L.E. Labuschagne and W. A. Majewski, Dynamics on noncommutative Orlicz spaces

Referee report

The paper deals with the possibility of extension of certain quantum Markov maps on von Neumann algebras to a class of non-commutative Orlicz spaces defined and discussed in previous works of the authors. Such Orlicz spaces are claimed to form the proper framework for statistical physics of regular systems and the present results complete these works by inclusion of a description of dynamics on such systems.

Basic questions.

- 1. Are the main results new? (YES)
- 2. Are the main results interesting? (SUFFICIENTLY)
- 3. Are the statements and the proofs of the basic results explained clearly and without redundancies? (NO, see the explanation below)
- 4. is the paper accurate in its citations? (SUFFICIENTLY)
- 5. Which priority do you suggest for publication? (I do not recommend the paper for publication)

Overall evaluation.

The paper contains several mathematical results that may very well be sound, interesting and even important, but the problem is that a reader, especially one not extensively familiar with the previous works of the authors, has little chance to appreciate or understand them. My main objections are as follows:

- 1. The paper is lacking a proper introduction. I understand that this is a continuation of several other papers, but a brief motivation and explanation of the context, with exact references to these works should be given, pointing out the paper where this is done properly. A discussion of similar works by other authors should also be included.
- 2. The paper uses advanced techniques of several fields (interpolation theory, Orlicz spaces, von Neumann algebras, etc). Therefore, careful thought should be given to what parts have to be introduced here, or perhaps postponed to appendices, and where just give references. This must be done not only for the purpose of the proofs, but also for better understanding

of the reader. In the present paper, this is done quite arbitrarily: e.g. the Dodds, Dodds and de Pagter construction is discussed to some extent, but the construction in type III case of [16], which is the context used here, is hardly even defined; the extension of positive normal maps to Haagerup L_p -spaces, which does not seem to be needed, is quoted from [12] but the extension to Orlicz spaces in the semifinite case is just stated as fact, without any description, in the proof of Prop. 4.6, etc. It should be possible for a reader with some background to understand at least the idea and structure of the proofs without having to study all the other works by the authors, this is hardly possible here.

3. statements like Theorem 2.3 and 2.4

The dual pair ... provides the basic mathematical ingredient for a description of a general classical/quantum regular system

This is not a mathematical theorem. Without proper context, this is hardly a meaningful sentence. It should be explained in the introduction what properties should a space have to be "the basic mathematical ingredient". Then the authors can (if they wish) state as a theorem that the given Orlicz spaces have these properties.

4. The exposition is not very well structured and difficult to follow. Besides, there are many omissions, lacking notations, etc, see also the specific remarks below.

Specific remarks. These are just a sample choice, there are many more problematic places.

- the meaning of Ψ^* is not yet clear on p. 3 (definition is given much later).
- This might be a matter of taste, but I would suggest to refrain from using emphasis and exclamation marks (apart from their proper places). A good paper does not need any advertisement slogans.
- It is not clear what is σ in the formula (3.1). The reader has to know what a crossed product is in order to understand your definition, but in that case, why give it?
- Def. 4.2., it is not clear what is $L_1(\mathfrak{M})$ (I guess this is the Haagerup L_p -space? later to be denoted by L_p)
- Theorem 4.3: why π_{α} ?
- I do not understand the purpose of Remark 4.4.
- What happened to the condition $T \circ \sigma_t^{\nu} = \sigma_t^{\nu} \circ T$ in Prop. 4.6, 4.7 and Coro. 4.13, 4.14? But perhaps I am missing something (in that case see objection 2 above).

- Def. 5.3: this is the extension of the previous section, applied to the map satisfying the DBC (right?).
- It is claimed in the abstract that:

As a consequence, we obtain that quantum dynamics in the form of Markov semigroups described by some Dirichlet form, naturally extends to the context proposed in [21].

But the extension of such maps is mentioned only in the last point of Sec. 6, stating that "this topic... exceeds the scope of this paper".

To come to some conclusion: I may be old-fashioned, but I am of the opinion that the purpose of a scientific paper is to communicate and explain some results to others with necessary background and similar interests, aiming at being as understandable as possible. The present paper does not fulfill this purpose, therefore cannot be recommended for publication.