In this paper, the authors obtain entropic uncertainty relations for a class of measurements consisting of interactions with a quantum process at multiple times and a final measurement. These relations include previously known results for measurements on quantum states and channels.
The results are obtained using the framework of quantum combs as well as the techniques of majorization lattice, applied in a previous work concerned with measurements on quantum channels. For specific pairs of measurements, compatible with the common cause resp. direct cause causal structures, it is shown that the obtained bound is tight and the uncertainty trade-off is analogous to the position and momentum measurements.

The results in this paper are timely and important, since the interactive measurements describe a general class of measurements that can be applied to all quantum processes that have a definite causal structure. The authors also provide a nice and easily understandable operational interpretation of the lower bound, that can be obtained from success probabilities in a guessing game they call a quantum roulette. The main text is written an a concise and accessible style, with illustrative diagrams that are easy to interpret.

I have only a few minor comments, listed below.

Main text:
- p.1, col. 1: ''Could such a fundamental...'' something seems missing in this sentence

- Fig. 1: ''the an'', ''dynamica''

- p. 2. col.1, line 16 from below: -the state of- H_{2k+1}

- p.2, col. 2, line 6 from below: ''log(1/c)'' it is not clear what ''c'' is (define or describe)

- Theorem 1: ''has'' - > have

- p. 3, col. 2: ''direct-case'' - > cause

- Eq. (3): it is not clear what $H(T_i)$ is

- last sentence ends in ..

Supplemental material:

- p.1, last line: ''player''?

- p.2: better define nonsignaling

- p.2: better define T_E (partial transpose)

-p.4, last sentence: ''though'' -> if (better)

- p.5 beginning: flatness process somewhat unclear (what happens if x is already nonincreasing?)

- p.5, line 2 in Sec. I.C: ''proving'' -> probing

- p.5 (and elsewhere) ''successful probability'' -> success probability

- p.6: better define p oplus q

- p.8: better give some reference for the characterization of quantum combs used in Eq. (49)

- p.9: One should be more precise here. Q is a subset of the unordered P_n^d, here majorization is just a preorder, so it cannot be a lattice. The preorder defines an equivalence relation, the quotient of P_n^d with respect to this is isomorphic to the ordered P_n^d and the preorder becomes a partial order. So, in fact, (vee Q) is an equivalence class.

- p.9 the notation (vee Q_k) is a bit confusing. It took me some time to realize that this is not the LUB of some set Q_k, but the k-th element of the sequence (vee Q). It would be better to explain this.

- p.11, Eq.(75): Similarly, ''4 vee Q_2'' looks like the LUB of 4 and some Q_2.

-p.11, below Eq. (77): ''families of infinite unitaries'' ?

- p.11, Sec. IV.A: sentence ending as ''...bipartite unitary.'' What is the bipartite unitary here?

- p.11, Eq. (84): why the direct-cause circuit cannot be of the form rho_A otimes E_{B\to FC}?

- p.12, just below Eq. (85) ''B to C'' -> B to F.

- p.12, line 4 from below: Phi_{AE} -> phi_{AE}

- p.13: it seems that the unitary operator U and the corresponding unitary channel are denoted by the same letter. Better change this.