Report of the Referee -- AH11863/Jencova

In this paper the authors discuss how to construct objects known as PR-boxes, using different general probabilistic theories. Most of their results indeed recover already known ones, but at the end they apply their methods to particular case they refer to as “qubit quantum channels”. When I received the paper I thought it would make an interesting read, but as soon as I dove into it I was proven wrong. The paper is very inaccessible, even for an expert like myself. More concretely:

- The review on GPTs (section II) is too technical: someone who does not know GPTs will not be introduced to them by it, and someone who knows them will not learn anything new.
- Section III is called “Calculation”, which is not informative of what is being discussed or presented, specially since the other sections also include heavy calculations. Moreover, the authors jump straight into the maths with little-to-no motivation of what they are doing: what calculation is this, why is it important, how does it fit in the big picture, why should the reader find it worth it working through it?
- Section IV is called “The case of classical Channels”, which is not informative of what the authors aim to discuss there, since there is no build-up of motivation towards it. Why do you want to interpret the set of classical channels as a state space? What is the scenario that you have in mind?
- Section V: like in the case of Section IV, the title is not informative, the motivation is lacking, and there’s no discussion of what the setup is. The authors mention “measurements on quantum channels”, which is not a widely known concept, so I suggest they spend some sentences describing what they mean by that. Also, in the last sentence of the section they say that “it’s straightforward to certify that the channel and measurements violate CHSH maximally”, which is not clear and raises the following questions: which is the setup that you have in mind? What is the system being measured, the state of the system, and the measurements performed? This may be clear in the authors’ minds, but it’s not mentioned in the text, hence it’s not clear what scenario they are studying. It may be worth adding a figure to depict which are the inputs/outputs, measurement instruments, parties, etc.
- So far the authors have only recovered known results from their framework: why is this reformulation relevant and worth pursuing? What is the new insight that comes from their complex mathematical viewpoint?
- Section VI is the one that presents the new result that the authors derive. However, I find it very obscure. Why is this question relevant/important in the field? What would we accomplish if we answer it? What have you achieved or unveiled with the answer you found?
- General comment: the manuscript is very heavy with mathematics, and given the little motivation in the text it’s hard to envision where the calculations are leading up to. I suggest the authors to change the presentation, by mentioning first the Proposition/ Lemma to be proven, and then the proof of the statement (see, e.g., Prop. 3).

In summary, I believe that this manuscript may present some complex mathematical formalism, but its relevance or the motivation for this does not come across. In its present form, this manuscript is not suitable for publication in Physical Review A.

I suggest the authors to work on the narrative of the manuscript, and link all the sections so that the narrative and motivation are not lost. Maybe some major restructure should happen, where heavy mathematical proofs that cut the flow of narrative are moved to an appendix. After the authors address these concerns, and make their paper more accessible, I suggest

they resubmit to Physical Review A.
Some specific comments:

- In Section II you use the letter “A” to denote two things: A(K) the linear space of functions, and a measurement A. This is very confusing, and I recommend the authors to change it.
- Second paragraph of section II-B: you say “All these sets...”. What does “these” refer to? I suggest the authors to rephrase the sentence, it’s ambiguous.
- Section II-B: the terminology “real tensor product” is not standard ion the GPT community, and is misleading (it makes you think of real numbers). I suggest the authors to choose a less confusing name.
- Section II-B: In the last equation the authors write P_x(B,A=1) = P_x(B’, A=1) as a “no-signaling” condition. This contradicts the notation they use until then, where Alice’s measurements are A,A’, and Bob’s are B,B’. I suggest the authors to change it to P_x(A,B=1) = P_x(A’, B=1) and remain consistent throughout the manuscript.
- Section III, at the end of paragraph 2: the word “measurements” appears twice.
- Section III: The authors work with a pair of measurements A and B, and these can be applied by both Alice and Bob. This is confusing, and goes against the convention they set up in Section II. I suggest they choose other labels for the measurements, which have no connotation towards Alice or Bob.
- Page 4: The sentence before Proposition 2 is confusing. I believe the authors mean the following, in which case I’d suggest them to rephrase their sentence: “The condition in Prop. 1 is trivial is the GPT composes through the max tensor product. Hence, the statement only becomes non-trivial for GPTs with restricted composition rules.”
- Page 5, first equation of Section IV: the symbol F_{s,f} is used without saying what it is in the text. I suggest the authors to introduce it in the previous sentence (e.g., “… each effect F_{s,f} on …”.
- Page 5, last sentence of Section IV: The text says “Prop. 1 and 1”, the authors should fix the typo.
- Discussion: the last paragraph is very perfunctory. I suggest the authors to give some substantiation for the claims.