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Print decision

Revise and Resubmit

The journal has requested that this manuscript be revised and resubmitted.

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Publication Decision from *Quantum*

"Spectral resolutions in effect algebras"

Decision made on February 23rd, 2022

Editorial board's determination

Revise and resubmit

Comments from the editor

Dear Anna and Sylvia,

Thank you for your recent submission “Spectral resolutions in effect algebras” to Quantum. I have received one pretty thorough review, which you will find attached. The referee’s recommendation, which I second, is that this paper really belongs in a mathematics journal (Alg. Univ. comes to mind). The paper certainly makes a real contribution to the pure theory of effect algebras, but how significant a contribution this is to the foundations of quantum theory is less clear. (In particular, to be publishable in Quantum it would be helpful if the paper contained some examples besides the “usual suspects”, or else to establish that spectrality leaves little room for exotic examples.)

Neither the referee nor I wish to reject the paper outright, since it is a serious piece of mathematics that could perhaps be (extensively) reworked to be suitable for Quantum. But this might be quite a substantial undertaking, so, again, my advice would be to consider submitting the paper to a mathematics journal instead. If you do wish to revise the paper for Quantum, please make every effort to address the referee’s comments, which I think you will find helpful in any case.

? Help

(Ordinarily, we solicit two reports for each submitted paper, but as I doubt that a second review wo recommendation, I have thought it best to communicate this to you now.)

With best regards,
Alex

Alexander Wilce
Editor at Quantum

If you would like to resubmit the manuscript itself, please upload the **revised version** and a **response letter file** to the points raised by the referees through Scholastica. It is in your own interest to ensure that the Referees and Editor can easily **retrace your revisions** (e.g., by attaching a detailed list of changes or a version of the manuscript with the changes highlighted). There is no need to update the arXiv version at this stage. If you have **confidential information** for the editor, please upload it as a **separate file**.

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Reviewer 1

Open response questions

Summary: what are the main questions posed by the manuscript and how does it answer them?

The authors adapt the previously studied notions of compression bases, the projection cover property, and b-comparibility, in order to give a definition of a spectral effect algebra. They show that in such an effect algebra any element has a spectral resolution of projections, which generalises spectral resolutions in more familiar objects like operator algebras or order unit spaces. They prove a couple of properties that one would expect such a generalisation to have, and show that in effect algebras with more structure (being either divisable or convex), their definition recovers previously used definitions.

What is your assessment of the paper? If you recommend acceptance, make a case that this work does indeed make a significant contribution to scholarship.

The paper is written in a clear mathematical style, and the results are sophisticated and non-trivial. I can however not recommend acceptance in Quantum, as I feel the paper is too technical and feels somewhat incremental in an already specialised field. I think the paper is in a good enough state to probably be accepted as-is in a more specialised mathematics-oriented venue. If the authors are however adamant to have this published in Quantum, I have below a list of changes, which might make it more suitable (but would probably require a lot of work to implement).

To what extent have you checked the technical correctness of the paper?

I have not checked the proofs in detail, but what I've seen all makes sense.

Comment on the presentation of the paper. Is it well written? Are the main results clearly laid out? Does the manuscript clearly describe assumptions and limitations? Is the literature review adequate?

See below

If the submission includes numerical or physical experiments, does it provide sufficient details such that they could be reproduced by readers? This includes for example source code, documentation, experimental data, experimental setup specifications, etc.

N/A

Suggested changes, corrections, and general comments.

There are three main changes I'd like to see, which all probably require new results to be proven or at least more examples to be found. Then I follow this with a list of minor remarks.

- The main definition, a spectral effect algebra, is highly technical. Just unrolling it all we have:
  an effect algebra, with
- A compression base, which is
  - a normal sub-effect algebra (which means it is closed under some additional sums) * of projections, of which the retraction satisfies a certain condition
  - has the projection cover property, which means that every element has a 'least' cover in the compression base
  - the b-comparibility property, which means that
    - it has the b-property (which says an element commutes with a projection iff its bicommutant commutes with a projection)
    - and the compressions 'order-separate' the elements in a particular way.

I think at least giving intuition for why each of these properties is necessary would be helpful to give the reader an intuitive understanding of the main definition. For instance, what goes wrong if the compression base is not a *normal* sub-effect algebra? What do you lose if you don't have the projection cover property? Could you have the b-comparibility property without the b-property? What is the intuition behind the b-property? What is an example effect algebra where it fails to hold?
Also, just making this list of all the needed property took quite some time, as everything is spread out throughout the paper. Putting all the properties together, so that the reader can easily see what ingredients actually go into a spectral effect algebra would make it more clear what is actually required for spectrality.
Something that might also help to motivate the definition in retrospect: Can you show that an effect algebra which has some set of projections satisfying the conditions of Theorem 5.5 must be spectral? If not, what is it that is 'extra' in a spectral effect algebra that is not captured by this list of properties of the spectral resolution?

