Editor

Dear Martin,

We have received the referee reports for your paper "On the properties of spectral effect algebras". You may find them enclosed.

Based on their recommendation, we are likely to accept the paper after you carefully **revise and resubmit** your manuscript, addressing the points raised by the referees.

Of special importance are that you improve the presentation by making it more self-contained, as per the suggestions of one of the reviewers.

Please upload the **revised version** and a **response letter file** to the points raised by the referees through Scholastica. It is in your own interest to ensure that the Referees and Editor can easily **retrace your revisions** (e.g., by attaching a detailed list of changes or a version of the manuscript with the changes highlighted). There is no need to update the arXiv version at this stage. If you have **confidential information** for the editor, please upload it as a **separate file**.

Best regards, Chris

Chris Heunen Editor at Quantum

The Editorial Policies of Quantum can be found here. By submitting your work to Quantum you have agreed to its terms and conditions.

Reviewer 1

Summary: what are the main questions posed by the manuscript and how does it answer them?

This paper proves a conjecture concerning spectral effect algebras. The problem solved is whether there are spectral effect algebras with a finite number greater than one of contexts. This is an interesting problem and the authors give an elegant solution. They also discuss the existence of some combinations of spectral effect algebras which is also of interest.

What is your assessment of the paper? If you recommend acceptance, make a case that this work does indeed make a significant technical or conceptual contribution to scholarship (including experimental methods and/or mathematical tools).

They give good motivation for studying spectral effect algebras and make some significant contributions to this field.

To what extent have you checked the technical correctness of the paper?

I have checked the work fairly carefully.

Comment on the presentation of the paper. Is it well written? Are the main results clearly laid out? Does the manuscript clearly describe assumptions and limitations? Is the literature review adequate?

The presentation is good and the paper is clear.

Suggested changes, corrections, and general comments.

I have no suggestions

Reviewer 2

Summary: what are the main questions posed by the manuscript and how does it answer them?

What is your assessment of the paper? If you recommend acceptance, make a case that this work does indeed make a significant technical or conceptual contribution to scholarship (including experimental methods and/or mathematical tools).

In my opinion, the paper is certainly accessible to anyone working in quantum foundations or quantum information. The material is self contained and rather well organised. The results are technical but the mathematical content of the paper is simple and I think that the results could be interesting for the quantum foundation community.

To what extent have you checked the technical correctness of the paper?

I did not analyse the proofs in detail (however, everything seems to be in order).

Comment on the presentation of the paper. Is it well written? Are the main results clearly laid out? Does the manuscript clearly describe assumptions and limitations? Is the literature review adequate?

I believe that a more detailed presentation of the concepts is needed. Regarding this issue, I have some suggestions that I would like the authors to consider.

Suggested changes, corrections, and general comments.

1. The introduction is well written. However, I would suggest to include a reference to arXiv:1511.01181, which is devoted to spectral decompositions in probabilistic theories.
2. I think that the paper would benefit by a review of working examples of the main notions like effect algebras, sharp and extremal effects, contexts, etc... Combining the abstract definitions with concrete examples from quantum theory, classical theory or PR-boxes would surely help a reader to familiarise himself with the mathematical framework used in the paper.
3. In section 3 (below Definition 7) the sets of states \hat{a}_i are introduced. Since they are a technical tool which plays a significant role in the proofs, I would emphasise their definition by placing it inside Definition 7 (or maybe adding a further definition). Similarly, the last paragraphs of section 3, where E-exposed faces are discussed, are a bit hasty.
4. Definition 9 and 10 lack the accuracy and the clarity which one would expect from mathematical definitions. I am not saying that they are incorrect or sloppy but I think that their narrative style is definitely not appropriate. As I wrote before, I suggest to provide concrete examples of

direct product and direct sum effect algebras. Finally, the comment below definition 10 is unclear to me. Looking at Ref. [19, def. 6], it seems to me that the effect algebra of the direct sum of state spaces is the direct product of effect algebras as in def. 9 of the present paper. I apologise if overlooked some relevant piece of information. However, I think it would be interesting to have a more detailed discussion about the state-effect duality.