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Report on the PhD thesis by Nidhin Sudarsanan Ragini : Labeling outcomes of Quantum Devices and Higher-order distinguishability

In the thesis, two problems concerning higher order quantum testers are studied. In the first part, the task of labeling of a quantum measurement (also called an observable or a POVM) is discussed. This can be cast as a measurement discrimination problem, where the ensemble consists of measurements that are obtained from all possible labelings (permutations) of a given set of effects summing up to identity, with equal probabilities. In this setting, perfect discrimination, minimum error and unambiguous discrimination is considered in various regimes: one shot or multiple-shot, using simple, entanglement-assisted or adaptive schemes. In the second part, the role of incompatibility of quantum testers as a resource in discrimination tasks for quantum combs is demonstrated by a generalization of the main result of the paper [SŠC19] by Skrzypczyk, Šupić and Cavalcanti from POVMs to quantum testers.

The problem studied in the first part is quite natural and although it is rather simple, the obtained results are interesting and nontrivial. The work in this part well demonstrates the capabilities of the student for working with the formalism of quantum combs and testers. The results in the single shot case were already published in Phys. Rev. A, which is a highly respected journal in the field of quantum information theory. The multiple-shot case is published as a preprint in arXiv. The second part follows the ideas of the original paper [SČS19] very closely, almost step-by-step, adding necessary modifications. Nevertheless, the obtained generalization is valuable as it brings a much wider applicability to many different settings. The results of this part are published in an arXiv preprint as well.

At this point, it could be said that the thesis contains enough results to be accepted for awarding the PhD title, though only one paper has been published so far. But there are some issues that have to be addressed here.

The first is the thesis presentation. What one notices very quickly is the somewhat strange writing style, with long, complicated sentences, often grammatically incorrect, that are difficult to decipher. It is also very repetitive, explaining basic concepts over and over again. A little less would be much more here. On the other hand, there are some places where it would be appropriate to write more calculations explicitly, e.g. between Eq.(5.25) and Eq.(5.27) and the text below them, or large parts of Chap. 6 (see also below).

Another thing is a rather large number of typos and the use of semicolon (;) instead of fullstop (.) at the end of sentences. This could have been easily avoided by using spellcheck and simple proofreading.

There are further instances where the thesis could be improved. Just a few examples:

- The first part of Proposition 2.1.1 is obviously false (just put $X = Y = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$)
- partial transpose is missing in the link product Eq.(2.33)
- Corollaries 3.2.1 and 3.2.2 are given as immediate corollaries of Proposition 3.2.1. But Prop. 3.2.1 is formulated (and proved) as a necessary condition, whereas Cor. 3.2.1 is a necessary and sufficient condition, Cor. 3.2.2 gives a sufficient condition.
- Theorem 4.2.2: the maximal absolute eigenvalue of X is the usual operator norm $\|X\|$, I do not understand why it is called, and denoted, as the operator 2-norm
- There is a mistake somewhere in Lemma 4.3.1, it works only if $\operatorname{Tr}\Pi_k = 1$
- Prop. 4.3.1: In fact, we have $\alpha_{\mathsf{M}} = \max_j \|M_j\|$
- Eq.(5.2) and the sentence below: strictly speaking, the summands are not positive semidefinite (but Eq.(5.3) holds, since the whole sum is block-diagonal, so each block must be 0) Similar wrong argumentation is used repeatedly.

- Theorem 4.2.1 is immediate, without use of Prop. 3.2.1, since if both M and $I-M$ are full rank, then both Choi matrices are full rank as well.

But the most serious problems are the following two:

1. It is claimed repeatedly that in the binary multiple-shot scenario, the error probability is increasing with the number of shots (e.g. at the end of Sec. 5.1.2, beginning of Sec. 5.2 or in the concluding section). But the multiple-shot scenario naturally contains single shot: we can use one shot for discrimination and discard all the other. So the error probability certainly cannot increase with the number of shots. Something must be wrong here, either in Eq.(5.27), which was derived here in detail only for $n = 2$, or in the subsequent paragraph, where it is simply stated that: "...we find that the error is increasing with more number of shots", without any further explanation or argument.

2. Chapter 6 is not readable. There are notations that are not introduced anywhere (e.g. $\mathsf{Q}_{\vec{a}}$, $d_{\vec{a}}$, $\tilde{\mathsf{Q}}_{\vec{a}}$, $\tilde{\Theta}$, etc.) The text is taken from the arXiv preprint arXiv:2405.20080, with some shortenings that removed some important parts. But the preprint itself is not well written. In particular, the paragraph between Eq.(6.35) and Eq.(6.36) makes little sense, moreover, I would say that the dual SDP in Eq.(6.36) is wrong. I still believe that the main result, Theorem 6.2.1, is true, but the proof should be improved substantially.

In conclusion, based on the present thesis, the PhD title can be recommended provided the issues described in the points 1. and 2. above are sufficiently addressed in the defense.

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