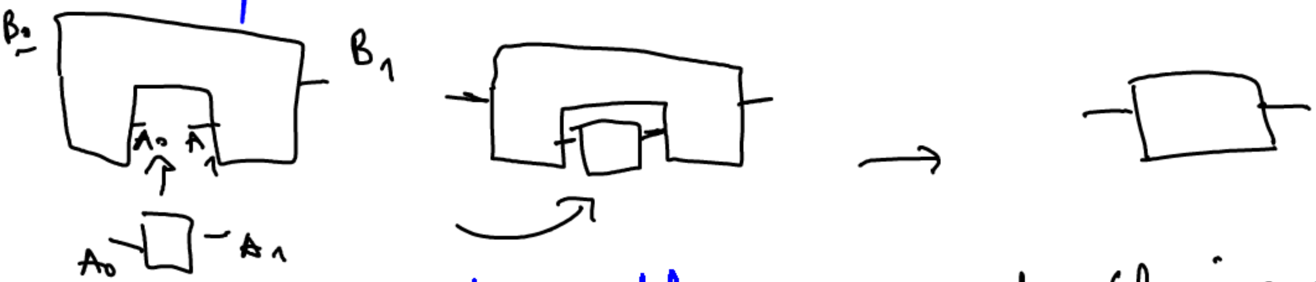
INTRO: interesting references

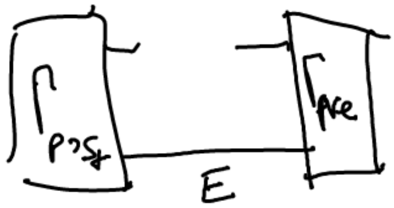
Prelim: Structure of the set of supermaps.

$\boxed{\text{It can be done better:}}$ Choi iso $\equiv$ $\underbrace{\text{wire bending}}_{\text{see ?}}$

Superchannels: $\overset{(?)}{=}$ 2-combs



1) **structure theorem**

- for Choi matrix
- for the associated map $A \to B$
- 

(As in D'Ariano et al) (semi-causal maps)

- Rewrite this with WB?

Entropies: min.entropy, conditional min entropy

$\boxed{\text{SDP and dual}}$ → relation to my base norms paper

(smoothed version — in general? what is it??

support function of the set of q. channels

Noisy superchannels (special types of superchannels)

- Random unitary

Convex combinations of unitary (isometric) superchannels:

$$\Phi \longmapsto \mathcal{U}_{post} \circ \Phi \circ \mathcal{U}_{pre}$$

unitary (isometric) conjugations on inputs and outputs

- Doubly stochastic:

both Θ and Θ^* are superchannels

condition: $\frac{d_{A_0}}{d_{A_1}} = \frac{d_{B_0}}{d_{B_1}}$

characterization
- Choi matrix
- $\Gamma_{pre}, \Gamma_{post}$

- Completely uniformity preserving (definition below)

Noisy channel (does not decrease noise)

must map max. mixed ↦ max. mixed

unitary

simulating → noisy superchannel

- must preserve the uniform ("most noisy") channel $X \mapsto (\mathrm{Tr} X)\,\textcircled{u} \to$ max. mixed

- but also it must be complete with this property:

  if Φ^{AC} is marginally uniform on A (the A-marginal is uniform)

  then $(\textcircled{u} \otimes \mathbb{1}^C)(\Phi_{AC})$ must be marginally uniform (on B) also.

- characterization: Choi; Γ post

Double stoch $\Rightarrow$ CPUP $\uparrow$
(with equal i-o dimensions)

- Completely unital channel preserving

  noisy channel = unital (as above)

  noisy superchannel: must map noisy channels to noisy (unital preserving)

  completely: $\Theta \otimes 1_C$ is $\uparrow$

- characterisation: Choi, Γ_{Θ}

Double stoch (with one equal dimensions.)?
- has one condition, but not enough
$\rightarrow$ <u>not</u> in general.

The entropy of a q-channel

now can be defined

Entropy: $f : \mathcal{L}_+^A \to \mathbb{R}$ such that

(cp maps $A_0 \to A_1$)

1, monotone under random unitary superchannels

$$f(\Phi) \le f(\Theta(\Phi))$$

what about sub-additivity?

2) additive under $\otimes$

3) $f(N^A) = \log d_{A_1}$ (uniform channel)

$$f(X \mapsto \mathrm{Tr}X \cdot |\psi\rangle\langle\psi|) = 0$$

pure state

• Extended min entropy

for channels

$$H_{min}^{ext}(A)_\Phi := H_{min}(A_1|A_0)_{J_\Phi / d_{A_0}}$$

(cond. entropy of the choi matrix density)

– is an entropy (as above)

• Extended conditional min entropy

for bipartite channels

as SDP

$$H_{min}^{ext}(B|A)_\Omega := -\log_2 \min \mathrm{Tr}\, \gamma^{AB_0}$$

$$\gamma^{AB_0} \otimes I^{B_1} \geq \omega^{AB}$$

$$\gamma^{A_0 B_0} = u^{A_0} \otimes \gamma^{B_0}$$

has a dual

Relation to base-norms??

base norm w.r. to superchannels??

↳ there are some general relations, following from my results on base-section norms (but no one will ever cite those).

Properties ... do these also follow? ??

Operational interpretation: (only if B is classical (or B_1 is classical))

this is lame even for cond. min. entropy

do it better!

Comparison of quantum channels

$$\Psi_1^A, \dots, \Psi_n^A \xrightarrow{\text{CQ}?} \Phi_1^B, \dots, \Phi_n^B$$

↳ special case of a problem for bipartite channels Ψ^{RA} Φ^{RB}

→ ordering on bipartite channels

$$\Phi^{RB} \prec_q \Psi^{RA}$$

(for the main one)

- Extension of divergences to channels (the obvious one)
  - DPI

- Characterization of $\prec_q$ (1)

comparison of $H_{\min}^{ext}$ for some special bipartite channels

compare this to my comparison paper.

- use this for channel comparison (Ψ^A_{n-1}, Ψ^A_n) and (Φ^B_{n}, Φ^B_n)

- <u>Everything</u> can be done using SDP

- Application to thermodynamics
(resource theory)