Yuan Li, Shuhui Gao, Cong Zhao, Nan Ma: On spectra of some completely positive maps

Referee report

This paper concerns normal completely positive maps Φ on $\mathcal{B}(\mathcal{H})$ such that the Kraus operators $A_1, A_2, \ldots$ satisfy both $\sum_i A_i^* A_i$ and $\sum_i A_i A_i^*$. It is proved that such a map Φ preserves the subspace $\mathcal{K}(\mathcal{H})$ of compact operators and the restriction to this subspace has the same spectrum as the original map Φ. Further, peripheral eigenvalues and the corresponding eigenvectors for the restricted map are characterized. This is based on the previous results obtained in references [7] and [8] on the fixed points of the restricted map. Furthermore, the spectra of two examples of such maps related to the unilateral shift are described.

Overall assesment

The results of this paper are mathematically correct and sound, the proofs are clearly written and readable. The characterization of spectra of normal completely positive maps is and important problem with many applications. However, the results of the present paper do not seem to be strong enough for publication. The proofs follow by standard manipulations, basically from those in Reference [7]. It is also not clarified why the properties of the restriction to compact operators are important or interesting.

Some further comments

1. p. 2, line 6 from below: "is the commutants" (typo)
2. p. 2 "...the Poisson boundary (the set of all fixed points of compact operators)" it seems that this should be "the set of compact operators in the Poisson boundary..."
3. p. 11 lines 7 and 8 from below: why is the dagger here?
4. The notation $\mathbb{T}$ in Example 1 is a bit confusing, since this notation is commonly used for the unit circle group: $\mathbb{T} = \{z \in \mathbb{C} \ : \ |z| = 1\}$, which is somewhat complementary to the present notation.
5. What is the meaning or purpose of the two final Remarks 2 and 3? Remark 2 concerns unitary operations and I doubt it is new. As for Remark 3, it also does not seem to be new, see E. Thorp, Projections onto the subspace of compact operators, Pacific J. Math. 10 (1960), 693-696 (the proof there even seems somewhat similar).