Linear algebra for AI and ML

- Left inverse / right inverse
- If a matrix is left & right invertible, then it is invertible.
- If A is invertible, then every left inverse is a right inverse.
- If A is invertible, inverse of A is unique.
- Use of left / right inverse to "<u>compute</u>" solution.

$$Ax = b \quad , \quad A \in \mathbb{R}^{n \times n} \quad \text{and} \quad b \in \mathbb{R}^n \quad \text{and } A \text{ invertible.}$$

$$x = A^{-1} b$$

LU decomposition:

$$A = LU$$

$$A = \begin{bmatrix} \diagdown & 0 \\ & \diagdown \end{bmatrix} \begin{bmatrix} \diagdown & \\ 0 & \diagdown \end{bmatrix}$$

$$Ax = b$$

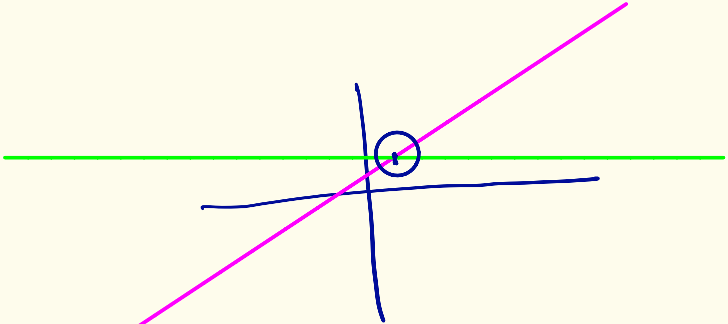
$$L\underbrace{Ux}_{y} = b$$

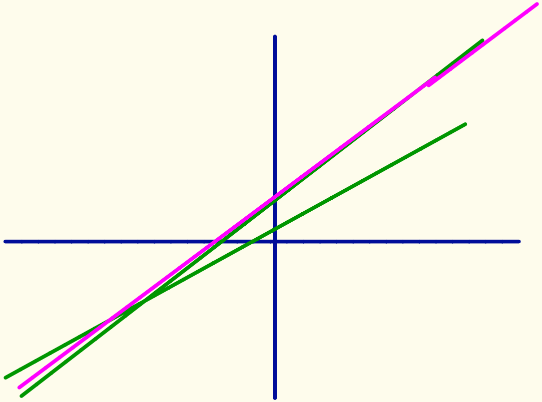
Step 1: $Ly = b$

back substitution.

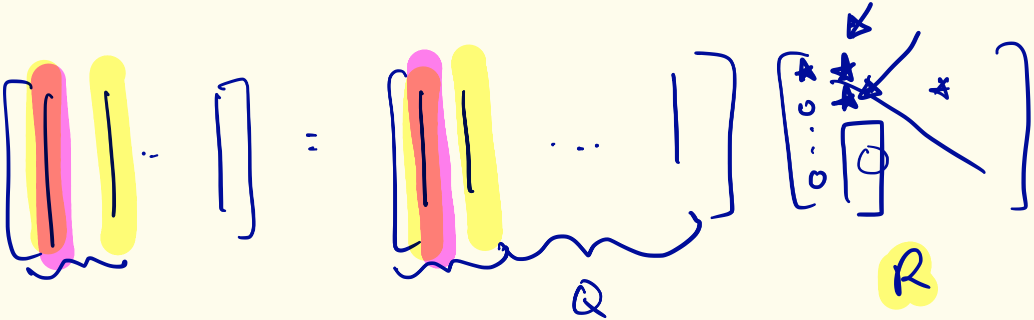
Step 2: $Ux = y$

$\mathbb{R}^2$





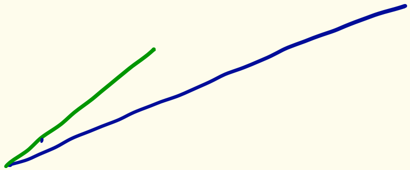
$$A = QR$$



$$Ax = b$$

$$QR\,x = b$$

$$\boxed{Rx = Q^T b}$$



Given $A \in \mathbb{R}^{m \times n}$

$B = A^T A \in \mathbb{R}^{n \times n} \to$ Gram matrix.

$B^T = B \qquad (A^T A)^T = A^T (A^T)^T = A^T A$

Given any $A \in \mathbb{R}^{m \times n}$, is $(A^T A)$ invertible??

$A^T A$ is invertible if and only if columns of A are linearly independent.

$\to$ Cols of A are lin. indep.

Let $x \in \mathbb{R}^n$ be such that

$$(A^T A) x = 0$$

$$0 = x^T 0 = x^T (A^T A) x = (x^T A^T)(Ax) = \|Ax\|_2^2$$

$$\Rightarrow Ax = 0 \quad \Rightarrow x = 0$$

For converse, assume that cols. of A are lin. dep.

to prove (A^TA) is <u>NOT</u> invertible.

$\exists\ x \neq 0$ s.t. $Ax = 0$

$$0 = A^T 0 = A^T (Ax) = (A^TA)x$$

$$(A^TA)x = 0$$

$\blacksquare$

$c = (A^TA)^{-1} A^T$ ← pseudo-inverse