Linear algebra for AI & ML

Frobenius norm:

$A \in \mathbb{R}^{m \times n}$, $\|A\|_F = \sqrt{\sum_{i,j} a_{ij}^2}$ $A = [a_{ij}]_{\substack{1 \le i \le m \\ 1 \le j \le n}}$

Matrix norm:

- all the properties of vector should be satisfied.
- Submultiplicative property: $(A, B \in \mathbb{R}^{n \times n})$

$$\|AB\|_F \le \|A\|_F \|B\|_F$$

Induced norm:

$T_A : \mathbb{R}^n \longrightarrow \mathbb{R}^m$

$$\|A\|_2 = \max_{x \neq 0} \frac{\|Ax\|_2}{\|x\|_2} = \text{maxmag}(A)$$

Az

$\|z\|_2 = 1$

$\|Az\| = \|v\|$

$\|A(\alpha z)\| = \alpha Az = \|\alpha v\| = |\alpha| \, \|v\|$

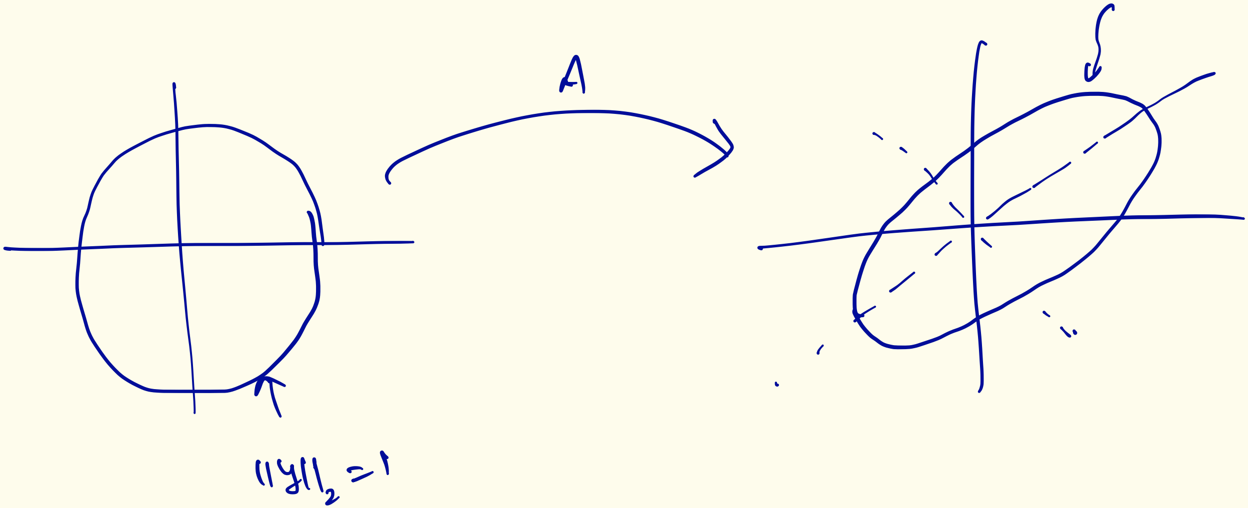
$$\|A\|_2 = \max_{x \neq 0} \frac{\|Ax\|_2}{\|x\|_2}$$

$$= \max_{x \neq 0} \left\| A \frac{x}{\|x\|_2} \right\|_2$$

$$= \max_{\|y\|_2 = 1} \|Ay\|_2$$

$$\text{minmag}(A) = \min_{y \neq 0} \|Ay\|_2$$

$$= \{Ay : \|y\|_2 = 1\}$$



$$\text{minmag}(A) = 0 \iff A \text{ has non-trivial null space.}$$

$$\exists\, x \neq 0 \ni : \quad Ax = 0$$

$$\Rightarrow \quad \frac{1}{\|x\|_2} Ax = 0 \Rightarrow A\left(\frac{x}{\|x\|_2}\right) = 0$$

$$\Rightarrow Ay = 0 \quad \text{for } y = \frac{x}{\|x\|_2}$$