Ting Zhang, Xiaofei Qi: Sandwiched Rēnyi relative entropy on density operators

Referee report

The authors prove a characterization of all surjective maps on the positive cone of the trace class operators on a Hilbert space that preserve the sandwiched Rényi relative entropy D_α for some $\alpha \in (0,1)$. Their result is a generalization of previous results for finite dimensional Hilbert spaces [5] and finite C*-algebras [9]. The paper is a contribution to the theory of preservers of quantum divergence measures, developed mostly by the author of [5] and [9] and connected to the famous Wigner's theorem on representation of quantum symmetries.

The paper is well written and good to read, generalizing an important result to infinite dimensional Hilbert spaces and the generalization is nontrivial. I have only a few minor remarks to be considered by the authors:

1. It might be remarked in the Introduction that there are different quantum versions of the Rényi relative entropy. Perhaps most notably the one called the standard Rényi relative entropy, with important operational interpretations, for which the preserver problems were also solved.

2. The authors are not the first to extend the sandwiched Rényi relative entropies to infinite dimensions, there are works extending D_α even to general von Neumann algebras. It would be appropriate to cite some of them.

3. page 3, line 10: "Particularly, in the case... the definition... is given independently in [1]" I am a bit confused by this statement, since the definition was not introduced in [1] and it is also not claimed there. Also the next two sentences should be somehow reorganized, since the term "sandwiched" is used in the first and introduced in the second.

4. proof of Prop. 3.6: the operators T_1 and T_2 should be assumed nonzero. In the final argument, one should say that the contractions D_i and D_j can always be chosen so that $T_1^{\frac{1}{2}}(D_i - D_j)T_2^{\frac{1}{2}} \neq 0$.

5. page 8, line 6 in Claim 3: an exponent $1/\alpha$ is missing

6. page 8: "independent with" independent of

7. page 11, reference [9]: the name of the author is misspelled (the accent is at a wrong place)