# Extended Abstract: Compression of quantum shallow-circuit states

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## 1 Background

Shallow quantum circuits are a focus of recent research, for they are arguably the most accessible resources with genuine quantum features and advantages. At the fundamental level, shallow quantum circuits with constant depth have been shown to be hard to simulate classically (unless BQP $\subseteq$ AM) [1], and they outperform their classical counterparts in certain computational tasks [2, 3]. In practice, variational shallow circuits [4, 5, 6, 7] will remain a core ingredient of quantum algorithms in the noisy and intermediate-scale quantum (NISQ) era [8]. Efficient methods of learning shallow and bounded-complexity quantum circuits have recently been proposed [9, 10, 11].

Here we ask a fundamental question: Given $N$ copies of an *unknown* $n$-qubit state and the promise that it is generated by a shallow circuit, is there a faithful compression protocol that encodes the $N$-copy state into a memory of fewer (qu)bits and then decodes it up to an error vanishing at $N \to \infty$? Processing quantum states in the many-copy form is important for extracting, storing, and distributing quantum information. Tasks where many-copy states serve as a fundamental resource, to list a few, include quantum metrology [12, 13, 14], quantum state tomography [15, 16] and shadow tomography [17, 18], quantum cloning [19, 20, 21, 22], and quantum hypothesis testing [23, 24, 25, 26]. Quantum algorithms, such as quantum principle component analysis [27], may also require states in the many-copy form. As such, compression of quantum states in the many-copy form is a basic and crucial protocol required for their storage and transmission. In the literature, compression of many-copy states was first studied for the simple case of pure qubits by Plesch and Bužek [28], experimentally demonstrated in Ref. [29], and later generalized in a series of works to mixed qudits [30, 31, 32, 33]. However, regarding states generated by shallow quantum circuits, the existing results are not applicable, for they all assume the state to be in a fixed-dimension space. Here, instead, we consider states in a growing-dimension ($D = 2^n$) space with complexity constraints. Therefore, studying the compression of shallow-circuit states not only requires better understanding of this important family of states but also demands new techniques of asymptotic quantum information processing.

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## 2 Overview of main results

Given a set $\mathsf{S}$ of quantum states, the task of faithful $N$-copy compression is to design a protocol that consists of an encoder $\mathcal{E}_N$ and a decoder $\mathcal{D}_N$ such that the compression error vanishes for large $N$:

$$\lim_{N\to\infty} \sup_{\rho\in\mathsf{S}} d_{\mathrm{Tr}}\left(\mathcal{D}_N \circ \mathcal{E}_N(\rho^{\otimes N}), \rho^{\otimes N}\right) = 0. \tag{1}$$

Here $d_{\mathrm{Tr}}$ denotes the trace distance between quantum states. The encoder $\mathcal{E}_N$ and the decoder $\mathcal{D}_N$ are dependent on $N$ but are independent of the input state. The memory cost is characterized by the dimension of the Hilbert space spanned by $\{\mathcal{E}_N(\rho^{\otimes N})\}$. The goal of compression is to reduce the memory cost

$$M := \log_2 \left|\mathsf{Supp}\left\{\mathcal{E}_N(\rho^{\otimes N})\right\}_{\rho\in\mathsf{S}}\right|, \tag{2}$$

i.e., the number of (qu)bits required for storing $\mathcal{E}_N(\rho^{\otimes N})$, while respecting the faithfulness condition (1).

Here, we are interested in the set of shallow-circuit states $\mathsf{S}_{\mathrm{sc}}$, which contains all $n$-qubit pure states that can be generated from $|0\rangle^{\otimes n}$ by circuits of depth no more than a constant $d$. As a proof-of-principle example, we focus on the most representative case of brickwork shallow circuits and consider the set of *shallow-circuit states*

$$\mathsf{S}_{\mathrm{sc}} := \left\{|\psi\rangle \;:\; |\psi\rangle = U_{\mathrm{sc}}|0\rangle^{\otimes n} \,\exists\, U_{\mathrm{sc}}\right\}, \tag{3}$$

where $U_{\mathrm{sc}}$ is a brickwork circuit with bounded depth ($\leq d$).

