Linear Algebra for AI & ML

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Texts:

1) Introduction to Applied Linear Algebra — Boyd & Vandenberg
   Vectors, Matrices & Least Squares
2) Linear algebra and learning from data — G. Strang

TAs: Dipayan, Anupam, Ashraf, Atif
(CoEAI scholars)

Linear space and subspaces over the real field, linear dependence and independence, basis and dimension, linear and affine maps, inner product and norm of vectors, matrices as linear transformation, composition of linear maps and matrix multiplication, application in linear multilayer fully connected neural network, computational complexity

Systems of linear equations (square, overdetermined and underdetermined), existence and uniqueness of solutions, invertibility of matrices, left and right inverses, Gram matrix and its invertibility, QR decomposition of matrices

Least squares problem, formulation, existence and uniqueness of the solution, geometrical interpretation, least squares data fitting, least squares classification, multi-objective least squares problem, regularized least squares, constrained least squares

Sensitivity of the system of linear equations, condition number, geometrical interpretation, SVD, sensitivity of the least squares problem, generalization of the concept of condition number to nonlinear functions, relation to constructing adversarial examples(?) in NN

Revisit to SVD, dimensionality reduction and PCA, relation to linear autoencoders, Low rank approximation (LRA) and structured low rank approximations (SLRA), computation with SVD, alternating minimization approach, application of LRA in sparse NN learning, SLRA of Sylvester matrix with application to computation of approximate GCD, SLRA of Hankel matrix with application to estimation of model order of an autoregressive model, SLRA of Toeplitz matrix and application to image deblurring

Low rank matrix completion problems, why are data matrices always low rank?, low rank completion via convex optimization, nuclear norm minimization and non-convex optimization formulation, application to recommender systems,

Eigenvalues and eigenvectors of a matrix, computation using power method, application to the page ranking problem, inverse eigenvalue problem, application to the fastest mixing Markov chains on graph

Sparse solutions to the underdetermined systems of linear systems, basis pursuit algorithms, application to dictionary learning, k-svd algorithm, equivalent NN architecture to learn dictionaries

Introduction to tensors: N-way tensors, Metricization of tensors, fibers and slices, Frobenius norm of a tensor, tensor multiplication, n-mode product, Khatri-Rao product, Kronecker product, Hadamard product, tensor rank, CP-decomposition, Low rank tensor approximation, image and video compression using tensor decomposition, low rank tensor decomposition inspired sparse learning in DNNs

$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, $y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \in \mathbb{R}^2$

$\alpha, \beta, \gamma \in \mathbb{R}$

1) $x + y = y + x$ commutativity of $+$

2) $x + (y + z) = (x + y) + z$ associativity of $+$

3) $\exists\ 0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ s.t. additive identity w.r.t. $+$

$x + 0 = 0 + x = x$

4) $\forall\ x \in \mathbb{R}^2$, $\exists\ y \in \mathbb{R}^2$ additive inverse w.r.t. $+$

s.t. $x + y = y + x = 0$

5) $\alpha(x + y) = \alpha x + \alpha y$

6) $(\alpha + \beta) x = \alpha x + \beta x$

7) $(\alpha\beta) x = \alpha(\beta x)$

8)

$\mathbb{R}^n$, $\leftarrow$ vector space

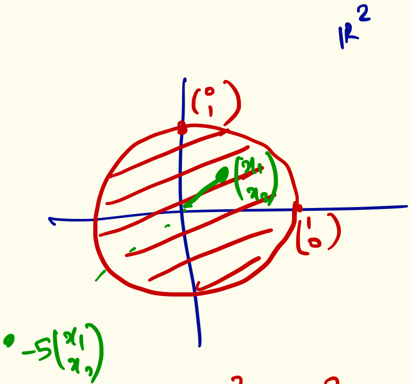
$\mathbb{R} \leftarrow$ scalars / field

Linear algebra
a geometric
approach

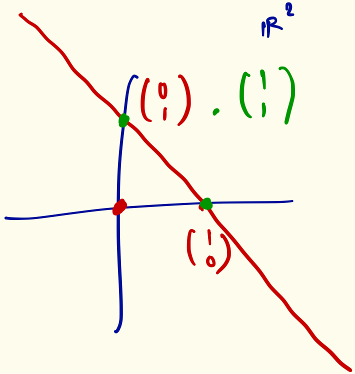
S. Kumaresan

A subspace of $\mathbb{R}^n$ is a subset of $\mathbb{R}^n$ which is a vector space in its own right.

(The subset is a subspace if it is "closed" under addition and scalar multiplication).



$$S' = \left\{ \begin{pmatrix}x_1\\x_2\end{pmatrix} \,\middle|\, x_1^2 + x_2^2 \leq 1 \right\}$$



$$\left\{ \begin{pmatrix}x_1\\x_2\end{pmatrix} \,\middle|\, x_1 + x_2 = 1 \right\}$$

Subspaces of $\mathbb{R}^2$

1) $\{0\}$

2) $\mathbb{R}^2$

3) All lines passing through origin.