## Instructions for the Applicant

## 

This template shall be used to fill in the project description by the *applicant under the call 09I03-03-V04 – Fellowships for excellent researchers R2-R4.* The structure of this form **must be maintained.** It is prepared to allow an efficient, transparent and fair evaluation of each application. Parts 1, 2 and 3 are identical to the evaluation criteria. Only complete applications, which address all the mandatory parts set out in this template and in the call, will be included in the evaluation process. If an application is approved, major changes in the content of the project will not be possible.

The description of the project shall not exceed 30 pages. All tables, numbers, references or other relevant information must be included in the project description and count towards the total page limit. The maximum limit must not be circumvented by inserting external hyperlinks. Content that exceeds the allowed page limit will not be taken into account and will not be subject to peer review, so it is recommended to comply with the maximum page limit.

Formal requirements for the description of the project:

The prescribed font to use in the description of the project is Times New Roman or Arial, minimum size 11. A font size of 10 is allowed in the tables; the applicant should not circumvent the page limit by artificially replacing the text with tables.

The size of the page is A4 with edges of at least 1.5 cm. Page numbering should be maintained.

Delete all instructions in this template highlighted in grey before the application is submitted.

Project description (template) for call 09I03-03-V04 Fellowships for excellent researchers R2-R4

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Title of the project: Quantization by Internalization

Short title of the project/Acronym: QuInt

Category of researcher: R2

Researcher’s job type (full-time, part-time – in %):

Type of research[[2]](#footnote-3): independent, basic, industrial

Identification of the entity involved in the implementation of the project:

|  |  |  |  |
| --- | --- | --- | --- |
|  | Official name of the entity | Abbreviated name of the entity  Please indicate the short name of the entity that you will use throughout the application, max. 15 characters | Role in the project |
| 1 |  |  | Applicant/host organisation |

## 1. Excellence

*Specific aspects that are relevant for this section*:

*The quality and adequacy of the proposed objectives of the project.*

*Relevance of the problems/needs the project is focused on.*

*How the project goes beyond the currently available solutions, procedures, etc. (“beyond the state of the art”).*

*Appropriateness, timeliness and relevance of the proposed methodology to the objectives of the project.*

*The quality and adequacy of the researcher’s professional experience, expertise, competences and skills.*

*The quality and adequacy of the host organisation in relation to the project and the researcher.*

*The quality and adequacy of the conditions that the host organisation will ensure and provide for the researcher (e.g., additional training, supervision/mentoring, possibilities to build its own research team, etc.).*

*The quality of two-way knowledge transfer between the researcher and the host organisation.*

### 1.1 PROJECT OBJECTIVES

Describe objectives of the project – they should be clearly defined, realistic, measurable and achievable in the implementation of the project. For each objective, please also indicate how it will be verified and evaluated.

### 1.1.1 INTRODUCTION

Many set-theoretic structures can be defined in terms of categorical constructions within the context of the categories **Set** and **Rel**. If some categorical construction that defines in such a way a structure also exists in another category **C**, then that allows us to transfer the definition of this structure to **C**, in which case we say that the structure can be *internalized* in **C**. For example, groups can be defined in terms of finite products in **Set,** and in the same way internal groups can be defined in any category **C** that also has all finite products, such as the category **Top** of topological spaces and continuous maps. Internal groups in the latter category are precisely the topological groups. The importance of the process of internalization lies in the fact that it allows for generalizations, transference of properties to other categories, and cross-disciplinary connections.

A program relevant to this project in which internalization plays an essential role is *categorical quantum mechanics[[3]](#footnote-4)[[4]](#footnote-5)[[5]](#footnote-6)*, in which one studies the foundations of quantum physics and quantum information theory in terms of dagger compact categories such as the category **FdHilb** of finite-dimensional Hilbert spaces. In particular, one tries to describe quantum mechanical phenomena as internal structures in **FdHilb**, for instance orthonormal bases, which can be characterized[[6]](#footnote-7) as commutative special Frobenius algebras in **FdHilb**. Also other dagger compact categories are studied as toy models (**Rel**) or as more elaborated models for quantum physics (such as the category **FdCStar** of finite-dimensional C\*-algebras and completely positive maps).  
  
Our primary motivation for exploring internalization lies in its connection with *quantization*, i.e., the process of finding noncommutative generalizations of mathematical structures: internalization in the order-enriched dagger category **WRel** of Neumann algebras and Weaver's quantum relations[[7]](#footnote-8) can be regarded as a form of quantization. However, since **WRel** is not compact, which is desirable for categorical quantum mechanics, we prefer to work instead with its full dagger compact subcategory **qRel** of *hereditarily atomic* von Neumann algebras, i.e., von Neumann algebras isomorphic to a (possibly infinite) sum of matrix algebras, which are less complicated to work with, while still having sufficient structure for most applications in quantum computing and quantum information theory. Following Kornell[[8]](#footnote-9), we call the process of quantization by internalization in **qRel** *discrete* quantization, since the objects in **qRel** have a discrete character, and are in particular useful to quantize discrete mathematics. The host, Gejza Jenča, is interested is internal structures in another order-enriched dagger compact category, namely the category **RelPosInv** of involutive posets and monotone relations, because effect algebras, which form an important class of quantum structures, can be described as Frobenius algebras in this category.

