# B. COECKE, T. FRITZ & R. SPEKKENS

# A MATH. THEORY OF RESOURCES

**In Short:**

- A resource theory (RT) $\equiv$ symmetric monoidal category (SMC)

- Partitioned theory of processes $\equiv (\mathcal{C}, \mathcal{C}_{free})$

  where $\mathcal{C}$ is a SMC

  $\mathcal{C}_{free} \hookrightarrow \mathcal{C}$ a sub SMC

  containing all objects of $\mathcal{C}$

FROM $(\mathcal{C}, \mathcal{C}_{free})$, some RT can be constructed:

1) A RT of states $S(\mathcal{C}, \mathcal{C}_{free})$:

- objects: elements of $\mathcal{C}(I, A)$ 

  ($I$-unit object)

  $\downarrow$ states

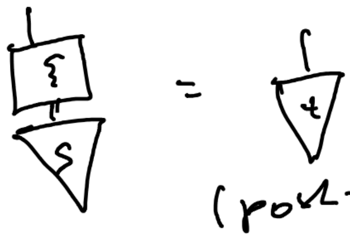
  free states: $\in \mathcal{C}_{free}(I, A)$

costlices of $\mathcal{C}$

(restricted morphisms)

• morphisms : $\xi \in C_{free}(I, A)$

obvious composition




$s \xrightarrow{\xi} t$

(post-composition)

• unit $I$ $\quad C(I,I) \ni id_I$

• $\otimes$ inherited from $C$

## 2) RT of parallel combinable processes

$PC(C, C_{free})$

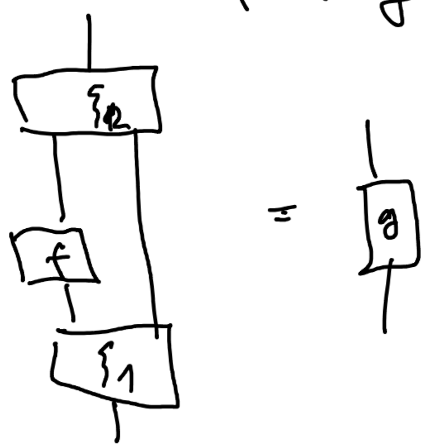
• objects : all morphisms in $C$ (processes)

• morphisms : $f \to g$

$\xi_1, \xi_2 \in C_{free}$

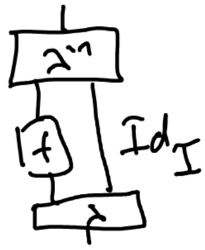
↓

1-combs (or equivalence classes of morphisms)




something like "twisted arrow" construction

one needs a strict SMC for this to be a category → what is the identity arrow?



- unit $id_I$
- composition : as combs
- $\otimes$ on objects → from $C$
  on arrows → obvious

3) RT of universally combinable processes

- objects : tuples of arrows in $C$
  $(f_1, \ldots, f_n)$ (not ordered!)
- morphisms : tuples of n-combs (here)
  not ordered? ?→ consisting of elements in $C_{free}$
- unit object : $id_I$
- composition : as n-combs
- monoidal structure :
  on objects ≡ concatenation
  on arrows ≡ concatenation of combs

symmetric multicategory

- Theories of resource convertibility

$(R, +, \geq, 0)$ : $R$ a set
$+$ binary operation
$\geq$ preorder
$0$ distinguished element

s.t. $(a+b)+c \simeq a+(b+c)$ | $a+0=0+a=a$
$(a+b) \simeq (b+a)$

$a \geq b,\ c \geq d \Rightarrow a+c \geq b+d$

→ obtained from a RT $(D, \otimes, \circ, I)$:

$R := |D|$ - objects of $D$ $\quad 0 := I$
$+ := \otimes$
$\geq$ hom-preorder

- catalyst for $a, b \in R$: an element $c \in R$, s.t. $a \not\geq b$ but $a+c \geq b+c$

- catalyst-free theory
- no-cloning
- disposable process : $a \geq 0$ ? not always true

- Monotones: $M : R \to \mathbb{R}$, monotone under $\succeq$

  complete set of monotones:

  $\{M_i\}_{i \in J}$:

  $M_i(a) \leq M_i(b) \;\; \forall i \in J \iff a \preceq b$

  $\hookrightarrow$ always exists

- Convertibility rates

  $R(a \to b) = \sup\{\frac{m}{n} \mid n\,a \succeq m\,b\}$

  $\to$ (can be also inf sometimes)