq \cancel{\sum_{i=1}^{n} W_i x_i^2} + \cancel{W_i \sum_{i=1}^{n} y_i^2}$

$+ \cancel{2} \sum_{i=1}^{n} W_i (x_i y_i)$

To prove :- $\left(\sum_{i=1}^{n} w_i x_i^2\right)\left(\sum_{i=1}^{n} w_i y_i^2\right) \geq \left(\sum_{i=1}^{n} w_i (x_i y_i)\right)^2$

Apply Cauchy-Schwarz inequality to LHS :-

$$\left(\sum_{i=1}^{n} (\underbrace{\sqrt{w_i}\, x_i}_{a_i})^2\right)\left(\sum_{i=1}^{n} (\underbrace{\sqrt{w_i}\, y_i}_{b_i})^2\right)$$

$$\geq \left(\sum_{i=1}^{n} \left(\sqrt{w_i}\, x_i \times \sqrt{w_i}\, y_i\right)\right)^2$$

$$= \left(\sum_{i=1}^{n} w_i (x_i y_i)\right)^2 \quad \boxed{\text{Proved}}$$

Since, $\|\cdot\|_W$ follows the 3 conditions for norm,

it defines a norm called the weighted norm. $\boxed{\text{Hence Proved}}$

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3. (a) $\boxed{\text{Approach 1}}$:-

Steps :-

(i) Add A and B :-

$\because A, B \in \mathbb{R}^{m \times n}$,

addition involves $m \times n$ operations.

(ii) Add x and y :-

$\because x, y \in \mathbb{R}^n$,

addition involves n operations.

(iii) Multiply $(A+B)(x+y)$:-

$A+B \in \mathbb{R}^{m \times n}$, $x+y \in \mathbb{R}^n$.

The matrix-vector multiplication involves $\overset{\text{no. of rows}}{m}(\underset{\text{products}}{n} + \underset{\text{addition}}{n-1})$

$= (2mn - m)$ oper^{n}s.

Total :- $mn + n + 2mn - m$

Approach 1 :- $\boxed{(3mn + n - m) \text{ operations}}$

(b) $\boxed{\text{Approach 2}}$:-

Steps :-

(i) Compute Ax :-

$A \in \mathbb{R}^{m \times n}$, $x \in \mathbb{R}^n$.

Matrix-vector multiplication involves $(2mn - m)$ ops.

(ii) Compute Ay :-

$A \in \mathbb{R}^{m \times n}$, $y \in \mathbb{R}^n$

Involves $(2mn - m)$ oper^{n}s.

(iii) Compute Bx :-

$B \in \mathbb{R}^{m \times n}$, $x \in \mathbb{R}^n$

Involves $(2mn - m)$ oper^{n}s

(iv) Compute By :-

$B \in \mathbb{R}^{m \times n}$, $y \in \mathbb{R}^n$

Involves $(2mn - m)$ oper^{n}s.

(v) Add A_x, A_y, B_x, B_y :-

$A_x, A_y, B_x, B_y \in \mathbb{R}^m$.

Addition involves $3 \times m$ operⁿs.

$\therefore$ Approach 2 :→ $4(2mn - m) + 3m$

$= \boxed{(8mn - m) \text{ oper}^n\text{s}}$

Now, for approach 2 to be computationally more efficient than approach 1,

$$8mn - \cancel{m} < 3mn + n - \cancel{m}$$

$$\Rightarrow \quad 5mn - n < 0$$

$$\Rightarrow \quad \boxed{n(5m - 1) < 0}$$

(Reqd. condition in 'm' and 'n')

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Let $X \in \mathbb{R}^n$ be a given vector, and let $X_K = K^{th}$ element of X.

