D. Lachman: Tensor products in the category of effect algebras

Referee report

The paper elucidates some aspects of the tensor product in the category of effect algebras and its relation to the tensor product in the category of partially ordered abelian group with order unit. It is shown that the construction of the universal group preserves the tensor product and the corresponding functor is strong monoidal. This is used to show that the tensor product of effect algebras does not preserve RDP, which was a question discussed and left open in previous works. Moreover, the colimits preserved by the tensor products are characterized.

The results of the paper give a significant insight into the structure of the tensor product in effect algebras, which is generally found rather complicated. The proofs are based on several clever ideas by the author, which are very interesting in themselves an likely to inspire further research on related subjects. The paper is generally well organized and fun to read, the ideas of the proofs are clear and easy to follow.

The only criticism I have pertains some language issues and the relatively high number of typos, especially in the introductory parts. (This becomes much better in the main part of the paper.) These issues do not decrease readability of the paper very much, but they are annoying and, in my opinion, degrade an otherwise excellent paper. I give a rather long list below, but perhaps the author should use some means of a language check. The list also gives a few suggestions where some parts might be a bit expanded to increase accessibility.

Comments, suggestions and typos

- page 1, last line: operator -> operators. Also, the sentence starting with "Whereas..." would be better formulated as "Whereas the spectrum of a projection is a subset of $\{0, 1\}$, the spectrum of an effect is a subset of the real interval $[0, 1]$."
- page 2, line 4: measurement -> measurements
- page 2, line 8: "yield attention" perhaps replace by "gained attention"
- page 2, line 9: Frobenious -> Frobenius
- page 2, line 15: "...concept of tensor product..." -> "concept of a tensor product"
- page 2, line 16: smalles -> smallest.
- page 2, end of second paragraph: "establishes the whole existence..." maybe replace "whole" by "mere", or just remove it
- page 2, line 26: "best sound" -> perhaps "most sound", or "soundest"
- page 2, last paragraph: "the tensor product of effect algebras does not preserve ordinal sums" This may be some folklore, but it would be better to give a reference, or sketch a counterexample.
- page 2, line 4 from below: "not closed monoidal" -> "not a closed monoidal"
- page 3, line 8: sentence starting with "So far,..." perhaps better rewrite as "So far, thanks to various researchers, we have..." (or maybe skip the "thanks...").

- page 3, first sentence of the last paragraph: whenever -> whether
- page 4, last sentence of Sec. 1: Something seems to be missing, better reformulate the sentence.
- page 4, Definition 2.1: Where -> Here
- page 4, line 8 from below: omomorphism -> homomorphism. Also algebra -> algebras
- page 5, Lemma 2.3: $\ominus$ is not defined
- page 5, Definition 2.5: "its tensor product" -> "their tensor product". Also "satisfying following" -> "satisfying the following"
- page 5, Theorem 2.6: "yields tensor product" this does not seem to be a good word here. Perhaps "has a tensor product", or better "...the tensor product exists for each pair..."
- page 5, second line below Theorem 2.6: "as a tensor product" -> "as the tensor product". Also the next sentence: "...the tensor product behaves functorially" (or "is functorial")
- page 5, line below Eq. (2.2): "implies..." -> "implies that...". Also next sentence: "for fixed" -> "for a fixed"
- page 6, Definition 2.7: add "consisting" between the tuplet and "of a category $\mathcal{C}$..."
- page 6, line 7 from below: po-groups -> po-group
- page 6, line 5 from below: $x \leq n \cdot u$
- page 7, second diagram of (2.6): $A \otimes_{\mathcal{C}} F(I)$ -> $F(A \otimes_{\mathcal{C}} I)$
- page 7, Lemma 2.13: "A category $\mathcal{C}$..." -> "The category $\mathcal{E}$..."
- page 10, line 16 of the proof of Theorem 3.4: $d \downarrow \mathcal{D}^{\bullet}$ -> $d \downarrow \mathcal{D}$
- page 10, line above the last displayed equation: Those -> Thus
- page 11, beginning of Section 4: "well known adjunction..." the construction is indeed well known, but it would be perhaps better either give a reference where it is formulated as an adjunction, or elaborate on this a bit more
- page 11, line above Lemma 4.1: "is an epimorphism"
- page 12, Definition 4.5: "its tensor product" -> their
- page 13, proof of Corollary 4.8: "very same" -> "the same" is appropriate here
- page 13, Eq. (4.3) in the first brackets $[,]$, replace u by v
- page 14, line below Eq. (4.4): it is better to write "As $\Gamma(g)$ is just a restriction of g..."
- page 14, line 5 in Section 5: $Gr(E)$ -> $Gr(F)$
- page 16, line line 6 above Proposition 6.1: "the elements of form..." -> "elements of the form..."

- page 17: ”To achieve the naturality...”-> ”To prove naturality...” would be better
- page 18, paragraph above Theorem 6.2: ”we subtle” -> perhaps ”we refine”(?)
- page 19, Theorem 6.5: ”Where..”-> ”Here...”