# A note on the cone $L_p(\mathcal{M})^+$

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**Lemma 1.** *Let $\varphi \in \mathcal{M}_*^+$ be faithful. For any $0 < p < \infty$, $h_\varphi^{\frac{1}{2p}} \mathcal{M}^+ h_\varphi^{\frac{1}{2p}}$ is dense in $L_p(\mathcal{M})^+$, in the (quasi)-norm $\|\cdot\|_p$.*

*Proof.* By [**?** , Lemma 1.1], $\mathcal{M} h_\varphi^{\frac{1}{2p}}$ is dense in $L_{2p}(\mathcal{M})$ for any $0 < p < \infty$. Let $y \in L_p(\mathcal{M})^+$, then $y^{\frac{1}{2}} \in L_{2p}(\mathcal{M})$, hence there is a sequence $a_n \in \mathcal{M}$ such that $\|a_n h_\varphi^{\frac{1}{2p}} - y^{\frac{1}{2}}\|_{2p} \to 0$. Then also

$$\|h_\varphi^{\frac{1}{2p}} a_n^* - y^{\frac{1}{2}}\|_p = \|(a_n h_\varphi^{\frac{1}{2p}} - y^{\frac{1}{2}})^*\|_p = \|a_n h_\varphi^{\frac{1}{2p}} - y^{\frac{1}{2}}\|_p \to 0$$

and

$$\|h_\varphi^{\frac{1}{2p}} a_n^* a_n h_\varphi^{\frac{1}{2p}} - y\|_p = \|(h_\varphi^{\frac{1}{2p}} a_n^* - y^{\frac{1}{2}}) a_n h_\varphi^{\frac{1}{2p}} + y^{\frac{1}{2}}(a_n h_\varphi^{\frac{1}{2p}} - y^{\frac{1}{2}})\|_p$$

Since $\|\cdot\|_p$ is a (quasi)-norm, the above expression goes to 0 by Hölder.

□