1.1.2 OBJECTIVES

Our main objective is internalizing structures in **qRel** and **RelPosInv** simultaneously, and in order-enriched dagger compact categories in general, which is reflected in three specific goals:

Our second objective concerns a specific structure we want to quantize, namely topological spaces. Topological tools are widely used to study mathematical structures; hence it is desirable to have a concept of quantum topology to study quantum structures. C\*-algebras form the standard approach to quantum topology, but are only noncommutative generalizations of locally compact Hausdorff spaces, whereas many relevant topologies are not locally compact or Hausdorff, which we plan to resolve:

**Objective O1** (Categorical quantum mechanics): Achieve the systematic development of the theory **qRel** and **RelPosInv** as dagger compact categories in the program of categorical quantum mechanics, and investigate the role of order enrichment in this program.

**Objective O2** (Quantization): Find a suitable quantum version of topological spaces that allows for the quantization of topological spaces that are not necessarily locally compact Hausdorff spaces.

*Bicategories of relations*[[9]](#footnote-10) are bicategories generalizing **Rel**, and include RelPos and **RelPosInv** as examples. Due to its quantum character, **qRel** is not an example because its wide monoidal subcategory **qSet** of internal functions fails to be cartesian. One could question how to adjust the definition of bicategories of relations so that it includes **qRel** as an example. Any such adjustment might be obtained by *double categories[[10]](#footnote-11)*, which can be regarded as large versions of internal categories in the category **Cat** of small categories and functors. Double categories form another 2-dimensional categorical structure that is the ideal setting to combine two different categories with the same objects, such as **Rel** and **Set**. In our case, this concerns **qRel** and **qSet**, and **RelPosInv** and **PosInv**, leading to our final objective:

**Objective O3** (2-dimensional categories): Investigate quantum relations and monotone relations in the context of bicategories and double categories.

1.2 RELEVANCE, QUALITY AND NOVELTY OF THE PROJECT

*Please briefly describe the current state in the area the project will focus on. How will the planned activities of the project address problems and challenges in this area? Why is necessary/appropriate to deal with such a project?*

*How does the project go beyond the currently available solutions, procedures, etc.? What makes it original and innovative?*

*Describe the link between the project and its activities with the European Research Area.*

### 1.2.1 STATE OF THE ART

Since locally compact Hausdorff spaces are dual to commutative C\*-algebras via Gelfand duality, the category of all C\*-algebras is traditionally regarded as the dual of a category of `formal' noncommutative locally compact Hausdorff spaces. The origin of quantum relations on von Neumann algebras lies in the work of Weaver and Kuperberg on the quantization of metric spaces[[11]](#footnote-12), which includes quantum generalizations of metric spaces that are not locally compact, and hence examples of quantum topological spaces beyond locally compact Hausdorff spaces. Quantum relations were also used to give a description of quantum graphs and to introduce quantum posets7. An alternative description of hereditarily atomic von Neumann algebras in terms of collections of finite-dimensional Hilbert spaces, called *quantum sets*, was given by Kornell[[12]](#footnote-13). In the same article, Kornell also showed that **qRel** is dagger compact, and explored the categorical properties of the wide subcategory **qSet** of internal functions in **qRel**, which he proved to be dual to the category of hereditarily atomic von Neumann algebras and normal unital \*-homomorphisms. The properties of the category **qPos** of quantum posets and monotone functions were investigated by Kornell, Mislove and me[[13]](#footnote-14) where we also showed the existence of a quantum power set monad on **qSet** for which it is essential that **qRel** is compact closed.

Discrete quantization has proven its value because of a very important application in the *denotational semantics* of quantum programming languages, i.e., the translation of any phrase in a programming language to a mathematical function in such a way that the function is the composition of the functions corresponding to the phrase's subphrases. Since it is virtually impossible to debug quantum programs, it is pertinent to find different tools for the verification of quantum programs, such as denotational semantics. For ordinary programming languages with recursive types, denotational models are often constructed in terms of *complete partial orders (cpos)*, i.e., posets in which any monotonically ascending sequence has a supremum. Together with Kornell and Mislove, based on our previous work on quantum posets, I used discrete quantization in order to quantize cpos, and used the thus-obtained quantum cpos to construct a sound and computationally adequate model for the current state-of-the-art quantum programming language, Proto-Quipper-M extended with recursive types[[14]](#footnote-15). Monotone relations between posets (or preorders) were studied under various names[[15]](#footnote-16). The category **RelPos** of posets and monotone relations is compact closed, but not a dagger category. However, its full subcategory **RelPosInv** of involutive posets is dagger compact.  
Recently, Jenča proved that effect algebras can be represented as certain commutative non-special dagger-Frobenius monoids in **RelPosInv** yielding an interesting link between the fields of algebraic quantum logic and categorical quantum mechanics. Using the methods of discrete quantization, Jenča and I found a quantum generalization of **RelPos**, which we used to construct a quantum version of lower set monad whose algebras we call *quantum suplattices*, since complete lattices (also called suplattices) are algebras of the ordinary lower set monad. We proved the quantum counterparts of several theorems on suplattices such as the Knaster-Tarski fixpoint theorem[[16]](#footnote-17). Finally, there are links between our program and *fuzzification*, i.e., the process of generalizing mathematics to fuzzy set theory, where the membership relation takes values in the unit interval, or more generally, in a commutative quantale *V*, since fuzzification can be regarded as an internalization process in the order-enriched dagger compact category *V*-**Rel** of sets and *V*-valued relations. Related to *V*-**Rel** is the order-enriched dagger compact category *V-*Prof of *V*-enriched categories and *V*-enriched profunctors, which generalizes **RelPos**.