Assume that X can be written as :-

$$X = X_s + X_a$$

where, X_s = symmetric
X_a = antisymmetric

$$\Rightarrow X_K = (X_s)_K + (X_a)_K$$

where, $(X_s)_K = K^{th}$ element of X_s
$(X_a)_K = K^{th}$ element of X_a

Since, X_s is symmetric,

$$(X_s)_K = (X_s)_{n-K+1} = a \text{ (say)}$$

Similarly, since X_a is antisymmetric,

$$(X_a)_K = -(X_a)_{n-K+1} = b \text{(say)}$$

Now, $(X)_K = (X_s)_K + (X_a)_K$

$$\Rightarrow X_K = a + b \longrightarrow (1)$$

$$X_{n-K+1} = (X_s)_{n-K+1} + (X_a)_{n-K+1}$$

$$X_{n-K+1} = a - b \longrightarrow (2)$$

Solving for 'a' and 'b' gives :-

$$a = \frac{1}{2}(X_K + X_{n-K+1}), \quad b = \frac{1}{2}(X_K - X_{n-K+1})$$

~~X can be decomposed into :-~~

$\therefore X = X_s + X_a$,
(symm.) (antisymm.)

where,

$$(X_s)_K = K^{th} \text{ element of } X_s = \frac{1}{2}(X_K + X_{n-K+1})$$

$$(X_a)_K = K^{th} \text{ element of } X_a = \frac{1}{2}(X_K - X_{n-K+1})$$

Also, the given expressions for X_s and X_a imply that this decomposition is unique.

(Proved)

6 **Given** :- $A \in \mathbb{R}^{n \times m}$.

P.T. :- Left inverse of A exists $\Leftrightarrow$ Columns of A are linearly independent.

'$\Rightarrow$' (Forward Proof) :-

Let A has a left inverse $C \in \mathbb{R}^{m \times n}$ such that :-

$$CA = I_m$$

We now prove by contradiction that A's columns are linearly independent.

Suppose, A's columns are linearly dependent, ie;

$\exists$ some $x \neq 0$ such that :-

$$Ax = 0 \qquad (x \in \mathbb{R}^m, 0 \in \mathbb{R}^n)$$

Multiply both sides by C,

$$\Rightarrow CAx = C0 = 0$$

$$\Rightarrow \underbrace{(CA)}^{I_m} x = 0$$

$$\Rightarrow \boxed{x = 0} \quad \text{(Contradiction)}$$

$\boxed{\therefore \text{ Left inverse of } A \text{ exists} \Rightarrow A\text{'s columns are linearly independent}}$ — (i)

(Proved)

'$\Leftarrow$' (Reverse Proof) :-

We first prove the following statement :-

("If columns of A are linearly independent, $A^T A$ is invertible.")

Proof by Contradiction :-

Suppose A's columns ~~of~~ are linearly independent, but $A^T A$ is not invertible.

ie; $\exists$ some $x \in \mathbb{R}^n$ s.t :-

$$x \neq 0 \text{ and } (A^T A)x = 0$$

Multiply both sides by x^T,

$$\Rightarrow x^T(A^TA)x = x\cancel{0} = 0$$

$$\Rightarrow (x^TA^T)\,Ax = 0$$

$$\Rightarrow (Ax)^T(Ax) = 0 \qquad (\text{As } (AB)^T = B^TA^T)$$

$$\Rightarrow \|Ax\|_2^2 = 0 \qquad (\text{As, } x^Tx = \|x\|_2^2)$$

By property of norm,

$$Ax = 0$$

$\boxed{\therefore \text{ Even though A's columns are linearly independent, } \exists \text{ some } x \neq 0 \text{ s.t. } Ax = 0 \text{ (Contradiction)}}$

$\boxed{\therefore \text{ If A's columns are linearly independent, } \Rightarrow (A^TA) \text{ is invertible.}}$

Since, A^TA is invertible,

we can define A^+ (Pseudoinverse) as :-

$$A^+ = (A^TA)^{-1}A^T.$$

Now, $A^+A = (A^TA)^{-1}(A^TA)$

$$\boxed{A^+A = I} \Rightarrow (\because A^+ \text{ is a left inverse of } A).$$

~~If A's columns are linearly independent,~~

$\boxed{\Rightarrow \text{ There exists a left inverse of } A}$ (Proved) $\longrightarrow$ 2

From 1 & 2, we get

$\boxed{\text{Left inverse of } A \text{ exists} \Leftrightarrow \text{A's columns are linearly independent.}}$

(Proved)

(7) Given :- $A \in \mathbb{R}^{(n+1)\times(n+1)}$ s.t. :-

$$A = \begin{bmatrix} I_n & X \\ X^T & 0 \end{bmatrix}$$ (where, $X \in \mathbb{R}^n$, $I_n \rightarrow$ identity matrix)

Let, $X = [X_1 \ X_2 \ \cdots \ X_n]^T$.

