Dear Editors,

We thank the referees for a careful reading of the paper, and for their several valuable suggestions. The paper has been extensively revised, and is now considerably shorter, despite adding more requested background and motivation of the final results concerning the relation between the Petz recovery map, its dial the Accardi Cechini coarse graining operator, and conditional expectations.

In particular, we have deleted what were sections 3 and 5. While we had felt that results obtained proving our stability theorem provided a particularly clean approach to classifying the cases of equality, the complete classification is in fact well known, and the novelty of this section, as well as section 5 on the relation with strong sub-additivity, was indeed, much lower. Therefore, we have deleted these sections entirely, and of course adjusted the introduction accordingly.

We have also carefully gone over the whole paper for clarity. In addition to finding a number of typos that have been corrected, we have cleaned up and improved a number of items. For example, we have given a better proof of the main inequality that the defines the Accardi-Cechini coarse graining operator; see (1.18) to (1.20). The proof here is better in that it would also work, as it is, in finite dimensions; it uses the Lieb Concavity Theorem, and otherwise is short and elementary. The previous proof was valid only in finite dimensions. (By the way, the Lieb concavity Theorem is not used in the paper of Accardi-Cechini; the short argument presented here that would work in their setting too is new.)

We have dealt with all the points that were raised. Starting with Referee 2:

(1) Right before Sec 1.2 Pets -> Petz. This has been fixed.

(2) Mosonyi's name is misspelt in Sec 1.2. This has been fixed.

(3) There is a sentence repeated after Definition 1.3. This has been fixed; we deleted the initial instance before the definition, keeping the one after.

(4) We have added the reference to H. Fawzi and O. Fawzi regarding the referee's point that the exact quantum analog of the classical inequality (1.12)cannot possibly hold.

(5) The referee also asked for more motivation for the results in what were section 4, and in particular asked for "something slightly less vague" that the discussion of this the section summarizing the main results. The penultimate paragraph in Section 1.2, which discusses these topics, has been completely re-written and expanded. We hope that this will provide the clarity the referee is seeking.

(5) For the final point, we we agree that section 5 is less novel, recapitulating some known results, and have eliminated it.

Turning now to the comments of referee 3:

(1) We are glad that the referee appreciated the results of Section 2 as they are. We agree that Section 3, in contrast, dealt with issues that are already well-discussed in the literature, and so we have entirely

eliminated what was Section 3. In this revision, we do not discuss the structure of the sets of Solutions to Petz's equations, and only refer to published work on this question.

(2) In what was Section 4, now Section 3, we have added a reference to Wolf's "Guided Tour", as suggested by the referee, as well as some discussion of what may be found there. The interesting approach to conditional expectations in Wolf is based on the well-known structure theory for finite dimensional non Neumann algebras as a finite direct sum of full matrix algebras.. However, we do not find there an elementary proof of Takeski's Theorem on the obstacle to the existence of expectation preserving conditional expectation in the quantum setting. Dealing with this obstacle was the motivation for Accardi and Cechini and it is was led them to introduce their coarse graining operator, which turns out to be nothing other than the dual of the Petz recovery map. Hence the nature of this obstacle is essentially the raison d'etre of the Petz recovery map, and for this reason we believe it is worthwhile to have an elementary analysis of it. This is what we provide in the new Section 3, and the main Theorem there provides somewhat more information than simply the finite dimensional version of Takesaki's Theorem, so that not only is the simple proof likely to be of interest to people who want to understand the origins of the Petz recovery map, the full result itself is new in part, not subsumed by any previous results.

Sincerely,
Eric Carlen and Anna Vershynina