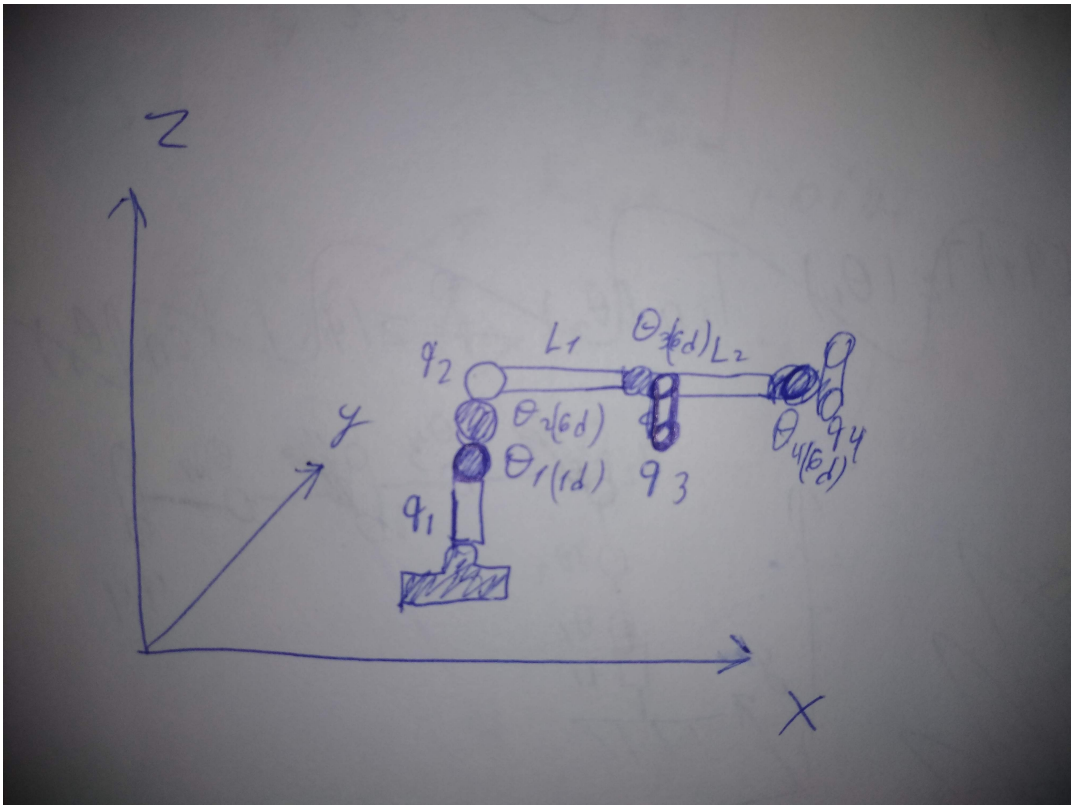
Stiffness model for Tripteron robot

1 Virtual Joints model

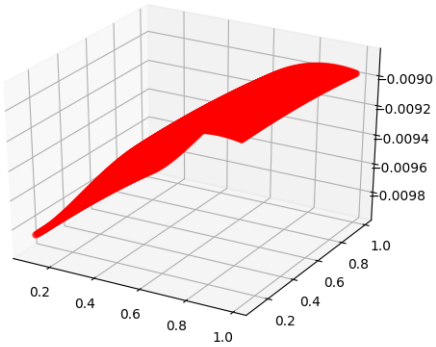
- Consider each of three legs separately. The model for them are same the only difference is axis of rotations and translations of links and joints.
  Model scheme:



- Inverse kinematics for one leg:
  - z coordinate defined by a prismatic joint position q_1 = z
  - q_2 and q_3 can be found as inverse kinematics for planar 2d manipulator:

$$q_3 = \arccos(\frac{x^2+y^2-L_1^2-L_2^2}{2L_1L_2})$$
$$q2 = \arctan(\frac{y}{x}) - \arctan(\frac{L_2\sin(q_3)}{L_1+L_2\cos(q_3)})$$