$\therefore$ A can be written as :-

$$A = \left[\begin{array}{ccccc|c} 1 & 0 & \cdots & & 0 & X_1 \\ 0 & 1 & 0 & \cdots & 0 & X_2 \\ 0 & 0 & 1 & \cdots & & \\ \vdots & & & & \vdots & \vdots \\ 0 & \cdots & & 0 & 1 & X_n \\ \hline X_1 & X_2 & X_3 & \cdots & X_n & 0 \end{array}\right]$$

Note, for 1st 'n' rows of A, the i^{th} element (1) is the 1st non-zero i^{th} row element. $(i = 1, 2, \cdots, n)$

Now, If A is invertible, it must be full-rank.

$\therefore$ Conversion of A to row-echelon form should give a matrix with some non-zero element present in $(n+1)^{th}$ row.

i.e. Apply $R_{n+1} \rightarrow R_{n+1} - X_1 R_1$:-

$$A_1 = \left[\begin{array}{ccccc|c} 1 & 0 & \cdots & & 0 & X_1 \\ 0 & 1 & 0 & \cdots & 0 & X_2 \\ & & & & & \\ 0 & \cdots & & 0 & 1 & X_n \\ \hline 0 & X_2 & X_3 & \cdots & X_n & -(X_1^2) \end{array}\right]$$

After successive row operations, $R_{n+1} \rightarrow R_{n+1} - X_i R_i$, $(i = 1, 2, \cdots, n)$

we get :-

$$A_{\text{row\_echelon}} = \left[\begin{array}{ccccc|c} 1 & 0 & \cdots & \cdots & 0 & x_1 \\ 0 & 1 & \cdots & \cdots & 0 & x_2 \\ 0 & 0 & 1 & \cdots & 0 & \vdots \\ \vdots & & & & \vdots & \vdots \\ 0 & \cdots & \cdots & 0 & 1 & x_n \\ \hline 0 & 0 & \cdots & \cdots & 0 & -\left(\sum_{i=1}^{n} x_i^2\right) \end{array}\right]$$

$\because$ A is full-rank, last element of last row of $A_{\text{row-echelon}} \neq 0$.

$\Rightarrow \quad +\left(\sum_{i=1}^{n} x_i^2\right) \neq 0$

$\Rightarrow \quad \sum_{i=1}^{n} x_i^2 = \underline{\underline{\|x\|_2^2 \neq 0}} \Rightarrow \underline{\underline{x \neq 0}}$.

$\therefore$ By property of norm,
Condition on x for invertibility of A :— (Ans.)
$\boxed{x \neq 0}$

Let $\quad A^{-1} = \left[\begin{array}{c|c} P & Q \\ \hline R & S \end{array}\right]$.

where, $P \in \mathbb{R}^{n \times n}$, $Q \in \mathbb{R}^{n \times 1}$, $R \in \mathbb{R}^{1 \times n}$, $S \in \mathbb{R}^{1 \times 1}$.

We have, $\quad A^{-1} A = I_{n+1}$

$$\Rightarrow \left[\begin{array}{c|c} P & Q \\ \hline R & S \end{array}\right] \left[\begin{array}{c|c} I_n & x \\ \hline x^T & 0 \end{array}\right] = I_{n+1} = \left[\begin{array}{c|c} I_n & 0 \\ \hline 0 & 1 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{c|c} PI_n + Qx^T & Px \\ \hline RI_n + Sx^T & Rx \end{array}\right] = \left[\begin{array}{c|c} I_n & 0 \\ \hline 0 & 1 \end{array}\right]$$

Eqⁿs :-

$\Rightarrow \quad PI_n + Qx^T = I_n \longrightarrow (1)$

$\Rightarrow \quad Px = 0 \longrightarrow (2)$

$\Rightarrow \quad RI_n + Sx^T = 0 \longrightarrow (3)$

$\Rightarrow \quad Rx = 1 \longrightarrow (4)$

Multiply both sides of (1) by X,

$\Rightarrow \cancel{PX}^{0} + Q(X^T X) = X$

Now, $X^T X \in \mathbb{R}^{1\times 1}$ (constant). $\Rightarrow$ (Invertible).

