J. Pitrik, D. Virosztek: QUANTUM HELLINGER DISTANCES REVISITED

Referee report

The authors study quantum versions of the Hellinger distance known form classical statistics and information theory. They propose a class of generalized quantum Hellinger distances, based on the family of Kubo-Ando means of positive matrices. This paper can be seen as an extension or complement to a recent paper by Bhatia et al. [7], where the distance obtained from the geometric mean $\#$ of Pusz and Woronowicz was studied. The introduced distances are not new, it is proved that they belong to the class of maximal f-divergences, which also implies some of their good properties. The barycenters with respect to these distances are studied. In particular, it is proved that the claim in [7], that the barycenter coincides with the weighted power mean, holds for commuting matrices but is not true in general.

In my opinion, this last observation is the most important result of the paper, together with the characterization of the barycenters. Also the relation to Bregman divergences (Claim 2) is potentially of interest. However, the Part 4 on the commutative case does not contain any citations. I think the authors should do some search through the literature, since this characterization may be already known in the classical case. For example, S. Amari,Integration of Stochastic Models by Minimizing α-Divergence, Neural Computation 10, 2780-2796, 2007 contains the results of Example 8.

After taking this and further comments below into account, the paper could be recommended for publication in LMP.

Further comments

1. I think the notation λ is somewhat overused in the paper (weight parameter, integration parameter), this might be confusing.
2. Remarks on p. 4, first paragraph: is there some *substantial* property of the generalized Hellinger distances that is not shared by all f-divergences? The "counter-intuitive phenomena" mentioned here can be remedied by adding a linear term.
3. A question to Part 4: assume that all A_j are contained in some MASA. It seems natural to ask whether in that case the barycenter is also in that MASA, so it has the form as in Prop. 7.