L. Molnár, Quantum Rényi relative entropies on density spaces of C^*-algebras: their symmetries and their essential difference

Referee report

The paper deals several possible extensions of the classical Rényi relative entropies to the noncommutative framework of C^*-algebras with a finite trace. Two questions are answered here: the possible form of a surjective map between the positive cones preserving the considered extensions is found and, secondly, it is proved that if such a surjective map exists that transforms one of the forms to another, then the algebras are necessarily commutative.

The quantum Rényi relative entropies are some of the fundamental quantities in quantum information theory and it is highly important to understand their properties and essential differences. I consider the present paper a valuable contribution to this research. Moreover, the proofs are sophisticated and elegant, using some of the deep properties of the positive cones of C^*-algebras, though I must admit I did not check all the details, but I trust the authors expertise in this area.

In conclusion, I recommend this paper for publication in Journal of Functional Analysis, with only a few comments and suggestions that might help the author:

There are some repetitions that seem unnecessary and make the paper less readable. For example

1. The description of the map ϕ, with a Jordan *-isomorphism J, central element C, etc, appears at many places. It would be better to display such a map in some definition or displayed equation and then refer to it (so the reader who wants to get a picture quickly does not have to check whether it is really the same all the time).

2. Also the arguments in the proofs are repeating. I would suggest to insert the following or similar statement as a lemma:

   Let $\phi : \mathcal{A}_+^{-1} \to \mathcal{B}_+^{-1}$ be a surjective map which is positively homogeneous and such that for any $A, B \in \mathcal{A}_+^{-1}$, we have $g(A) \leq g(B)$ if and only if $f(\phi(A)) \leq f(\phi(B))$, where f and g are either $x \mapsto \log(x)$ or $x \mapsto x^t$ for some $t > 0$. Then either $\mathcal{A}$ is commutative or $f = g$ and in the latter case,

   $$\phi(A) = g^{-1}(CJ(g(A))C)$$

   *where C is central and J is a Jordan *-isomorphism.*

Instances of this statement appear repeatedly through the proofs and I thing referring to such a lemma would make the proofs more effective and readable.

Some misprints and minor points:

- page 2, line 7: "...similar happens to the operational interpretation..." something is missing here
- page 2, line 8: "non-sufficiently good behaviour" ??
- page 2, line 16: "Farenick etal." better include a reference here
- the "conventional" Rényi entropy is often called "standard"
- page 4, line 8: "It might worth mentioning..." "be"?
- page 4, line 9: As far as I can see, the preservers are linear maps, why "sort of" linear?
- page 20, line 14, displayed equation: $\tau((YXY)^{\alpha-1}) \to \tau(YXY)^{\alpha-1}$