$\boxed{\therefore Q = (X^T X)^{-1} X}$

$\boxed{\therefore P = I_n - (X^T X)^{-1} X X^T}$

Multiply both sides of (3) by X,

$\Rightarrow \cancel{RX}^{1} + S(X^T X) = 0$

$\Rightarrow S(X^T X) = -1$

Since $X^T X$ is a constant, it is invertible.

$\Rightarrow \boxed{\therefore S = -(X^T X)^{-1}}$

$\boxed{\therefore R = (X^T X)^{-1} X^T}$

$$\boxed{\therefore A^{-1} = \left[\begin{array}{c|c} I_n - (X^T X)^{-1} X X^T & (X^T X)^{-1} X \\ \hline (X^T X)^{-1} X^T & -(X^T X)^{-1} \end{array}\right]}$$ (Ans.)

8 **Given :-**

$A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$ and $\hat{x} \in \mathbb{R}^n$ is the least squares soln to :-

$$Ax = b.$$

To prove :- (i) $(Ay)^T b = (Ay)^T (A\hat{x}) \quad \forall \, y \in \mathbb{R}^n.$

(ii) $$\frac{(A\hat{x})^T b}{\|A\hat{x}\|_2 \|b\|_2} = \frac{\|A\hat{x}\|_2}{\|b\|_2}$$

Proof :-

We know,

Least sqrs. soln $\hat{x}$ to the system of equations $Ax = b$ satisfies the normal equation :-

$$\boxed{(A^T A)\hat{x} = A^T b} \rightarrow (1)$$

Now, $(Ay)^T (A\hat{x}) \qquad (y \in \mathbb{R}^n)$

$\Rightarrow y^T \{(A^T A)\hat{x}\} \qquad$ (As, $(AB)^T = B^T A^T$)

From (1), we have :-

$\Rightarrow (y^T A^T) b$

$\Rightarrow (Ay)^T b. \qquad$ (As, $B^T A^T = (AB)^T$)

(i) $\boxed{\therefore (Ay)^T b = (Ay)^T (A\hat{x})} \quad \forall \, y \in \mathbb{R}^n$ (Proved)

In the prev. eqn, put $y = \hat{x}$.

$$\Rightarrow (A\hat{x})^T b = (A\hat{x})^T (A\hat{x}) = \|A\hat{x}\|_2^2$$

$$\Rightarrow \frac{(A\hat{x})^T b}{\|A\hat{x}\|_2} = \|A\hat{x}\|_2$$

Divide both sides by $\|b\|_2$,

(ii) $$\boxed{\therefore \Rightarrow \frac{(A\hat{x})^T b}{\|A\hat{x}\|_2 \|b\|_2} = \frac{\|A\hat{x}\|_2}{\|b\|_2}}$$ (Proved)

(9) **Let** :- $u_1, u_2, \ldots, u_T$ and $y_1, y_2, \ldots, y_T$ (Given)

and, $y_t \approx \hat{y}_t = \sum_{j=1}^{n} h_j u_{t-j+1}$; $t = 1, 2, \ldots, T$

Given:- $\|Ah - b\|_2^2 = (y_1 - \hat{y}_1)^2 + (y_2 - \hat{y}_2)^2 + \cdots + (y_T - \hat{y}_T)^2$

$$\Rightarrow \|Ah - b\|_2^2 = \left\| \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \vdots \\ \hat{y}_T \end{bmatrix} - \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_T \end{bmatrix} \right\|_2^2$$

Let, $Ah = \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \vdots \\ \hat{y}_T \end{bmatrix}$ and $\boxed{b = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_T \end{bmatrix}}$ (Ans.)