Without compression, the memory cost of storing the input state equals $N \cdot n$ qubits. Our main contribution is to show that a faithful $N$-copy compression exists for $\mathsf{S}_{\mathrm{sc}}$, as long as $N$ grows at least as a polynomial of $n$ with a high enough degree. The memory cost of the compression is linear in $n$ and logarithmic in $N$, i.e., $M = O(n \cdot \log_2 N)$, achieving an exponential memory reduction in terms of $N$. Moreover, the memory does not have to be fully quantum. Instead, one may use a classical-quantum hybrid memory, where the ratio between the number of qubits and the number of classical bits decreases as $O(\log_2 n / \log_2 N)$. That is to say, when $N$ is large, the memory consists mainly of classical bits, while a fully classical memory doesn't work. More details can be found in the technical manuscript.

Following the main result, it is natural to ask if the memory cost can be further reduced. We prove that a memory of size $\Omega(n \cdot \log_2 N)$ is required for keeping the compression faithful. In this sense, our compression protocol is optimal in the scaling of $n$ and $N$.

To establish the compression protocol, we develop novel tools for quantum information processing in the asymptotic regime of many copies, including a method of parameterizing shallow-circuit states in a small neighborhood with only $\mathsf{poly}(n)$ parameters and a correspondence between copies of a low-complexity state and a multi-mode coherent state. These tools can be further applied to other information processing tasks involving complexity-constrained quantum states.

## 3 Discussions

We have shown that $N$ copies of an $n$-qubit shallow-circuit state can be optimally compressed to $\Theta(n \cdot \log_2 N)$ qubits. Intriguingly, the two key parameters $n$ (the number of qubits per copy) and $N$ (the number of copies) take distinct positions in the compression rate. We may give an interpretation to this phenomenon: $n$ is the parameter of informativeness, as it is proportional to the number of free parameters of a shallow-circuit state. On the other hand, $N$ is the parameter of accuracy, since $1/\sqrt{N}$ is the error scaling of tomography, i.e., of how well can we learn the information in the state. Our result shows that the $N$-copy state can be exponentially compressed only in the parameter of accuracy.

Besides memory efficiency, one may also be curious about the computational efficiency of shallow-circuit state compression. Unfortunately, the compression protocol in this work, despite being memory-efficient, is not computational efficient. The main obstacle is that the protocol requires searching over a covering mesh of shallow-circuit states, whose cost is exponentially large (in $n$). It is noteworthy that this is also the key step of converting a part of the memory to classical bits. It is thus intriguing to conjecture that any protocol using a hybrid memory is computationally inefficient. On the other hand, there exist compression protocols using fully quantum memory [30, 31] that do not require searching, and there remains hope that these protocols could inspire a computationally efficient protocol for shallow-circuit states.

As we focused on the most fundamental case, there is plenty room for extension. For example, one may consider shallow-circuit states with a 2D structure, and the techniques developed here should apply. Moreover, the circuit depth $d$ is treated as a constant throughout this work, but from the derivation of results it can be seen that the compression will still be faithful when $d$ grows very slowly (e.g., $d \ll \log n$) with $n$. In particular, it would be interesting to cover pesudorandom quantum states [34], which are low-depth states processing approximate Haar-randomness and are thus of particular interest in quantum cryptography. At last, one may even take into account the effect of noise and consider the compression of noisy shallow-circuit states. While similar results are expected there, some techniques in this work do not immediately generalize to mixed states and require moderate adaptation.

This work serves as the first step of establishing a new direction of coherent quantum information processing where the complexity of resources determines the rate and performance of processing, which goes beyond the existing literature that focused on incoherent information processing [9, 10, 11]. For future perspectives, it is our goal to consider more tasks such as cloning [19, 20, 21, 22] and gate programming [35, 36, 37, 38, 39] and, ultimately, to re-examine the entire quantum Shannon theory established in the past decade from the new perspective of the NISQ era.

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