

**Reviewer Recommendation and Comments for Manuscript Number FSS-D-21-00091R1****Fuzzy observables and the universal family of fuzzy events**

Revision Number 1  
 Anna Jencova **Reviewer 2**

[Back](#)[Edit Review](#)[Print](#)[Submit Review to Editorial Office](#)**Recommendation:** **Accept****Transfer Authorization****Response**

If this submission is transferred to another publication, do we have your consent to include your identifying information?

Yes

If this submission is transferred to another publication, do we have your consent to include your review?

Yes

If this submission is transferred to another publication with "Open Peer Review", do we have your consent to publish your review in a pre-publication history?

Yes

**Reviewer Comments to Author**

The authors addressed all my comments in a satisfactory way. Their motivations become clear and justified, the proofs are much more readable. The paper can be recommended for publication.  
 A few remarks:

- the notion of a "fuzzy observable" is often mentioned, but as far as I can see, never really defined. This term may be familiar to readers of FSS but perhaps not for all researchers in the field.

- p. 4 (and also p. 20): "proof... has been proved" it is better to write "proof... has been given" or "proof ... can be found" etc.

- p. 5, line 3 from below: "than" -> then

- Def. 3 and below: note that an orthogonal POVM may be not projection valued if it is not normalized. Usually a POVM is assumed normalized, but according to your Def. 1 it is not necessarily so.

- p. 12, line 9 from below: here " $x_{[0,1]^\omega}$ " is missing in the expression for  $I^{\{n\}}_l$

- p. 12, line 3 from below: the lower indices in  $i^{\{n\}}_1$  and  $i^{\{n\}}_2$  should probably be  $j$  and  $l$

- p. 13, line 4:  $D_k \rightarrow d_k$  and  $E^F \rightarrow E^F_0$

- p. 16, first displayed equation: better specify the limit. (It seems to me this is actually a supremum of an increasing sequence of projections (??))

- p. 22, line 8 from below: "denotes" -> denote

- p. 24, line 3 from below: better give some explanation or reference for the existence of the Borel function  $f$

- just a question: The (weak ) Markov kernels can be seen as POVMs, so it is natural to ask whether the functional subordination can be interpreted as some of the usual preorders of POVMs, such as post- or (more likely) preprocessings.

**Reviewer Confidential Comments to Editor:**

REFeree REPORT FOR THE AUTHOR  
 (Feel free to upload a pdf file instead)

**General evaluation**

Please rate the following criteria using only one of the five following words: BAD, WEAK, FAIR, GOOD, EXCELLENT

- a) Relevance to Fuzzy Sets and Systems:   good
- b) Originality:   good
- c) Significance; usefulness:   good
- d) Technical soundness:   good
- e) Reference to the related literature:   good
- f) Presentation:   good
- g) Linguistic quality:   good

Summary of the paper and appreciation of the contribution (please comment on novelty, significance, appropriateness for the journal):

2 ?

The paper studies commutative POVMs (fuzzy observables) and their relation to PVMs (sharp observables). It is known that any commutative POVM is a randomization (or "smearing") of a PVM, the authors give a general construction of a universal Markov kernel performing the

randomization for each such POVM on Borel subsets of a Hausdorff locally compact second countable space  $X$ . In this way, the paper extends and completes previous works in the area and answers some questions that were left open.

Technical comments to the authors (please explain in which way the paper should be revised, including presentation, technical deficiencies, connection to related work, etc.):

See the comments above

-----  
CONFIDENTIAL PART

Type of paper

- a) ☒ Research paper (with original results)  
b) ☐ Application paper  
(experimentation ☐ / case study ☐  
c) ☐ Survey of recent advances  
d) ☐ Position paper  
e) ☐ Other. Please specify \_\_\_\_\_

Main reasons for your recommendation:

- For choice accept please indicate:

- 1) ☒ accept because of the originality (good ideas, sound presentation)  
2) ☐ accept because of the quality of the proposed synthesis (useful review on recent advances)

- For choice reject please indicate:

- 1) ☐ reject because it is not relevant to Fuzzy Sets and Systems  
2) ☐ reject because of the presentation  
3) ☐ reject because the content is too premature  
4) ☐ reject because of lack of originality (results already known; similar overview already published)  
5) ☐ reject because of major errors

Your level of expertise:

- a) ☐ I am an expert of the field and know the relevant literature  
b) ☒ I understand the problem and I have some knowledge of the field  
c) ☐ I only have superficial knowledge of the issues

Could you indicate whether:

- a) ☐ you would prefer seeing the paper again after revision,  
or  
b) ☒ you do not think it necessary to check the revision yourself.

Please note that your recommendation and reviewer report are expected to cover the Highlights and Graphical Abstract if submitted with the manuscript.

Additional comments for the Editors on the paper:

---

[Back](#)

[Edit Review](#)

[Print](#)

[Submit Review to Editorial Office](#)