

Review 1

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Review of “On the convex characterisation of the set of unital quantum channels”

for *Quantum*, completed on Apr 22, 2022

Author will not see the reviewer's name or the date completed.

Overview questions

 only editors will see response

Overall rating

★★★★★

For this manuscript I recommend...

Revise and resubmit

Comments to editor

The paper addresses an important problem of the structure of extremal unital channels. Some specific family is studied, but significance of the results is not clear. The extremality conditions are obtained in a straightforward manner and no particular insight seems to be gained.

It seems that some new examples of extremal unital channels are obtained, but it is not clear what is special or new about their structure (extremal unital qutrit channels of rank 3 and 4 are already known to exist).

Perhaps if the authors could explain and emphasize the importance of the problem and significance of their results, the paper would be more suitable for publication.

Open response questions

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ok

Summary: what are the main questions posed by the manuscript and how does it answer them?

 intended for the author

The topic of the paper is the convex structure of the set of unital channels, more precisely the description of extreme points of this set. The authors restrict to a family of unital channels of rank equal to the dimension, such that the expression of the Kraus operators with respect to the Heisenberg-Weyl basis has a certain form. For this family, extremality conditions are given. Specific examples of such channels are presented in the qutrit case. Apart from these, a new example of a qutrit extremal unital channel of rank 4 is also found.

What is your assessment of the paper? If you recommend acceptance, make a case that this work does indeed make a significant contribution to scholarship.

 intended for the author

The paper mostly consists of straightforward applications of known conditions to elements in the specified family. It is not clear why this family was chosen and what significance the results have. Existence of extremal unital channels that are not unitary or even not extremal in the set of channels nor unital maps is already known, established in the works cited also in this paper. It is a question whether the present results might yield some significant insight into the structure of the extreme boundary of the set of unital channels, and this question is not addressed in the paper.

As for the qutrit case: three examples of members of the family are presented, one of which is not extremal and one is related to previously known examples by unitary conjugations. As for the third example, it is extremal in the set of unital channels, but as far as I can see, it is not clear whether it is not also extremal in the set of all channels or all unital maps, which would make it less interesting. The rank 4 maps considered here are obtained from the rank 3 maps in the family by adding some zero rows to the Kraus operators.

From the maps resulting from the three studied examples, only one is an extremal unital channel. Since it is rank 4, it cannot be an extremal channel or unital map, which makes in a genuine example.

To what extent have you checked the technical correctness of the paper?

 *intended for the author*

I have checked the formulas to some details and all seem correct.

I did not check the computations for the examples.

Comment on the presentation of the paper. Is it well written? Are the main results clearly laid out? Does the manuscript clearly describe assumptions and limitations? Is the literature review adequate?

 *intended for the author*

I think that the presentation of the paper could be improved.

- This concerns already in the Introduction: there is some very general and rather uninteresting paragraph concerning quantum channels, but the relevance and importance of the problem studied in the paper is not described at all. Clearly, it is an interesting mathematical problem, but why the unital channels, and their extremal points, are important in quantum information? The authors should emphasise this more.
- Further, as already written above, the choice of the family of maps is not explained. Is there some intuition, or importance of such maps, or is it just some technical reasons? Why extremal unital channels of this form would be interesting?
- The main result on extremality conditions, Theorem 6, is formulated as a sufficient condition for extremality, but in fact used as a necessary and sufficient condition. Also the proof is very strange, since what is really proved there are sufficient conditions for extremality in the set of CPT maps and UCP maps separately! It is not difficult to see, using Theorem 4, that the Theorem can be proved in a very similar manner also for UCPT maps (and that the condition is necessary and sufficient), but the proof should be rewritten.
- As an important work on the convex structure of unital channels, I think a reference to arXiv:0806.2820 would be appropriate.

If the submission includes numerical or physical experiments, does it provide sufficient details such that they could be reproduced by readers? This includes for example source code, documentation, experimental data, experimental setup specifications, etc.

 *intended for the author*

No experiments are included.

Suggested changes, corrections, and general comments.

 intended for the author

Apart from the suggestions already given above, here is a list of further comments and typos:

1. p.2, l.3: “and and”
2. p.2. last paragraph of the Introduction: “Kraus operators are given by *convex* sums of elements of...” (convex ?)
3. p.3: it is not explained in what sense the sets of CPT and UCP maps are dual. Though probably clear to readers, it would be better to write it explicitly.
4. p.4, beginning of Sec. 3: “...Kraus operators are given by linear combinations of elements of the Heisenberg-Weyl basis.” This is strangely formulated (all operators are linear combinations of elements of the HW basis). Something like “...Kraus operators have a special form with respect to the HW basis” would be better.
5. p.4, Eq.(8): U instead of u_{ij}
6. p.5, Eq.(13) better introduce the Kronecker delta before its first use (on p. 4)
7. p.6, Eq. (21) $\alpha_{ib} \rightarrow \alpha_{ij}$??
8. Example 10: the equations (54)-(56) are exactly the same as those in (42)-(44). what is the point in repeating this?
9. Sec. 4.3.1: it does not seem very useful to write down the general matrices M_1 and N_1 . It would be enough just to write that $M_2=M_1^*$ and $N_2=N_1^*$ so that it is enough to establish the ranks of $(M_0|N_0)$ and $(M_1|N_1)$.
10. As already indicated above, it is important to check that the map E_a is not extremal also in the set of CPT or UCP maps, that is, at least one of the matrices M_0 , M_1 and one of N_0 , N_1 is not of full rank.
11. Sec. 4.3.2. is not very well written. It seems that the idea is exactly the same as in the proof of Theorem 6, only some zero columns are removed from the matrices M_1' and N_1' . This should be better explained. Again, it does not make much sense to write down explicitly all the matrices in Eq. (69)-(76).
12. p. 17, Conclusions: “We also illustrated the relations between the examples presented and well known examples...” it is shown that E_c is related to the Landau-Streater, resp. Werner-Holevo channels. Is there any other relation to extremal channels in your family to other examples, e.g. in [6] or [7]?

Would you be willing to referee an updated version of this work before a final decision is made?

 only editors will see response

Yes.

Do you think this is an outstanding work that deserves to be highlighted? If this work is accepted, would you be willing to write a short Perspective (opinion piece similar to a viewpoint or editorial) based on your report? For examples, see: <http://quantum-journal.org/category/Editorial,Perspective/>

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Not in the present form.

How was your reviewing experience? Is there anything you would like us to improve?

 only editors will see response

Ok.