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Uncertainty relations based on state-dependent norm of commutator

Referee report

In the paper, two uncertainty relations are proposed, based on two generalizations of the Böttcher-Wenzel (BW) inequality. The BW inequality holds for the Frobenius norm of the commutator of any pair of operators and the generalization deals with a state-dependent version of this norm. The new uncertainty relations are compared to the known uncertainty relations (the Robertson, Schrödinger, and the Luo-Park uncertainty relations), averaging over all pairs of observables on a qubit system, and over all pairs of mutually unbiased observables on a qudit system. It is proved that the two new relations outperform the both Robertson and Schrödinger bounds. The Luo-Park bound is better than both new bounds in the qubit case, but the second of the new bounds outperforms the Luo-Park bound in the case of mutually unbiased observables.

The first of the generalized BW inequalities was proved in a previous paper of the authors' [62], where the second inequality is conjectured. This second inequality is shown to be tight, but there is no proof that it actually holds, the authors claim "numerical evidence" for the inequality.

These inequalities and the corresponding uncertainty relations, especially the conjectured one, may well be worth further investigation. The present paper does contain some results that would support such a study. However, I do not think that this paper brings enough new information. In my view, it can be seen as some example extending or supporting a part of the previous paper [62], which is not enough for a separate publication. Also the computation methods are quite standard and not particularly insightful.

Some further comments

1. p. 2: "Observing that the bounds in (7) and (8) are composed of the commutator between observables, thus, our relations are detecting a trade-off of non-commutative observables that has been unidentified by conventional uncertainty relations." I am not sure that I understand this remark, since all the uncertainty relations are based on the commutator between observables, so this is no special feature of the new bounds.

2. p. 3, paragraph 2:"...(not necessarily normalized) density matrix" - this seems strange. Such a matrix is simply a psd matrix. It might be better to work with a density matrix, and then remark that (obviously) everything holds also for any psd matrix.

3. p.3, column 1, paragraph 3: descending -> ascending

4. p. 4, paragraph under Eq. (19): ϵ_{ijk} is not defined

5. Appendix B, the title - proof of *generalized* BW inequality