### 1.2.2 MAIN INNOVATIVE ASPECTS AND APPLICATIONS

There are often several inequivalent ways to internalize the same structure in a category. However, for categories with sufficient structure, there are procedures to internalize several structures simultaneously and consistently. For instance, in topoi all constructive mathematics can be internalized. Every topos has an associated bicategory of relations, hence by studying the order-enriched dagger compact categories as generalizations of bicategories of relations, we obtain a better understanding of how to internalize structures in these categories, and thus how to consistently quantize several structures simultaneously. We expect to find connections with fuzzification (which has many applications in modern technology) because it can also be regarded as an internalization process in an order-enriched dagger compact category. Furthermore, our project concerns models of categorical quantum mechanics that have not been studied in this context before. In particular, the category **qRel** is not just a toy model of categorical mathematics, but a category with genuine applications in quantum computing (see Section 1.2.1 STATE OF THE ART). The addition of order enrichment to the program of categorical quantum mechanics is relevant, because **FdCStar** is order enriched, but there is not yet a framework that integrates order-enrichment and dagger compact categories.

Traditionally, C\*-algebras form the standard approach to quantum topology, but are only noncommutative generalizations of locally compact Hausdorff spaces. One of the project's most captivating objectives is to quantize topological spaces beyond locally compact Hausdorff spaces such that one can quantize specific topologies that are associated to structures that previously have been quantized, such as posets and graphs. This is particularly intriguing as many mathematical structures are illuminated by their associated topological spaces, which are not always locally compact Hausdorff—such as the Scott topology on a cpo.

Applications in the denotational semantics of quantum programming languages form one of the reasons why we are interested in the quantization of topological spaces. The quantum cpo model mentioned in Section 1.2.1 STATE OF THE ART lacks an important feature, namely support for probabilistic computation. Because of this, it is impossible to describe the preparation of states in the model. The problem would be resolved by finding a monad for probabilistic computation on the category **qCPO** of quantum cpos, but this problem turns out to be very difficult. Classically, valuations are used to construct such a monad, but valuations turn out to be very difficult to quantize. Topology offers a different route, but because the Scott topology on a cpo is not locally compact Hausdorff, the quantization of topological spaces that are not necessarily locally compact Hausdorff will be essential.

A second application to the denotational semantics of quantum programming languages we anticipate concerns differential programming languages, which are used in deep learning. The models of these languages are given by differential categories[[17]](#footnote-18), which include **Rel** and the category **Sup** of complete lattices and supremum-preserving maps as examples. Hence, we expect that the quantum counterparts of these categories, **qRel** and **qSup**, respectively, will be examples, too. Here, the latter category is precisely the category of quantum suplattices previously introduced by Jenča and me. If these categories indeed are differential categories, this might lead to applications of categories of discrete quantum structures to quantum differential languages, hence to quantum deep learning.

1.3 METHODOLOGY

*Describe how the project will be implemented. Describe the methods and procedures used in each activity and their appropriateness and interconnection. Describe the concepts, models, assumptions underlying the proposed project methodology.*

*Describe how the proposed methods and procedures will ensure the achievement of the project’s objectives.*

*Describe possible challenges in the implementation of the project in relation to the methodology and the proposed way to overcome them.*

*Describe the use and management of research data and other research outputs within the project. If the project collects data and/or other research outputs (except publications), how the data/outputs of the research will be managed. Describe how open science principles are integrated in the project.*

*Describe how the following aspects are taken into account in the project: multi- and interdisciplinary approach, principles of open science, FAIR access to research data, gender equality in research.*

This project is a multidisciplinary project concerning research in mathematical physics with applications in computer science. Within mathematical physics, this research employs elements and methods from various fields, in particular operator algebras, category theory, order theory, fuzzy mathematics, and monoidal topology. Our main structures of interest are order-enriched dagger compact categories. The main examples of such categories that we will consider are **Rel**, **qRel**, **RelPosInv**, **FdCStar**,*V***-Rel,** *V***-Prof**.

