Stiffness model for Tripteron robot

1 Virtual Joints model

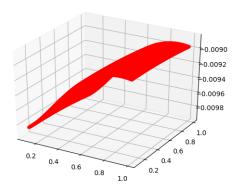
- Consider each of three legs separately. The model for them are same the only difference is axis of rotations and translations of links and joints.
- Inverse kinematics for one leg:
 - z coordinate defined by a prismatic joint position $q_1 = z$
 - $-q_2$ and q_3 can be found as inverse kinematics for planar 2d manipulator:

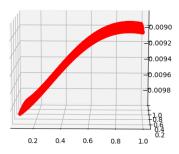
$$q_3 = \arccos(\frac{x^2 + y^2 - L_1^2 - L_2^2}{2L_1 L_2})$$

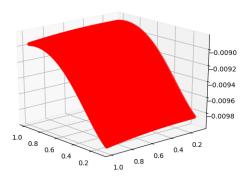
$$q_2 = \arctan(\frac{y}{x}) - \arctan(\frac{L_2 \sin(q_3)}{L_1 + L_2 \cos(q_3)})$$

- For every leg there are 4 virtual joints. One is 1d (after active prismatic joint) and 3 6d (after each link).
- So the direct kinematics for this robot can be written as follows: $T = T_z(q_1)T_z(\theta_1)T_{6d}(\theta_2)R_z(q_2)T_x(L_1)T_{6d}(\theta_3)R_z(q_3)T_x(L_2)T_{6d}(\theta_4)R_z(q_4)$
- After that we construct K_{θ} matrix base on arbitrarily chosen values of stiffness properties of links and active joints
- Then build deflection map for the robot by measurement end-effector displacement in some set of points x, y (z chose as constant since end-effector move on one plane). X and Y were taken in range from 0.1 to 1 with step 0.01:
 - 1. For each point compute inverse kinematics of each leg assuming that virtual joints positions are 0 (since they are small)
 - 2. Compute Jacobians by virtual joints positions J_{θ} and passive joints J_{q} for each leg
 - 3. Compute stiffness matrix K for each leg
 - 4. Compute stiffness matrix K_C of whole robot by taking sum of stiffness of all three legs

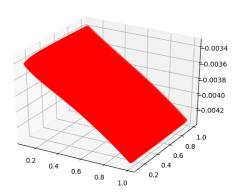
- 5. Compute end-effector displacement: $\Delta t = K_C^{-1} W$
- \bullet Deflection map on x axis plot when force applied on x (dependent on x, y position of end-effector):

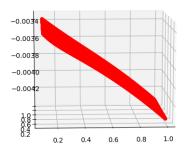




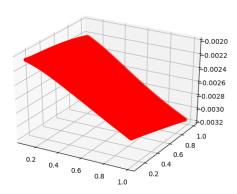


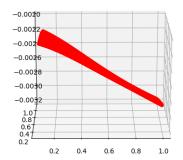
• Deflection map by y axis plot when force applied on y (dependent on x, y position of end-effector):





• Deflection map by z axis plot when force applied on z (dependent on x, y position of end-effector):





2 Github link:

 ${\rm https://github.com/jenamax/Robotics}_S y stems/tree/master/Homework 1$