Mohit Lal Bera and Manabendra Nath Bera: Quantum Bayes' rule affirming consistency in measurement inferences in quantum mechanics

Referee report

The authors propose an input-output causal relation based on a quantum Bayes' rule, applicable to situations with causes/effects in coherent superpositions or parts of joint systems. The difference to classical causal relations and Bayes' rule is demonstrated on some examples and the use of the new relations is proposed as a remedy to some paradoxes appearing in the literature, namely the Frauchiger-Renner and the Hardy paradox.

Overall assesment: In my opinion, the paper does not contain enough relevant new material to be published. See the comments below.

1. It is not clear what are the results of this paper. What are the questions asked, and what is the answer? Some definitions are introduced (Defs 1 and 2), but restricted to the case of unitary processes and pure states, when the relations seem quite trivial. (Note also that the equalities (8) and (9), resp. (13) and (14) are fully equivalent, so one of them is always redundant.) It is then shown on concrete examples that the obtained relations differ from the classical causal relations, which is not at all surprising.

2. I am not sure about the relevance of this approach to the Frauchiger-Renner and Hardy paradoxes. Note that the reasoning in [4] is based on the Born rule and is not explicitly using classical causal relations as described in the present paper. In any case, this is not sufficiently explained.

3. Definition 1 is based on previous works (Ref. [7]). Definition 2 may be new, but there is little explanation and no analysis of its consequences. In particular, consider the condition $\tau_S = Tr_B[\tau_{RB}]$, below Eq. (10). Note that the channel Λ_M defines a channel $\Phi : B(H_S) \to B(H_R \otimes H_B)$ given as $\tau_S \mapsto \Lambda_M(\tau_S \otimes |0_A\rangle\langle 0_A|)$. Let $\Phi_R := Tr_B \circ \Phi$ and $\Phi_B = Tr_R \circ \Phi$, then Φ_R and Φ_B are compatible channels and according to the definition, for the effect τ_B to be predicted by the cause τ_S, we must have $\Phi_R(\tau_S) = Tr_R[\tau_{RB}] = \tau_S$ and $\Phi_R(\tau_S) = Tr_B[\tau_{RB}] = \tau_R$. But this restricts the sets of possible causes and/or effects rather severely. For example, if we accept any state τ_S as a possible cause, then Φ_R would have to be identity (note that $H_R = H_S$). But then the compatible channel Φ_B would have to be a replacement channel, mapping all states to a single one, so a single possible effect. In general, this definition restricts the sets of possible causes and effects to mutually orthogonal sets of states (or convex combinations thereof). This might be a desirable property of the setup, but no explanation is given here.