C. Duarte, L. Catani, R.C. Drummond, Relating compatibility and divisibility of quantum channels

Referee report

As the title suggests, the aim of the paper is to connect two important relations of a pair of quantum channels, namely their compatibility and, secondly, the fact that one channel divides the other (in other words, the latter channel is a concatenation of the former with another channel). Two conjectures are stated: conjecture 1 says that divisibility implies compatibility, while conjecture 2 states the converse. It is shown that the conjectures are not true in general, but hold under some additional assumptions of one of the channel being (anti)degradable and both conjectures hold if one of the channels is self-degradable.

Frankly, I do not understand why these "conjectures" would be of interest. Both are quite obviously false, as the authors themselves explain and demonstrate on very simple examples. The main results for (anti)degradable channels follow straightforwardly from the results of [13] (Prop. 4), stated in the present paper as Theorem IV.2, giving the characterization of compatibility in terms of divisibility by a *conjugate* channel. In particular, Theorem IV.5 (ii) is a direct consequence of the observation in [13], Sec. 4.3, that the antidegradable channels are self-compatible (in fact, these are exactly the self-compatible channels).

The paper is written very clearly and is easy to read and understand. On the other hand, the value of the results is questionable. I agree that the (anti)degradable channels are very interesting for various reasons, but the authors do not sufficiently explain why the relation to the above conjectures is of importance. In fact, very little is gained from the paper which is not already contained in [13]. Therefore I cannot recommend the paper for publication.