Response for Review Report

First of all, we would like to thank the anonymous referee for the detailed review. We have addressed all the comments listed in the report. Below is the our detailed response to each point in the list.

1. Changed to "$d\lambda < C_{a,b} d\mu(\lambda)$ on (a, b)" as suggested.
2. Corrected
3. Corrected
4. We simplify the proof of Lemma 4.17 using direct verification in finite dimensions, which should be more accessible.
5. The independence of choices of vector representation are now emphasised in Definition 5.1. The independence is verified in Proposition 5.2.
6. Corrected
7. An explanation for "$\langle h_\rho^{1/2} | f(\Delta(\sigma, \omega)) | h_\rho^{1/2} \rangle$ is convex over σ and concave over ω" is added.
8. Yes. Thm 3 of [BST18] requires a stronger assumption that $\rho \le c\sigma$ for some $c > \infty$. The original assumption $\tilde{D}_\alpha < \infty$ is also sufficient by Proposition 3.8 of [Jen18]. This point is now clarified. For the general case, we add an argument via approximation $D(\rho||\sigma + \varepsilon\rho) \to D(\rho||\sigma)$.
9. Corrected
10. Yes, the generalized conditional expectation is the Petz map of the inclusion as a normal UCP map. This is corrected.
11. Yes, the assumption $s(\rho) \le e$ is added.
12. We modified the proof. As pointed out in review report, we only need $aV_\rho = V_\rho a$ and we can simply choose $\boldsymbol{\omega} = V_\rho(h_{\omega_\mathcal{N}}^{1/2})$. The argument using spatial derivative is removed.
13. Yes, you are right. It should be E_ρ. Corrected
14. Corrected.
15. Corrected.
16. Actually, here we don't need polar decomposition. We only need $V_\rho h_{\omega_\mathcal{N}}^{1/2} = u h_{\omega_\mathcal{N} \circ E_\rho}^{1/2}$ with some unitary $u \in \mathcal{M}$. This is corrected.
17. We add the explanation that Eq. (25) remains valid in Haagerup L_p-space because its proof in [CV20a] only rely on Hölder inequality.