Our first objective **O1** concerns the interaction between order enrichment and dagger compactness. We are interested in the identification of the categorical structure that assures that the homsets of order-enriched dagger compact categories are complete orthomodular lattices, which is the case for our first two examples. The homsets of our last two examples seem to be only ortholattices or orthomodular lattices if *V* is an ortholattice or an orthomodular lattice, respectively. Homsets of **FdCStar** do not have an orthocomplementation, and seem to be only bounded-directed complete, not complete. A starting point will be the work of Heunen and Kornell on the reconstruction of Hilbert spaces in terms of dagger categories[[18]](#footnote-19), which relies on reconstructing orthomodular lattices from the categorical structure of the category of Hilbert spaces. The category **FdCStar** can be obtained by applying the so-called CP\*-construction[[19]](#footnote-20) to the dagger compact category **FdHilb** of finite-dimensional Hilbert spaces, hence this construction adds completely positive maps to the picture. The order of these maps is traditionally given by a variation of the Löwner order, hence a question we will investigate is whether this order generalizes to any dagger compact category obtained by applying the CP\*-construction to some dagger compact category. The CP\*-construction relies on the concept of a dagger Frobenius algebra in a dagger compact category; any such algebra is in particular a monoid. Hence, it will be imperative to characterize the monoids, comonoids and dagger Frobenius algebras in our categories **qRel** and **RelPosInv**. A starting point for this will be the work of Jenčová and Jenča[[20]](#footnote-21) of on monoids in **Rel**. We will also calculate the dagger limits in **qRel** and **RelPosInv**, for which the work of Heunen and Karvonen[[21]](#footnote-22) will be the starting point.

Objective **O2** concerns quantization for which the main strategy we will follow is to find categorical constructions in **Rel** that describe the structures that we want to quantize, lifting these constructions to **qRel**, and try to show that the quantized structures are subject to similar theorems. The basis of discrete quantization and its techniques was developed by Kornell, and applied to posets resulting in a quantum version of the power set monad. For the specific goal of quantizing topological spaces, we will also rely on the previous quantization of suplattices by Jenča and me, since a topology on a set is in particular a complete sublattice of the power set of the set. We also want to investigate another possible quantum version of topological spaces based on the program of monoidal topology[[22]](#footnote-23), which was used for the fuzzification of topological spaces. In this program, one characterizes topological spaces as lax algebras of the Barr extension of the ultrafilter monad to **Rel**. There are several possible ways to generalize the ultrafilter monad to the quantum setting, and we will investigate whether these generalizations coincide. When found a satisfactory quantum ultrafilter monad on **qSet**, we will attempt to find whether we can find a quantum version of the Barr extension of a monad, and we will investigate whether its lax algebras coincide with the quantum topological spaces defined in terms of quantum sublattices of the quantum power set monad. As an application of discrete quantization to the denotational semantics of quantum programming languages, we will work on constructing a monad on the category **qCPO** of quantum cpos that describes probabilistic computation. A method of obtaining such a monad for classical programming languages is obtained via *K-completions[[23]](#footnote-24)*; which are topological completions of posets that yield cpos. We will investigate whether our definition of quantum topological spaces allow us to carry over this construction to the quantum setting. The second application to the denotational semantics of quantum programming languages concerns differential categories. We aim to show that the categories **qRel** and **qSup** are differential categories, for which the biggest step will be characterizing the cocommutative comonoids in both categories. For **qRel** this characterization is part of objective **O1**.

For objective O3, we will investigate **qRel** in terms of bicategories and double categories. The starting point is the work of Shulman[[24]](#footnote-25), who introduced the notion of a *framed bicategory*, which refines double categories. We will verify whether **qRel** and **qSet** form a double category or even a framed bicategory. Another possible route to an appropriate 2-dimensional categorical structure of relations that includes **qRel** as an example, consists of attempting to relax the definitions of bicategories of relations and/or of allegories. Any modular lattice can be regarded as a one-object allegory, so based on quantum nature of **qRel**, one might expect that orthomodular lattices are one-object examples of an appropriate generalization of allegories. A relevant reference might be the work of Heunen and Tull[[25]](#footnote-26) on role of bicategories of relations in the framework of categorical quantum mechanics.

1.4 EXCELLENCE OF THE RESEARCHER

Describe the quality and adequacy of the researcher’s professional experience and expertise in relation to the proposed project. Please support the description by the CV below.

