

# Answer to reviewer's report on Quantum $f$ -Divergences in von Neumann Algebras

Dear Reviewer:

I am very grateful to the reviewer for careful reading of the manuscript and helpful suggestions. In the revised version I have addressed all the reviewer's comments as explained below.

1. Sec. 3.3: the index  $\alpha$  is missing in  $\tilde{D}_\alpha$  and  $\tilde{D}_\alpha$  in some places in Sec. 3.3;

I can't find this typo. But I found a place (in Lemma 3.18) where  $\alpha$  is missing in  $\tilde{Q}_\alpha$ , and corrected it.

2. Lemma 4.19:  $\varphi_t$  should be  $\sigma_t$ ;

Corrected.

3. Lemma 5.6: it seems that  $s$  should be replaced by  $t$  (and vice versa) in some of the expressions  $s \in (0, \infty)$  in displayed equations;

Thank you. Corrected.

4. page 78: it seems that  $S_{h_s}$  should be  $S_{g_s}$  and  $S_{\varphi_s}$  should be  $S_{h_s}$

Corrected.

5. Chapter 7: some introduction is needed at the beginning of this chapter, explaining what is going to happen;

Thank you for this. At the beginning of Chapter 7 I have added the following (Reference numbers are changed in the new version):

This chapter is concerned with approximate reversibility (sufficiency) for a sequence of quantum operations  $\alpha_k : M_k \rightarrow M$  (or quantum channels with input  $M$  and outputs  $M_k$ ). Our main problem is to characterize the approximate reversibility of  $(\alpha_k : M_k \rightarrow M)_{k=1}^\infty$  for  $\psi, \varphi \in M_*^+$  in terms of the convergence  $S_f(\psi \circ \alpha_k \| \varphi \circ \alpha_k) \rightarrow S_f(\psi \| \varphi)$ . In particular, we are concerned with the case where  $\alpha_k$ 's are measurement operations with commutative  $M_k$ 's (or quantum-classical channels). The problem was formerly investigated by Petz [101] (also [94, Sec. 9]). We will revisit Petz' results with some refinements.

6. Lemma 7.1: some more explicit comments on the relation to the results in [95] and [88] are needed, also for other results in this chapter;

Thank you for this comment. In a few places I have written the relation to the results in [101] and [94] ([95,88] are [101,94] in the new version). By the way, I have expanded Section 7.1 to cover all the results of Petz in [101].

7. p. 93, first line of Sec. 7.2: a parenthesis ( is missing;  
I have deleted a parenthesis ) in the right side.
8. p. 95, the statement of Lemma 7.4, last line: the full stop between “unitary” and “Them” should be a comma;  
Corrected. By the way, this lemma is slightly changed and is moved in Section 7.1.
9. p. 97, last line:  $S_f(\alpha\|\varphi)$  should be  $S_f(\psi\|\varphi)$ ;  
Corrected.
10. p. 99, line 6:  $L^1(M)_+$  should be  $L^1(M)$ ;  
Corrected.
11. p. 99, before Definition 8.2, maybe recall the definition of an operator connection here, or refer to Appendix D;  
Thank you for this suggestion. I have added this before Definition 8.1.
12. Definition 8.2: we have to assume again that  $(\psi, \varphi) \in (M_*^+ \times M_*^+)_{\leq}$ ;  
I have added this assumption.
13. page 99–100: it would be good to give the relation to  $\hat{S}_f$  (Prop. D.11) somewhere before Thm. 8.4  
Thank you for this suggestion. I have added this properly. Please see the paragraph including Eq. (8.4).
14. page 103, Eq. (8.7):  $s$  is missing on the left hand side;  
Corrected.
15. I found some strange English phrases, like “definition is well defined” p. 36, “question holds true” p. 85, “problem holds true” p. 98, p. 107  
Thank you. I have corrected these phrases properly.

On this occasion, I have changed several places with additional material. In particular, Section 7.1 has been expanded to cover all the results of Petz in his 1994 JFA paper. Also, Section A.8 and Appendix B have largely been revised.

Sincerely yours,  
The author