E.A. Carlen, A. Vershynina: Recovery map stability for the Data Processing Inequality

Referee report

The authors provide a sharpening of the data processing inequality (DPI) for the relative entropy of quantum states with respect to a restriction to a subalgebra. It has been known already since the results of Petz [27] that equality in DPI implies that both states are recovered by the Petz recovery map. This is an important result, for example it can be used to characterize Markov chains. However, the approximate version was missing until the break-through paper by Fawzi and Renner [13]. Since then, a number of DPI lower bounds has been obtained in the literature, but the bound in the present paper has the advantage that it is expressed directly in terms of the Petz recovery map and can be made independent on one of the states. Besides, the proof of this result is also remarkably simple and elegant.

In the second part of the paper, the authors discuss the case of equality in the DPI. Some conditions on this equality are provided, but the problem is that all these conditions were already obtained in the literature and the relevant papers are not cited. The equality conditions were studied in a number of works, also in the more general case of a quantum channel (not just the trace-preserving conditional expectation) and in infinite dimensions. As mentioned above, it was obtained already by Petz that equality is equivalent to the so-called reversibility (or sufficiency) of the channel with respect to a set of states. A long list of equivalent conditions for reversibility is known at present, including equality in DPI for a large class of quasi-entropies (or quantum f-divergences) and a description of the structure of states. It seems that the authors are not familiar with (or simply ignored?) these papers. In particular, Theorem 1.9 is well known, as well as Corollary 1.10 and further remarks on f-relative quasi entropies.

In conclusion, the lower bound on DPI clearly deserves publication, although a more detailed comparison to other known bounds could be added. But the second part is not acceptable and cannot be published in the present form.I would strongly suggest that the authors study the papers arXiv:1008.2529 and arXiv:1604.03089, and references therein, before submitting a revision. The result of Theorem 1.9 and a lot of results on conditional expectations in finite dimensions can be also found in M. M. Wolf. Quantum Channels and Operations - Guided Tour. Online lecture notes, 2012.