This proposal requires a good understanding of several mathematical disciplines, namely operator algebras, category theory, orthomodular structures, logic, and domain theory, where the latter is the underlying mathematical theory for denotational semantics, which I obtained during my PhD project at Radboud University in Nijmegen with Klaas Landsman, and my postdocs at Tulane University in New Orleans with Michael Mislove, Johannes Kepler University in Linz with Thomas Vetterlein, and currently the Mathematical Institute of the Slovak Academy of Sciences in Bratislava with Anna Jenčová. During these years, I built a network that includes Andre Kornell (Dalhousie University, expertise on discrete quantization), Klaas Landsman, Michael Mislove (Tulane University) and Xiaodong Ji (Hunan University), both experts in domain theory and the semantics of programming languages, JS Lemay (Macquarie University, expertise on differential categories), Vladimir Zamdzhiev (Loria, expertise on quantum programming languages and their denotational semantics), and Isar Stubbe (Université du Littoral-Côte d'Opale), expertise on quantaloids, a kind of order-enriched category that includes both **qRel** and **RelPosInv**.  
During my initial postdoctoral position within the MURI project 'Semantics, Formal Reasoning, and Tool Support for Quantum Programming' at Tulane University Orleans, I collaborated with Andre Kornell and Michael Mislove. Our work centered on quantizing cpos to subsequently apply the thus-obtained quantum cpos to the denotational semantics of quantum programming languages\textsuperscript{\ref{fn:qCPO}}: in terms of quantum cpos, we constructed the first (and so far only) sound and computational adequate denotational model for extension of the state-of-the art quantum programming language, Proto-Quipper-M, with recursive types. It was during this collaboration that Kornell introduced me to the technique of discrete quantization, which also yielded a publication on the properties of the category **qPos** of quantum posets and monotone maps\textsuperscript{\ref{fn:qPos}}.  
I am excited by the open problem of finding support for probabilistic computation in our denotational model. This concerns proving the existence of a quantum probabilistic power domain monad, a challenging task even in the classical scenario relying on topological tools. Hence, I am keen on quantizing topological spaces, to which Jenča and I already made a first step by quantizing complete lattices\textsuperscript{\ref{fn:qSup}}, demonstrating our ability to collaborate effectively.

Curriculum Vitae

**Personal information**

First and last name: Albertus Johannis Lindenhovius

Identifier: 0000-0001-5380-4705 (Orcid)

Date of birth:October 20, 1984

Nationality: Netherlands

Website (if relevant):

**Education**

07/2016 – PhD

Institute for Mathematics, Astrophysics and Particle Physics, Radboud University, the Netherlands

07/2011 – Master

Faculty of Science, University of Amsterdam, the Netherlands

**Current position/positions**

06/2023 – postdoc

Institute of Mathematics, Slovak Academy of Sciences, Slovakia

**Previous positions**

01/2021 – 05/2023 – postdoc

Institute for Mathematical Methods in Medicine and Data Based Modeling, Johannes Kepler University, Austria

08/2016 – 12/2020 – postdoc

Department of Computer Science, Tulane University, United States of America

**Teaching activities (if applicable)**

2019 – Introduction to Discrete Math (both spring and fall), Tulane University, United States of America

2018 – Operator Algebras, Tulane University, United States of America

2017 – Calculus III, Tulane University, United States of America

**Organisation of scientific meetings (if applicable)**

2023 – co-organizer of the Dutch Mathematical Congress, the Netherlands, with 150 participants. I was in particular responsible for the organization around the PhD student prize.

**Major collaborations (if applicable)**

- John Harding, Bohrification, Department of Mathematical Sciences, New Mexico State University, United States of America

- Chris Heunen, Bohrification, School of Informatics, University of Edinburgh, United Kingdom

- Gejza Jenča, Quantization, Faculty of Civil Engineering, Department of Mathematics and Descriptive Geometry, Slovak University of Technology, Slovakia

- Andre Kornell, Quantization, Department of Mathematics and Statistics, Dalhousie University, Canada

- Michael Mislove, Denotational Semantics of Quantum Programming Languages/Quantization, Department of Computer Science, Tulane University, United States of America.

- Vladimir Zamdzhiev, Denotational Semantics of Quantum Programming Languages, Inria/Saclay, France

**Overview of the researcher’s most important projects in the last 5 years** (max. 5)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Project name/identification** | **Source of funding** | **Budget (EUR)** | **Project period** | **The role of the researcher in the project** |
|  |  |  |  |  |
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**Overview of the researcher’s most important outputs** (max. 5)

|  |  |  |  |
| --- | --- | --- | --- |
| **Output name/identification** | **Type of output** *(e.g., publication, dataset, software, patent, service, product, etc.)* | **Short description** | **The role of the researcher** |
|  |  |  |  |
|  |  |  |  |
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Briefly describe your most important research and/or innovation achievements.

1.5 EXCELLENCE OF THE APPLICANT/HOST ORGANISATION

Describe the applicant/host organisation and its excellence.

