Review 2

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Review of "Integral formula for quantum relative entropy implies data processing inequality"

for *Quantum*, completed on Jun 19, 2023

Author will not see the reviewer's name or the date completed.

Overview questions

✏ *only editors will see response*

Overall rating
★★★★★

For this manuscript I recommend...
Accept

Comments to editor
In the new version, readability and impact of the paper is much increased and I
am happy to recommend publication.

Open response questions

Note: if you prefer to submit a free-form review instead of filling in this form,
simply reply to the invitation email with your report as an attachment. Due
to Scholastica's limitations, please make sure that you send it from the

same address at which you received the invitation. If that's not possible, you can email the review to info@quantum-journal.org . Otherwise, just write "ok" in the reply to this question and proceed to the rest of the form.

🖉 *only editors will see response*

ok

Summary: what are the main questions posed by the manuscript and how does it answer them?

👤 *intended for the author*

See below

What is your assessment of the paper? If you recommend acceptance, make a case that this work does indeed make a significant contribution to scholarship.

👤 *intended for the author*

See below

To what extent have you checked the technical correctness of the submission? This includes for example examining mathematical proofs or source code.

👤 *intended for the author*

See below

> Comment on the presentation of the paper. Is it well written? Are the main
> results clearly laid out? Does the manuscript clearly describe assumptions
> and limitations? Is the literature review adequate?

👤 *intended for the author*

See below

> If the submission includes software, numerical or physical experiments,
> does it provide sufficient details such that they could be reproduced by
> readers? This includes for example source code, documentation,
> experimental data, experimental setup specifications, etc.

👤 *intended for the author*

Not Applicable

> Suggested changes, corrections, and general comments.

👤 *intended for the author*

In the new version, the author has added some explanations as requested,
especially for the derivation of the basic formula. The proof is also changed
slightly and readability is increased. Moreover, the author added two further
sections indicating possible future research directions connected to the basic
formula, which I find very promising and highly interesting. Namely, the author
suggest a similar formula that is an extension of the classical Renyi divergence
and has the desirable properties of such an extension (monotonicity, strict
positivity). Furher, the formula can be easily extended to General probabilistic
theories, giving a divergence measure on states that is monotone under positive
maps. While the results given in these sections are easily shown from the main
results, one can see that this paper opens a large space for further investigations

using this potentially powerful approach.

A few comments and questions:

- there are still some places where the author could be more helpful to the reader at a cost of a few words or lines of space. As a random example: in the paragraph under Theorem 15, the argument that the map rho -> S(rho)+D(rho||sigma) is affine is quite clear, but the one additional line of computations showing that this indeed gives the expression for the Holevo quantity in terms of entropies would be helpful. My suggestion to the author is to try to identify other such places in the text.
- Related to the above example (paragraph under Thm. 15), the entropy can be identified as S(rho)=-D(rho||u), where u is (a multiple of) the center of the euclidean unit ball K. Could one define such a notion of entropy for any centrally symmetric state space (i.e. isomorphic to a unit ball of a norm)? Such a possibility, and the relation to other notions of entropy in GPT is another of (many) interesting question suggested by your approach.
- still related to the first point above: the properties of the generalized Renyi divergences stated at the beginning of p. 15 may deserve a short proof.

> Would you be willing to referee an updated version of this work before a
> final decision is made?

✎ only editors will see response

Yes

Do you think this is an outstanding work that deserves to be highlighted? If this work is accepted, would you be willing to write a short Perspective (opinion piece similar to a viewpoint or editorial) based on your report? For examples, see: http://quantum-journal.org/category/Editorial,Perspective/

✎ only editors will see response

Yes, I do think this is an outstanding work.

How was your reviewing experience? Is there anything you would like us to improve?

✎ only editors will see response

Ok