

A. Łuczak: Strong subadditivity of Segal's entropy

Referee report

This paper deals with trace preserving conditional expectations on a semifinite von Neumann algebra, forming a commuting square. Such commuting squares are characterized using the Segal entropy of normal states. This extends a result of Ref. [1], where this characterization was proved in the finite dimensional case. One of the consequences of this characterization is the strong subadditivity of quantum entropy.

The proofs in the present paper closely follow those in [1] and the techniques necessary in infinite dimensions are quite standard manipulations, based on the previous results of the author concerning Segal's entropy. On the other hand, various versions of SSA related to entropic uncertainty relations and their extensions were recently intensely studied in quantum information theory, but most of the results are restricted to finite dimensions. An extension to arbitrary semifinite von Neumann algebras is therefore timely and valuable.

Some comments

1. The title is somewhat misleading. The SSA is obtained as a consequence in the case of finite von Neumann algebras and the reader is directed to Ref. [7] for the general semifinite case.
2. page 1, last line: "normal states" - better stress that the "states" here are not assumed to be normalized.
3. page 3, lines 5-6: "...nonnegative for a normalised state and finite trace". In fact, in finite dimensions, the usual trace is finite and the expression for the entropy as given in line 3 is nonpositive for normalized states. It seems that the author means that the trace is also normalized, hence a tracial state.
4. page 4, Lemma 2: it would be better to add that x is affiliated with M .
5. page 7, line 4 from below: Ref. [2] refers to an unpublished preprint by the present author, with no further information. The preprint should be made available, or another reference should be given.
6. Thm. 7: Ref. [6] gives another equivalent condition for equality in monotonicity of the relative entropy, namely that there is a recovery map, given by the Petz dual (generalized conditional expectation). A form specific for SSA was given in (P. Hayden et al., Structure of states which satisfy strong subadditivity of quantum entropy with equality, Commun. Math. Phys., 246(2):359-374, 2004). It might be useful to consider this condition in case of commuting squares.
7. Thm. 10: Better write the specific form of Eq. (8) for independent subalgebras.