Linear algebra for AI & ML

$Ax = \lambda x$ for some $x \neq 0$, $\lambda \in \mathbb{C}$.

If $\lambda_1 \neq \lambda_2$ are eigenvalues, then

x_1 & x_2 are lin. indep.

where x_1 & x_2 are eigenvectors corresponding to eigenvalues λ_1 & λ_2.

$$A x_1 = \lambda_1 x_1$$
$$A x_2 = \lambda_2 x_2$$

$$X = \begin{bmatrix} x_1 & \cdots & x_n \end{bmatrix}$$

$\underline{A x_i = \lambda_i x_i}$ $\forall i = 1, 2, \ldots, n$

All the λ_i's are distinct.

$$AX = \begin{bmatrix} \lambda_1 x_1 & \lambda_2 x_2 & \cdots & \lambda_n x_n \end{bmatrix}_{n \times n}$$

$$AX = X \Lambda$$ where $\Lambda = \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_n \end{bmatrix}$

$$\Rightarrow A = X \Lambda X^{-1}$$

$$x_{k+1} = A x_k \qquad x_k \in \mathbb{R}^n \qquad k = 0, 1, 2, \dots$$

$$A \in \mathbb{R}^{n \times n}$$

$$x_0 \in \mathbb{R}^n$$

(initial condition)

$$x_m = A^m x_0$$

$$\downarrow$$

$$0$$

$$x_m = (X \Lambda^m X^{-1}) x_0$$

$$\Lambda^n = \begin{bmatrix} \lambda_1^m & & & \\ & \lambda_2^m & & \\ & & \ddots & \\ & & & \lambda_n^m \end{bmatrix}$$

$$\begin{cases} x_{k+1} = A x_k + b \\ x_1 = A x_0 + b \\ x_2 = A(A x_0 + b) = A^2 x_0 + Ab + b \\ x_3 = A(A^2 x_0 + Ab + b) = A^3 x_0 + \underbrace{A^2 b + Ab + b} \end{cases}$$