Chengkai Zhu, Zhiping Liu, Chenghong Zhu, and Xin Wang, Limitations of classically-simulable measurements for quantum state discrimination

Referee report

The topic of the paper is quantum state discrimination by using POVMs with nonnegative Wigner functions (PWF). The importance of this problem lies in the fact that such POVMs can be effectively simulated classically. The authors show that there are pairs of mutually orthogonal states that cannot be unambiguously distinguished by such POVMs, even if an arbitrary number of copies of the states is given. This is done using a property the authors call (strong) PWF unextendability of certain subspaces. Further results are proved on the minimum error discrimination of the strange state and its orthogonal complement, and relations to data hiding are suggested. This situation is somewhat similar to entanglement and discrimination of states by LOCC measurements, but there are distinctions that are also discussed and illustrated in the paper. **Overall evaluation**

The results of the paper are not particularly surprising and the techniques applied are not really involved. On the other hand, the question of limitations of classically simulable measurements is surely important and it seems that the problem of their use in state discrimination was not considered before. The similarities and relations to entanglement and LOCC shown in the paper, as well as the PWF robustness and the connection to data hiding, are intriguing and surely deserve investigation.

The paper is quite well written, there are only a few small remarks and suggestions to make:

1. for the sake of the general readers, it would be good to define the magic states
2. page 2, column 1, line 22: "the unitary ...operators... is defined..." are defined
3. in the same sentence as above: it would be better to indicate that $\oplus$ is the addition in $\mathbb{Z}_d$
4. There is some confusion concerning the use of the expression "POVM" throughout the paper and also the Supplementary material. There are sentences like "Let E be a POVM...", but actually E is an operator $0 \leq E \leq I$, such operator is usually called an effect. A POVM is a collection of effects summing up to identity, used in description of measurements. Any effect defines a two-valued POVM $\{E, I - E\}$. So an effect E is PWF if $W(E|\mathbf{u}) \geq 0$, but the corresponding POVM is PWF if also $I - E$ has this property, so that $0 \leq W(E|\mathbf{u}) \leq 1$. It would be better to clarify this, to avoid confusion.
5. p.3, paragraph below Theorem 4: "Theorem 4 is broad applicability..," perhaps "of" is missing
6. p. 3, paragraph above prop. 5: "We remain the dual SDP..." better replace "remain" with some other verb
7. Supplementary material, proof of Lemma 2: better give some arguments showing that $\mathrm{Tr}_1\sigma$ and σ' are PWF (like the property 5. of the operators $\{A_{\mathbf{u}}\}$)
8. Eq. (S12): the factor $\frac{1}{d}$ seems missing.

9. SM, proof of Prop. 3: "...that contains only magic states." perhaps "supports" would be better instead of "contains"
10. Main text and SM: both notations A_0 and $A_{\mathbf{0}}$ are used, better pick one.
11. The arguments between Eq.(S28c) and (S28d) are difficult to understand
12. Eq. (S35): Perhaps the factor $\frac{1}{d}$ is missing here? Or should there be $W(P_{\mathcal{S}}^{\perp}|\mathbf{u})$? These notations are somewhat inconsistent.
13. Better explain (or cite) the equality in (S44).
14. Eq. (S29) $a_{\mathbf{u}}$ or $\hat{a}_{\mathbf{u}}$?