*What is the comparative advantage of implementing this project within the given host organisation, why is the applicant the ideal hosting organisation to implement the project?*

*Describe the conditions that the applicant/host organisation will create and provide for the researcher for the implementation of the project. In the case of supervision/mentoring, describe the person of the supervisor/mentor, his/her quality and adequacy in relation to the researcher and the project.*

*Describe the conditions the researcher will be provided with to build his/her own team (particularly relevant for R3 and R4 researchers’ categories).*

*Describe the quality and how two-way knowledge transfer between the applicant/host organisation and the researcher will be ensured.*

## Impact

*Specific aspects that shall be taken into account in this section:*

*The credibility of the proposed procedures, the likelihood that the project will achieve the expected results and will have the expected impact.*

*The assumption of a positive impact on the further career of the researcher, the assumption of a positive impact on the applicant/host organisation.*

*The significance of the expected impact – on the given area of knowledge and the scientific community, on the economy, on society, on the environment.*

*Adequacy of expected results and impact of the project – qualitative and quantitative.*

*The appropriateness and quality of the proposed measures to maximise the results and impact of the project.*

*The quality of the proposed intellectual property rights management strategy for project results (if applicable).*

### 2.1 THE WIDER IMPACT OF THE PROJECT

Describe the expected impact of the project in the short, medium and long term. What impact will the implementation of the project have beyond its direct scope and after completion of its implementation?

Describe the impact of project implementation on the researcher’s further career and the development of his/her skills.

Describe the impact of the project implementation on the applicant/host organisation.

Identify the individual target groups that will benefit from the activities and achievement of the project objectives, describe the impact of the project on these groups.

Describe the direct and relevant scientific, economic, environmental, societal impact of the project (or other, if relevant).

Identify and describe the potential negative impact of the project and what the proposed measures to eliminate/mitigate it are.

Describe the specific expected results and impacts of the project activities (qualitative and quantitative), which will bring significant and direct benefits measurable within the monitored data.

Monitored data may include, for example:

* + number of excellent students, PhD candidates and researchers implementing the project;
  + number of patent applications;
  + number of publications;
  + number of collaborations (international, with private sector, application sphere),
  + Others.

When designing project results and impact as part of the monitored data, describe the basis on which the estimate, benchmarks, statistical data, etc. were made.

* Potential obstacles to the planned impact of the project

Describe potential barriers, conditions (e.g., legislative, competition or others that go beyond the scope and duration of the project) that may affect the desired results and impact. Identify whether these factors can evolve over time and the ways you address them.

(This does not include the implementation risks of the project, which will be described below)

### 2.2 MEASURES TO MAXIMISE IMPACT – DISEMINATION AND COMMUNICATION, EXPLOITATION OF RESULTS

*What tools and measures do you choose to maximise the impact of project results and deliverables? Describe what communication and sharing tools you will use, list the planned communication activities and target audience that will be targeted both during and after the project.*

*(In the planned communication activities, take into consideration the target groups and how to reach them through different tools)*

*How will possible technology transfer, commercialisation of project outputs, etc. be ensured?*

Describe the measures to exploit the results of the project even after its completion. Describe the measures for the use of research data and other research outputs after the completion of the project implementation.

*If relevant, describe the strategy for managing intellectual property rights in relation to the results of the project. How will their protection and the possibility of commercial use be ensured? Please briefly describe what requirements will need to be met in order for the results of the project to be exploited by intellectual property and how you intend to meet these conditions.*

## Implementation

*Specific aspects that shall be taken into account in this section*:

Quality and efficiency of the project plan, feasibility of planned activities.

The coherence and logical framework of the work packages and the adequacy of the resources allocated to them, the adequacy of the proposed milestones and deliverables.

Estimation of implementation risks, quality of proposed measures.

Capacities (personnel, professional, technical, infrastructure, other) of the applicant/host organisation.

3.1 PROJECT PLAN AND DELIVERABLES

Describe the overall structure of the project plan, which consists of individual work packages, their interconnection, logical and chronological relation.

Describe the timeline of the project – specify project duration and the time frame for the implementation of each work package (e.g., Gantt diagram).

Describe the planned work packages (template of the table below) and add a separate table for each work package. The number of work packages should reflect the scale and complexity of the project.

3.1.1 Work packages

Template of the table for the work package (1 work package = 1 table):

|  |  |
| --- | --- |
| Work package number | 1 |
| Title of the work package | CQM |
| **Start of implementation of the work package (Mx Month[[26]](#footnote-27))** | M1 |
| End of implementation of the work package (Mx month) | M11 |
| **Involvement (expressed in Person Months)[[27]](#footnote-28)** | 11 |
| **Personnel costs (in EUR)[[28]](#footnote-29)** | 46673 |
| Other eligible costs, excluding personnel costs (in EUR excluding VAT) |  |
| Objectives | |
|  | |
| Description of the work package | |
| Where appropriate, please also provide a breakdown per task level. | |
| Deliverables | |
| Identify deliverables in numbered list and describe each deliverable in more detail | |

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| --- | --- |
| Work package number | 2 |
| Title of the work package | Provide the name of the work package, which will have a maximum of 20 characters. This name should be stated same throughout the application and its annexes. |
| **Start of implementation of the work package (Mx Month[[29]](#footnote-30))** | M6 |
| End of implementation of the work package (Mx month) | M18 |
| **Involvement (expressed in Person Months)[[30]](#footnote-31)** |  |
| **Personnel costs (in EUR)[[31]](#footnote-32)** |  |
| Other eligible costs, excluding personnel costs (in EUR excluding VAT) |  |
| Objectives | |
|  | |
| Description of the work package | |
| Where appropriate, please also provide a breakdown per task level. | |
| Deliverables | |
| Identify deliverables in numbered list and describe each deliverable in more detail | |

