Linear algebra for AI and ML (September-23)

LS data fitting problem:
$$\chi^{(1)}, \ldots, \chi^{(N)} \in \mathbb{R} \quad \text{and} \quad y^{(1)}, \ldots, y^{(N)} \in \mathbb{R}$$

$$f: \mathbb{R} \rightarrow \mathbb{R}$$

$$f(\chi) = y$$

$$f: \mathbb{R} \rightarrow \mathbb{R}$$

Gracin basis:
$$\{f_1, f_2, \dots, f_p\}$$

hinear in $\Rightarrow f = d_1f_1 + \dots + d_p f_p$
 $x_i \leq i \Rightarrow f = d_1f_2 + \dots + d_p f_p$
 $x_i \leq i \Rightarrow f = d_1f_2 + \dots + d_p f_p$
 $x_i = x_i \Rightarrow x_i = x_i \Rightarrow x_i = x_i \Rightarrow x_$

min $\sum_{i=1}^{N} (y^{(i)} - \hat{f}(x^{(i)}))^2$ = min | 112 | 2

The classification:

$$x^{(1)}, \dots, x^{(N)}$$

and

 $y^{(1)}, y^{(2)}, \dots, y^{(N)}$

inhele/

 $f: IR \rightarrow IR$

only two values

 $f(x) = y$
 $TRUE/FALSE$
 yES/NO
 $0/1$
 $-1/1$

: labels/

classification:

binary/ two clases/ f: R - {-1,1} Boolean davitication.

Find $\hat{f}: \mathbb{R} \longrightarrow \{-1, 1\}$ $\hat{y} = \hat{f}(x)$

2(1), x(2), ..., x(N) ER spam detection/ Disease detection 0,1,2,..,9 MNIST {13 {0,2/3,4,..,9} y(1),y(2),...,y(N) Class 2 : labels 1=784 (28×28) maje f(x) = -1 if image = 1 if x = 1 $\downarrow R = 1$ $\downarrow R = 1$

prediction error: i=1,2,-, N Confusion positive: positive: y= outcome

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Ntb + Ntv
error rate:
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true positive rati:

NFu

Specificity/true negative:

preciaim:

Nth + Nth

ordinary Ls fitting We first carry out the out comes. of the bacis P= : f1, f2, --, fp Choose parameters: d, d2, ..., dp f = difi + d2 f2 + ...+ Apf4 min $\sum_{i=1}^{N} \left(y^{(i)} - f(x^{(i)})\right)^2$ for any all R sign (a) = +1 signla)=-1 $\hat{f}(x) = sign(\hat{f}(x))$

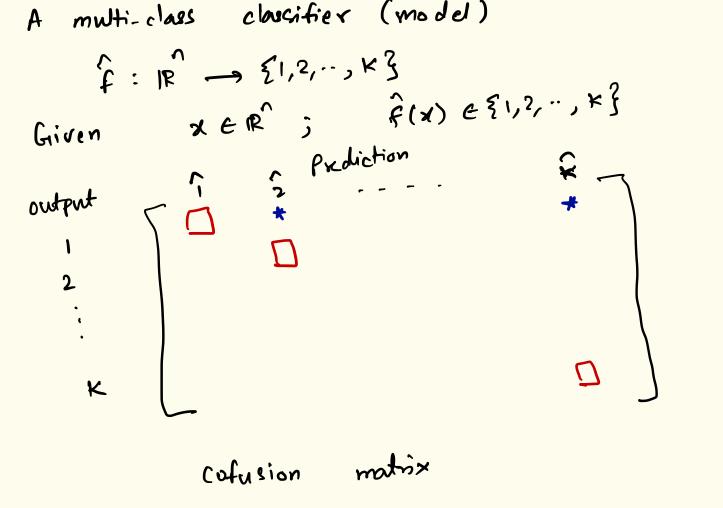
$$\hat{f}(x) = sign(x^Tp + v)$$

$$f(x_{(i)}) = 0.8$$

$$\mathbf{\hat{f}}(\mathbf{x}^{(ij)}) = + 1$$

f (x(1)) = +1

US multi-class classification: $x^{(1)}, x^{(2)}, \dots, x^{(N)}$ and $y^{(1)}, y^{(2)}, \dots, y^{(N)}$ $f: \mathbb{R}^3 \rightarrow \{1,2,\dots,K\}$ where K is



2-class classifier
$$7 \text{ K-class classifier}$$
.

$$\hat{f}(x) = \underset{k=1,2,.,k}{\operatorname{arg max}} \hat{f}_{k}(x)$$

$$k=1,2,.,k$$
Where $f_{k}(x)$ is the least squares regression model for label k against the where $[k] = [l,..,k]$
others.

$$f_{i}(x) \qquad [k] \qquad [k] \qquad [k] \sim [l]$$

$$f_{i}(x) \qquad [k] \sim [l]$$

$$f_{k}(x) \qquad [k] \sim [l]$$

$$f_{k}(x) \qquad [k] \sim [l]$$

$$F_{\kappa}(x) = -0.7 \quad \leftarrow \text{ does NoT belong to } \Sigma 1$$

$$F_{\kappa}(x) = -0.7 \quad \leftarrow \text{ does NoT belong to } \Sigma 2$$

$$F_{\kappa}(x) = 0.2 \quad \leftarrow \text{ belongs to } \Sigma 2$$

$$F_{\kappa}(x) = 0.8 \quad \leftarrow \text{ belongs to } \Sigma 2$$

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$$F_{\kappa}(x) = 0.8 \quad$$

 $\{1,2,3\} = [k] = [3]$

K = 3

$$f(x) = \frac{d_1 x_1 + d_2 x_2 + \dots}{- + d_n x_n + d_n x_n$$