Dear Editor,

I would like to thank all 4 reviewer for their very careful reading and for the detailed reviews and the numerous suggestions to improve the paper. I implemented all of their suggestions, and this improved the paper significantly. Below I respond to all the specific comments by the referees. Since I implemented all the typos found by all 4 referees, I respond below only to the suggestions by the reviewers for the more significant changes.

I believe the paper is now suitable for publication in IEEE.

Please let me know in case you would like me to make additional changes.

Sincerely,
Gilad Gour

Response to Reviewer 1

Reviewer: "As I was reading the paper, I kept wondering what happened to the von Neumann entropy. I know that min entropy is commonly used in one-shot operations and quantum thermodynamics, but I was confused by the fact that it was often the only entropy discussed. Then my question was answered in the conclusion: the i.i.d. version of this theory remains open. This might be made clearer from the beginning, to remove the element of suspense."

Author: This is a very good point. I clarified it at the end of the introduction.

Reviewer: -P.11 "While they form a strictly larger set of channels than the set of random unitary channels, if a state ρ can be converted into σ via a doubly stochastic channel, then this transformation can also be achieved by random unitary channels." Is this supposed to be obvious, or is there a non-trivial proof/reference somewhere I can look up?

Author: I added a reference to Lemma 10 in Ref.[33] (of the new version).

Reviewer: -p27 "Note that replacing |ηi> with a mixed state" Do you mean with a completely mixed state, or with any state that is not pure?

Author: Yes, any state that is not pure. I made it clearer in the text.

Response to Reviewer 2

Reviewer: Mistakes: there is a number of them. These are not serious, mostly just typos, but can be confusing and make the reading more difficult, in particular when one needs to keep track of the various input and output spaces. Some of these are listed below, but the author should carefully re-read and check the paper.

Author: Thank you for pointing these mistakes. I fixed them all, and also all the errors pointed out by the other referees. I believe the paper is now free of errors.

Reviewer: Proofs: it often happens that in a proof of equivalence of two statements, only one direction is proved. It would be better to say at least that the converse is easy. Or, better, add something like “The converse follows from the form of the Choi matrix”, etc.

Author: I went through all the proofs and added these comments and in some cases additional proofs whenever they were missing.

Reviewer: Notations: it happens at some places that the same symbols denote different things, even within the same proof on the same page, see the detailed comments below.

Author: This was also pointed out by the other referees. I changed notations in some places to make the presentation less confusing and clearer.

Reviewer: A question: is there an operational interpretation to the conditions in Theorem 7 or Corollary 1?

Author: I do have some ideas on the operational interpretations of these conditions (particularly in the classical case), but at the moment I was not able to fully establish it. Note that in the case in which all channels are quantum state (i.e. 1-d inputs), then the conditions becomes equivalent to quantum majorization which is a generalization of the Lorenz-curves of the classical case.

Author: I liked a lot the last suggestion by the referee, particularly that Delta_\Theta is basically the lifting of \Theta. However, I decided not to include it in this paper as this paper is already very long, but in future work I will be using this VERY nice observation! Also, I added the reference suggested.

Response from Reviewer 3

Reviewer: On page 20, the author defined the extended min-entropy of a quantum channel via the normalized Choi matrix of a channel. It is the normalized version of the min-entropy of the Choi matrix in the following paper (see, Theorem 2).
[DW16] Duan, R. & Winter, A. No-Signalling-Assisted Zero-Error Capacity of Quantum Channels and an Information Theoretic Interpretation of the Lovász Number. *IEEE Trans. Inf. Theory* **62,** 891–914 (2016).
In my opinion, [DW16] is a very relevant paper, but it is not mentioned in the current manuscript. The authors of [DW16] provided an operational meaning of the min-entropy of the Choi matrix as the classical simulation cost of the quantum channel, whose smoothed version was recently introduced in the following paper.
[FWTB18] Fang, K., Wang, X., Tomamichel, M. & Berta, M. Quantum Channel Simulation and the Channel's Smooth Max-Information. arXiv:1807.05354 (2018).

Author: I agree with the referee that these papers are very relevant. I did not know about them before. I added them to the reference list, and referred to them appropriately in the paper (see Remark 3 on page 20).

Reviewer: The structure of bipartite quantum channels and its application were previously studied in [DW16] and [LW15] D. Leung and W. Matthews, "On the Power of PPT-Preserving and Non-Signalling Codes," *IEEE Trans. Inf. Theory*, 62, 4486–4499 (2015). For instance, Eq. (8) in this paper seems to be related to Proposition 7 in [DW16]. Moreover, Eq. (10) in this paper is in the same sense as the equation after Eq. (11) in [LW15].

Author: Again, I agree with the referee that these papers are very relevant. I added them to the reference list, and referred to them appropriately in the paper (see first sentence below Eq.(1))).

Reviewer: As the conditional min-entropy of a point-to-point channel has an operational meaning as the simulation cost under no-signaling assistance, will the conditional min-entropy of a bipartite channel also have a similar operational meaning?

Author: I believe it will, but I do not have a proof for that.

<u>Response from Reviewer 4</u>

Reviewer: It will be better if the author outlines the need for studying superchannels. As an abstract concept, the idea of superchannels is indeed interesting. However, there are some connections which can be made between Shannon theoretic tasks and superchannels. It would be nice to bring out the details in a couple of paragraphs.

Author: I added a few words about it in the introduction, including new references to the use of superchannels in very recent works on resource theories of quantum processes.

Reviewer: I would prefer if E_{ab} is changed to $E_{a_0 a_1}$. That is the small indices characterizing the basis should refer to the Hilbert space that is being considered. Also, I would prefer if A_0 and A_1 are not condensed into A and similar for B.

Author: Thanks, that's a good point. I have made the change in the notation throughout the paper.

Reviewer: Please change the notation of $\Phi A\tilde{A}$. This is a standard for maximally entangled state and can get confusing at times.

Author: I now use \Upsilon instead of \Phi

Reviewer: Also, summarize the physical meaning of the conditions in equation 43 in a paragraph. As of now, it is spread across a couple of pages. This would make this section more accessible.

Author: I added a paragraph below the conditions.

Reviewer: Shouldn't there be a constraint corresponding to $\alpha^{A_1B_0} = I^{A_1B_0}$ in equation 82 from equation 61? This constraint turns out to be $\alpha^{A_1}_{x|y} = I^{A_1}$, where the second α is with respect to the $x|y$ definition below equation 81?

Author: Note that the condition

$\alpha^{A_1B_0} = I^{A_1B_0}$

does NOT imply that

$\alpha^{A_1}_{x|y} = I^{A_1}$

but instead

$$\sum_{x} \alpha^{A_1}_{x|y} = I^{A_1}$$

Which is equivalent to (82) of the old version –now it is Eq.(84).

Author: I followed all the other suggestions by the referee.