H. Arai, M. Hayashi: Derivation of Standard Quantum Theory via State Discrimination

Referee report to revised version

The authors have corrected the mistake in Theorem 5, the statement is now correct. However, my main concern has not been resolved, namely the novelty and significance of their results.

The main result, Theorem 7, follows straightforwardly from the statement formulated in the present version as Corollary 9. I never meant that this statement was trivial, but I insist that it is an easy consequence of well known facts about base-normed spaces, see e.g. Chap. 1 in the book E.M. Alfsen, F.W. Shultz, State Spaces of Operator Algebras: Basic Theory, Orientations, and C*-products. It is really well known that $D_G(\rho_0, \rho_1) = \frac{1}{2}\|\rho_0 - \rho_1\|_G$, where $\|\cdot\|_G$ is the base norm with respect to the base $\mathcal{S}(G)$ (see e.g. Thm. 3.43 in Ref. [21]). If ρ is a quantum state not belonging to $\mathcal{S}(G)$, then it must be expressed in the form

$$\rho = (1+\lambda)\rho_1 - \lambda\rho_2,$$

where $\rho_1, \rho_2 \in \mathcal{S}(G) \subseteq \mathcal{S}(QT)$ are such that $\|\rho_1 - \rho_2\|_G = \|\rho_1\|_G + \|\rho_2\|_G = 2$ and $\lambda \geq 0$ (see Props. 1.25 and 1.26 in the above mentioned book). But if the condition (ii) in Corollary 9 of the present paper holds, then we also have $\|\rho_1 - \rho_2\| = 2$ for the trace norm $\|\cdot\|$, which means that ρ_1 and ρ_2 are quantum states with orthogonal supports. Hence $\lambda = 0$ and $\rho = \rho_1 \in \mathcal{S}(G)$. Note, by the way, that a similar argument can be used not only for QT but for any GPT, even for compact convex subsets in locally convex spaces, not necessarily finite dimensional.

As for the other results in the paper, the results of Thms 5 and 6 can be seen as new, but I do not think that this warrants publication. The results are obtained by straightforward computation or an easy separation argument. I am also not convinced about the significance of the results of the paper from the point of view of physics.