

R. Beneduci, T. Gentile: Fuzzy observables and the universal family of fuzzy events

Referee report

The most general description of quantum mechanical observables is given by positive operator valued measures (POVMs), while the spectral measures corresponds precisely to projection valued measures (PVMs). It is now well known that a POVM F is commutative (that is, ranges in a commutative subalgebra) if and only if it is obtained from a PVM (called the sharp reconstruction of F) by a "randomization" or "smearing". This result and characterizations of the sharp reconstruction and of the randomization were studied in a number of papers, including several works of one of the present authors. The paper under consideration gives a construction of a universal Markov kernel μ^U , such that any commutative POVM F defined on the σ -algebra $B(X)$ of a Hausdorff locally compact second countable topological space X is obtained from a real PVM by a randomization described by μ^U . It is also shown that any other such randomization, given in general by a weak Markov kernel, can be obtained as a function of the universal Markov kernel μ^U .

The paper is based on a transferring principle, which is a modification of a classical result by Jessen and consists of finding a bijective mapping f of $[0, 1]^\omega$ onto $[0, 1]$ with certain properties. The construction is not very well described and also the subsequent proofs are confusing and difficult to follow. However, this construction seems very similar to one that appeared already in the paper [14] by Cattaneo and Nistico and the basic ideas of the proofs closely follow the previous works [4,6,7] of one of the authors dealing with the special case of real POVMs.

While the results might be of interest for readers of FSS, the proofs seem the same as in the real case and it is hard to find any novel ideas. The paper is not well written and difficult to read. So in the present form, the paper cannot be recommended for publication.

I suggest that the authors rewrite the paper, focusing on the following points:

1. It should be clearly explained what is **essentially** new here compared to the previous works. What are the main technical obstacles the authors had to tackle to obtain the present results?
2. The constructions leading to the transferring principle should be written in a clearer and more explicit way, so that the computations can be more easily understood.
3. The proofs should be better written. For example at many places, the authors use subindices like r_l that are in fact treated as pairs of indices (as on p. 12, where some expressions with index r_l^k are summed over r and k , etc), this is quite confusing. There is also a number of typos.

With the suggested changes made, the paper might be resubmitted and considered again.