

S. Plosker, C. Ramsey, Bistochastic operators and quantum random variables

Referee report

I regret to say that I am still not convinced about the significance of this paper. The authors made some improvements and added some lines to the introduction, but only one sentence referring to the motivation, pointing to Ref. [11]. This is certainly not enough to convince any reader to go on.

This is not to say that the paper does not contain any results. The proposed L_1 space and its norm seems very reasonable, but I would expect some more thorough and systematic analysis of this norm and its properties. For example, the fact that for a positive f the L_1 -norm is just the norm of the integral is repeatedly used but not even stated (though it is probably easy to see). I would be interested in a discussion of properties that are similar and those that are different from the classical theory, but there is little of it. Instead, the authors discuss (and dismiss) some other proposals for a norm that do not work, which is only confusing and distracting and could be placed into some side remark.

As the authors state, the driving force of the work is the goal to extend some results on majorization into this context. But in the full generality, the majorization (via bistochastic operators) is merely defined here. The main result is proved only in a special case, where the integral is defined "entrywise" with respect to a (classical) measure, and the operator is a classical bistochastic operator acting on the "classical" part. A characterization of majorization in this setting is obtained by combining and adjusting classical results of [2] and arguments of [18]. This is done in a nice way, but as I see it, the above introduced general theory of L_1 spaces with respect to a POVM is not really needed in the statement nor in the proof.

This result is of some interest, but somewhat lost within the general framework. In all, I do not think a reader would gain much information or insight from this paper.