Review 2

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Review of “Integral formula for quantum relative entropy implies data processing inequality”

for *Quantum*, completed on Feb 21, 2023

Author will not see the reviewer's name or the date completed.

Overview questions

only editors will see response

Overall rating

For this manuscript I recommend...

Revise and resubmit

Comments to editor

The main result of this paper is a new integral formula for the quantum relative entropy. The formula is used mostly to obtain an alternative proof to known data processing inequality, but I am convinced that it is a significant result that definitely should be published.

My recommendation is to revise and resubmit, mostly to give the author an opportunity to improve the presentation and increase the impact of the paper.

Open response questions

Note: if you prefer to submit a free-form review instead of filling in this form, simply reply to the invitation email with your report as an attachment. Due to Scholastica's limitations, please make sure that you send it from the same address at which you received the invitation. If that's not possible, you can email the review to info@quantum-journal.org . Otherwise, just write "ok" in the reply to this question and proceed to the rest of the form.

only editors will see response

ok

Summary: what are the main questions posed by the manuscript and how does it answer them?

intended for the author

In this paper, a new integral formula is proved for relative entropy for a pair of density matrices. This formula is applied to obtain an analogous integral expression for higher derivatives of the entropy function. These formulas lead to easy proofs of some properties of these quantities, such as convexity and nonnegativity, as well as the data processing inequality of the relative entropy with respect to (not necessarily completely) positive trace preserving maps. Moreover, some properties of the Holevo quantity are re-established using these results.
In the last part of the paper, it is shown that for a general divergence on density matrices satisfying cetrain data processing inequality, the infimum of the value of the divergence on all pairs of density matrices with a fixed L_1-distance is attained on binary classical states.

What is your assessment of the paper? If you recommend acceptance, make a case that this work does indeed make a significant contribution to scholarship.

intended for the author

Integral formulas for relative entropies and other related quantities
in quantum information theory are very useful, expressing the quantity in question in terms of simpler expressions. This is often helpful in proving important properties such as positivity, convexity or data processing inequality, or for finding expresions for derivatives and their properties., etc.The integral formula presented in the paper is obtained by techniques which are rather classical, but in any case different from those used previously (based on integral expressions of operator convex functions). I find the resulting formula very interesting and important, since the relative entropy is expressed in terms of quantities that can be directly related to the L_1 distance and success probabilities in state discrimination problems.
This connection is especially important in view of the operational interpretation of the relative entropy. Together with the expressions for the higher derivatives, it can also be

potentially useful for some technical considerations, higher order asymptotics or for studying some related quantities (e.g. such as the monotone metrics (quantum Fisher information) obtained by second derivatives of the relative entropy.)

The author applied these formulas mostly to reprove previously known results. In particular, the title and the introduction suggest that the main application of the integral formula would be the proof of the data processing inequality for the relative entropy on density matrices, in its most general form. But such a DPI was already proved before, using quite different methods, and was extended to much more general infinite dimensional settings where the methods of this paper do not seem applicable. Anyway, I find that the potential impact of the results is much higher.

To what extent have you checked the technical correctness of the paper?

intended for the author

I read through all the proofs and find them correct. The paper mostly uses classical methods of linear algebra and complex analysis. The proofs could be written more clearly in some parts.

Comment on the presentation of the paper. Is it well written? Are the main results clearly laid out? Does the manuscript clearly describe assumptions and limitations? Is the literature review adequate?

intended for the author

The paper is mostly well written, but the proofs could be somewhat expanded to improve readability for readers not so much used to the presented techniques. In the introduction, the relevance and significance of the results could be more stressed. The last part on the generalized divergences is lacking some explanation of its significance: why the infimum of a divergence on states of a fixed L1-distance would be interesting? Also a discussion/conclusions section is missing.

If the submission includes numerical or physical experiments, does it provide sufficient details such that they could be reproduced by readers? This includes for example source code, documentation, experimental data, experimental setup specifications, etc.

intended for the author

Not applicable.

Suggested changes, corrections, and general comments.

intended for the author

A summary of suggestions which may improve the paper:

- In the introduction, elaborate some more on the significance and relevance of the results for quantum information theory, e.g. by pointing out the relation of the expressions in the integral formula to quantum hypothesis testing, or suggesting some further possible applications.
- A Conclussions/Discussion section is missing. Some hints on possible further lines of research or open questions would increase the impact of the paper.
- Add more explanation and/or motivation in Sec. 6
- It would be good to expand the proofs in Sec. 2 and 3 a little. (For example, stress that A(t) is a function of complex variable, or explain that, in the first displayed equation on p.5, r is a function of t and r^+ is its positive part). It would be also good to summarize briefly the idea of the proof at the beginning of Sec. 3.

Would you be willing to referee an updated version of this work before a final decision is made?

only editors will see response

Yes.

Do you think this is an outstanding work that deserves to be highlighted? If this work is accepted, would you be willing to write a short Perspective (opinion piece similar to a viewpoint or editorial) based on your report? For examples, see: http://quantum-journal.org/category/Editorial,Perspective/

only editors will see response

It depends on the final paper.

How was your reviewing experience? Is there anything you would like us to improve?

only editors will see response

Ok
