S. Hollands: Variational approach to relative entropies (with application to QFT)
Referee report.

In this paper, a new divergence is proposed in the general setting of normal states of a von Neumann algebra. The divergence is based on a variational expression involving the fidelity of states and is similar to Kosaki's formula for quantum relative entropy. By construction, it is related to sandwiched Rényi $\alpha$-divergence for $\alpha \in (1/2, 1)$ and is its upper bound, and it reduces to fidelity (similarly as the sandwiched Rényi entropy) in the limit $\alpha \to 1/2$.

The divergence cannot be seen as a new version of classical Rényi divergence, since it reduces to a multiple of fidelity in the commutative case. Nevertheless, it is proved that this quantity has some desirable properties (DPI, ordering) and as an upper bound of sandwiched Rényi divergence it might provide a useful technical tool. Moreover, an application in QFT is proved, showing its relation to the Jones index. Further, an entropic certainty relation is given in terms of the relative entropy and the sandwiched divergences.

While the results are of interest and are possibly worth publication, the way of presentation is unsatisfactory. There is a lot of confusion, omissions, typos, the arguments are mostly rather sketchy and imprecise. It seems more like a first private sketch of a paper, certainly not a manuscript suitable for publication. Some remarks and typos are listed below, but it is not an exhaustive list. Therefore my recommendation is to reject the paper. I suggest that the author resubmits the paper, but it should be carefully rewritten before it can be considered for publication anywhere.

**Some comments:**

1. page 1, line -3: a channel is (usually) defined as a unital map, without such an assumption, the DPI in Eq. (2) is not true

2. p.1, footnote: why the mix-up with the Schrödinger/Heisenberg picture? Also, $\rho[T]$ should be $T[\rho]$.

3. p. 2: a reference should be given for the operational interpretation of sandwiched Renyi entropies for $\alpha \in [1/2, 1)$

4. p. 3, line -6: the support of $\Delta_{\psi,\zeta}$ is $\pi^{\mathcal{M}}(\psi)\pi^{\mathcal{M}'}(\zeta)$
5. p. 4: in all the references [20,24,34], as well as [18], it is assumed that $\mathcal{N}$ is a subfactor
6. p. 5 and 6: it is not clear how the exposition in the last paragraph on p. 5 and beginning of p. 6 is related to Appendix A
7. Eq. (22) some primes are missing
8. p. 8, just below Eq. (29): why $|\zeta\rangle \in \mathcal{D}(\Delta^{-1/2})$? (Note that in [Lemma 5.9, 33] $\zeta$ is a representing vector for $\psi$.)
9. p.8, last line of Prop. 1: $a \leftrightarrow b$
10. p. 9, line 2: "But then..." which variational formula do you have in mind? (There is a variational formula for $\|\zeta\|_{p,\psi}$, but with no obvious relation to fidelity.)
11. p. 10, first paragraph: Coro. 4 clearly follows from the variational formula and Eq. (38), why are the norms considered here?
12. p. 10, Coro. 5: why the assumption that $\mathcal{M}$ and $\mathcal{N}$ are properly infinite?
13. p. 14, there seems to be a lot of typos, I basically stopped reading here.
14. Footnotes: there is quite a lot. I suggest to avoid footnotes and/or include them into the text.