Referee report on Martin Plávala's PhD dissertation

Non-classical effects on generalized quantum channels

Generalized Probabilistic Theories (GPTs) provide a general framework for the statistical description of physical phenomena, based on a few basic concepts: the state space, which is a compact convex set of a normed space, measurements, which are affine maps of states to probability measures on the set of possible measurement outcomes, state transformations (or channels) that are affine maps between state spaces, and a rule to build these ingredients for composite systems from those of its subsystems. Two such theories stand out due their physical significance: classical theory, where the state space is a simplex, and quantum theory, where the state space is the set of density operators on a Hilbert space. One of the key differences between the two is that quantum theory admits incompatible measurements, i.e., measurements that cannot be simultaneously performed on a system. This implies various other non-classical features of quantum theory, like the non-existence of measurements that do not disturb the measured state, or that quantum states cannot be copied, or broadcast, which in turn have been recognized to be at the heart of various important applications of quantum theory, like quantum cryptography. One of the primary aims of the study of GPTs is to explore what characteristics of a statistical theory (for instance, in terms of the geometry of its state space) lead to such non-classical features, and to find operational characterizations that single out classical or quantum theory.

The results presented in this thesis advance the theory in various interesting and important directions, revolving mainly around the concept of compatibility. The main part of the thesis consists of five papers (co)authored by the candidate; the first three in Phys. Rev. A, and the last two in Quantum, both of which are high reputation (though not leading) journals in theoretical physics. The first and the third papers are single-author papers, the second and the last one are coauthored with Anna Jenčová, the supervisor of the candidate, and the fourth one is coauthored with Teiko Heinosaari and Leevi Leppäjärvi.

In the first paper it is showed that any non-classical GPT admits a pair of incompatible 2-outcome measurements, thus giving a characterization of the classical theory. The second paper investigates a quantitative version of this problem by studying the degree of incompatibility for 2-outcome measurements, which is a measure of the minimal amount of noise that has to be added to the two measurements to make them compatible. It has been shown in [P. Busch, T. Heinosaari, J. Schultz, and N. Stevens, Europhys. Lett. 103, 10002 (2013)] that a pair of maximally incompatible 2-outcome measurements exist in a GPT if its state space contains a square whose opposite edges lie in parallel exposed faces of the state space. The main result of the second paper is that this condition is also necessary, thus completing the characterization of GPTs with maximally incompatible 2-outcome measurements.

The third paper considers the more general concept of the compatibility of channels, introduces the concepts of steering by channels and Bell non-locality of channels in GPTs, and shows (among others) that compatible pairs of channels are not useful for steering or creating Bell nonlocality. Although the concepts are introduced in a rather abstract mathematical form, they are justified by the fact that they reduce to the more familiar operational concepts when the channels are measurements.

The notion of compatibility of channels also features in the fourth paper, where three sets of measurements are analyzed in GPTs: $\mathcal{T}_1$, the set of trivial measurements that output the same probability distribution on all states, $\mathcal{T}_2$, the set of measurements that are compatible with the identity channel, which can be seen as non-disturbing measurements, and $\mathcal{T}_3$, the set of measurements compatible with any other measurements. In the quantum theory, these sets coincide, and the equality of the first two is called the "no information without disturbance" principle (NIWDP), while the equality of the first and the last one the "no-free-information" principle (NFIP). It is shown in the paper by concrete examples that there exist non-classical theories where these equalities do not hold, and hence NIWDP does not imply NFIP, while it is also shown that measurements in $\mathcal{T}_2$ and $\mathcal{T}_3$ must have in some sense simple structures.

The last paper ventures in a slightly different, but related direction, and investigates some properties of the recently introduced concept of spectral effect algebras [S. Gudder, Convex and sequential effect algebras, arXiv:1802.01265]. This can be seen as a generalization of the set of POVM elements in quantum theory, in that each element can be written as the barycenter of a context, which in turn is a generalization of a collection of rank 1 projections summing to the identity. The main result of the paper is that there can be either only one, or uncountably many different contexts in a spectral effect algebra.

These results give important contributions to the intensively researched fields of generalized probabilistic theories and foundations of quantum mechanics. Their significance is clearly indicated, among others, by the fact that the above five papers, all published in the past three years, have already attracted at least 26 independent citations, according to google scholar. A particular strength of the thesis is that it contains two single-author papers by the candidate, clearly demonstrating that he can already work as an independent researcher, while the fourth paper is proof that he can also successfully collaborate with other researchers apart from his supervisor. From a mathematical point of view, the proofs are often not particularly complicated; however, they require a strong command of convex analysis and the theory of tensor products, which the candidate has clearly mastered.

My only critical comment concerns the presentation of the results. Since the contents of the papers are very closely related, they all come with a few pages of very similar introductions of basic definitions and examples, making the presentation highly redundant, and requiring an unnecessary amount of effort from the reader. The candidate made a good effort to give a detailed mathematical introduction into the theory of GPTs in the first half of the thesis; it would have made a much more appealing work and a better reading experience, in my opinion, if the results of the papers had been presented in a unified and coherent way after this, instead of just pasting together the papers as they were published. This, however, is a minor thing, and does not decrease the scientific value of the thesis.

Taking all the above into account, I consider the results presented in the thesis an excellent scientific achievement for a PhD dissertation, and I recommend the highest grade (A) for it without the slightest hesitation.

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