- For every leg there are 4 virtual joints. One is 1d (after active prismatic joint) and 3 6d (after each link).
- So the direct kinematics for this robot can be written as follows: $T = T_z(q_1)T_z(\theta_1)T_{6d}(\theta_2)R_z(q_2)T_x(L_1)T_{6d}(\theta_3)R_z(q_3)T_x(L_2)T_{6d}(\theta_4)R_z(q_4)$
- After that we construct K_θ matrix base on arbitrarily chosen values of stiffness properties of links and active joints
- Then build deflection map for the robot by measurement end-effector displacement in some set of points x, y (z chose as constant since end-effector move on one plane). X and Y were taken in range from 0.1 to 1 with step 0.01:
  1. For each point compute inverse kinematics of each leg assuming that virtual joints positions are 0 (since they are small)
  2. Compute Jacobians by virtual joints positions J_θ and passive joints J_q for each leg
  3. Compute stiffness matrix K for each leg
  4. Compute stiffness matrix K_C of whole robot by taking sum of stiffness of all three legs
  5. Compute end-effector displacement: $\Delta t = K_C^{-1}W$
- Deflection map on x axis plot when force applied on x (dependent on x, y position of end-effector):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0.0092
0.0094
0.0096
0.0098
0.2
0.4
0.6
0.8
1.0
0.2
0.4
0.6
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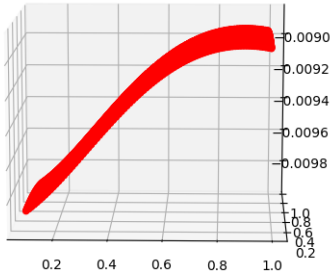
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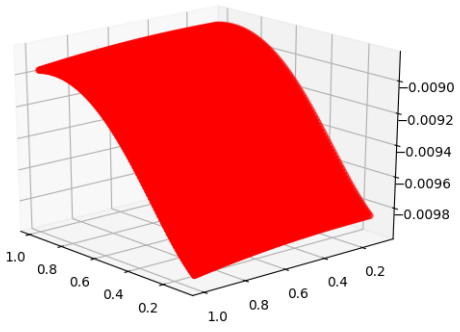
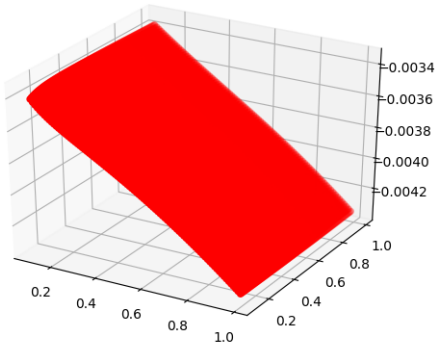
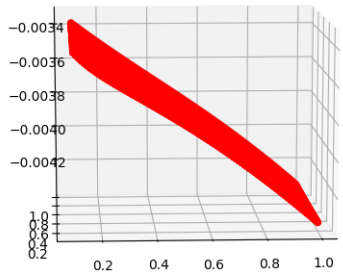
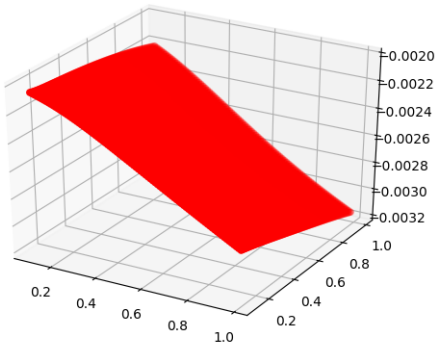


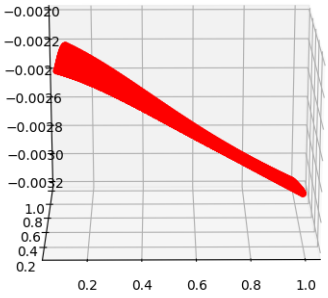
- Deflection map by y axis plot when force applied on y (dependent on x, y position of end-effector):





- Deflection map by z axis plot when force applied on z (dependent on x, y position of end-effector):





2 Github link:

https://github.com/jenamax/Robotics$_S ystems/tree/master/Homework$1