M. Weiner: Realization of an arbitrary structure of perfect distinguishability of states in general probability theory

Referee report

This paper studies the problem of perfect distinguishablity of states in a generalized probabilistic theory (GPT), where physical state spaces are represented by convex sets. A set of states $s_1, \dots, s_n$ in a given state space S is jointly perfectly distinguishable if there is some measurement with n-outcomes which gives the output j with probability 1 if the true state is s_j, for all j. Equivalently, there is an affine map of S to the simplex Δ_n that maps each s_j to the j-th vertex (in other word, this means that the afffine map from the simplex Δ_n to S mapping each j-th vertex to s_j is a section).

More generally, for a given set set of states $s_1, \dots, s_n$, the system of subsets of the set of indices $[n] = \{1, \dots, n\}$ such that the corresponding subset of states is perfectly distinguishable, is an independence system. The result of this paper is that any independence system of subsets of $[n]$ can be realized in this way. More precisely, there is a convex set (in fact, a convex polytope) S and states $s_1, \dots, s_n \in S$ such that the independence system describes precisely the jointly perfectly distinguishable subsets of $\{s_1, \dots, s_n\}$.

Overall evaluation

Of course, it is a natural question whether the systems of perfectly distinguishable subsets in arbitrary GPTs have any further properties other than being an independence system, this question was answered here in the negative. But, frankly, the paper does not seem very interesting and it is not clear what a reader would gain. The result seems not really surprising, and the method of proof is not particularly involved or illuminating, and does not provide much insight.

If the author could argue some more on the importance of this result and its consequences from the point of view of physics or mathematics or convex geometry, or perhaps share some interesting geometric ideas behind the proof, I would gladly reconsider my evaluation of the paper.

Small remarks and typos

- p.3, secod paragraph: we shal conclude... We... (sentence should begin with a capital letter)
- p.3: Is P_e^* here the average? Otherwise it is not necessarily ≤ 1.
- p.4, Eq. (2.3): k should be m (?)
- p.5, last line: $\Phi(j)$ should be $\Phi(s_j)$
- p.6, line 2: better write that ...still having the property that it maps s_j to the j-th vertex...
- p. 7, line 11 from below: Δ_m should be Δ_n
- p. 7, line 7 from below: $\hat{\phi}$ sould be $\hat{\Phi}$
- p. 7, line 5 from below: $\hat{\Phi}(S)$