S. Hollands: Variational approach to relative entropies (with application to QFT)

(Revision)

Referee report.

The changes in the paper are satisfactory, the paper is now much more readable. I have only one point: the proof of the DPI for  $\Phi_s$  (Cor. 5) works only for properly infinite von Neumann algebras and it is not clear whether this property holds in general. The author should provide a more general proof if available, or point out specifically that DPI holds only in that case. The usefulness of the proposed divergences is limited without DPI in e.g. the finite dimensional case, which is common in quantum information theory.

Nevertheless, the proposed application in QFT, connecting  $\Phi_s$  and in particular the fidelity with the index of subfactors is worth pursuing further and the paper should be published, taking into account the above comment.

## Some minor points:

- 1. the Abstract: the entropic certainty relation (Cor. 1 and 3) is proved for subfactors, not "arbitrary von Neumann subalgebras" as stated in the last but one sentence.
- 2. p.2, last line: expectatation
- 3. In Eqs. (29) and (31), we clearly need to have  $|\zeta\rangle \in \mathcal{D}(\phi)$
- 4. Eq. (32) is obtained using the assumptions on  $\zeta$  and  $\phi$ . Some more arguments are needed for removing these assumptions and obtaining (33).