H. Arai, M. Hayashi: Derivation of Standard Quantum Theory via State Discrimination

Referee report

This paper claims to give a characterization of quantum theory among GPTs (with certain dimension condition) by an operational condition, namely in terms of error probabilities in state discrimination problems. This is done in the following way: under the dimension condition, the state space of the theory in question can be mapped into the set of quantum states by an affine isomorphism, it is shown that the image is a strict subset (and hence the GPT is different from the quantum theory) if and only if the error probabilities in some state discrimination problem are smaller than the quantum ones. I have the following comments:

1. Inequality Eq. (7) stated in Theorem 5 is easily seen to be wrong unless $p = 1/2$, in fact, the equality denoted as (d) in the proof of this theorem does not hold. To find a counterexample, consider the state discrimination problem for two quantum states ρ_0, ρ_1, with $p \neq 1/2$. Let $P_\pm$ denote the projection onto the support of $(p\rho_0 - (1-p)\rho_1)_\pm$ and put $M_0 = sP_+ + rP_-$, with $0 \leq r < s \leq 1$. Then $\{M_0, I - M_0\}$ is a valid quantum measurement and it is easy to compute that

   $$\mathrm{Err}(\rho_0; \rho_1; p; M) = \frac{1}{2} - \frac{1}{2}(s-r)\|p\rho_0 - (1-p)\rho_1\|_1 - \frac{1}{2}(2p-1)(s+r-1).$$

   We clearly may choose $s + r > 1$ if $p > 1/2$ and $s + r < 1$ if $p < 1/2$, to violate the inequality Eq.(7). Nevertheless, Theorem 7 in fact holds (note that only the case $p = 1/2$ in Theorem 5 is used in the proof).

2. The inequality Eq.(2) holds in any GPT, with the trace norm replaced by the base norm corresponding to the state space. The main result of the paper just gives the fact that the base norm increases if it is computed with respect to a smaller base, this is well known and easy to see.

3. An operational characterization of quantum theory would be an intrinsic property of states and measurements of a GPT that singles out the quantum theory. This is not given in the present paper. The main result is just an easy and well-known step away from stating that a state space is not quantum if and only if it is not affinely isomorphic to a set of quantum states, which is trivial.

4. There are a lot of misspellings, incomplete or grammatically incorrect sentences, etc. I do not point them out, since it would not improve the paper in any significant way.