Linear algebra for AI and ML

Given a matrix $A \in \mathbb{R}^{p \times q}$ and $b \in \mathbb{R}^{p}$

find $x \in \mathbb{R}^{q}$ such that

$$Ax = b$$

$$\begin{bmatrix} | & | & & | \\ a_1 & a_2 & \cdots & a_q \\ | & | & & | \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_q \end{bmatrix}$$

<u>Existence</u>: If $b \in \text{span}\{a_1, \ldots, a_q\}$

<u>uniquness</u>: Existence and columns of A are linearly independent.

$$\text{rank}(A) = \text{rank}\,[A \,\vdots\, b]$$

$3x = 2$

<u>Left inverse</u>: Let A be a given matrix. Then a matrix X is called as a left inverse of A if

$$XA = I$$

$A \in \mathbb{R}^{p \times q}$

$X \in \mathbb{R}^{q \times p}$

Ex: $A \in \mathbb{R}^{n \times 1}$ s.t. $A \neq 0$.

$A = \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_n \end{bmatrix}$; let $A_i \neq 0$ consider

$X = \frac{1}{A_i} e_i^T$

In particular, $A = e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$

Ex: $\begin{bmatrix} e_1 & e_2 & \cdots & e_{p-1} \end{bmatrix} \in \mathbb{R}^{p \times (p-1)}$

Fact: Let A be a matrix such that left inverse of A exists. Then columns of A are linearly independent.

Consider a linear combination of columns of A

$$Ax = 0$$

Then we want to prove $x = 0$.

$$0 = C0 = C(Ax) = (CA)x = Ix = x$$