

Reply to the referees and a list of changes

We are grateful to the referee for careful reading of the manuscript and providing helpful comments. All the comments were respected in the revision. Accordingly, the following changes were made:

- References [18] and [40] were added and cited in the Introduction, regarding the classical notion of sufficient statistics.
- In the proof of Lemma 2.6, a sentence was added explaining that the equality $a_1 = a_2$ is obtained from strict operator convexity of $t \mapsto t^{-1}$ on $(0, \infty)$.
- In the last sentence of Remark 4.4, it is specified that only positivity is assumed in Proposition 4.3.
- References to sections, equations and theorems were corrected.
- Definition of the generalized s -number was added at the beginning of Sec. 5.1.
- In the second paragraph of Sec. 7 (Concluding remarks), it is specified that by the "proper form" of conditions for equality in DPI we mean that it is stated as reversibility (sufficiency) of the channel, in the sense of Def. 4.1. This remark is added to distinguish our results from other equality conditions known in finite dimensions, that do not have such a clear interpretation.
- All typos and/or grammatical mistakes pointed out by the referee were corrected.

Additional changes to the manuscript:

- The reference [16] citing previous related work is added.
- Theorem 2.4 is restated, in the sense that the expressions holds for a larger set of parameters, namely for $0 < \alpha < 1$ and all $z > 0$, and for $\alpha > 1$ and $z \geq \alpha/2$. The proof is modified to cover these larger sets. Consequently, appropriate changes in the Introduction and in some parts of the subsequent text are made.
- Reference [36] is updated.
- Some further typos are corrected.