L. Lami, D. Goldwater, G. Adeso: A Post-Quantum Associative Memory

Referee report

The paper studies the limitations of an associative memory in the setting of general probabilistic theories (GPTs). More precisely, the problem is to find the smallest dimension of a GPT such that there exists a set of 2^m-states such that all subsets of cardinality N are jointly perfectly distinguishable, where m and N are given. The problem is solved for $N = 2$, where it is shown that the hypercubic GPT of dimension $m + 1$ is optimal. This is in contrast with the classical and quantum cases, where the required dimension is exponentially larger. For $N > 2$ some constructions are suggested but the problem is left open, even for $N = 3$.

General assessment

The problem addressed in the paper is well explained and has a significance in understanding the possibilities of associative memories beyond classical and quantum theories. The paper is very well written and readable, easy to understand also for people not working in GPTs or convex geometry. The solution for $N = 2$ is based on an old geometric result by Danzer and Grunbaum in [37] and I find the presented proof very appealing. For $N > 2$ some possibilities are suggested, such as restriction to pure states or the formulation of perfect distinguishability as a convex program, but it does not go far beyond well known or straightforward ideas. The concept of using an algorithm to find maximal cliques in a hypergraph may be more interesting, but the whole method crucially depends on the structure of N-tuples of perfectly distinguishable elements in a given GPT, for which little insight is provided.

What I find lacking is more examples of jointly distinguishable subsets, e.g. an example for jointly distinguishable triple for hypercubic state spaces. At present, only a nonexample is given. Can one describe all such triples? The geometric interpretation of perfectly distinguishable pairs is quite clear, but what about $N > 2$? Can one use the fact that all N-wise distinguishable subsets are in fact pairwise distinguishable? etc. I think the addition of examples would provide more insight into the problem and the paper would become more valuable as a starting point in an important investigation.

Small remarks

1. page 3, last sentence: ”that the with the”

2. page 11, notation $\bar{A}$ for the Minkowski symmetrisation is not good, since it might be confused with topological closure (especially when interior is treated)
3. page 13, proof of Thm. 14, part (iii): the sets $\frac{x+B}{2}$, $x \in X$ are not disjoint: for $x, y \in X$, $\frac{x+y}{2}$ is clearly in both $\frac{x+B}{2}$ and $\frac{y+B}{2}$. (But the interiors are, which is enough).
4. page 21, second point in the frame: ”tjese”