

# K. Hatano, Y. Ueda, Pusz-Woronowicz functional calculus revisited

## Referee report

The aim of this paper is to show that operator means and operator perspectives can be obtained from the functional calculus for sesquilinear forms by Pusz and Woronowicz.

There is a quite large literature about operator means and related quantities, but apart from the geometric mean, it does seem that the Pusz-Woronowicz approach is not so widely known. Though the proofs are not really involved, this is a nice observation that could be of value, if a more pedagogical approach was chosen. As it is, the presentation of the paper is not satisfactory:

1. For example, in the rather informally written introduction, the second sentence is: "However, it seems that Pusz-Woronowicz's approach has not been paid much attention, except [21], [15,16], [3], [22,23], etc."

This is a bit strange. The authors should explain more clearly what they find missing in the previous literature and what they aim to amend by the present paper.

2. The paper would be more readable if the basic definitions were given here, e.g. the Pusz-Woronowicz functional calculus or the operator perspectives. As it is, the central object of the paper is not exactly defined and its purpose is not clearly stated. For example, at the beginning of Sec. 4.1, it is written: "It is rather easy to see that Pusz-Woronowicz's functional calculus  $f(A, B)$  is an example of regular map in the sense of [7, Definition 2.1]."

Why not give a definition of a regular map and explain its relation to the aims of the paper? Also the purpose of Proposition 8 should be clarified. And this is just one example, there are other places where it is not clear what is going on.

In conclusion: I quite like the ideas of the paper and believe they deserve to be published. But not in the present form. I would expect a clearer, more pedagogical exposition, explaining the important notions and the aims of the paper. I encourage the authors to rewrite the paper along the above lines.