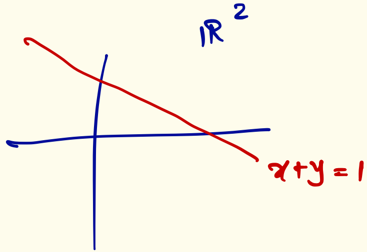
Linear Algebra for AI & ML

$\mathbb{R}^n$ — vector space, $\mathbb{R}$ - scalars

$\mathbb{R}^3$ - vector space



All subspaces of $\mathbb{R}^3$

- $\{0\} \subseteq \mathbb{R}^3$ $\quad 0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

- $\mathbb{R}^3$

- planes passing thru' origin.

- lines passing thru' origin.

$A = \{v_1, v_2, \dots, v_k\} \subseteq \mathbb{R}^n$

$$\mathcal{L}(A) = \{\alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_k v_k \mid \alpha_1, \alpha_2, \dots, \alpha_k \in \mathbb{R}\}$$

Linear combination of $v_1, \dots, v_k$

Linear span of A

$x, y \in \mathcal{L}(A)$

$x = \alpha_1 v_1 + \dots + \alpha_k v_k$

$y = \beta_1 v_1 + \dots + \beta_k v_k$

$\Rightarrow$ $x+y$

$= \sum_{i=1}^{k} (\alpha_i + \beta_i) v_i$

Linear dependence and independence.

Let $\{v_1, \dots, v_k\} \subseteq \mathbb{R}^n$. We call $v_1, \dots, v_k$ linearly dependent if there exists scalars $\alpha_1, \alpha_2, \dots, \alpha_k$ not all zero such that

$$\alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_k v_k = 0$$

Let $\alpha_j \neq 0$.

$$\alpha_j v_j = -\alpha_1 v_1 - \alpha_2 v_2 - \alpha_{j-1} v_{j-1} - \alpha_{j+1} v_{j+1} \dots - v_k v_k$$

$$v_j = \left(-\frac{\alpha_1}{\alpha_j}\right) v_1 + \left(-\frac{\alpha_2}{\alpha_j}\right) v_2 + \dots + \left(-\frac{v_k}{\alpha_j}\right) v_k$$

$$v_j = \sum_{\substack{i=1 \\ i \neq j}}^{k} \left(-\frac{\alpha_i}{\alpha_j}\right) v_i$$

Linear independence

Let $\{v_1, \dots, v_k\} \subseteq \mathbb{R}^n$. $v_1, \dots, v_k$ are linearly independent vector if

$$\alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_k v_k = 0$$

if and only if

$$\alpha_1 = \alpha_2 = \dots = \alpha_k = 0$$