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| --- | --- |
| Work package number | 3 |
| Title of the work package | Provide the name of the work package, which will have a maximum of 20 characters. This name should be stated same throughout the application and its annexes. |
| **Start of implementation of the work package (Mx Month[[32]](#footnote-33))** | M14 |
| End of implementation of the work package (Mx month) | M24 |
| **Involvement (expressed in Person Months)[[33]](#footnote-34)** |  |
| **Personnel costs (in EUR)[[34]](#footnote-35)** |  |
| Other eligible costs, excluding personnel costs (in EUR excluding VAT) |  |
| Objectives | |
|  | |
| Description of the work package | |
| Where appropriate, please also provide a breakdown per task level. | |
| Deliverables | |
| Identify deliverables in numbered list and describe each deliverable in more detail | |

3.1.2 List of work packages (template):

|  |  |  |  |
| --- | --- | --- | --- |
| Work package number | Title of the work package | **Start of activities** *(specify month of project implementation)* | **End of activities** *(specify month of project implementation)* |
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3.1.3 List of deliverables (template):

Mandatory deliverables shall be at least:

* Interim report on the implementation and achievements of the project submitted at mid-term of project implementation
* Final report on the achievements of the project presented at the end of the project implementation
* Researcher’s publications in scientific and/or professional journals
* Outputs in the conference proceedings with the active participation of the researcher
* Submission/award of a research project(s)/grant(s) with the involvement of the researcher

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| --- | --- | --- | --- | --- | --- | --- |
| Deliverable number | Deliverable | Work package number | Type | Access and dissemination | Method of verification | Delivery (project implementation month) |
|  |  |  | Report, publication, prototype, software, patent, other (please specify) ... | P= public  N= non-public, limited only to team/host organisation, grant provider for reporting purpose |  |  |
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The applicant shall define the number of planned mandatory deliverables and define other deliverables relevant to its project.

3.1.4 List of milestones (template):

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Milestone number | Milestone | Work package number | Method of verification | Expected time to reach the milestone (project month) |
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3.2 IMPLEMENTATION RISKS AND PROPOSED MEASURES

Describe the approach to risk management in the implementation of the project.

3.2.1 Risks of implementation (template):

|  |  |  |
| --- | --- | --- |
| **Description of the risk of implementation[[35]](#footnote-36)** | **Work package** *(one or more)* | Proposed measures for risk mitigation or elimination |
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3.3 OPERATIONAL CAPACITY OF THE APPLICANT/HOST ORGANISATION

Describe the capacities (staff, professional, technical, infrastructure and others) of the applicant/host organisation that are necessary for the successful implementation of the project. Describe their relevance to the project and how they will be made available to the researcher or how the access to them will be ensured during the implementation of the project.

3.3.1 Description of the research/innovation infrastructure of the applicant/host organisation that is necessary for the implementation of the project (template):

|  |  |
| --- | --- |
| Name of infrastructure or equipment | Short description |
|  |  |
|  |  |
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3.3.2 List of the five most important projects of the applicant/host organisation and their relevance to the proposed project (in the last 5 years) (template):

|  |  |  |
| --- | --- | --- |
| Project name/identification | Programme/scheme/grant provider | Short description |
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3.3.3 List of maximum five most important outputs of the applicant/host organisation relevant to the submitted project (Template):