- It is not clear to me what is exactly gained by generalising this notion of spectrality. It is impressive that the authors managed to give a definition in such an abstract setting that actually seems to work, but they don't give any examples of spectral effect algebras where this level of generality is actually needed. They give three examples: Hilbert space effects (which is convex, so that a more intuitive definition could have been used), orthomodular posets (as a non-example, since they must then be Boolean), and monotone sigma-complete MV-algebras (which are Archimedean, and hence by Corollary 3.30 belong to a C(X,[0,1]) so that we are again almost in the convex setting). In addition, in the most abstract setting, the authors only prove one main property: that in a spectral effect algebra you actually have a well-defined rational spectral resolution. However, any other properties seem to need additional assumptions to be proved. For instance, in Theorem 4.10 you assume it has a separating set of states, but in that setting the effect algebra again embeds into a convex effect algebra, so is this level of generality then really needed?
Maybe it will turn out that spectrality enforces so much structure that you automatically go back to the more concrete setting. But if this is the case then this would be a nice result to have. For instance, I would imagine that if you have a divisable effect algebra that is monotone sigma-complete, that the algebra would be convex. So maybe you can combine some abstract properties that imply divisability or sigma-completeness in order to recover the standard convex framework of spectrality?
In addition, I think the authors are doing themselves a disservice by not speculating on future work in the end. It is not clear to me where this work could be taken next. Are the authors hoping to prove more properties of spectral effect algebras? Study them in more specific cases? Try to simplify the definition?

- One thing that bothers me about the main definition is that all the results for spectral effect algebras seem to depend on the chosen compression base. Is there some kind of result like "If E is spectral with regards to a compression base P, as well as to P', then P'=P" or something like that, like an inclusion, or the existence of a maximal compression base, that would make the choice of a compression base be less arbitrary? Maybe something like this uniqueness will follow in the convex strongly Archimedean case due to Theorem 6.5?

And now for some small things:

- p.1: I would also cite here https://arxiv.org/abs/1904.03753 (https://arxiv.org/abs/1904.03753)
- Top line on p.4: I think there is a word missing here somewhere. Also, "categorically equivalent" sounds a bit weird to me here. Do you mean to say equivalent in the sense of category theory?
- Definition 3.1: A retraction is a monotone idempotent map. I would view monotone idempotent maps as a natural class of study as well. There's probably good reasons to define retractions in this way, but it might be nice to share some examples for why to choose this more restrictive definition.
- p.5: "that he set".
- I know coexistent is a standard property to study in effect algebras, but it would be helpful to give some intuition here for why you single this out as something important (I believe it had something to do with commutativity in the operator algebra case?)
- Above Definition 3.3: Can you give some intuition for when a subalgebra is 'normal'? For instance, what are the normal subalgebras of E(H)? What about for a general interval effect algebra? I think having this intuition is helpful considering the crucial Definition 3.3 requires the subalgebra to be normal.
- Below Definition 3.3: you say here 'pairwise compatible'. Do you mean 'Mackey compatible' and hence 'coexistent'? In that case I would just use 'coexistent'.
- Example 3.5: How do we know that the join of a and p always exists?
- In Definition 3.20 you say an *effect algebra* has the b-comparibility property, but whether the b-comparibility property holds depends on the compression base we've chosen right?
- Remark 3.27: I was a bit confused when reading the first sentences whether his holds in all effect algebras with a compression base, or only those with b-comparibility. It is the latter, right?

- At the start of Section 3.4, I don't see why G = G_p \oplus G_{p'}. Is this a special property of an interpolation group?

- At the start of Section 4 there is a nice list of examples. I think it would be good to also include some remarks on how these definitions work out in the cases of convex and/or sequential effect algebras. In particular, in Example 2.6 you note that the sharp elements of a sequential effect algebra make a compression base. If the sequential effect algebra is sigma-monotone complete then it also has the projection cover property. What then remains to be proven for it to be spectral? In https://quantum-journal.org/papers/q-2020-12-24-378/ (https://quantum-journal.org/papers/q-2020-12-24-378/) they find a spectral theorem for normal sequential effect algebras. How does this fit into your definitions?

- It would be a service to the reader if the authors give some intuition on what U_{m,n}, c_{m,n} and d_{m,n} represent. Perhaps with a picture (or just using words) demonstrating what they are in C(X,[0,1]). I'm assuming they are the indicator functions for where the element a is between a certain interval of values.

- The proof of Theorem 4.5 uses notation that is only introduced in the Appendix. So I would either say to define these notations in the main text, or otherwise moving the proof of this statement to the appendix. Just reading the text, it seems like the appendix is crucial reading for understanding the overal argument, so the authors might consider including (parts of) the appendix in the main text anyway.

- Top of Section 5: "(\lambda, a) \mapsto \lambda a" has the obvious properties. You might want to reference that it is an *effect module* (see e.g. Jacobs, 2015) over [0,1]\cap \mathbb{Q}, as that notion captures what these obvious properties are.

- Your definition of divisibility requires the dividing effect to be unique. This precludes the study of the 'almost-convex' algebras studied in https://quantum-journal.org/papers/q-2020-12-24-378/ (https://quantum-journal.org/papers/q-2020-12-24-378/), which were also shown to have a spectral theorem. How much of the results can you prove here without assuming the uniqueness of the division? I.e. where you could have elements x and y such that nx = ny, without necessarily having x=y.

- Again in Theorem 5.3: General comparibility is only defined in the appendix.

- In the proof of Theorem 5.5 it looks like p_\lambda and p_\lambda' are being used through each other without consistency (but maybe I'm just misunderstanding the proof).

- In Example 6.9 you talk about generalised spin factors. By Alfsen and Shultz we know that if a space is suitably spectral then it is a JBW-algebra, so in this setting I would expect it to reduce to an actual spin factor, and not a generalised one. How far away is a generalised spin factor with your type of spectrality removed from just being a JBW-algebra/spin-factor? Perhaps the author's other paper on the comparison between the two types of spectrality already sheds light on this.

- In Eq. (10) in the appendix, the symbol a^o is used for the projection cover, while in Definition 3.12 you use a^\circ.
- Some kind of typo in [27].

Jump to section