M. Scandi, P. Abiuso, D. De Santis, and J. Surace, Physicality of evolution and statistical contractivity are equivalent notions of maps

Referee report

The Fisher information is an important information measure, that can be seen as a local distinguishability measure on probability distributions/quantum states. Both classical and quantum versions have a wide range of applications, though in the quantum case, only few of the large family of QFI's are used. It is therefore of importance to understand the structure and properties of QFI.

The family of QFI is selected among all Riemannian metrics on the manifold of density matrices as those that are contractive under physical maps, that is, quantum channels. The present paper contributes to the study of QFI by proving that also conversely, any metric in the family will select the physical maps among all "reasonable" linear maps, as those that cannot increase the metric.

This is an interesting observation, though it is based on a very simple idea, namely that while any QFI is bounded on any closed line segment contained in the interior of the positive cone, it diverges to infinity when approaching the boundary. Basically the same result is also contained in a recent longer preprint [14], where the proof is also given. The authors do not give any further applications of their observation, though there are some lines in the concluding section. In [14], the result is used in the study of Markovianity of evolutions.

Overall evaluation

I am not entirely convinced that this paper contains enough results to be published, though the observations are of potential interest in studying quantum evolutions or in quantum foundations. It may also be redundant, since it does not go beyond the results contained in a recent preprint by the same authors. It is also not well written, in any case it should be revised before publication. See the remarks below.

Some specific comments and suggestions

1. The family of quantum Fisher informations was characterized by Petz in [23], but the fact that contrary to the classical case, a monotone metric is not unique in the quantum case was observed by Chentsov and Morozova (Chentsov, N. N. and Morozova, E. A. (1990). Markov invariant geometry on state manifolds, Itogi Nauki i Tekhniki 36, 69–102). In fact, to my knowledge this was the first paper where the approach to QFI as a monotone Riemannian metric was considered. It may be appropriate to cite this paper.

2. Corollary 3.2: the condition Eq.(13) should be required for states such that $\Phi(\rho), \Phi(\sigma) \in \mathcal{S}_d^\circ$. (Otherwise the condition already presupposes that Φ is positive, since H_g is defined only on positive matrices).

3. Proof of Theorem 3 is not well written and confusing, especially the part after "Then, choose a perturbation..." is hard to understand and needs to be revised. As far as I can see from Eq. (18), the only thing that is needed is to find some matrix $\delta\rho_\eta$ for each η such that $|\langle\psi_\eta|\Phi(\delta\rho_\eta)|\psi_\eta\rangle| > \alpha$ for some $\alpha > 0$ (the requirement that it is "positive and finite" is rather puzzling for me). Then why not take $\delta\rho_\eta = \pi$ for all η? Since by the assumption $\Phi(\pi)$ is positive definite, $\langle\psi|\Phi(\pi)|\psi\rangle$ is bounded from below by the smallest eigenvalue of $\Phi(\pi)$, for any unit vector $|\psi\rangle$.

4. Below Eq.(19), the sentence "...the assumption that Φ contracts the Fisher metric for any two points in $\mathcal{P}_d^\circ$" is strange, since the requirement is for any point in $\mathcal{P}_d^\circ$ that is mapped to $\mathcal{P}_d^\circ$, and any tangent vector (that means for any hermitian matrix). So why "any two points"?