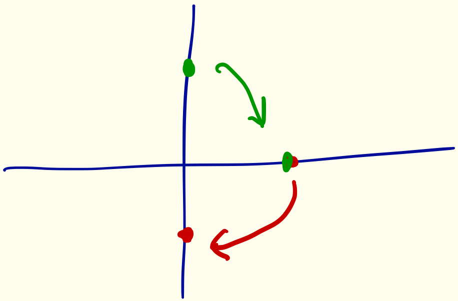
Linear algebra for AI & ML

Special class of L.T.s

$T : \mathbb{R}^n \longrightarrow \mathbb{R}^n \quad (n=2)$

$$T = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -x_1 \end{bmatrix}$$

$\mathbb{R}^2$



$a, b \in \mathbb{R}^n$

$$\cos\theta = \frac{a^T b}{\|a\|_2 \|b\|_2}$$

$$\cos\theta = \frac{\langle a,b \rangle}{\|a\|_2 \|b\|_2}$$

a & b are orthogonal if

$a^T b = 0$

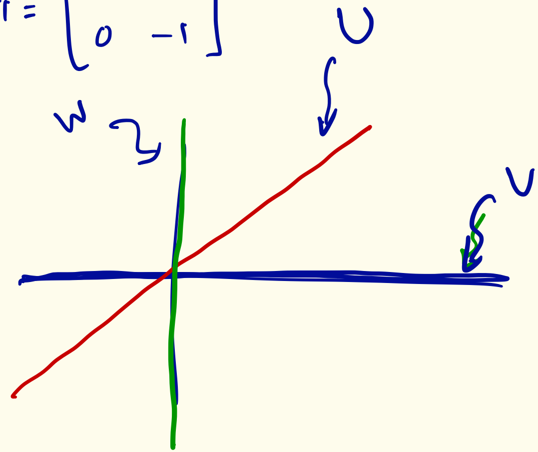
$\langle a,b \rangle = 0$

A matrix $Q \in \mathbb{R}^{n \times n}$ is an orthogonal matrix if $Q^T Q = Q Q^T = I$

$$Q = \begin{bmatrix} q_1 & q_2 & \cdots & q_n \end{bmatrix}$$

Ex:

$$T = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$$



$$V = \left\{ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \;\middle|\; x_2 = 0 \right\}$$

$$T(V) = V$$

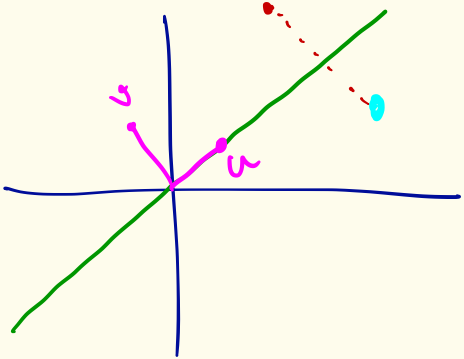
$$W = \left\{ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \;\middle|\; x_1 = 0 \right\}$$

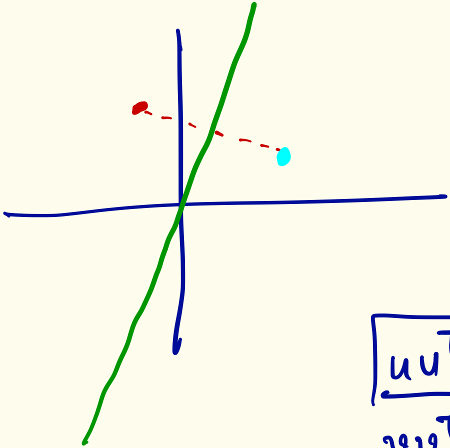
$$T(W) = W$$

$$U = \left\{ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \;\middle|\; x_1 = x_2 \right\}$$

$$T(U) \neq U$$

Ex: Reflection :





$\boxed{uu^T}$

vv^T

$T(u) = u$

$T(v) = -v$

For any $x \in \mathbb{R}^2$, $\exists$ unique α & β s.t.

If T is linear!!

$$x = \alpha u + \beta v$$

$$T(x) = T(\alpha u + \beta v) = \alpha T(u) + \beta T(v) = \alpha u - \beta v$$

$$A = uu^T$$

$$Au = (uu^T)u = u(u^T u) = u$$

$$Av = (uu^T)v = u(u^T v) = 0$$

$I - 2uu^T$