Now, we need to find suitable A such that :-

$Ah = \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \vdots \\ \hat{y}_T \end{bmatrix}$ (where, $\hat{y}_t = \sum_{j=1}^{n} h_j u_{t-j+1}$, $t = 1, 2, \ldots, T$)

There are 2 cases for A :-

Case 1 :- $(T > n-1)$

$$\boxed{Ah = \underbrace{\begin{bmatrix} u_1 & 0 & 0 & \cdots & 0 \\ u_2 & u_1 & & & \vdots \\ \vdots & u_2 & \ddots & & 0 \\ \vdots & \vdots & & & u_1 \\ \vdots & \vdots & & & \vdots \\ u_T & u_{T-1} & \cdots & & u_{T-n+1} \end{bmatrix}}_{A \text{ (similar to Toeplitz matrix)}}} \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_n \end{bmatrix} = \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \vdots \\ \hat{y}_T \end{bmatrix}$$

(Ans.)

Case 2 :- (If $T \leq n-1$) :-

$$Ah = \underbrace{\overbrace{\begin{bmatrix} u_1 & 0 & \cdots & & & 0 \\ u_2 & u_1 & \cdots & & & 0 \\ \vdots & \vdots & \ddots & & & \vdots \\ \underbrace{u_T \quad u_{T-1} \quad \cdots \quad u_1}_{T} & & & \underbrace{0 \quad \cdots \quad 0}_{n-T} \end{bmatrix}}^{n}}_{A} \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_n \end{bmatrix} = \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \vdots \\ \hat{y}_T \end{bmatrix}$$

A (Ans.)

(similar to Toeplitz matrix)

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Given :-

(i) N vectors $x_1, x_2, \ldots, x_N \in \mathbb{R}^n$ (input)
(ii) K vectors $z_1, z_2, \ldots, z_K \in \mathbb{R}^n$ (cluster representatives)

(a) In step 1 :-

We obtain the cluster assignment c_i for each x_i by:-

$$c_i = \underset{j}{\operatorname{argmin}} \|x_i - z_j\|_2 \qquad (j = 1, 2, \ldots, K)$$

For each of N vectors $x_1, x_2, \ldots, x_N$:

⇒ This involves comparing distances with K cluster representatives.

Also,
Distance calculation ($\|x_i - z_j\|_2$) involves :- ($x_i, z_j \in \mathbb{R}^n$)

(i) 'n' subtractions.
(ii) 'n' squaring opers.
(iii) 'n-1' additions
(iv) 1 square root opern.

Total $= n + n + n - 1 + 1 = (3n)$ operns (per distance comparison).

Total distance comparisons $= NK$.

∴ Total computational complexity for step 1 :-

$= \boxed{(3NKn) \text{ operations}}$ (Ans.)

$\approx \boxed{O(nNK)}$

(b) In step 2 :-

Suppose a cluster 'm' has 'p_m' vectors.

Updating cluster representatives involves averaging these 'p' vectors, each of shape $\in \mathbb{R}^n$.

⇒ Total additions per component $= (P_m - 1)$.
~~And~~ ∴ Total operations per cluster $(J_m) = n(P_m - 1)$.

∴ Total operations for step 2

$$= \sum_{m=1}^{K} J_m$$

$$= (n) \sum_{m=1}^{K} (P_m - 1)$$

But, we know

$$\sum_{m=1}^{L} P_m = N \quad (\text{Total input vectors})$$

$\therefore$ Total ops for step 2 $\approx (n)(N-K)$.

$\therefore$ Computational complexity for step 2

$$\approx n(N-K) \text{ ops.}$$

$$\approx \boxed{nN \text{ ops.}}$$

(Ans.)

(c) Total computations involved per iteration

$$\approx nN(3K+1) \text{ oper}^{n}\text{s.}$$

$\because$ Total no. of iterations $= 10$,

$\therefore$ Total computations involved to obtain cluster assignment

$$\approx \boxed{10nN(3K+1) \text{ oper}^{n}\text{s}}$$

(Ans.)

Question 4 :

The Google Colab link for the solution notebook to Qn. 4 of Assignment 1, involving 2D Convolution, is given below :-

https://colab.research.google.com/drive/1nRcNhrCdcNt-bpjEA4K86n3718ezF7m4#scrollTo=6ws1PjfzPxO8

Question 11 :

The Google Colab link for the solution notebook to Qn. 11 of Assignment 1, involving the K-Means Algorithm and its variations, is given below :-

https://colab.research.google.com/drive/1Vd-hIabFg3xS08odtp8Y2p3AnR-34G3Z