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| --- | --- | --- |
| Output name/identification | **Type of output** *(e.g., publication, dataset, software, patent, service, product, etc.)* | Short description |
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1. After completion of the document, update the content. [↑](#footnote-ref-2)
2. Choose one type of research. [↑](#footnote-ref-3)
3. S. Abramsky, B. Coecke, *A categorical semantics of quantum protocols*, Proceedings of LiCS'04 (2004) [↑](#footnote-ref-4)
4. B. Coecke, A. Kissinger, *Picturing Quantum Processes: A First Course in Quantum Theory and Diagrammatic Reasoning*, Cambridge University Press (2017) [↑](#footnote-ref-5)
5. C. Heunen, J. Vicary, *Categories for Quantum Theory*, Oxford University Press (2019) [↑](#footnote-ref-6)
6. B. Coecke, D. Pavlovic, J. Vicary, *A new description of orthogonal bases*, Math. Struct. Comput. Sci. 23(3), 555--567 (2013). [↑](#footnote-ref-7)
7. N. Weaver, *Quantum Relations*, Memoirs of the American Mathematical Society, Vol. 215, No. 1010 (2012) [↑](#footnote-ref-8)
8. A. Kornell, *Discrete Quantum Structures*, arXiv:2004.04377 (2020). [↑](#footnote-ref-9)
9. A. Carboni, R.F.C. Walters, *Cartesian bicategories I*, J. Pure Appl. Algebra, Vol. 49, Issue 1–2, Pp. 11-32 (1987) [↑](#footnote-ref-10)
10. C. Ehresmann, *Catégories structurées*, Annales scientifiques de l’École Normale Supérieure, Vol. 80. No. 4. (1963) [↑](#footnote-ref-11)
11. G. Kuperberg, N. Weaver, *A von Neumann algebra Approach to Quantum Metrics*, Memoirs AMS, Vol. 215, No. 1010 (2012) [↑](#footnote-ref-12)
12. A. Kornell, *Quantum Sets*, J. Math. Phys. 61, 102202 (2020) [↑](#footnote-ref-13)
13. A. Kornell, B. Lindenhovius, M. Mislove, *A category of quantum posets*, Indag. Math., 33 (6), 1137-1171 (2022) [↑](#footnote-ref-14)
14. A. Kornell, B. Lindenhovius, M. Mislove, *Quantum CPOs*, Proceedings Quantum Physics and Logic 174--187 (2020) [↑](#footnote-ref-15)
15. A. Kurz, A. Moshier, A. Jung, *Stone Duality for Relations* (2019) [Preprint](https://arxiv.org/abs/1912.08418) [↑](#footnote-ref-16)
16. G. Jenča, B. Lindenhovius, *Quantum suplattices*, to appear in the Proceedings of the 20th International Conference on Quantum Physics and Logic, 58--74 (2023) [Preprint](http://dx.doi.org/10.4204/EPTCS.384.4) [↑](#footnote-ref-17)
17. R. F. Blute, J. R. B. Cockett, and R. A. G. Seely, *Differential categories*, Math. Struct. Comput. Sci. 16(06), 1049–1083 (2006 [↑](#footnote-ref-18)
18. C. Heunen, A. Kornell, *Axioms for the category of Hilbert spaces*, Proc Natl Acad Sci U S A. (2022) Mar 1;119(9):e2117024119 [↑](#footnote-ref-19)
19. B. Coecke, C. Heunen, A. Kissinger, *Categories of quantum and classical channels*, QIP. 15, 5179–5209 (2016) [↑](#footnote-ref-20)
20. A. Jenčová, G. Jenča, *On Monoids in the Category of Sets and Relations*, Int. J. Theor. Phys. 56, 3757-–3769 (2017) [↑](#footnote-ref-21)
21. C. Heunen, M. Karvonen, *Limits in dagger categories* (2018) [Preprint](https://arxiv.org/abs/1803.06651) [↑](#footnote-ref-22)
22. D. Hofmann, G. Seal, W. Tholen (Eds.), *Monoidal Topology: A Categorical Approach to Order, Metric, and Topology*, Cambridge University Press (2014) [↑](#footnote-ref-23)
23. X. Jia, M. Mislove, *Completing simple valuations in K-categories*, Topology and its Applications, Volume 318, 108192 (2022) [↑](#footnote-ref-24)
24. M. Shulman, *Framed bicategories and monoidal fibrations*, Theory Appl. Categories 20(18), 650–738 (2008). [↑](#footnote-ref-25)
25. C. Heunen, S. Tull, *Categories of relations as models of quantum theory*, (2015) [Preprint](https://arxiv.org/abs/1506.05028) [↑](#footnote-ref-26)
26. The months are indicated ascending from the start of the project, i.e., the month in which the project started is M1. [↑](#footnote-ref-27)
27. In case of involvement of the research team in the project implementation, it is necessary to identify the individual positions, identify the category of researcher R1-R4 for researchers and determine the level of involvement for all members of the research team. [↑](#footnote-ref-28)
28. Where the research team is involved in the project implementation, the personnel costs will include personnel costs of the researcher and the members of the research team. [↑](#footnote-ref-29)
29. The months are indicated ascending from the start of the project, i.e., the month in which the project started is M1. [↑](#footnote-ref-30)
30. In case of involvement of the research team in the project implementation, it is necessary to identify the individual positions, identify the category of researcher R1-R4 for researchers and determine the level of involvement for all members of the research team. [↑](#footnote-ref-31)
31. Where the research team is involved in the project implementation, the personnel costs will include personnel costs of the researcher and the members of the research team. [↑](#footnote-ref-32)
32. The months are indicated ascending from the start of the project, i.e., the month in which the project started is M1. [↑](#footnote-ref-33)
33. In case of involvement of the research team in the project implementation, it is necessary to identify the individual positions, identify the category of researcher R1-R4 for researchers and determine the level of involvement for all members of the research team. [↑](#footnote-ref-34)
34. Where the research team is involved in the project implementation, the personnel costs will include personnel costs of the researcher and the members of the research team. [↑](#footnote-ref-35)
35. Indicate the probability of risk occurrence (low, medium, high) and the severity of the risk (low, medium, high) [↑](#footnote-ref-36)